C. Burgess, Physics beyond the SM

Exercise: Find the unique dimension-5 operator consistent with Lorentz Inv. + $SU(3) \times SU(2) \times SU(1)$ + show that it is unique

* SM as EFT

* Operator classification + "technical" naturalness

SM as an EFT

1) Partical content
2) Symmetries

Partical content (SM):

Spin 2 $g_{\mu\nu}$

1 W^a_μ B_μ G^α_μ

1/2 Q_i, U_i, D_i, L_i, E_i

? → 0 H

3 options → 1) SM fields only
2) SM " but no Higgs
3) SM " + extras

Symmetries:

1) Only gauge symmetries:
- general covariance
- $SU(3) \times SU(2) \times U(1)$

- Not Baryon or lepton conservation
(Not B, L, ...)

Why should the SM be an EFT?

1) Because it is most general
Renormalizable (w/o gravity)
Possible given the particles

2) gravity exists: non renorm → EFT

$$G_N = \frac{1}{M_p^2} \qquad (\hbar = c = 1)$$

$$M_p = 1.2 \times 10^{19}\ \text{GeV}$$

$$m_p \simeq 1\ \text{GeV}$$

$$M_W = 80\ \text{GeV}, \ ...$$

$$G_N / G_F \simeq 10^{-33} \rightarrow M_\odot \simeq \frac{M_p^3}{m_p^2} \simeq 10^{38} M_p$$
$$\simeq 10^{33}\ g$$

Warning: $G_F^{-1/2} \simeq \sqrt{10^5}\ \text{GeV} \simeq 300\ \text{GeV}$

$$M_W \simeq 80\ \text{GeV} \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

This can also happen to gravity!

3) ν mass <u>may</u> point to scales

like $10^{10} - 10^{15}$ GeV

4) SM couplings <u>may</u>* unify

* with new particles, ...

at energies $\simeq 10^{16}$ GeV

1) With the SM fields + imposing only gauge sym. What are the most general interactions?

$$\mathcal{L}_{eff} = \sqrt{g} \sum_k \frac{c_k}{M_k^{d_k - 4}} \underbrace{\mathcal{O}_k (H, B_\mu, Q}_{\text{dimension } (\text{mass})^{d_k}}$$

$\sqrt{-g} = \sqrt{-\det g_{\mu\nu}}$. Because of $\sqrt{g}$ in front

The O's are Lorentz + coord. scalars, $SU_3 \times SU_2 \times SU_1$ invariants

Dimension 0: $c_0 M_0^4 I$

→ the acceleration of the Universe

→ $c_0 M_0^4 \simeq (10^{-2} eV)^4$

Because of Lorentz inv.

Dim 1: nada

2: $c_2 M_2^2 H^+ H$

3: nada

Dim 4 : $G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$

$$\frac{g_3^2 \, \theta_3}{64\pi} \epsilon^{\mu\nu\lambda\rho} G^\alpha_{\mu\nu} G^\alpha_{\lambda\rho}$$

$y_{ij} \bar{Q}_i \gamma_L U_j H \,, \quad \lambda^2 (H^+ H)^2$

$y_e \simeq 10^{-6} \qquad \theta_3 < 5 \times 10^{-10}$

$\Delta L = \pm 2$
$\Delta B = 0$

Dim 5 : $\dfrac{(c_5)_{ij}}{M_5} (L_i \gamma_L L_j) HH + cc$

[show this is unique]

$H = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$

$$\mathcal{L}_{eff} = \frac{(c_5)_{ij}}{\sqrt{2} M_5} (\nu_i \gamma_L \nu_j) v^2$$

Neutrino mass matrix $m_{\nu_{ij}} \simeq c_{5_{ij}} \dfrac{v^2}{M_5} \simeq 10^{-2}\,\text{eV}$

