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Quantum Interference Effects in Graphene

Alex Savchenko

- Weak Localisation in Mono-layer and Bi-layer graphene.
- Transition from WL to AWL in graphene.
- UCF in graphene.

Benasque 2009

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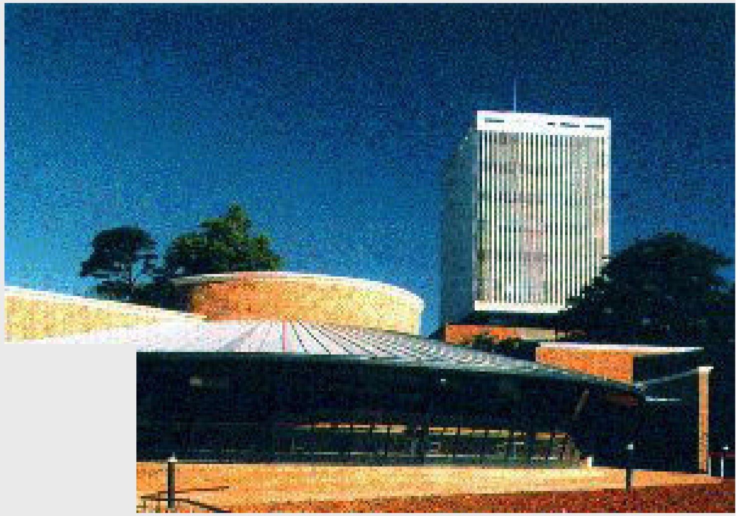


Experiment:

F.V. Tikhonenko

D.W.Horsell

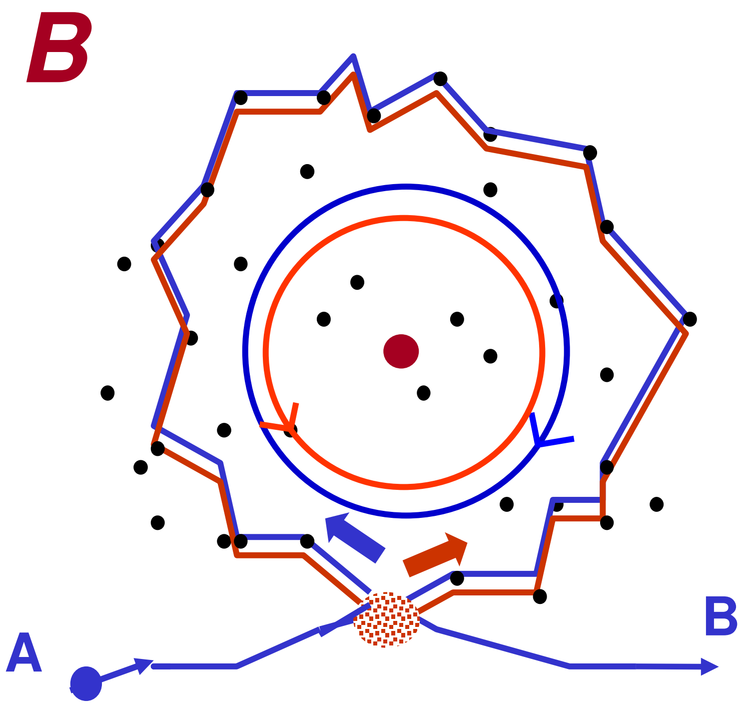
A.A. Kozikov

R.V. Gorbachev

Funding:

EPSRC Engineering and Physical Sciences Research Council

Quantum interference in conductance



AWL is usually due to strong *s-o* scattering

Size of trajectories is limited by $\tau_\phi(T)$.

WL: Constructive interference

⇩

Decrease of conductance

⇩

Positive Magneto Conductance

AWL: Destructive interference

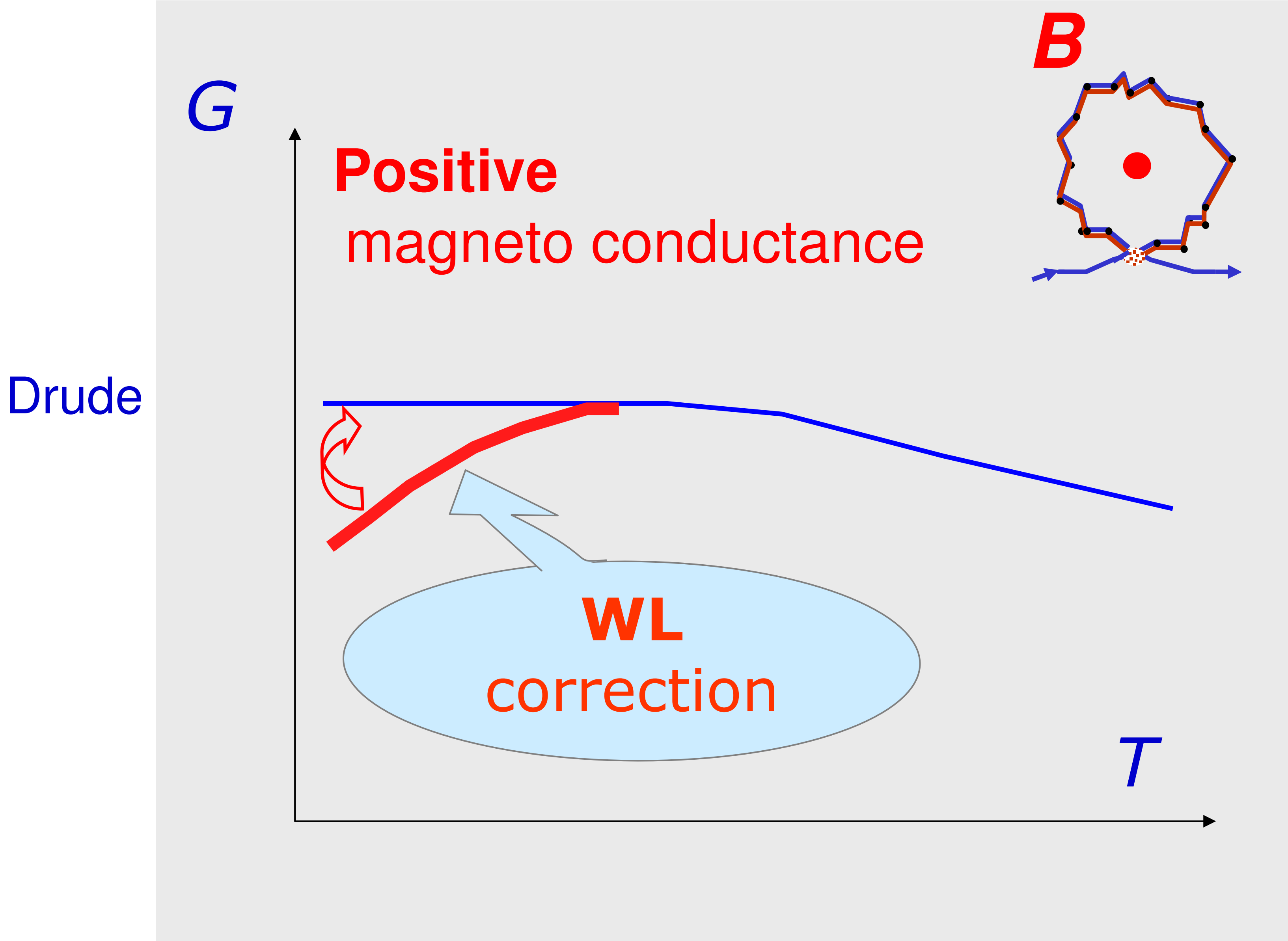
⇩

Increase of conductance

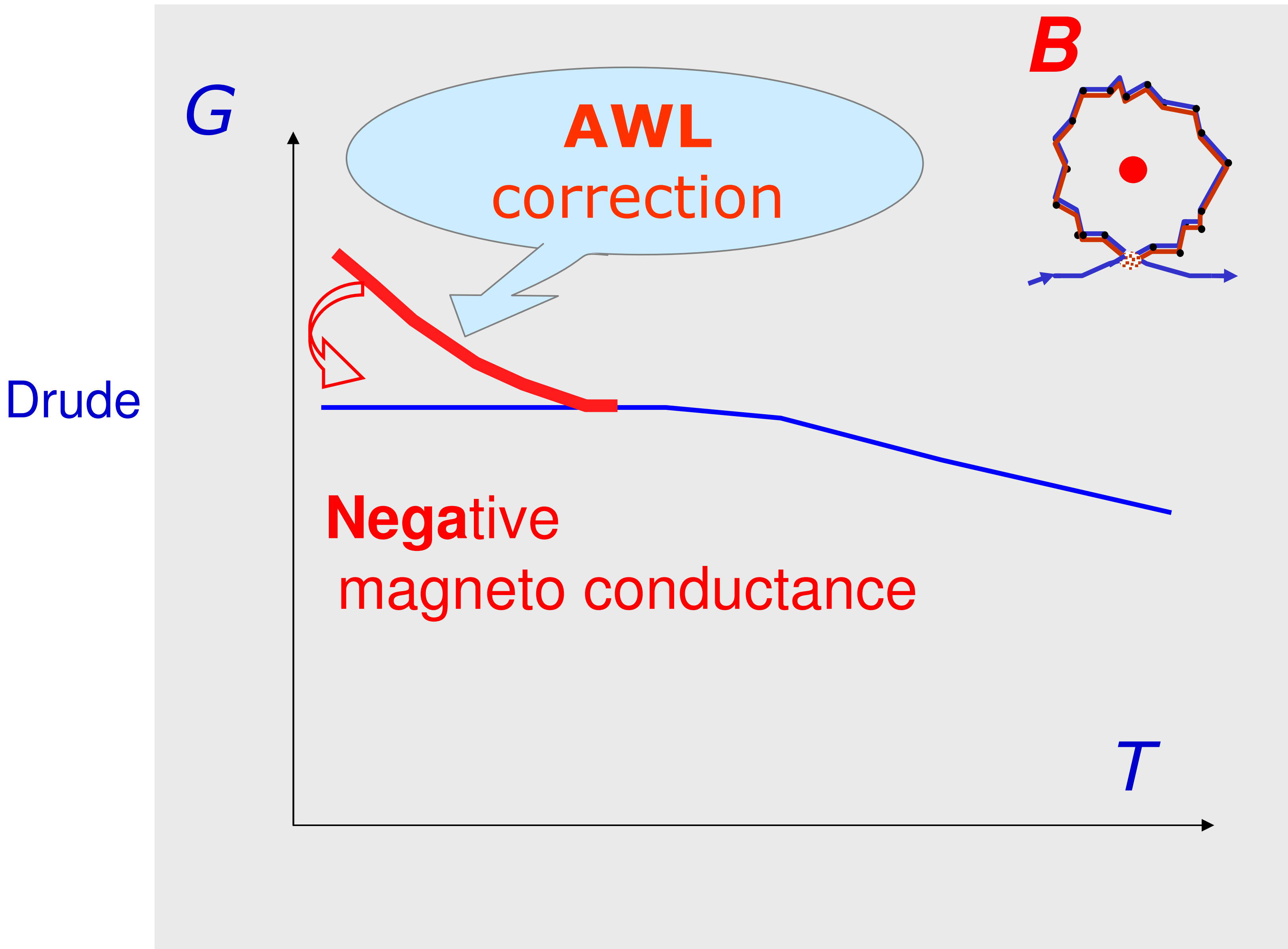
⇩

Negative Magneto Conductance

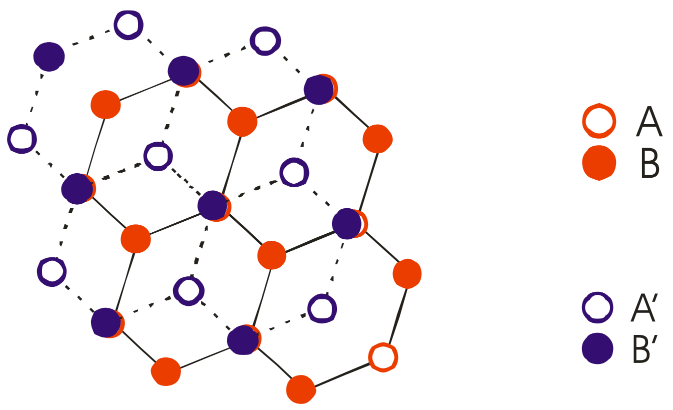
Quantum interference in conductance

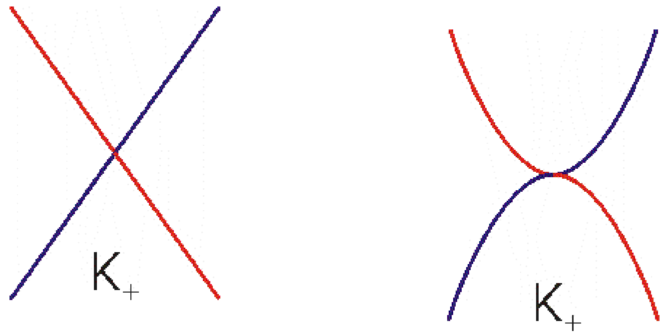


Quantum interference in conductance



Carriers in graphene and bi-layer



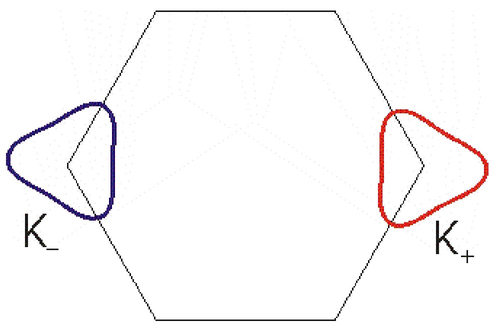


Massless Massive

Two valleys with

Chiral carriers:

Berry phase π Berry phase 2π



Trigonal warping of energy spectrum
(stronger in bi-layer).

Quantum Interference in graphene

Sensitivity to *elastic* scattering:

- *Intra-valley* scattering by topological defects breaking chirality, τ_t , *Morpurgo, Guinea, PRL (2006)*
- *Intra-valley* scattering in the presence of trigonal warping, τ_w . *McCann et al., PRL (2006)*

Such scattering, τ_*^{-1} , *destroys* quantum interference in one valley.

- *Inter-valley* scattering by sharp defects, τ_i^{-1} .

Inter-valley scattering *restores* interference destroyed in one valley and results in WL.

