

# Quantum Hall Effect and Electron-electron interaction in graphene

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# Outlines

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- Review of graphene Quantum Hall effect
  - \* Non-interacting picture
  - \* Interaction mediated SU (4) symmetry breaking of zero LL
- Bulk gap measurements
  - \* IR Spectroscopy
  - \* Transport measurement using Corbino geometry
- SU(8) Symmetry breaking in bilayer graphene sample
- Fractional Quantum Hall Effect in Suspended Graphene

# Quantum Hall Effect in Graphene (2005)

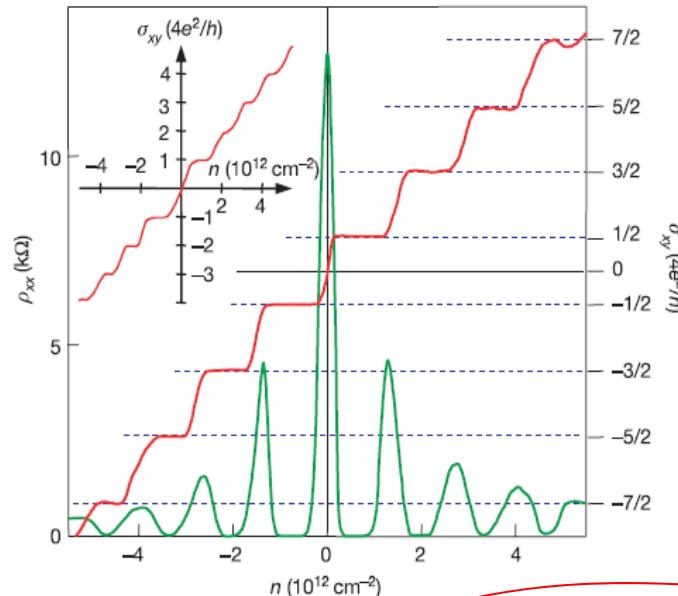
Vol 438 | 10 November 2005 | doi:10.1038/nature04233

nature

LETTERS

## Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov<sup>1</sup>, A. K. Geim<sup>1</sup>, S. V. Morozov<sup>2</sup>, D. Jiang<sup>1</sup>, M. I. Katsnelson<sup>3</sup>, I. V. Grigorieva<sup>1</sup>, S. V. Dubonos<sup>2</sup> & A. A. Firsov<sup>2</sup>



**Quantization:**

$$R_{xy}^{-1} = 4 \left( n + \frac{1}{2} \right) \frac{e^2}{h}$$

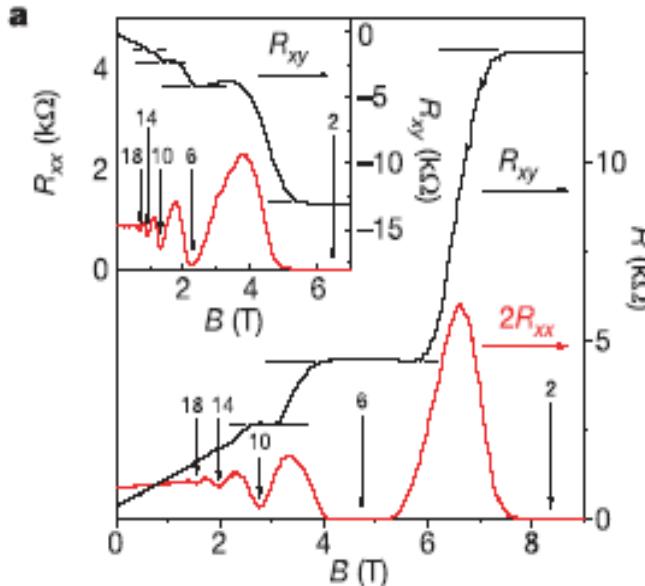
Vol 438 | 10 November 2005 | doi:10.1038/nature04235

nature

LETTERS

## Experimental observation of the quantum Hall effect and Berry's phase in graphene

Yuanbo Zhang<sup>1</sup>, Yan-Wen Tan<sup>1</sup>, Horst L. Stormer<sup>1,2</sup> & Philip Kim<sup>1</sup>



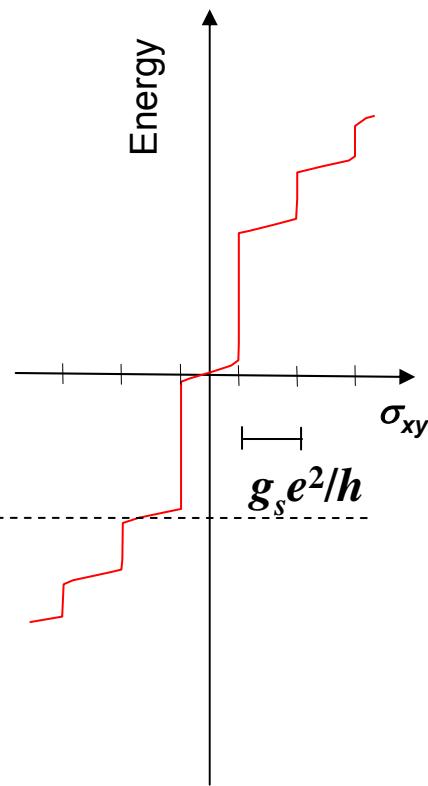
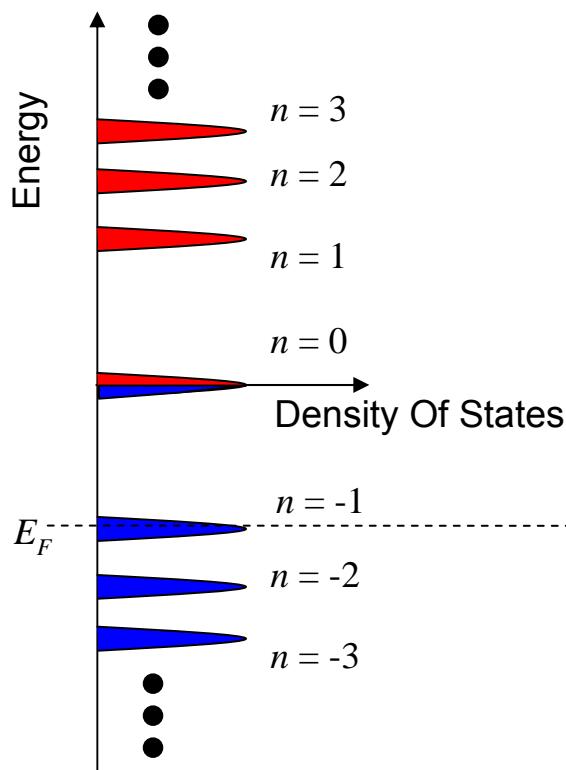
spin (2) X valley (2)