$v = 246$ GeV

$\dfrac{M_5}{c_5} \simeq 10^{15}$ GeV

Dim 6 : $(\bar{\Psi}_i \Gamma \Psi_j)(\Psi_k \Gamma \Psi_l)$

all but 6 preserve B, L	$(H^+ H)^3 , \; (H^+ \tau_a H) \bar{\Psi} \tau_a \not{D} \Psi$
other 6 $\Delta B = \pm 1$	
$\Delta B = \Delta L$	$(\bar{\Psi} \tau_a \gamma^{\mu\nu} \gamma_L \Psi) H W^a_{\mu\nu}$
$M_6 > 10^{16}$ GeV	$(H^+ H) G^\alpha_{\mu\nu} G^{\alpha\,\mu\nu} + \dots$

Claim: there is usually a pattern in the size of the coefficients

1) For dim ≥ 5, the lightest particle that can contribute dominates

2) For dim ≤ 3, the heaviest part. that can contribute dominates

$$\sum_i g_i M_i^a \quad \text{is dominated by } M_{max}$$

$$\sum_i g_i \frac{1}{M_i^p} \quad \text{'' \quad '' \quad ''} \quad M_{min}$$

if $p > 0$

Warning: If a selection rule (eg $\Delta L = 2$) is at work only the lightest of the $\Delta L \neq 0$ particle contribute.

Lecture II

$$\mathcal{L}_{EFT} = \sqrt{-g} \sum_k \frac{c_k}{M_k^{d_k - 4}} O_k (H, \dots)$$

Quest.: How big are the M_k's and c_k's as functions of parameters in BSM physics?

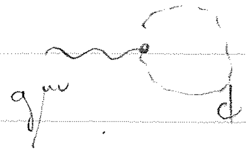
Cartoon underlying theory: $\mathcal{L}_{int}(\phi, SM) = -\sqrt{-g}\, \overset{\text{coupling}}{h^2}\, \phi^* \phi\, H^* H$

E

M_ϕ — ϕ is a complex scalar field of mass $M_\phi \gg$ SM scale

TeV

SM



$$c_0 M_0^4 \simeq \# \int \frac{d^4 l}{(l^2+M_\phi^2)} \left[l_\mu l_\nu , M_\phi^2 \right]$$

$$\simeq \Lambda^4$$

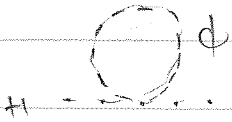
$$\int e^{iS(l,h)} \mathcal{D}l \, \mathcal{D}h = \int_{E<\Lambda} e^{i\Gamma(l)} \mathcal{D}l$$

$$e^{i\Gamma(l)} = \int_{E>\Lambda} e^{iS(l,h)} \mathcal{D}h$$

Burgess argues: Cutoff Λ is stupid ...

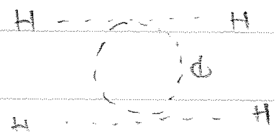
Dim. Reg. much better ...

$$\delta\left(c_0 M_0^4\right) \simeq \frac{k M_\phi^4}{16\pi^2}$$ Exercise: Compute k

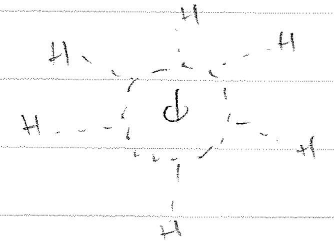


$$\delta\left(c_0 M_0^4\right) \simeq h^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+M_\phi^2)}$$

$$\simeq \frac{\tilde{k} h^2 M_\phi^2}{16\pi^2}$$



$$\delta\lambda \simeq g^4 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+M_\phi^2)^2} \simeq \frac{\hat{k} g^4}{16\pi^2} \ln\left(\frac{M_\phi^2}{\mu^2}\right)$$



$$\delta\left(\frac{c_6}{M_6^2}\right) \simeq g^6 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+M_\phi^2)^3} \simeq \frac{\hat{k}' g^6}{16\pi^2 M_\phi^2}$$

For many such ϕ's: $\delta\left(\frac{c_6}{M_6^2}\right) \simeq \sum_l \frac{k'_l \, g_l^6}{16\pi^2 M_l^2}$ smallest wins

