Stefano Frixione

Introduction to perturbative QCD and LHC phenomenology

Benasque, 22 – 24/7/2008

My aim is that of giving you sufficient information to understand the complicated *phenomenology* which will emerge from the LHC. This includes a small vocabulary in a sort of technical language

All the techniques I shall discuss are based on a *perturbative approach*

I shall deal with the *physics of hard processes*. The properties of any given hadron species will be irrelevant

Plan

- Motivations; basics of perturbative QCD
  $\sim 1^{st}$ lecture
- Fixed-order and resummed results
  $\sim 2^{nd}$ lecture
- Parton Shower Monte Carlos
  $\sim 3^{rd}$ lecture
- Outlook

Minimal bibliography

Textbook stuff

Ellis, Stirling, Webber, *QCD and Collider Physics*, Cambridge Press (1996)

Recent and current work

CTEQ Summer Schools: `www.cteq.org`

Les Houches writeups: `hep-ph/0604120`, `hep-ph/0403100`, `hep-ph/0403045`, `hep-ph/0204316`, `hep-ph/0005114`

Check CERN Academic Training lectures (several speakers)

Strong interactions: why bother?

- We have a theory (QCD – QuantumChromoDynamics) that describes the interactions of hadrons in a satisfactory manner; it is established (2004 Nobel prize)
- It is a one-parameter theory: $\alpha_s(M_Z)$ tells you everything about QCD, and we know it well
- It has nothing to say about some of the problems that keep us busy: origin of the EWSB, origin of mass, CP violation,...

Strong interactions: ubiquitous

- High-energy collisions
  - Fixed target experiments (pN, πN, γN)
  - DIS (HERA)
  - Hadronic colliders (Tevatron, LHC)
- Hadron properties
  - Hadron masses
  - Hadron decays
- High-density media
  - Heavy-ion collisions (RHIC, LHC)
  - Star formation and evolution
  - Early history of the universe

For accelerator particle physics in particular

LHC will have for many years a crucial importance for the field

LHC is a proton-proton collider: it is obvious that we have to understand hadron interactions well

So as far as we are concerned, the most important questions are rather:

- Do we really need to understand strong interactions to make discoveries*?

- If yes, do we master the theory of strong interactions well enough to guarantee a successful LHC physics program?

* I have in mind here "direct" detection of Higgs, sparticles, ...

The LHC

pp collisions at very large c.m. energy: $\sqrt{S} = 14$ TeV (10 TeV in 2008)
(Tevatron: $p\bar{p}$, $\sqrt{S} = 1.96$ TeV)

Immense integrated luminosity: $\int dt\mathcal{L} = 10 - 20$ fb^{-1}/yr (2009–2010)
$\longrightarrow$ 100 fb^{-1}/yr (2011–2014) $\longrightarrow$?? 1000 fb^{-1}/yr (after 2016 - SLHC)
(Tevatron: $\int dt\mathcal{L} = 3.8$ fb^{-1} in spring 2008)

Potential to discover almost all BSM scenarios
(SUSY, ED, little Higgs, TC,...)

$\Longrightarrow$ How soon?
What about the underlying theory?

Answers depend (also) on understanding of QCD

Road to discovery and beyond discovery

Most likely, we'll go through the following steps

0. Re-discover the SM. Experiments calibrate detectors and perform first "simple" measurements: W, Z, $t\bar{t}$, jets – 2008*(optimistic)*-2009
1. Discovery of BSM signals: easy (e.g. $Z'/G \to l\bar{l}$), intermediate (e.g. $\tilde{g}$ with "kind" SUSY), difficult (e.g. light SM Higgs)
2. Figure out/constrain the underlying theory

Accurate QCD predictions very important for 0 and 2;
from almost irrelevant to crucial for 1

"Do we need to understand strong interactions to make discoveries?" Basically YES

… and also not to claim fake discoveries (e.g. compositness, sparticles, …)

We've had "BSM signals" which haven't been confirmed, and a vast majority of cases where BSM signals were never found. Why?

- A Because accurate SM predictions had been used
- B Because even not-so-accurate SM predictions are sufficient to exclude new physics

When designing strategies for searches, not-so-accurate SM predictions are the not-so-unlikely choices (e.g. multijet backgrounds...)

In order not to make unjustified claims of discoveries, it is necessary to understand to which point one can trust SM computations and, if necessary, how to improve them

This is why we now set out to study QCD and its implications

Strong interactions = QCD

QuantumChromoDynamics is

- A non-abelian gauge theory, with gauge group SU(3)
- There are 8 spin-1 massless *gluons* that carry the interaction (adjont representation of SU(3)):

$$A^a, \qquad a = 1, \ldots 8$$

- There are $3 \times N_F$ spin-1/2 *quarks*, the matter fields (fundamental representation of SU(3)):

$$\psi_i^{(f)}, \qquad i = 1, 2, 3\,, \quad f = 1, \ldots N_F$$

"Chromo" since $(1, 2, 3) = (r, g, b)$ are called colours

f are the *flavours*. Their number N_F depends on which physics one considers. We call $1, \ldots 6 \longrightarrow$ up, down, strange, charm, beauty, top

QCD interactions are flavour blind; differences among quarks are due to EW interactions

The QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_{f=1}^{N_F} \bar{\psi}_i^{(f)} \left(i\gamma_\mu D^\mu_{ij} - m_f \delta_{ij} \right) \psi_j^{(f)} + \mathcal{L}_{GF} + \mathcal{L}_{ghost}$$

Covariant derivative:

$$D^\mu_{ij} = \delta_{ij} \partial_\mu + igt^a_{ij} A^a_\mu$$

Gluon strengh tensor:

$$G_a^{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

Plug the term fAA into the Lagrangian, and you'll get gluon 3- and 4-gluon self interactions – it makes all the difference wrt to QED. This term has a fundamental importance for the very existence of hadrons

SU(3) colour algebra

t^a and T^a are the SU$(N_c = 3)$ generators (in the fundamental and adjoint representations), with

$$[t^a, t^b] = if^{abc}t^c, \;\; [T^a, T^b] = if^{abc}T^c, \;\; (T^a)_{bc} = if^{abc}$$

Choosing the normalization

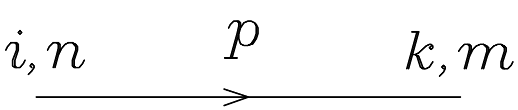
$$\mathrm{Tr}\left(t^a t^b\right) = T_R \delta^{ab} \equiv \frac{1}{2}\delta^{ab}$$

one has

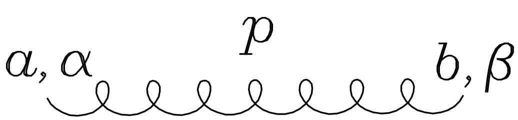
$$\sum_a t^a_{ij} t^a_{jk} = C_F \delta_{ik}\,, \;\; \mathrm{Tr}\left(T^a T^b\right) = C_A \delta^{ab}$$

$$C_F = \frac{N_c^2 - 1}{2N_c} \equiv \frac{4}{3}\,, \;\; C_A = N_c \equiv 3$$

Feynman rules

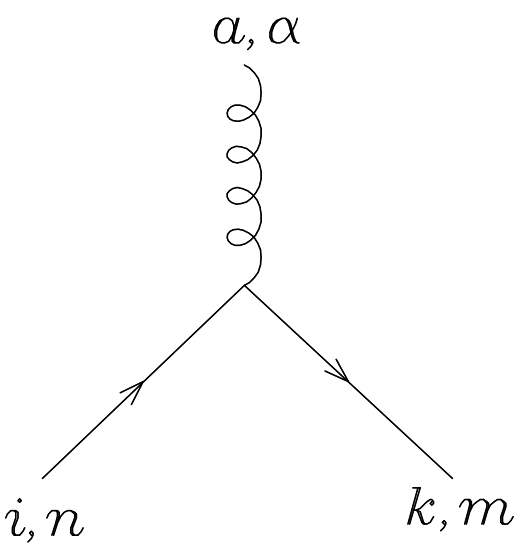


$$i\delta_{ik}\frac{(\gamma_\mu p^\mu + m_f)_{nm}}{p^2 - m_f^2 + i\epsilon}$$



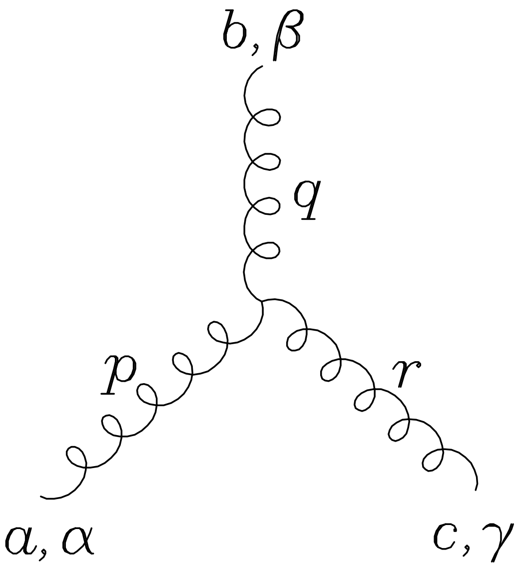
$$\frac{i\delta_{ab}}{p^2 + i\epsilon}\left[-g^{\alpha\beta} + (1-\lambda)\frac{p^\alpha p^\beta}{p^2 + i\epsilon}\right]$$

$$\frac{i\delta_{ab}}{p^2 + i\epsilon}\left[-g^{\alpha\beta} + \frac{p^\alpha n^\beta - p^\beta n^\alpha}{n \cdot p} - n^2\frac{p^\alpha p^\beta}{(n \cdot p)^2}\right]$$

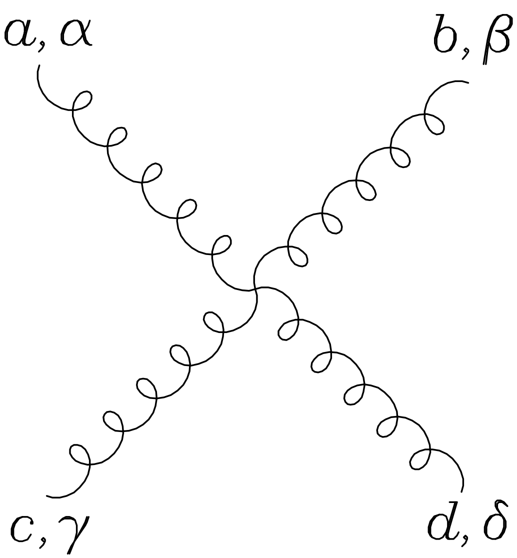


$$-igt^a_{ki}\gamma^\alpha_{mn}$$

Feynman rules



$$-gf^{abc}\left[g^{\alpha\beta}(p-q)^{\gamma}+g^{\beta\gamma}(q-r)^{\alpha}+g^{\gamma\alpha}(r-p)^{\beta}\right]$$

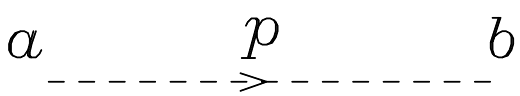


$$-ig^2 f^{eac} f^{ebd}\left(g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}\right)$$
$$-ig^2 f^{ead} f^{ebc}\left(g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}\right)$$
$$-ig^2 f^{eab} f^{ecd}\left(g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}\right)$$

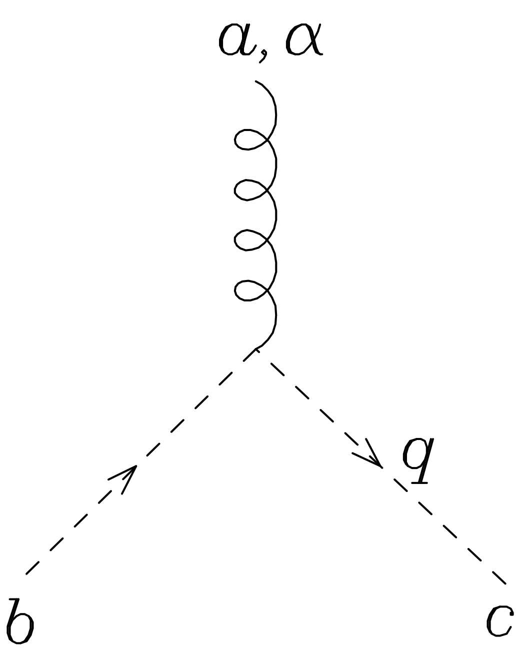
Feynman rules

In axial gauges the gluon has only physical (ie transverse) polarization states $\Longrightarrow$ simpler, intuitive physical picture. Drawback: involved computations become, well, more involved

In covariant gauges of non-abelian theories, ghosts must be included to cancel unphysical polarization states of gluons



$$i\delta_{ab}\frac{1}{p^2 + i\epsilon}$$



$$gf^{abc}q^{\alpha}$$

Light-quark symmetries

$$\mathcal{L}_{matter} = i \sum_{f=1}^{N_F} \left(\bar{\psi}_L^{(f)} \gamma_\mu D^\mu \psi_L^{(f)} + \bar{\psi}_R^{(f)} \gamma_\mu D^\mu \psi_R^{(f)} \right) - \sum_{f=1}^{N_F} m_f \left(\bar{\psi}_L^{(f)} \psi_R^{(f)} + \bar{\psi}_R^{(f)} \psi_L^{(f)} \right)$$

$$\psi_L^{(f)} = \frac{1}{2}(1-\gamma_5)\psi^{(f)}\,, \qquad \psi_R^{(f)} = \frac{1}{2}(1+\gamma_5)\psi^{(f)}$$

There is a huge symmetry when $m_f = 0$ *(chiral)*

$$\psi_L^{(f)} \longrightarrow e^{i\phi_L} U_L^{ff'} \psi_L^{(f')}\,, \qquad \psi_R^{(f)} \longrightarrow e^{i\phi_R} U_R^{ff'} \psi_R^{(f')}$$

$$\mathrm{SU}_L(N_F) \otimes \mathrm{SU}_R(N_F) \otimes \mathrm{U}_L(1) \otimes \mathrm{U}_R(1)$$

Chiral symmetry is not apparent in the observed spectrum; and, quantization effects may also distroy classical symmetry

- $\mathrm{SU}_L(N_F) \otimes \mathrm{SU}_R(N_F) \longrightarrow \mathrm{SU}_V(N_F)$, isospin; $\mathrm{SU}_A(N_F)$ is spontaneously broken, with Goldstone bosons = light mesons (π's for $N_F = 2$, π's+K's+η for $N_F = 3$)
- $\mathrm{U}_L(1) \otimes \mathrm{U}_R(1) \longrightarrow \mathrm{U}_V(1)$, baryon number conservation; $\mathrm{U}_A(1)$ spoilt by quantum effects ($\mathcal{L}_\theta$)

The QCD Lagrangian should surprise you a lot – which is OK, since it took several *decades* of efforts to establish it

First of all: why is QCD a quantum field theory, like the Standard Model? There's no parameter which is naturally small in strong interactions

- Typical cross sections are $\mathcal{O}$(10 mb)$\simeq 1/M_s^2$
- Baryon size $\simeq 1/M_s$
- Hadronic widths $\simeq M_s$

with $M_s = \mathcal{O}$(100 MeV)

This is a long way from a weakly-coupled pointlike vertex

Once a candidate theory is available, one can test it in all phenomena involving strong interactions

To formulate a candidate theory is another story. We can say that

- Matter content: motivated by “static” experiments, ie hadron spectroscopy
- Gauge content: motivated by scattering experiments able to probe the dynamics (scaling in DIS)

We have a framework, the QCD Lagrangian, to work with

Scaling and hadron spectrospy have led to the QCD Lagrangian.
The minimal requirements are that:

1) QCD implies approximate scaling, and predicts scaling violations

2) QCD explains why we don't see free colour charges, and the bound states we postulated coincide with observable hadrons

If you think it's trivial, think twice:

- Gross, Politzer, and Wilczek got the 2004 Nobel prize for 1);
- and 2), for which we have solid evidence but no proof as that of 1), will be another Nobel prize

Memo on RGE and beta functions

Suppose A is a dimensionless quantity which depends on a single large energy scale $Q \gg m$, with m any mass. If the limit $m \to 0$ exists, then by dimensional analysis A is independent of Q

$$A = A(Q/m, \alpha_S) \stackrel{m \to 0}{\longrightarrow} A(\alpha_S)$$

This elegant derivation does not survive quantization. Because of the presence of ultraviolet divergences, the theory must be renormalized, and this always introduce an arbitrary mass scale μ (in A and α_S renormalized)

$$A \stackrel{\text{quantization}}{\longrightarrow} A(Q^2/\mu^2, \alpha_S)$$

The scale μ is arbitrary, and physical results cannot depend on it

$$\frac{d}{d\mu^2} A(Q^2/\mu^2, \alpha_S) = \left(\frac{\partial}{\partial \mu^2} + \frac{\partial \alpha_S}{\partial \mu^2} \frac{\partial}{\partial \alpha_S} \right) A = 0$$

which is a Renormalization Group Equation

In order to solve RGE's, one defines

$$t = \log \frac{Q^2}{\mu^2}\,, \qquad \beta(\alpha_S) = \mu^2 \frac{\partial \alpha_S}{\partial \mu^2}$$

$$\left(-\frac{\partial}{\partial t} + \beta(\alpha_S)\frac{\partial}{\partial \alpha_S}\right) A = 0$$

The *running coupling* $\alpha_S(Q)$ is then introduced

$$t = \int_{\alpha_S}^{\alpha_S(Q^2)} da \frac{1}{\beta(a)}\,, \qquad \alpha_S(\mu^2) = \alpha_S$$

from which it follows that

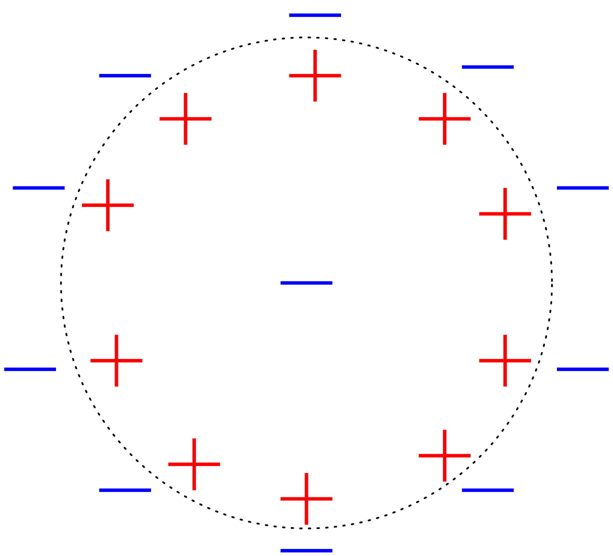
$$A(Q^2/\mu^2, \alpha_S) = A(1, \alpha_S(Q^2))$$

Thus, the scale dependence of A is known if that of $\alpha_S(Q^2)$ is known

The computation of β functions in QFTs has profound implications

The case of QED...

...is relatively simple, and allows a graphical explanation of the running coupling



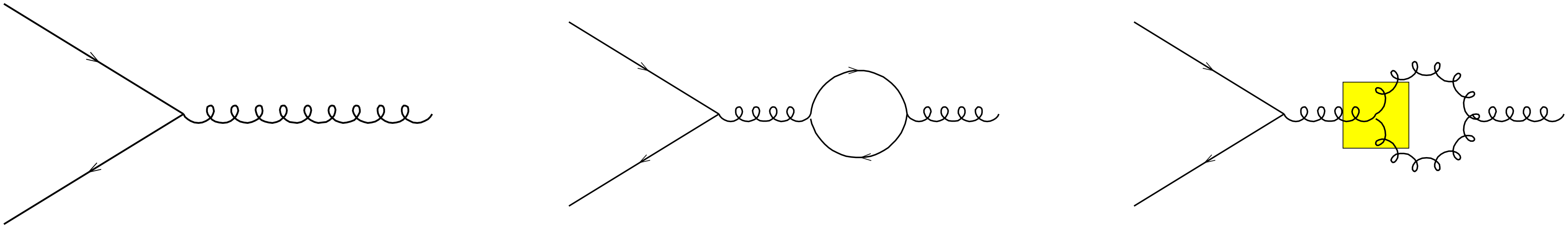
In a relativistic framework, an electron is surrounded by a cloud of virtual electrons and positrons. From the distance, one may not see their charges. By looking closer (probe with larger momenta), one starts to resolve them, and electron charge appears larger

$$Q^2 \frac{d\alpha}{dQ^2} = \beta_{QED}(\alpha)\,, \quad \beta_{QED}(\alpha) = \frac{\alpha^2}{3\pi} + \mathcal{O}(\alpha^3) \quad \Longrightarrow \quad \alpha(Q^2) = \frac{1}{137 - \frac{1}{3\pi}\log(Q^2/m_e^2)}$$

Since $\alpha \to \infty$ for $Q^2 \to e^{411\pi} m_e^2$, Landau (1954) thought QED was ill-defined

The case of QCD

In QCD there are additional contributions from gluon self-interaction...



that have a dramatic effect on the β function

$$\beta_{QCD}(\alpha_S) = -\beta_0 \alpha_S^2 + \mathcal{O}(\alpha_S^3)\,, \quad \beta_0 = \frac{11C_A - 2N_F}{12\pi}\,, \quad C_A = N_C \equiv 3$$

Basically, the gluonic contribution to the vacuum polarization reverses the sign of the β function, in such a way that $\alpha_S(Q^2)$ *decreases* when Q^2 *increases* (for $N_F \leq 16...$)

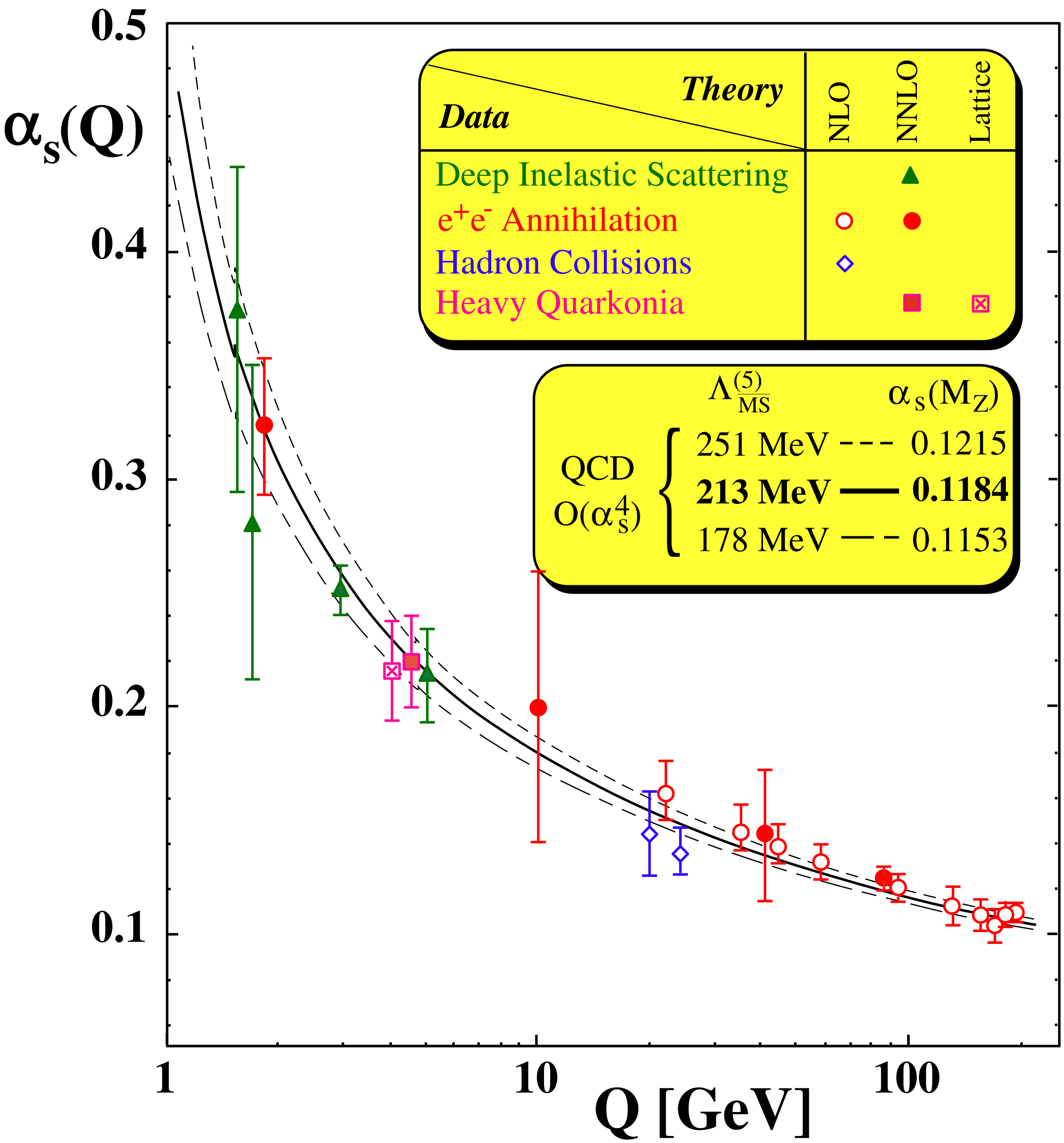
This is called Asymptotic Freedom

Gross, Politzer, Wilczek Nobel prize 2004

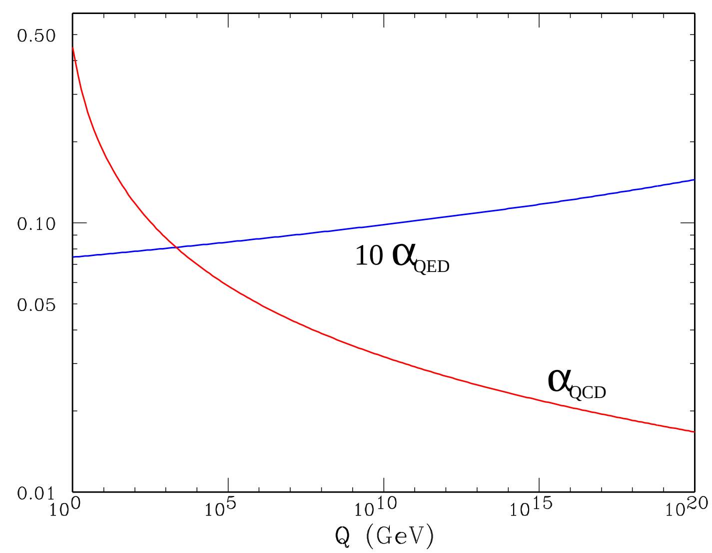
This is the opposite as in QED, which implies that QCD is not an effective low-energy theory of something unknown

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 + \alpha_S(\mu^2)\beta_0 \log(Q^2/\mu^2)}$$

Comparisons with data



There is nowadays a very solid evidence that α_S runs as predicted by QCD with $N_C = 3$



Thanks to asymptotic freedom, we understand why the quarks behave as free particles in DIS (and elsewhere)

This allows one to use standard perturbation techniques, as the case of β_{QCD} determination spectacularly shows

We also have *hints* on why quarks/gluons cannot be seen in isolation (i.e. confinement). Naively, large distances $\equiv$ small scales $\Longrightarrow$ inter-parton force grows

Lattice gives further (solid) evidence

A short summary

- ♦ In certain kinematic regimes, strong interactions are weakly coupled: asymptotic freedom allows us to use the perturbative machinery
- ♦ We know (we suspect) that QCD can describe physical hadrons and explain confinement

This is not sufficient for us to give predictions for physical observables. What we can compute (quark and gluon reactions) is non-observable, and what is observable (hadrons) we cannot compute

We need three additional concepts to proceed:

- ▶ Hadron-parton duality
- ▶ Infrared safety
- ▶ Factorization theorems

Hadron-parton duality

Inclusive hadronic observables can be expressed in terms of quark-gluon degrees of freedom. More precisely

$$\int ds\, w(s)\, O^{hadron}(s) = \int ds\, w(s)\, O^{parton}(s)$$

with $w(s)$ a weight function of some energy scale s, peaked at $s = s_0$ (s_0 is a characteristic scale of the process). In practice one always uses *local* hadron-parton duality, for which

$$w(s) = \delta(s - s_0)$$

In other words: compute your observables in terms of quarks and gluons, and assume the results would be the same if you were able to perform a hadron-level computation

Infrared safety

An observable $\mathcal{O}$ is infrared safe if the functions $\mathcal{O}_n(k_1, \cdots, k_n)$ that define it in terms of parton momenta have the following properties:

$$\mathcal{O}_n(k_1, \cdots, k_i, \cdots, k_n) \xrightarrow{E_i \to 0} \mathcal{O}_{n-1}(k_1, \cdots, k_n)$$

$$\mathcal{O}_n(k_1, \cdots, k_i, \cdots, k_j, \cdots, k_n) \xrightarrow{k_i \| k_j} \mathcal{O}_{n-1}(k_1, \cdots, k_i + k_j, \cdots, k_n)$$

Translation: an observable must be insensitive to the emission of soft partons, or to the collinear splittings of partons

- IR-safe observables: thrust, p_T of single-inclusive and hardest jet,...
- IR-unsafe observables: number of gluon jets, y of the hardest jet,...

