Lectures on the Soft-Collinear Effective Theory

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Problem 1) Practice with Operators in HQET

In this problem you will derive the HQET Lagrangian to $\mathcal{O}(1/m_Q)$.

a) In lecture we made a change of variables to write the QCD Lagrangian as

$$\mathcal{L}_{\rm QCD} = \bar{h}_v(iv\cdot D)h_v - \bar{H}_v(iv\cdot D + 2m_Q)H_v + \bar{Q}_v(i\not{D}_T)H_v + \bar{H}_v(i\not{D}_T)h_v\,. \tag{1}$$

We identified the first term as the leading order HQET Lagrangian. To obtain the terms at $\mathcal{O}(1/m_Q)$ at tree level we can integrate out the antiparticle field H_v by deriving its equation of motion from Eq. (1). Derive the equation of motion for H_v, and use it to find an operator expression relating H_v to h_v, and to derive a tree level expression for $\mathcal{L}_{\rm QCD}$ that is entirely in terms of h_v.

b) Expand your result to $\mathcal{O}(1/m_Q)$ and identify a term involving the kinetic operator $\bar{h}_v(iD_T)^2h_v$, and a term involving the chromomagnetic operator $\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v$ where $G^{\mu\nu}$ is the gluon field strength. What symmetries of the leading HQET Lagrangian do these two terms violate? Consider Feynman diagrams that contain both the h_v and H_v fields of Eq. (1) and explain diagrammatically what you have done when you integrated out H_v in the above manner.

Problem 2) SCET Operators with Collinear Quarks and Wilson Lines

a) Start with the QCD Lagrangian for a massive quark and decompose $\not{D}$ in terms of n, $\bar{n}$, and $\perp$ components. As in lecture, write $\psi = \xi_n + \zeta_{\bar{n}}$ where $\not{n}\xi_n = 0$ and $\not{\bar{n}}\zeta_{\bar{n}} = 0$ and determine which products of fields are non-zero. Keeping all the non-zero terms, integrate out the field $\zeta_{\bar{n}}$ to generate an effective action for the massive collinear quark ξ_n.

[With power counting $m \sim p_\perp \sim Q\lambda \ll Q$ this is the starting point to derive the action for a massive collinear quark, ie. prior to decomposing the gluon field into collinear and ultrasoft pieces and prior to distinguishing between large and small momenta. The remaining steps are the same as those discussed in lecture except that you keep the mass. The mass terms that you have derived are important for considering how a collinear Lagrangian of light quarks u, d, s explicitly breaks chiral symmetry. They are also relevant for discussing an energetic jet initiated by a massive quark, when the jet energy $Q \gg m$.]

b) To get more familiar with Wilson lines lets consider the current for a $b \to u$ transition. In QCD $J = \bar{u}\Gamma b$. For SCET we did a matching calculation to find the leading order current

$$J^{(0)} = \bar{\xi}_n W\Gamma h_v\,, \tag{2}$$

where W included terms involving the order λ^0 collinear gluon field $\bar{n}\cdot A_n$. In lecture we explicitly computed the term in W with one $\bar{n}\cdot A_n$ field and wrote down the result for any number of $\bar{n}\cdot A_n$ fields. Do the matching computation for two $\bar{n}\cdot A_n$ fields (by expanding QCD diagrams with offshell propagators). Verify that the result for one and two $\bar{n}\cdot A_n$ fields agree with the momentum space Feynman rules derived from the position space Wilson line

$$W(y^+) = P\exp\left(ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_n(s\bar{n}+y^+)\right),$$

where P is path-ordering.