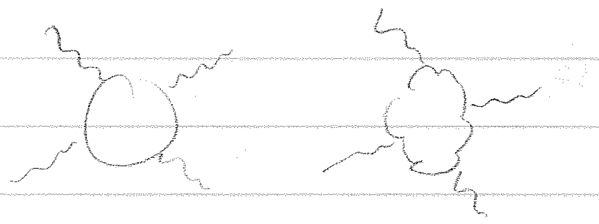
Real examples where this happens in everyday life:

Integrate out W in SM:

$$\mathcal{L}_{int} \simeq \frac{G_F}{2} (\nu \gamma^\mu \gamma_L \nu)(l \gamma_\mu \gamma_L l) \simeq \frac{1}{M_W^2}$$

If we integrate out e^-:

$$\mathcal{L}_{eff} \simeq \frac{\alpha^2}{180\pi^2 m_e^4} \left[5 (F_{\mu\nu} F^{\mu\nu})^2 - 14 F_{\mu\nu} F^{\nu\lambda} F_{\lambda\rho} F^{\rho\mu} \right]$$



$$\rightarrow \int \frac{C_8}{M_8^4} \simeq \frac{\alpha^2}{M_W^4}$$

Some important consequences for phenomenology:

$\nu\nu \rightarrow \gamma\gamma$ $\nu\nu \rightarrow \gamma\gamma\gamma$

at $E \ll m_e$: $\Rightarrow$ $\mathcal{L}_{eff} \simeq \frac{4 G_F}{\pi M_W^2 \sqrt{2}} \left(1 + \frac{4}{3} \ln \left(\frac{M_W^2}{\mu^2} \right) \right)$

$$+ (\nu \gamma_\mu \ldots \nu) F^{\mu\lambda} F^{\nu}{}_{\lambda}$$

$$\sigma \simeq \frac{G_F^2 \alpha^2}{M_W^4} E^6$$

$$\mathcal{L}_{eff} \simeq \frac{e\alpha \left(\frac{1}{2} + 2 s_W^2 \right)}{90\pi^2 m_e^4} \frac{G_F}{\sqrt{2}} \left[5 N_{\mu\nu} F^{\mu\nu} F_{\lambda\rho} F^{\lambda\rho} - 14 N_{\mu\nu} F^{\nu\lambda} F_{\lambda\rho} F^{\rho\mu} \right]$$

where $N_{\mu\nu} = \partial_\mu [\nu \gamma_\nu \gamma_L \nu] - (\mu \leftrightarrow \nu)$

$$\sigma \simeq \frac{G_F^2}{m_e^8} E^{10}$$

Since our goal is phys. beyond SM we should focus on dim ≤ 3 interactions:

$$\sqrt{-g}\,[c_0 M_0^4 + c_2 M_2^2 H^* H]$$

Because the are the most sensitive to heaviest particles.

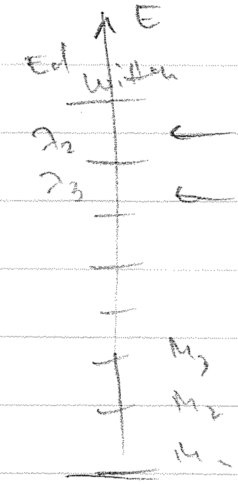
<u>Technical Naturalness:</u>

How do hierarchies of scale arise in EFT's?

If $\lambda \simeq \frac{M_0}{M_\phi}, \frac{M_2}{M_\phi}$ $\quad \alpha \simeq \frac{1}{137}$

are small, in which EFT is it small?

$\lambda \ll 1$ is technically natural



$\leftarrow \lambda_{EW} \ll 1$

$\leftarrow$ <u>and</u> $\lambda_2 \ll 1$

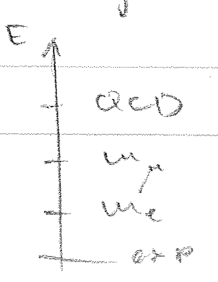
<u>and</u> $\lambda_3 \ll 1$

and $\lambda_{exp} \ll 1$

λ_{exp}

experimental

Why are nuclei small comp to atoms?



