Soft - Collinear Effective Theory

An effective field theory for energetic hadrons & jets

$$E \gg \Lambda_{\rm QCD}$$

Lecture 4

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So far

Lecture I

- Introduction to SCETI, SCETII
- Collinear & Soft degrees of freedom
- Construction of HQET

Lecture II

- SCETI propagators, field power counting
- Leading Lagrangian and heavy-light current
- Gauge symmetry and reparameterizations in SCET

Lecture III

- Wilson coefficients & hard-collinear factorization
- Field redefinition & ultrasoft-collinear factorization
- One-Loop ultrasoft and collinear graphs, IR divergences
- Renormalization group evolution & Sudakov logs

Lecture 4 Outline

- $B \to X_s\gamma$ Factorization Theorem
- More on large logs, Evolution with Convolutions

- SCETII, building blocks, exploiting SCETI
- Factorization for $B \to D\pi$, $B \to \pi\ell\bar{\nu}$
- eg. of power corrections in SCETI

- Jet Production $e^+e^- \to J_n J_{\bar{n}} X$

SCET$_{\rm I}$

Construction of operators (using power counting, ultrasoft & collinear gauge invariance, RPI)

We built gauge invariant operators with nice power counting:

eg. LO heavy-to-light current

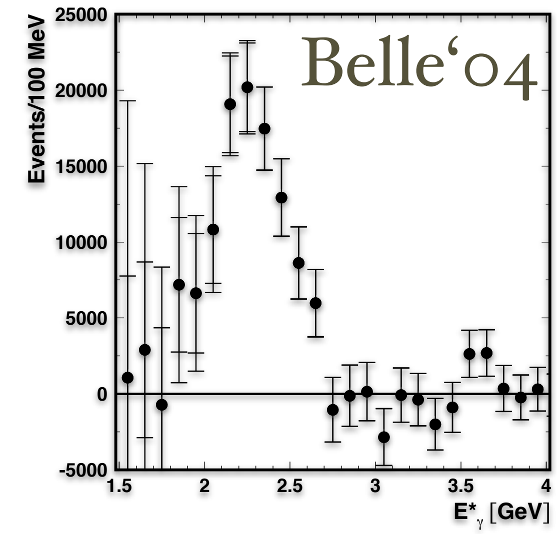
$$J^{(0)} = \int d\omega\, C(\omega,\mu)\, \Big[(\bar\xi_n W)\delta(\omega-\bar{\mathcal{P}}^\dagger)\Gamma(Y_n^\dagger h_v)\Big] = \int d\omega\, C(\omega,\mu)\ \bar\chi_{n,\omega}\,\Gamma\,\mathcal{H}_v^n$$

eg. a subleading current suppressed by λ

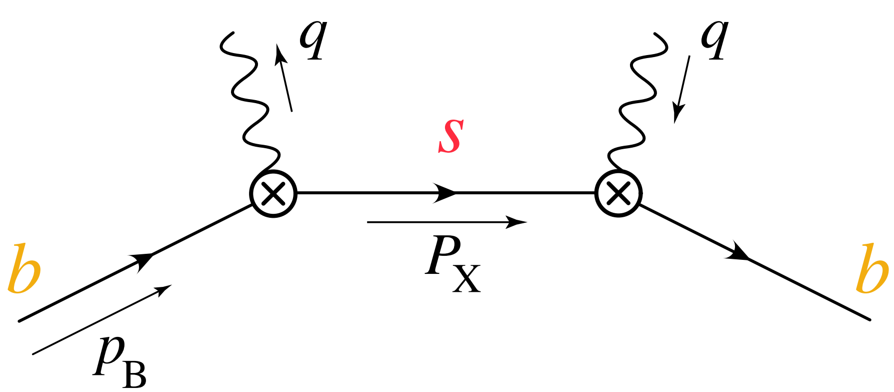
$$J^{(1)} = \int d\omega\, d\omega'\, C^{(1)}(\omega,\omega',\mu)\ \bar\chi_{n,\omega}\, ig\not{\mathcal{B}}^\perp_{\omega'}\,\Gamma\,\mathcal{H}_v^n$$

$$\begin{aligned} ig\mathcal{B}^{\perp\mu}_{\omega'} &= \frac{1}{\bar{\mathcal{P}}} W\big[i\bar n\!\cdot\! D_n, iD_n^{\perp\mu}\big]W^\dagger\ \delta(\omega'-\bar{\mathcal{P}}^\dagger) \\ &= gA^{\perp\mu}_{n,\omega'} + \ldots \end{aligned}$$

Endpoint $B \to X_s\gamma$



Optical Thm: $\Gamma \sim \mathrm{Im} \int d^4x\, e^{-iq\cdot x} \langle B | T\{J^\dagger_\mu(x) J^\mu(0)\} | B\rangle$



	$P_X^2 = m_B(m_B - 2E_\gamma)$
standard OPE	$\sim m_B^2$
endpoint region	$\sim m_B \Lambda_{QCD}$
resonance region	$\sim \Lambda^2_{QCD}$

For EndPoint: $E_\gamma \gtrsim 2.2\,\mathrm{GeV}$, X_s collinear, B usoft, $\lambda = \sqrt{\frac{\Lambda_{QCD}}{m_B}}$

We want to prove that the
Decay rate is given by factorized form

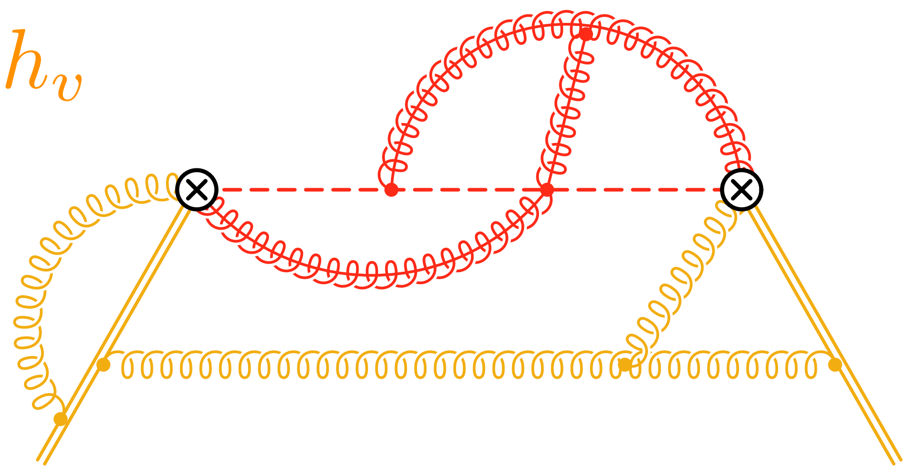
$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu) \int_{2E_\gamma - m_b}^{\bar\Lambda} dk^+\, S(k^+,\mu)\, J(k^+ + m_b - 2E_\gamma, \mu)$$

Match: $\bar{s}\Gamma_\mu b \to e^{i(m_b v-\mathcal{P})\cdot x} C(\bar{\mathcal{P}})\bar{\xi}_{n,p}\, W \gamma_\mu^\perp P_L\, h_v$

$$T_\mu^\mu = \int d^4x\, e^{i(m_b \frac{\bar{n}}{2} - q)\cdot x} \left\langle B \middle| T J_{\rm eff}^\dagger(x) J_{\rm eff}(0) \middle| B \right\rangle$$

label conservation
$\bar{\mathcal{P}} \to m_b$

Factor usoft: $\bar{\xi}_n W \Gamma_\mu h_v \to \bar{\xi}_n W \Gamma_\mu Y_n^\dagger h_v$



$$T_\mu^\mu = \left|C(m_b)\right|^2 \int d^4x e^{i(m_b \frac{\bar{n}}{2} - q)\cdot x} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle$$

$$\times \left\langle 0 \middle| T[W^\dagger \xi_n](x)[\bar{\xi}_n W](0) \middle| 0 \right\rangle \times [\Gamma_\mu \otimes \Gamma^\mu]$$

$$= \left|C(m_b)\right|^2 \int d^4x \int \frac{d^4k}{(2\pi)^4}\, e^{i(m_b \frac{\bar{n}}{2} - q - k)\cdot x} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle$$

$$\times J_P(k) \times [\Gamma_\mu \otimes \Gamma^\mu]$$

Convolution $J_P(k) = J_P(k^+)$

$$\mathrm{Im}\, T^\mu_\mu = \left|C(m_b)\right|^2 \int d^4x \int \frac{d^4k}{(2\pi)^4}\, e^{i(m_b \frac{\bar{n}}{2} - q - k)\cdot x} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle$$
$$\times \mathrm{Im} J_P(k^+)$$
$$= \left|C(m_b)\right|^2 \int dk^+ \left[\int \frac{dx^-}{4\pi}\, e^{i(m_b - 2E_\gamma - k^+)x^-/2} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle \right]$$
$$\times \mathrm{Im} J_P(k^+)$$
$$= \left|C(m_b)\right|^2 \int dk^+ S(2E_\gamma - m_b + k^+) \mathrm{Im} J_P(k^+)$$

as desired

calculable	calculable	nonpert. shape function

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu) \int dk^+ J(k^+,\mu)\; S(2E_\gamma - m_b + k^+,\mu)$$

