Baryon Chiral Perturbation Theory

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Outline (1)

Part I: Basics

- construction of the meson-baryon Lagrangian
- power counting and its failure
- heavy-baryon ChPT and infrared regularisation

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Part II: πN σ-term and strangeness in the nucleon (1)

- quark mass dependence of the nucleon mass
- sigma term and πN scattering

Outline (2)

Part III: Strangeness in the nucleon (2)

- Strangeness form factors and parity-violating $e^- p$ scattering
- Chiral perturbation theory: a failure

Part IV: Isospin-violating form factors

- Why? isospin violation and strangeness
- Chiral perturbation theory: a success

Part VI: Crimes and omissions

- The role of the $\Delta(1232)$ resonance
- Two- and more-nucleon systems

Questions, anyone?

I reserve the use of irony without warning.

Part I: Basics

Chiral perturbation theory with baryons

- H. Leutwyler's lectures: ChPT for (pseudo) Goldstone bosons
- perhaps the most important extension: inclusion of nucleons [chiral SU(2)] / the baryon ground state octet [SU(3)]
- objective: understand the properties of nucleons
- long-term goal: put nuclear physics on a more systematic footing

Chiral perturbation theory with baryons

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- objective: understand the properties of nucleons
- long-term goal: put nuclear physics on a more systematic footing
- problem (as we shall see later):
  nucleon mass is a new, heavy mass scale that does not vanish in the chiral limit,

$$\lim_{m_q \to 0} m_N \approx m_N$$

- idea: view nucleons as (massive) matter fields coupled to pions and external sources
  3-momenta ought to remain small $\sim M_\pi$
- number of baryons conserved
  no baryon–antibaryon creation / annihilation
  here: consider only processes with 1 baryon

Construction of the meson–baryon Lagrangian (1)

- proceed as before:
  - ▷ choose suitable representation / transformation law for baryons under $\mathsf{SU}(N)_L \times \mathsf{SU}(N)_R$
  - ▷ construct effective Lagrangians according to increasing number of momenta
- remember: $U \longmapsto V_L U V_R^\dagger$ for the Goldstone boson fields
- introduce a new field u according to $u^2 = U$

$$
\begin{aligned}
u \longmapsto \sqrt{V_L U V_R^\dagger} &= V_L\, u\, K^\dagger(V_L, V_R, U) \\
&= K(V_L, V_R, U)\, u\, V_R^\dagger
\end{aligned}
$$

$K(V_L, V_R, U) \in \mathsf{SU}(N)$ compensator field

- for $\mathsf{SU}(N)_V$ transformations ($V_L = V_R$), reduces to $K(V_L, V_R, U) = V_L = V_R$

Construction of the meson–baryon Lagrangian (2)

- particularly convenient representation for nucleons / baryons:

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad \longmapsto \quad \psi' = K\psi \qquad [\mathsf{SU}(2)]$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} \quad \longmapsto \quad B' = KBK^{-1} \quad [\mathsf{SU}(3)]$$

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- introduce a covariant derivative:

$$D^\mu = \partial^\mu + \Gamma^\mu = \partial^\mu + V^\mu$$

Γ^μ chiral connection (vector)

$$\Gamma^\mu = \frac{1}{2}\left(u^\dagger(\partial^\mu - i\,r^\mu)u + u(\partial^\mu - i\,l^\mu)u^\dagger\right)$$

transforms according to

$$\Gamma^\mu \longmapsto K\Gamma^\mu K^\dagger - (\partial^\mu K)K^\dagger$$

such that

$$D^\mu\psi \longmapsto KD^\mu\psi$$

Construction of the meson–baryon Lagrangian (3)

- chiral vielbein (axial vector)

$$u^\mu = i\left(u^\dagger(\partial^\mu - i\,r^\mu)u - u(\partial^\mu - i\,l^\mu)u^\dagger\right)$$

transforms according to

$$u^\mu \longmapsto K u^\mu K^\dagger$$

- finally, rewrite quark mass term $\chi = 2B(s + i\,p) = 2B\mathcal{M} + \ldots$

$$\chi_+ = u^\dagger \chi u^\dagger + u\chi^\dagger u$$

such that

$$\chi_+ \longmapsto K\chi_+ K^\dagger$$

$\Rightarrow$ everything transforms in the same way

- power counting: $\Gamma^\mu,\ u^\mu = \mathcal{O}(p)$, $\chi_+ = \mathcal{O}(p^2)$

The leading-order chiral meson-baryon Lagrangian

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U\chi^\dagger\rangle$$

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$$\mathcal{L}_{\pi N}^{(1)} = \bar\psi\left(i\gamma_\mu D^\mu - m + \frac{g_A}{2}\gamma_\mu\gamma_5 u^\mu\right)\psi$$

$$\mathcal{L}_{\phi B}^{(1)} = \langle \bar B\,(i\gamma_\mu D^\mu - m)\,B\rangle + \frac{D}{2}\langle \bar B\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \frac{F}{2}\langle \bar B\gamma_\mu\gamma_5[u^\mu, B]\rangle$$

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Remarks:

- $i\gamma_\mu D^\mu$, $m = \mathcal{O}(p^0)$ both individually large, *but* $i\gamma_\mu D^\mu - m = \mathcal{O}(p)$
- in meson ChPT, Lagrangians came in even powers of momenta (Lorentz invariance)
- here, due to spin (Dirac structures), odd powers possible:

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \ldots$$

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new parameters:

- m: nucleon (baryon) mass in the chiral limit
- g_A: expand $u_\mu = 2a_\mu + \mathcal{O}(\pi) \quad\Rightarrow$ axial vector coupling known from neutron beta decay, $g_A = 1.26$
- D/F: two axial vector couplings in SU(3), can be determined from semileptonic hyperon decays, SU(2) constraint $D + F = g_A$ ($D \approx 0.80, F \approx 0.46$)

Goldberger-Treiman relation

- setting $v_\mu = a_\mu = 0$, the chiral vielbein is $u_\mu = -\frac{\partial_\mu \pi}{F_\pi} + \mathcal{O}(\pi^3)$

  find the $\pi N N$ vertex

$$\mathcal{L}_{\pi N}^{(1)} \to -\frac{g_A}{2F_\pi} \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \pi \psi$$

