Further exercises for the lectures by H. Leutwyler

4. Show that the classical field theory belonging to the QCD Lagrangian in the presence of external fields is invariant under

$$\begin{aligned} v'_\mu + a'_\mu &= V_{\rm R}(v_\mu + a_\mu)V_{\rm R}^\dagger - i\partial_\mu V_{\rm R} V_{\rm R}^\dagger \\ v'_\mu - a'_\mu &= V_{\rm L}(v_\mu - a_\mu)V_{\rm L}^\dagger - i\partial_\mu V_{\rm L} V_{\rm L}^\dagger \\ s' + i\,p' &= V_{\rm R}(s + i\,p)V_{\rm L}^\dagger \\ q'_{\rm R} &= V_{\rm R}\,q_{\rm R}(x) \\ q'_{\rm L} &= V_{\rm L}\,q_{\rm L} \end{aligned}$$

where $V_{\rm R}, V_{\rm L}$ are space-time dependent elements of U(3).

5. Evaluate the pion mass to NLO of χPT . Draw the relevant graphs and verify the representation

$$M_\pi^2 = M^2 + \frac{2\,\ell_3 M^4}{F^2} + \frac{M^2}{2F^2}\,\frac{1}{i}\Delta(0, M^2) + O(M^6)$$