$\alpha m_e \ll m_N$

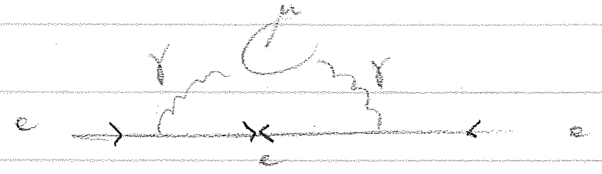
$\lambda_1 = \alpha \quad \lambda_2 = \frac{m_e}{m_N}$ why small?

If $E > \Lambda_{QCD}$: $N \to 8$

$\alpha, m_e \ll \Lambda_{QCD}$ $\lambda_1 = \alpha$ $\lambda_2 = \frac{m_e}{\Lambda_{QCD}}$

Ask: how does λ_1, λ_2 change as we pass through m_μ?

$m_e^{EFT} \simeq m_e^{FT}$



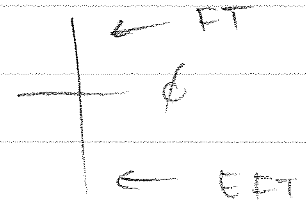
$$+ \# \frac{\alpha^2}{(16\pi^2)^2} m_e \ln\left(\frac{m_\mu}{m_e}\right)$$

Why was $\delta m_e \propto m_e$? if $m_e = 0$ $e \to e^{i\alpha\gamma_5} e$ is a symmetry

$\Rightarrow$ Natural small: $\frac{m_e}{m_N}$

Similarly for α.

But for $\sqrt{-g}\left[c_0 M_0^4 + c_2 M_2^2 H^\dagger H\right]$



$$\underbrace{c_0 m_0^4{}_{EFT}}_{(10^{-2}\,eV)^4} = c_0 m_0^4{}_{FT} + \frac{k}{16\pi^2} M_\phi^4$$

$$\underbrace{c_2 m_2^2{}_{EFT}}_{(100\,GeV)^2} = c_2 m_2^2{}_{FT} + \frac{\tilde{k} g^2}{16\pi^2} M_\phi^2$$

i) Cosmological constant problem

Why do particles with $m > 10^{-2}$ eV not contribute small amounts to the vacuum energy?

Crisis: Known particles are not completely understood

(*) ii) Electroweak hierarchy (hierarchy prob.)

Why doesn't new physics make

$c_2 m_2^2 \simeq (\text{biggest scale poss})^2$?

Lecture III

(*) The 3 possible approaches to solving the EW hierarchy prob:

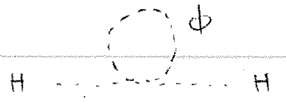
1) Choose particle content such that their exists a sym which protect M_2, such as susy.

2) Deny the existence of H at high energy, such as composite ϕ models.

3) Deny the existence of fundamental scales larger than ~TeV
Q: How does one understand M_p ?

Supersymmetric option:

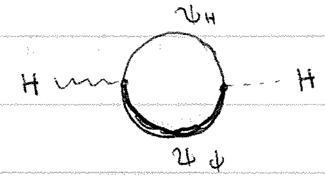
$$\mathcal{L}_{int} \cong -g^2 \phi^* \phi H^* H$$



$$\simeq \delta(c_2 M_2^2) \simeq \frac{k g^2}{16\pi^2} M_\phi^2$$

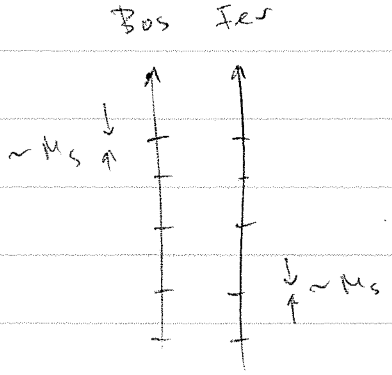
ψ_ϕ: $\mathcal{L}_{int} \simeq -g(\psi_\phi \gamma_L \psi_H) H + c.c.$