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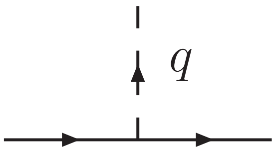
- Feynman rule:



$$\frac{g_A}{2F_\pi}\not{q}\gamma_5\vec{\tau} = V_{\pi NN}$$

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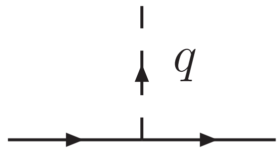

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- $N \to \pi N$ transition amplitude:

$$T_{\pi NN} = -i\bar{u}(p')V_{\pi NN}u(p) = -i\frac{g_A m_N}{F_\pi}\bar{u}(p')\gamma_5 u(p)\vec{\tau}$$

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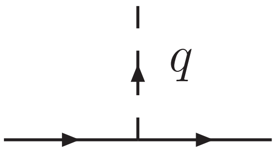
  find the $\pi N N$ vertex

$$\mathcal{L}^{(1)}_{\pi N} \to -\frac{g_A}{2F_\pi}\bar{\psi}\gamma^\mu\gamma_5\partial_\mu\pi\psi$$

- Feynman rule: 

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- compared to the canonical amplitude $-\,i\,g_{\pi N}\bar{u}(p')\gamma_5 u(p)\vec{\tau}$:

$$\boxed{g_{\pi N} = \frac{g_A m_N}{F_\pi}}$$

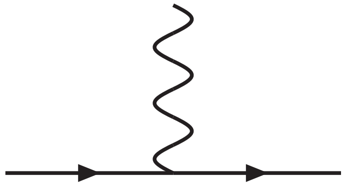
  remarkable for relating weak and strong interactions
  numerically: $13.1\ldots13.4 = 12.8$

Some other familiar Feynman rules

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}\left(i\gamma_\mu D^\mu - m + \frac{g_A}{2}\gamma_\mu\gamma_5 u^\mu\right)\psi\,, \qquad D^\mu = \partial^\mu + \Gamma^\mu$$

- expand Γ_μ, u_μ in powers of the pion fields:

$$\begin{aligned} i\,\Gamma_\mu &= v_\mu + \frac{i}{8F^2}[\pi, \partial_\mu \pi] - \frac{1}{8F^2}\left[\pi, [\pi, v_\mu]\right] + \ldots \\ u_\mu &= 2a_\mu - \frac{\partial_\mu \pi}{F} + \frac{i}{2F}[v_\mu, \pi] + \ldots \end{aligned}$$



$$ie\,\not{\epsilon}\,\frac{1}{2}(1+\tau^3)$$

γ–N coupling (electric)

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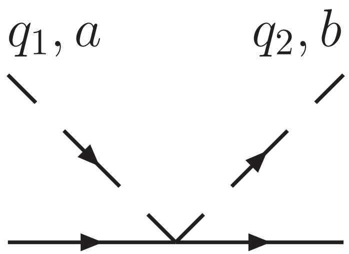

$$\frac{1}{4F^2}(\not{q}_1 + \not{q}_2)\,\epsilon^{abc}\tau^c$$ Weinberg–Tomozawa term

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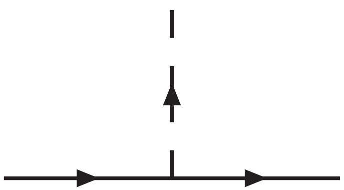

$$\frac{g_A}{2F}\,\not{q}\,\gamma_5\,\tau^a$$

(axial vector) πNN coupling

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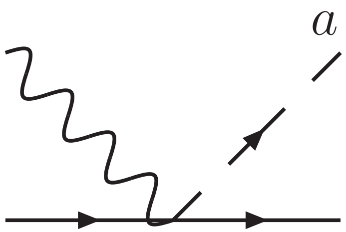

$$\frac{ieg_A}{2F}\,\not\epsilon\gamma_5\,\epsilon^{a3b}\tau^b$$

Kroll–Rudermann term

Weinberg's power counting argument for pions (1)

- consider an arbitrary loop diagram based on

$$\mathcal{L}_{\text{eff}} = \sum_d \mathcal{L}^{(d)}$$

with L loops, I internal lines, V_d vertices of order d:

$$\mathcal{A} \propto \int (d^4p)^L \frac{1}{(p^2)^I} \prod_d (p^d)^{V_d}$$

- let $\mathcal{A}$ be of chiral dimension ν

$$\nu = 4L - 2I + \sum_d dV_d$$

- use topological identity for L to eliminate I

$$L = I - \sum_d V_d + 1 \quad \Rightarrow \quad \boxed{\nu = \sum_d V_d(d-2) + 2L + 2}$$

Weinberg's power counting argument for pions (2)

$$\nu = \sum_d V_d(d-2) + 2L + 2$$

- chiral Lagrangian starts with $\mathcal{L}^{(2)}$, i.e. $d \geq 2$,
  i.e. right-hand-side is a sum of non-negative terms
  $\Rightarrow$ for fixed ν, there is only a finite number of combinations L, V_d
- $+2L$: each loop suppressed amplitude by two orders
  in the momentum expansion

Weinberg's power counting for the 1-baryon sector

- similar derivation as for the meson sector:
  chiral dimension ν of an arbitrary L-loop diagram
  with $V_d^{\pi\pi}$ meson–meson vertices of order d
  and $V_{d'}^{\pi N}$ meson–baryon vertices of order d':

$$\nu = 2L + 1 + \sum_d V_d^{\pi\pi}(d-2) + \sum_{d'} V_{d'}^{\pi N}(d'-1)$$

  note again: $d \geq 2$, $d' \geq 1$

- therefore:
  $\mathcal{O}(p^1)$, $\mathcal{O}(p^2)$: tree only
  $\mathcal{O}(p^3)$, $\mathcal{O}(p^4)$: tree + one-loop
  $\mathcal{O}(p^5)$, $\mathcal{O}(p^6)$: tree + one-loop + two-loop
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- and the problem is: it doesn't work ...