Theory of WL in graphene layers

Single layer

*McCann et al., PRL **97**, 146805 (2006)*

$$\Delta\sigma(B)=\frac{e^2}{\pi\hbar}\left[F\left(\frac{\tau_B^{-1}}{\tau_\phi^{-1}}\right)-F\left(\frac{\tau_B^{-1}}{\tau_\phi^{-1}+2\tau_i^{-1}}\right)-2F\left(\frac{\tau_B^{-1}}{\tau_\phi^{-1}+\tau_i^{-1}+\tau_*^{-1}}\right)\right]$$

$$F(z)=\ln z+\Psi(1/2+1/z);\qquad \tau_B^{-1}=\frac{4De}{\hbar}B,\qquad \tau_*^{-1}=\tau_w^{-1}+\tau_t^{-1}.$$

- □ **No** intra-valley suppression and **no** inter-valley scattering ($\tau_\phi^{-1} >> \tau_*^{-1}, \tau_i^{-1}$) ➡ AWL
- ◘ Intra-valley suppression but **no** inter-valley scattering ($\tau_*^{-1} >> \tau_\phi^{-1} >> \tau_i^{-1}$) ➡ 0
- ■ Intra-valley suppression **and** inter-valley scattering ($\tau_*^{-1} >> \tau_\phi^{-1} \sim \tau_i^{-1}$) ➡ 0.5 WL

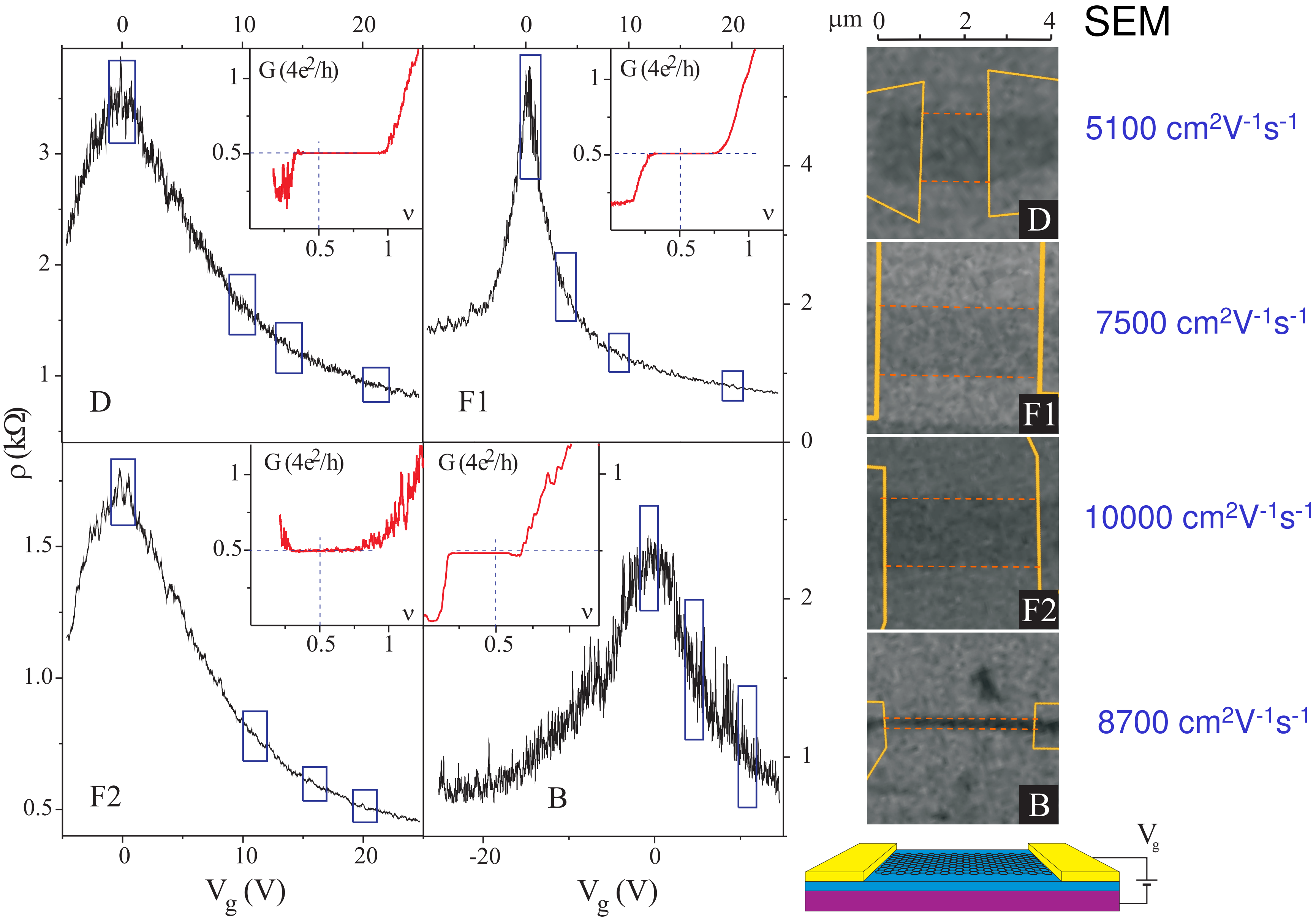
Bilayer

*Kechedzhi et al., PRL **98**, 176806 (2007)*

$$\Delta\sigma(B)=\frac{e^2}{\pi\hbar}\left[F\left(\frac{\tau_B^{-1}}{\tau_\phi^{-1}}\right)-F\left(\frac{\tau_B^{-1}}{\tau_\phi^{-1}+2\tau_i^{-1}}\right)+2F\left(\frac{\tau_B^{-1}}{\tau_\phi^{-1}+\tau_i^{-1}+\tau_*^{-1}}\right)\right]$$

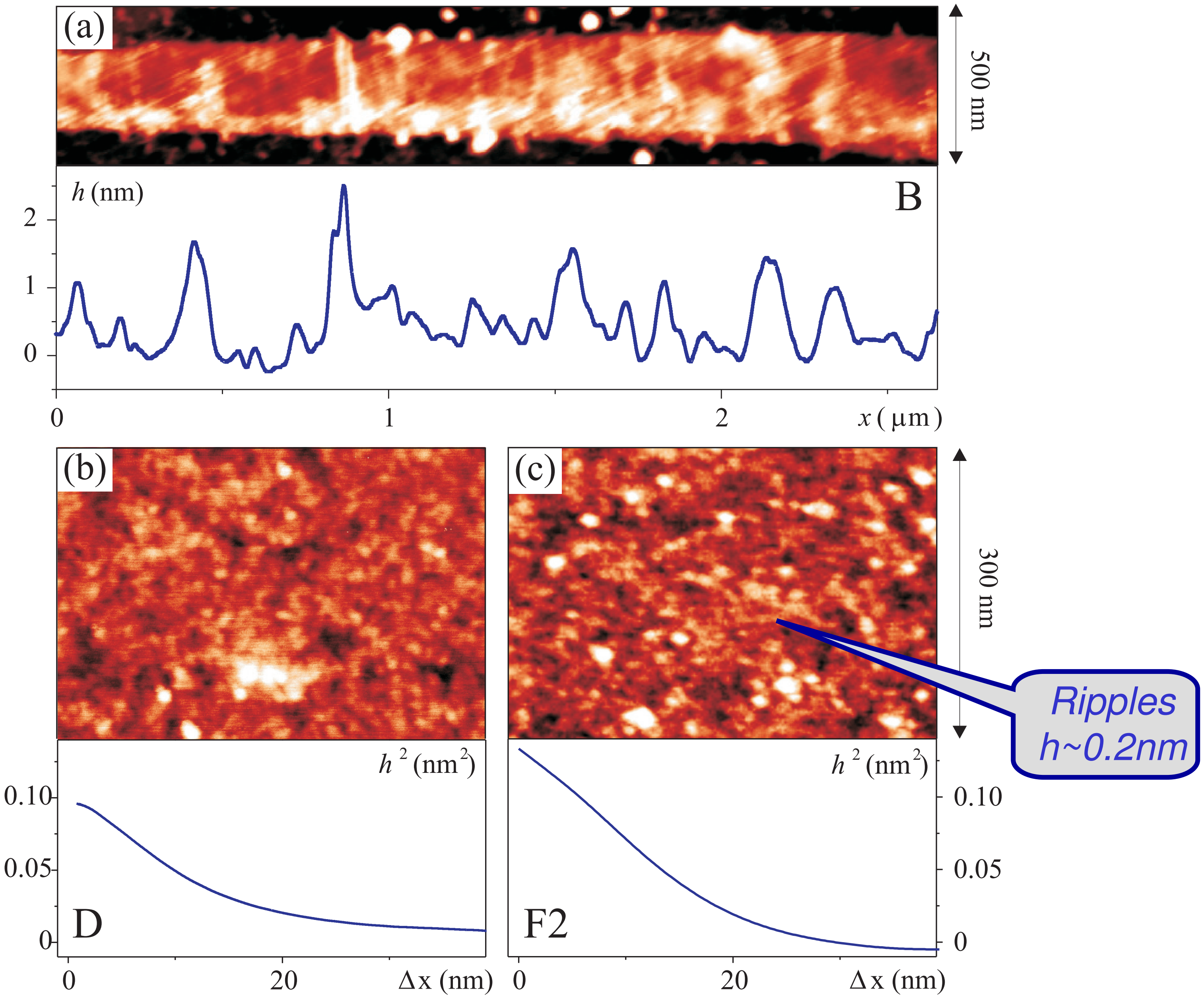
- □ **No** intra-valley suppression and **no** inter-valley scattering ($\tau_\phi^{-1} >> \tau_*^{-1}, \tau_i^{-1}$) ➡ WL

Single-layer samples



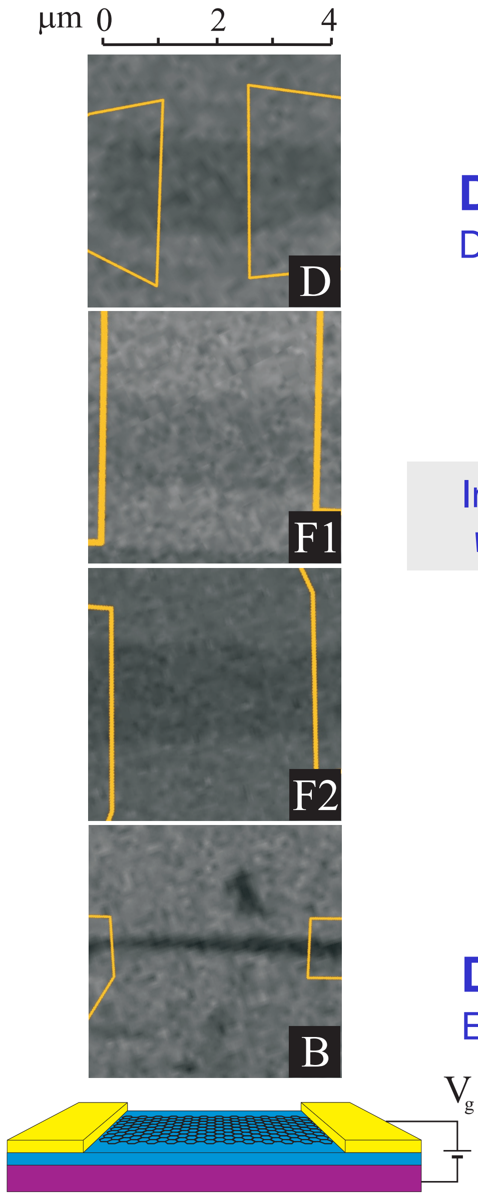
*Tikhonenko, Horsell, Gorbachev, Savchenko, PRL **100**, 056802 (2008)*

AFM images



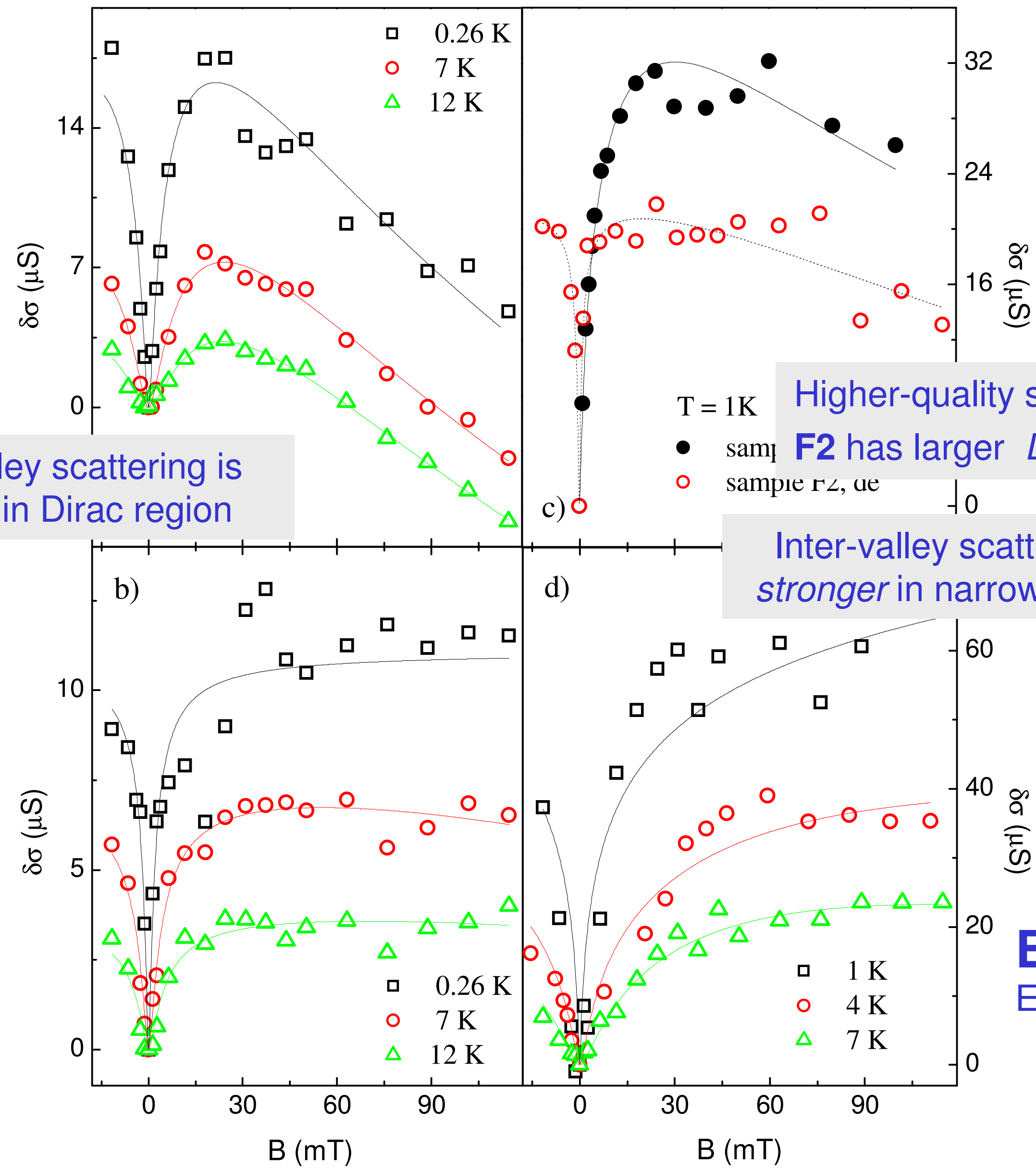
(a)
500 nm
h (nm)
B
2
1
0
0
1
2
x (μm)
(b)
(c)
300 nm
Ripples
h~0.2nm
h 2 (nm2)
h 2 (nm2)
0.10
0.05
0
D
F2
0
20
Δx (nm)
0
20
Δx (nm)
0.10
0.05
0

