# Lectures on Quark Flavor Physics: Exercises

(Matthias Neubert)

Benasque Summer School on Flavor Physics, July 2008

1. **Yukawa couplings, CKM matrix, and unitarity triangles:**

   a) Show that flavor non-diagonal kinetic terms in the Standard Model Lagrangian can always be diagonalized and brought into standard form by field redefinitions. To this end, study the Lagrangian

   $$\mathcal{L}_{\text{kinetic}} = \bar{Q}_L\, Z_Q\, i\not{D}\, Q_L + \bar{u}_R\, Z_u\, i\not{D}\, u_R + \bar{d}_R\, Z_d\, i\not{D}\, d_R\,,$$

   where all fields are 3-component vectors in generation space, and $Z_A$ are non-negative, hermitian $3\times 3$ matrices.

   b) Show that an arbitrary complex matrix $Y$ can be diagonalized by a biunitary transformation:

   $$W^\dagger\, Y\, U = \lambda\,,$$

   where $U$, $W$ are unitary matrices, and $\lambda$ is a real, diagonal matrix with non-negative eigenvalues. (*Hint:* Consider the matrices $YY^\dagger$ and $Y^\dagger Y$.)

   c) Derive the number of mixing angles and physical (i.e., observable) phases of the CKM matrix for the Standard Model with $N$ fermion generations.

   d) Show that the Jarlskog determinant $J$ defined as

   $$\text{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right) = J \sum_{m,n} \epsilon_{ikm}\,\epsilon_{jln} \quad (i\neq k,\, j\neq l)$$

   is invariant under phase redefinitions of the quark fields, and calculate its value in terms of the Wolfenstein parameters to leading nontrivial order in $\lambda$.

   e) Show that all unitarity triangles have the same area $J/2$.