pseudo-spin origin

# Graphene Landau Level and Half Integer QHE

I. I. Rabi, Z. Phys. 49, 507 (1928); McClure, Phys Rev. (1957), Haldane, Phys. Rev. Lett. (1988)

Landau Level  $E_n = \pm \sqrt{2e\hbar v_F^2 |n| B}$



Landau Level Degeneracy  
 $g_s = 4$   
2 for spin and 2 for sublattice

**Quantized Condition**

$$R_{xy}^{-1} = \pm g_s \left(n + \frac{1}{2}\right) \frac{e^2}{h}$$

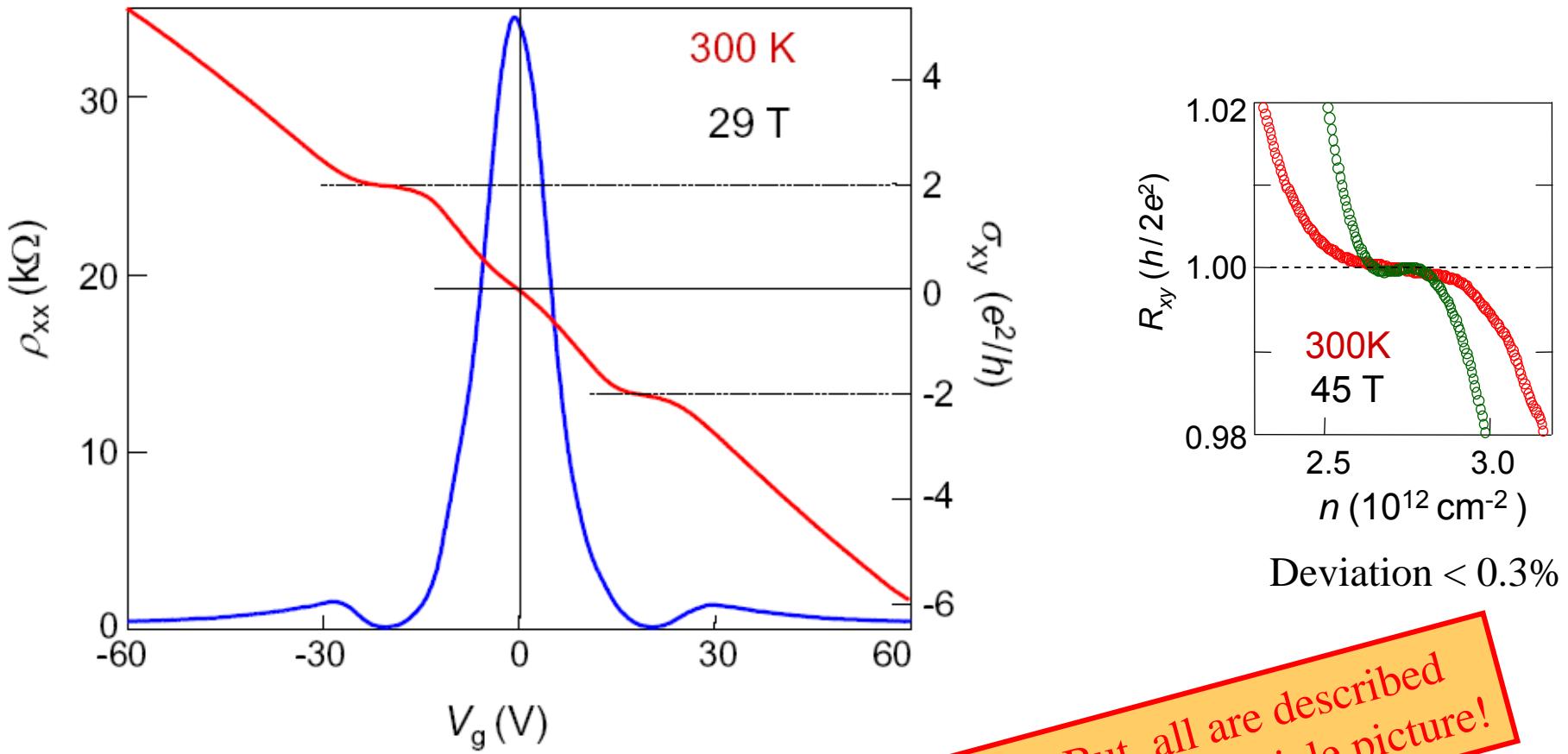
$$\nu = \pm g_s (n + 1/2)$$

LL filling factor

T. Ando et al (2002)

$$E_1 \sim 300K [B(T)]^{1/2}$$

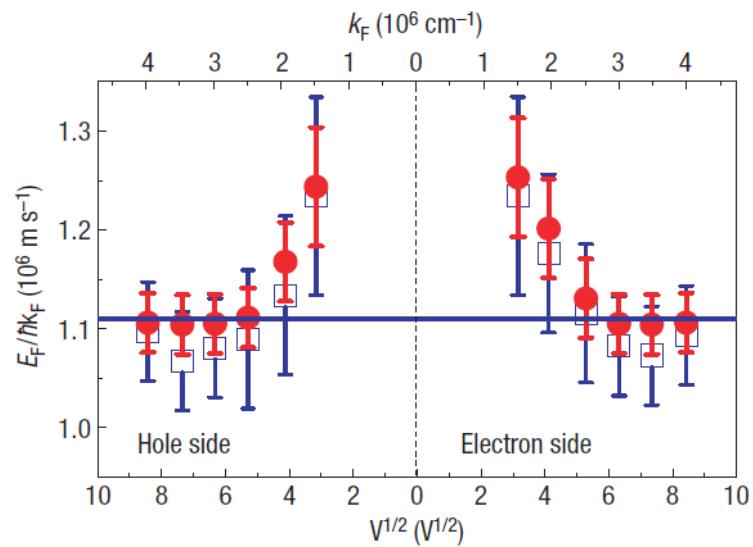
# Room Temperature Quantum Hall Effect



# What is the role of electron-electron interaction in graphene?

Fermi velocity renormalization

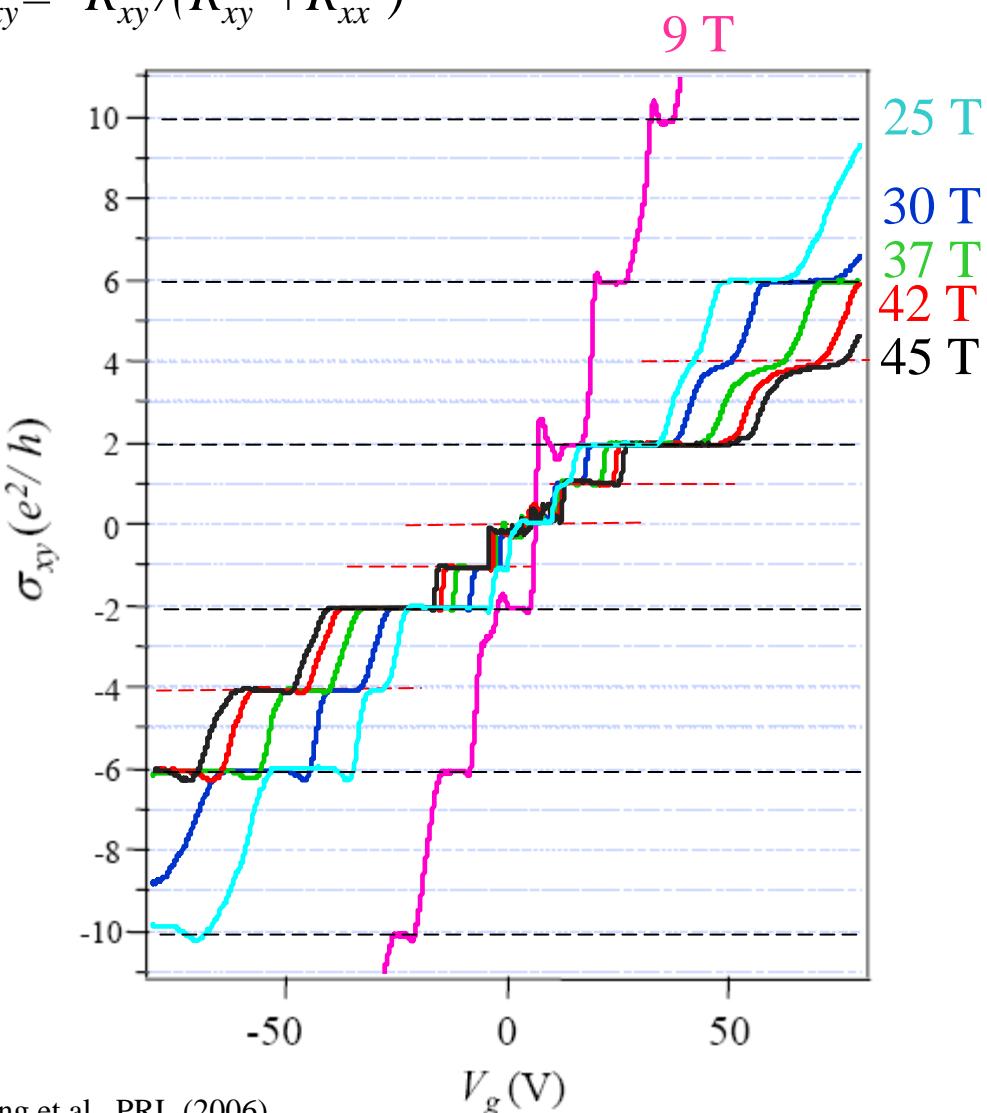
Dirac charge dynamics in graphene by infrared spectroscopy  
