Exercises for lecture by H. Leutwyler

1. Evaluate the positive frequency part of the massless propagator

$$\Delta^+(z,0) = \frac{i}{(2\pi)^3}\int \frac{d^3k}{2k^0}\, e^{-ikz}\,, \quad k^0 = |\vec{k}|$$

for $\mathrm{Im}\, z^0 < 0$. Show that the result can be represented as

$$\Delta^+(z,0) = \frac{1}{4\pi\, i\, z^2}$$

2. Evaluate the d-dimensional propagator

$$\Delta(z,M) = \int \frac{d^dk}{(2\pi)^d}\frac{e^{-ikz}}{M^2 - k^2 - i\,\epsilon}$$

at the origin and verify the representation

$$\Delta(0,M) = \frac{i}{4\pi}\Gamma\left(1-\frac{d}{2}\right)\left(\frac{M^2}{4\pi}\right)^{\frac{d}{2}-1}$$

How does this expression behave when $d \to 4$?

3. Leading order effective Lagrangian:

$$\begin{aligned}
\mathcal{L}^{(2)} &= \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\rangle + h_0 D_\mu\theta D^\mu\theta \\
D_\mu U &= \partial_\mu U - i(v_\mu + a_\mu)U + i\,U(v_\mu - a_\mu) \\
\chi &= 2\,B\,(s + ip) \\
D_\mu\theta &= \partial_\mu\theta + 2\langle a_\mu\rangle \\
\langle X\rangle &= \mathrm{tr}X
\end{aligned}$$

- Take the space-time independent part of the external field $s(x)$ to be isospin symmetric (i. e. set $m_u = m_d = m$):

$$s(x) = m\,\mathbf{1} + \tilde{s}(x)$$

- Expand $U = \exp i\,\phi/F$ in powers of $\phi = \vec{\phi}\cdot\vec{\tau}$ and check that, in this normalization of the field ϕ, the kinetic part takes the standard form

$$\mathcal{L}^{(2)} = \tfrac{1}{2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi} - \tfrac{1}{2}M^2\vec{\phi}^2 + \ldots$$

  with $M^2 = 2mB$.

- Draw the graphs for all of the interaction vertices containing up to four of the fields $\phi, v_\mu, a_\mu, \tilde{s}, p, \theta$.