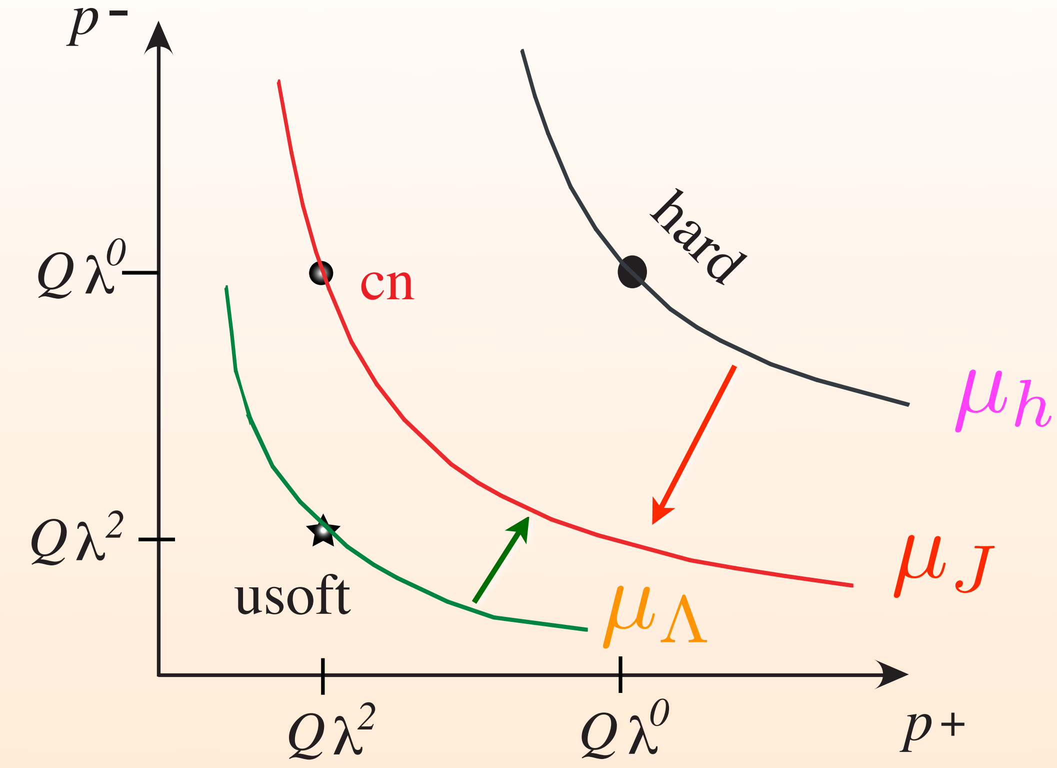
$p^2 \sim m_b^2$	$p^2 \sim m_b \Lambda_{\mathrm{QCD}}$	$p^2 \gtrsim \Lambda_{\mathrm{QCD}}^2$
$\sim \mu_h^2$	$\sim \mu_J^2$	$\sim \mu_\Lambda^2$

To minimize large logs we want to evaluate these functions at different μ's

- our result from last lecture for the RGE for C, allows us to write

$$H(m_b, \mu_J) = H(m_b, \mu_h)\, U_H(m_b, \mu_h, \mu_J)$$

- need to be able to run the shape function up to μ_J



or we could run the jet and hard functions down to μ_Λ

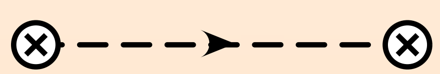
Lets consider the jet function & its RGE

The Jet Function

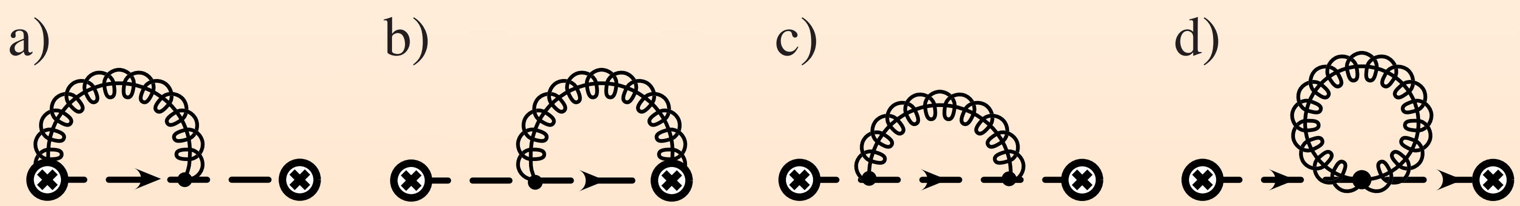
$$\sum_{X_n} \frac{1}{4N_c} \mathrm{tr}\, \langle 0 | \bar{n}\!\!\!/ \chi_n(x) | X_n \rangle \langle X_n | \chi_{n,Q}(0) | 0 \rangle = Q \int \frac{d^4 r_n}{(2\pi)^3} e^{-ir_n \cdot x} J_n(Q r_n^+, \mu)$$

$$J_n(Q r_n^+, \mu) = \frac{-1}{8\pi N_c Q} \,\mathrm{Disc} \int d^4x\, e^{ir_n \cdot x} \langle 0 | \mathrm{T}\, \bar{\chi}_{n,Q}(0) \bar{n}\!\!\!/ \chi_n(x) | 0 \rangle$$

tree level:



one loop:



RGE:

$$\mu \frac{d}{d\mu} J(s,\mu) = \int ds'\, \gamma_J(s-s',\mu)\, J(s',\mu)$$

solution

$$J(s,\mu) = \int ds'\, U_J(s-s',\mu,\mu_0)\, J(s',\mu_0)$$

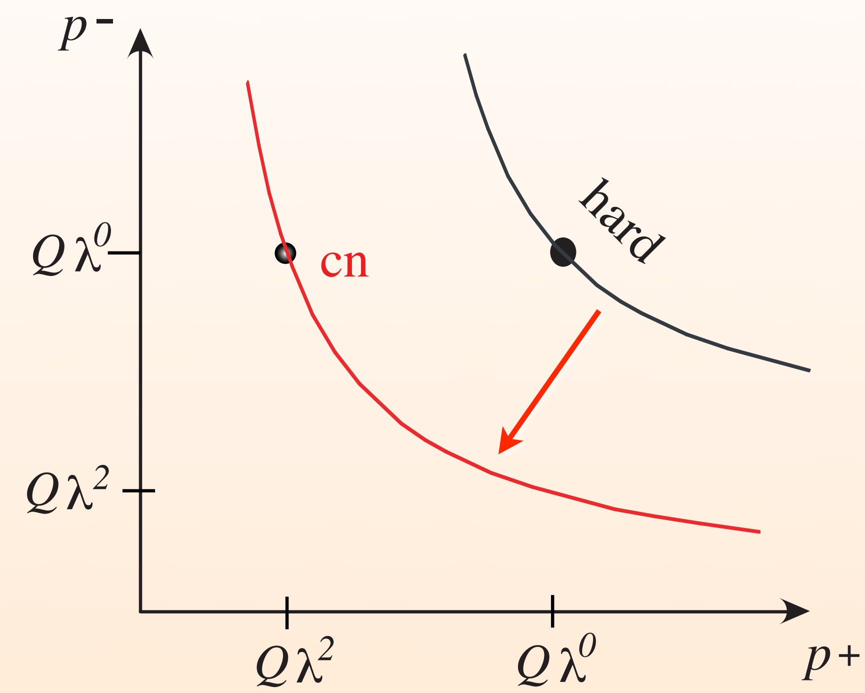
$$U_J(s-s',\mu,\mu_0) = \frac{e^K \left(e^{\gamma_E}\right)^{\omega}}{\mu_0^2\, \Gamma(-\omega)} \left[\frac{(\mu_0^2)^{1+\omega} \theta(s-s')}{(s-s')^{1+\omega}} \right]_+$$