Z. Q. Li et al., *Nature Physics* **4**, 532-535 (2008)



In a Landau level:  $E_{e-e} \sim \frac{e^2}{\epsilon \ell_B}$

# Splitting of Landau Levels in High Magnetic Fields

$$\sigma_{xy} = -R_{xy}/(R_{xy}^2 + R_{xx}^2)$$



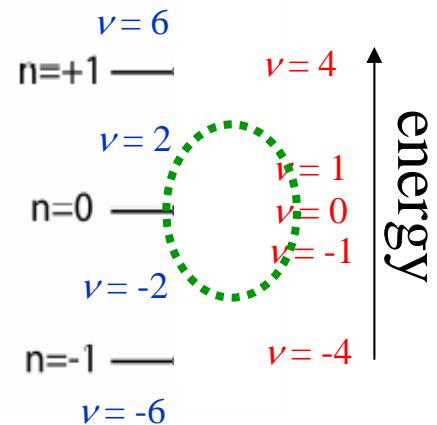
Low fields ( $B < 10$  T)

$$v = \pm 2, \pm 6, \pm 10, \dots$$

High fields ( $B > 20$  T)

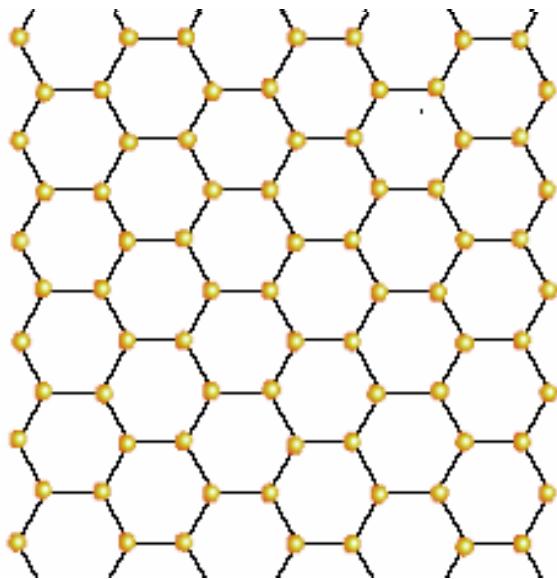
$$v = 0, \pm 1, \pm 2, \pm 4, \pm 6, \dots$$

Landau Level  $E_n = \text{sgn}(n)\sqrt{2e\hbar v_F^2 |n| B}$



Spin & valley symmetry lifted!

# How to break sub-lattice symmetry?



SU(4) Symmetry:  
spin/pseudo spin

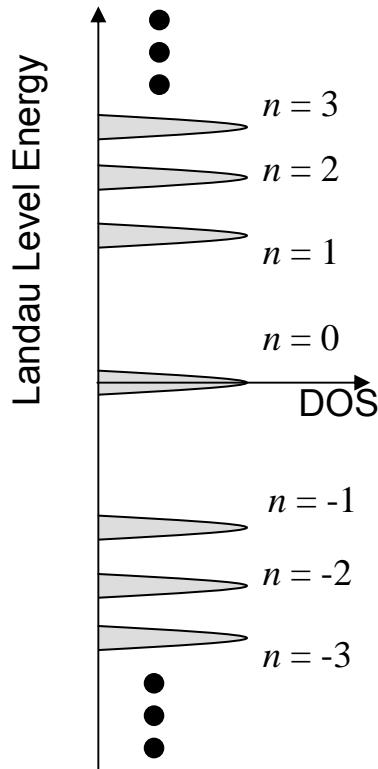
Spontaneous Symmetry Breaking  
Charge density wave, Spin density wave,  
Skyrmions, excitons, and etc