Factorization theorems

$$d\sigma_{H_1H_2}(P_1,P_2) = \sum_{ij}\int dx_1 dx_2 f_i^{(H_1)}(x_1,\mu^2) f_j^{(H_2)}(x_2,\mu^2) \times d\hat{\sigma}_{ij}(x_1P_1,x_2P_2;\alpha_S(\mu^2),\mu^2)$$

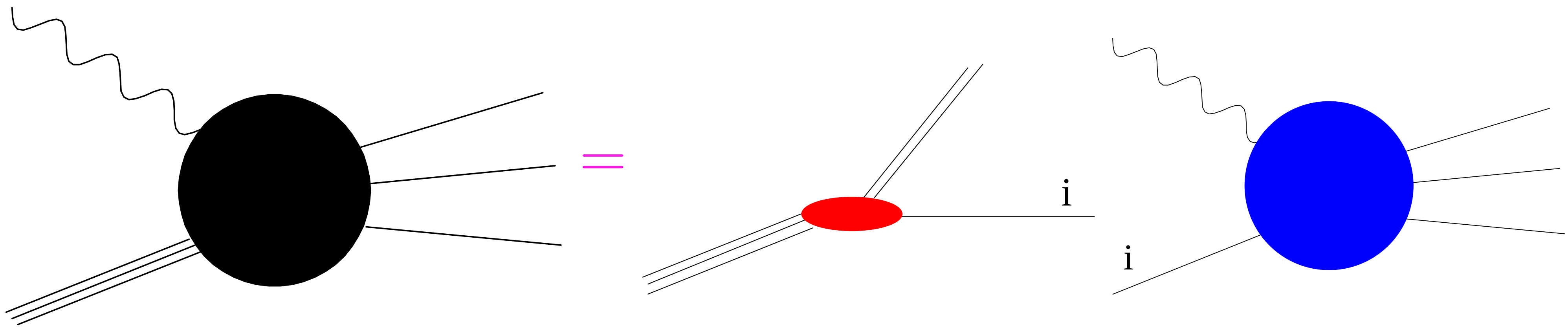
$$d\sigma_{eH}(P) = \sum_{i}\int dx f_i^{(H)}(x,\mu^2) d\hat{\sigma}_{ei}(xP;\alpha_S(\mu^2),\mu^2)$$

- The partonic cross sections $d\hat{\sigma}_{ij}$, $d\hat{\sigma}_{ei}$ are computable in perturbation theory
- The PDFs f_i must be extracted from data

Intuitive physical picture (Born & Oppenheimer): phenomena at different time scales (hadronization and hard scattering) factorize

Factorization theorems are, apart from the case of DIS, formally unproved. They are however largely accepted, and stand countless tests

$$d\sigma_{eH}(P) \;=\; \sum_i \int dx\, f_i^{(H)}(x,\mu^2)\, d\hat{\sigma}_{ei}(xP;\alpha_S(\mu^2),\mu^2)$$



The timescale $1/M$ for binding the hadron is much larger than the timescale $1/Q$ for the hard scattering $\Longrightarrow$ incoherent scatterings

- μ arbitrarily separates hard from soft scales
- In practice: pull out a parton with a random fraction z of the hadron momentum, scatter it with the photon. Ignore the hadron remnants
- There are "leakages", ie corrections of type $(1/Q)^p$
- Intuitively clear that f doesn't depend on the nature of hard scattering

■ Why is it not *identical* to the parton model?

Summary so far

- Understanding QCD is crucial for the LHC discovery programme
- QCD is an asymptotically-free QFT, supported by hadron spectroscopy and high-energy experiments
- Perturbative techniques can be used, but are not sufficient: large-distance effects are always present
- To deal with them, one must introduce (at least) hadron-parton duality, infrared safety, factorization theorems

EXAMPLES OF PERTURBATION THEORY AT WORK

$e^+e^- \longrightarrow$ hadrons, jets

DIS and the problem of initial-state divergences

Scale dependence of PDFs

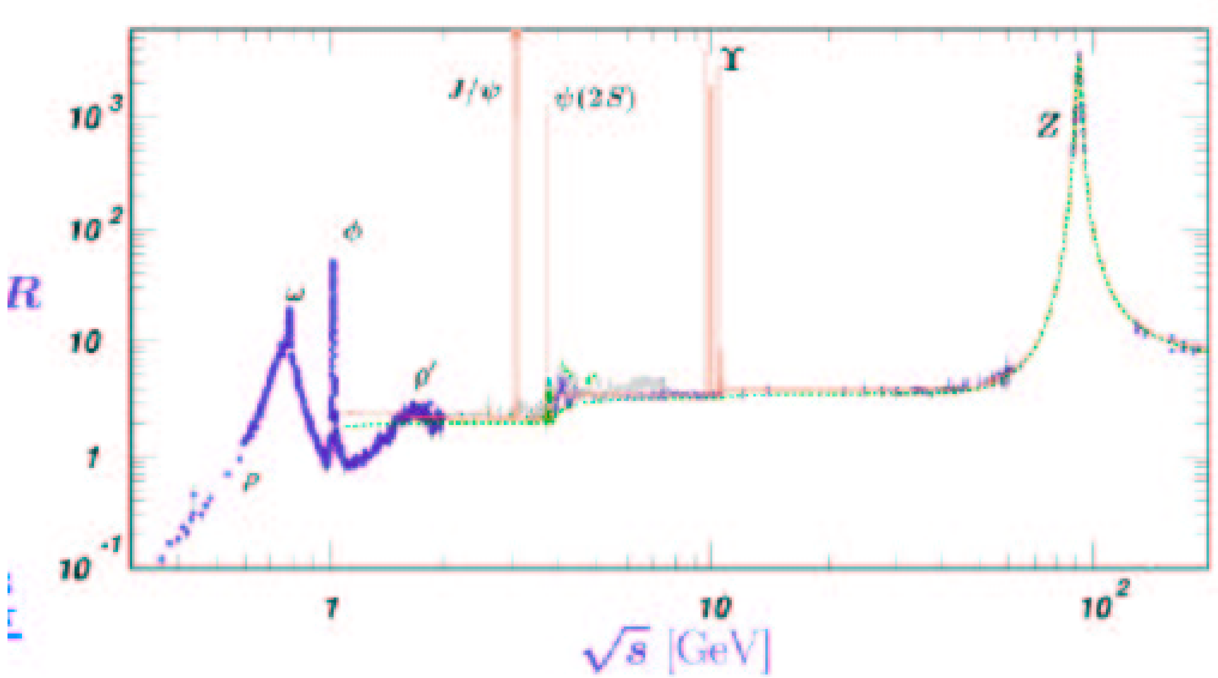
Let's see in practice the way in which hadron-parton duality, infrared safety and factorization theorems work

The simplest case is the total hadronic rate in e^+e^- collisions

- Hadron-parton duality $\Longrightarrow$ compute the total *partonic* rate
- Total rate is (trivially) infrared safe

It's actually customary to give the results as

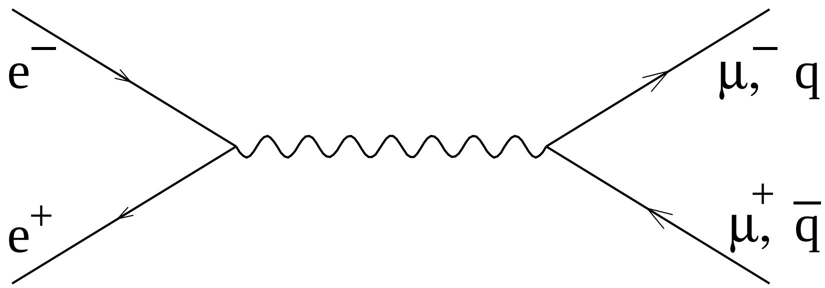
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



At the lowest order in perturbation theory (of α_S)

$$R = \sum_{i,f} \frac{\sigma(e^+e^- \to q_i^{(f)}\bar{q}_i^{(f)})}{\sigma(e^+e^- \to \mu^+\mu^-)} \longrightarrow \sum_{i=1}^{N_C}\sum_{f=1}^{N_F} e^2(f)$$

since numerator and denominator are the same diagram



$$\Longrightarrow \quad R = N_C\left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right]$$

which is also a test of colour and charge assignments (here for u, d, and s quarks)

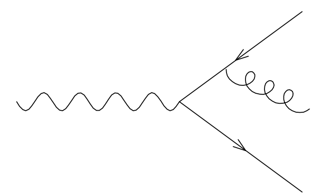
For larger c.m. energies, just add more quark flavours

- Is this result systematically improvable, in the sense of perturbation theory? This is what we expect from the β_{QCD} computation

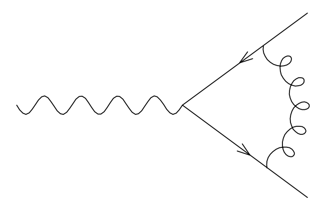
Perturbative corrections to R

At the first order beyond Born (*next-to-leading order*, NLO), there are two classes of corrections (as in QED)

- Real contribution: all Feynman diagrams with an additional (wrt Born) parton in the final state



- Virtual contribution: all one-loop Feynman diagrams that can be obtained from Born diagrams

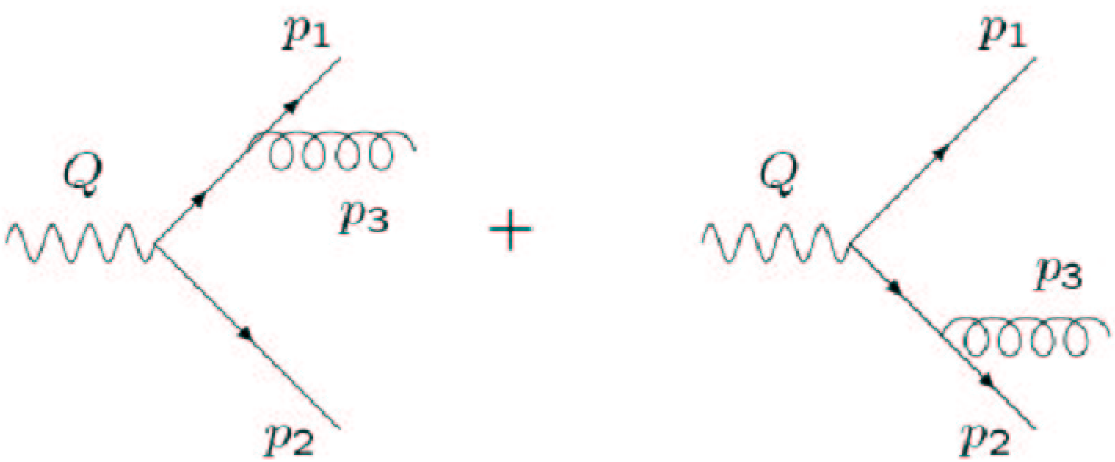


R and V don't interfere: diagrams have different number of legs

$$\text{real} = g_S \mathcal{A}_R \qquad \text{virtual} = g_S^2 \mathcal{A}_V$$

$$|\mathcal{A}_{NLO}|^2 = |\mathcal{A}_{LO}|^2 + \alpha_S \left(|\mathcal{A}_R|^2 + 2\Re(\mathcal{A}_{LO}\mathcal{A}_V^\star) \right) + \mathcal{O}(\alpha_S^2)$$

Real contribution



$$x_i = \frac{2p_i \cdot Q}{Q^2} = \frac{2E_i}{\sqrt{s}}$$

$$p_1 + p_2 + p_3 = Q \Longrightarrow$$

$$x_1 + x_2 + x_3 = 2$$

Phase space and matrix element:

$$d\Phi_{q\bar{q}g} = \frac{s}{32(2\pi)^5}\delta(2 - x_1 - x_2 - x_3)dx_1dx_2dx_3d\Omega$$

$$|\mathcal{A}_R|^2 = |\mathcal{A}_{LO}|^2 \, C_F \frac{\alpha_S}{2\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

which lead to

$$\sigma_R = \int d\Phi_{q\bar{q}g} \, |\mathcal{A}_R|^2 = \infty$$

It is instructive to see why this is divergent

$$
\begin{aligned}
1 - x_1 &= x_2 \frac{E_3}{\sqrt{s}}(1 - \cos\theta_{23}) = \frac{(p_2 + p_3)^2}{Q^2} \\
1 - x_2 &= x_1 \frac{E_3}{\sqrt{s}}(1 - \cos\theta_{13}) = \frac{(p_1 + p_3)^2}{Q^2}
\end{aligned}
$$

The divergences of the matrix elements are at

$$
\begin{aligned}
x_1 \longrightarrow 1 \;\&\; x_2 \longrightarrow 1 \quad &\Longleftrightarrow \quad E_3 \longrightarrow 0 \quad \text{soft} \\
x_1 \longrightarrow 1 \quad &\Longleftrightarrow \quad \theta_{23} \longrightarrow 0 \quad \text{collinear} \\
x_2 \longrightarrow 1 \quad &\Longleftrightarrow \quad \theta_{13} \longrightarrow 0 \quad \text{collinear}
\end{aligned}
$$

This clarifies that the divergences are not physical: we are pushing pQCD beyond its range of applicability, since parton energies or parton-pair invariant masses are comparable to hadron masses $\Longrightarrow$ confinement effects can't be neglected

In other words: we are trying to resolve partons in a regime where the concept of parton is not particularly meaningful

- Go home and throw hadron-parton duality (and pQCD) in the bin?

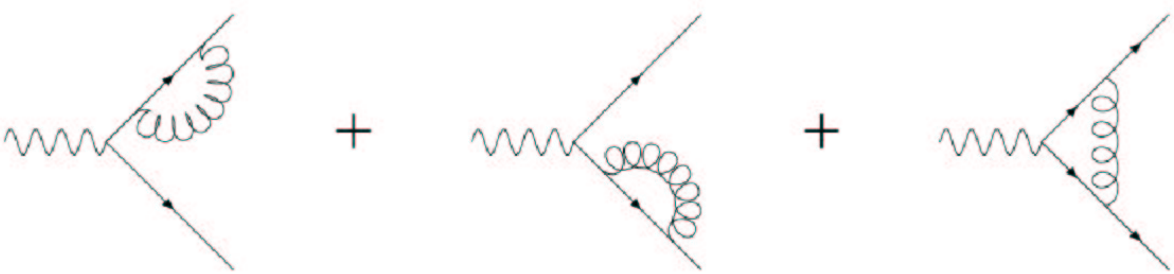
Not yet: what the previous computation tells us is that the cross section for the production of $q\bar{q}g$ is not a meaningful quantity in perturbation theory

But this cross section is just one of the contributions to $e^+e^- \longrightarrow$ hadrons at $\mathcal{O}(\alpha_S)$ – we still have to consider the virtual contribution

So before throwing everything away, we have to prove that soft/collinear emissions are dominant also after adding virtual corrections

Note that what we've got is not peculiar of QCD: you get the same if you compute $\mu^+\mu^-\gamma$ production in QED

Virtual contribution



$$x_i = \frac{2p_i \cdot Q}{Q^2} = \frac{2E_i}{\sqrt{s}}$$

$$p_1 + p_2 = Q \Longrightarrow$$

$$x_1 = 1, \; x_2 = 1$$

One can easily see that

$$\sigma_V = \int d\Phi_{q\bar{q}} \Re(\mathcal{A}_{LO}\mathcal{A}_V^{\star}) = -\infty$$

- Physical meaning: we are trying to compute the probability of having *exactly* two quarks in the final state
- As in QED, this quantity diverges order-by-order in PT. The result to all orders, however, is not the same as in QED, owing to the different behaviour of the running coupling

$$\sigma_R + \sigma_V = \infty - \infty = ?$$

! *Regularize* R and V contributions before summing them $\longrightarrow$ in QCD, this usually means computing the integrals in $d = 4 - 2\epsilon$ dimensions

$$\int^1 \frac{dx}{1-x} = -\log(0) \quad \xrightarrow{\text{regularization}} \quad \int^1 \frac{dx(1-x)^{-2\epsilon}}{1-x} = -\frac{1}{2\epsilon}$$

$$\sigma_R = \sigma_{LO} C_F \frac{\alpha_S}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2\right) + \mathcal{O}(\epsilon)$$

$$\Longrightarrow \qquad \sigma_V = \sigma_{LO} C_F \frac{\alpha_S}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right) + \mathcal{O}(\epsilon)$$

$$\lim_{\epsilon \to 0} (\sigma_R + \sigma_V) = \frac{\alpha_S}{\pi}\sigma_{LO}$$

The singularities are gone! So we can obtain

$$R = N_C \sum_f Q_f^2 \left(1 + \frac{\alpha_S}{\pi}\right) + \mathcal{O}(\alpha_S^2)$$

This is a small correction ($< 5\%$), and improves the comparison to data – we have proven that the total rate is insensitive to soft/collinear emissions

Physical meaning: soft/collinear real configurations are kinematically degenerate with virtual configurations. Thus, it looks like finite quantities are obtained by summing over degenerate (ie non-resolvable) partonic configurations

This is true to all orders:

Kinoshita-Lee-Nauenberg (KLN) theorem: in the computation of inclusive (enough) quantities, infrared divergences cancel, and the result is finite

And this can indeed be checked by explicit computations $\longrightarrow$

$$R = R_{LO}\left[1 + \frac{\alpha_S}{\pi} + 1.411\left(\frac{\alpha_S}{\pi}\right)^2 - 12.8\left(\frac{\alpha_S}{\pi}\right)^3\right] + \mathcal{O}(\alpha_S^4)$$

The new terms improve further the agreement with data

This is a huge success! Keep in mind we have used several highly non trivial ingredients

- *Asymptotic freedom*
- *Hadron-parton duality*
- *Infrared safety*

and we have also verified that the KLN theorem works

Speaking of which: how can one prove such an all-order statement?

What to take home

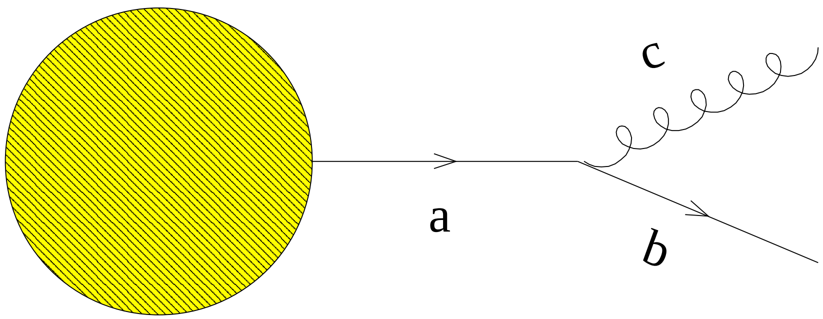
- When considering perturbative corrections, IR divergences appear
- Certain observables are finite, ie insensitive to the IR sector. For this to happen, real and virtual contributions to the perturbative corrections must both be considered at the NLO
- Perturbative corrections are larger than in QED, but still under control; a pQCD program makes sense...
- ... but one always needs hadron-parton duality, infrared safety, factorization theorems (large distance unavoidable)

Do we have to prove, observable by observable, the cancellation of IR singularities of real and virtual origin?

How can we prove all-order statements?

- The two questions are closely related

Divergences are actually observable independent, and "universal"; can be easily computed in a physical gauge



$$k_b = zk_a + k_T + \zeta_b n$$

$$k_c = (1-z)k_a - k_T + \zeta_c n$$

$$k_b^2 = 0 \;\Rightarrow\; \zeta_b = -\frac{k_T^2}{2zn\cdot k_a}$$

$$k_c^2 = 0 \;\Rightarrow\; \zeta_c = -\frac{k_T^2}{2(1-z)n\cdot k_a}$$

$$d\sigma_R = \frac{\alpha_S}{2\pi}\int dk_T^2 dz\, C_F \frac{1+z^2}{1-z}\frac{1}{k_T^2} d\sigma^{(0)}(k_a) + \text{non singular}$$

Again the collinear ($k_T \to 0$) and soft ($z \to 1$, with $k_T \to (1-z)\hat{k}_T$) divergences. They arise when parton a goes on shell $\Longrightarrow$ the propagator diverges

These IR divergences will cancel when adding virtual corrections

The quantity associated with the divergence depends only on parton flavours and kinematics. At the LO, we have the following cases

$$q \to q(z)g(1-z) \quad \Longrightarrow \quad P_{qq}(z) = C_F \frac{1+z^2}{1-z}$$

$$g \to q(z)\bar{q}(1-z) \quad \Longrightarrow \quad P_{qg}(z) = T_R \left(z^2 + (1-z)^2\right)$$

$$q \to g(z)q(1-z) \quad \Longrightarrow \quad P_{gq}(z) = C_F \frac{1+(1-z)^2}{z} = P_{qq}(1-z)$$

$$g \to g(z)g(1-z) \quad \Longrightarrow \quad P_{gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$

$$C_F = \frac{4}{3}\,, \quad C_A = 3\,, \quad T_R = \frac{1}{2}$$

which are the (unsubtracted) *Altarelli-Parisi splitting kernels*

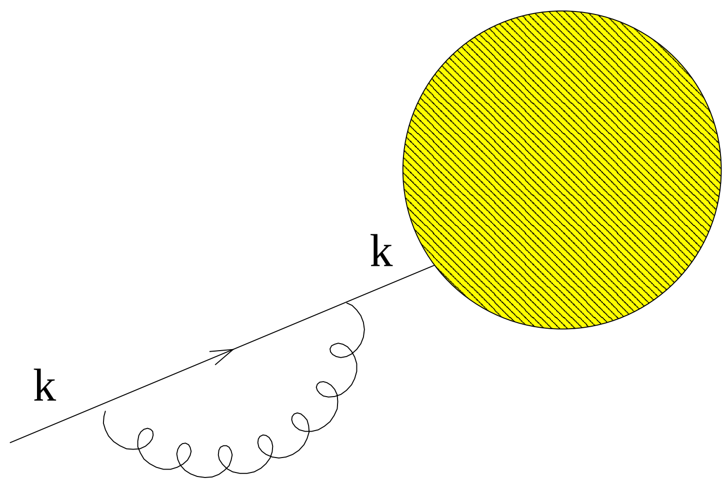
Consider now the case a process with an initial-state hadron: DIS

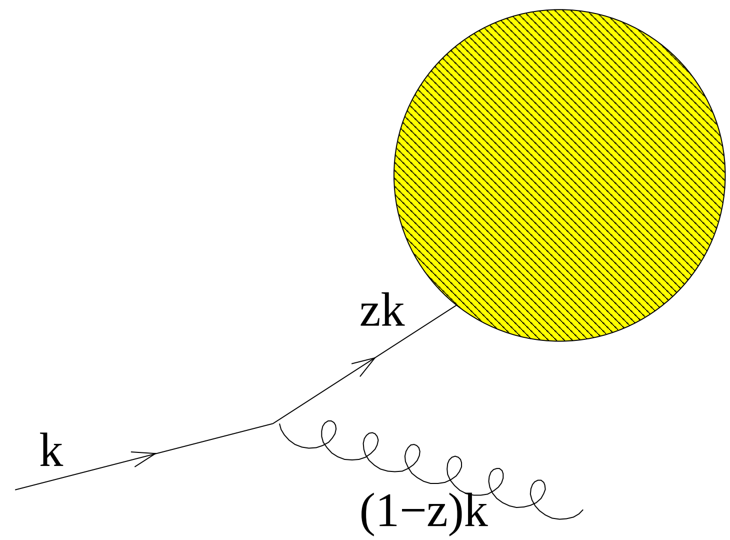
The leading order is well known: it's Feynman parton-model formula

$$d\sigma_{ep}(K) = \sum_q \int dx f_q(x) d\sigma_{eq}(xK)$$

with $d\sigma_{eq}$ the LO cross section for $eq \to eX$

Following what done before, we consider NLO corrections to $d\sigma_{eq}$





$$d\sigma_R + d\sigma_V = \frac{\alpha_S}{2\pi}\int dk_T^2 dz\, C_F \frac{1+z^2}{1-z}\frac{1}{k_T^2}\left(d\sigma^{(0)}(zk_a) - d\sigma^{(0)}(k_a)\right)$$

Finite for $z \to 1$ (soft), but *divergent* for $k_T \to 0$ (collinear)!