More examples which involve convolutions

twist 2 operators

label on 2nd block of fields is fixed by mom.cons. in m.elt.

$$J^{(0)} = \int d\omega\, C(\omega,\mu)\, \underbrace{\bar{\chi}_{n,\omega}\, \bar{n}\!\!\!/\, \chi_n}_{\mathcal{O}(\omega)}$$



Matrix Elements

- π light-cone distrib. $\langle \pi_n(p_\pi^-)|J^{(0)}|0\rangle = \int d\omega\, C(\omega,\mu)\, \phi_\pi(\omega/p_\pi^-,\mu) = p_\pi^- \int_0^1 dx\, C(xp_\pi^-,\mu)\, \phi_\pi(x,\mu)$

- DIS p.d.f

$$\langle p_n(p^-)|J^{(0)}|p_n(p^-)\rangle = \int d\omega\, C(\omega,Q,\mu)\, f_{i/p}(\omega/p^-,\mu)$$

$$= \frac{Q}{x}\int_x^1 d\xi\, C\!\left(\frac{Q\xi}{x},Q,\mu\right) f_{i/p}(\xi,\mu)$$

$$p^- = \frac{Q}{x}$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu)\int dk^+ J(k^+,\mu)\ S(2E_\gamma - m_b + k^+,\mu)$$

$$+\dots$$

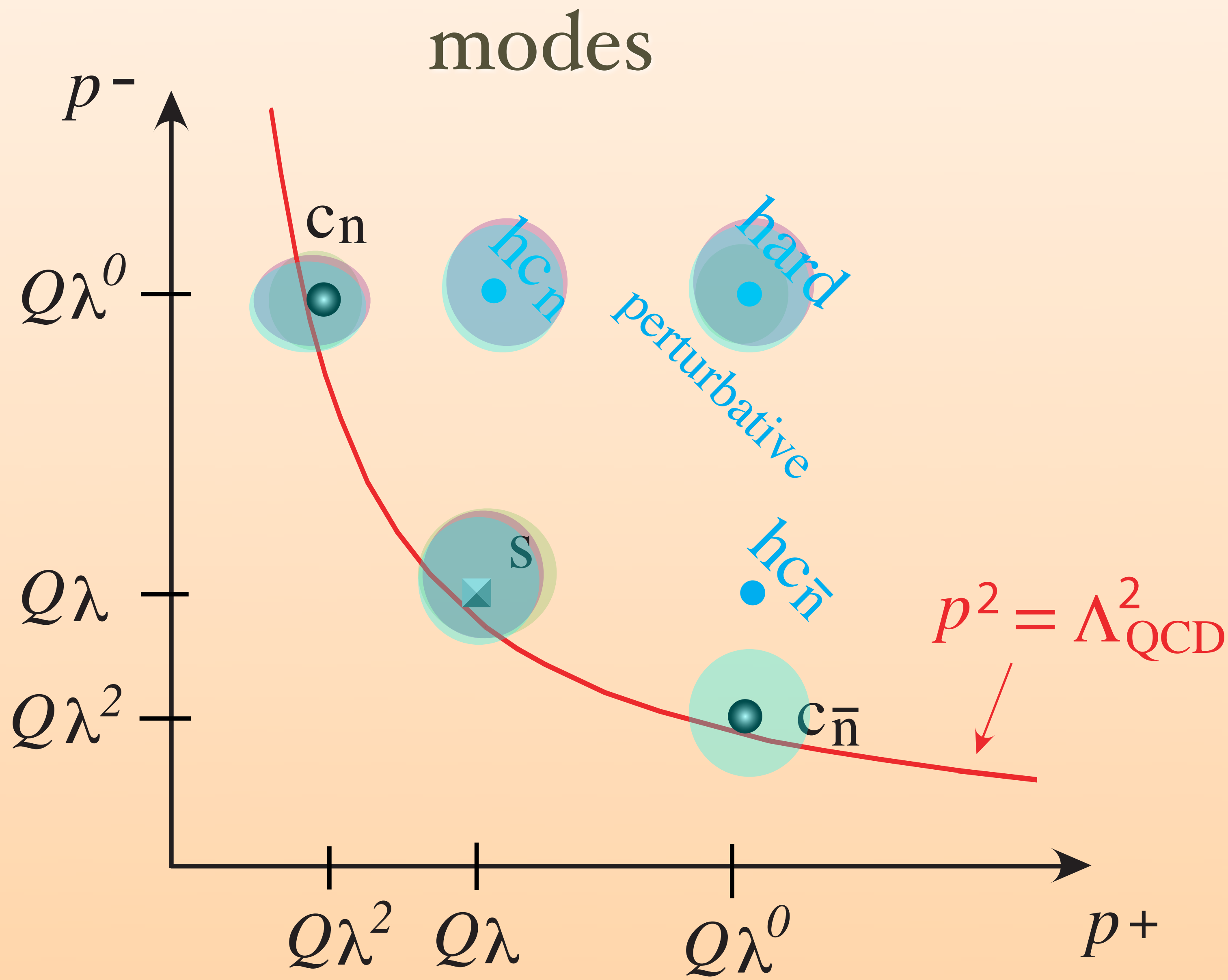
Factorization formulas of this type have also been derived for the power corrections using SCET

$\mathrm{SCET_{II}}$

$$\lambda = \frac{\Lambda}{Q}$$

- So far we have considered inclusive processes with jets, or processes with only one identified hadron like DIS
- $\mathrm{SCET_{II}}$ allows us to treat cases with two or more hadrons

  eg. $B \to D\pi$, $B \to \pi \ell \bar{\nu}$, $B \to \pi\pi$

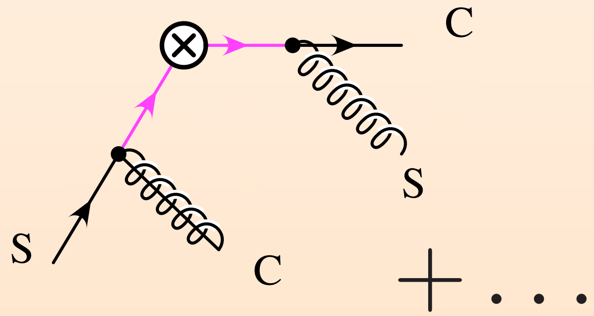


Constructing $\mathrm{SCET_{II}}$ Operators

- For simplicity consider a collinear (c_n) and a soft (s) mode

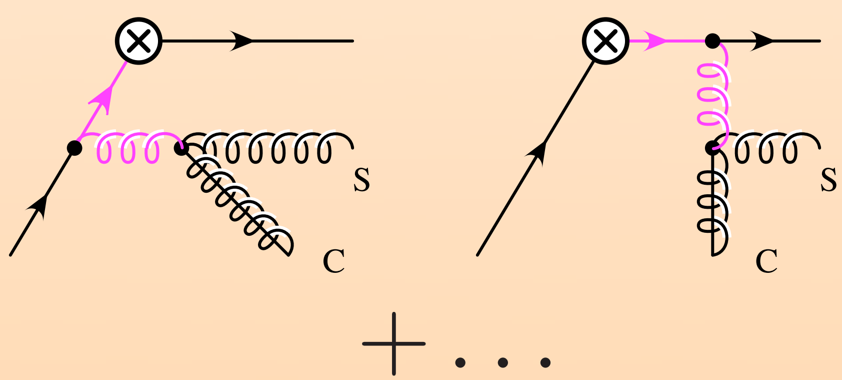
We can construct operators directly from QCD by integrating out the offshell modes

$$q = q_s + q_n \sim Q(\lambda, 1, \lambda) \quad \text{in h.c.} \qquad q^2 \sim Q\lambda \gg \Lambda^2$$