## Theory Reference list (partial)

- [13] K. Nomura, A.H. MacDonald, Phys. Rev. Lett. 96 (2006) 256602.
- [14] M.O. Goerbig, R. Moessner, B. Doucot, Phys. Rev. B 74 (2006) 161407.
- [15] J. Alicea, M.P.A. Fisher, Phys. Rev. B 74 (2006) 075422.
- [16] Kun Yang, S. Das Sarma, A.H. MacDonald, Phys. Rev. B 74 (2006) 075423.
- [17] Dmitry A. Abanin, Patrick A. Lee, Leonid S. Levitov. [cond-mat/0611062](#).
- [18] V.P. Gusynin, V.A. Miransky, S.G. Sharapov, I.A. Shovkovy, Phys. Rev. B 74 (2006) 195429.
- [19] V.P. Gusynin, V.A. Miransky, S.G. Sharapov, I.A. Shovkovy. [cond-mat/0612488](#).
- [20] I.F. Herbut, Phys. Rev. B 75 (2007) 165411.
- [21] M. Ezawa, [cond-mat/0609612](#); [cond-mat/0606084](#).
- [22] D.V. Khveshchenko, Phys. Rev. Lett. 87 (2001) 206401.
- [23] Jean-Noël Fuchs, Pascal Lederer, Phys. Rev. Lett. 98 (2007) 016803; [cond-mat/0612386](#).

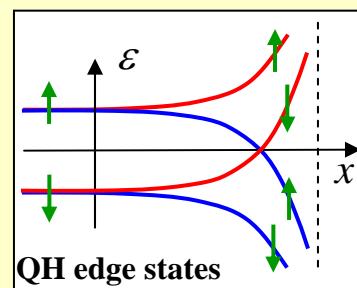
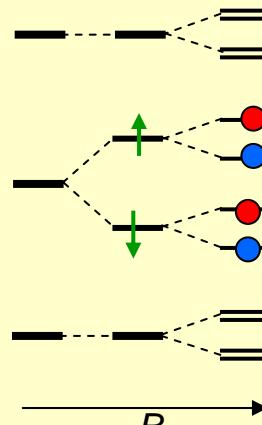
# Quantum Hall Insulator OR Quantum Hall Ferromagnet?

Low magnetic field

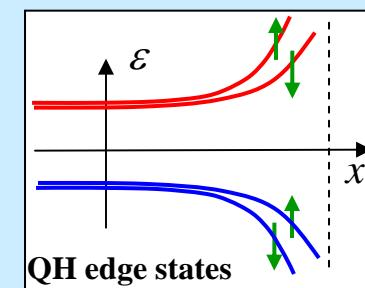
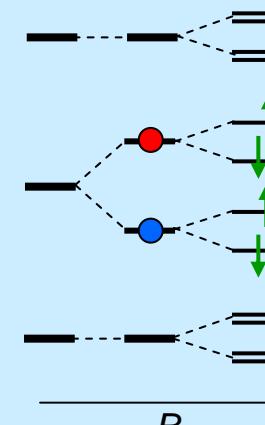


High magnetic field degeneracy break: two scenarios

QHE Ferromagnet  
Spin  $\rightarrow$  Pseudo Spin



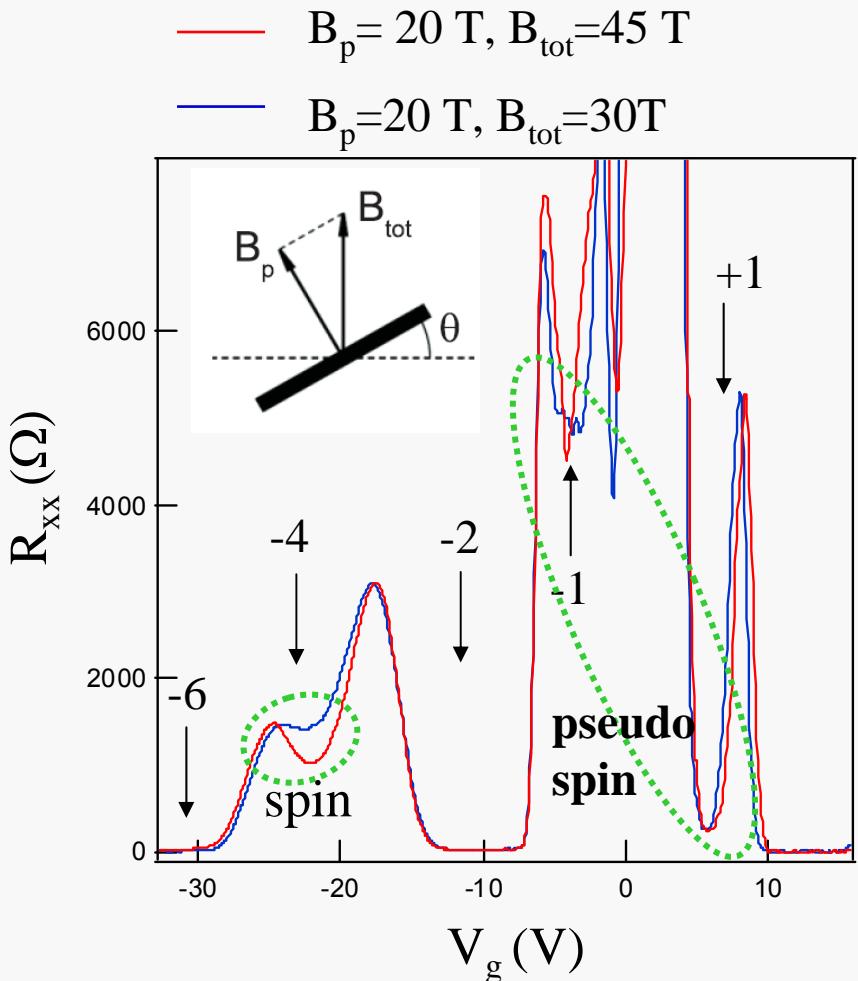
QHE Insulator  
Pseudo Spin  $\rightarrow$  Spin



