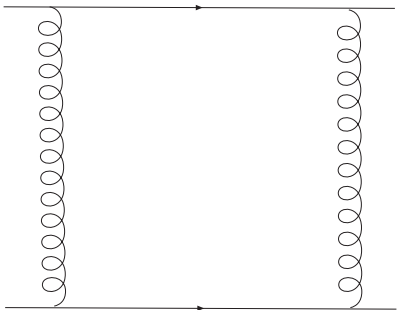
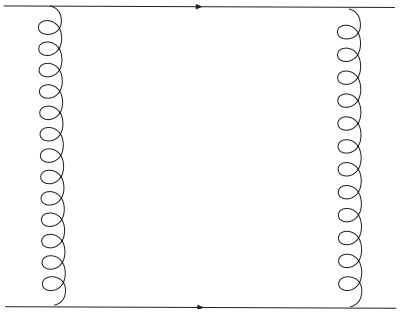
Exercises for the lectures of NRQCD at weak coupling. Antonio Pineda

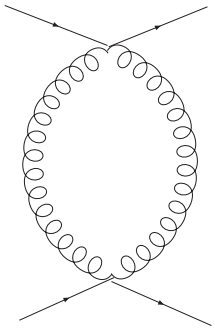
1) Compute the logarithmic divergent term of the following diagram in QCD with the quarks at rest $P^\mu = (m, \mathbf{0})$ and deduce the associated contribution to d_{ss}, d_{vs}, d_{sv} and d_{vv}.



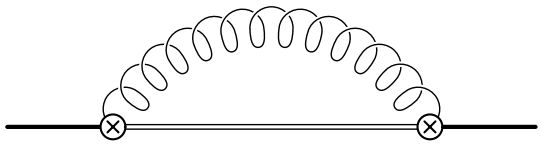
2)
2.1) Obtain the Feynman rules of NRQCD/NRQED at $\mathcal{O}(1/m)$.
2.2) Draw all the diagrams that contribute to the potential up to $\mathcal{O}(\alpha_s/m^2)$.

2) Obtain the logarithmic behavior of the following diagrams (plus crossed) of $\mathcal{O}(1/m^2)$ in NRQED with static heavy quark propagators. Work in the Coulomb gauge. In the first diagram one of the gluons is transverse and the other is longitudinal. In the second diagram both gluons are transverse. Deduce the associated corrections to the potential.



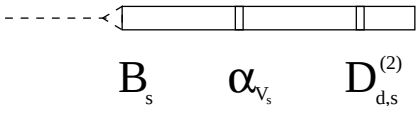


3) Compute the ultraviolet (logarithmic) behavior of the following diagram in pNRQCD. Absorb those divergences in the potentials and deduce the ultrasoft renormalization group equation of the potentials.



$1/(E - V_o^{(0)} - \mathbf{p}^2/m)$

4) Compute the logarithmic divergent term of the following diagram in pNRQCD



and obtain its contribution to the renormalization group equation of

$$\nu_p \frac{d}{d\nu_p} B_s \,. \tag{1}$$

5) Using that

$$\mathrm{Im} \int_0^\infty \mathrm{d}t \, e^{-t/\alpha_s} \, \delta B[m_{\mathrm{OS}}](t) \sim \Lambda_{QCD} \tag{2}$$

obtain the coefficients b and c_1 if (we define $u = \frac{\beta_0 t}{4\pi}$)

$$\delta B[m_{\mathrm{OS}}](t(u)) = N_m \nu \frac{1}{(1-2u)^{1+b}} \left(1 + c_1(1-2u) + c_2(1-2u)^2 + \cdots\right). \tag{3}$$