Baryon Chiral Perturbation Theory: Exercises

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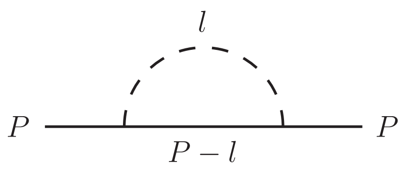
Problem 1



(a) Consider the nucleon self-energy diagram (full line: nucleon, dashed line: pion) in dimensional regularisation, neglecting all momentum-dependent vertices and Dirac structures, i.e. just the scalar loop integral

$$I(P^2) = \frac{1}{i}\int \frac{d^d l}{(2\pi)^d} \frac{1}{(M^2 - l^2)(m^2 - (P-l)^2)} , \tag{1}$$

where $m = m_N$ and $M = M_\pi$ are the nucleon and pion masses, respectively. Evaluate $I(m^2)$ (corresponding to the mass renormalisation) explicitly. What power in $\alpha = M/m$ would you expect $I(m^2)$ to have according to naive power counting arguments? Show that this expectation is not met.

<u>Hints</u>:

The standard formula for dimensional regularisation is

$$\frac{1}{i}\int \frac{d^d l}{(2\pi)^d} \frac{1}{(\Delta - l^2)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(n - d/2)}{\Gamma(n)} \Delta^{d/2-n} .$$

You may furthermore find the following integral useful:

$$\int_0^1 dx \log\big(\alpha^2(1-x) + x^2\big) = \alpha\sqrt{4-\alpha^2}\arctan\frac{\sqrt{4-\alpha^2}}{\alpha} + \alpha^2\log\alpha - 2 .$$

(b) Now do the same integral (in the special kinematics needed for mass renormalisation) in the non-relativistic (or heavy-baryon) limit: with $P \to m\,v + \mathcal{O}(m^0)$, $v^2 = 1$, replace the nucleon propagator according to

$$m^2 - (P-l)^2 \to 2m\,v\cdot l + \ldots . \tag{2}$$

<u>Hint</u>:

A maybe slightly less familiar Feynman parametrisation useful here is

$$\frac{1}{AB} = \int_0^\infty dx \frac{2}{(A + 2xB)^2} .$$

Show that the integral now obeys naive power counting rules. Why does the result have to be finite (i.e. non-divergent for $d \to 4$)? Compare the leading non-analytic (in the *quark* mass) pieces in parts (a) and (b).

Problem 2

The theoretical description of semileptonic hyperon decays $B \to B' e^- \bar{\nu}_e$ requires knowledge of the hadronic matrix elements

$$\langle B'(p_2)|V_\mu|B(p_1)\rangle = \bar{u}(p_2)\big\{\gamma_\mu F_1^{BB'}(t) + \ldots\big\}u(p_1) \ , \quad t = (p_1 - p_2)^2 \ , \tag{3}$$

with $V_\mu = \bar{u}\gamma_\mu s$. [There is also a piece due to the axial current $A_\mu = \bar{u}\gamma_\mu\gamma_5 s$ which we do not consider here.] The part of the total decay width due to the vector current is given by

$$\Gamma_V \;=\; G_F^2\,|V_{us}|^2\,\frac{\Delta m^5}{60\pi^3}\left\{\Big(1-\frac{3}{2}\beta\Big)|F_1(0)|^2 + \mathcal{O}(\beta^2)\right\} ,$$

where $\Delta m = m_1 - m_2$, $\beta = \Delta m/m_1$, $m_{1/2}$ are the masses of B and B', respectively. I.e. up to next-to-leading order in the small parameter β (what is the order of β in the chiral power counting?), Γ_V is determined by the form factor F_1 at vanishing momentum transfer; momentum dependence and other form factors (indicated by the ellipses in Eq. (3)) are suppressed. Obviously, only if you know $F_1(0)$ with sufficient precision, you can hope to extract $|V_{us}|$ from semileptonic hyperon decays.

In the SU(3) symmetry limit, the "charges" $F_1^{BB'}(0)$ are given by simple Clebsch-Gordan coefficients $g_V^{BB'}$,

$$g_V^{\Lambda p} = -\sqrt{\frac{3}{2}}\ , \quad g_V^{\Sigma^0 p} = -\frac{1}{\sqrt{2}}\ , \quad g_V^{\Sigma^- n} = -1\ , \quad g_V^{\Xi^-\Lambda} = \sqrt{\frac{3}{2}}\ , \quad g_V^{\Xi^-\Sigma^0} = \frac{1}{\sqrt{2}}\ , \quad g_V^{\Xi^0\Sigma^+} = 1\ .$$

The *Ademollo–Gatto theorem* guarantees that there are no corrections to the SU(3) limit *linear* in the symmetry-breaking parameter,

$$F_1^{BB'}(0) = g_V^{BB'}\Big\{1 + \mathcal{O}\big((m_s - \hat{m})^2\big)\Big\} \tag{4}$$

(we assume isospin symmetry, $\hat{m} = m_u = m_d$).

(a) Imagine you perform a chiral calculation of the symmetry-breaking corrections in $F_1(0)$. Draw typical tree and one-loop diagrams for this calculation (i.e., just with a few example intermediate states from the meson and baryon octets).

<u>*Remark:*</u>

The meson-baryon couplings in SU(3) are determined by the axial vector coupling constants D, F in analogy to the Goldberger–Treiman relation $g_{\pi N} = g_A m_N/F_\pi$. The latter are known to reasonable accuracy, $D \approx 0.80$, $F \approx 0.46$.

(b) At what order in the chiral power counting would you find *analytic/polynomial* contributions to the symmetry-breaking terms in $F_1(0)$? What loop order does this correspond to?

(c) At what order in the chiral power counting do you expect the *leading* contributions? What is the structure of those contributions? To how many orders can chiral perturbation theory therefore *predict* the symmetry-breaking terms in $F_1(0)$? [You can safely assume the low-energy constants accompanying the polynomial terms from (b) to be unknown.]