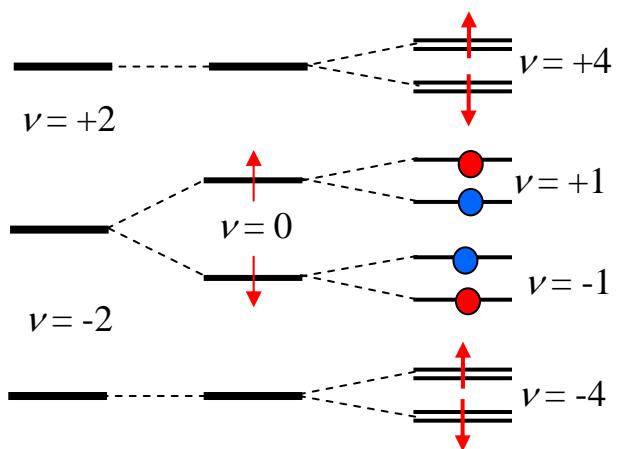
Spin & Pseudo-spin degenerate

# Spin or Pseudo Spin Splitting?

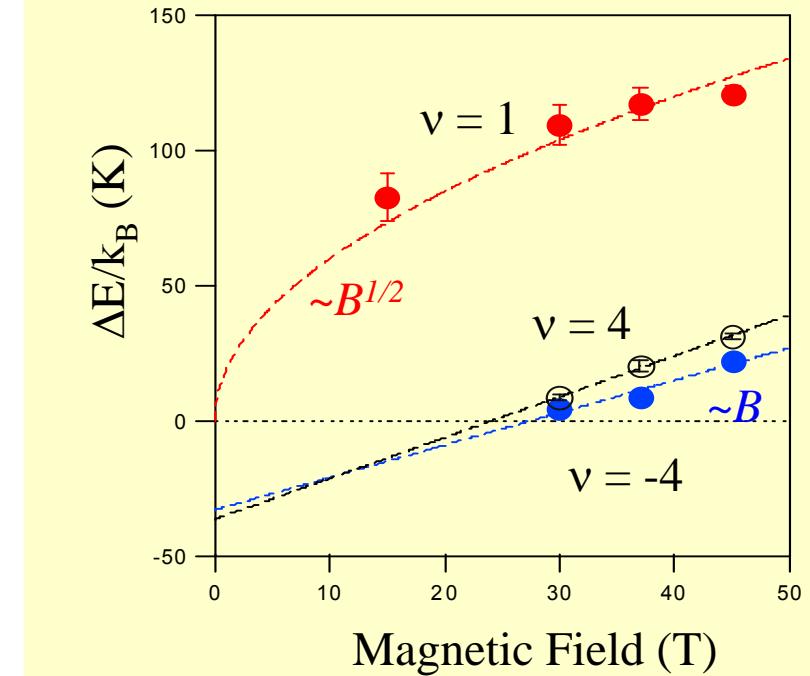
## Tilted Magnetic Field



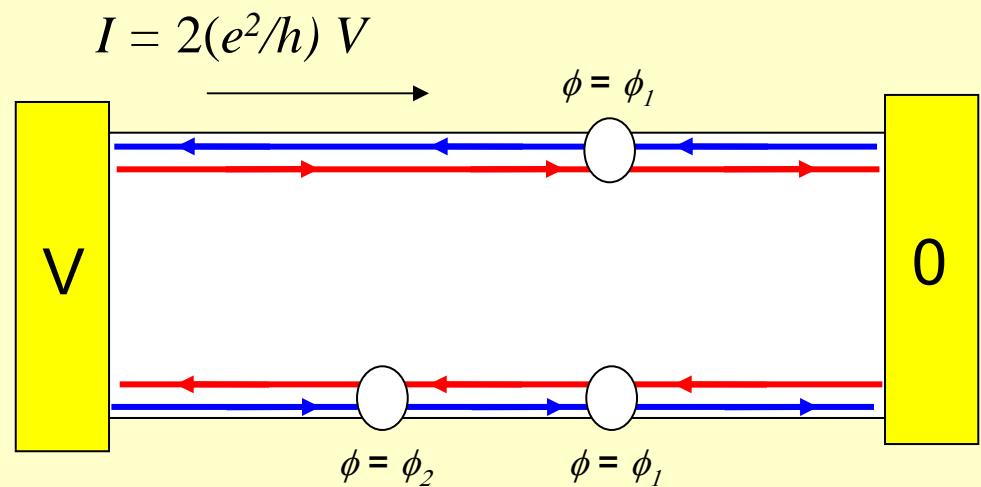
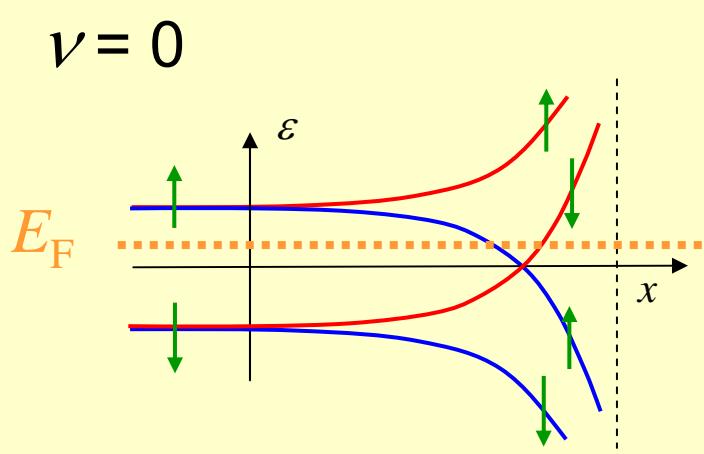
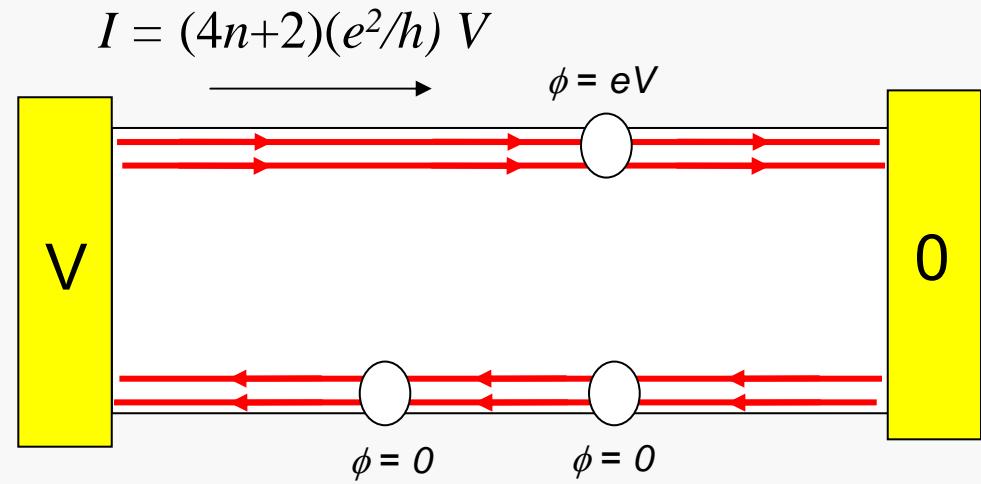
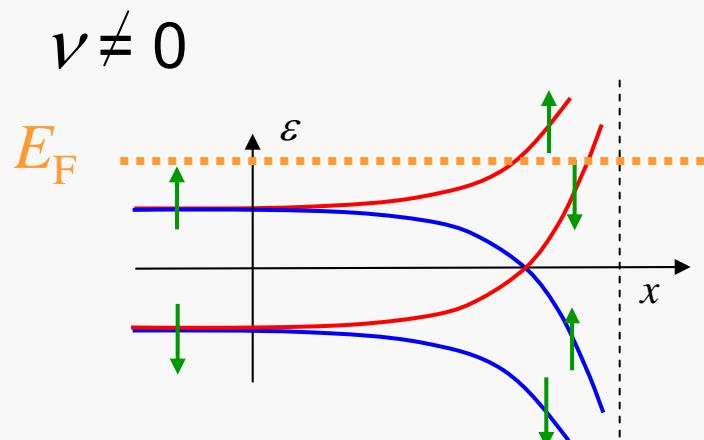
Quantum Hall Ferromagnet!