Magnetoconductance of graphene at different T



D
Dirac

D
Electron



Intra-valley scattering is *weaker* in Dirac region

F1, F2
Electron

Higher-quality sample
F2 has larger L_ϕ and L_i

Inter-valley scattering is *stronger* in narrow sample

B (narrow)
Electron

Tikhonenko, Horsell, Gorbachev, Savchenko, PRL ***100****, 056802 (2008)*

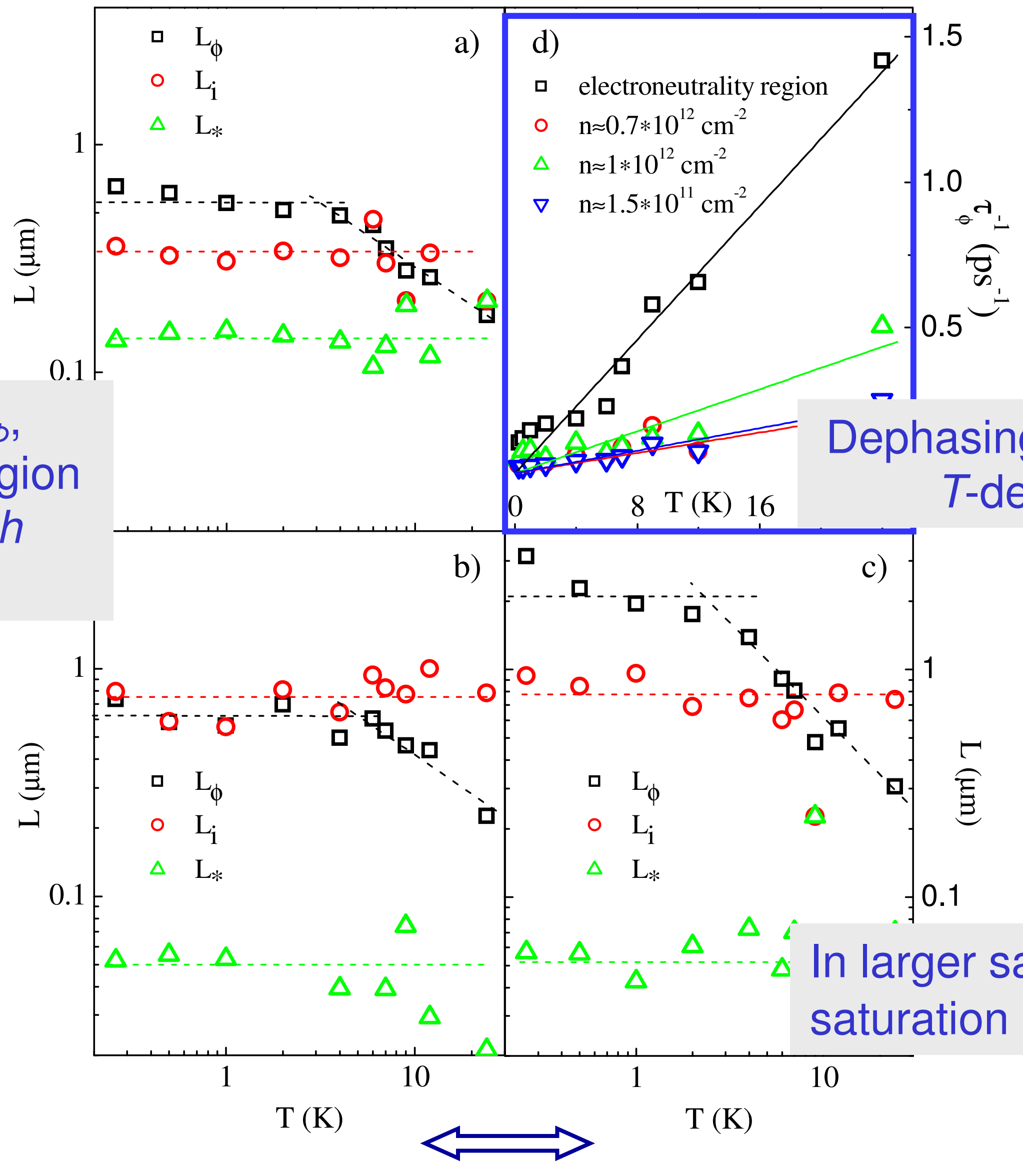
L_ϕ as a function of T

B. L. Altshuler, A. G. Aronov and D. E. Khmelnitsky, J. Phys. C 15, 7367 (1982)

D
Dirac region

Saturation of L_ϕ, smaller in Dirac region (formation of *e-h* puddles?)

D
Electron region



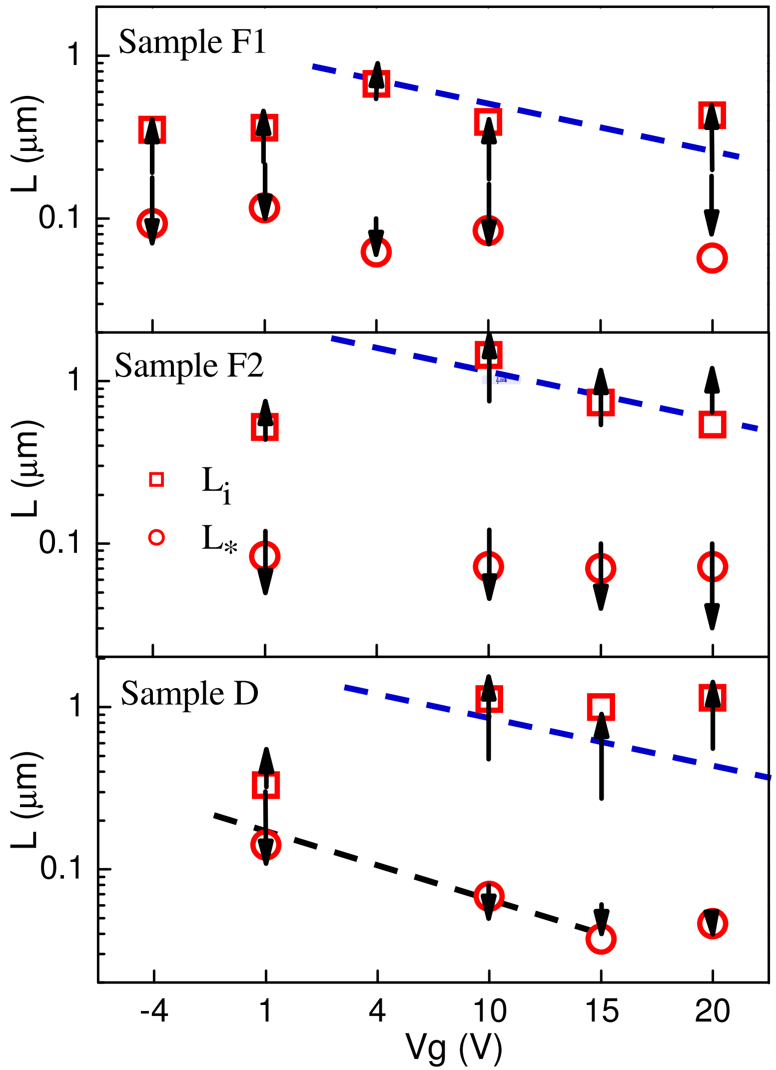
D

Dephasing rate has usual *T*-dependence

F2
Electron region

In larger sample **F2** saturation L_ϕ is larger

L_i and L^* in different n regions



- τ_i decreases with n (DOS increases with E).
- τ^* decreases with n (warping increases with n ?)

Estimations of $\tau_w^{-1} = 2\tau_p \left(\eta\varepsilon^2 / \hbar v_F^2\right)^2, \ \eta = \frac{\gamma_o a^2}{8\hbar^2}$:

$\tau_w \approx 720$ ps for $\varepsilon_F = 30$ meV

$\tau_w \approx 2$ ps for $\varepsilon_F = 130$ meV

Experiment: $\tau_w < 1$ ps .

Strong intra-valley suppression of WL (small τ^*)

× *Trigonal warping:* *McCann et al., PRL (2006)*

$$\tau_w^{-1} \approx 2\tau_p \left(\mu \varepsilon_F^2 / \hbar v_F^2\right)^2 , \quad \text{where} \quad \mu = \gamma_0 a^2 / \hbar^2 .$$

× *Sharp defects breaking chirality,* but $L_i >> L^*$!

× *Dislocations:* *Marpurgo, Guinea, PRL (2006)*

$\tau_d^{-1} \approx v_F / k_F \xi^2$, where ξ is the distance between dislocations .

× *Ripples:* *Morozov, et al. (2006).*

$$B_{eff} \approx \frac{\gamma_o}{e v_F L_\phi} \left(\frac{h}{d}\right)^2$$

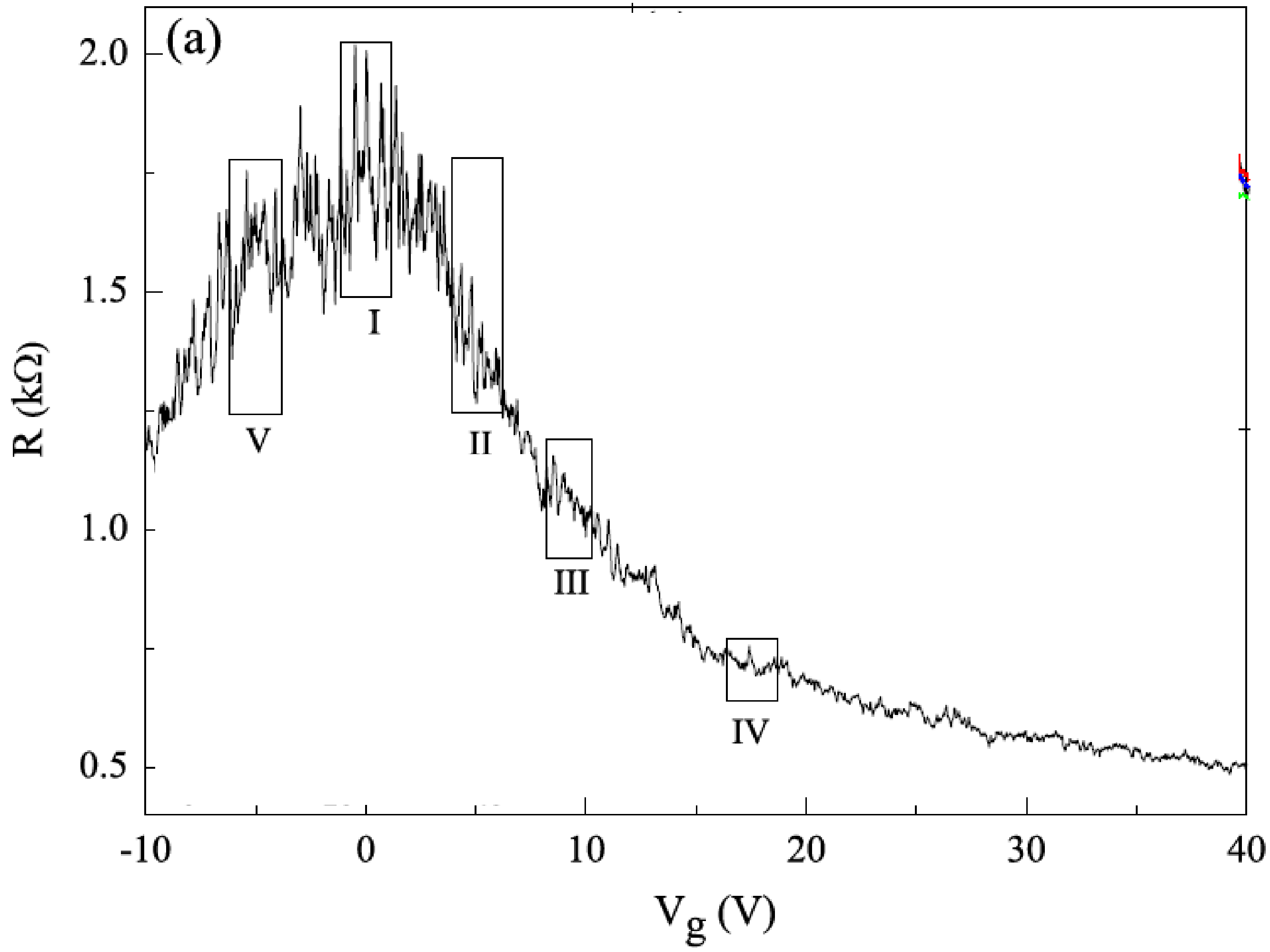
× *Potential gradients:* *Marpurgo, Guinea, PRL (2006)*

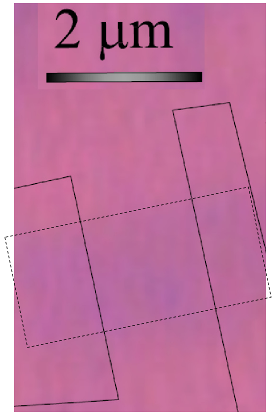
$$\tau_{grad}^{-1} \approx \tau_p^{-1} (k_F a)^2$$

Origin of strong *WL* suppression in one valley ?