builds up $\bar{\xi}_n \, S_n^\dagger \, \Gamma \, W \, q_s$

soft Wilson line



switches order $\bar{\xi}_n \, W \, \Gamma \, S_n^\dagger \, q_s$

Soft & Collinear Gauge Invariant

Soft-Collinear Factorization

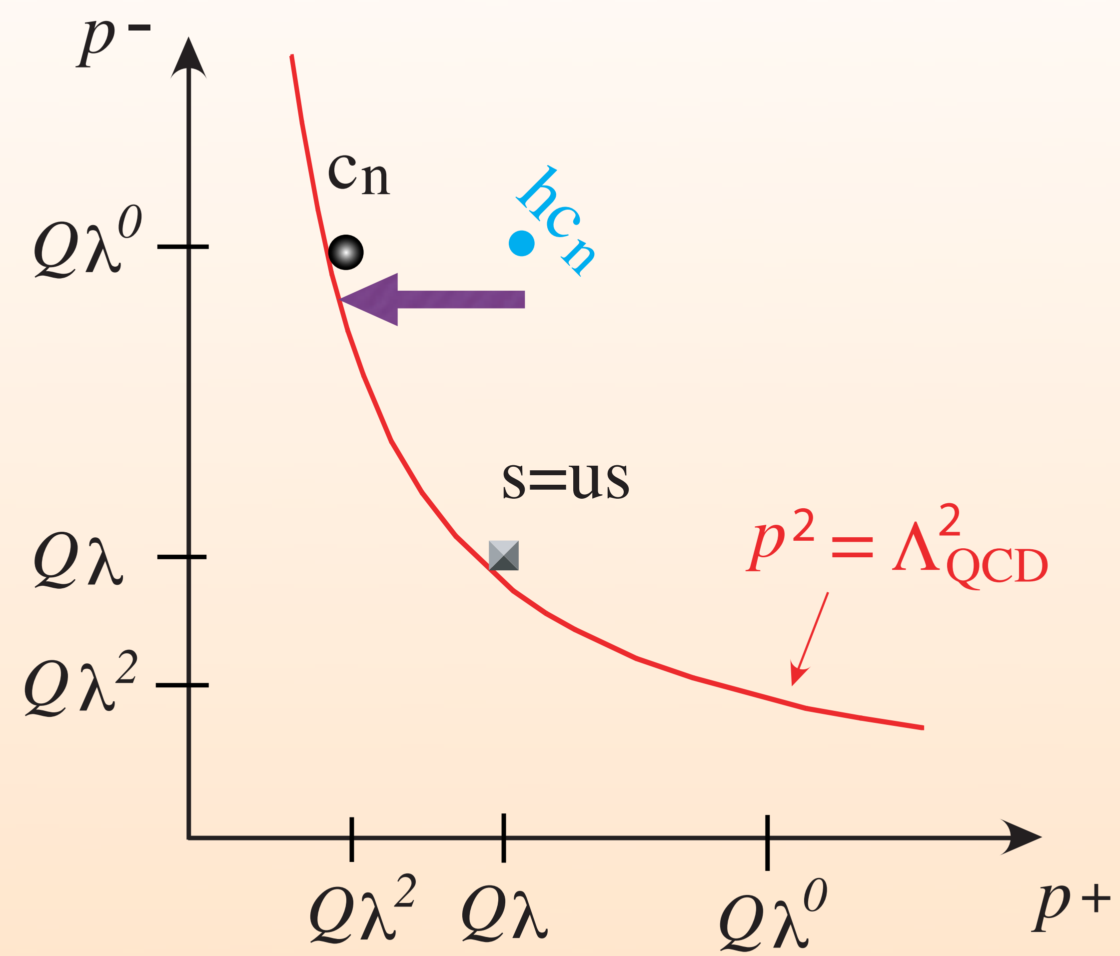
A Simpler Method: use factorization in SCETɪ

1) Match QCD onto $\mathrm{SCET_I}$

2) Factorize usoft with field redefinition

3) Match onto $\mathrm{SCET_{II}}$

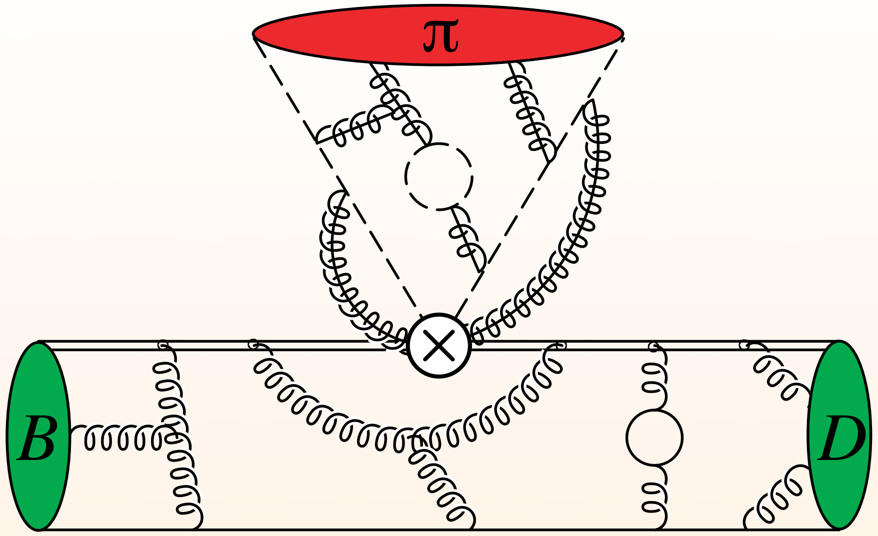
$\{\mathrm{hc}_n, \mathrm{us}\} \longrightarrow \{\mathrm{c}_n, \mathrm{s}\}$



eg.
$$J = (\bar{\xi}_n W)\Gamma h_v$$
$$= (\bar{\xi}_n W)\Gamma(Y_n^\dagger h_v)$$
$$\longrightarrow J = (\bar{\xi}_n W)\Gamma(S_n^\dagger h_v)$$

In this matching, the power of λ can only increase and does so due to change in scaling to uncontracted fields

Exclusive Example $B \to D\pi^-$



Steps

- Match at $\mu^2 \sim Q^2$ onto SCET$_\mathrm{I}$ [Decouple $\xi \to Y\xi^{(0)}$]

$$\left.\begin{matrix}[\bar c\, b]\,[\bar d\, u] \\ [\bar c\, T^A b]\,[\bar d\, T^A u]\end{matrix}\right\} \Longrightarrow \left\{\begin{matrix}[\bar h_{v'}^{(c)}\ h_v^{(b)}]\,[\bar\xi_{n,p'}^{(0)} W^{(0)} C_0(\bar{\mathcal{P}}_+) W^{(0)\dagger}\xi_{n,p}^{(0)}] \\ [\bar h_{v'}^{(c)}\ Y T^A Y^\dagger\, h_v^{(b)}]\,[\bar\xi_{n,p'}^{(0)} W^{(0)} C_8(\bar{\mathcal{P}}_+) T^A W^{(0)\dagger}\xi_{n,p}^{(0)}]\end{matrix}\right.$$

- Match at $\mu^2 \sim Q\Lambda$ onto SCET$_\mathrm{II}$

Factorized!

$$[\bar h_{v'}^{(c)}\ h_v^{(b)}]\,[\bar\xi_{n,p'} W C_0(\bar{\mathcal{P}}_+) W^\dagger \xi_{n,p}]$$

$$[\bar h_{v'}^{(c)}\ S T^A S^\dagger\, h_v^{(b)}]\,[\bar\xi_{n,p'} W C_8(\bar{\mathcal{P}}_+) T^A W^\dagger \xi_{n,p}]$$

⟵ octet m.elt. will vanish

- Take matrix elements

$$\langle \pi_n | \bar\xi_{n,p'}^{(0)} W^{(0)} C_0(\bar{\mathcal{P}}_+) W^{(0)\dagger}\xi_{n,p}^{(0)} | 0\rangle = \frac{i}{2} f_\pi E_\pi \int dx\, C[2E_\pi(2x-1)]\phi_\pi(x)$$

$$\langle D_{v'} | \bar h_{v'} \Gamma_h h_v | B_v\rangle = F^{B\to D}(0)$$

$$\boxed{\langle D\pi | \bar c b \bar u d | B\rangle = N\, F^{B\to D} \int_0^1 dx\, T(x,\mu)\, \phi_\pi(x,\mu)}$$