## Activation Energy Gap Measurements



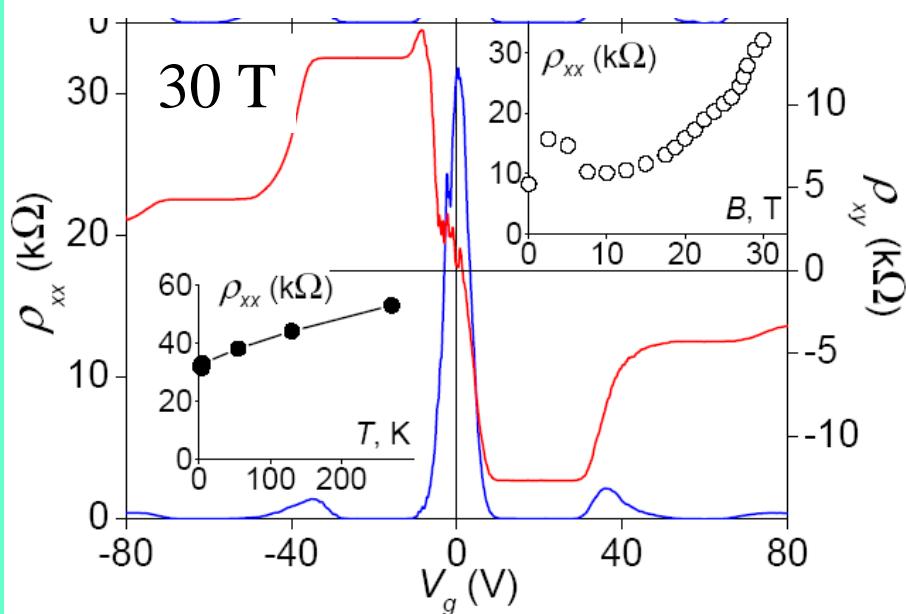
# Graphene QH Edge States for Quantum Hall Ferromagnet



We expect metallic states all gate voltages!

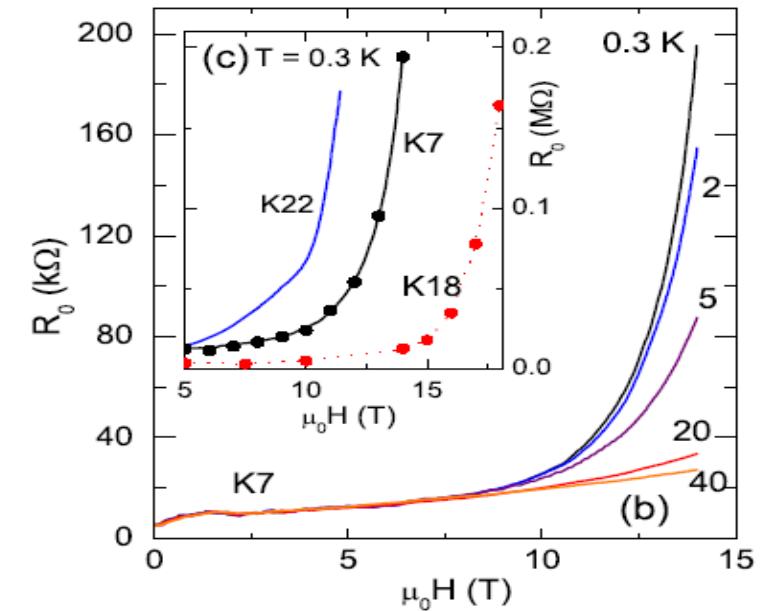
Abanin, et al., Phys. Rev. Lett. 98, 196806 (2007)

# Resistance Maximum for $\nu = 0$ Quantum Hall State



Abanin, et al., Phys. Rev. Lett. 98, 196806 (2007)

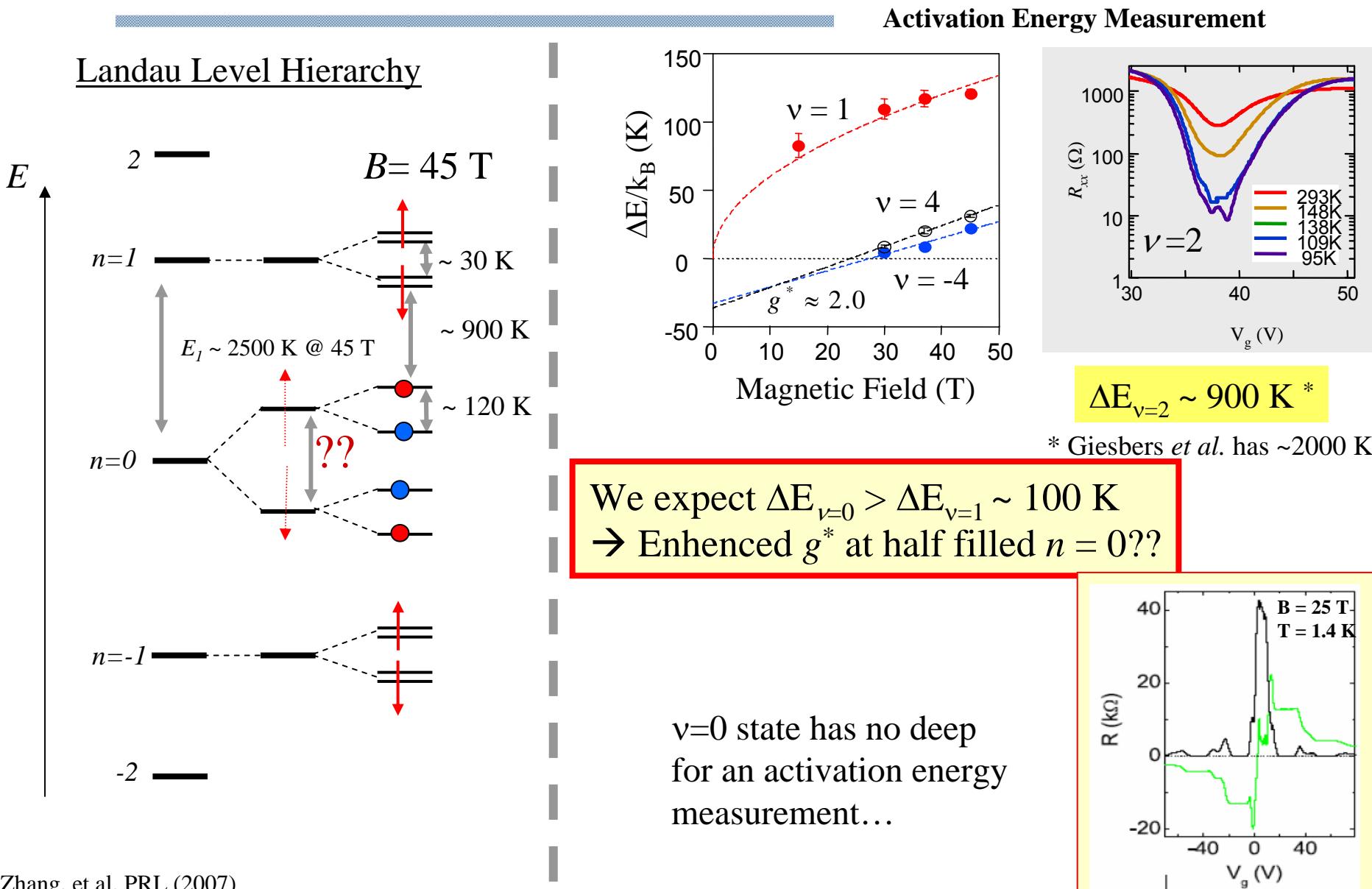
Metallic temperature behavior  
 $\rho_{xx} < 40$  kΩ @ 30 T