*Tikhonenko et al., PRL **100**, 056802 (2008) - Supplementary. Material*

Bilayer sample





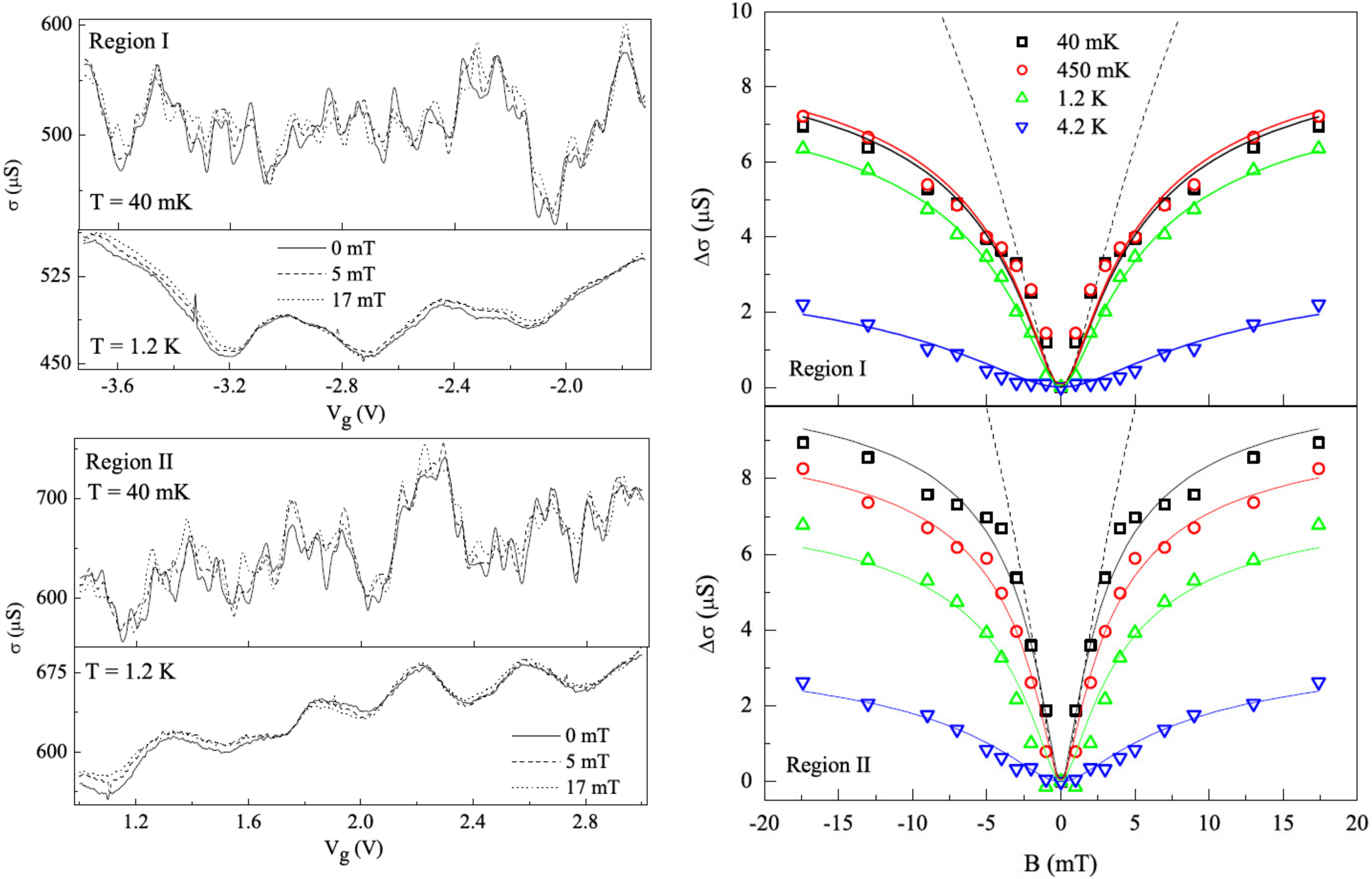
2-terminal: *L*=1.46 μm, *W*=1.84 μm;

R_c≈175 Ohm;

Mobility: 8000 $cm^2V^{-1}s^{-1}$

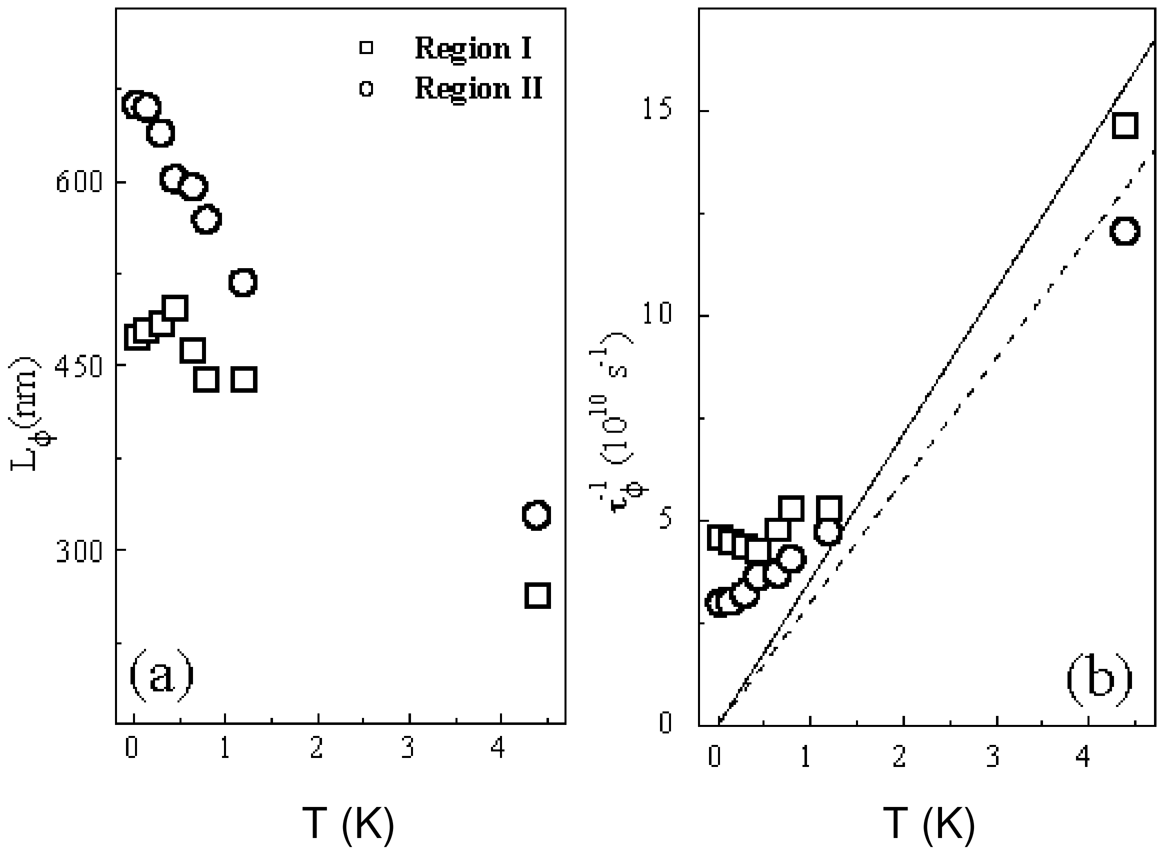
Gorbachev, Tikhonenko, Mayorov, Horsell, Savchenko, PRL ***98****, 176805 (2007)*

Magnetoconductance of bilayer at different *T*



Gorbachev, Tikhonenko, Mayorov, Horsell, Savchenko, PRL ***98****, 176805 (2007)*

Decoherence rate as a function of T



Dephasing rate at T >1K is due to *e*-*e* interaction in the diffusive regime:

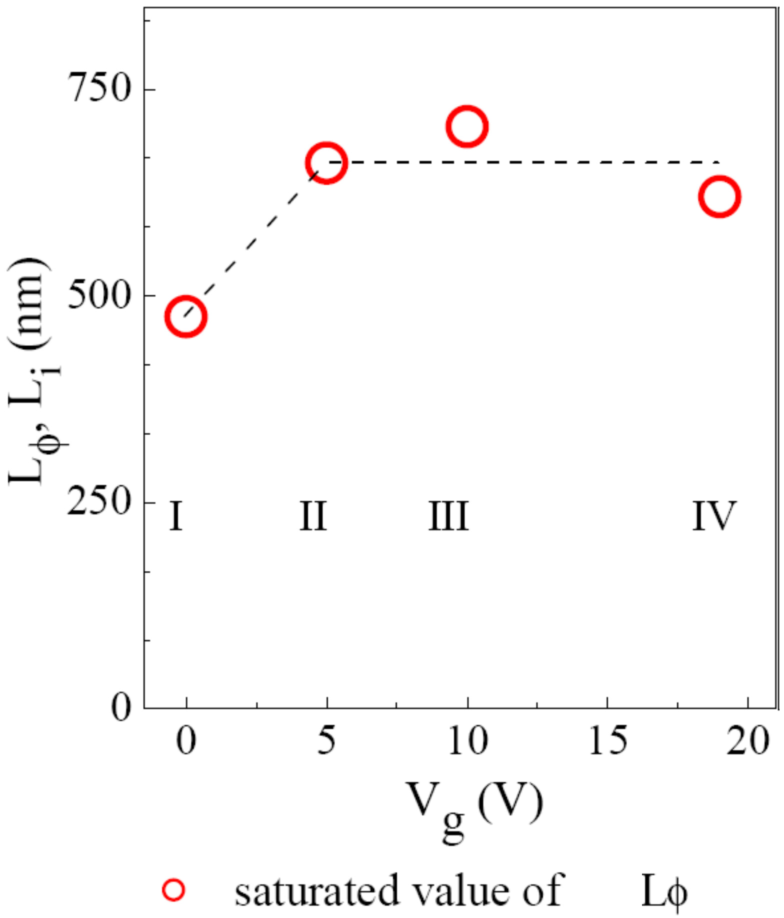
$$\tau_\phi^{-1} = \frac{\beta \ln g}{\hbar g} k_B T,$$

where $\beta \approx 1.2$.

$$\frac{\tau_p}{\tau_T} = \frac{\tau_p k_B T}{\hbar} \approx 0.001 \dots 0.1$$

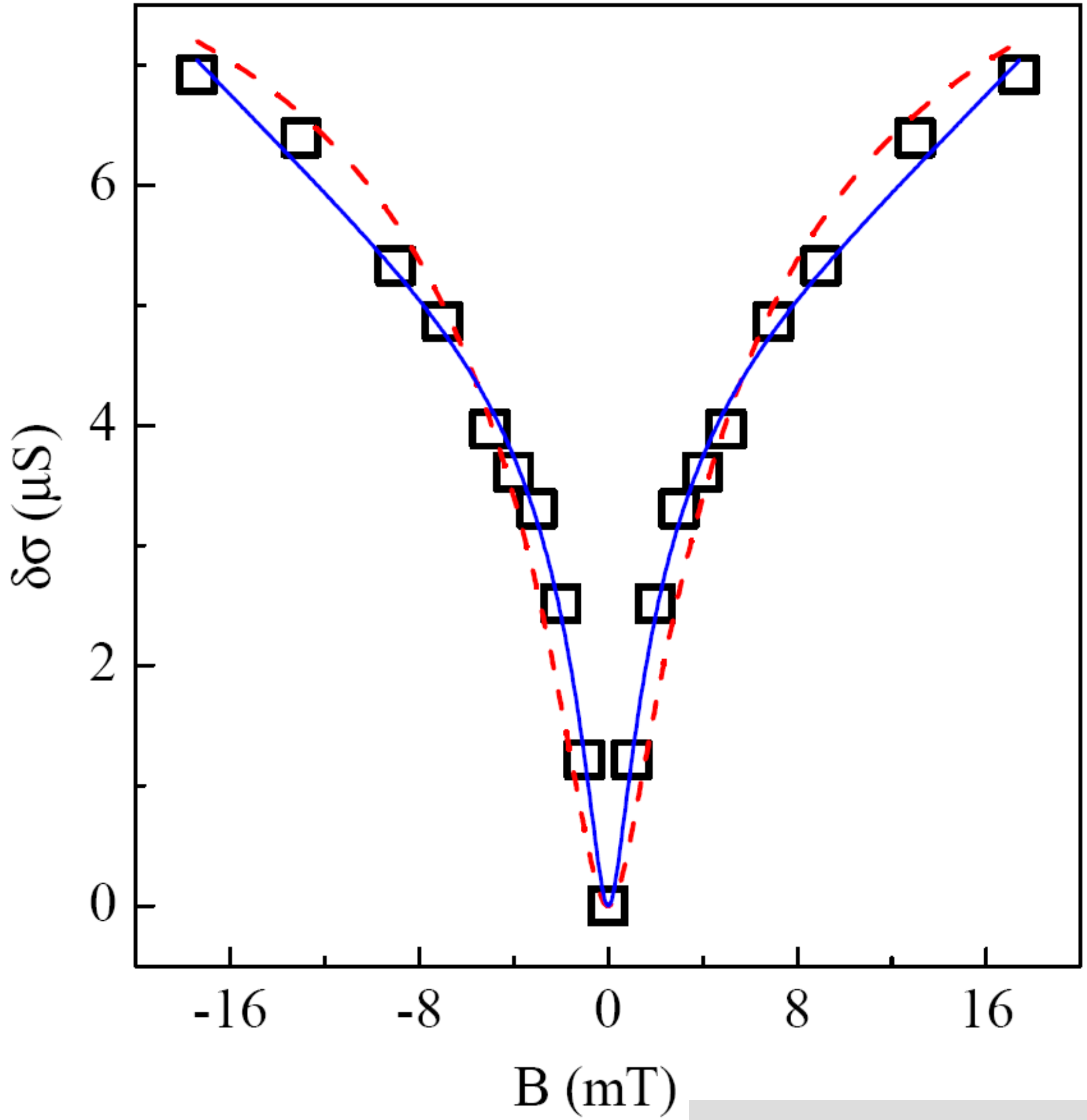
B. L. Altshuler, A. G. Aronov and D. E. Khmelnitsky, J. Phys. C., 15, 7367 (1982).

Saturation L_ϕ at different carrier densities



Smaller L_ϕ in the electro-neutrality region is due to *e-h* puddles, which are not totally transparent

Intra-valley suppression of WL in bilayer



with $\tau_*^{-1} = \tau_t^{-1} + \tau_w^{-1}$

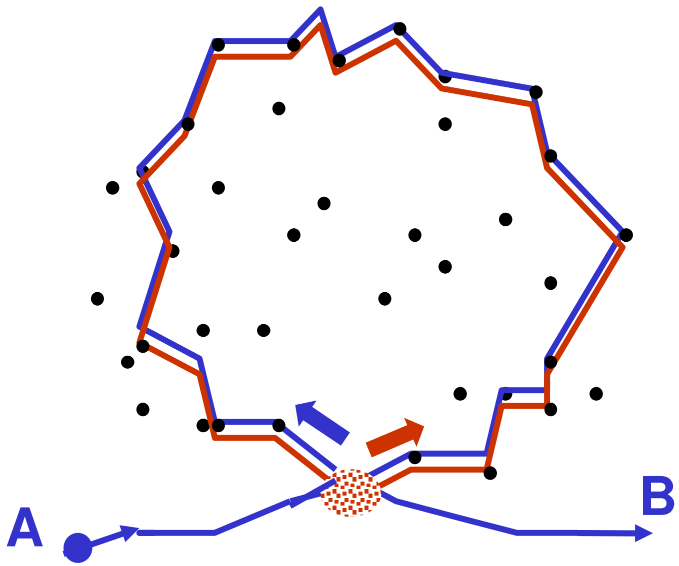
Experiment: $\tau_* \leq 0.5$ ps

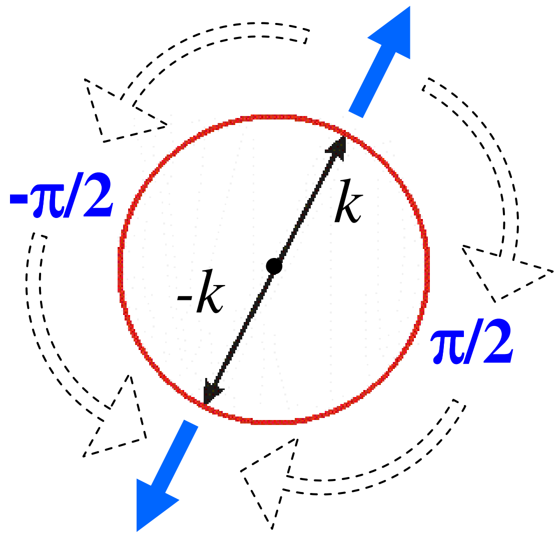
Theory: $\tau_w \approx \tau_p$

Kechedzhi et al. (2007)

In bilayer graphene *warping* (τ_w^{-1}) dominates in intra-valley suppression of WL.