The failure of power counting in meson–baryon ChPT

- loop integrals cover all energy scales
- Goldstone boson sector: all mass scales "small"
  in a mass-independent regularisation scheme (like dimensional regularisation), naive power counting has to work
- with baryons: new mass scale $m_N \approx \Lambda_\chi \approx 1$ GeV
  loop integration picks up momenta $p \sim m_N$

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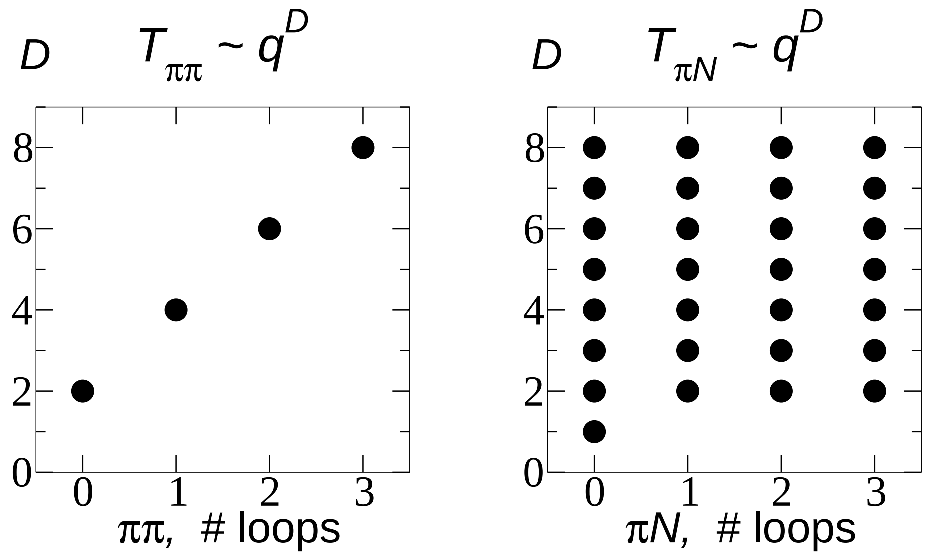
- schematically:

Gasser, Sainio, Švarc 1988



$\Rightarrow$ higher-order loops renormalise lower-order couplings

Remedies (1): Heavy-baryon ChPT

Jenkins, Manohar 1991

Bernard, Kaiser, Meißner 1995

- in close analogy to heavy-quark EFT:
  decompose baryon momentum according to

$$p_\mu = \underbrace{m_N v_\mu}_{\text{large}} + \underbrace{l_\mu}_{\text{residual}} , \quad v^2 = 1 , \quad v \cdot l \ll m_N$$

- nucleon propagator in the heavy-baryon limit:

$$\frac{1}{p^2 - m_N^2} \rightarrow \frac{1}{2m_N} \frac{1}{v \cdot l} + \mathcal{O}\left(1/m_N^2\right)$$

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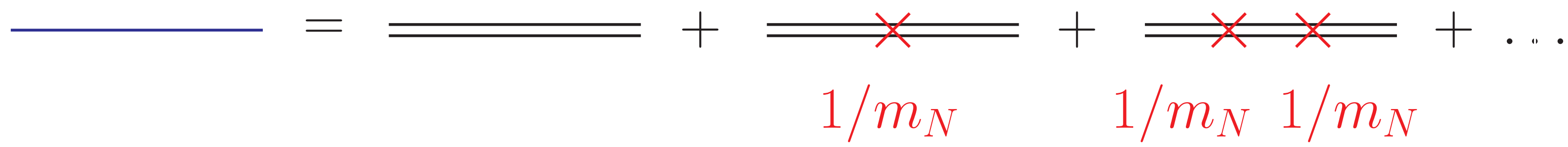

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- eliminates mass scale m_N from propagator
  re-enters as parametrical suppression factor

- two-fold expansion $\left(\frac{p}{\Lambda_\chi}\right)^n, \left(\frac{p}{m_N}\right)^n$ $(m_N \approx \Lambda_\chi)$

Heavy-baryon ChPT (2)

- decompose ψ into velocity eigenstates:

$$\begin{aligned} \text{"big":} \quad H_v(x) &= e^{im_N v\cdot x} P_v^+ \psi(x) \\ \text{"small":} \quad h_v(x) &= e^{im_N v\cdot x} P_v^- \psi(x) \end{aligned}$$

- projectors $P_v^{\pm} = \frac{1}{2}(1 \pm \not{v})$
- exponential "rotates away" the large mass term from time evolution of the field H_v

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- projectors $P_v^\pm = \frac{1}{2}(1 \pm \not{v})$
- exponential "rotates away" the large mass term from time evolution of the field H_v
- $\mathcal{L}_{\pi N}^{(1)}$ becomes:

$$\mathcal{L}_{\pi N}^{(1)} = \bar{H}_v \big(iv \cdot D + g_A S \cdot u \big) H_v + \mathcal{O}\big(1/m_N\big)$$

Pauli-Lubanski spin vector $S_\mu = \frac{i}{2}\gamma_5 \sigma_{\mu\nu} v^\nu$

$\Rightarrow$ nucleon mass gone from $\mathcal{L}_{\pi N}^{(1)}$

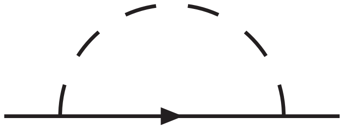
$\Rightarrow$ Dirac structure massively simplified

- $1/m_N$ corrections can be constructed systematically on Lagrangian level à la Foldy–Wouthuysen

Remedies (2): Infrared regularisation

Becher, Leutwyler 1999

consider (relativistic) nucleon self-energy graph (at threshold):



$$= \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}(d-3)} \frac{m_N^{d-3} + M_\pi^{d-3}}{m_N + M_\pi}$$

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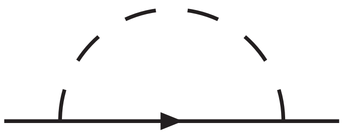


$$= \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}(d-3)} \frac{m_N^{d-3} + M_\pi^{d-3}}{m_N + M_\pi}$$

"regular" part

- fractional powers in m_N, regular in M_π, p
- violates naive power counting
- can be expanded as polynomial in M_π, p

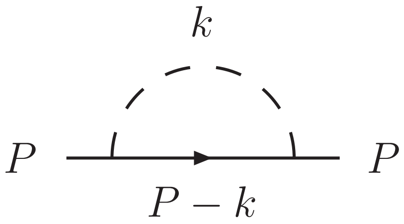
$\Rightarrow$ can be absorbed by re-definition of contact terms

"infrared" part

- fractional powers in M_π, p
- obeys power counting rules
- non-analytic terms, imaginary parts ...