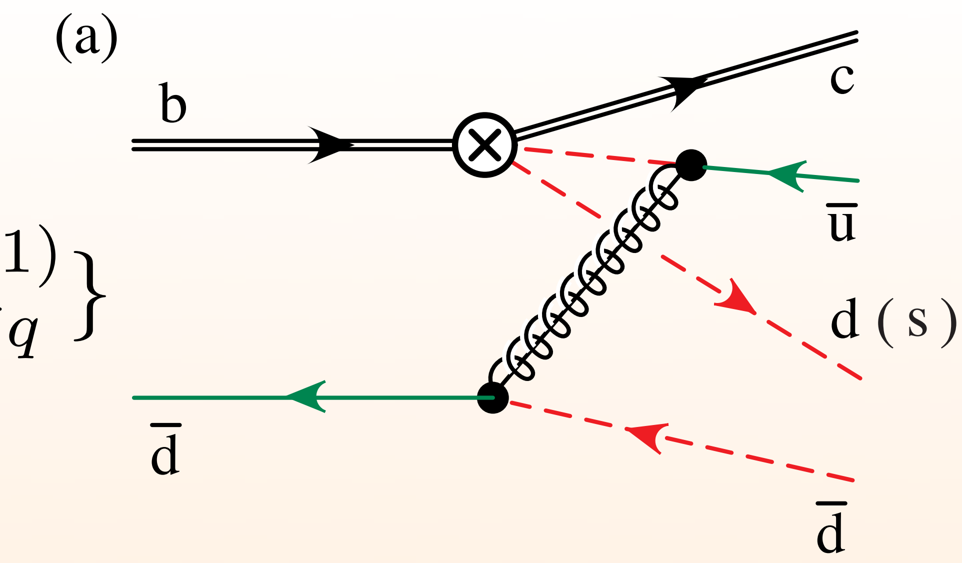
+ power corrections

Power Corrections & Color Suppressed Decays

$$\bar{B}^0 \to D^0 \pi^0 \; ,$$
$$\bar{B}^0 \to D^{*0} \pi^0$$

$$T\{O^{(0)} \mathcal{L}^{(1)}_{\xi q} \mathcal{L}^{(1)}_{\xi q}\}$$

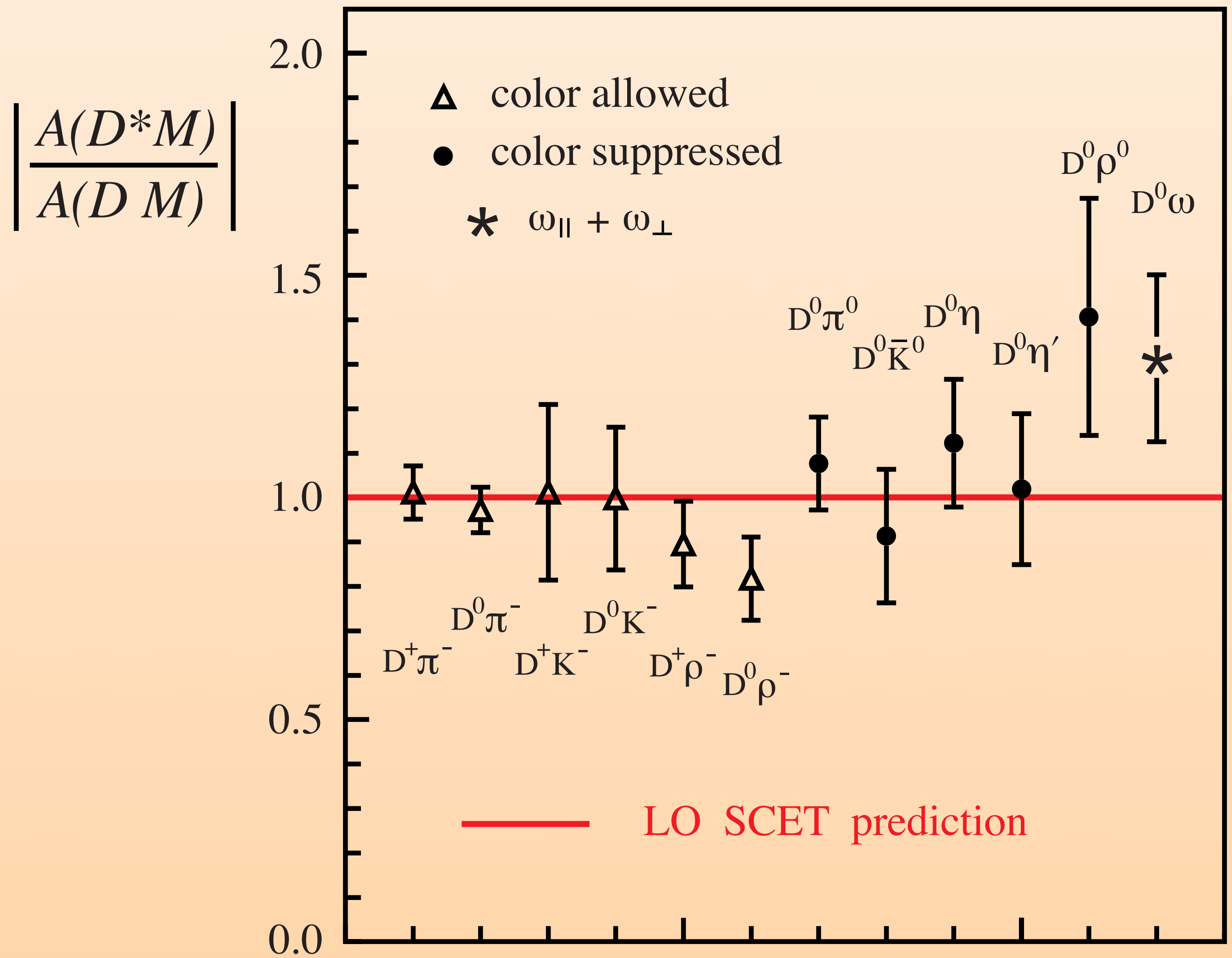
$$\mathcal{L}^{(1)}_{\xi q} = (\bar{q} Y) i g \not{B}^{\perp}_{n,\omega'} \chi_n$$



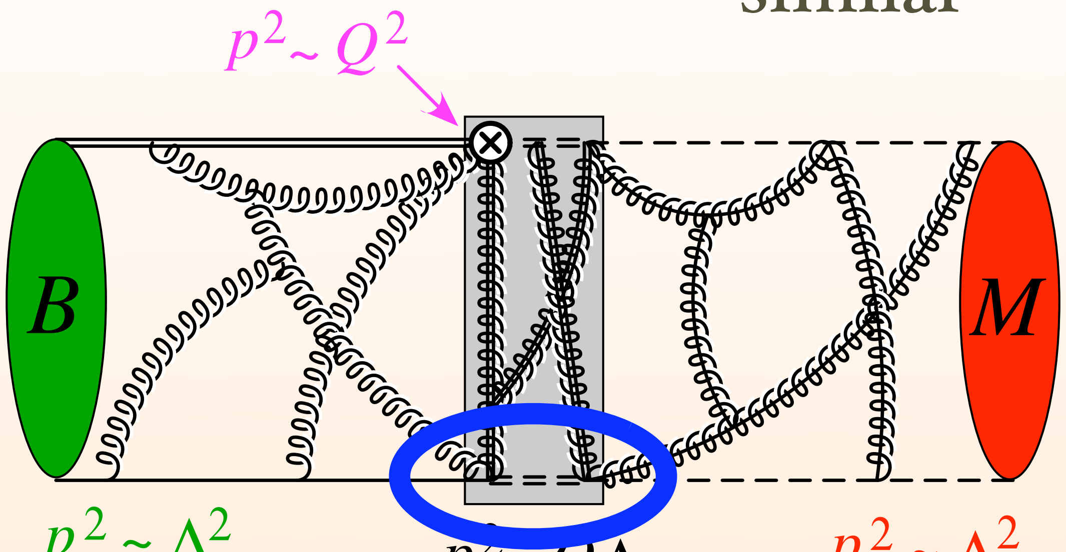
$$A^{D^{(*)}}_{00} = N^{(*)}_0 \int dx\, dz\, dk_1^+ dk_2^+ \; T^{(i)}(z) \; J^{(i)}(z, x, k_1^+, k_2^+) \; S^{(i)}(k_1^+, k_2^+) \; \phi_M(x)$$