AWL in graphene





Due to the Berry phase π, *destructive* interference occurs:

⇨ AWL **without** strong ***s-o*** **coupling**.

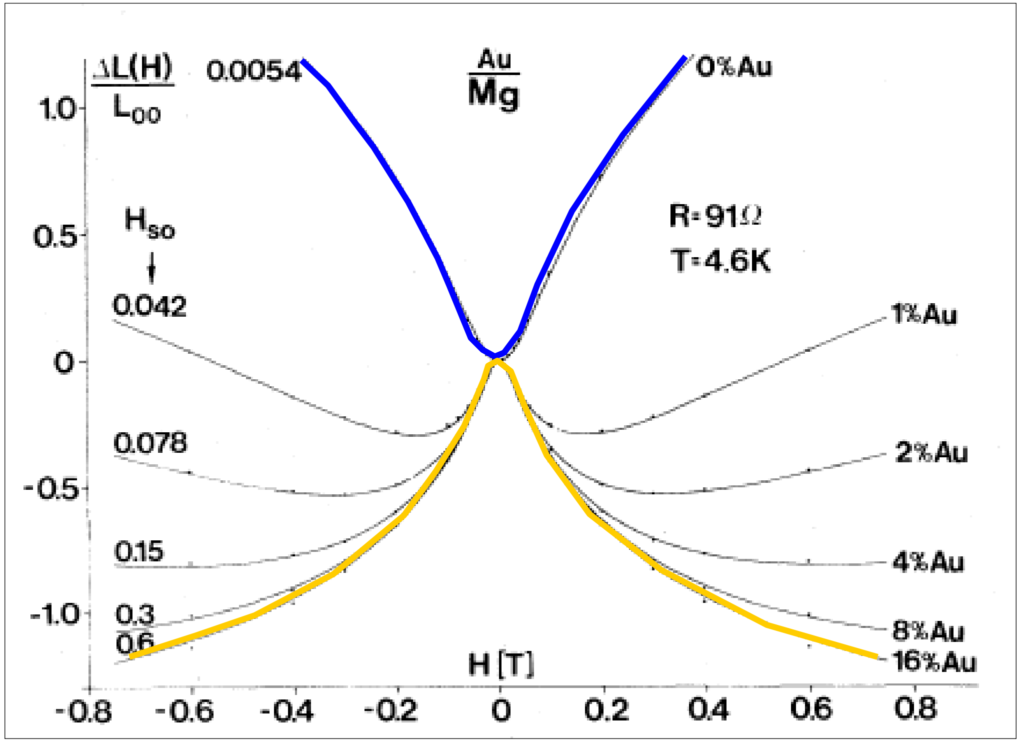
Anti Weak Localisation in graphene

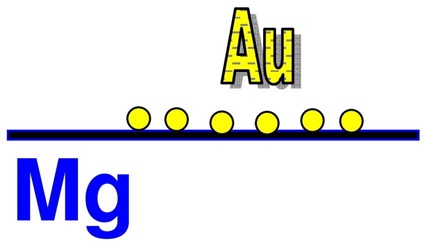
Its origin in conventional 2D systems: spin-orbit coupling and spin flip at scattering on impurities.

PHYSICAL REVIEW LETTERS 12 April 1982

Influence of Spin-Orbit Coupling on Weak Localization

Gerd Bergman





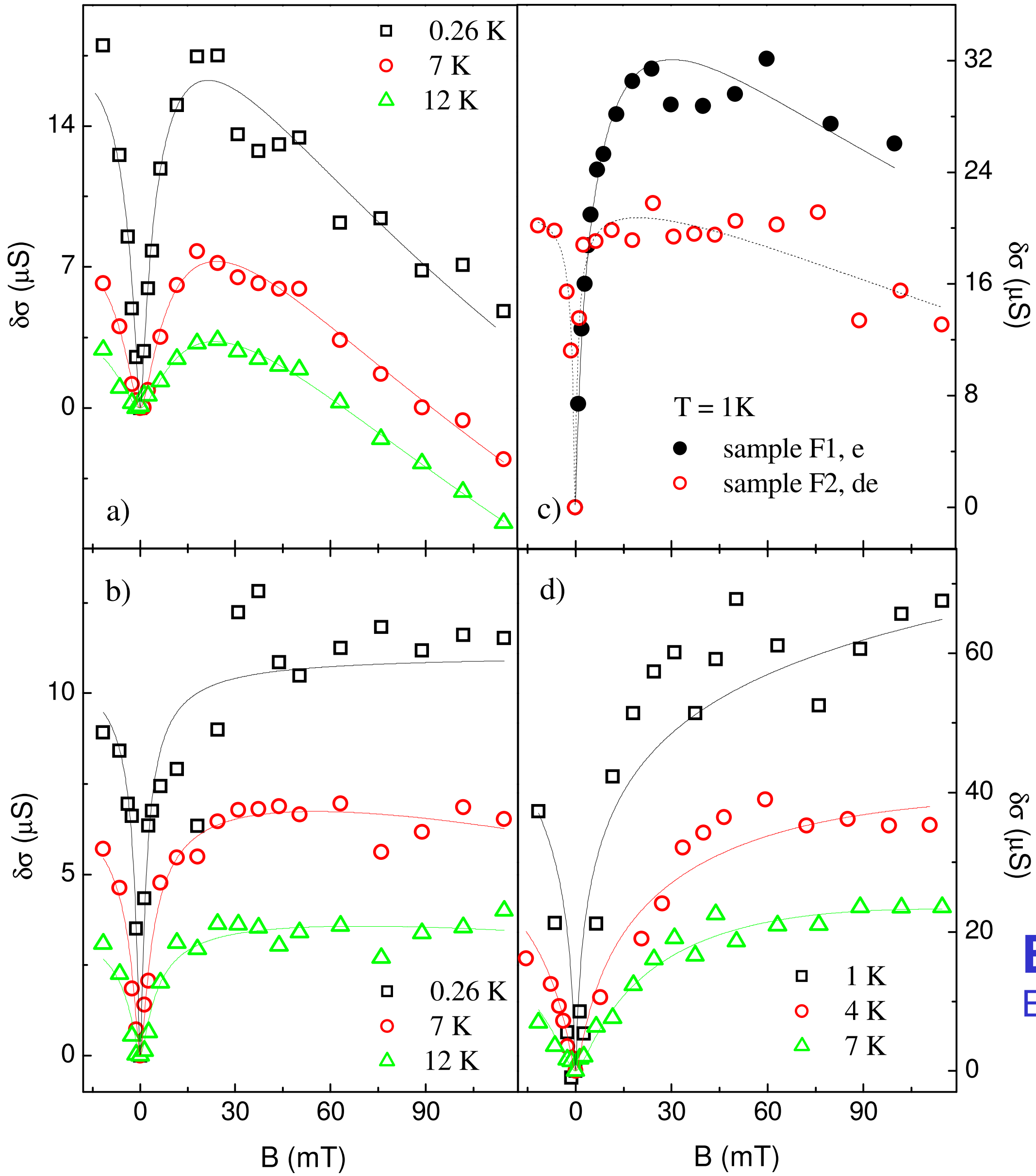
Magnetoconductance of graphene at different *T*

D
Dirac

F1, F2
Electron

D
Electron

B (narrow)
Electron

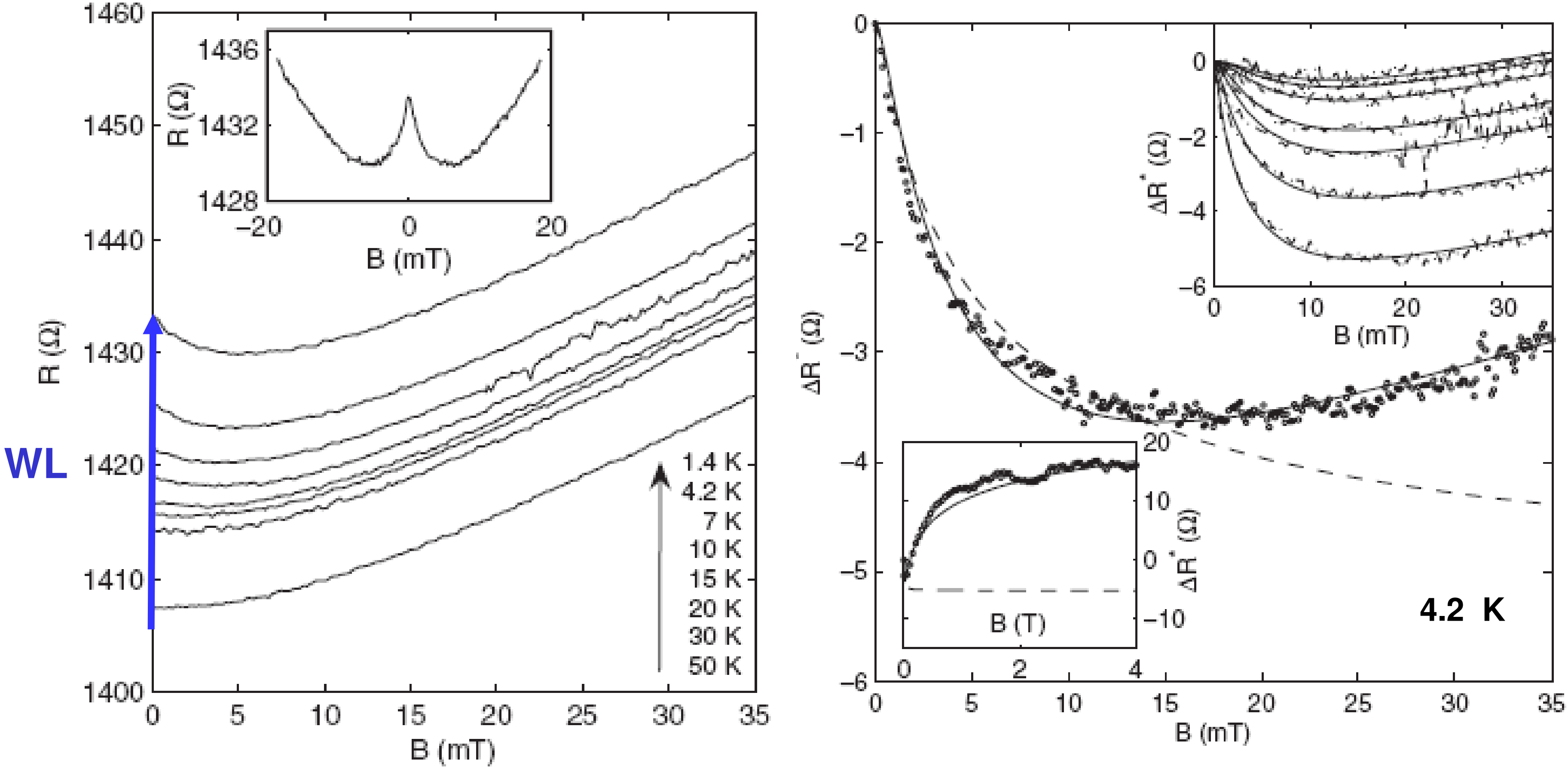


*Tikhonenko, Horsell, Gorbachev, Savchenko, PRL **100**, 056802 (2008)*

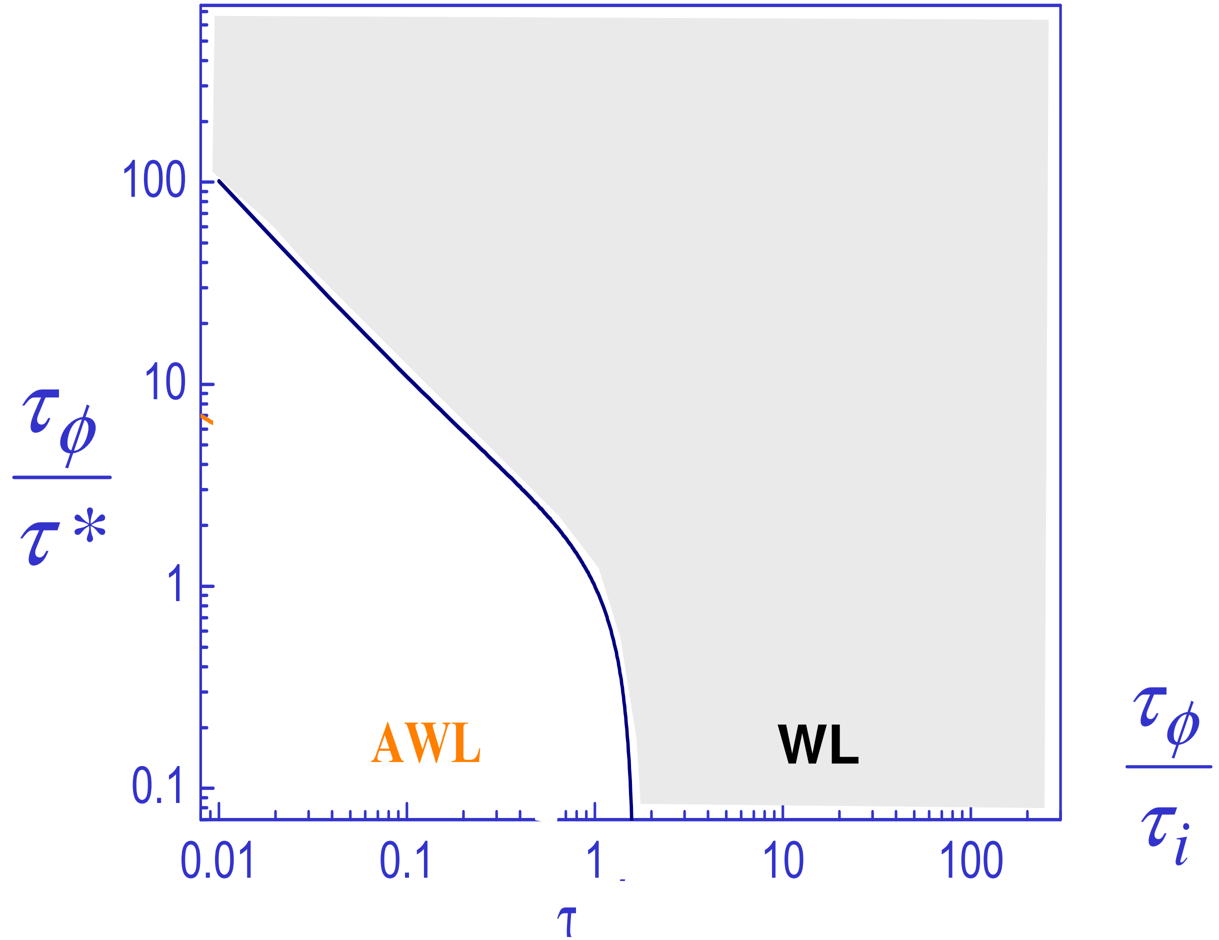
PRL **98,** 136801 (2007)