$\Rightarrow$ all "interesting" loop contributions

Infrared regularisation (2)



$$a = M_\pi^2 - k^2 - i\epsilon\,, \quad b = m_N^2 - (P-k)^2 - i\epsilon$$

$$H = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{ab} = \int_0^1 dz \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{[(1-z)a + zb]^2}$$

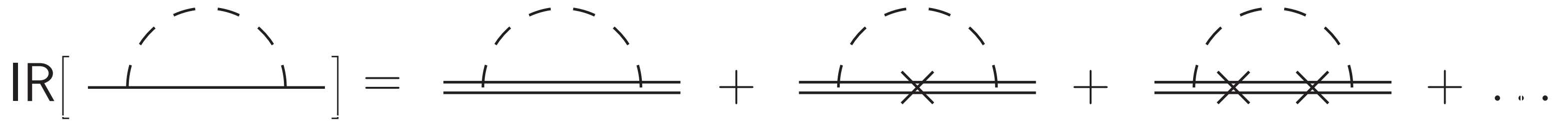
- Landau equations: singularity structure

$z = \frac{M_\pi}{m_N + M_\pi}$	leading (pinch) singularity	$\leftrightarrow$	$P^2 = (m_N + M_\pi)^2$
$z = 0$	endpoint singularity	$\leftrightarrow$	$M_\pi^2 = 0$
$z = 1$	endpoint singularity	$\leftrightarrow$	$m_N^2 = 0$

- $z = 1$ not a "low-energy" singularity
- obtain infrared part by avoiding the $z = 1$ endpoint singularity:

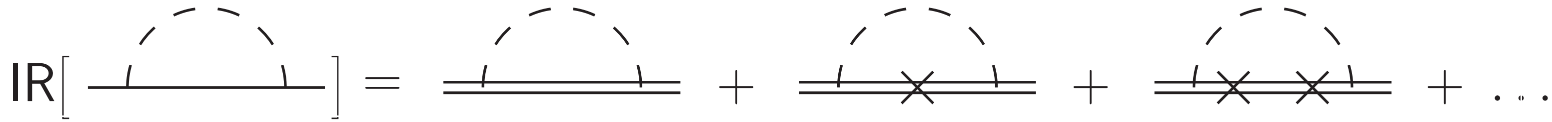
$$IR = \int_0^{\boxed{\infty}} dz \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{[(1-z)a + zb]^2}$$

Relation infrared regularisation $\leftrightarrow$ heavy-baryon

- 

  $\Rightarrow$ resummation of all $1/m_N$ corrections of a certain diagram

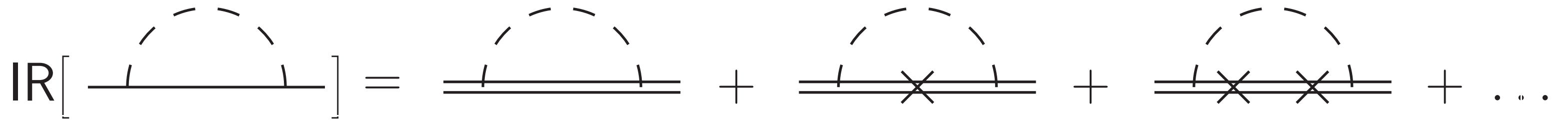
Relation infrared regularisation ↔ heavy-baryon

- 

  $\Rightarrow$ resummation of all $1/m_N$ corrections of a certain diagram

- wait — isn't this just what we did for the heavy-baryon propagator in the first place?

Relation infrared regularisation $\leftrightarrow$ heavy-baryon

- 

  $\Rightarrow$ resummation of all $1/m_N$ corrections of a certain diagram

- wait — isn't this just what we did for the heavy-baryon propagator in the first place?

- yes —
  but we interchanged summation and (highly irregular) integration
  the difference is the "regular" part

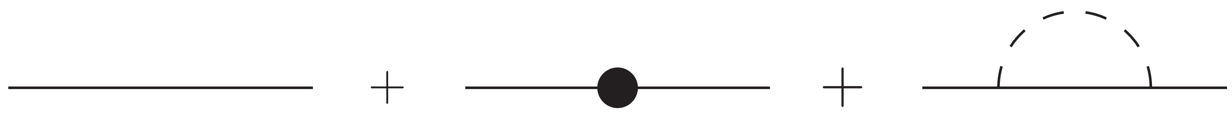
Part II: πN σ-term and strangeness in the nucleon (1)

Quark mass dependence of the nucleon mass

- calculate m_N up to $\mathcal{O}(p^3)$:

$$\mathcal{L}_{\pi N}^{(2)} = c_1 \bar{\psi} \langle \chi_+ \rangle \psi + \ldots \text{(6 more terms)}$$

+ one-pion loop:



$$\boxed{m_N = m - 4c_1 M_\pi^2 - \frac{3 g_A^2 M_\pi^3}{32 \pi F_\pi^2} + \mathcal{O}(M_\pi^4)}$$

- pion loop yields non-analytic term $\propto M_\pi^3 \propto \hat{m}^{3/2}$
- leading correction term comes with unknown low-energy constant c_1

The πN σ-term

- define scalar form factor of the nucleon:

$$\langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle = \sigma(t)\bar{u}(p')u(p)$$

$$\sigma \equiv \sigma(0) = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

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$$\sigma \equiv \sigma(0) = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

- Feynman–Hellman theorem:

$$\boxed{\sigma = \hat{m}\frac{\partial m_N}{\partial \hat{m}} = -4c_1 M_\pi^2 - \frac{9g_A^2 M_\pi^3}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^4)}$$

The πN σ-term

- define scalar form factor of the nucleon:

$$\langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle = \sigma(t)\bar{u}(p')u(p)$$

$$\sigma \equiv \sigma(0) = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

- relate σ to strangeness content of the nucleon!

$$\sigma = \frac{\hat{m}}{2m_N}\frac{\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle}{1-y}\,, \quad y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

now $(m_s - \hat{m})(\bar{u}u + \bar{d}d - 2\bar{s}s)$ is the part of $\mathcal{L}_{\text{QCD}}$ that produces SU(3) mass splittings:

$$\sigma = \frac{\hat{\sigma}}{1-y}\,, \quad \hat{\sigma} = \frac{\hat{m}}{m_s - \hat{m}}\big(m_\Xi + m_\Sigma - 2m_N\big) \simeq 26 \text{ MeV}$$

higher-order corrections: $\hat{\sigma} \to (36 \pm 7)$ MeV

- if we know σ, we know y $\Rightarrow$ strangeness in the nucleon

The σ-term and πN scattering (1)