The real kinematic is not degenerate with the virtual one in the collinear limit. This does not happen in the case of final-state emissions

Tentative conclusion: the parton model *does not survive* radiative corrections

If so, pQCD can only be used for final-state hadrons

But there is a way out, which implies replacing the naive parton model by its QCD equivalent, the factorization theorem

Before going into that, a bit of notation

Plus distributions

Redefine the qq Altarelli-Parisi kernel as follows (a distribution)

$$P(z) = C_F \left(\frac{1+z^2}{1-z} \right)_+$$

This notation introduces the "plus prescription":

$$\int_0^1 dz h(z) (g(z))_+ = \int_0^1 dz (h(z) - h(1)) g(z)$$

The NLO corrections to the parton cross section can therefore be written in a much more compact form

$$d\sigma_R(k_a) + d\sigma_V(k_a) = \frac{\alpha_S}{2\pi} \int \frac{dk_T^2}{k_T^2} dz P(z) d\sigma^{(0)}(z k_a)$$

The $+$ reminds to subtract the $z = 1$ singularity $\Longleftarrow$ includes part of the virtual corrections

Recovering the parton model

Exclude the collinear divergence with a cutoff $\mu_0 \ll Q$. Inserting the partonic cross section into the parton model we get after the k_T integration

$$d\sigma^{(1)}(K) = \frac{\alpha_S}{2\pi} \log \frac{Q^2}{\mu_0^2} \int dy dz f(y) P(z) d\sigma^{(0)}(yzK)$$

and with some algebra

$$d\sigma(K) \equiv d\sigma^{(0)}(K) + d\sigma^{(1)}(K) = \int dy \hat{f}(y, \mu^2, \mu_0^2) d\hat{\sigma}(yK, \mu^2, Q^2)$$

with $\mu_0 \ll \mu \sim Q$

$$\begin{aligned} \hat{f}(y, \mu^2, \mu_0^2) &= f(y) + \frac{\alpha_S}{2\pi} \log \frac{\mu^2}{\mu_0^2} \int_y^1 \frac{dz}{z} P(z) f(y/z) \\ d\hat{\sigma}(K, \mu^2, Q^2) &= d\sigma^{(0)}(K) + \frac{\alpha_S}{2\pi} \log \frac{Q^2}{\mu^2} \int_0^1 dz P(z) d\sigma^{(0)}(zK) \end{aligned}$$

Note: it is $\hat{f}$ that is usually denoted by f

It is now manifest that the *divergence is independent of the process* (as for final-state emissions). Consequences

- PDFs acquire a dependence upon mass scales: scaling violations
- PDFs cannot be expanded in perturbation theory
- Parton cross sections do have a perturbative expansion

The key assumption: Nature will kill the $\log \mu_0$ divergence in the PDFs (smearing typical of long-distance phenomena). We cannot compute PDFs, but we can extract them from data

Parton model is formally recovered. An all-order proof of these QCD-improved formulae gives a *factorization theorem*

If one derives the PDFs wrt the hard scale μ

$$\frac{\partial \hat{f}(y,\mu^2,\mu_0^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi}\int_y^1 \frac{dz}{z} P(z)\hat{f}(y/z,\mu^2,\mu_0^2) + \mathcal{O}(\alpha_S^2).$$

The cutoff dependence is entirely in $\hat{f}$ $\Longrightarrow$ sensible to assume that the r.h.s. is the first order of a well-behaved perturbative expansion

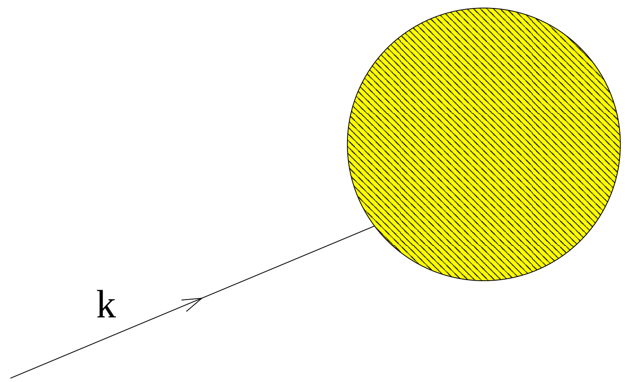
One therefore arrives at the Altarelli-Parisi equations (1977), being careful enough to include *all possible splitting* types

$$\begin{aligned} \frac{\partial \hat{f}_a}{\partial \log \mu^2} &= \sum_b P_{ab} \otimes \hat{f}_b \\ P_{ab} &= \alpha_S \bar{P}_{ab}^{(0)} + \alpha_S^2 \bar{P}_{ab}^{(1)} + \alpha_S^3 \bar{P}_{ab}^{(2)} + \ldots \end{aligned}$$

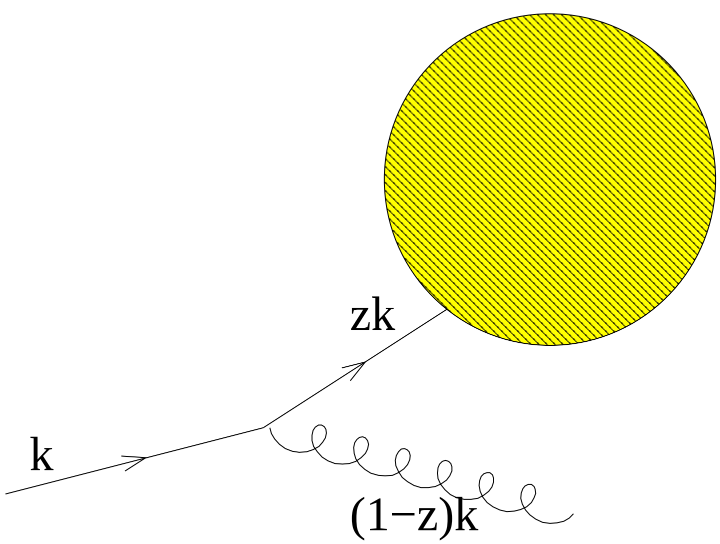
I introduced another frequently used notation

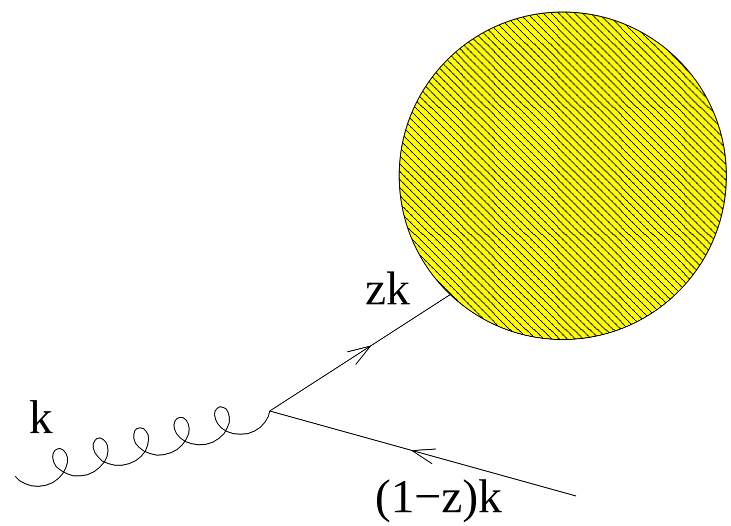
$$f = g \otimes h \quad \Longleftrightarrow \quad f(x) = \int_0^1 dy dz \delta(x - yz) g(y) h(z)$$

Note: the necessity of considering all splitting types is a consequence of perturbative corrections. The LO diagram



has the following real correction diagrams





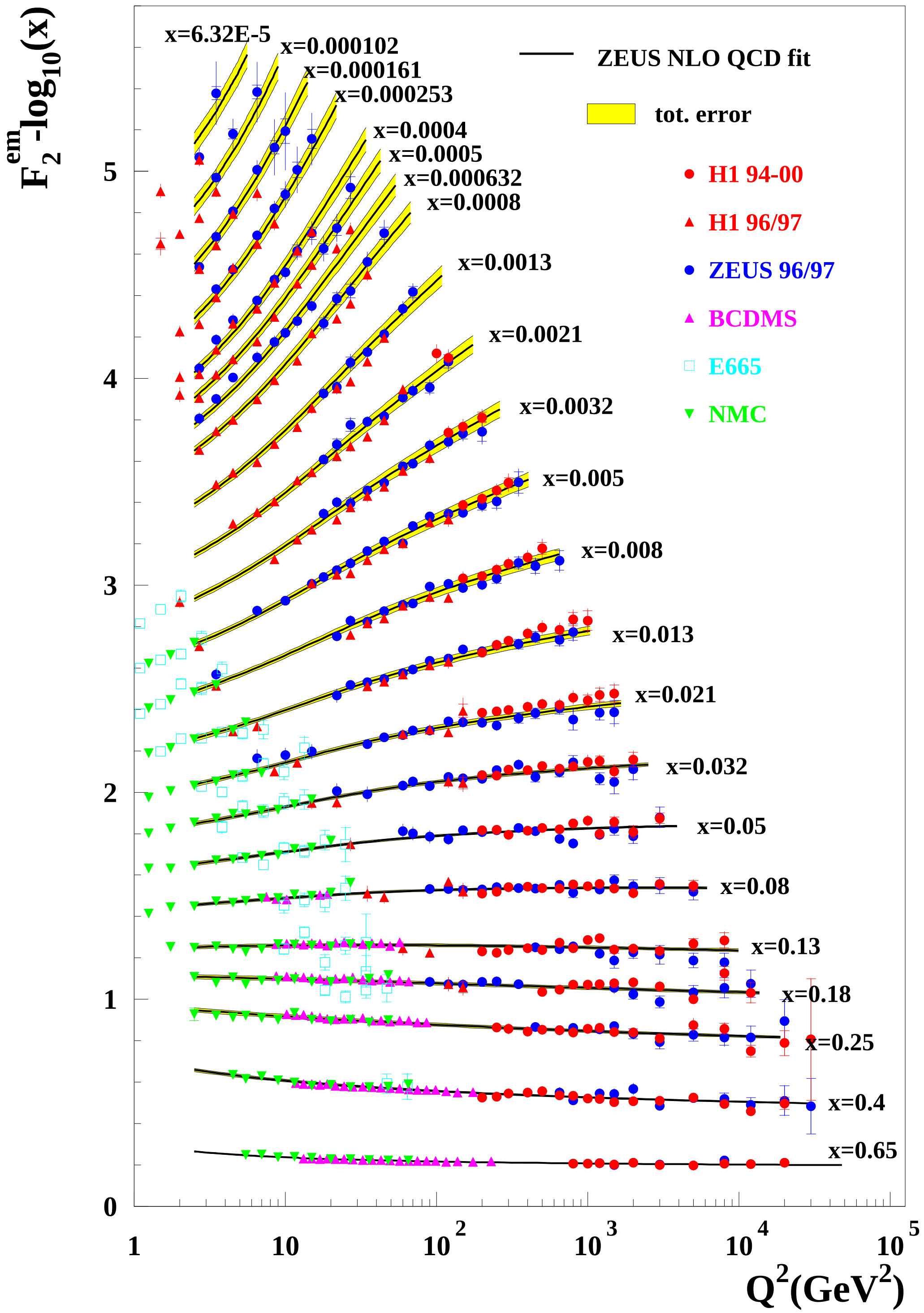
Thus, just by following QCD perturbative rules, we must admit the possibility that gluons can also act as partons. The previous RGE gets therefore generalized to:

$$\frac{\partial f_a}{\partial \log \mu^2} = \sum_b P_{ab} \otimes f_b$$

$$P_{ab} = \sum_i \left(\frac{\alpha_S}{2\pi}\right)^i P_{ab}^{(i)}$$

which are the Altarelli-Parisi equations

- Even if we cannot compute the parton densities, we can predict perturbatively their scale dependence
- Asymptotically, gluons carry more than half of a proton momentum
- Once again, large-scale behaviour is related to the infrared regime



$$F_2^{\mathrm{NC}} = x \sum_f e^2(f) \left[q^{(f)} + \bar{q}^{(f)} \right] + \mathcal{O}(\alpha_S)$$

An excellent fit already at the NLO

History of AP kernels

- $P_{ab}^{(0)}$: Altarelli, Parisi (1977)
- $P_{ab}^{(1)}$: Curci, Furmanski, Petronzio (1980)
- $P_{ab}^{(2)}$: Moch, Vermaseren, Vogt (2004)

The calculation of $P_{ab}^{(2)}$ is the toughest ever performed in perturbative QCD, with 10^6 lines of dedicated algebraic code, and 20 man-year of work

- One loop $\Longrightarrow$ 18 Feynman diagrams
- Two loops $\Longrightarrow$ 350 Feynman diagrams
- Three loops $\Longrightarrow$ 9607 Feynman diagrams

We are on the right track for an exact determination of PDFs at the NNLO

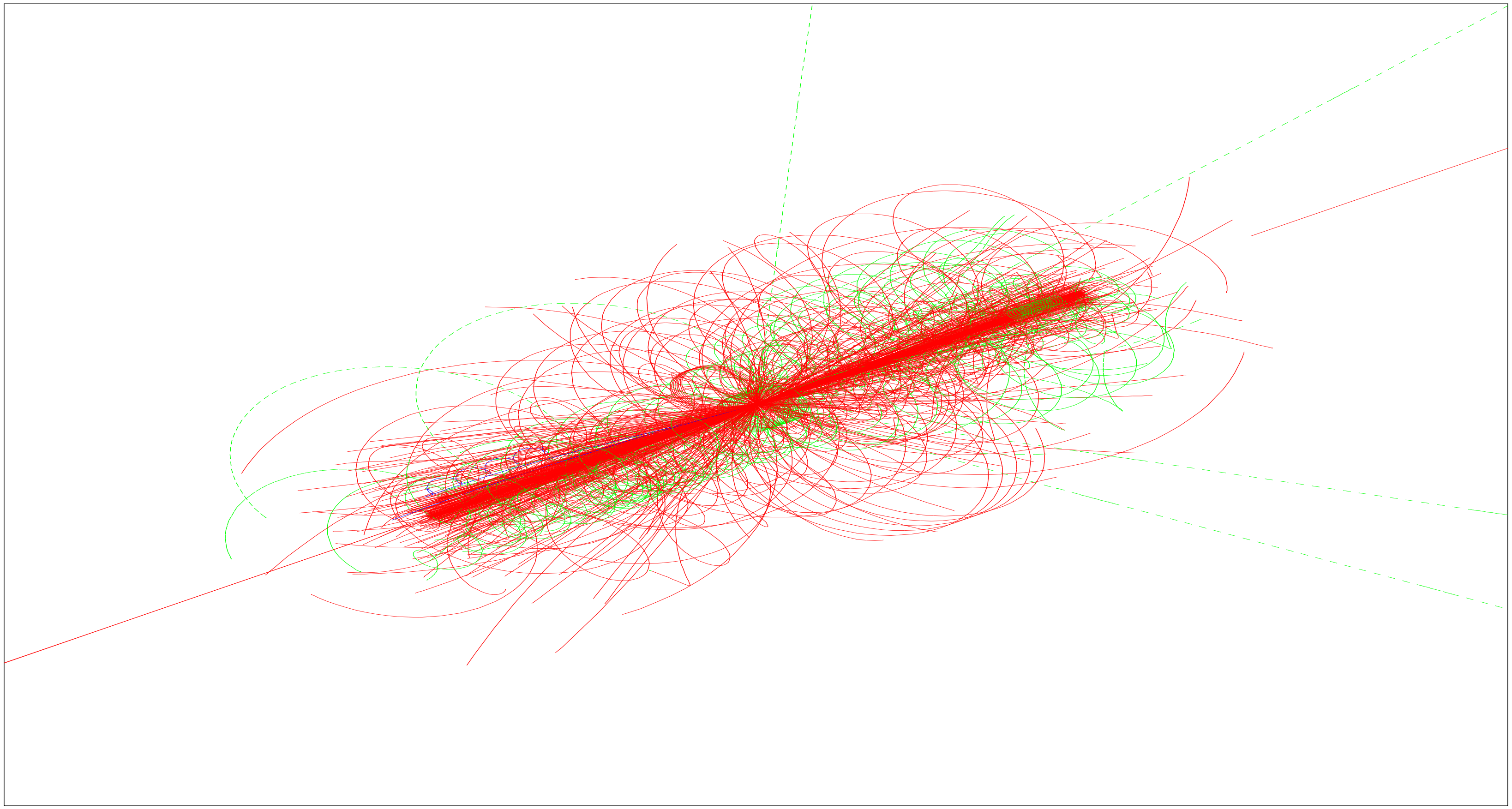
Summary on pQCD

- We are able to describe *some aspects* of a world of hadrons in terms of quarks and gluons
- The perturbative machinery works, if supplemented by non-perturbative inputs (PDFs)
- Intuitive ideas (parton model and factorization) survive in QCD, at the price of certain complications

This framework must be able to stand the challenge posed by data, and we can now say that it does it in an excellent way – the days of QCD *tests* are over; precision physics is possible

We can therefore confidently tackle the problem of predictings SM processes at the LHC. Which remains a very difficult problem...

A "typical" pp event



$H \to ZZ \to 4\mu$ as simulated by ATLAS

- Straight, dashed lines: μ's, i.e. the signal
- The rest: a big mess, due to the fact that hadrons are complicated objects

A complete description must account for two ingredients:

1) the hard process: all the high-p_T stuff, plus particles at small relative p_T or with small energies

2) the rest: this is generally low-p_T stuff, and includes

- the underlying event;
- the pile-up, ie other pp collisions

Truth be told, there's no unambiguous separation between 1) and 2), since to a certain extent it is always definition dependent

Before going into that, some order of magnitude estimates

Final state	σ
Total	~ 100 mb
$W \to e\nu$	20 nb
$Z \to e^+e^-$	2 nb
$b\bar{b}$	0.8 mb
$t\bar{t}$	800 pb
H ($m_H = 200$ GeV)	20 pb

So what is the bulk of the cross section made of?

The vast majority of what happens at a hadronic collider has small momentum transfer. The so-called *minimum bias events* have particles with low-p_T (a few hundreds MeV), and lots of them (< 10 per unit of rapidity)

The hard-scattering picture is further blurred by *secondary interactions*, which takes place between the hadronic remnants, ie partons which are not involved in the primary (hard) interaction

Such secondary interactions are power-suppressed. The probability for producing an extra pair of jets with transverse momentum equal to p_T is roughly

$$\frac{\alpha_S^2(p_T^2)}{p_T^2 \times (1 \text{ fm})^2} \simeq \frac{\Lambda^2 \alpha_S^2(p_T^2)}{p_T^2}$$

All low-p_T phenomena which are not part of the hard scattering process are collectively denoted as *underlying event*

A complete description must account for two ingredients:

1) the hard process: all the high-p_T stuff, plus particles at small relative p_T or with small energies

2) the rest: this is generally low-p_T stuff, and includes

- the underlying event;
- the pile-up, ie other pp collisions

Two different approaches

♦ Event Generators: aim at giving a description as realistic as possible, including all the details of 1) and 2)

Examples: HERWIG, PYTHIA, ARIADNE, ...

♦ Cross Section Integrators: don't include 2), and are only able to give predictions for infrared-safe observables resulting from 1)

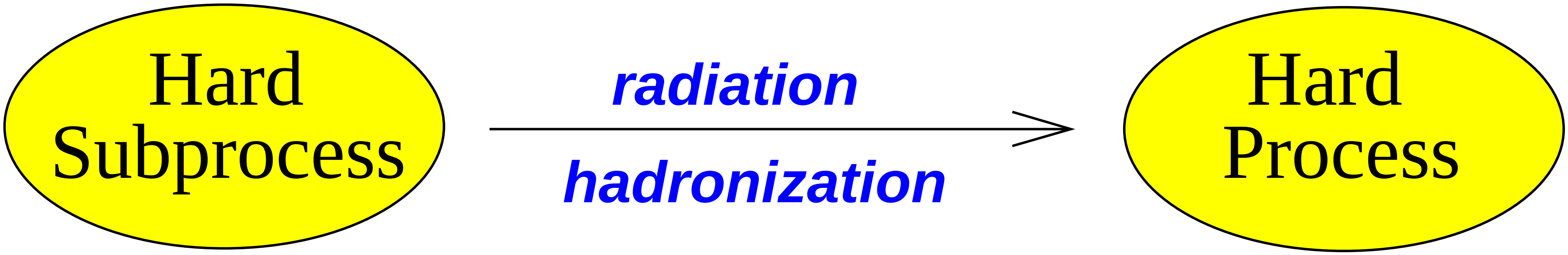
Examples: MCFM, ResBos, ...

A bit of terminology:

- Event Generators are frequently called Parton Shower Monte Carlos – not really correct, but not wrong either (we'll see why)

- Cross Section Integrators are called Monte Carlos (by theorists) – this is due to the fact that they use numerical monte carlo methods to carry out the necessary integrations

For both Event Generators and Cross Section Integrators, the simulation of the hard process proceeds schematically as follows



- Hard subprocess: *only* large-p_T particles, parton-level. Two partons pulled out of the incoming hadrons scatter and produce few (2–6) particles

- Radiation: adds more partons. Equivalent to considering *higher-order corrections* in perturbative QCD

- Hadronization: converts incoming partons into scattering hadrons, and outgoing partons into observed particles

Strategies

- For Hadronization
  1. Use factorization theorems $\longrightarrow$ Cross Section Integrators
  2. Use phenomenological models at mass scales where pQCD is not applicable $\longrightarrow$ Event Generators

- For Higher-order Corrections
  1. Compute exactly the result to a given order in α_S
  2. Estimate the dominant effects to all orders in α_S

  Cross Section Integrators may implement 1, 2, or a combination of the two. Event Generators always implement 2, possibly combined with 1

Summary so far

- It is convenient to separate high- from low-p_T phenomena
- High-p_T (ie hard) processes are *predicted*, low-p_T ones are *modeled* (and fitted to data)
- Cross Section Integrators will neglect the problem of low-p_T stuff if not associated with high-p_T particles
- Event Generators and CSIs both start from simulating a *hard subprocess*. They differ in the way radiation and hadronization are described

The hard subprocess may be seen as a zero-order approximation in the description of the hard process in CSIs

As an example, consider

$$H_1 H_2 \longrightarrow W + X$$

which gets contributions from the leading-order hard subprocesses

$$q\bar{q}' \to W$$

Almost trivial

- kinematically: $p_T(W) = 0$
- dinamically: at the LHC, one expects to have gluons around

In spite of this, not unrealistic for total rate and rapidity

Cross Section Integrators

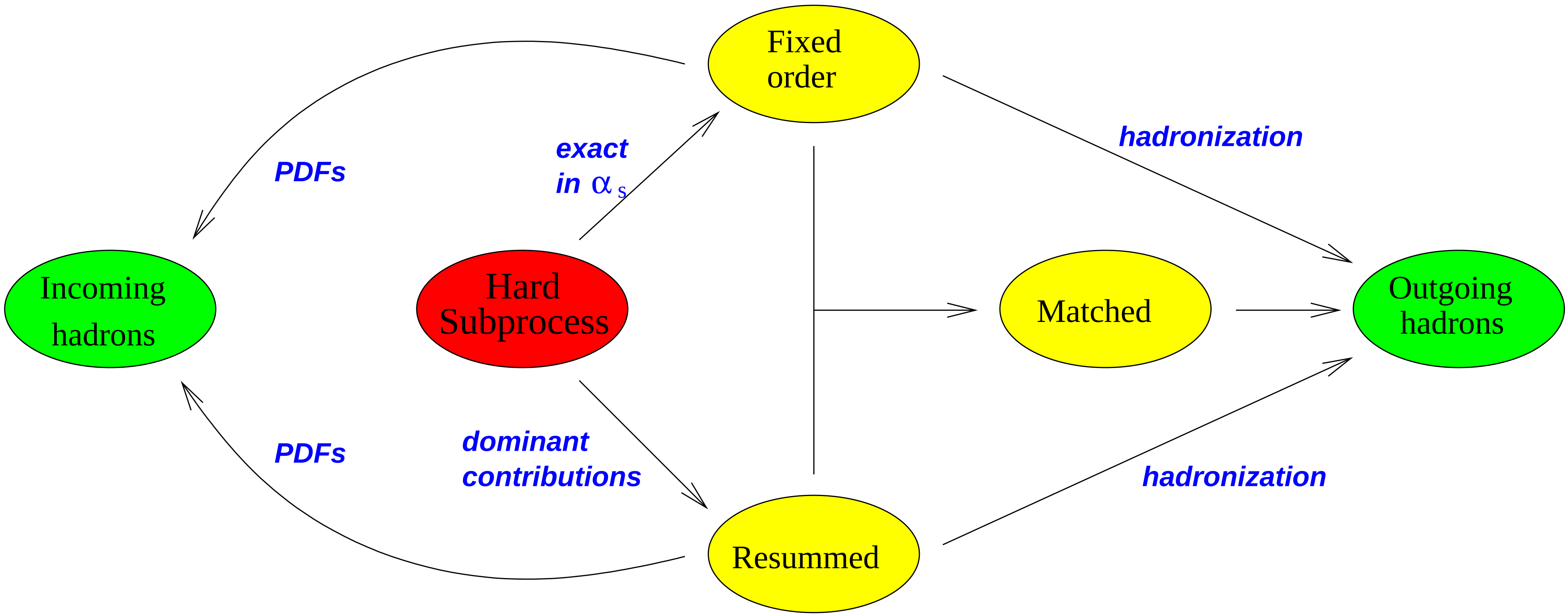
Keep in mind that

- CSIs do a good job in dealing with the hard process, computed using perturbative techniques
- CSI are basically *parton-level computations*
- Hadrons in the initial/final state are obtained by convoluting parton results with PDFS/fragmentation functions
- Unweighted unbiased events are in general not available beyond LO

CSIs can be broadly divided into two classes (which can be combined)

- Fixed-order (eg MCFM) $\longleftarrow$ exact to some α_S^k
- Resummation (eg ResBos) $\longleftarrow$ dominant effects to all orders in α_S

The making of the hard process with CSIs



Convolution with PDFs

The master formula is always the factorization theorem

$$d\sigma_{H_1H_2}(P_1,P_2) = \sum_{ij}\int dx_1 dx_2 f_i^{(H_1)}(x_1,\mu^2) f_j^{(H_2)}(x_2,\mu^2) \times d\hat{\sigma}_{ij}(x_1P_1, x_2P_2; \alpha_S(\mu^2), \mu^2)$$

In order to obtain a theoretical prediction, one computes $d\hat{\sigma}_{ij}$, then uses the formula above taking the PDFs from available repositories*

The classification of CSIs is equivalent to the classification of the short-distance parton cross sections $d\hat{\sigma}_{ij}$

Thus typically one deals with parton cross sections, understanding the convolution with the PDFs

* or to fit PDFs to data

Hadronization (fragmentation)

The idea: partons produced in the hard collision move fast away from each other. Each of them will eventually pick up (at large p_T) the missing colour and flavour from the vacuum to create an observable hadron

Example: b hadroproduction. The single-inclusive p_T spectrum of the b-flavoured hadron is:

$$\frac{d\hat{\sigma}_{ij\to H_b}}{dp_T(H_b)} = \int \frac{dz}{z} D^{b\to H_b}(z,\epsilon) \frac{d\hat{\sigma}_{ij\to b}}{dp_T(b)}, \qquad p_T(H_b) = z p_T(b)$$

- $d\hat{\sigma}_{ij\to H_b}$ is convoluted with the PDFs to get $H_1 H_2 \longrightarrow H_b$
- The *fragmentation function* $D^{Q\to H_Q}$ is analogous to the PDFs: it cannot be computed in pQCD, but is universal
- One tipically uses e^+e^- to fit the parameter(s) ϵ; the functional form in z must be guessed (Peterson, Kartvelishvili,...)