Weak Antilocalization in Epitaxial Graphene: Evidence for Chiral Electrons

Xiaosong Wu,[1] Xuebin Li,[1] Zhimin Song,[1] Claire Berger,[1,2] and Walt A. de Heer[1]



AWL: *dR/dT* > 0

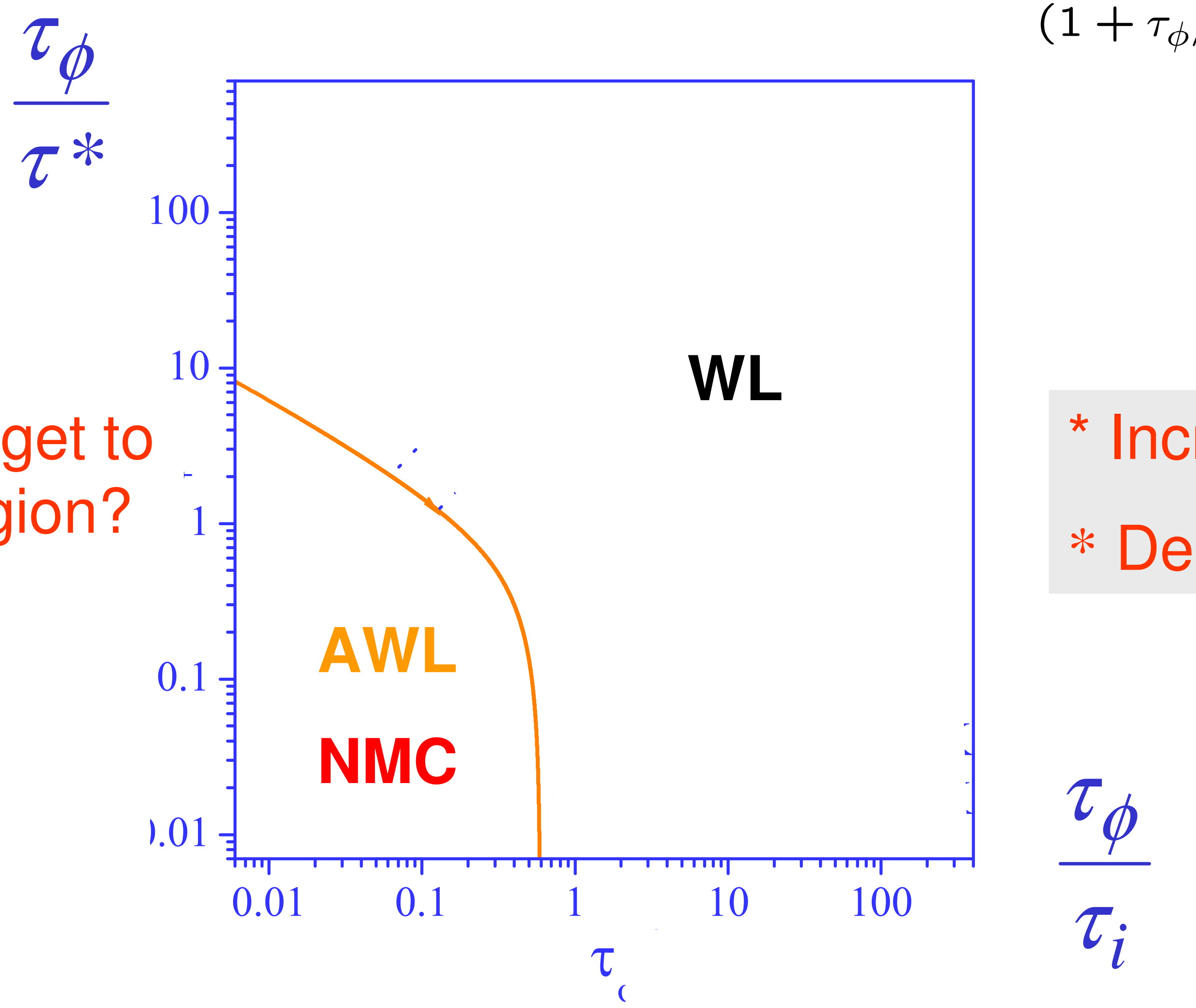


$$\frac{\delta\rho(0)}{\rho^2} = -\delta g = \frac{e^2}{\pi h}\left[\ln(1+2\frac{\tau_\varphi}{\tau_i}) - 2\ln\frac{\tau_\varphi/\tau_{tr}}{1+\frac{\tau_\varphi}{\tau_*}}\right]$$

McCann et al., PRL (2006)

AWL : Positive MC at small *B*

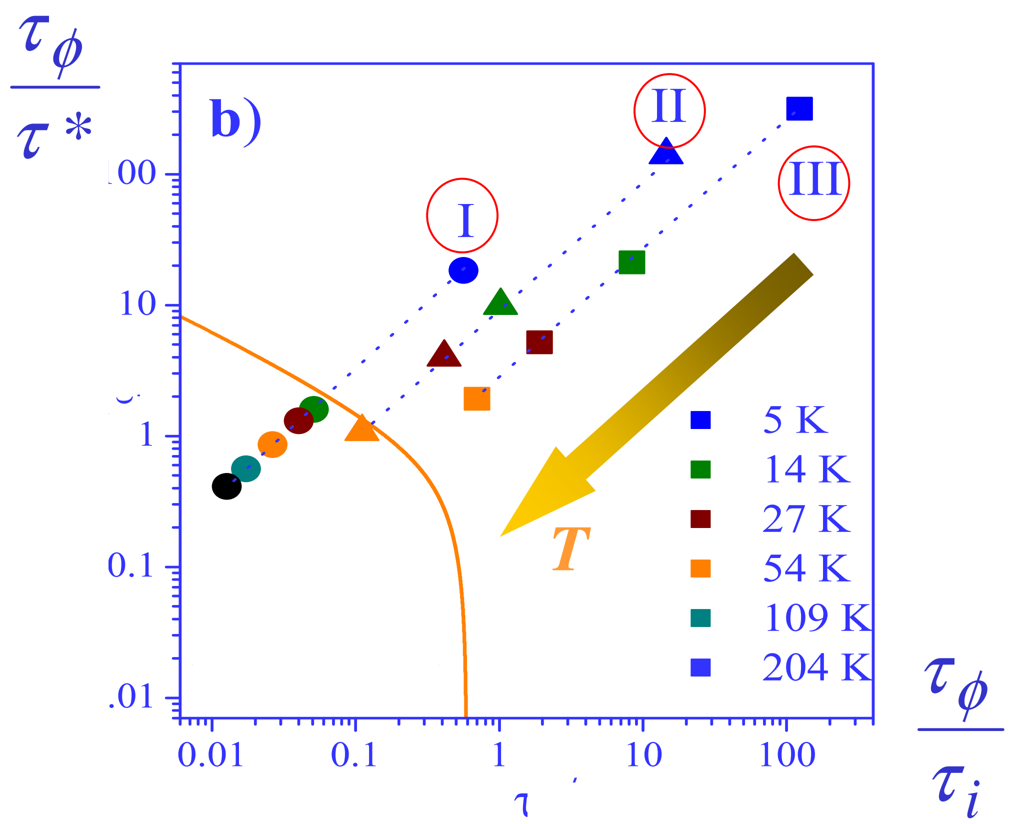
$$\Delta\sigma(B) = \frac{e^2}{24\pi h}\cdot\left(\frac{4eDB\tau_\phi}{\hbar}\right)^2\left(1-\frac{1}{(1+2\tau_\phi/\tau_i)^2} - \frac{2}{(1+\tau_\phi/\tau_i+\tau_\phi/\tau_*)^2}\right).$$

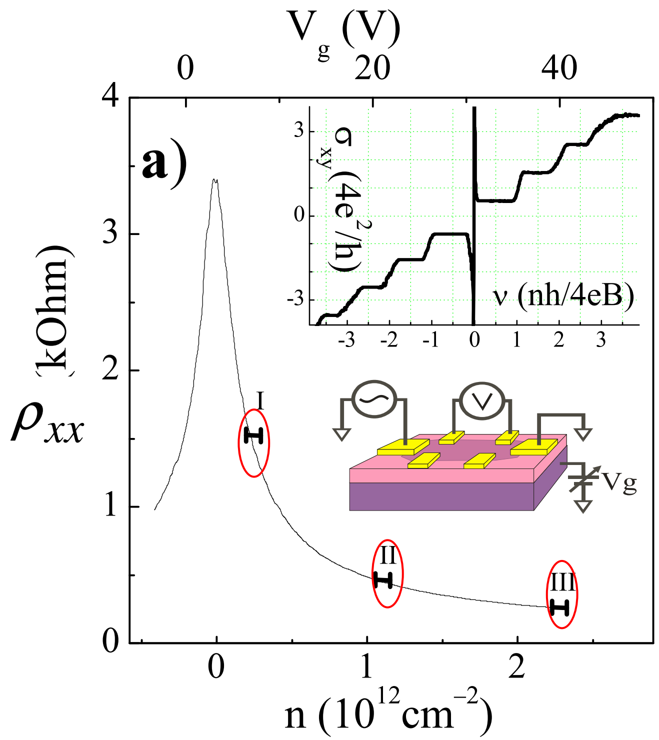


How to get to this region?

* Increase *T*;
* Decrease *n*.

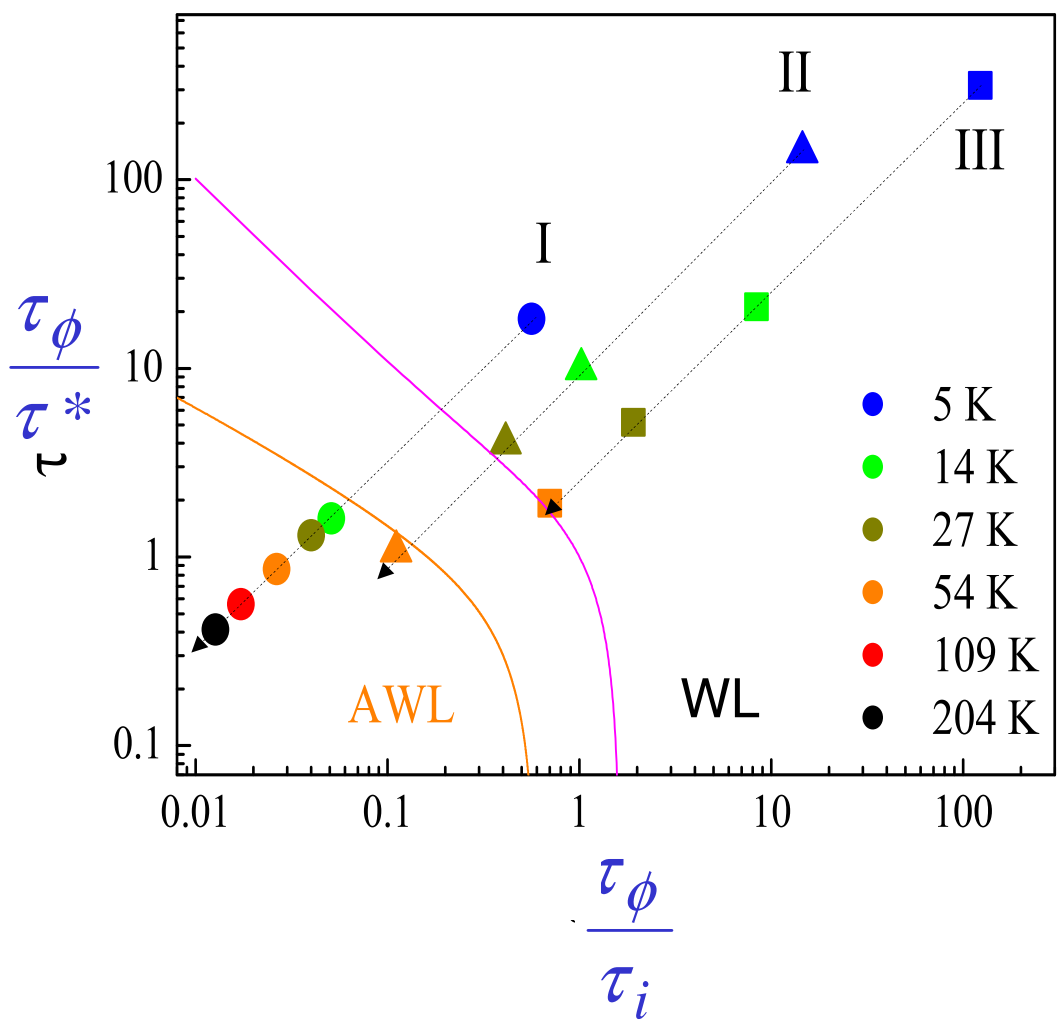
Experimental scattering times at different *n*