J. Chekelsky, L. Li, N. P. Ong, PRL (2007) PRB (2008)

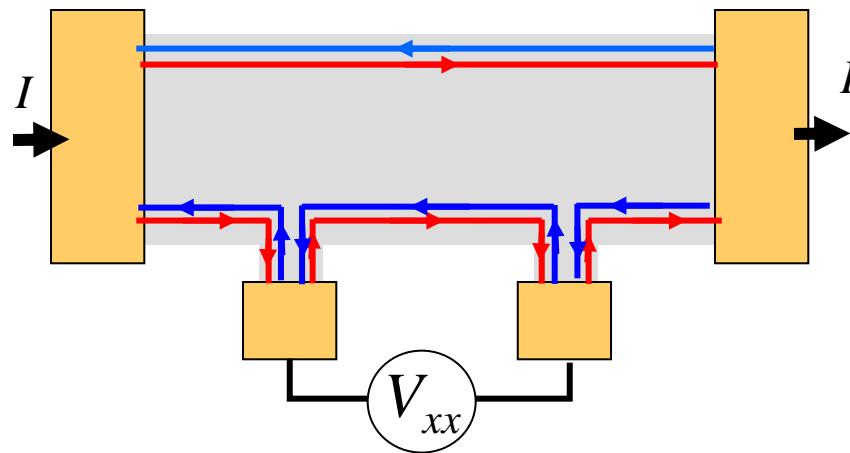
Insulator like behaviors for clean samples  
at high magnetic field 30 T.

# Probing the Nature of $\nu=0$ QH state : Energy Gap



# Transport Gap Measurement at the Dirac Point

## Hall Bar Measurement



In the Quantum Hall regime

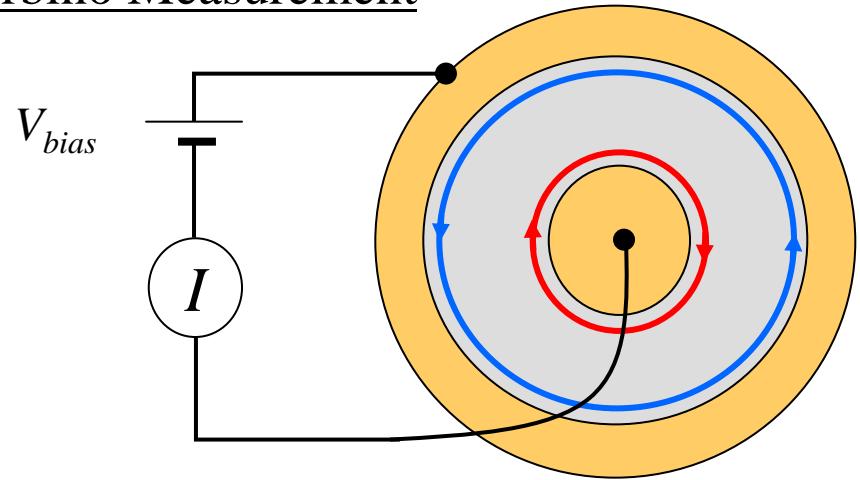
Filling factor

$$\nu \neq 0 : R_{xx} = V_{xx}/I = 0$$

$$\nu = 0 : R_{xx} \neq 0$$

(only in graphene, see Abanin *et al.*)

## Corbino Measurement



In the Quantum Hall regime

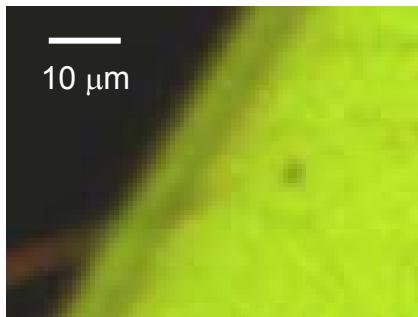
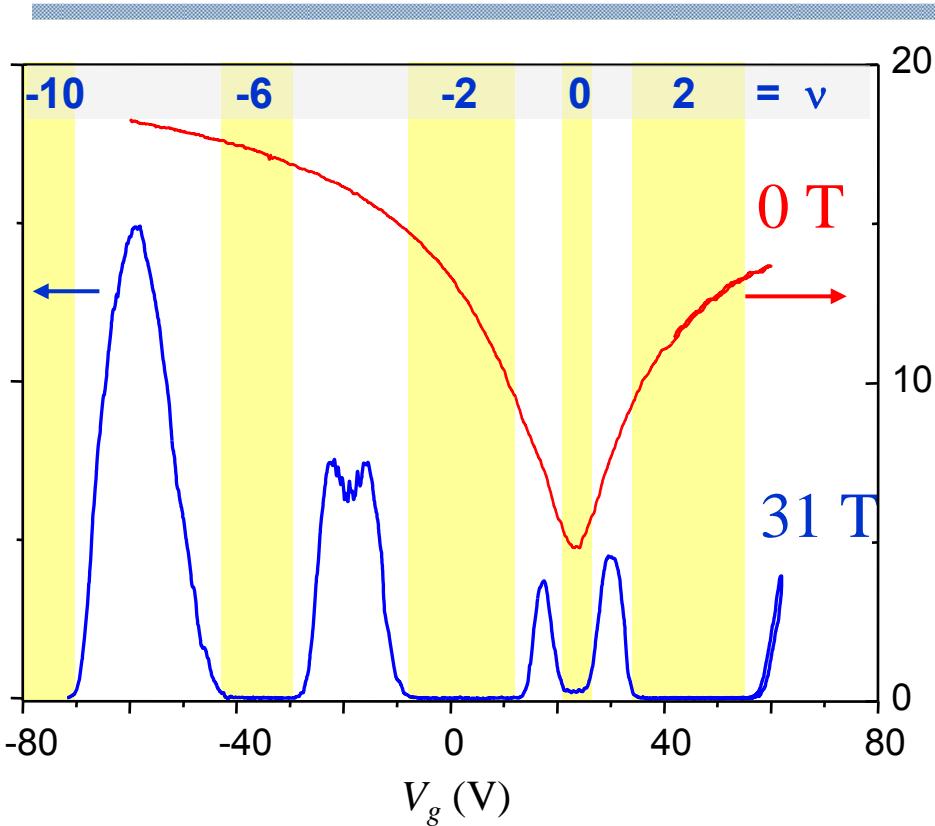
$$\sigma_{xx} = I / V_{bias} = 0$$