From hard subprocess to hard process

Through the inclusion of hadronization effects and higher-order corrections, the hard event is converted into the "physical" event, ie the best approximation of what happens in the detector according to the chosen method of computation

- More particles are present in the final state wrt the hard subprocess

  Still a small number, say less than 10 for CSIs

- It gives the W something to recoil against, and thus $p_T(W) > 0$

Higher-order corrections, however, pose a problem

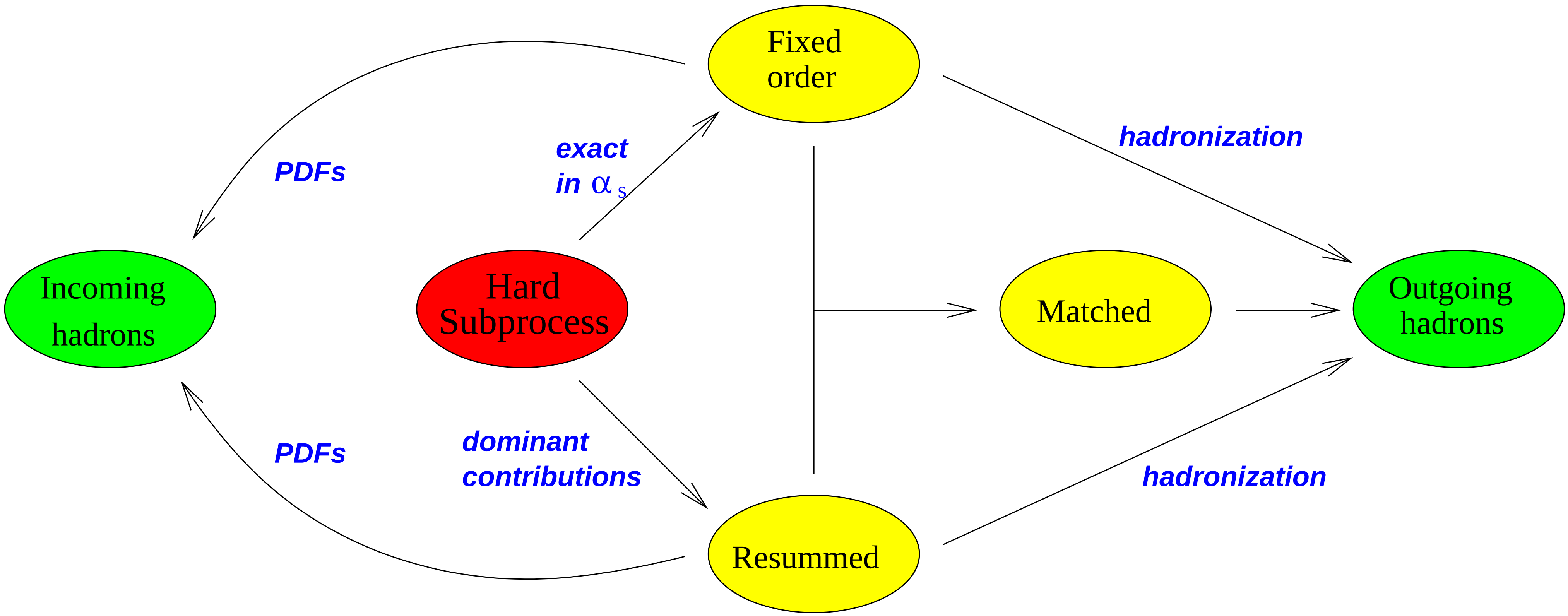
- In the context of Cross Section Integrators, *unweighted events do not exist*. They can be defined only by introducing unphysical cutoffs, which bias observables

Unweighted physical events are only meaningful in Event Generators

Summary on CSIs

- Aim at giving an accurate description of hard processes
- Parton-level results (except for fragmented partons). Small final-state multiplicities (< 10)
- Unsuited for detector simulations. Best tools for "precision" tests, PDF extractions, α_S measurements
- May incorporate exact perturbative results up to α_S^k
  $\longrightarrow$ Fixed-order CSIs
- May incorporate approximate perturbative results to all α_S^n
  $\longrightarrow$ Resummed CSIs

The making of the hard process with CSIs



Fixed-order CSIs

Implement the computation of the production process of interest at a given perturbative order

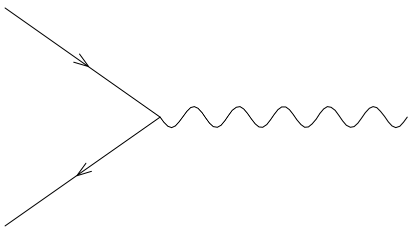
Why? Consider again W production

- More realistic kinematics $(p_T(W) \neq 0)$ owing to real corrections
- Non-trivial jet structure
- More parton channels $(qg \to Wq,\, gg \to Wq\bar{q}',\ldots)$
- Improve estimates of total rate and reduce scale dependence

This is promising, since results can be improved systematically in perturbation theory

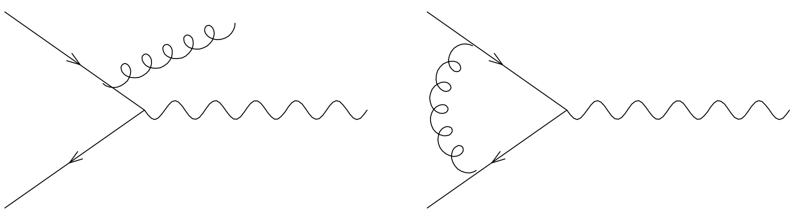
The problem: computations rapidly become very difficult

LO



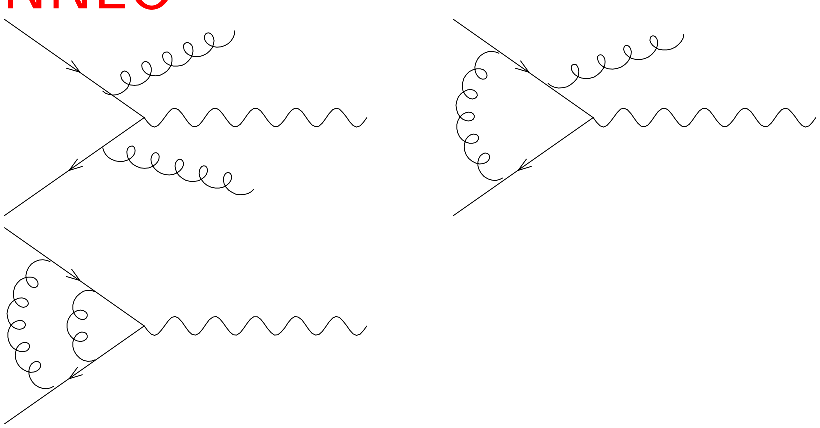
$$\left|\mathcal{A}^{(0)}_{q\bar{q}'}\right|^2$$

NLO



$$\left|\mathcal{A}^{(0)}_{q\bar{q}'}\right|^2, \;\; \Re(\mathcal{A}^{(0)}_{q\bar{q}'}\mathcal{A}^{(1)^\star}_{q\bar{q}'})$$

NNLO



$$\left|\mathcal{A}^{(0)}_{q\bar{q}'}\right|^2, \;\; \Re(\mathcal{A}^{(0)}_{q\bar{q}'}\mathcal{A}^{(1)^\star}_{q\bar{q}'}),$$
$$\Re(\mathcal{A}^{(0)}_{q\bar{q}'}\mathcal{A}^{(2)^\star}_{q\bar{q}'}), \;\; \left|\mathcal{A}^{(1)}_{q\bar{q}'}\right|^2$$

and so forth...

The inclusion of *all* the diagrams contributing to a given order in α_S leads to LO, NLO, NNLO,... results

There are two main technical obstacles in these computations

- Virtual corrections are difficult to obtain
- Cancellation of the infrared divergences

Present situation $\longrightarrow$

NLO

- We know how to cancel infrared divergences in a process- and observable-independent way, for any number of external legs. The problem is fully solved in the context of the so-called *universal formalisms*
  - Subtraction (dipole is a subtraction formalism)
  - Slicing
- Virtual corrections can be computed with standard methods for up to 4 external legs. Some 5-leg results also available ($pp \to 3j$, $pp \to H + 2j$, $pp \to Ht\bar{t}$, $pp \to W/Z + 2j$, $e^+e^- \to H\nu\bar{\nu}$) The current frontier: find numerical or semi-numerical methods to compute multi-leg one-loop amplitudes

What's on the market

can be found at the HEPDATA web site

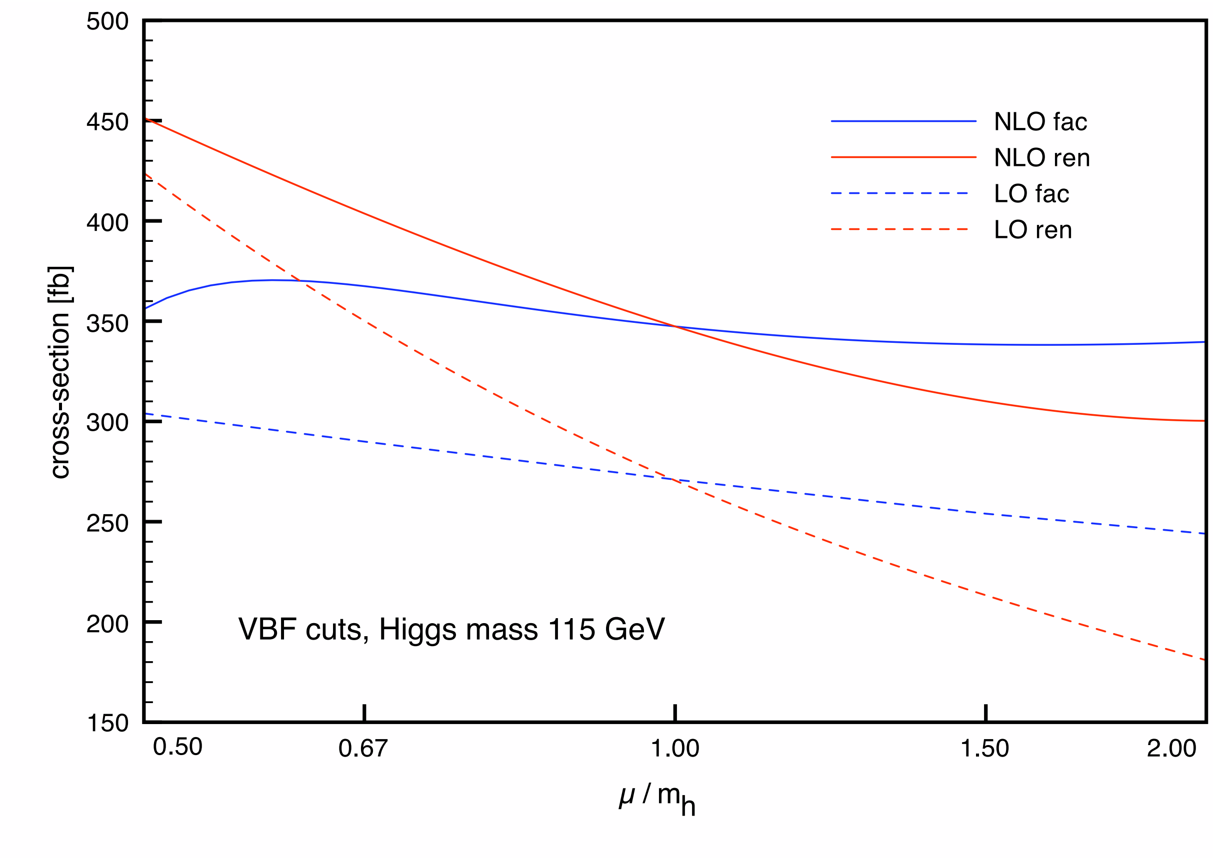
http://www.cedar.ac.uk/hepcode/

Examples are

- MCFM: a *lot* of processes
- NLOJET++: $pp \to 3j$
- PHOX: $pp \to \gamma + j, \gamma\gamma$
- MNR: $pp \to Q\bar{Q}$
- H+QQ: $pp \to Ht\bar{t}$
- AYLEN: $pp \to WW, WZ, ZZ, W\gamma, Z\gamma$

NLO with semi-numerical 1-loop

Promising approach by K. Ellis, Giele, Zanderighi. Previously-unknown results obtained for six-gluon amplitude and $H+4$ partons in gg fusion



- Singularity cancellation checked for a couple of points, then assumed
- One-loop computation fast enough to be used in a cross section integrator (Higgs)

Alternative approaches by Anastasiou, Daleo; Ossola, Papadopoulos, Pittau; Binoth, Guillet, Heinrich, Pilon, Schubert; Feroglia, Passera, Passarino, Uccirati; Denner, Dittmaier; the GRACE group; Giele, Kunszt, Melnikov

NNLO

- We don't know a general algorithm to cancel divergences as in the NLO case. Physical results are available for Higgs and dilepton hadroproduction, and are obtained with brute force or with methods difficult to extend to arbitrarily large multiplicities

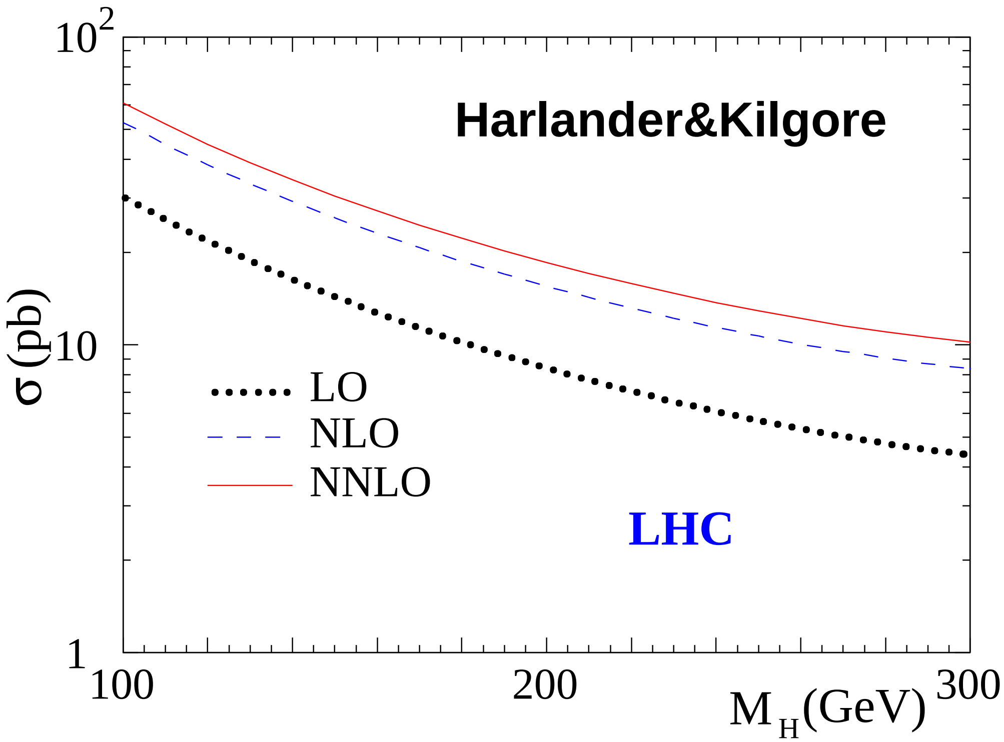
  The current frontier: find general cancellation algorithms analogous to those at NLO

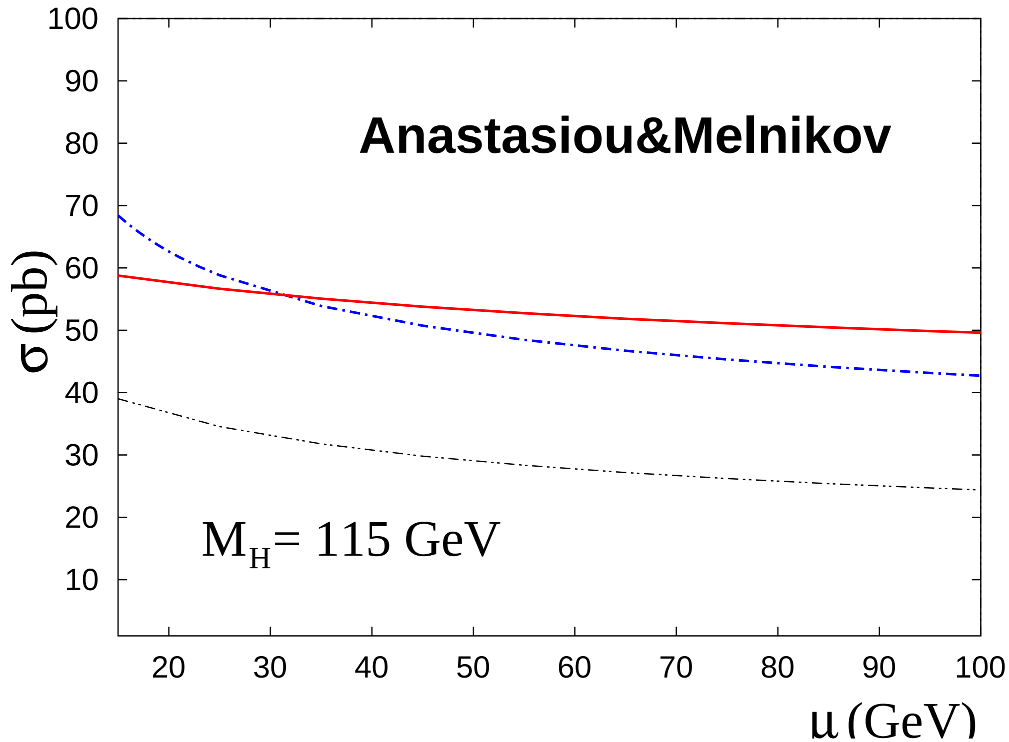
- Two-loop amplitudes available for 3 and 4 legs, one of which may be massive

  The current frontier: more (massive) legs. $Q\bar{Q}$ highly desirable

The enormous complexity of these computations implies that costs and benefits must be carefully assessed before starting. It is easy to become perverse (and irrelevant)

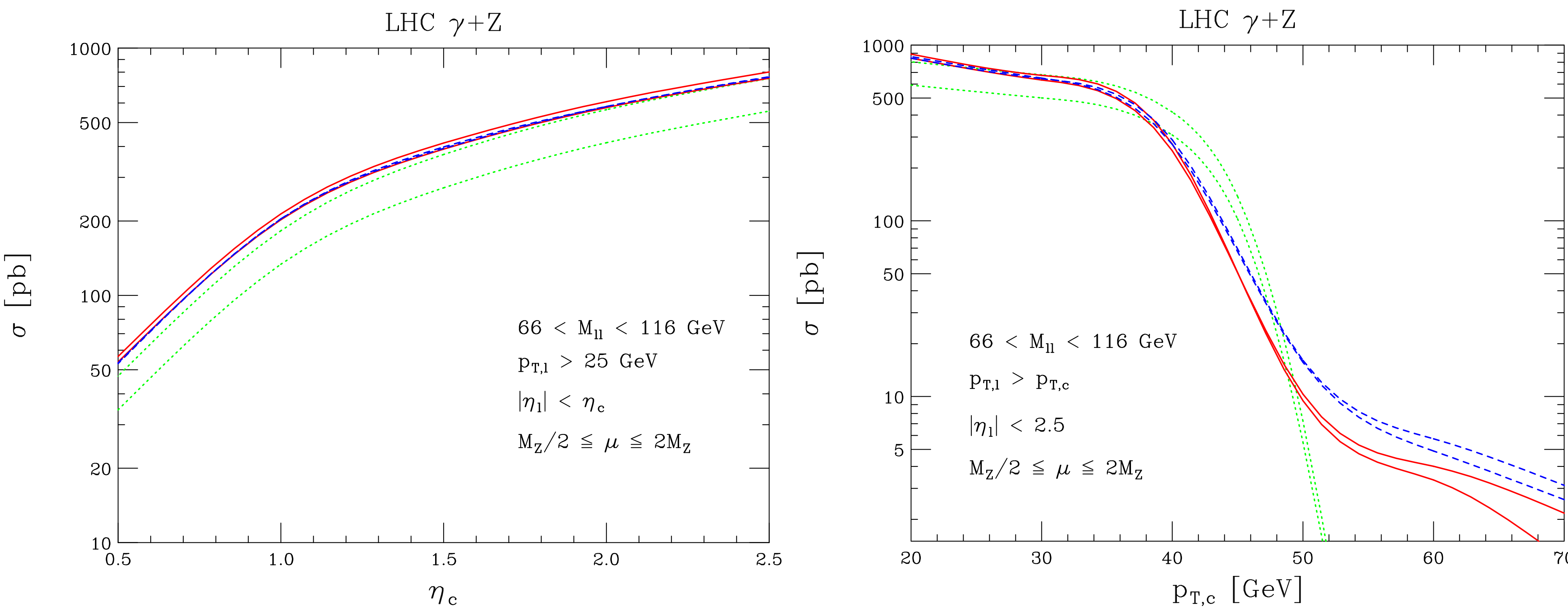
$pp \to H$





- NLO=1.8 LO; NNLO=1.15 NLO. It looks like the series is well behaved
- The scale dependence improves as we expect
- This is in the $m_{top} \to \infty$ limit, but it gives us confidence that we control the corrections to $gg \to H$ at the LHC

$pp \to Z/\gamma \to l^+l^-$ (Melnikov, Petriello)



green: LO; red: NLO; blue: NNLO

- Most complex final state available at NNLO
- Scale dependence invisible at NNLO
- Larger effects in the $p_T(l)$ spectrum – but this is NLO in pQCD

NNNLO and beyond

....

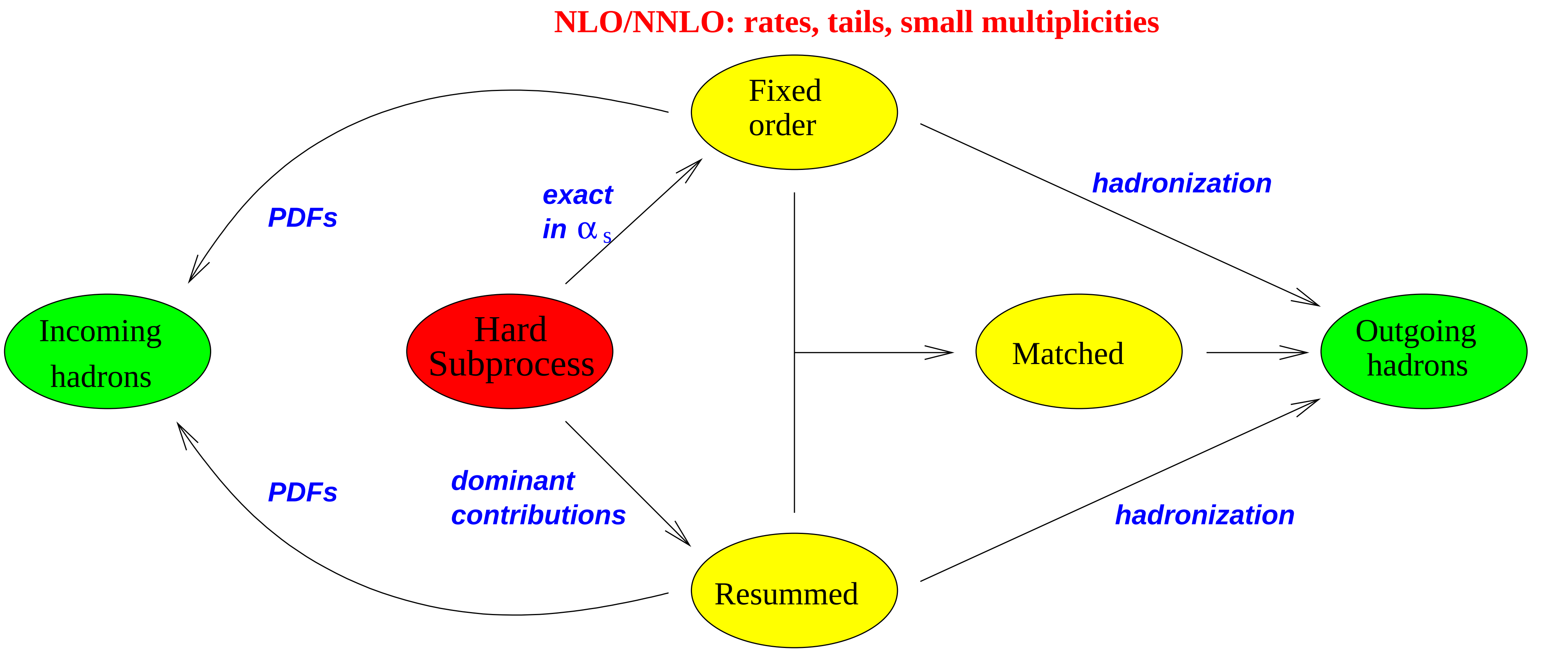
Summary on fixed-order CSIs at NLO/NNLO

- Parton level
- Don't have unweighted events
- Up to 3-particles final states at the NLO, Higgs and dilepton production at the NNLO

Must be used to

- Improve predictions for rates and inclusive shapes
- Study theoretical systematics through scale dependence

Why you should care: higher-order corrections affect signals and backgrounds in different manners: *analysis significance may change*



NLO/NNLO: rates, tails, small multiplicities
Fixed order
exact
in α_s
hadronization
PDFs
Incoming hadrons
Hard Subprocess
Matched
Outgoing hadrons
PDFs
dominant
contributions
hadronization
Resummed

What is it reasonable to expect for the first years of LHC running?

- Multileg NLO results should become available: impact on searches dominated by many-jet events (SUSY the primary example)
- $e^+e^- \to 3j$ ($\longrightarrow \alpha_S$ determination), $pp \to 2j$
- Exact NNLO PDFs
- General cancellation algorithms at NNLO; the bottleneck will then be the computation of two-loop amplitudes (again)

The experience with higher-order results at LEP and Tevatron has been a very positive one. The physics of LHC is more demanding, and a lot of work remains to be done

It may take a while before completing all the (N)NLO computations needed for phenomenology...

But some issues cannot wait. The typical example is indeed that many-jet final states (a serious problem even for SM studies: $W + 4j$ is a huge background for $t\bar{t}$ production)

Key observation: we are able to compute in a highly-automated manner *real-emission* (ie tree level) amplitudes up to a very large number of external legs $(8 - 10)$

$\Longrightarrow$ Keep only real contributions in fixed-order computations

Cannot work! KLN theorem tells us that the result will diverge

Solution: avoid infrared divergences by cutting them out by hand

In hadronic collisions, this is typically equivalent to imposing

- $\Delta R_{ij} \equiv \sqrt{(\varphi_i - \varphi_j)^2 + (\eta_i - \eta_j)^2} \geq R_{cut}$
  $\Longrightarrow$ will avoid final-state collinear divergences

- $p_{Ti} \geq p_{Tcut}$
  $\Longrightarrow$ will avoid initial-state collinear and soft divergences

The cut parameters R_{cut} and p_{Tcut} are *arbitrary*, and in general physical observables will depend on them

On the other hand, it is sufficient to find values of the cut parameters which do not affect the observables we aim to study. If such values do not exist, this approach is simply bound to fail

CSIs implementing the solution above are known as *Matrix Element Generators* – and one should actually add *Tree-Level* to their names

Lacking virtual corrections, MEGs are basically leading-order computations for many-leg processes. As such, the scale dependence is that typical of a LO result (ie very large)

Side effect: the matrix elements are bounded (the upper bound is likely be obtained by computing the MEs at $\Delta R_{ij} = R_{cut}$ and $p_{Ti} = p_{Tcut}$), and thus unweighted events can be obtained

Clearly, these unweighted events are *biased* by R_{cut} and p_{Tcut}, but according to the strategy outlined above one should use them in such a way that the bias will not affect the physics

There are two classes of MEGs $\longrightarrow$

Matrix element generators for specific processes

Feature a pre-defined list of partonic processes, for which phase-space sampling is optimized

Here's a *non-exahustive* list of codes

- AcerMC
- ALPGEN
- GR@PPA
- MadCUP
- VECBOS

There are substantial differences in the number of processes simulated, and in the techniques used to compute the matrix elements!

