Lectures on the Soft-Collinear Effective Theory

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Problems for lecture 3

Problem 3) Decoupling of Soft and Collinear Gluons in SCET$_{\rm II}$

Consider an operator with one collinear quark and one soft quark in SCET$_{\rm II}$. This case differs from SCET$_{\rm I}$ in that soft gluons knock collinear quarks offshell and collinear gluons knock soft quarks offshell. To be definite lets make the soft quark a heavy quark from HQET, h_v, and the energetic quark a massless collinear quark, ξ_n. The SCET operator will be

$$(\bar\xi_n W_{\bar n})\Gamma(S_n^\dagger h_v) \tag{1}$$

where W is the Wilson line we saw in problem 2, and S_n is an exact analog of the Y from lecture, but with A_s gluons rather than ultrasoft gluons. To find Eq. (1) requires tree level matching (to determine the direction and gluon components appearing in the Wilson lines), and gauge invariance (to ensure that loops do not spoil the structure so obtained). The first non-trivial term in perturbation theory have one soft and one collinear gluon. By computing the QCD graphs in Figure 1 expanded to LO, verify that the three-gluon interactions are responsible for putting the gluons in the right order in Eq. (1).

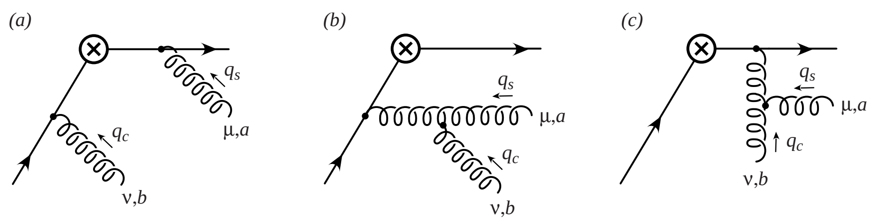

FIG. 1: QCD graphs with collinear, q_c, and soft, q_s, momenta.

Problem 4) SCET Loops for Two-Jet Production

Consider the two-jet production process through a virtual photon in SCET, namely $e^+e^- \to J_n J_{\bar n} X_{us}$ where J_n is a jet in the $n = (1,0,0,-1)$ direction, $J_{\bar n}$ is a jet in the $\bar n = (1,0,0,1)$ direction, and any remaining particles in the final state are ultrasoft, contained in X_{us}.
a) Write down two collinear quark Lagrangians, one for ξ_n fields and one for $\xi_{\bar n}$ fields. Interactions between these two types of collinear fields are hard, and so do not effect your analysis. What are the Feynman rules for the ultrasoft gluon coupling to each of these collinear quarks?
b) Start with $J^{\rm QCD} = \bar\psi\gamma_\mu\psi$ and determine the appropriate LO SCET current $J^{\rm SCET} = \bar\xi_n \cdots \xi_{\bar n}$, ie. fill in the dots with appropriate collinear Wilson lines and Dirac structure.
c) Draw the five one-loop Feynman diagrams that are non-zero for $e^+e^- \to q_n \bar q_{\bar n}$ (use Feynman gauge for all gluons when determining which graphs are zero). Here q_n has n-collinear momentum p, and $\bar q_{\bar n}$ has $\bar n$-collinear momentum $\bar p$ and you should work in the CM frame. All graphs but one can be directly read off using the loop computations done in lecture (or given in the handout notes), as long as you use the same IR regulator. That is, you should keep both collinear quarks offshell, $p^2 \neq 0$ and $\bar p^2 \neq 0$. Compute the divergent terms in the one remaining ultrasoft graph using dimensional regularization in the UV.
d) Add up the $1/\epsilon$ terms from the graphs in c) and determine the lowest order anomalous dimension equation for C the Wilson coefficient of $J^{\rm SCET}$. Solve this equation keeping only the $\ln\mu/Q$ term and using a fixed coupling α_s, and then with a running coupling $\alpha_s(\mu)$. (Voilá, Sudakov double logs resummed.)