

Lectures on Quark Flavor Physics: Exercises

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1. Yukawa couplings, CKM matrix, and unitarity triangles:

a) Show that flavor non-diagonal kinetic terms in the Standard Model Lagrangian can always be diagonalized and brought into standard form by field redefinitions. To this end, study the Lagrangian

$$\mathcal{L}_{\text{kinetic}} = \bar{Q}_L Z_Q i\not{D} Q_L + \bar{u}_R Z_u i\not{D} u_R + \bar{d}_R Z_d i\not{D} d_R,$$

where all fields are 3-component vectors in generation space, and Z_A are non-negative, hermitian 3×3 matrices.

b) Show that an arbitrary complex matrix Y can be diagonalized by a biunitary transformation:

$$W^\dagger Y U = \lambda,$$

where U, W are unitary matrices, and λ is a real, diagonal matrix with non-negative eigenvalues. (*Hint:* Consider the matrices YY^\dagger and $Y^\dagger Y$.)

c) Derive the number of mixing angles and physical (i.e., observable) phases of the CKM matrix for the Standard Model with N fermion generations.

d) Show that the Jarlskog determinant J defined as

$$\text{Im} \left(V_{ij} V_{kl} V_{il}^* V_{kj}^* \right) = J \sum_{m,n} \epsilon_{ikm} \epsilon_{jln} \quad (i \neq k, j \neq l)$$

is invariant under phase redefinitions of the quark fields, and calculate its value in terms of the Wolfenstein parameters to leading nontrivial order in λ .

e) Show that all unitarity triangles have the same area $J/2$.