Comparison to Data

$$\delta(D\pi) \;=\; 30.4 \pm 4.8^\circ$$
$$\delta(D^*\pi) \;=\; 31.0 \pm 5.0^\circ$$



Another Exclusive Example $B \to \pi \ell \bar{\nu}$ $(B \to \pi\pi)$ similar



Requires a power suppressed interaction

SCETI

needs time-ordered products of

$$Q^{(0)} = \bar{\chi}_{n,\omega} \Gamma \mathcal{H}_v^n$$

$$Q^{(1)} = \bar{\chi}_{n,\omega} i g \mathcal{B}^{\perp}_{n,\omega'} \Gamma \mathcal{H}_v^n$$

with

$$\mathcal{L}^{(1)}_{\xi q} = (\bar{q} Y) i g \mathcal{B}^{\perp}_{n,\omega'} \chi_n \quad ,\ldots$$

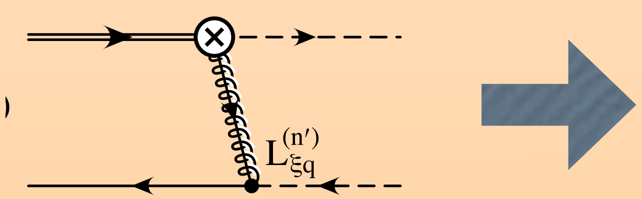
$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E) + C(E)\, \zeta^{BM}(E)$$

same functions in $B \to \pi\pi$
universality at $E\Lambda$

SCETII (further factorization)

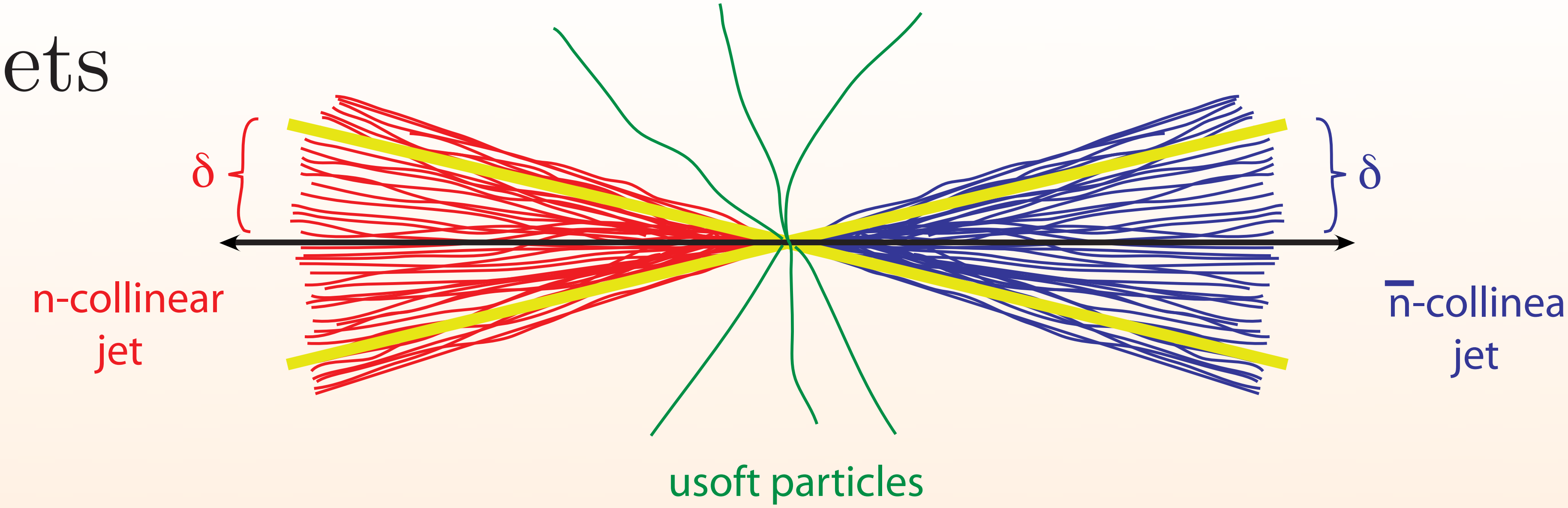
$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z,x,k^+,E) \phi_M(x) \phi_B(k^+)$$

$\zeta^{BM} = ?$ has endpoint singularities



$$\int_0^1 dx\, \frac{\phi_\pi(x)}{x^2}$$

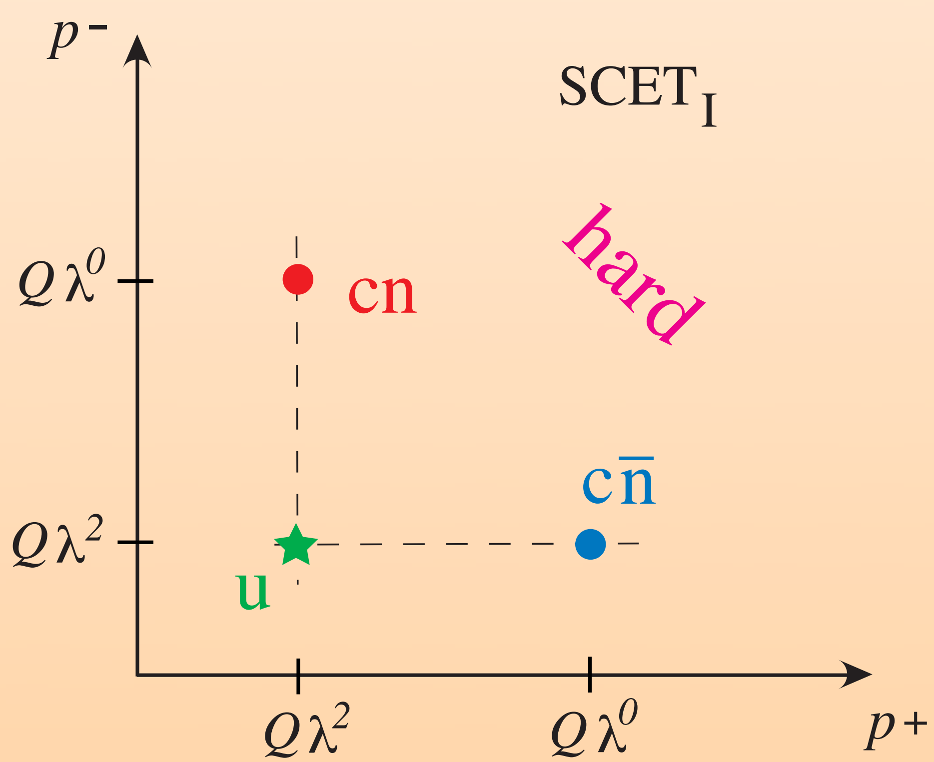
eg. $e^+e^- \to 2$ jets



event shapes in two jet region

$$\frac{d\sigma}{de} = \frac{1}{Q^2} \sum_X L_{\mu\nu} \langle 0 | J^{\dagger\nu}(0) | X \rangle \langle X | J^{\mu}(0) | 0 \rangle \delta(e - e(X)) \delta^4(q - p_X)$$