- remember: learnt about M_π^2 from $\pi\pi$ scattering
- isospin even / odd πN scattering amplitudes:

$$T^{\pm}_{\pi N} = \frac{1}{2}\big[T(\pi^- p \to \pi^- p) \pm T(\pi^+ p \to \pi^+ p)\big]$$

- decompose $T^{\pm}$ further (spin-flip / non-flip amplitudes ...)
  $\Rightarrow$ consider specific combinations $\bar{D}^{\pm}$
- show in Chiral Perturbation Theory:

$$\Sigma \equiv F_\pi^2 \bar{D}^+ \big(\underbrace{s = u = m_N^2, t = 2M_\pi^2}_{\text{"Cheng-Dashen point"}}\big) = \sigma(2M_\pi^2) + \Delta_R$$

$\Delta_R = \mathcal{O}(M_\pi^4)$ is very small, $\Delta_R \lesssim 2$ MeV

The σ-term and πN scattering (2)

Procedure:

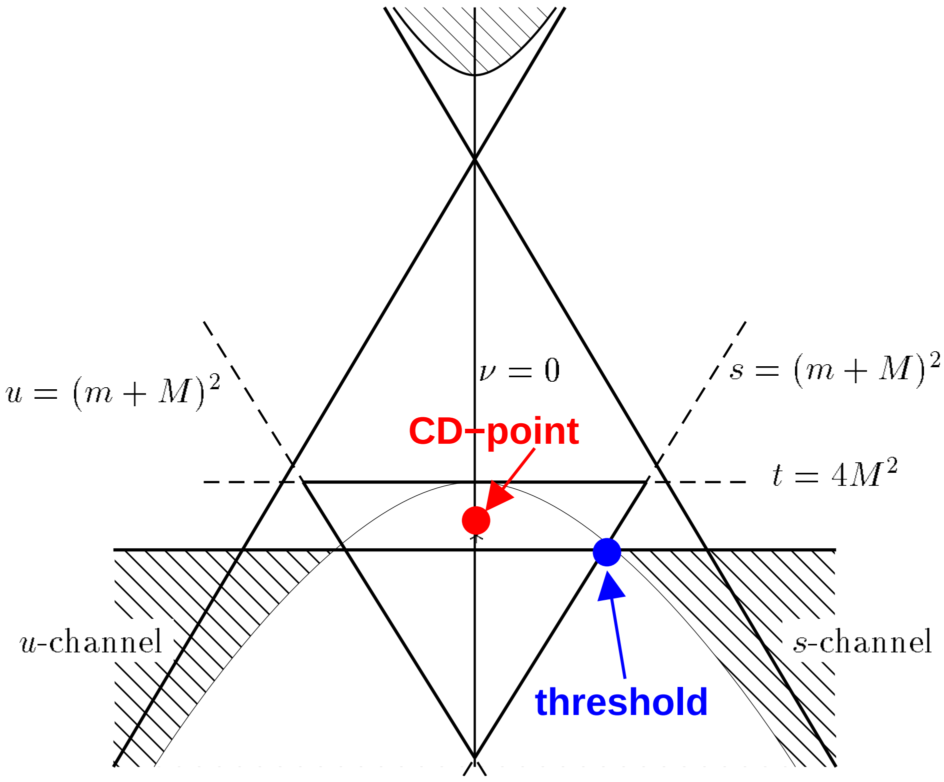
πN scattering $\xrightarrow{(1)}$ $\Sigma = F_\pi^2 \bar{D}^+_{CD}$ $\xrightarrow{(2)}$ $\sigma(2M_\pi^2)$ $\xrightarrow{(3)}$ $\sigma(0)$ $\xrightarrow{(4)}$ y

The σ-term and πN scattering (2)

Procedure:

πN scattering $\xrightarrow{(1)}$ $\Sigma = F_\pi^2 \bar{D}^+_{CD}$ $\xrightarrow{(2)}$ $\sigma(2M_\pi^2)$ $\xrightarrow{(3)}$ $\sigma(0)$ $\xrightarrow{(4)}$ y

(1)



- Cheng–Dashen point: $s = u = m_N^2$, $t = 2M_\pi^2$
- physical region: $s \geq (m_N + M_\pi)^2$, $t \leq 0$
- extrapolate using dispersion relations

$$\Sigma = 60 \pm 7 \text{ MeV}$$

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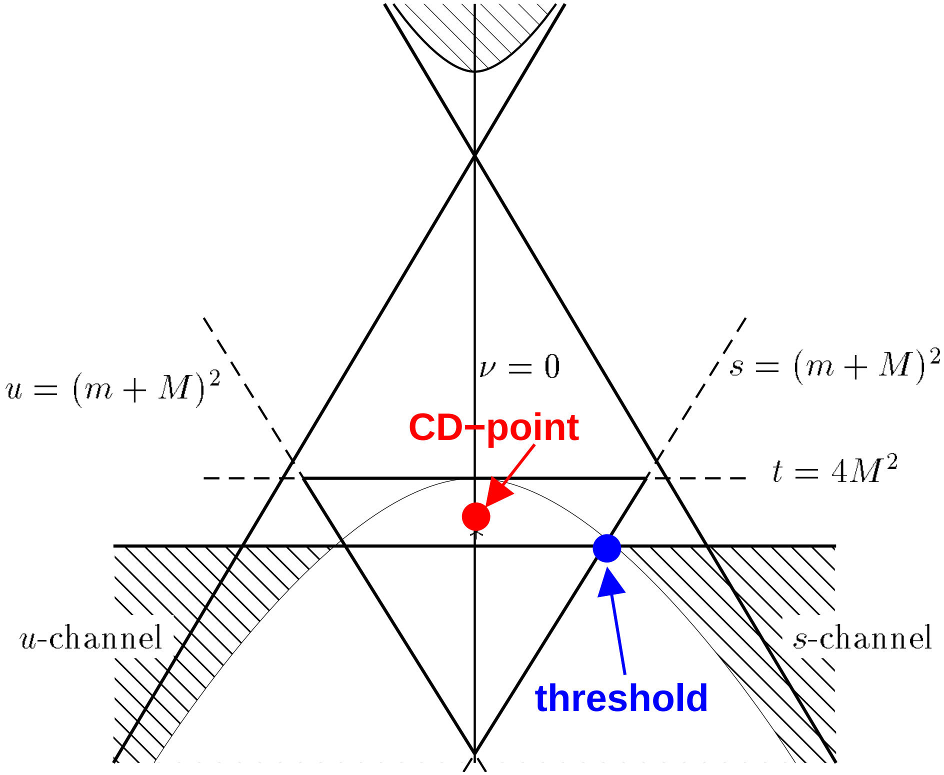
(1)