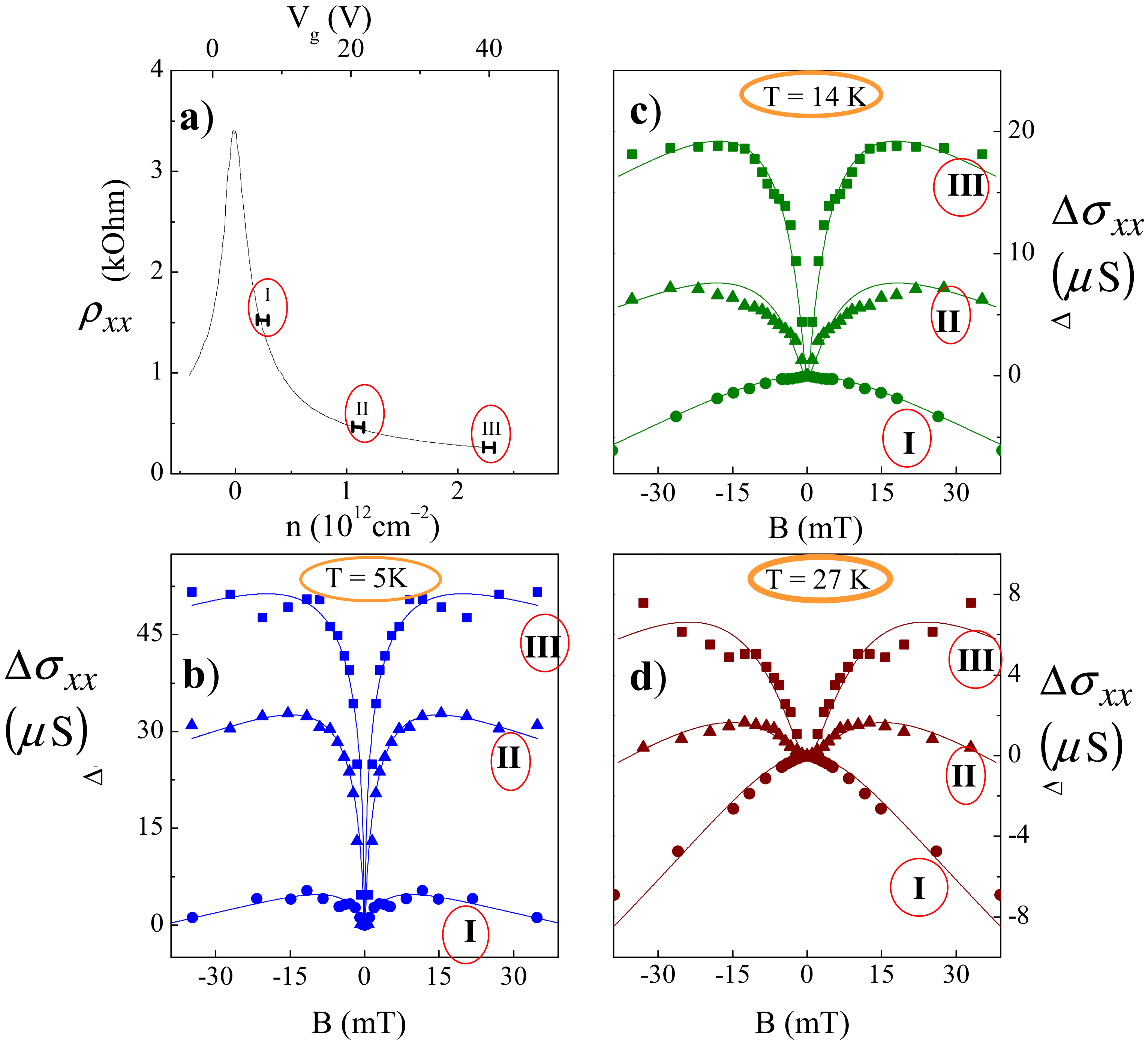




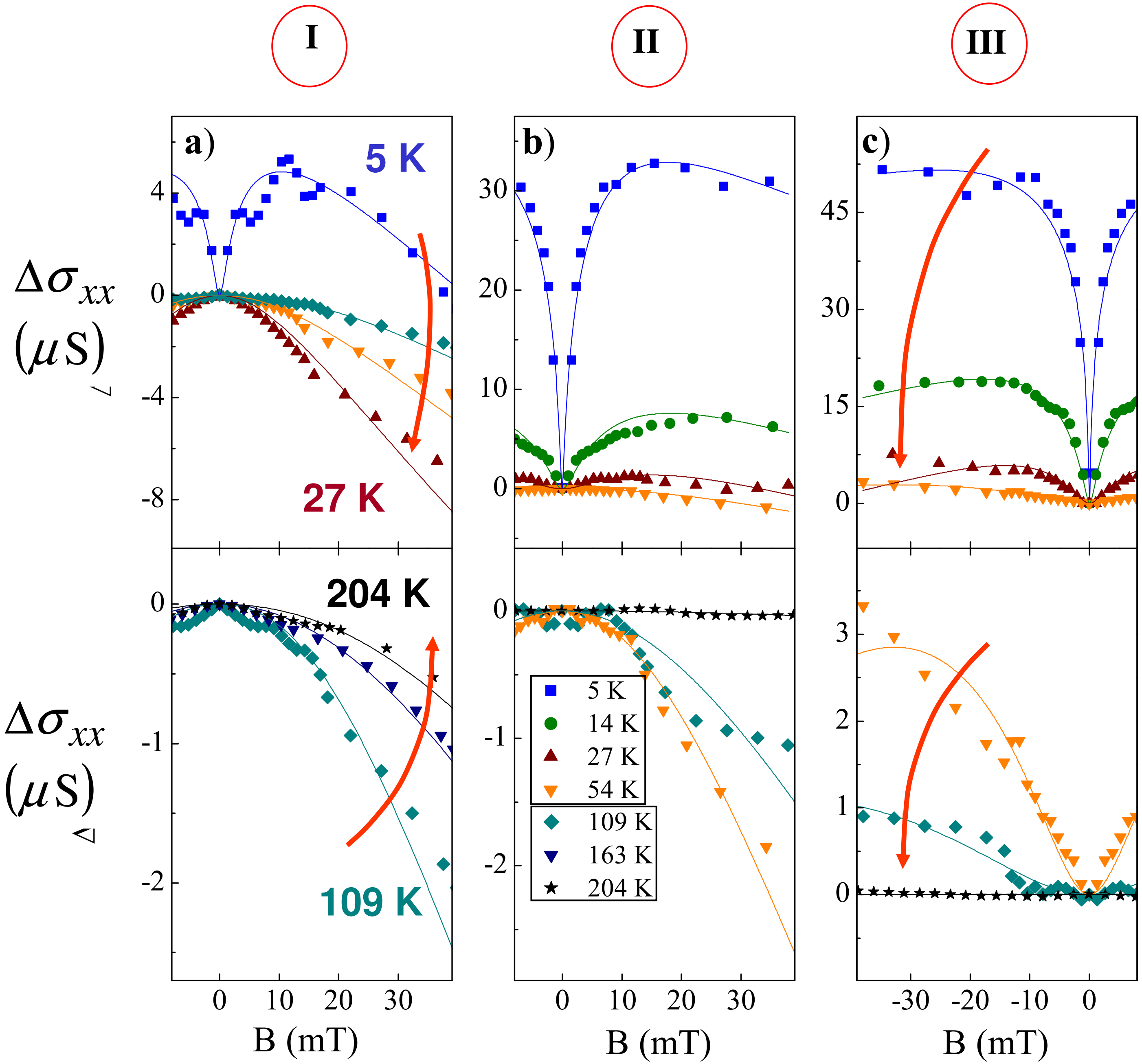
Tikhonenko, Kozikov, Savchenko, Gorbachev, arXiv: 0903.4489

Transition from WL to AWL



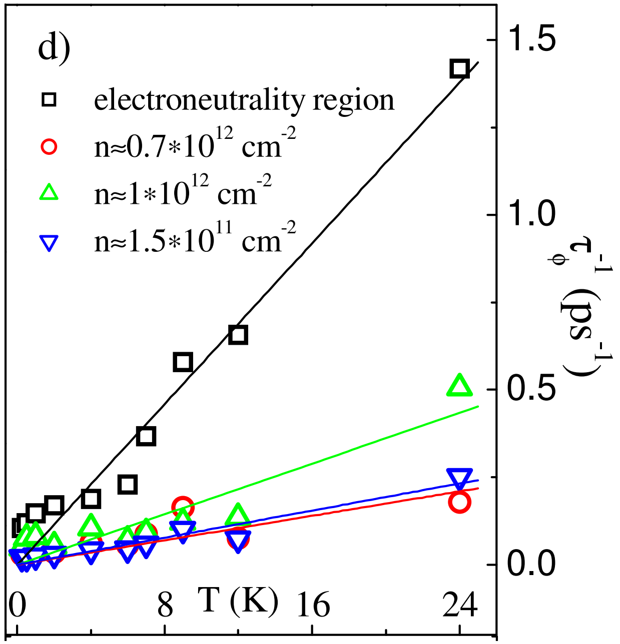


a)
b)
c)
d)
V_g (V)
ρ_{xx} (kOhm)
n (10^{12}cm^{-2})
I
II
III
T = 14 K
T = 5K
T = 27 K
$\Delta\sigma_{xx}$ (μS)
B (mT)



I
II
III
a)
5 K
27 K
b)
c)
204 K
109 K
$\Delta\sigma_{xx}$ (μS)
$\Delta\sigma_{xx}$ (μS)
B (mT)
B (mT)
B (mT)
5 K
14 K
27 K
54 K
109 K
163 K
204 K

Dephasing rate τ_ϕ^{-1}



Electron-electron scattering in 'diffusive' regime, $T\tau_p < 1$:

$$\tau_\phi^{-1} = \alpha \frac{k_B T}{2E_F\tau_p} \ln\left(\frac{2E_F\tau_p}{\hbar}\right) .$$

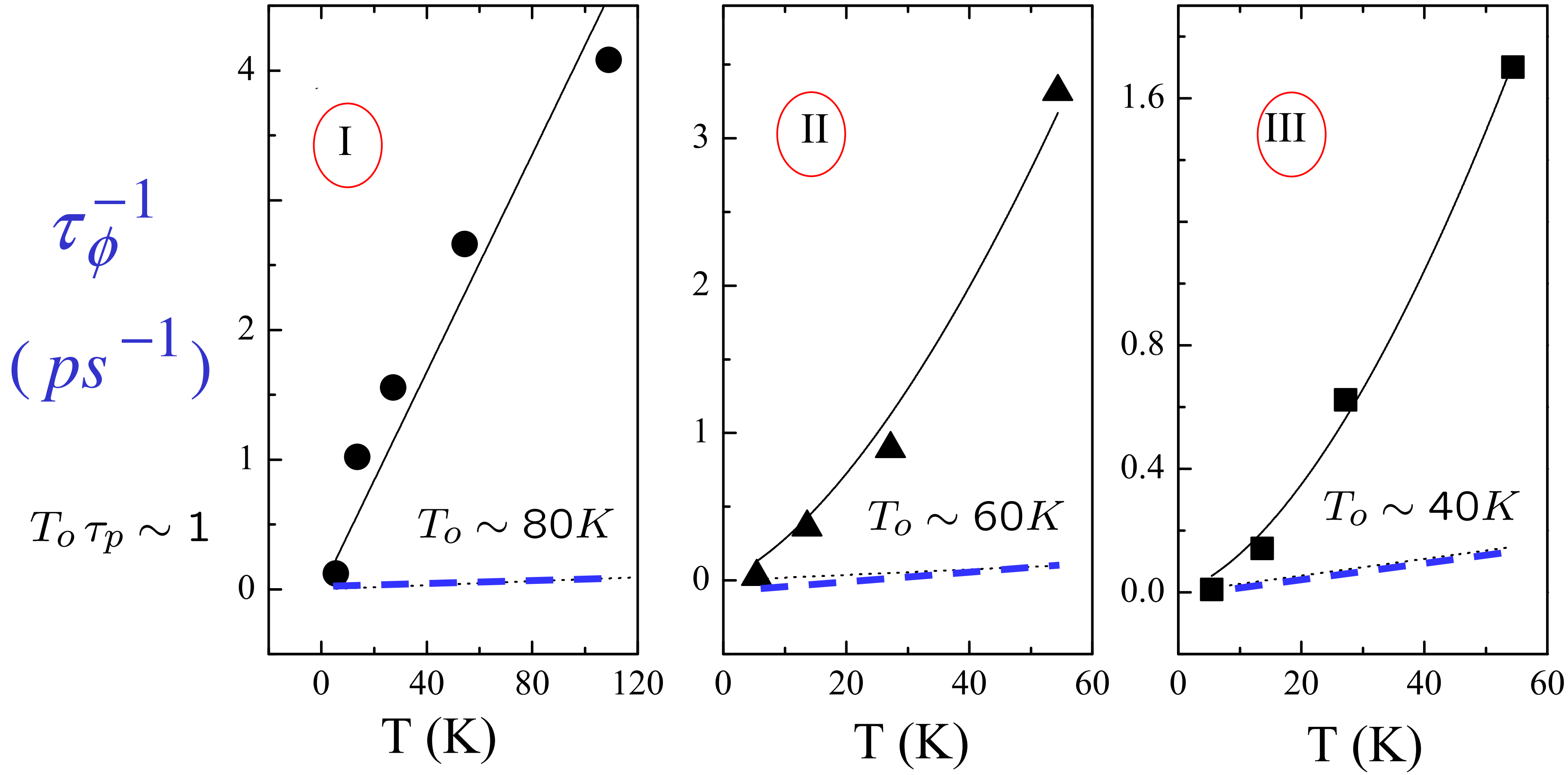
Electron-phonon scattering :

$$\tau_{e-ph}^{-1} = \frac{1}{\hbar^3}\frac{E_F}{4V_F^2}\frac{D_a^2}{\rho_m V_{ph}^2} k_B T .$$

Stauber, Peres, Guinea, PRB (2007)

Hwang, Das Sarma, PRB (2008)

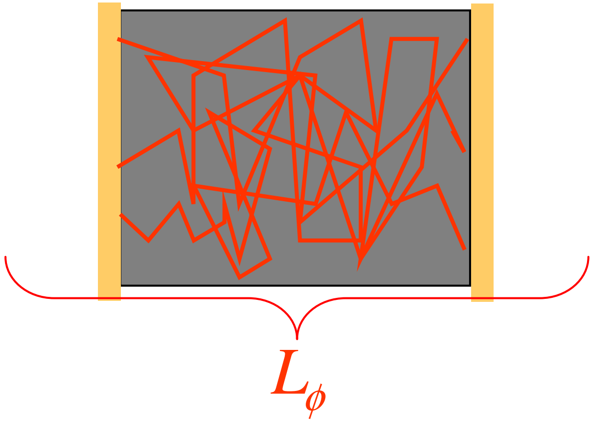
Dephasing rate



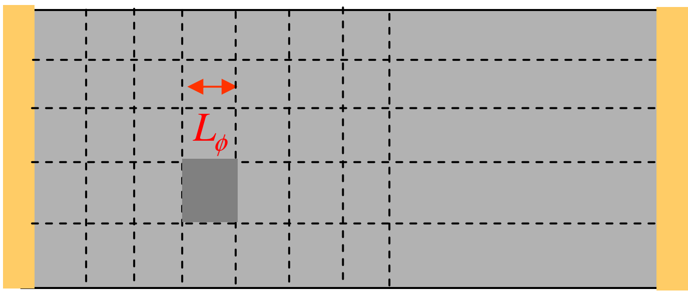
Electron-electron scattering in 'ballistic' regime, $T\tau_p > 1$:

$$\tau_\phi^{-1} = \beta \frac{\pi}{4} \frac{(k_B T)^2}{\hbar E_F} \ln\left(\frac{2E_F}{k_B T}\right) .$$

Reproducible conductance fluctuations, UCF



UCF, $\delta G \approx e^2/h$, are sample 'fingerprints'



$\delta G \approx e^2/h$

Self-averaging of UCF is determined by two lengths:

$L_\phi(T) = \sqrt{D\tau_\phi}$ - dephasing length;

$L_T(T) = \sqrt{\frac{D\hbar}{k_\mathbf{B}T}}$ - 'thermal' length'.