$\mathrm{SCET_I}$

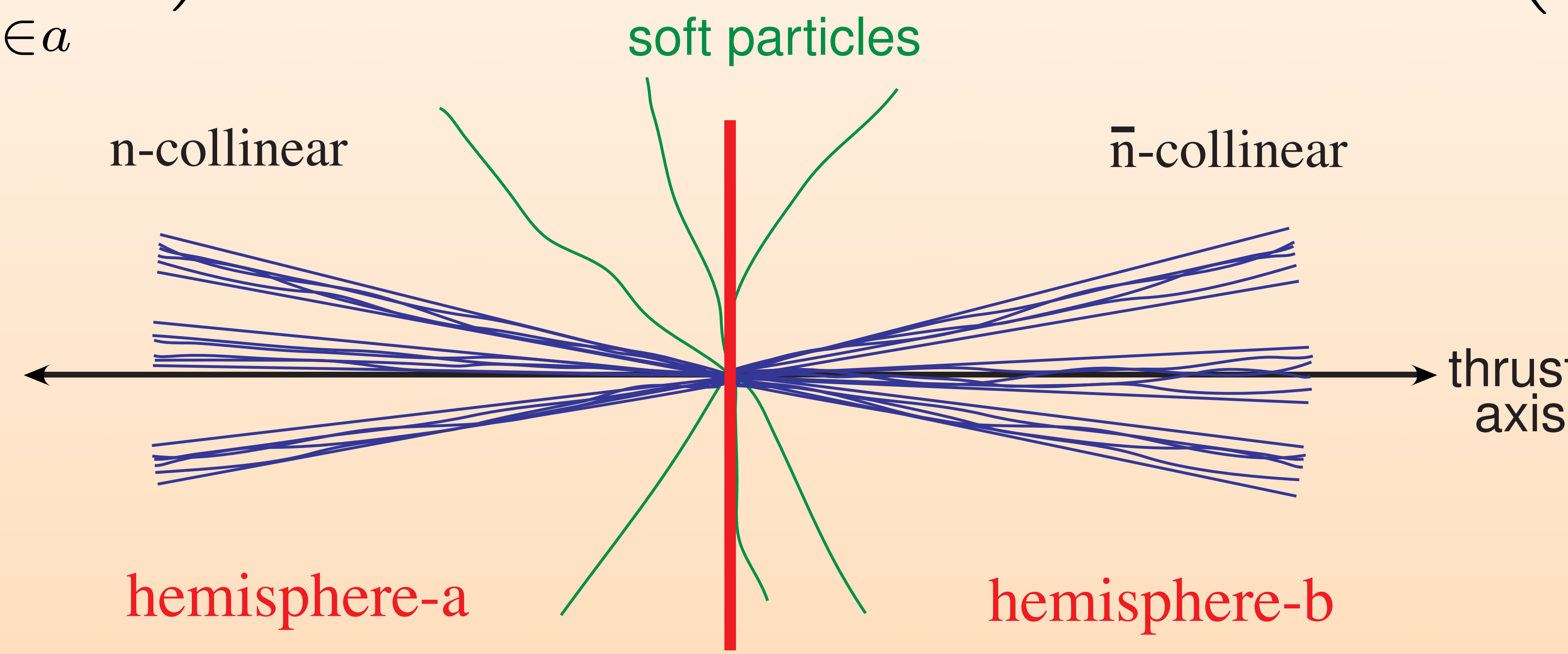


$$|X\rangle = |X_n X_{\bar{n}} X_{us}\rangle$$

What observable? $\dfrac{d^2\sigma}{dM^2\, d\overline{M}^2}$

Hemisphere Invariant Masses

$$M^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \qquad \overline{M}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$



Dijet region: $M^2, \overline{M}^2 \ll Q^2$

Let: $s \equiv M^2$

$\bar{s} \equiv \overline{M}^2$

In QCD: The full cross-section is

a restricted set of states: $s \equiv M^2 \ll Q^2$

$$\sigma = \sum_X^{res.} (2\pi)^4 \, \delta^4(q - p_X) \sum_{i=a,v} L^i_{\mu\nu} \, \langle 0 | \mathcal{J}_i^{\nu\dagger}(0) | X \rangle \langle X | \mathcal{J}_i^{\mu}(0) | 0 \rangle$$

lepton tensor, γ & Z exchange

by using EFT's we will be able to move these restrictions into the operators

In SCET:

$$\mathcal{J}_i^{\mu}(0) = \int d\omega \, d\bar{\omega} \, C(\omega, \bar{\omega}, \mu) J_i^{(0)\mu}(\omega, \bar{\omega}, \mu)$$

Wilson coefficient

SCET current

$$(\bar{\xi}_n W_n)_\omega \, Y_n^\dagger \Gamma^\mu Y_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}}$$

$$\equiv \bar{\chi}_{n,\omega} \, Y_n^\dagger \Gamma^\mu Y_{\bar{n}} \chi_{\bar{n},\bar{\omega}}$$

Momentum conservation:

$$\to C(Q, Q, \mu)$$

SCET cross-section:

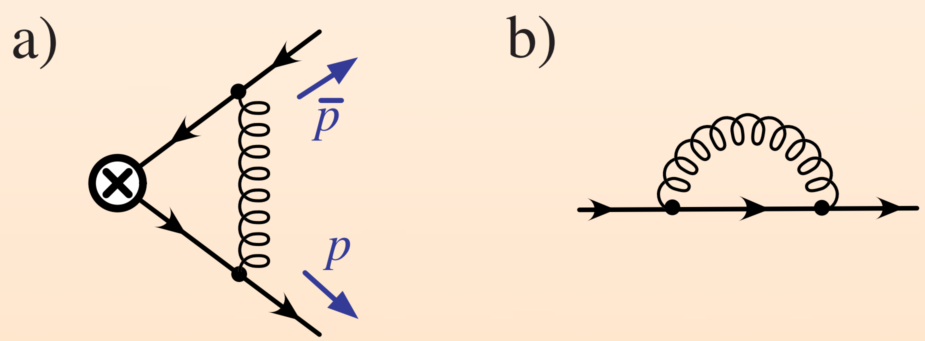
$$|X\rangle = |X_n X_{\bar n} X_s\rangle$$

$$\sigma = K_0 \sum_{\vec n} \sum_{X_n X_{\bar n} X_s}^{res.!} (2\pi)^4\, \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \langle 0|\overline{Y}_{\bar n}\, Y_n|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger|0\rangle$$

$$\times\ |C(Q,\mu)|^2\, \langle 0|\bar{n}\!\!\!/\,\chi_{n,\omega'}|X_n\rangle\langle X_n|\overline{\chi}_{n,\omega}|0\rangle \langle 0|\overline{\chi}_{\bar n,\bar\omega'}|X_{\bar n}\rangle\langle X_{\bar n}|n\!\!\!/\,\chi_{\bar n,\bar\omega}|0\rangle$$

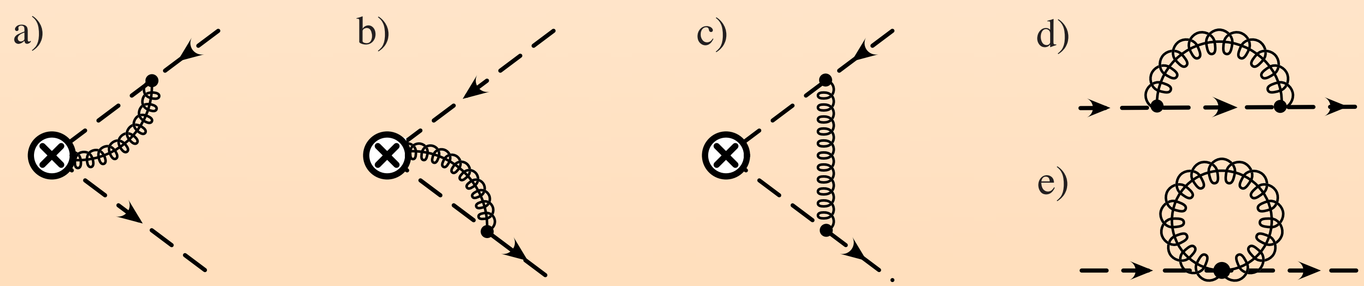
all-orders

QCD



one-loop

SCET



difference gives one-loop matching:

$$C(Q,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3\log\frac{-Q^2 - i0}{\mu^2} - \log^2\frac{-Q^2 - i0}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

Specify hemisphere invariant masses for the jets:

total soft momentum is the sum of momentum in each hemisphere

$$K_{X_s} = k_s^a + k_s^b \qquad \hat{P}_a\,|X_s\rangle = k_s^a\,|X_s\rangle, \quad \hat{P}_b\,|X_s\rangle = k_s^b\,|X_s\rangle$$

hemisphere projection operators

Insert: $$1 = \int ds\, \delta\Big((p_n + k_s^a)^2 - s\Big) \int d\bar{s}\, \delta\Big((p_{\bar{n}} + k_s^b)^2 - \bar{s}\Big)$$

expand: $$\delta\Big((p_n + k_s^a)^2 - s\Big) = \frac{1}{Q}\,\delta\Big(k_n^+ + k_s^{+a} - \frac{s}{Q}\Big)$$

$$\delta\Big((p_{\bar{n}} + k_s^b)^2 - \bar{s}\Big) = \frac{1}{Q}\,\delta\Big(k_n^- + k_s^{-b} - \frac{\bar{s}}{Q}\Big)$$