Phase-space sampling typically optimized process-by-process, to improve unweighting efficiency

Matrix element generators for arbitrary processes

Compute the matrix elements for any process given in input by the user (sort of automated matrix element generator authors...)

- AMEGIC++
- CompHEP
- Grace
- MadEvent/MadGraph

On average, the largest number of external legs is smaller than that obtained with MEGs for specific processes. Beyond-SM capabilities are being added to these codes

Phase-space sampling (where present) cannot be optimized process-by-process. Adaptive importance sampling techniques are used instead

X-sects (pb)	Number of jets						
$e^-\bar{\nu}_e + n$ QCD jets	0	1	2	3	4	5	6
ALPGEN	3904(6)	1013(2)	364(2)	136(1)	53.6(6)	21.6(2)	8.7(1)
AMEGIC++	3905(4)	1014(3)	370(2)				
CompHEP	3947.4(3)	1022.4(5)	364.4(4)				
GR@PPA	3906.37 (4)	1046.85 (5)					
HELAC/PHEGAS/JetI	3786(81)	1021(8)	361(4)	157(1)	46(1)		
MadEvent	3902(5)	1012(2)	361(1)	135.5(3)	53.6(2)		

X-sects (pb)	Number of jets				
$e^-\bar{\nu}_e + b\bar{b} + n$ QCD jets	0	1	2	3	4
ALPGEN	9.34(4)	9.85(6)	6.82(6)	4.18(7)	2.39(5)
AMEGIC++	9.42(5)	9.92(10)			
CompHEP	9.415(5)	9.91(2)			
HELAC/PHEGAS/JetI	9.88(11)	12.68(9)			
MadEvent	9.32(3)	9.74(1)	6.80(2)		

Good agreement among codes

Capabilities will increase with computer power

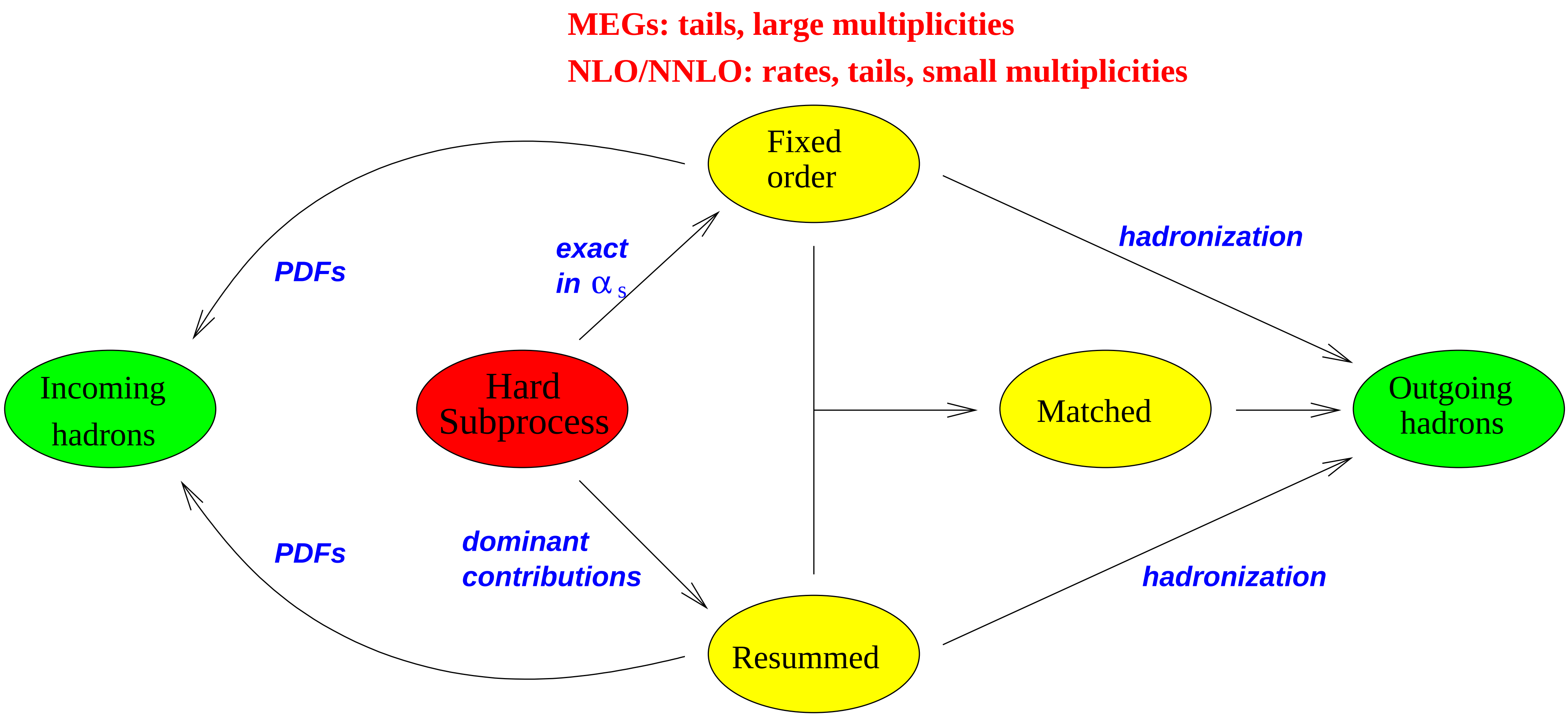
Summary on Matrix Element Generators

- Parton level and LO
- Have unweighted (biased) events
- Up to 8-particles final states
- Physical results must be proved independent of unphysical cutoffs used in the computations

Must be used to

- Improve predictions for shapes of many-jet observables

These codes can be included into Event Generators (see later)



MEGs: tails, large multiplicities
NLO/NNLO: rates, tails, small multiplicities
Fixed order
PDFs
exact in α_s
hadronization
Incoming hadrons
Hard Subprocess
Matched
Outgoing hadrons
PDFs
dominant contributions
hadronization
Resummed

Interlude: determination of PDFs

Key ingredients: factorization theorems and AP equations. Then (in Mellin space, where convolutions become ordinary products)

$$\sigma_{data} = f\sigma_{th} \implies f = \sigma_{data}/\sigma_{th}$$
$$\sigma_{data} = (f_1 f_2)\sigma_{th} \implies (f_1 f_2) = \sigma_{data}/\sigma_{th}$$

- Parametrize PDFs at a small scale $Q_0 = 1 - 4$ GeV

$$xf(x, Q_0) = Ax^{\delta}(1-x)^{\eta}(1 + \epsilon\sqrt{x} + \gamma x)$$

- Impose momentum conservation

$$\sum_a \int_0^1 dx x f_a(x, Q_0) = 1$$

- Evolve PDFs to relevant Q and compute σ_{th}

- Fit to data

Master equations for PDF determination

$$\sigma_{data} = f\sigma_{th} \implies f = \sigma_{data}/\sigma_{th}$$

$$\sigma_{data} = (f_1 f_2)\sigma_{th} \implies (f_1 f_2) = \sigma_{data}/\sigma_{th}$$

- PDFs are *non physical*: they have a perturbative accuracy which is that of σ_{th} – this is why we talk about LO, NLO, ... PDFs

- This is also why we need perturbative computations $\Longleftrightarrow$ CSIs

- The above implies a (subtraction) scheme dependence. Popular ones are $\overline{\rm MS}$ and $\rm DIS$ – different schemes are related by a convolution with a process-independent function

- A given σ_{data} will receive contributions from many different parton combinations (Z hadroproduction: $u\bar{u} \to Z$ $d\bar{d} \to Z$, $ug \to Zu$, ...).
  The problem: *disentangle* these contributions

Consider F_2 in Neutral Current DIS

$$F_2^{\rm NC}(x,Q^2) = x\sum_f e_f^2(q_f+\bar{q}_f) + \alpha_S\Big\{C_f\otimes(q_f+\bar{q}_f) + C_g\otimes g\Big\}$$

$$\alpha_S = \alpha_S(Q^2)\,, \qquad f = \text{flavours}$$

The coefficient functions C_i have a perturbative expansion, and:

$$\frac{\partial F_2^{(\rm sing)}}{\partial\log Q^2} = \alpha_S\Big[P_{qq}\otimes F_2^{(\rm sing)} + 2N_F P_{qg}\otimes g\Big] + \mathcal{O}(\alpha_S^2)$$

- DIS is the backbone of PDF determination; singlet easy
- At $\mathcal{O}(\alpha_S)$, also determines the gluon – but small-x only
- Current determinations also use low-Q^2 DY data ($\rightarrow \bar{u}-\bar{d}$), $\mu^+\mu^-$ low-energy data ($\rightarrow s$), $y(W^\pm\rightarrow l^\pm)$ @Tevatron ($\rightarrow u/d$ slope), jet @Tevatron ($\rightarrow$ large-x gluon)

■ What is the uncertainty affecting PDF determinations?

Evolution

Note: AP equations are a system of $2N_F + 1$ equations

$$\frac{\partial}{\partial \log \mu^2} \begin{pmatrix} q_i \\ g \end{pmatrix} = \frac{\alpha_S(\mu^2)}{2\pi} \begin{pmatrix} P_{q_i q_j} & P_{q_i g} \\ P_{g q_j} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_j \\ g \end{pmatrix}$$

The flavour structure of the kernel is trivial ($P_{q_i g} = P_{qg}$ and $P_{g q_j} = P_{gq}$) except for the quark-(anti)quark sector

$$P_{q_i q_j} = \delta_{q_i q_j} P_{qq}^{(0)} + \frac{\alpha_S}{2\pi} P_{q_i q_j}^{(1)}$$

with $q_{i,j}$ quarks or antiquarks. Flavour symmetry helps

$$\begin{aligned} P_{q_i q_k} &= \delta_{ik} P_{qq}^V + P_{qq}^S \\ P_{q_i \bar{q}_k} &= \delta_{ik} P_{q\bar{q}}^V + P_{q\bar{q}}^S \end{aligned}$$

with V and S denoting flavour *non-singlet* and *singlet* components

Solutions

Denoting by

$$P^{\pm} = P_{qq}^{V} \pm P_{q\bar{q}}^{V}\,, \qquad q_i^{\pm} = q_i \pm \bar{q}_i$$

one introduces the flavour combinations (for 5 flavours)

$$\begin{aligned} V_i &= q_i^- & T_3 &= u^+ - d^+ \\ T_8 &= u^+ + d^+ - 2s^+ & T_{15} &= u^+ + d^+ + s^+ - 3c^+ \\ T_{24} &= u^+ + d^+ + s^+ + c^+ - 4b^+ & \Sigma &= \textstyle\sum_i q_i^+ \end{aligned}$$

From AP equations we see that V_i and T's are *not* coupled to the gluon, and their NLO kernels are P^- and P^+. The gluon only couple to the singlet combination

$$\frac{\partial}{\partial \log \mu^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \frac{\alpha_S(\mu^2)}{2\pi} \begin{pmatrix} P^+ + N_F(P_{qq}^S + P_{q\bar{q}}^S) & 2N_F P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

Mellin space

Given a function $f(x)$, with $0 \leq x \leq 1$, one introduces the *Mellin moments*

$$\tilde{f}(j) = \int_0^1 dx x^{j-1} f(x)$$

We have

$$f(x) = g \otimes h(x) \implies \tilde{f}(j) = \tilde{g}(j)\tilde{h}(j)$$

ie convolutions are turned into ordinary products by Mellin transform

$$\begin{aligned} \int_0^1 dx x^{j-1} f(x) &= \int_0^1 dy dz x^{j-1} \delta(x - yz) g(y) h(z) \\ &= \int_0^1 dy y^{j-1} g(y) \int_0^1 dz z^{j-1} h(z) \end{aligned}$$

This is the reason why it is important to compute the Mellin moments of the Altarelli-Parisi kernels

Consider a flavour non singlet combination V. At the LO the evolution equation is

$$\frac{\partial \tilde{V}(j,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}\gamma_{qq}^{(0)}(j)\tilde{V}(j,\mu^2)\,, \qquad \gamma_{qq}^{(0)}(j) = \int_0^1 dx x^{j-1} P_{qq}^{(0)}(x)$$

The Mellin transforms of the AP kernels have a special name: *anomalous dimensions*. At α_S fixed

$$\tilde{V}(j,\mu^2) = \tilde{V}(j,\mu_0^2)\left(\frac{\mu^2}{\mu_0^2}\right)^{\frac{\alpha_S}{2\pi}\gamma_{qq}^{(0)}(j)}$$

ie V would have exact scaling if $\gamma(j) = 0$ for all j's. Using the one-loop expression for α_S we get the more realistic result

$$\tilde{V}(j,\mu^2) = \tilde{V}(j,\mu_0^2)\left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu^2)}\right)^{\frac{\gamma_{qq}^{(0)}(j)}{2\pi b}}$$

By computing the anomalous dimensions explicitly (do not forget the + prescription!) one gets

$$\gamma_{qq}^{(0)}(j+k) < \gamma_{qq}^{(0)}(j) < 0$$

This means that the large Mellin moments of the non-singlet distribution vanish faster at large scales wrt small moments

In turn, this implies that, when the scale increases, non-singlet distributions decrease at large x and increase at small x

Therefore, at large momentum transfers the fraction of the hadron momentum carried by the quarks decreases, in favour of that carried by the gluons

By comparing the second moments of the singlet and gluon distributions, we can actually compute this fraction analytically. At $Q^2 \to \infty$
$f_g = 16/(16 + 3N_F)$

■ UNCERTAINTIES: CTEQ and MRST use the Hessian method: find the 1σ errors on the parameters of the fit, and consider PDFs obtained by changing the parameters by $\pm 1\sigma$

The problem has the dimensionality d of the parameter space: 15 (CTEQ) or 20 (MRST)

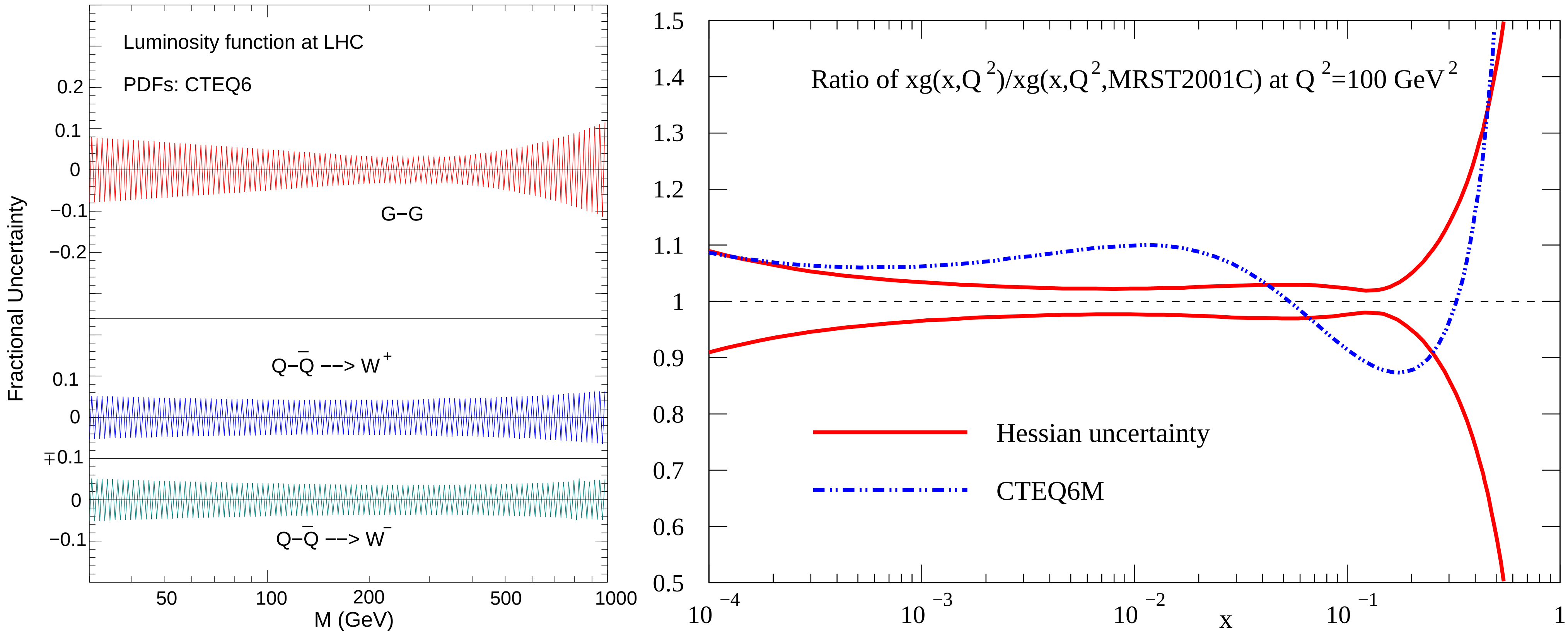
$$\chi^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}(D_i - T_i(a))\,\sigma_{ij}^{-1}\,(D_j - T_j(a))$$

$$\Rightarrow \Delta\chi^2 = \sum_{k=1}^{d}\sum_{l=1}^{d}\left(a_k - a_k^0\right) H_{kl} \left(a_l - a_l^0\right)$$

One then introduces $T^2 > \Delta\chi^2$, diagonalises H_{kl}, and defines $2d$ PDFs $S_k^{\pm}$, which correspond to displacements $\pm\sqrt{T}$ along the direction of the k^{th} eigenvector. Then

$$(\Delta\mathcal{O})^2 = \frac{1}{2}\sum_{k=1}^{d}\left(\mathcal{O}(S_k^+) - \mathcal{O}(S_k^-)\right)^2$$

T is called Tolerance



- Inconsistencies between the data sets imply underestimation of errors
- This also imply that $\Delta\chi^2 = 1$ rule cannot be imposed: the T is arbitrary (CTEQ and MRST defaults differ)
- Theoretical uncertainties, bias from parametrizations not included

Summary on PDFs with errors

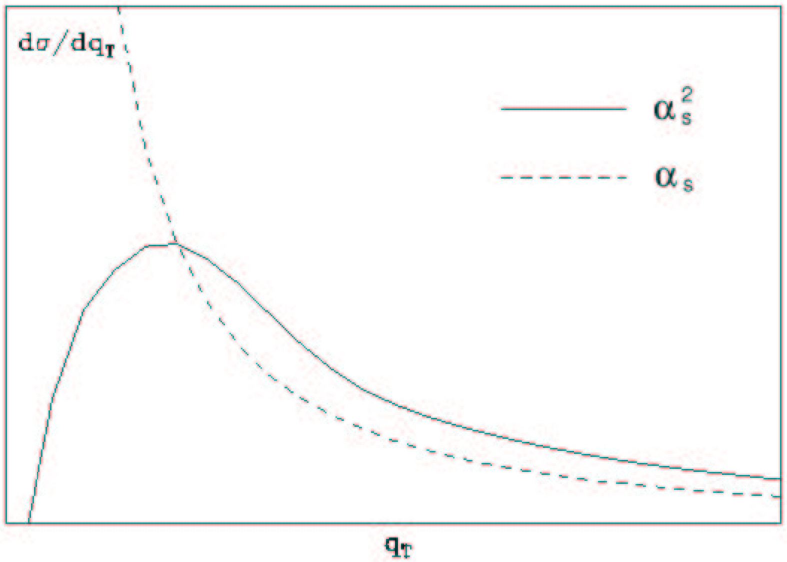
- CTEQ and MRST do have NLO PDFs with errors
- A "central" PDF set has 30 or 40 companion sets
- The central set is used to compute the main value of the observable chosen
- This computation has to be repeated 30 or 40 times, with the companion sets, to determine the uncertainty on the prediction due to uncertainties on PDFs

Keep in mind that

- There is a hidden dependence on Tolerance
- The method would suggest parallelization (parton cross sections don't change), but this is usually not done $\Longrightarrow$ computing intensive

Problems with fixed-order CSIs

Compute $p_T(W)$ at $\mathcal{O}(\alpha_S)$ and $\mathcal{O}(\alpha_S^2)$



Not exactly what you expect to see when LHC is turned on...

We know from KLN theorem that the problem would be less severe (or absent) by considering the integrated cross section in $0 \le p_T(W) \le$ a few GeV

Still, it is very instructive to see what happens to the perturbative expansion

$$\frac{d\sigma}{dp_T(W)} = \sum_n c_n \alpha_S^n \,, \qquad c_n = \frac{d}{dp_T(W)} \sum_{m=0}^{2n} d_{n,m} \log^m \frac{m_W^2}{p_T^2(W)}$$

- Up to two logs per power of α_S
- Logarithms grow large when $p_T(W) \to 0$, and spoil the perturbative expansion
- This is the typical situation in a multi-scale problem: here $p_T(W) \ll m_W$

Since all terms in the perturbative expansion are equally important, it would seem that perturbation theory is useless in this case

Let's understand where the large logs come from. Start at $\mathcal{O}(\alpha_S)$: we have $p_T(W) = p_T(g)$. For small $p_T(W)$, the gluon is thus soft and/or collinear

$$\frac{d\sigma}{dp_T(W)} = \int d\Phi_{Wg}\delta(p_T(W) - p_T)\mathcal{M}(q\bar{q} \to Wg)$$

$$\stackrel{p_T(W)\to 0}{\longrightarrow} \frac{\alpha_S}{2\pi}\int_0^{m_W^2}\frac{dp_T^2}{p_T^2}\delta(p_T(W) - p_T)\int_0^{1-\frac{p_T}{m_W}} dz\, C_F\frac{1+z^2}{1-z}\mathcal{M}(q\bar{q} \to W)$$

$$= \frac{\alpha_S}{2\pi}\mathcal{M}(q\bar{q} \to W)\frac{1}{p_T(W)}\left\{A_1 \log\frac{m_W^2}{p_T^2(W)} + B_1 + \mathcal{O}\left(\frac{p_T(W)}{m_W}\right)\right\}$$

with

$$A_1 = C_F \qquad B_1 = -\frac{3}{2}C_F$$

These values are entirely determined by the AP kernel P_{qq}.

Hence, you know the kernels, you know the logs

- Large logs result from soft and collinear emissions
- Soft and collinear emissions are universal

Strategy: include all pertubative orders in the computation, *keeping only the dominant logs*

One can prove that this is doable in practice, provided that *both* dynamics and kinematics factorize (ie can be expressed as products of simple building blocks)

More often than not, factorized kinematics is obvious when introducing a conjugate variable, e.g. in the case of $p_T(W)$

$$\delta\left(\vec{p_T}(W) - \sum_i \vec{p}_{Ti}\right) = \int \frac{d\vec{b}}{(2\pi)^2} \exp\left[i\vec{b}\cdot\left(\vec{p_T}(W) - \sum_i \vec{p}_{Ti}\right)\right]$$

$$= \int \frac{d\vec{b}}{(2\pi)^2} \exp\left(i\vec{b}\cdot\vec{p_T}(W)\right) \prod_i \exp\left(-i\vec{b}\cdot\vec{p}_{Ti}\right)$$

Using the conjugate variable, the large logs change form

$$L = \log \frac{m_W^2}{p_T^2(W)} \quad \longrightarrow \quad \tilde{L} = \log \left(b^2 m_W^2\right)$$

For the record, here's the form that results upon including all orders

$$\frac{d\sigma}{dp_T^2(W)} \propto \mathcal{M}(q\bar{q} \to W) \int_0^\infty db\, b J_0(b p_T(W)) e^{\mathcal{G}}$$

$$\mathcal{G} = -\int_{b_0^2/b^2}^{m_W^2} \frac{d\mu^2}{\mu^2} \left(A(\alpha_S(\mu^2)) \log \frac{m_W^2}{\mu^2} + B(\alpha_S(\mu^2)) \right)$$

$$= \tilde{L} g_1(\alpha_S \tilde{L}) + \sum_{n=2}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-2} g_n(\alpha_S \tilde{L})$$

$$A(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_n \,, \quad B(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_n \,,$$

There is a $-\infty$ at the exponent, which is therefore damped *(Sudakov suppression)*: the cross section is finite at $p_T(W) = 0$

A resummation is therefore just a re-organization of the perturbative series.
One starts from the fixed-order expression

$$\begin{aligned} \sigma_{FO} &= f_{00} + \alpha_S(c_{12}L^2 + c_{11}L + f_{10}) \\ &+ \alpha_S^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + \ldots + f_{20}) + \ldots \end{aligned}$$

When the logarithm L grows large, one rewrites this as

$$\sigma_{res} = \exp\left[Lg_1(\alpha_S L) + g_2(\alpha_S L) + \ldots\right](f'_{00} + \ldots)$$

The resummed expression can be systematically improved

- g_1 (ie A_1) Leading Logs
- g_2 (ie A_2, B_1) Next-to-Leading Logs
- ...

precisely as the perturbative expansion in α_S can be improved by computing the next contribution in α_S

- Rule of thumb:

  Soft *and* collinear emissions $\longrightarrow$ double logs

  Soft, large angle *or* collinear, hard emissions $\longrightarrow$ single logs

- The precise nature of the logs depends however on the observable studied

- Coefficients A_i and B_i are obtained from perturbative computations; typically N^{k}LL $\leftrightarrow$ N^{k+1}LO (DIS, Drell-Yan)

- It is best to count the logs at the exponent. In such a way, the condition for the validity of the expansion is $\alpha_S L < 1$

- If the exponent is expanded, g_i's mix, and the condition for the validity of the expansion is $\alpha_S L^2 < 1$

- State of the art: NLL, some NNLL results available

Example: logs in $Q\bar{Q}$ production

1) Observable-dependent logs: depend strictly on the kinematics of the final state (including cuts). Occur in specific regions of the phase space

$$\mathcal{Q} = \frac{p_T(Q)}{m_Q}, \qquad p_T(Q) \gg m_Q$$

$$\mathcal{Q} = \frac{p_T(Q\overline{Q})}{m_Q}, \qquad p_T(Q\overline{Q}) \simeq 0$$

$$\mathcal{Q} = 1 - \frac{\Delta\phi(Q\overline{Q})}{\pi}, \qquad \Delta\phi(Q\overline{Q}) \simeq \pi$$

2) Observable-independent logs

Threshold logs: occur when the c.m. energy is small

$$\mathcal{Q} = 1 - \frac{4m_Q^2}{\hat{s}}, \qquad \hat{s} \simeq 4m_Q^2$$

Small-x logs: occur when the c.m. energy is large

$$\mathcal{Q} = \frac{m_Q^2}{\hat{s}}, \qquad \hat{s} \gg m_Q^2$$

Roughly speaking, fixed-order and resummed computations apply to complementary kinematic regions

- Large-pt tails $\Longrightarrow$ fixed-order
- Peaks $\Longrightarrow$ resummation