In 2D for $L_T < L_\phi$: $\langle \delta G^2 \rangle \approx (e^2/h)^2 (W/L)(L_T/L)^2$.

UCF in graphene

Theory:

Kharitonov, Efetov (2008)
Kechedzhi, Kashuba, Falko (2008)

In addition to the usual lengths
$L_\phi = \sqrt{D\tau_\phi}$ and $L_T = \sqrt{D\hbar/k_B T}$,
fluctuations are sensitive to *elastic scattering*:

length $L_i = \sqrt{D\tau_i}$ (*inter*-valley) and
length $L^* = \sqrt{D\tau^*}$ (*intra*-valley).

If in a usual metal $\langle \delta G^2 \rangle = \mathcal{R}_1(T, B, L, W)\ (e^2/h)^2$,

in graphene $\langle \delta G^2 \rangle = \alpha \mathcal{R}_1(T, B, L, W)\ (e^2/h)^2$,

where α determines the number of 'channels':

$$\alpha = \begin{cases} 4 & \text{for} \quad L_i, L^* > L_\phi \ ; \\ 2 & \text{for} \quad L_i > L_\phi > L^* \ ; \\ 1 & \text{for} \quad L_\phi > L_i \ . \end{cases}$$

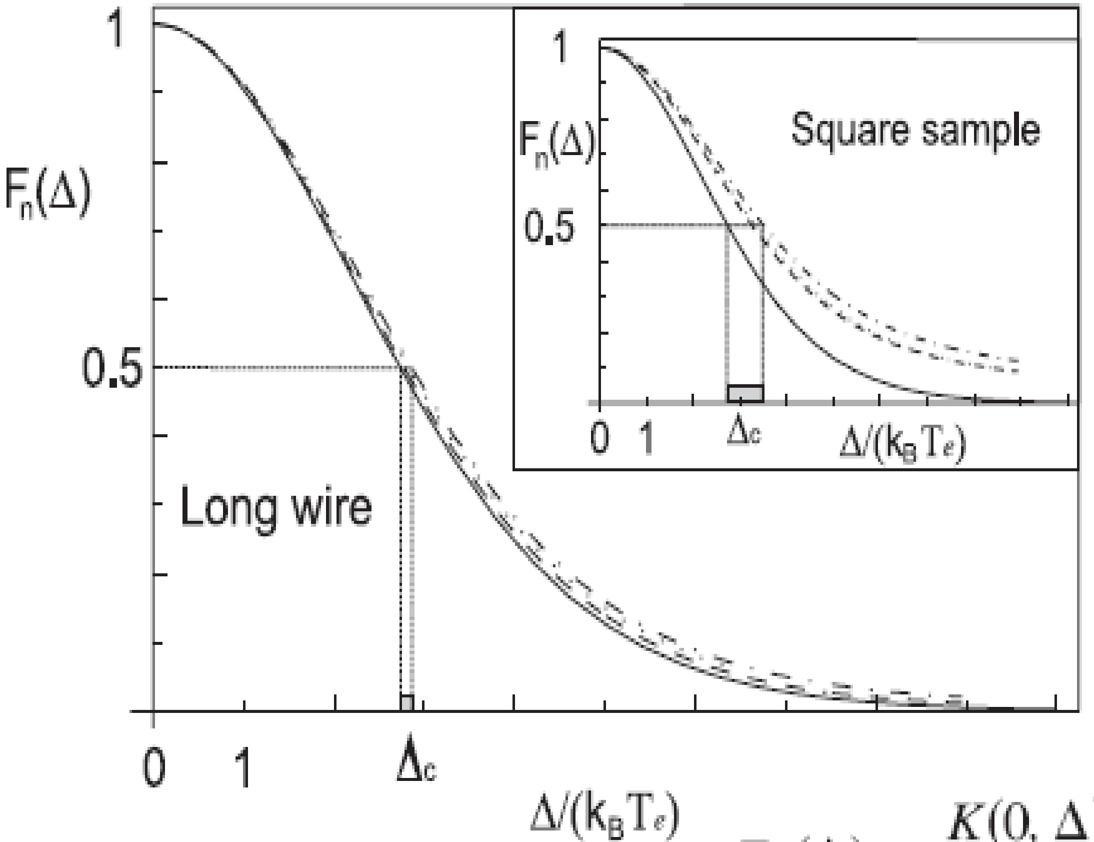
AWL

WL

Note: contrary to *WL*, fluctuations *decrease* with increasing *inter*-valley scattering (*decreasing* L_i).

The normalised correlation function, however, depends only on electron temperature, T_e .

Kechedzhi, Horsell, Tikhonenko, Savchenko, Gorbachev, Lerner, Fal'ko, PRL (2009)



$$F(\Delta) = \langle \delta G(\varepsilon_F)\delta G(\varepsilon_F + \Delta)\rangle$$

$$\Delta_c \approx 2.7\, k_B T_e\ ,$$

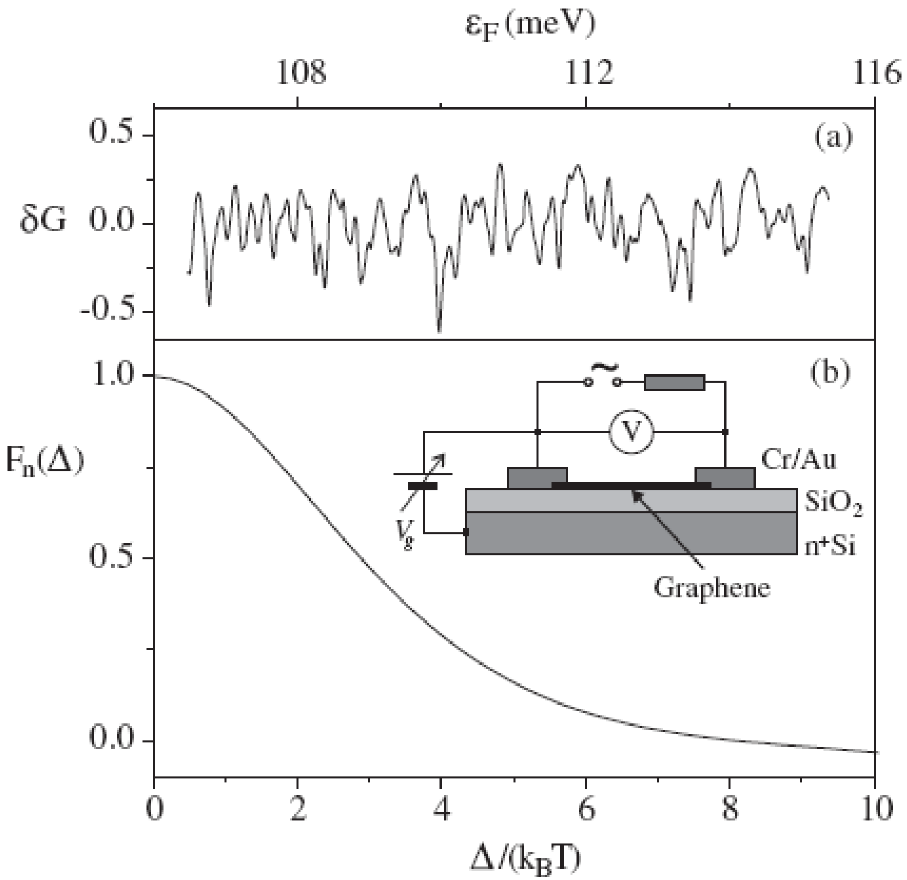
provided

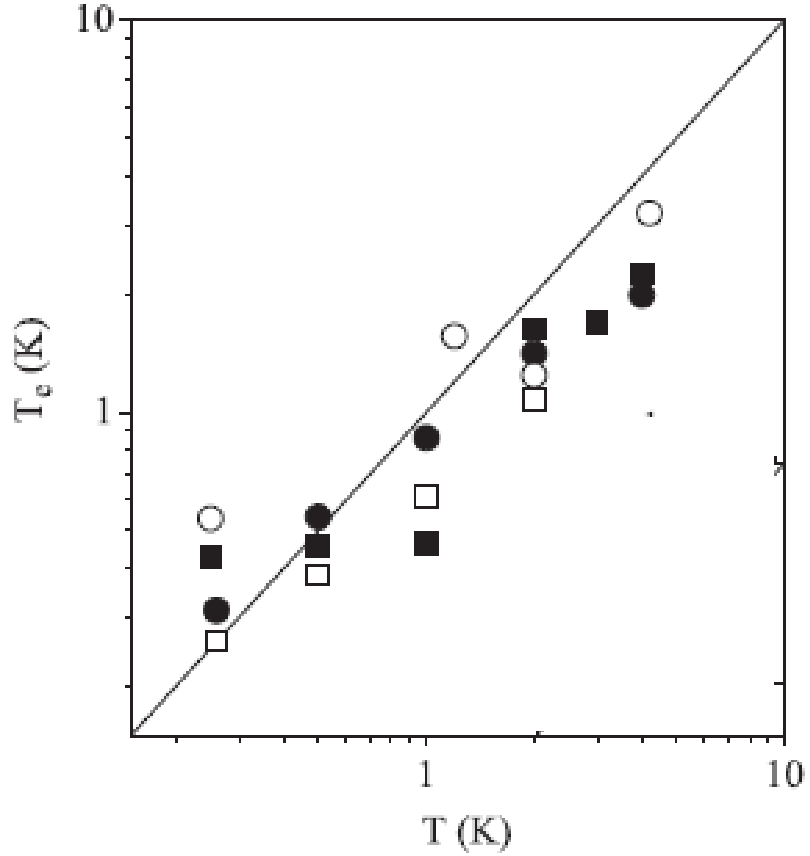
$$L_T \ll \min(L_\varphi, L_x),$$

$$L_y \ll \min(L_\varphi, L_x),$$

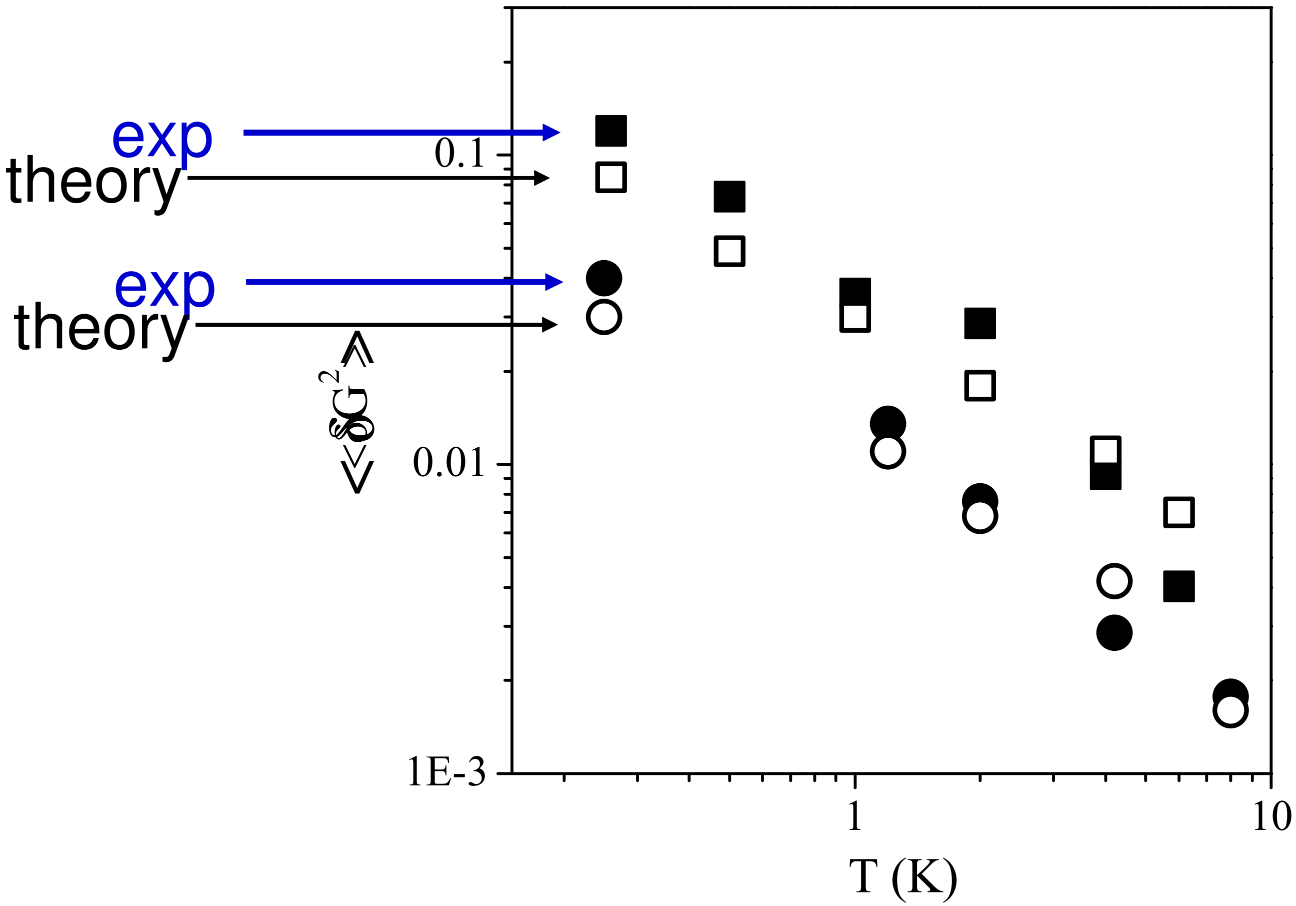
$$F_n(\Delta) = \frac{K(0,\Delta)}{K(0,0)} = \frac{3(\theta\coth\theta - 1)}{\sinh^2\theta}, \qquad \theta \equiv \frac{\Delta}{2k_B T_e}$$

Experimental testing





Temperature dependence of variance



Parameters L_Φ , L_i and L^* are found from experiments on WL.

Theory of conductance fluctuations amplitude gives α~1:

Kechedzhi, Horsell, Tikhonenko, Savchenko, Gorbachev, Lerner, Fal'ko, SSC (2009)

SUMMARY

- Weak localisation in graphene is sensitive to different *elastic* scattering mechanisms.
- By changing n and T one can achieve a transition from WL to AWL due to the Berry phase.
- Weak localisation in graphene exists at high T, with dephasing due to *e*-*e* interactions.
- UCF are also sensitive to *elastic* scattering, but the correlation function is only controlled by T_e.