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- physical region: $s \geq (m_N + M_\pi)^2$, $t \leq 0$
- extrapolate using dispersion relations

$$\boxed{\Sigma = 60 \pm 7 \text{ MeV}}$$

(2) $\Sigma = \sigma(2M_\pi^2) + \Delta_R\,, \quad \Delta_R \leq 2$ MeV $\quad \rightarrow$ okay

The σ-term and πN scattering (2)

Procedure:

πN scattering $\xrightarrow{(1)}$ $\Sigma = F_\pi^2 \bar{D}^+_{CD}$ $\xrightarrow{(2)}$ $\sigma(2M_\pi^2)$ $\xrightarrow{(3)}$ $\sigma(0)$ $\xrightarrow{(4)}$ y

(3)

- $$\sigma(2M_\pi^2) = \sigma(0) + \Delta\sigma$$

  $\Rightarrow$ understand the t-dependence of the scalar form factor $\sigma(t)$!

- crude estimate:
  - ▷ $\sigma(t) = \sigma(0)\left\{1 + \frac{1}{6}\langle r^2\rangle_\sigma\, t + \ldots\right\}$
  - ▷ assume $\langle r^2\rangle_\sigma \simeq \langle r^2\rangle_{\text{EM}} = 0.8\ \text{fm}^2 \quad \Rightarrow \quad \Delta\sigma \approx 3.5$ MeV

- one-loop ChPT: $\Delta\sigma = 4.6$ MeV

$$\boxed{\sigma \approx 55\ \text{MeV}}$$

The σ-term and πN scattering (2)

Procedure:

πN scattering $\xrightarrow{(1)}$ $\Sigma = F_\pi^2 \bar{D}^+_{CD}$ $\xrightarrow{(2)}$ $\sigma(2M_\pi^2)$ $\xrightarrow{(3)}$ $\sigma(0)$ $\xrightarrow{(4)}$ y

(4)

- $$y = 1 - \frac{\hat{\sigma}}{\sigma} = 1 - \frac{35 \text{ MeV}}{55 \text{ MeV}} \approx 0.4 \text{ !!}$$
- "σ term puzzle":
  300 MeV of nucleon mass due to strange quarks??

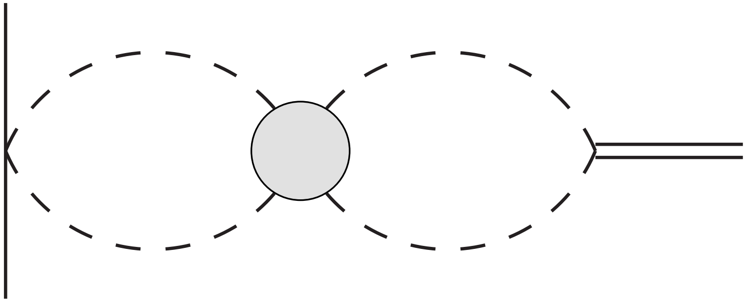
The σ-term and πN scattering (2)

Procedure:

πN scattering $\xrightarrow{(1)}$ $\Sigma = F_\pi^2 \bar{D}^+_{CD}$ $\xrightarrow{(2)}$ $\sigma(2M_\pi^2)$ $\xrightarrow{(3)}$ $\sigma(0)$ $\xrightarrow{(4)}$ y

(3)

- scalar form factor beyond one loop:



isoscalar s-wave $\pi\pi$ scattering $\Rightarrow$ strong!

- dispersive analysis, $\langle r^2 \rangle_\sigma \approx 2 \langle r^2 \rangle_{EM}$
large curvature $\Rightarrow \quad \Delta\sigma \approx 15$ MeV

$$\boxed{\sigma \approx 45 \text{ MeV}}$$

Gasser, Leutwyler, Sainio 1991

The σ-term and πN scattering (2)

Procedure:

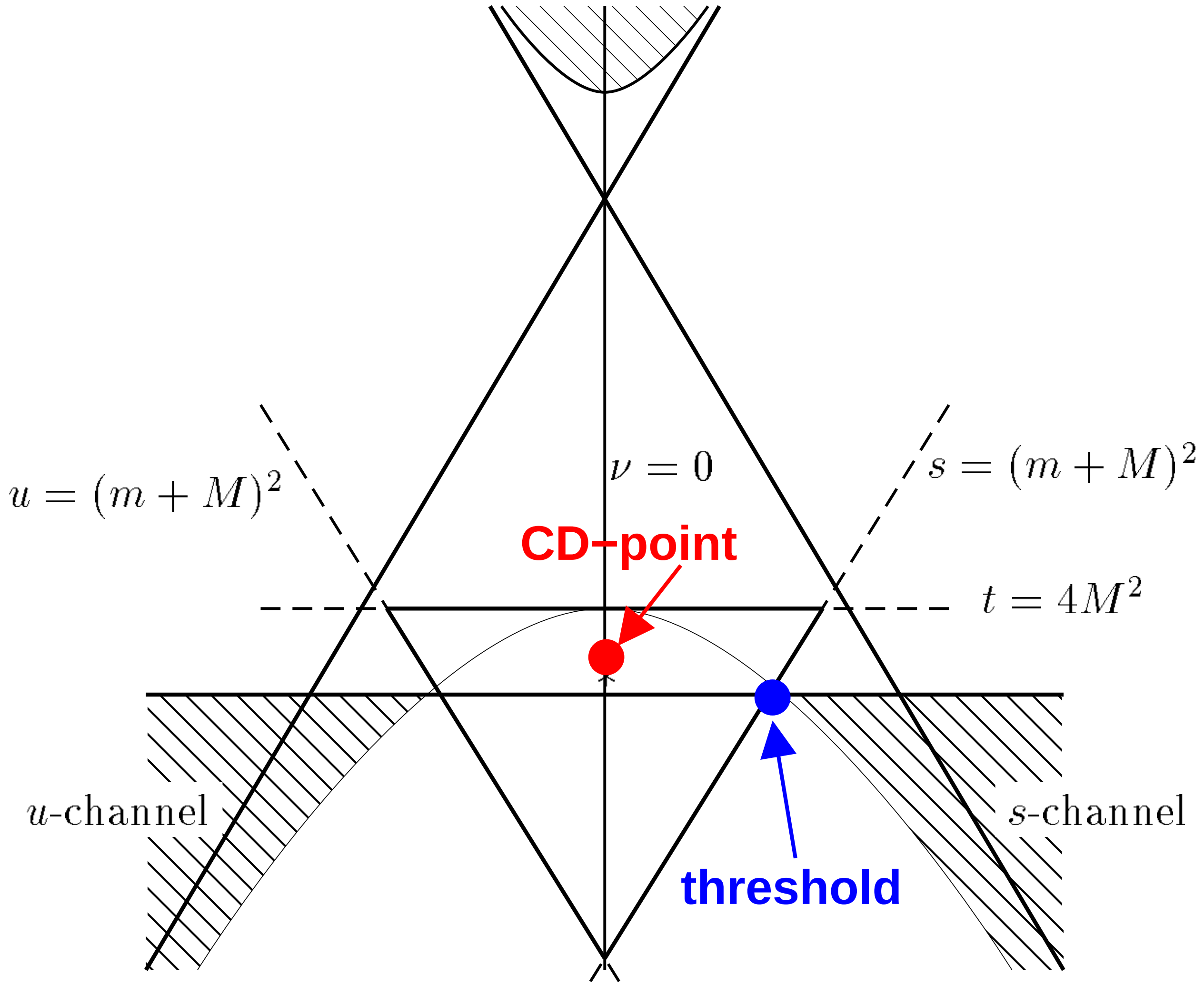
πN scattering $\xrightarrow{(1)}$ $\Sigma = F_\pi^2 \bar{D}_{CD}^{+}$ $\xrightarrow{(2)}$ $\sigma(2M_\pi^2)$ $\xrightarrow{(3)}$ $\sigma(0)$ $\xrightarrow{(4)}$ y

(4)

- $$y = 1 - \frac{\hat{\sigma}}{\sigma} = 1 - \frac{35\ \text{MeV}}{45\ \text{MeV}} \approx 0.2$$
- strangeness in the nucleon sizeable, but not outrageous

$$\langle N | m_s \bar{s} s | N \rangle \simeq 130\ \text{MeV}$$

πN scattering lengths



Büttiker, Meißner 1999

πN scattering lengths in ChPT

- at threshold: πN scattering lengths

$$a^{\pm} = \frac{1}{4\pi(1 + M_\pi/m_N)} T^{\pm}\left(s = (m_N + M_\pi)^2\right)$$

- in ChPT:

$$a^{-} = \frac{M_\pi}{8\pi(1 + M_\pi/m_N)F_\pi^2} + \mathcal{O}\left(M_\pi^3\right)$$

$$a^{+} = 0 + \frac{M_\pi^2(-g_A^2 + 8m_N(-2c_1 + c_2 + c_3))}{16\pi m_N(1 + M_\pi/m_N)F_\pi^2} + \mathcal{O}\left(M_\pi^3\right)$$

- $a^{-} = 8.0 \times 10^{-2} M_\pi^{-1}$ + small corrections
- a^{+} vanishes at leading order, several LECs, bad convergence
  $\Rightarrow$ the helpful guy for the σ term barely known
  $\Rightarrow$ chiral series very different for $a^{\pm}$

πN scattering lengths from pionic atoms (1)

- $\pi^- p$, $\pi^- d$ systems, bound by electromagnetism
  calculate energy levels as in quantum mechanics for the hydrogen atom!
- energy levels perturbed by strong interactions:
  ground state not stable, decays: $A_{\pi^- p} \to \pi^0 n,\ \gamma n,\ \ldots$
- ground state strong energy level shift ΔE^{str} and width Γ related to πN scattering amplitudes at threshold:

$$
\begin{aligned}
\Delta E^{\text{str}} &= -2\alpha^3 \mu_c^2 \mathcal{A}(\pi^- p \to \pi^- p) + \ldots \quad \propto \ a^+ + a^- + \ldots \\
\Gamma &= 8\alpha^3 \mu_c^2 p^* |\mathcal{A}(\pi^- p \to \pi^0 n)|^2 + \ldots \quad \propto \ -a^- + \ldots
\end{aligned}
$$

μ_c: reduced mass, p^*: CMS-momentum of π^0

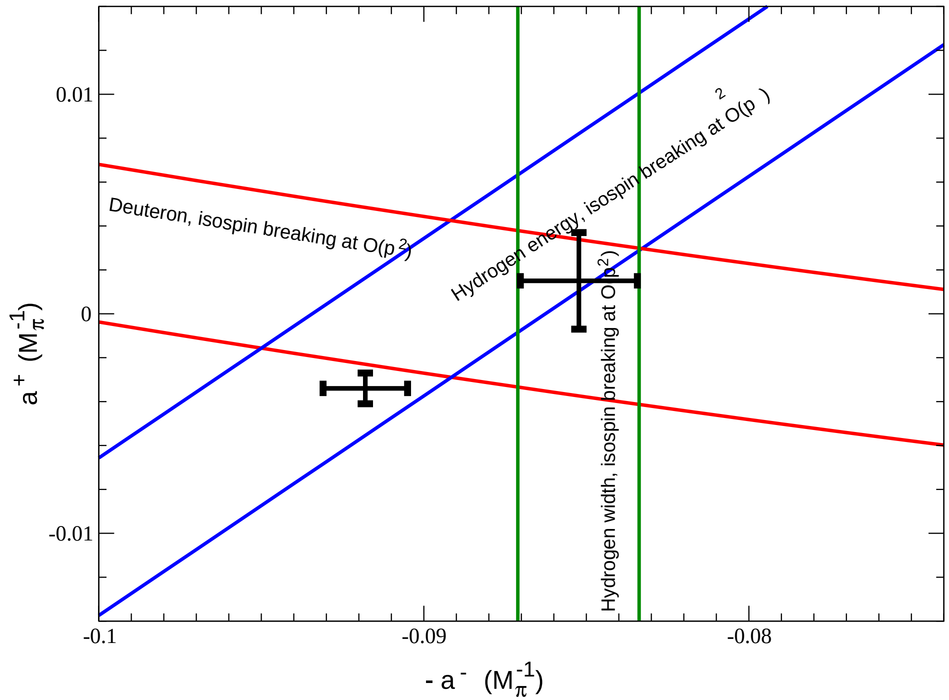
Deser, Goldberg, Baumann, Thirring 1954

Gasser, Lyubovitskij, Rusetsky 2008

- $\pi^- d$: additional information due to different isospin combinations

πN scattering lengths from pionic atoms (2)

from pionic hydrogen and deuterium measurements: PSI 1995-2006



$$a^- = (8.52 \pm 0.18) \times 10^{-2} M_\pi^{-1}$$
$$a^+ = (0.15 \pm 0.22) \times 10^{-2} M_\pi^{-1}$$