One can exploit the good features of each approach, and construct a *matched* result

$$\sigma_{matched} = \sigma_{FO} + \Big(\sigma_{res} - \sigma_{FO}\big|_{L\to\infty}\Big) G(\mathcal{Q})$$

$$L = \log \mathcal{Q}\,, \qquad G(x) = 1 + \sum_i a_i x^i$$

Problems:

- Resummed computations are observable-specific
- They are also lenghty, tedious, and most of all error-prone

There are alternative solutions to the problem posed by analytical resummation

- (Semi-)numerical approach (CAESAR)
- Parton Shower Monte Carlos

CAESAR (Banfi, Salam, Zanderighi) has been developed in the past few years, reproduces known analytical results, and computes results not available at the analytical level

PSMCs, which are at the core of Event Generators, have enjoyed and will keep on enjoying enormous success. They will keep us busy for the rest of these lectures

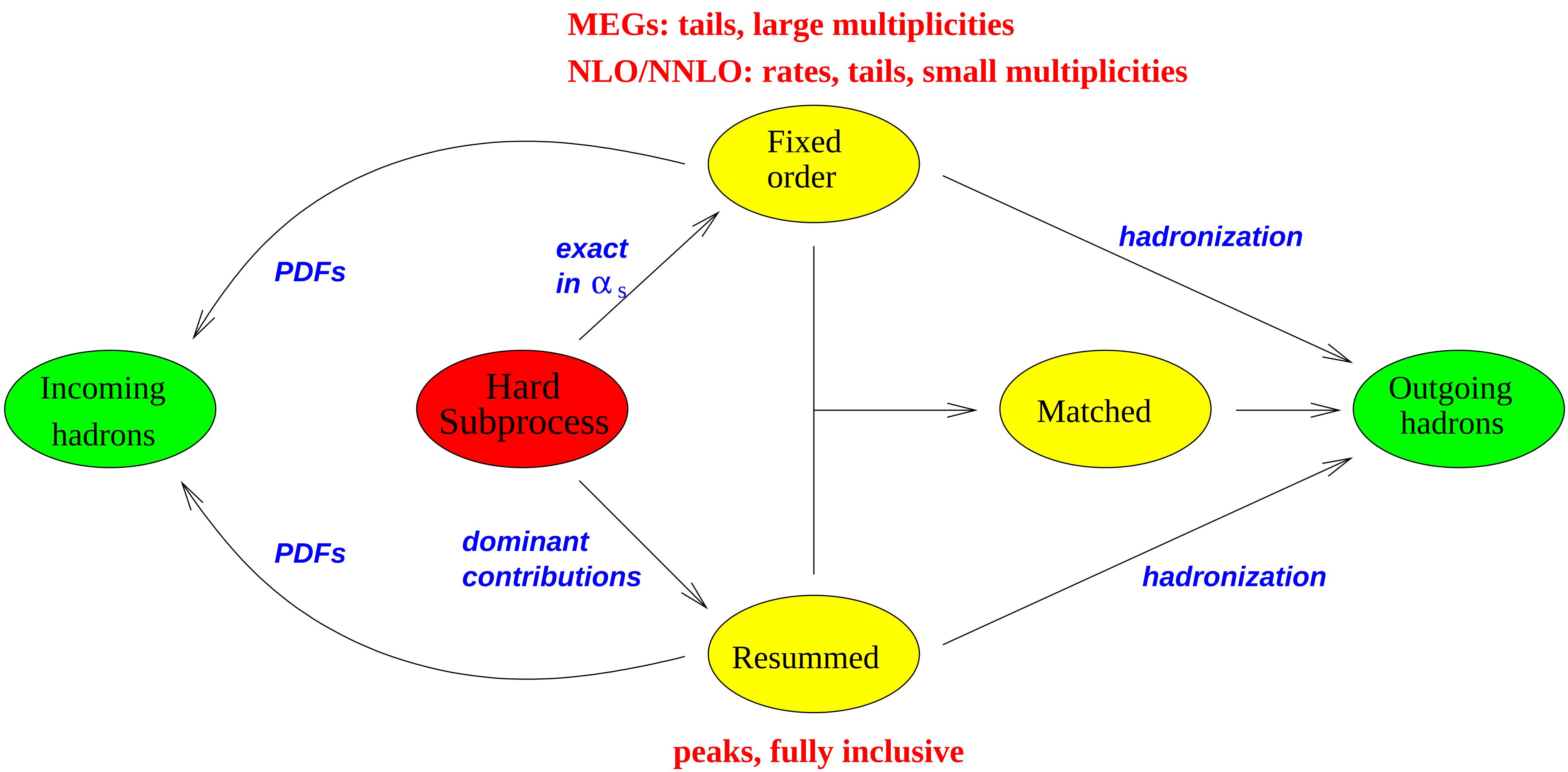
Summary on resummed computations

- Reorganize the perturbative expansion: $\alpha_S \longrightarrow \alpha_S L^k$
- Don't have unweighted events
- Can (and should always) be matched to fixed-order results

Must be used to

- Get sensible predictions for inclusive shapes at peaks
- Cross check the results of Event Generators

Why you should care: Event Generators perform the same kind of computation for inclusive variables, but with smaller nominal accuracy


MEGs: tails, large multiplicities
NLO/NNLO: rates, tails, small multiplicities
Fixed order
PDFs
exact in α_s
hadronization
Incoming hadrons
Hard Subprocess
Matched
Outgoing hadrons
PDFs
dominant contributions
hadronization
Resummed
peaks, fully inclusive

What to take home

- Fixed order: lots at NLO, a few at NNLO
- Highly-automated generation of tree-level diagrams
- High-accuracy resummed computations available for a few key observables
- Resummed and fixed-order results are complementary
- Progress being made in (semi)-numerical approaches to loop computations, resummations
- PDFs with errors must be considered for serious assessment of systematics. Computing intensive

Event Generators

Remind that an Event Generator aims at giving a *complete* description of collision processes

The core of Event Generators is the *Parton Shower* mechanism, which serves two main purposes:

- To provide estimates of higher-order corrections that are enhanced by large kinematic logarithms
- To generate high-multiplicity partonic states which can readily be converted into the observed hadrons

The Parton Shower is built on the same concept as resummations: logarithmically dominant contributions to the cross section are "universal". Power-suppressed and finite terms are neglected

Parton Showers are more flexible than (analytical or numerical) resummation results. This comes at a price, since more approximations need be made

Event Generators in a nutshell

- Infinite number of dominant Feynman diagrams

  Generate high-multiplicity parton final state: shower

- Models for hadronization, underlying event

  Convert partons into incoming and outgoing hadrons

- PDG information embedded

  Used to decay particles with correct branching ratios

Let's discuss the Parton Shower

Before going into that, let me stress that the problem of the sensible generation of the underlying event is a serious one, owing to

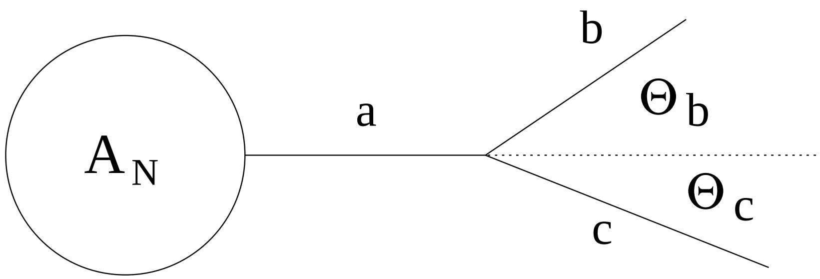
- its importance for all kind of physics simulations
- the still-poor theoretical understanding of its mechanisms

The process of checking the predictions of and of improving the models for the underlying event will start immediately after the LHC turn on

There is a lot of ongoing activity on this issue, which I won't report

Let's start by ignoring the problem of soft singularities

Collinear kinematics



$$\begin{aligned} z &= E_b/E_a \quad t = k_a^2 \\ \Theta &= \Theta_b + \Theta_c \\ &= \frac{\Theta_b}{1-z} = \frac{\Theta_c}{z} \\ &= \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}} \end{aligned}$$

Work in axial gauges

$$\begin{aligned} d\sigma_{N+1} &= d\sigma_N \frac{dt}{t}\frac{d\phi}{2\pi} dz \frac{\alpha_S}{2\pi} \left|K_{ba}(z)\right|^2 \\ d\bar{\sigma}_{N+1} &= d\bar{\sigma}_N \frac{dt}{t} dz \frac{\alpha_S}{2\pi} P_{ba}(z) \end{aligned}$$

as we already know from fixed-order and resummed computations

In the phase space, ϕ can be conveniently identified with the azimuthal angle between the plane of branching and the polarization of a

It is easy to iterate the branching process (splittings are called branchings in this context)

$$a(t) \longrightarrow b(z) + c\,, \quad b(t') \longrightarrow d(z') + e$$

$$d\bar{\sigma}_{N+2} = d\bar{\sigma}_N \frac{dt}{t} dz \frac{dt'}{t'} dz' \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ba}(z) P_{db}(z')$$

This is a *Markov process*, ie a random process in which the probability of the next step only depends on the present values of the random variables. In formulae

$$\tau_1 < \ldots < \tau_n \implies P\Big(x(\tau_n) < x_n | x(\tau_{n-1}), \ldots, x(\tau_1)\Big) = P(x(\tau_n) < x_n | x(\tau_{n-1}))$$

In our case, the probability of each branching depends on the type of splitting ($g \to gg$, ...), the virtuality t, and the energy fraction z

Following a given line in a branching tree, it is clear that enhanced contributions will be due to the strongly-ordered region

$$Q^2 \gg t_1 \gg t_2 \gg \dots t_N \gg Q_0^2$$

$$\sigma_N \propto \sigma_0 \alpha_S^N \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \dots \int_{Q_0^2}^{t_{N-1}} \frac{dt_N}{t_N} = \sigma_0 \frac{\alpha_S^N}{N!} \left(\log \frac{Q^2}{Q_0^2} \right)^N$$

Denote by

$$\Phi_a[E, Q^2]$$

the ensemble of parton cascades initiated by a parton a of energy E emerging from a hard process with scale Q^2. Also, denote by

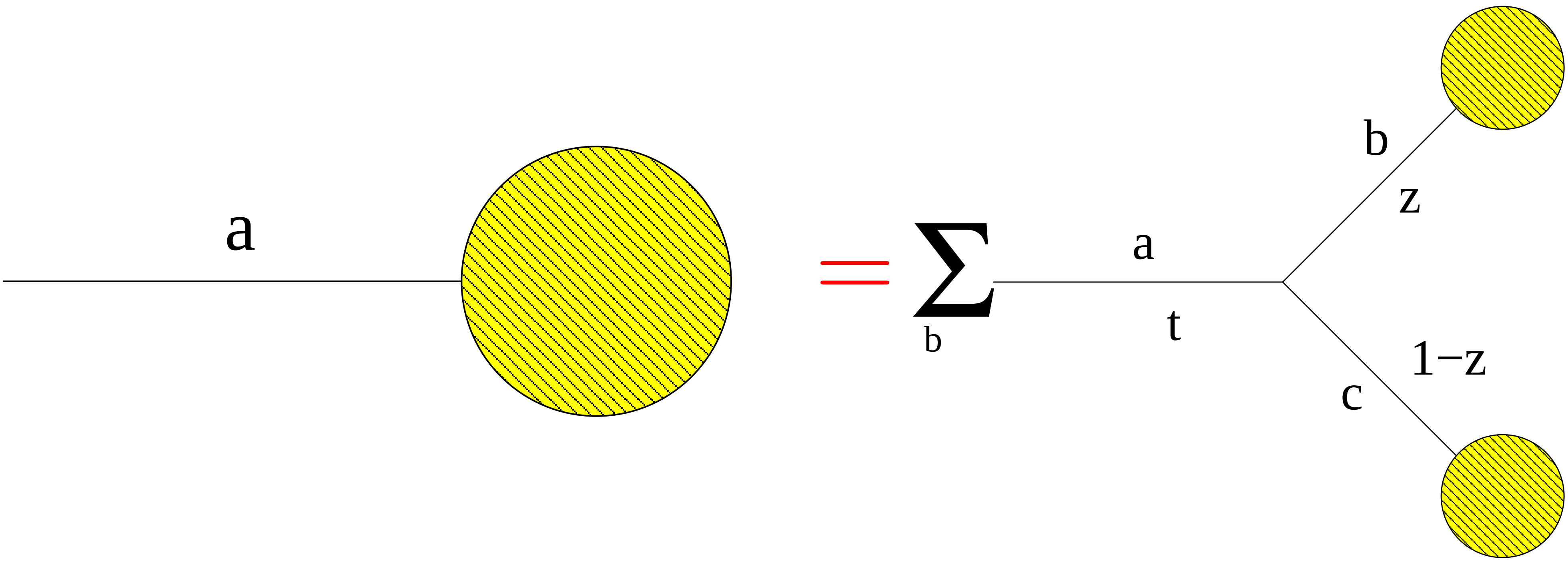
$$\Delta_a(Q_1^2, Q_2^2)$$

the probability that a does not branch for virtualities $Q_2^2 < t < Q_1^2$

With this, it is easy to write a formula that takes into account all the branches in a branching tree:

$$\begin{aligned}\Phi_a[E,Q^2] &= \Delta_a(Q^2,Q_0^2)\Phi_a[E,Q_0^2] \\ &+ \int_{Q_0^2}^{Q^2}\frac{dt}{t}\Delta_a(Q^2,t)\sum_b\int dz\frac{\alpha_S}{2\pi}P_{ba}(z)\Phi_b[zE,t]\Phi_c[(1-z)E,t]\end{aligned}$$

which has an immediate pictorial representation



Now simply impose that no information is lost during the parton shower: the sum of all the probabilities associated with the branchings of partons must be one. Therefore

$$1 = \Delta_a(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dt}{t} \Delta_a(Q^2, t) \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

which can be solved:

$$\Delta_a(Q^2, Q_0^2) = \exp\left(- \int_{Q_0^2}^{Q^2} \frac{dt}{t} \sum_b \int dz \frac{\alpha_S}{2\pi} P_{ba}(z) \right)$$

Note

- This Sudakov form factor looks familiar $\longrightarrow$ resummation
- *Some* virtual corrections must be included, otherwise unitarity couldn't be imposed!

It's clear that a Sudakov must appear: resummation and parton shower described the same physics

Double logs

Keep in mind: this treatment is valid only in the collinear limit. Choices which affect the behaviour away from this limit are equivalent

For example, the choice of the shower variable t affects the double-log structure

$$
\begin{array}{lllll}
t & = & z(1-z)\theta^2 E^2 \quad (\text{virtuality}) & \Longrightarrow & \frac{1}{2}\log^2\frac{t}{E^2} \\
t & = & z^2(1-z)^2\theta^2 E^2 \quad (p_T^2) & \Longrightarrow & \frac{1}{4}\log^2\frac{t}{E^2} \\
t & = & \theta^2 E^2 \quad (\text{angle}) & \Longrightarrow & \log\frac{t}{\Lambda}\log\frac{E}{\Lambda}
\end{array}
$$

owing to soft divergences. In MC's they are easy to locate:

$$
z \to 1 \quad \Longrightarrow \quad P_{qq},\ P_{gg} \sim \frac{1}{1-z}
$$

So the study of soft emission may give extra information on the proper choice for t

Note that when $t = \theta^2 E^2$ is chosen, the energy integral is infrared divergent

The results are fairly different from each other. Thus the choice of the shower evolution variable, although irrelevant in the context of the collinear approximation, is likely to have a dramatic impact for the predictions of physical observables

The differences arise in the soft region. It is therefore necessary to study soft emissions. This will help us finding the optimal choice for t

Soft emissions

Using soft-gluon techniques (Bassetto, Ciafaloni, Marchesini)

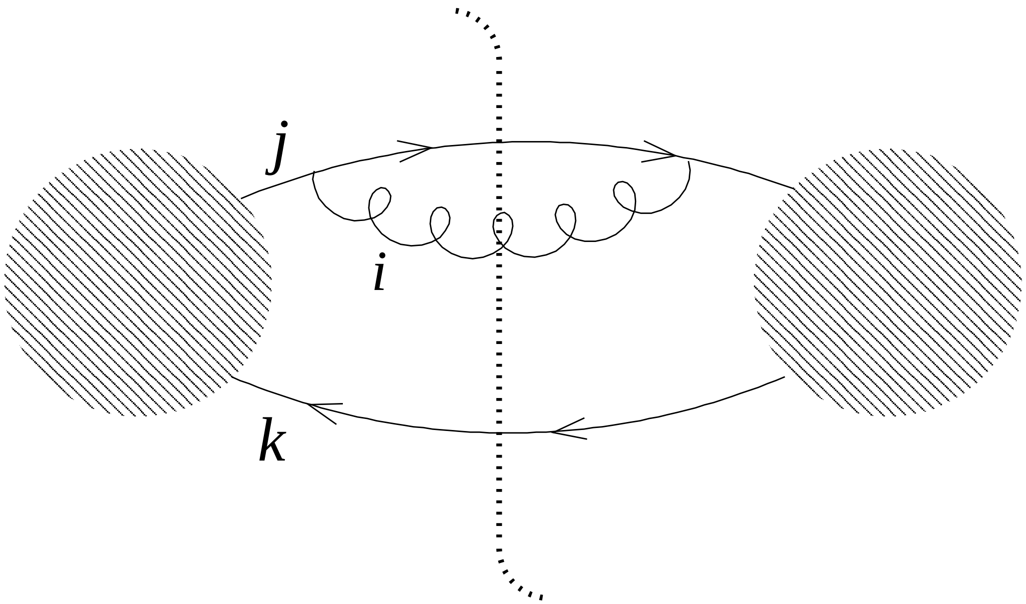
$$d\bar{\sigma}_{N+1} \quad = \quad -d\bar{\sigma}_N \frac{dE_i}{E_i} \frac{d\Omega_i}{2\pi} \frac{\alpha_S}{2\pi} \sum_{jk} \mathbf{T}_j \cdot \mathbf{T}_k \frac{\zeta_{jk}}{\zeta_{ij}\zeta_{ik}}$$

Gluon i has collinear singularities to j and k

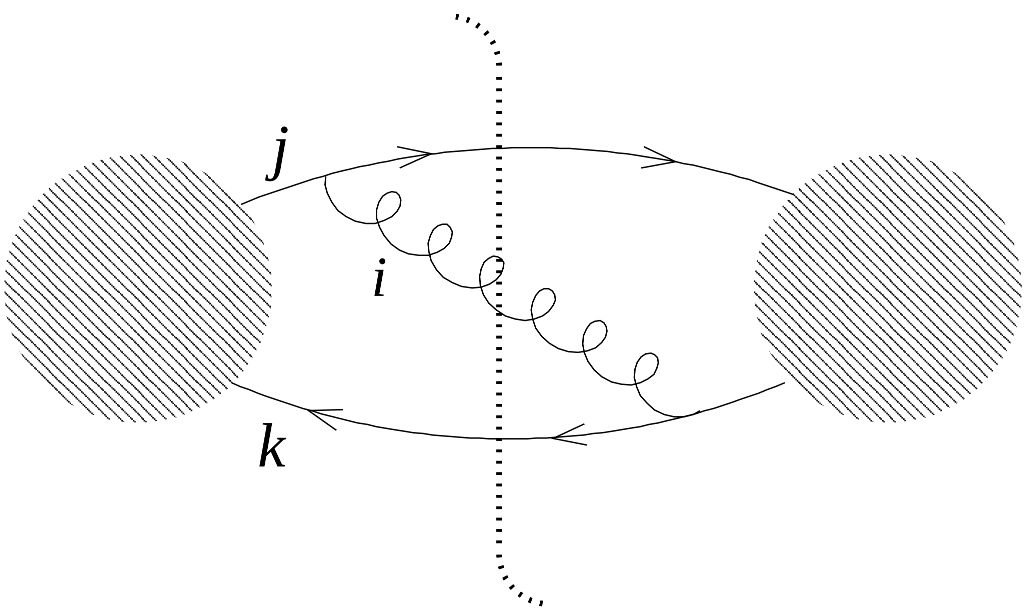
$$\begin{aligned} \zeta_{ab} &= \frac{k_a \cdot k_b}{E_a E_b} = 1 - \cos\theta_{ab} \\ \mathbf{T}_a &= \langle c_a | T^a \qquad \text{colour} - \text{charge operator} \\ \mathbf{T}_g^2 &= C_A\,, \quad \mathbf{T}_q^2 = C_F \end{aligned}$$

When iterating this formula to the next emission, one gets

- A non-positive definite expression (owing to interference)
- A non-Markovian structure (step 2 depends on step 1 *and* 0)



Collinear



Soft

Manipulate the radiation function

$$
\begin{aligned}
W_{jk} &= \frac{\zeta_{jk}}{\zeta_{ij}\zeta_{ik}} = W_{jk}^{[j]} + W_{jk}^{[k]} \\
W_{jk}^{[j]} &= \frac{1}{2}\left(\frac{\zeta_{jk}}{\zeta_{ij}\zeta_{ik}} + \frac{1}{\zeta_{ij}} - \frac{1}{\zeta_{ik}}\right)
\end{aligned}
$$

This decomposition has two remarkable properties

- It disentangles the collinear singularities
- It has *angular ordering*

$$
\int_0^{2\pi} d\phi_{ij} W_{jk}^{[j]} = \begin{cases} 1/\zeta_{ij} & \zeta_{ij} < \zeta_{jk} \\ 0 & \zeta_{ij} > \zeta_{jk} \end{cases}
$$

Angular ordering is a manifestation of (destructive) interference effects present in gauge theories – eg in QED

The radiation of a soft gluon is confined in the cone defined by the two partons that "exchange" the gluon

This looks like a frame-dependent statement, but it is not

In the rest frame of the emitting dipole, the emitters are back to back, and the gluon can be emitted at any angle

Now if a boost is performed, the emitters and the gluon will all be squeezed in the boost direction

This is equivalent to considering a soft emission in the boosted frame, where the angle between the two emitters is small

Angular ordering implies that after azimuthal average we have

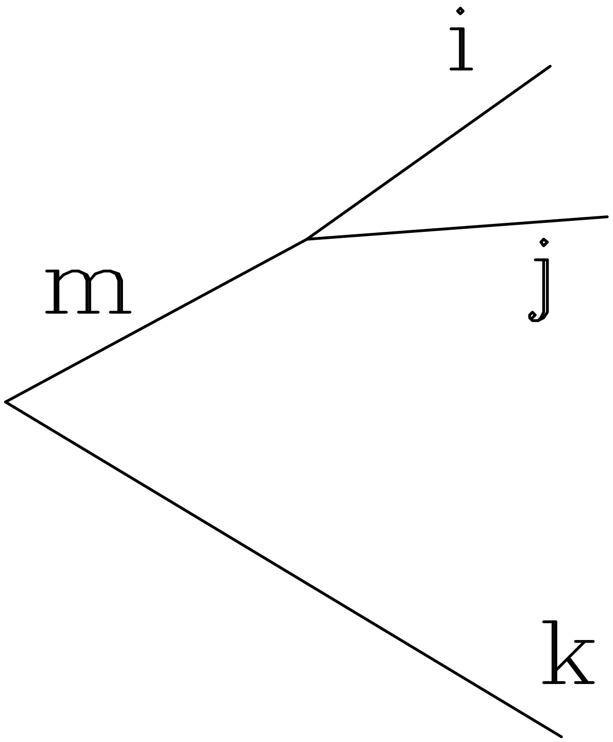
$$d\bar{\sigma}_{N+1} \;=\; -d\bar{\sigma}_N \frac{dE_i}{E_i} \frac{\alpha_S}{2\pi} \sum_{jk} 2\mathbf{T}_j \cdot \mathbf{T}_k \int_0^{\zeta_{jk}} \frac{d\zeta_{ij}}{\zeta_{ij}}$$

This looks promising: may be interpreted as

$$\ldots \longrightarrow j + k\,; \quad j \longrightarrow i + j' \,\ldots$$

The process is fully symmetric in $j \longleftrightarrow k$

In order to study the emission pattern in more details, we must at least consider the next branching



Consider the emission of a soft gluon from the *colour singlet* formed by the three partons i, j and k

The radiation pattern will be obtained by attaching a soft gluon to the three external legs i, j, k

$$W_{ijk} = -\mathbf{T}_i \cdot \mathbf{T}_j W_{ij} - \mathbf{T}_j \cdot \mathbf{T}_k W_{jk} - \mathbf{T}_i \cdot \mathbf{T}_k W_{ik}$$

Assuming that $\theta_{mk} \gg \theta_{ij}$ one gets

$$W_{ijk} = \mathbf{T}_i^2 W_{ij}^{[i]} + \mathbf{T}_j^2 W_{ij}^{[j]} + \mathbf{T}_k^2 W_{km}^{[k]} + \mathbf{T}_m^2 W_{km}^{[m]} \Theta(\theta_{mg} > \theta_{ij})$$

- Inside the cone (ij), the gluon is emitted by two independent charges $\mathbf{T}_i^2$ and $\mathbf{T}_j^2$
- Outside of this code, the gluon cannot resolve i and j, and only "sees" $\mathbf{T}_m^2 = (\mathbf{T}_i + \mathbf{T}_j)^2$

$\Longrightarrow$ A Markov structure has emerged: $(ijk) \equiv ((i+j)k) + (ij)$

Indeed, we can obtain

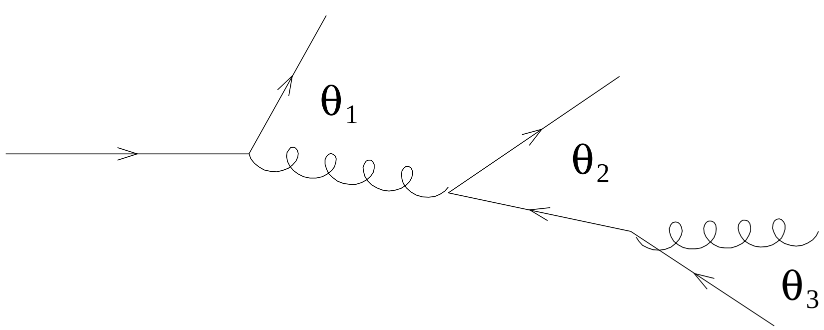
$$W_{ijk} = \mathbf{T}_i^2 W_{ij}^{[i]} + \mathbf{T}_j^2 W_{ij}^{[j]} + \mathbf{T}_k^2 W_{km}^{[k]} + \mathbf{T}_m^2 W_{km}^{[m]} \Theta(\theta_{mg} > \theta_{ij})$$

as a two-step branching process. First, attach the soft gluon to the pair (mk), ie

$$\mathbf{T}_k^2 W_{km}^{[k]} + \mathbf{T}_m^2 W_{km}^{[m]}$$

Note that m is on shell!. Next, after the branching $m \to ij$ with $\theta_{ij} < \theta_{mg}$, attach the soft gluon to the pair (ij), ie

$$\mathbf{T}_i^2 W_{ij}^{[i]} + \mathbf{T}_j^2 W_{ij}^{[j]}$$



Angular ordering

$$\theta_1 > \theta_2 > \theta_3$$

We have therefore obtained that, after an azimuthal average, soft emissions can be treated as a Markov process with probability

$$dP_i = 2\vec{Q}_i^2 \frac{\alpha_S}{2\pi} \frac{d\zeta}{\zeta} \frac{dE_i}{E_i}$$

with the pre-factor of 2 coming from the symmetrization over eikonals in the original formula

