Lectures on Quark Flavor Physics: Exercises

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1. **Yukawa couplings, CKM matrix, and unitarity triangles:**

   a) Show that flavor non-diagonal kinetic terms in the Standard Model Lagrangian can always be diagonalized and brought into standard form by field redefinitions. To this end, study the Lagrangian

   $$\mathcal{L}_{\rm kinetic} = \bar{Q}_L\, Z_Q\, i\not{D}\, Q_L + \bar{u}_R\, Z_u\, i\not{D}\, u_R + \bar{d}_R\, Z_d\, i\not{D}\, d_R\,,$$

   where all fields are 3-component vectors in generation space, and Z_A are non-negative, hermitian 3×3 matrices.

   b) Show that an arbitrary complex matrix Y can be diagonalized by a biunitary transformation:

   $$W^\dagger\, Y\, U = \lambda\,,$$

   where U, W are unitary matrices, and λ is a real, diagonal matrix with non-negative eigenvalues. (*Hint:* Consider the matrices $YY^\dagger$ and $Y^\dagger Y$.)

   c) Derive the number of mixing angles and physical (i.e., observable) phases of the CKM matrix for the Standard Model with N fermion generations.

   d) Show that the Jarlskog determinant J defined as

   $$\mathrm{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right) = J\sum_{m,n}\epsilon_{ikm}\,\epsilon_{jln}\quad (i\neq k,\, j\neq l)$$

   is invariant under phase redefinitions of the quark fields, and calculate its value in terms of the Wolfenstein parameters to leading nontrivial order in λ.

   e) Show that all unitarity triangles have the same area $J/2$.

2. **Matching of Wilson coefficients in the effective weak Hamiltonian:**

   Assume that, in addition to its standard interactions, the Z^0 boson has a small flavor-changing coupling to left-handed b and s quarks:

   $$\mathcal{L}_Z = \frac{g_2}{\cos\theta_W}Z^\mu\left\{\sum_f \bar{f}\,\gamma_\mu\left(T_f^3\,\frac{1-\gamma_5}{2} - Q_f\sin^2\theta_W\right)f + \left(\varepsilon_{bs}\,\bar{s}\,\gamma_\mu\,\frac{1-\gamma_5}{2}\,b + \text{h.c.}\right)\right\},$$

   where $|\varepsilon_{bs}|\ll 1$. The sum in the first term is over all Standard Model fermions. T_f^3 is the third component of weak isospin, Q_f the electric charge in units of e, g_2 the SU(2) gauge coupling, and θ_W the weak mixing angle.

   Calculate the contributions to the Wilson coefficients C_{3-10} in the effective weak Hamiltonian for $b\to s\bar{q}q$ transitions arising from tree-level Z-boson exchange, working to first order in ε_{bs}. Recall that $m_Z\cos\theta_W = m_W$ and $G_F/\sqrt{2} = g_2^2/8m_W^2$. Use the fact that $T_f^3 = 0$ for right-handed quarks, while $T_f^3 = Q_f - Y$ with $Y = 1/6$ for left-handed quarks.