Meißner, Raha, Rusetsky 2006

a^+ very small, quite sensitive to isospin breaking corrections

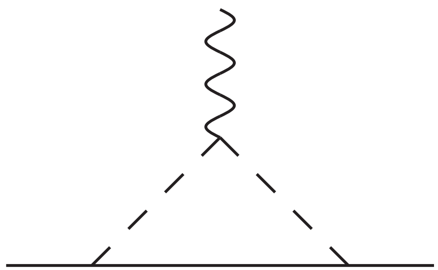
Spares

Heavy-baryon vs. infrared regularisation (1)

- consider the electromagnetic form factors of the nucleon:

$$\langle N(p')|J_\mu^{em}|N(p)\rangle = e\bar{u}(p')\left\{\gamma_\mu F_1^N(t) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2^N(t)\right\}u(p)$$

- important contribution to the spectral function $\mathrm{Im}\, F_1^v(t)$ [$F_1^v(t) = F_1^p(t) - F_1^n(t)$] from so-called triangle graph:



- "normal" threshold at $t = 4M_\pi^2$
- anomalous threshold at $t = 4M_\pi^2 - \frac{M_\pi^4}{m_N^2} \overset{\text{HB}}{=} 4M_\pi^2 + \mathcal{O}(M_\pi^4)$

$\Rightarrow$ analytic structure distorted

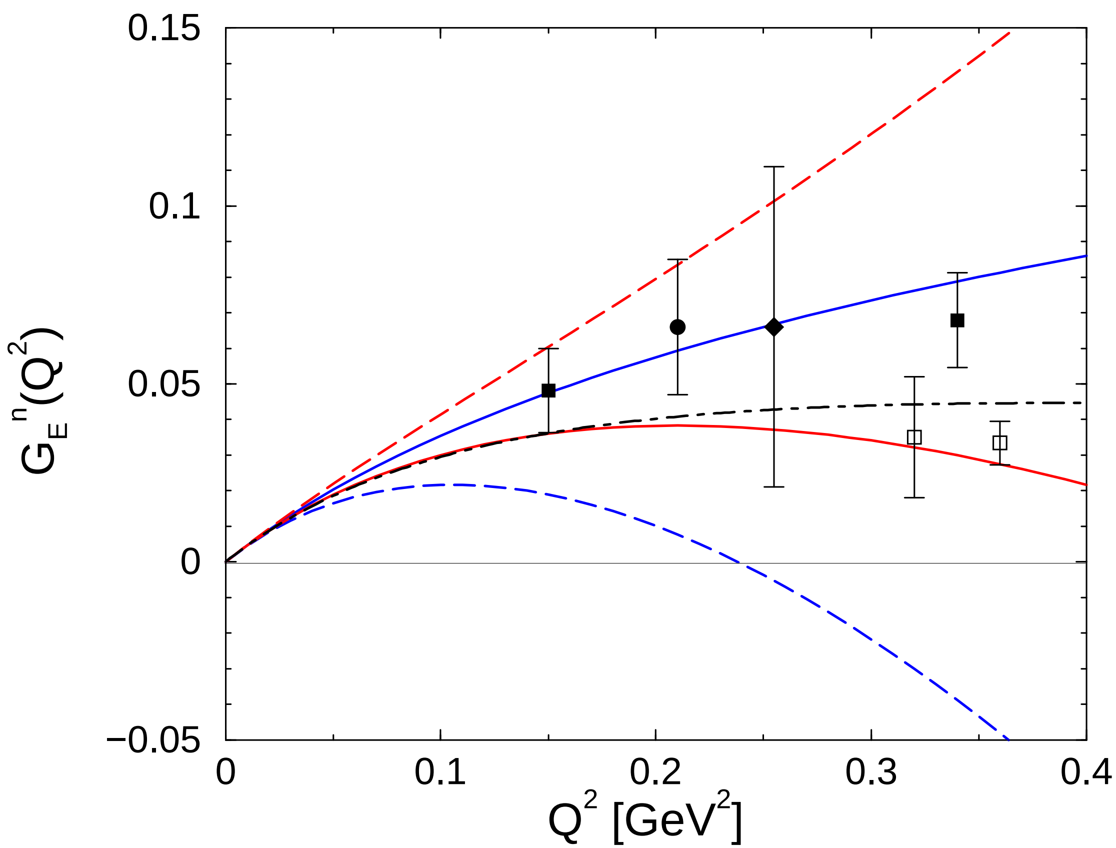
$$\mathrm{Im}\, F_1^v(t) \overset{\text{IR}}{=} \frac{g_A^2}{192\pi F_\pi^2}(4m_N^2 - M_\pi^2)\left(1 - \frac{4M_\pi^2}{t}\right)^{3/2} + \ldots \quad \text{p-wave}$$

$$\mathrm{Im}\, F_1^v(t) \overset{\text{HB}}{=} \frac{g_A^2}{96\pi F_\pi^2}(5t - 8M_\pi^2)\left(1 - \frac{4M_\pi^2}{t}\right)^{1/2} + \ldots \quad \text{wrong!}$$

Heavy-baryon vs. infrared regularisation (2)

- resummation of nucleon "recoil effects"
  ($\Rightarrow$ correct relativistic kinematics) sometimes helps:

$$G_E^n(t) = F_1^n(t) + \frac{t}{4m_N^2} F_2^n(t) \;, \quad Q^2 = -t$$



blue: $\mathcal{O}(q^3)$
red: $\mathcal{O}(q^4)$
full: IR
dashed: HB
(dot-dashed: disp.th.)

BK, Meißner 2000