Defining z such that

$$\frac{dE_i}{E_i} = \frac{dz}{z}$$

one observes that $2\vec{Q}_i^2/z$ is the leading-soft behaviour of the relevant Altarelli-Parisi kernel. This is all we need to guess the branching probability which describes soft and/or collinear emissions

Coherent branching

What done above can be combined with the collinear branching stuff. One arrives at a coherent branching formalism, which correctly incorporates collinear *and* soft enhancements to all orders

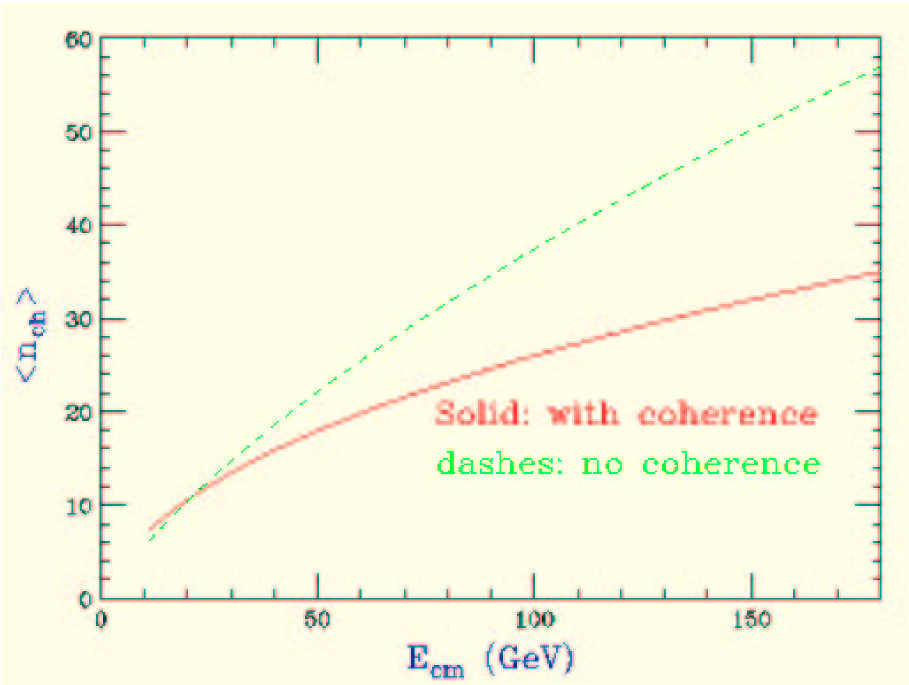
The most straightforward approach it that of replacing the shower variable t with $\zeta = 1 - \cos\theta$, and impose $\zeta_{n+1} < \zeta_n$. Iterated cross section formulae now read

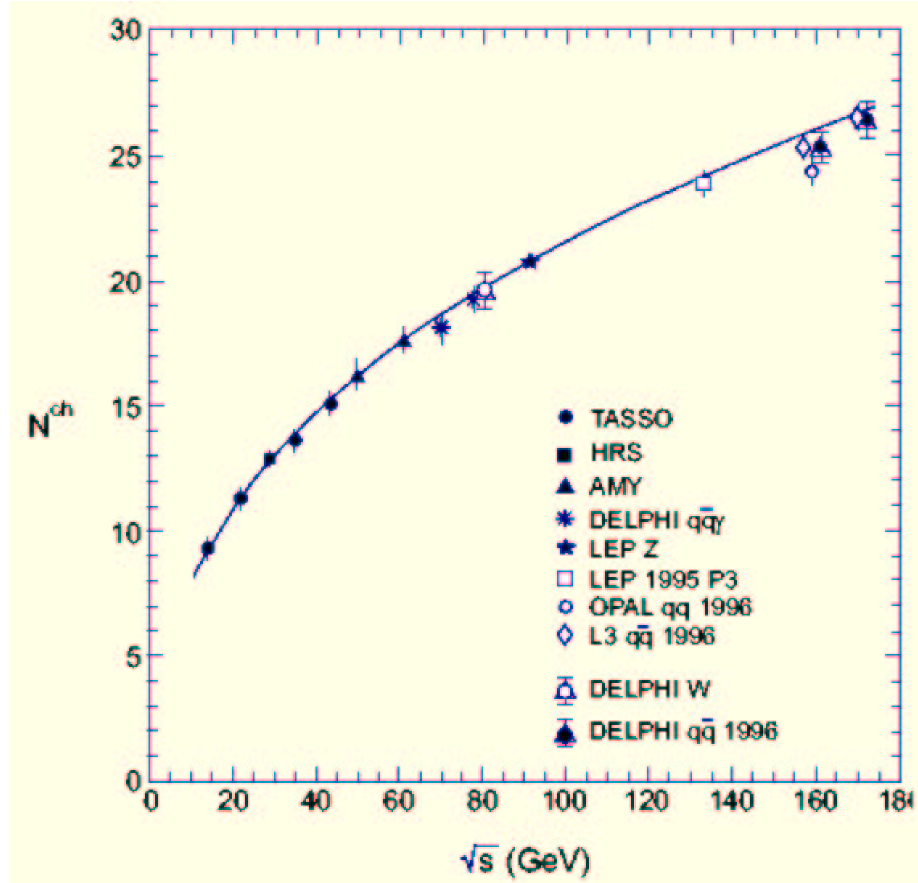
$$d\bar{\sigma}_{N+1} \quad = \quad d\bar{\sigma}_N \frac{d\zeta}{\zeta} dz \frac{\alpha_S}{2\pi} P_{ba}(z)$$

In practice, to take into account emission from non-zero-mass lines, it's more convenient to use as shower variable for $a \to bc$ (HERWIG)

$$Q_a^2 = E_a^2 \zeta_a \, ; \qquad \zeta_a > \zeta_b \quad \Longrightarrow \quad Q_b^2 < z_b^2 Q_a^2$$

There are non-accessible kinematic regions (dead zones)





Coherence can be seen in data

Note that coherence reduces the multiplicity wrt to what one would get from fully incoherent radiation

3 very successful implementations

HERWIG	PYTHIA	ARIADNE
$t = E^2\zeta$	$t = M^2$	$t = p_T^2$
hardest not first	hardest first	hardest first
coherent	coherence forced	coherent
dead zones	no dead zones	no dead zones
ISR easy	ISR easy	ISR difficult
$g \longrightarrow q\bar{q}$ OK	$g \longrightarrow q\bar{q}$ OK	$g \longrightarrow q\bar{q}$ difficult

Each has pros and cons: don't be lazy, try to use more than one

Summary on Event Generators

0) Start from a *leading order* hard subprocess

1) Let initial- and final-state partons branch

2) Iterate 1) (ie shower) till reaching a small scale Q_0

3) For final-state partons, use a model to convert partons into hadrons; for initial-state partons, force further branchings till valence flavours are generated, and fold with $f(x, Q_0)$

4) Add low-p_T stuff (underlying events, ...)

Always keep in mind

Parton Shower Monte Carlos are very flexible, essential tools for experimental physics. But:

- Each emission in a shower is based on a collinear approximation; matrix elements are leading order
- No K factors, no hard emissions
- Very good in peak regions, ie the bulk of the cross section
- Fairly poor in large-p_T tails, ie rare events

Implications

- There are large uncertainties in LO+LL QCD: one can go way too far beyond limits of applicability of the MC, without noticing it

- To stretch the theory to fit data may hide some interesting unknown physics

In general, weaknesses of MC's will be dramatically exposed at the LHC, both for SM physics and for BSM searches

Other troubles

It is a lot easier to misuse an MC than a CSI implementing resummation

Example: W hadroproduction. One may want to study not only W properties, but also consider the accompanying jets

- Can't do this with a CSI: it's inclusive in W, jets are simply not there
- Can do it with an MC: the partons against which the W recoils are available in the event record, and jets can be reconstructed

This is OK, if the jets are not too hard, and not too far from each other

But hard and well-separated jets can be generated by the MC – and no warning is given that the corresponding cross section is *totally wrong*

It is left to you to determine whether you are using an MC outside the range of validity of its approximation. It is a very common mistake to abuse of this freedom

The current frontier(s)

Go beyond LO (K factors and large-p_T tails)

Done

Go beyond LL

Not yet

How to improve Monte Carlos?

The key issue is to go beyond the collinear approximation

$\Longrightarrow$ use exact matrix elements of order higher than leading

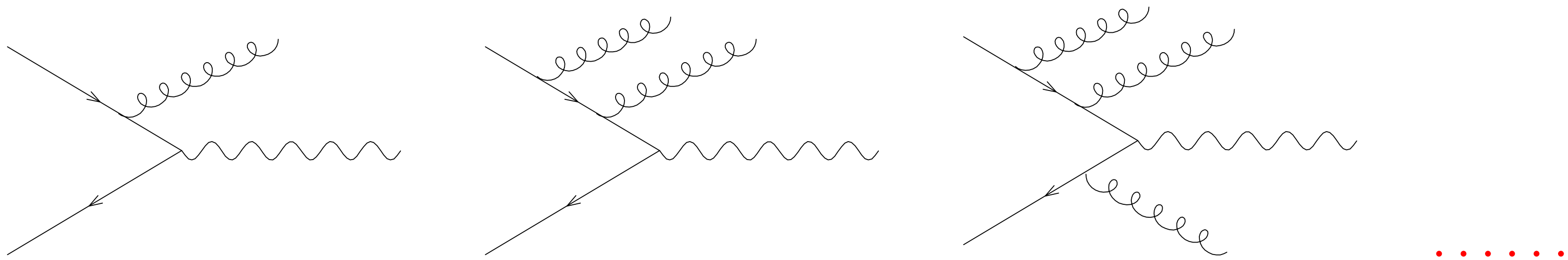
Which ones?

There are two possible choices, that lead to two vastly different strategies:

- Matrix Element Corrections $\longrightarrow$ tree level
- NLOwPS $\longrightarrow$ tree level and loop

Matrix Element Corrections

Compute (exactly) as many as possible real emission diagrams before starting the shower. Example: W production



Problems

- Double counting (the shower can generate the same diagrams)
- The diagrams are divergent

Solution

$\longrightarrow$ Catani, Krauss, Kuhn, Webber (2001), Lonnblad (2002), Mangano (2005)

How to achieve MEC

- Preliminary step: compute the real matrix elements

  Non trivial for high-multiplicities. Problem now fully solved and highly automatized (`AcerMC`, `ALPGEN`, `AMEGIC++`, `CompHEP`, `Grace`, `MadEvent`)

- The strategy: apply a cut δ_{sep} on matrix elements to avoid divergences

  For a fixed multiplicity n, this implies a large, unphysical δ_{sep} dependence

$$\sigma_n \sim \alpha_S^{n-2} \sum_k a_k \alpha_S^k \log^{2k} \delta_{sep}$$

Then reweight ME's and modify the shower to eliminate or reduce the δ_{sep} dependence

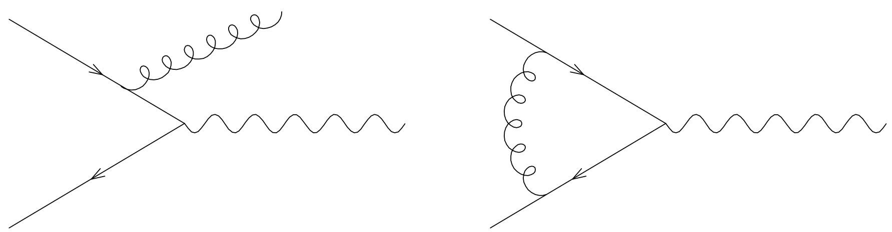
Following CKKW, one gets

$$\sigma_n \sim \alpha_S^{n-2} \sum_k a_k \alpha_S^k \log^{2k} \delta_{sep} \quad \longrightarrow \quad \alpha_S^{n-2} \left(\delta_{sep}^a + \sum_k b_k \alpha_S^k \log^{2k-2} \delta_{sep} \right)$$

NLOwPS

Compute all the NLO diagrams (and only those) before starting the shower.

Example: W production



......

Problems

- Double counting (the shower can generate *some of* the same diagrams)
- The diagrams are divergent

Solution

$\longrightarrow$

Proposals for NLOwPS's

- First working hadronic code (Z): Φ-veto (Dobbs, 2001)
- First correct general solution: MC@NLO (Frixione, Webber, 2002)
- Automated computations of ME's: grcNLO (GRACE group, 2003)
- Absence of negative weights (Nason, 2004; Frixione, Nason, Oleari, 2007) – POWHEG
- Showers with high log accuracy in ϕ_6^3 (Collins, Zu, 2002–2004)
- Proposals for $e^+e^- \to jets$ (Soper, Krämer, Nagy, 2003–2006)
- Shower and matching with QCD antennae (Giele, Kosower, Skands 2007) – VINCIA
- Within Soft Collinear Effective Theory (Bauer, Schwartz, 2006)
- With analytic showers (Bauer, Tackmann, Thaler, 2008) – GenEvA

Some of these ideas have passed the crucial test of implementation.
Two codes (MC@NLO and POWHEG) can be used to fully simulate hadronic processes

NLOwPS vs Matrix Element Corrections

NLOwPS are vastly different from MEC. MEC lack virtual corrections

This forces the use of an unphysical cutoff δ_{sep} in MEC, upon which physical observables depend $\longrightarrow$ matching systematics

NLOwPS are better than MEC since:

+ There is no δ_{sep} dependence (i.e., no matching systematics)
+ The computation of total rates is meaningful and reliable

NLOwPS are worse than MEC since:

− The number of hard legs is smaller

- The days of the universal tools are over. Choose the one that best suits your analysis. Typically: small/large number of *extra* legs $\Longrightarrow$ NLOwPS/MEC

The idea of embedding full NLO corrections into MC's is almost as old as MC's themselves. Why did it take so long?

- There is one crucial difficulty. KLN cancellation is inclusive. MC's are "exclusive" – which is possible because they include an hadronization mechanism

There no unique solution – as we have seen, at least two (MC@NLO, POWHEG) are working in practice in hadroproduction. However, in general:

- The key point: the cancellation of IR singularities in an observable- and process-independent manner (sort of "exclusive"), as done in the universal subtraction formalisms

  A similar understanding at NNLO would pave the way to NNLOwPS

With NLO corrections

- NLOwPS's are the only way in which K-factors can be embedded into MC's (rescaling is WRONG!)
- The scale dependence of observables is meaningful
- Realistic hadronization for NLO-accurate predictions
- Allow a fully-consistent determination of PDF uncertainties (PDF with errors are NLO fits), and of PDFs themselves
- Non-trivial dynamics beyond LO ($t-\bar{t}$ asymmetry, FCR vs FEX vs GSP in $b\bar{b}$, $qg \to Wq$, $Wt \leftrightarrow t\bar{t}$ interference, jet algorithms, ...)

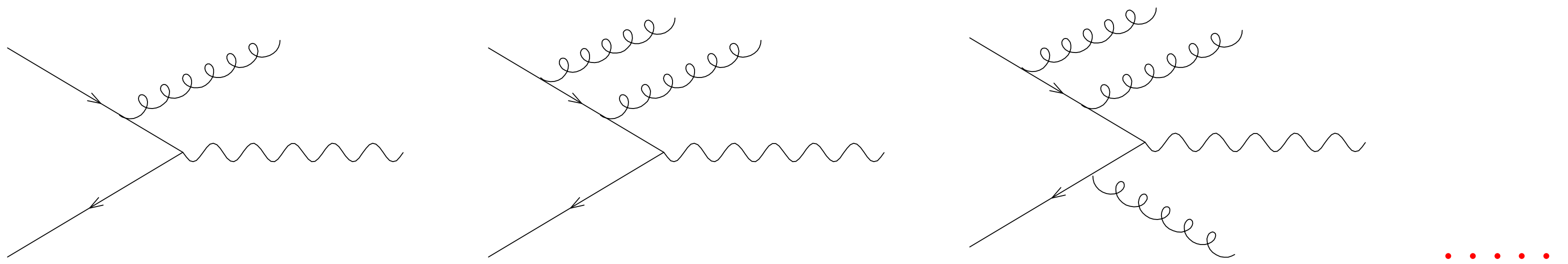
Summary on NLOwPS

- Event Generators including the typical benefits of NLO computations now exist
- Sensible predictions for total rates and large-p_T tails; it is meaningful to study scale dependence in a realistic experimental environment
- Absence of matching parameters, matching systematics (which may be introduced if needed). Increased predictive power wrt MEC
- Multileg NLO results are difficult to obtain. At present, MEC and NLOwPS are therefore complementary
- Next steps: more NLOwPS formalisms, extension of CKKW-like procedures at the NLO

AND HERE IS THE LONG VERSION...

Matrix Element Corrections

Compute (exactly) as many as possible real emission diagrams before starting the shower. Example: W production



Then use the kinematics configurations generated in this way as initial conditions for the shower

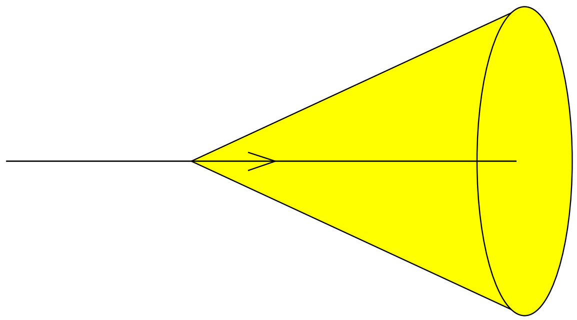
- The idea: large-p_T, well-separated partons will evolve into large-p_T, well-separated jets

Problems

- Double counting (the shower can generate the same diagrams)
- The diagrams are divergent

The two problems are connected: the matrix elements diverge in the soft/collinear regions, which are those "preferred" by showers

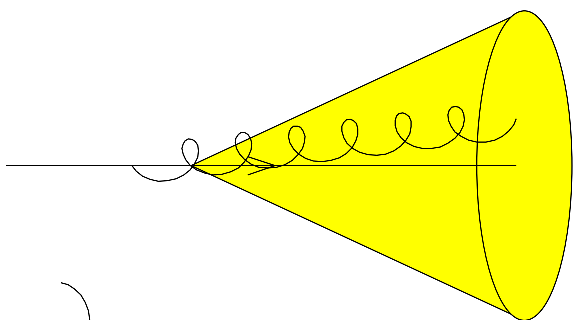
This suggests a (naive) solution: use some observable to decide if two partons are close to each other or not. If not, use the matrix elements, otherwise use the parton shower



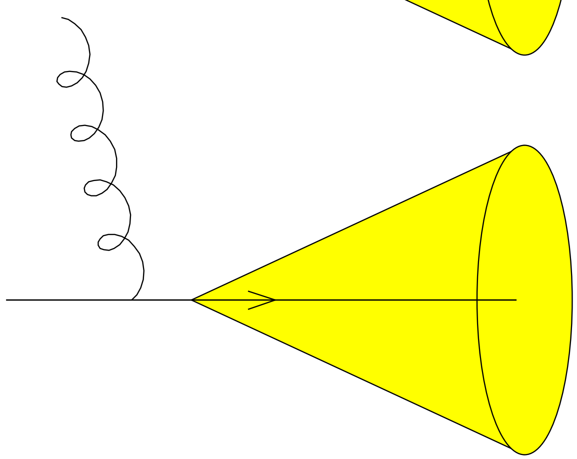
Simplest example: jet with cone algorithm

Close = in the cone

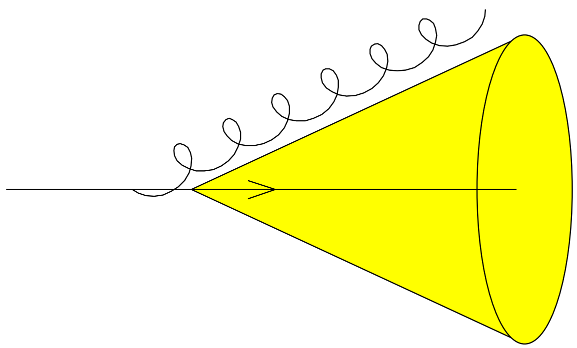
Far = outside the cone



Close $\Longrightarrow$ use PS



Far $\Longrightarrow$ use ME's



How about this one?

There are other obvious problems

- Partons emitted from far away parton which re-enter the cone
- Relative weight of ME's with different multiplicities unspecified
- What happens when changing cone size, or jet-finding algorithm?

The basic idea, however, is correct, and needs only be refined

I start discussing the approach of CKKW (Catani, Krauss, Kuhn, Webber), originally formulated for e^+e^- and then extended to hadronic collisions

Definition of interparton distance

It is conveniently suggested by a jet-finding algorithm, which *has nothing to do* with that used in the analysis. CKKW choose the k_T algorithm, where

$$\begin{aligned} d_i &= p_{Ti}^2 && \text{parton} - \text{beam distance} \\ d_{ij} &= \min\left(p_{Ti}^2, p_{Tj}^2\right) R_{ij}^2 && \text{parton} - \text{parton distance} \\ R_{ij}^2 &= (\varphi_i - \varphi_j)^2 + (\eta_i - \eta_j)^2 \end{aligned}$$

One then introduces a stopping value d_{ini} (typically, of $\mathcal{O}$(10 GeV))

Two partons i, j are close to each other, or one parton i is close to the beam (ie to an incoming parton), if

$$d_{ij} < d_{ini}\,, \qquad d_i < d_{ini}$$

If two partons are close, they can be recombined into one pseudo-parton. By iteration, one arrives at a set of partons and pseudo-partons all far away from each other

The prescription of CKKW

We are interested in $p_1 + p_2 \longrightarrow X + many\ jets$

1) Compute the probabilities

$$P_n^{(0)} = \sigma_n^{(0)} \Big/ \sum_{i=1}^{N} \sigma_i^{(0)}$$

with $\sigma_n^{(0)}$ the tree-level n-jet cross section for k_T-jets with resolution scale d_{ini}; use $\alpha_S = \alpha_S(d_{ini})$

$\sigma_n^{(0)} \longleftarrow$ n-parton matrix elements, with partons separated by d_{ini}

2) Choose a multiplicity $0 \leq \bar{n} \leq N$ with probability $P_{\bar{n}}^{(0)}$

3) Use the matrix elements $\mathcal{M}(p_1 + p_2 \rightarrow X + \bar{n}\ \mathrm{partons})$ to generate an $X + \bar{n}\ \mathrm{partons}$ kinematic configuration

We have now an $\bar{n}$ partons unweighted hard event

4) Cluster the $\bar{n}$ partons using the k_T-algorithm, and find the *nodal values*

$$d_1 > d_2 > \ldots d_{\bar{n}} > d_{ini}$$

at which 1, 2, ... $\bar{n}$ jets are resolved

The $\bar{n}$-parton configuration can now be depicted as a branching tree, with successive branchings at scales d_i

5) Apply a coupling reweighting factor

$$\alpha_S(d_1)\alpha_S(d_2)\ldots\alpha_S(d_{\bar{n}})\Big/\Big(\alpha_S(d_{ini})\Big)^{\bar{n}} \leq 1$$

Had we known the branching tree, we should have computed the ME's with these couplings

6) Apply a Sudakov reweighting factor

$$\Delta(d_{ini}, d_i)\Big/\Delta(d_{ini}, d_j)$$

to each line from a node with scale d_i to the next node with scale $d_j < d_i$. If the line is external, $d_j = d_{ini}$

7) Unweight again the hard configuration, ie accept it if the product of coupling and Sudakov reweighting factors is larger than a random number. Otherwise, start again from 2)

8) The accepted configuration is the initial condition for the parton shower. Branchings $a \to bc$ in the shower *must be vetoed* if $d_{bc} > d_{ini}$

When an emission is vetoed it does not take place, but the shower scale for the next branching is recomputed as if the branching had occurred

In words: what happens in CKKW

- A jet clustering algorithm is used to separate the ME-dominated from the PS-dominated regions
- In the ME-dominated regions the ME's are corrected, as if they were generated (kinematically) by the PS. The Sudakov factors make sure that a PS would not emit extra partons wrt those entering the ME's
- In the PS-dominated regions the PS does its job, but it's prevented, owing to the veto, from emitting large-p_T, well-separated partons

If one goes through this considerable mess, he/she would like to be sure that in the end the predicted IR-safe observables will *independent* of the choice of the *jet-clustering algorithm*, and of d_{ini}

Accuracy in CKKW

A formal statement has been given only for jet observables in e^+e^- collisions, but is believed to be correct also for hadronic observables