... Some Algebra ...

$$\frac{d^2\sigma}{ds\,d\bar{s}} = \frac{\sigma_0}{Q^2}\left|C(Q,\mu)\right|^2 \int dk_n^+\, dk_{\bar{n}}^-\, d\ell^+\, d\ell^-\, \delta\Big(k_n^+ + \ell^+ - \frac{s}{Q}\Big)\delta\Big(k_{\bar{n}}^- + \ell^- - \frac{\bar{s}}{Q}\Big)$$

$$\times \sum_{X_n} \frac{1}{2\pi}\int d^4x\, e^{ik_n^+ x^-/2}\ \mathrm{tr}\langle 0|\bar{\not{n}}\chi_n(x)|X_n\rangle\langle X_n|\bar{\chi}_{n,Q}(0)|0\rangle$$

$$\times \sum_{X_{\bar{n}}} \frac{1}{2\pi}\int d^4y\, e^{ik_{\bar{n}}^- y^+/2}\ \mathrm{tr}\langle 0|\overline{\chi}_{\bar{n}}(y)|X_{\bar{n}}\rangle\langle X_{\bar{n}}|\not{n}\chi_{\bar{n},-Q}(0)|0\rangle$$

$$\times \sum_{X_s} \frac{1}{N_c}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\mathrm{tr}\langle 0|\overline{Y}_{\bar{n}}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar{n}}^\dagger(0)|0\rangle$$

Factorization Theorem:

$$\frac{d^2\sigma}{ds\,d\bar{s}} = \sigma_0\, H_Q(Q,\mu)\int_{-\infty}^{+\infty} d\ell^+ d\ell^-\ J_n(s - Q\ell^+,\mu)\, J_{\bar{n}}(\bar{s} - Q\ell^-,\mu)\, S_{\mathrm{hemi}}(\ell^+,\ell^-,\mu)$$

Hard Function

$H_Q(Q,\mu) = |C(Q,\mu)|^2$

Quark Jet Function

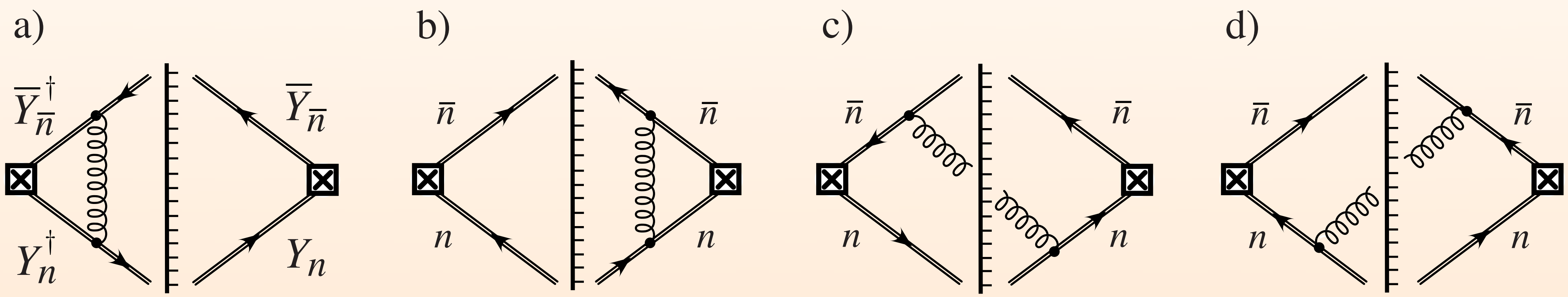
Anti-quark Jet Function

Soft radiation Function

universal

$S_{\rm hemi}(\ell^+,\ell^-,\mu)$

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$



Soft function is perturbative if $\ell^+,\ell^- \gg \Lambda_{\rm QCD}$

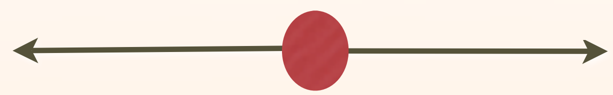
and is nonperturbative if $\ell^+,\ell^- \sim \Lambda_{\rm QCD}$

It is also universal, it appears in many different event shapes (thrust, heavy-jet mass, ...) for both massless and massive jets

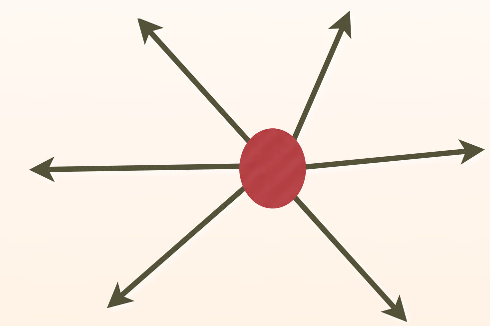
A very popular event shape is thrust $T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$

Thrust

$T = 1$ dijet



$T = \frac{1}{2}$



Insert: $$1 = \int dT\, \delta\Big(1 - T - \frac{s + \bar{s}}{Q^2}\Big)$$

Factorization theorem

$$\frac{d\sigma}{dT} = \sigma_0\, H(Q,\mu) \int ds\; J_T(s,\mu)\; S_{\rm thrust}\Big(Q(1-T) - \frac{s}{Q}, \mu\Big)$$

with $$S_{\rm thrust}(\ell,\mu) = \int_0^\infty d\ell^+ d\ell^-\; \delta(\ell - \ell^+ - \ell^-)\; S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

SCET is a field theory which:

- explains how soft & collinear degrees of freedom communicate with each other, and with hard interactions
- organizes the interactions in a series expansion in λ

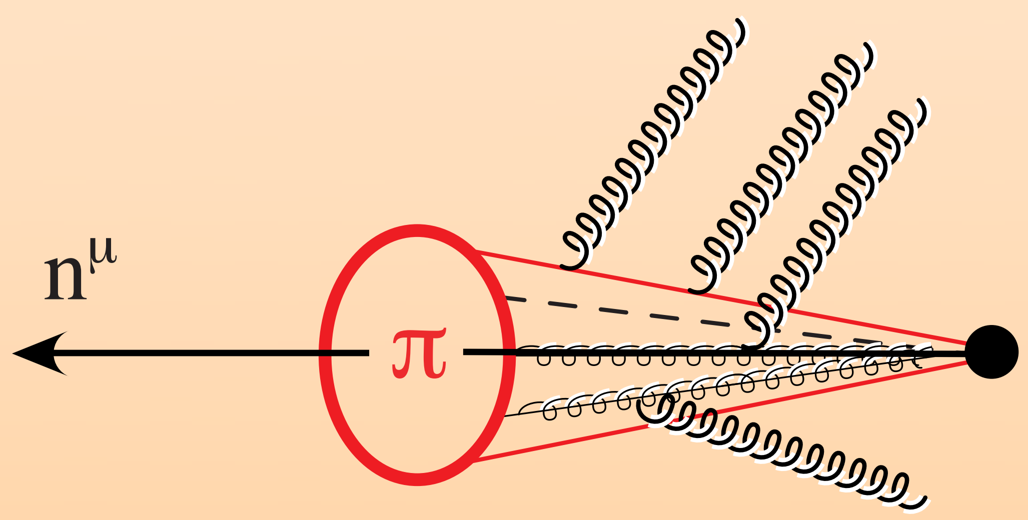
  which measures how collinear/soft the particles are

$$\lambda = \sqrt{\frac{\Lambda_{QCD}}{m_b}} \qquad \lambda = \frac{\Lambda_{QCD}}{m_b} \qquad \lambda^2 = \frac{m_X^2}{Q^2}$$

- provides a simple operator language to derive factorization theorems in fairly general circumstances

  eg. unifies the treatment of factorization for exclusive and inclusive QCD processes

- results are constrained by symmetries
- scale separation & decoupling



How is SCET used?

- cleanly separate short and long distance effects in QCD
  - ➞ derive new factorization theorems
  - ➞ find universal hadronic functions, exploit symmetries
  - ➞ predict decay rates and cross sections
- model independent, systematic expansion
  - ➞ study power corrections
- keep track of μ dependence
  - ➞ sum large logarithms

The End