ψ_H



$$\delta(c_2 M_2^2) \simeq -\frac{k g^2}{16\pi^2} M_{\psi_\phi}^2$$

Supersym. idea: Suppose that particles come in $\underset{\text{bose-fermi}}{\text{pairs}}$.
Then, if their coupling ~~s~~ + masses are equal then their is a new symmetry (susy) relating them, ~~s~~ and $\delta(c_2 M_2^2) = 0$ $\delta(c_0 M_0^4) = 0$ if the sym. is unbroken.
If SUSY is broken, failure to cancel is:



$$\delta(c_2 M_2^2) \simeq \frac{k}{16\pi^2}\Big[\underbrace{g_b^2 M_b^2 - g_F^2 M_F^2}_{M_S^2}\Big]$$

This is small enough if $M_S \lesssim$ TeV

EFT: SM + some superpartners (spin 1/2 in adjoint repr.)
λ^a corresponding to $W^a_{\mu\nu}$
(Scalars in the repr. of the known fermions) Higgs sector + friends.

these new particles are all allowed low-dim mass terms.

$$\mathcal{L}_3 = - m_{ab} (\lambda_a \gamma_L \lambda_b) + c.c.$$

$$\mathcal{L}_2 = - \tilde{\mu}_{ij} \overset{*}{\varphi}^i \varphi^j + \dots$$

We should expect these to be heavy

There are renormalizable interactions allowed by gauge symmetries which break B, L:

e.g. $\mathcal{L}_{int} = - \epsilon_{\alpha\beta\gamma} (u^\alpha \gamma_L u^\beta) u^\gamma + c.c.$

R-parity: $R = (-)^{F+3B-L}$ $= +$ for known particles

$= -$ for superpartners

R-parity invariance removes dim 4 $\Delta B, \Delta L \neq 0$ operators, but not at dim 5.

More about option 3:

Q: How does $M_p \gg$ TeV if there are no fundamental energy scales $M \gg$ TeV?

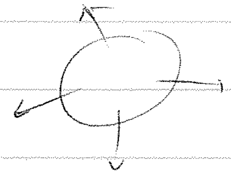
- Large extra dimension models.

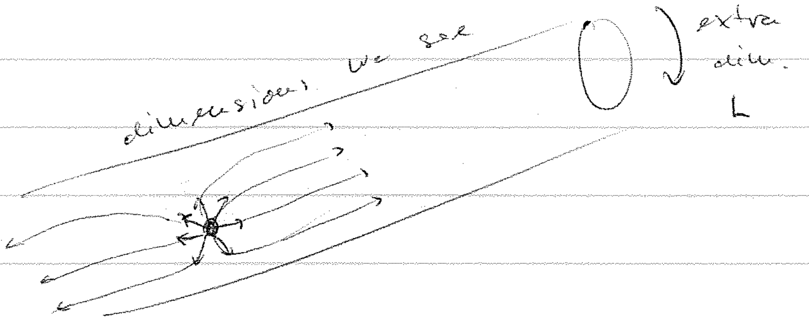
Observation: there can be surfaces in space-time on which all know particles (except graviton) are trapped.

n extra dimensions: (4+n in total)

Assume $G_{4+n} \simeq \frac{1}{M_*^{2+n}}$ where $M_* \sim$ TeV

$$\vec{F} = \frac{G_{4+n} m_1 m_2}{r^{2+n}} \vec{e}$$





if $r \ll L$ $\quad \vec{F} \simeq \frac{G_{4+n} M_1 M_2}{r^{2+n}} \vec{e}$

if $r \gg L$ $\quad |F_{eff}| \simeq \frac{G_{4+n} M_1 M_2}{L^n r^2}$

$$\frac{1}{M_p^2} \equiv G_4 = G_N \simeq \frac{G_{4+n}}{L^n} \simeq \frac{1}{M_*^{2+n} L^n}$$

$$M_p^2 = M_*^2 \cdot (M_* L)^n$$

Newtons law is tested only to 45 μm $\quad L \lesssim 10\,\mu m$.
if all other ints are trapped.

if $n \geq 2$ $\quad M_* \simeq$ TeV is consistent with $M_p \simeq 10^{19}$ GeV
and test of Newtons law.
if $n = 2$, $L \simeq 10\,\mu m$.
if $n > 2$, $L \lesssim 10\,\mu m$. $\quad E \approx \frac{1}{L} \simeq 10^{-2}$ eV