- The separation of the ME- and PS-dominated regions introduces a dependence

$$\sigma_n \sim \alpha_S^{n-2} \sum_k a_k \alpha_S^k \log^{2k} \frac{d_{ini}}{s}$$

 in the n-jet cross section

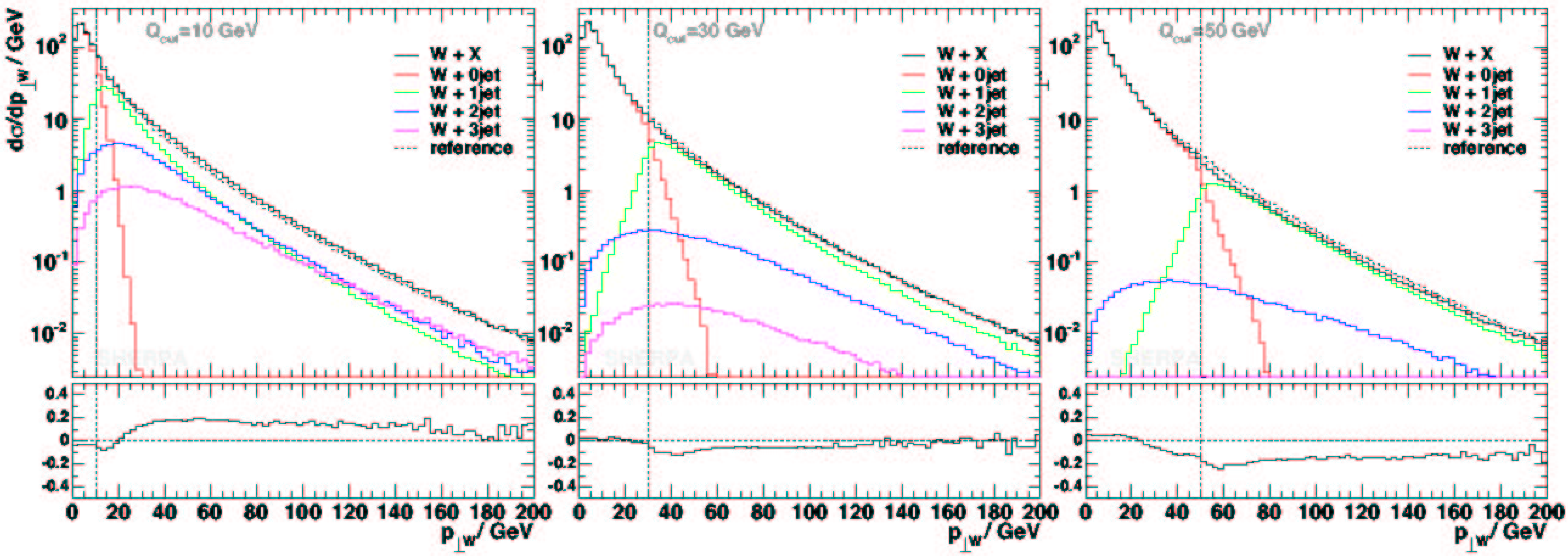
- At the end of the CKKW procedure, this is reduced to

$$\sigma_n \sim \alpha_S^{n-2} \left(\left(\frac{d_{ini}}{s} \right)^a + \sum_k b_k \alpha_S^k \log^{2k-2} \frac{d_{ini}}{s} \right)$$

 ie it is cancelled to NLL accuracy

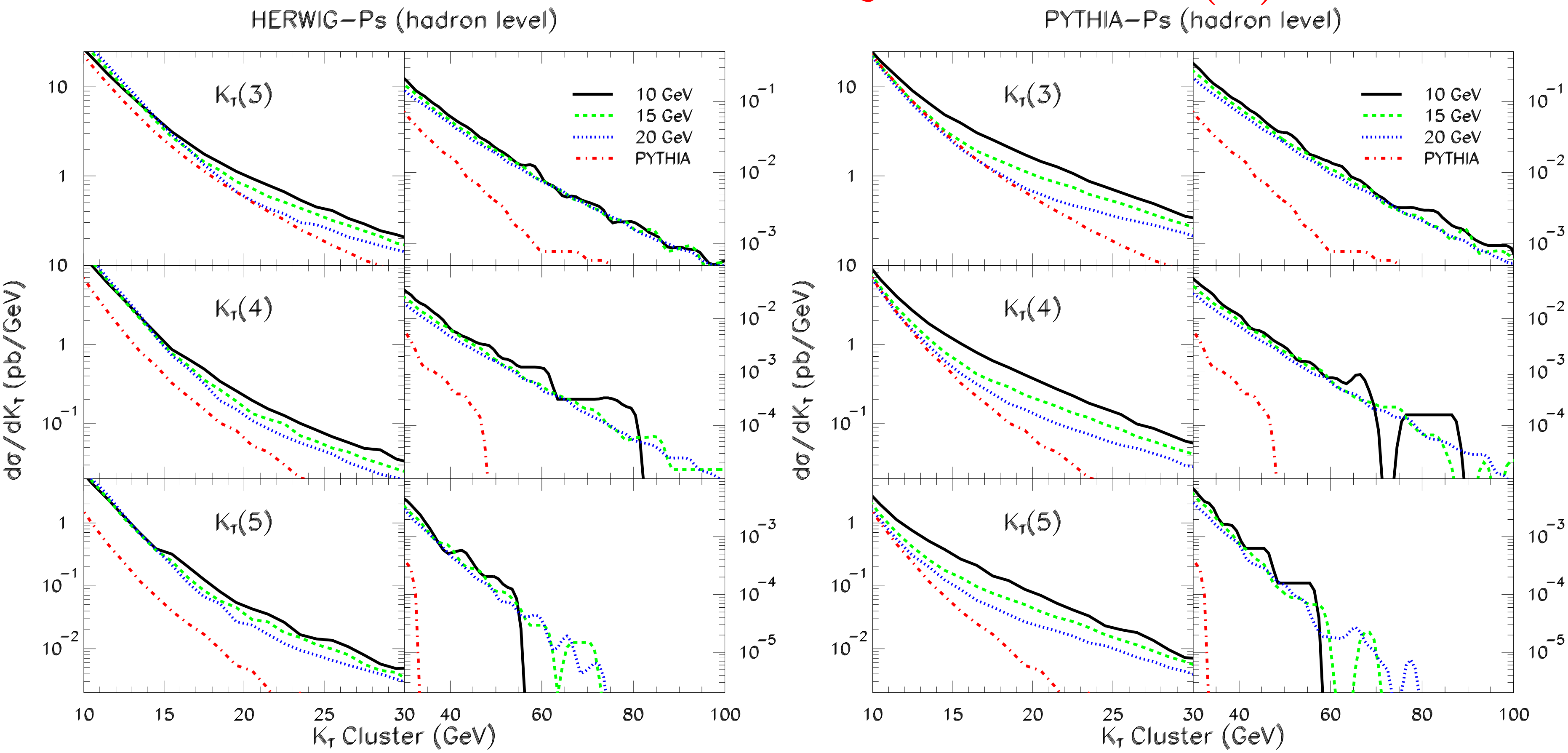
Is this good enough?

Test case: W+jets $\longrightarrow$ $p_T(W)$



- Here $Q_{cut} \equiv d_{ini}$
- The larger d_{ini}, the smaller the impact of high-multiplicity ME's
- A 20% bias is acceptable, and can be used to tune to the data

Test case: W+jets $\longrightarrow$ $k_T(n)$



- $k_T(n)$ is the value of the resolution scale at which an n-jet configuration becomes an $(n-1)$-jet one
- The dependence on d_{ini} is of the same order as that for $p_T(W)$
- Clear improvements wrt standard parton showers (black vs red lines)

CKKW is an *interpolation* procedure between a PS and the ME's. It defines a fremework, but there is a lot of freedom left, which can be used to reduce unphysical biases on observables

- Clustering algorithm and momentum-recombination scheme
- Sudakov definitions
- Scale choices
- Corrections due to $N < \infty$ (highest-multiplicity ME)

Never forget that the d_{ini} dependence can be reduced but not eliminated. So make sure, before embarking in extensive physics studies, that d_{ini} is properly chosen, and the biases are small

An alternative approach: MLM matching

Proposed by Mangano. This is now used in ALPGEN

1) Generate hard unweighted events with the ME's, imposing

$$E_T > E_T^{min}\,, \qquad R_{ij} > R_{min}$$

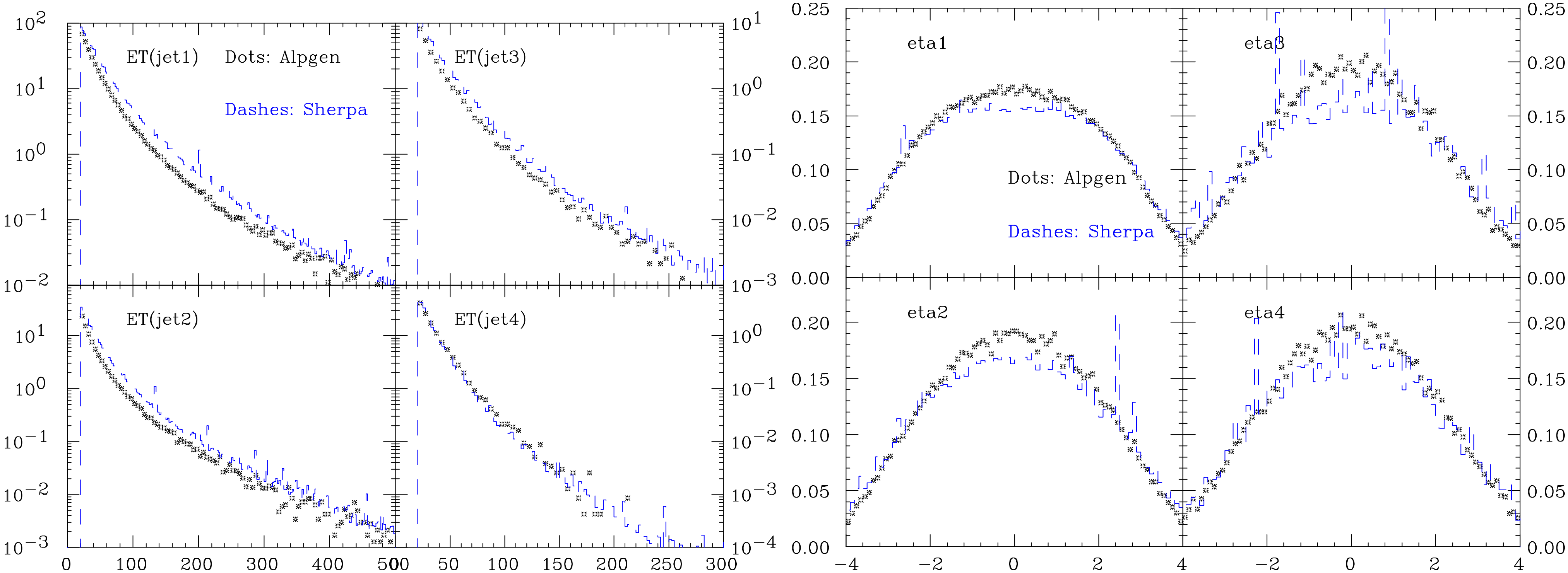
2) Define a branching-tree structure as done in CKKW, but using colour flows extracted from the ME calculations

3) Compute α_S at the nodal values found in 2), but do not apply any Sudakov reweight factors

4) Shower the hard event, without applying any veto; when done, find jets using a cone algorithm with (E_T^{min}, R_{min})

5) Require jets be matched to hard partons. Events with more jets than hard partons are rejected, except for the highest-multiplicity ME's

CKKW vs MLM for W+jets



- Differences in leading-jets distributions slightly larger than for $p_T(W)$
- Differences may be due to the matching algorithm, the shower (SHERPA for CKKW, HERWIG for MLM), or a combination of the two

Summary on Matrix Element Corrections

- Various approaches and implementations on the market; the use of standard PSMC for multi-jet studies cannot be justified any longer
- Overall, existing approaches are robust, and lead to tolerably small dependence on unphysical parameters, if these are cleverly chosen
- There are discrepancies among the different approaches; there is a lot of flexibility in implementation details
- Tuning to data is strongly recommended, and anyhow necessary to figure out the correct normalization: these are LO QCD computation!

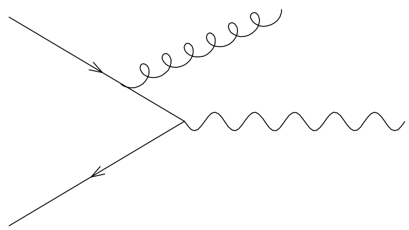
Matching parameter systematics must be assessed

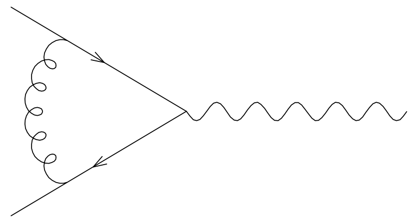
Try different codes and implementations

NLOwPS

Compute all the NLO diagrams (and only those) before starting the shower.

Example: W production





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Then use the kinematics configurations generated in this way as initial conditions for the shower

Problems

- Double counting (the shower can generate *some of* the same diagrams)
- The diagrams are divergent

- The problems are *almost* identical to those encountered in MEC. The solution, however, is completely different

Matching NLO with MC: NLOwPS

What do we want? Let's *define* it:

- Total rates are accurate to NLO
- Hard emissions are treated as in NLO computations
- Soft/collinear emissions are treated as in MC
- NLO results are recovered upon expansion of NLOwPS results in α_S. In other words: there is no double counting in NLOwPS
- The matching between hard- and soft/collinear-emission regions is smooth
- The output is a set of events, which are fully exclusive
- MC hadronization models are adopted

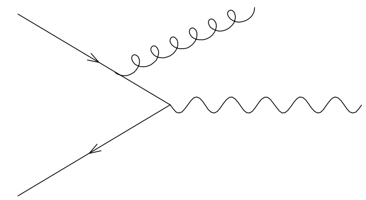
Note: in general, negative-weight events can be generated

NLO and MC computations

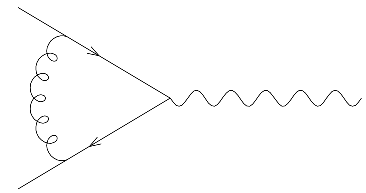
NLO cross section (based on subtraction)

$$\left(\frac{d\sigma}{dO}\right)_{subt} = \sum_{ab}\int dx_1\, dx_2\, d\phi_{n+1}\, f_a(x_1) f_b(x_2) \times$$

$$\Big[\delta(O - O(2\to n+1))\mathcal{M}^{(r)}_{ab} + \qquad \longleftarrow$$



$$\delta(O - O(2\to n))\Big(\mathcal{M}^{(b,v,c)}_{ab} - \mathcal{M}^{(c.t.)}_{ab}\Big)\Big] \qquad \longleftarrow$$



MC

$$\mathcal{F}_{\mathrm{MC}} = \sum_{ab}\int dx_1\, dx_2\, d\phi_n\, f_a(x_1) f_b(x_2)\, \mathcal{F}^{(2\to n)}_{\mathrm{MC}} \mathcal{M}^{(b)}_{ab}$$

- Matrix elements $\longrightarrow$ normalization, hard kinematic configurations
- δ-functions, $\mathcal{F}^{(2\to n)}_{\mathrm{MC}} \equiv$ showers $\longrightarrow$ observable final states

NLO + MC $\longrightarrow$ NLOwPS?

Naive first try: use the NLO kinematic configurations as initial conditions for showers, rather than for directly computing the observables

- $\delta(O - O(2 \to n)) \longrightarrow$ start the MC with n "hard" emissions: $\mathcal{F}_{\text{MC}}^{(2\to n)}$
- $\delta(O - O(2 \to n+1)) \longrightarrow$ start the MC with $n+1$ "hard" emission: $\mathcal{F}_{\text{MC}}^{(2\to n+1)}$

$$\mathcal{F}_{\text{naive}} = \sum_{ab} \int dx_1\, dx_2\, d\phi_{n+1}\, f_a(x_1) f_b(x_2) \times \left[\mathcal{F}_{\text{MC}}^{(2\to n+1)} \mathcal{M}_{ab}^{(r)} + \mathcal{F}_{\text{MC}}^{(2\to n)} \left(\mathcal{M}_{ab}^{(b,v,c)} - \mathcal{M}_{ab}^{(c.t.)} \right) \right]$$

It doesn't work:

- Cancellations between $2 \to n+1$ and $2 \to n$ contributions occur after the shower: hopeless from the practical point of view; and, unweighting is impossible
- $(d\sigma/dO)_{naive} - (d\sigma/dO)_{NLO} = \mathcal{O}(\alpha_S)$. In words: double counting

MC@NLO: formalism (SF, Webber (2002))

The naive prescription doesn't work: MC evolution results in spurious NLO terms

$\longrightarrow$ *Eliminate the spurious NLO terms "by hand": MC counterterms*

- The generating functional is

$$\mathcal{F}_{\text{MC@NLO}} = \sum_{ab} \int dx_1\, dx_2\, d\phi_{n+1}\, f_a(x_1) f_b(x_2) \times \Big[\mathcal{F}_{\text{MC}}^{(2\to n+1)} \left(\mathcal{M}_{ab}^{(r)} - \mathcal{M}_{ab}^{(\text{MC})} \right) + \mathcal{F}_{\text{MC}}^{(2\to n)} \left(\mathcal{M}_{ab}^{(b,v,c)} - \mathcal{M}_{ab}^{(c.t.)} + \mathcal{M}_{ab}^{(\text{MC})} \right) \Big]$$

$$\mathcal{M}_{\mathcal{F}(ab)}^{(\text{MC})} = \mathcal{F}_{\text{MC}}^{(2\to n)} \mathcal{M}_{ab}^{(b)} + \mathcal{O}(\alpha_S^2 \alpha_S^b)$$

There are *two* MC counterterms: they eliminate the spurious NLO terms due to the branching of a final-state parton, and to the non-branching probability

Let's have a look at the weight functions

$$w_{\mathbb{H}} = \mathcal{M}_{ab}^{(r)} - \mathcal{M}_{ab}^{(\mathrm{MC})}$$

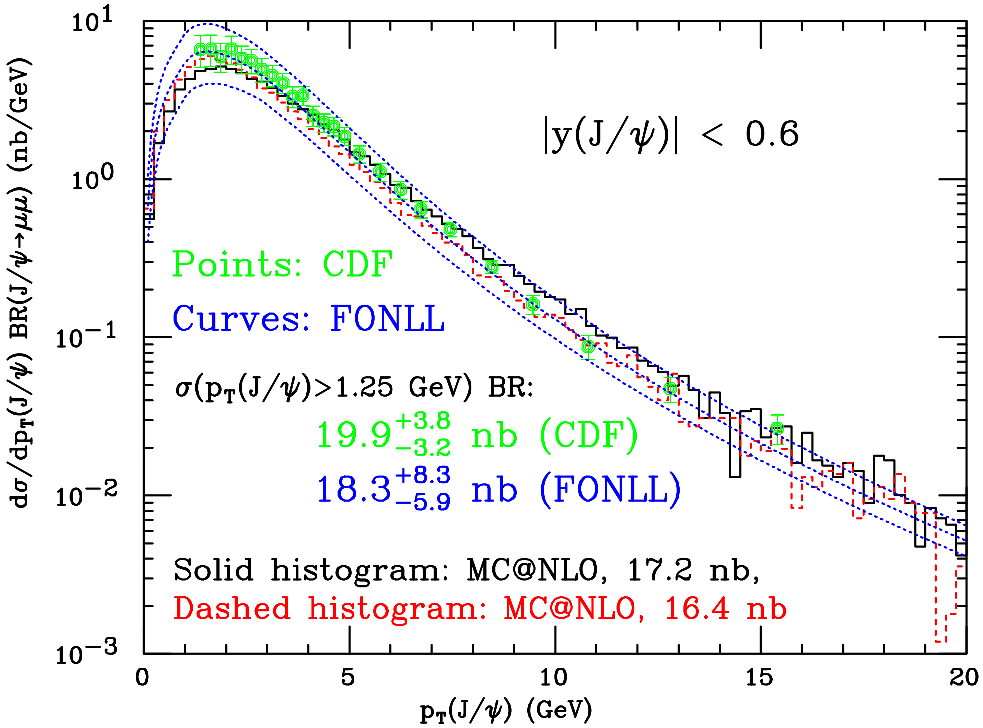
$$w_{\mathbb{S}} = \mathcal{M}_{ab}^{(b,v,c)} - \mathcal{M}_{ab}^{(c.t.)} + \mathcal{M}_{ab}^{(\mathrm{MC})}$$

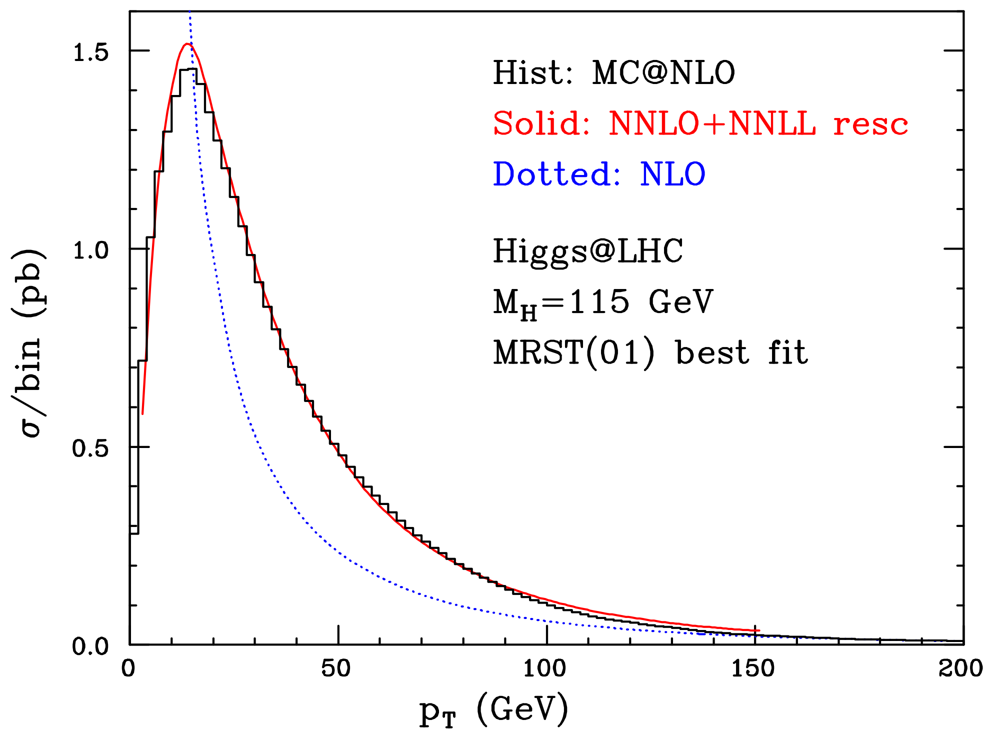
These are finite (i.e. don't diverge) for any phase-space point!

The MC provides local, observable-independent, counterterms $\Longrightarrow$ greater numerical stability, unweighting possible

By solving the problem of double counting, one also cancels the singularities at the level of *hard matrix elements* (i.e., with no reference to a specific observable). Configurations with different final states can then be showered independently

MC@NLO results





- Implements several hadroproduction processes; used by experimenters
- Left plot: we have another *predictive* way to show that b production is under control
- Excellent agreement with matched analytic computation of formally higher log accuracy

POWHEG (Nason (2004))

The proposal stems from the following *theorem*

A shower can be defined which has the largest-p_T emission at the first branching, and is equivalent (to LL accuracy) to the angular-ordered shower

Such a shower goes through the following steps

- Do the first branching as usual. It will define branching variables z and $t_0 < t < t_{ini}$, and a p_T
- Do a shower from each of the two legs from the first branching, with upper scales $z^2 t$ and $(1-z)^2 t$, and veto all emissions with a relative transverse momentum larger than p_T (vetoed showers)
- Do a further p_T-vetoed shower, with upper scale t_{ini} and lower scale t (vetoed truncated shower) $\longrightarrow$ restores coherence

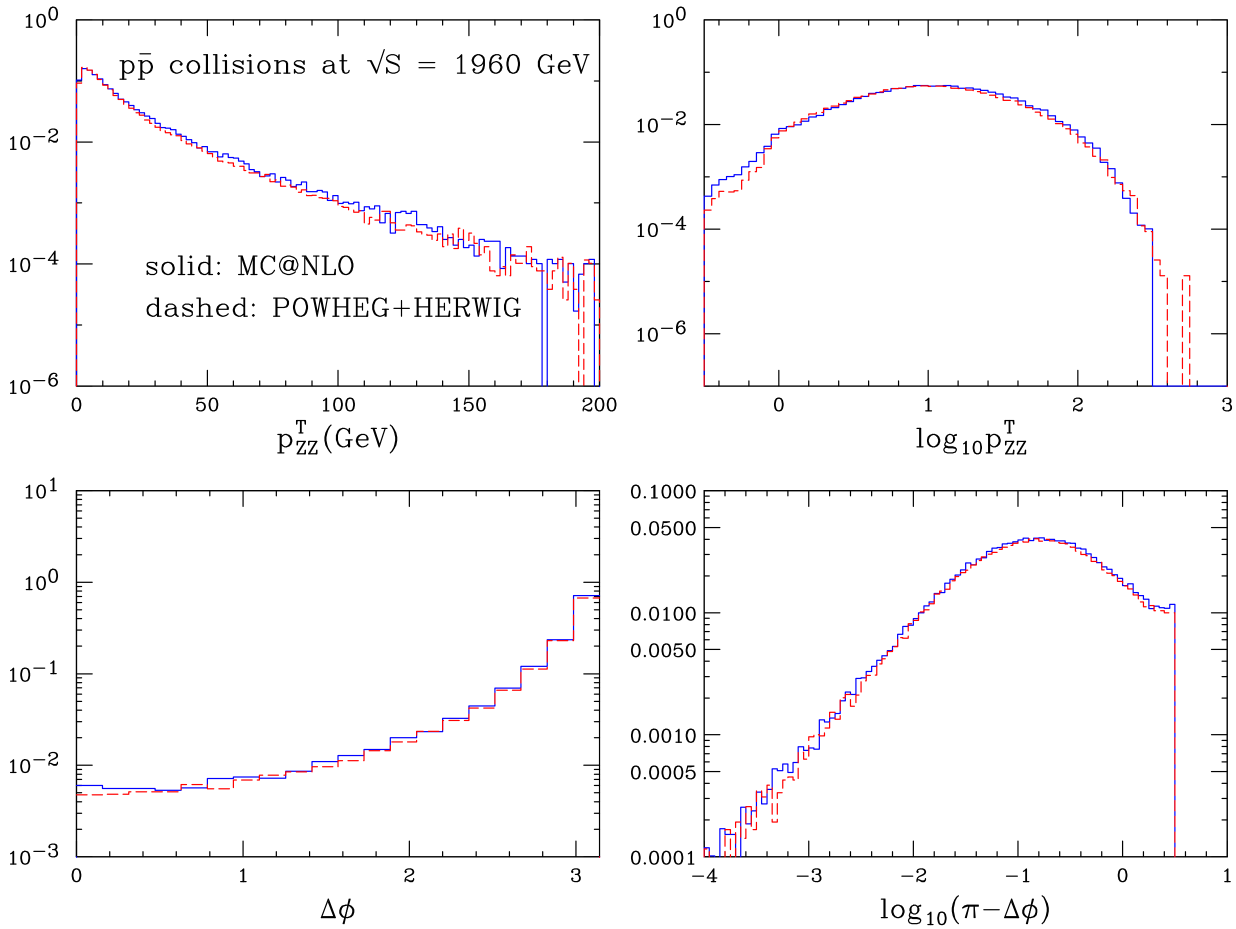
Proposal for POWHEG

The basic idea builds upon the previous theorem

Exponentiate the full real corrections into a Sudakov, and use that for the first branching. Then proceed as before, with vetoed and vetoed truncated showers

- MC must be capable of handling vetoed truncated showers; not the case at the moment
- May use a separate package for the vetoed truncated showers
- Beyond-LL structure changed: need for re-tuning?
- |MC@NLO-pMC@NLO|=$\mathcal{O}(\alpha_S^2)$ $\longleftarrow$ how large α_S^2 terms?

Implementations of $pp \to ZZ$ (Nason, Ridolfi) and $pp \to Q\bar{Q}$ (SF, Nason, Ridolfi), without vetoed truncated showers. General formulation now available (SF, Nason, Oleari)



- ZZ hadroproduction now implemented in POWHEG
- Vetoed-truncated showers not yet available
- Should not matter much for inclusive observable in ZZ: excellent agreement with MC@NLO

General formulation and other implementations are under way

Conclusions

This is the decade of hadron colliders – and the most exciting time in high-energy particle physics after 1984. We can't tell what lies ahead, and thus we must have reliable predictions for what we believe we know

QCD theorists have responded remarkably well, with major breakthroughs in the past few years *(many topics seemed unrealistic 5 years ago)*

I've tried to give you an overview on selected topics which will presumably have a strong impact on the LHC programme

- Fixed-order computations at tree-level, NLO and NNLO
- Resummed and matched computations
- PDFs
- Monte Carlos of the new generation

There are of course so many things I did not even mention

- Twistors
- Soft and semihard physics
- Small-x physics
- Quarkonia (NRQCD)
- Power corrections
- Diffraction

… therefore: where do we stand?

There is a lot of work to be completed, which is supposed to be relevant for LHC physics, both in the perturbative and non-perturbative domains

But a lot has already been achieved and, more importantly, I am confident that, thanks to what we have understood, we'll be able to solve the problems which is reasonable to expect from the LHC