## Graphene Corbino

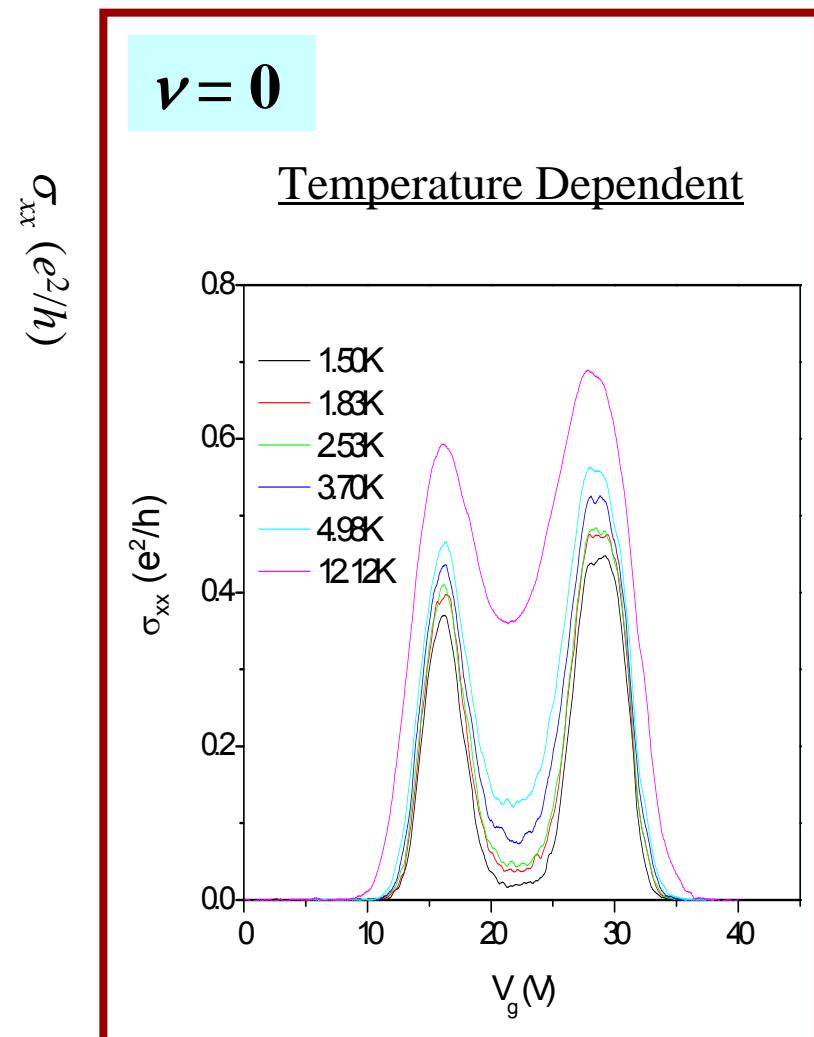
- Dielectric layer deposition
- Contact via fabrication
- Inner electrode contacted by a ground plane

# Quantum Hall Effect in Graphene Corbino Device



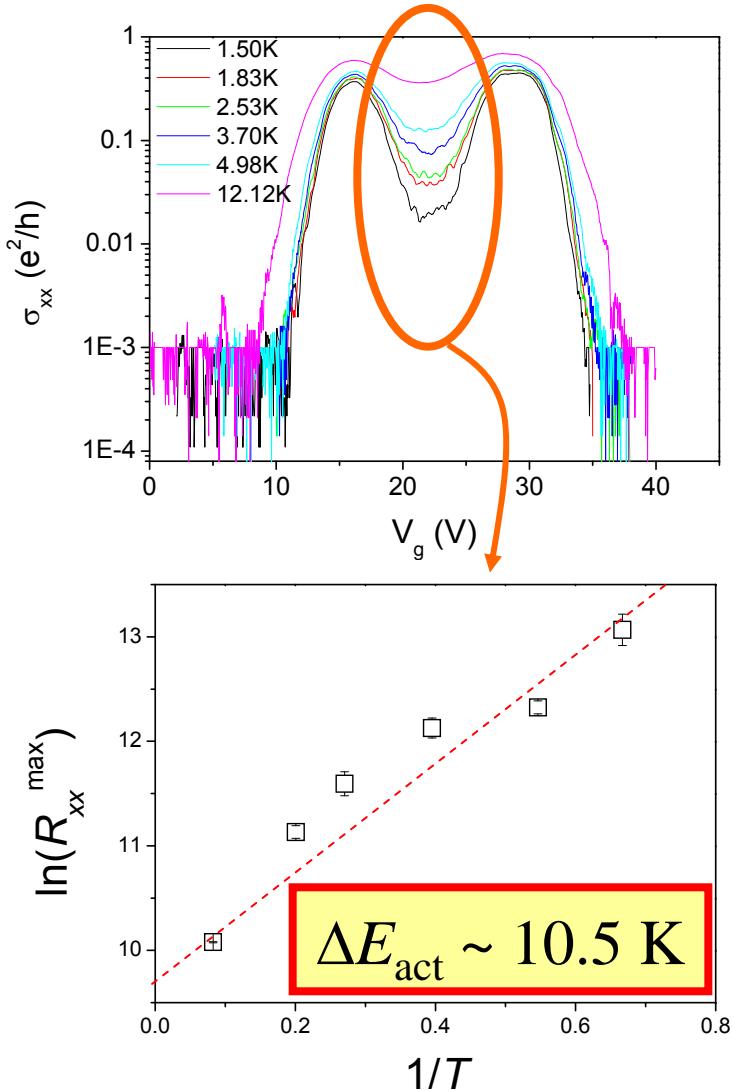
$T = 1.5 \text{ K}$

Mobility  $\sim 10,000 \text{ cm}^2/\text{Vsec}$

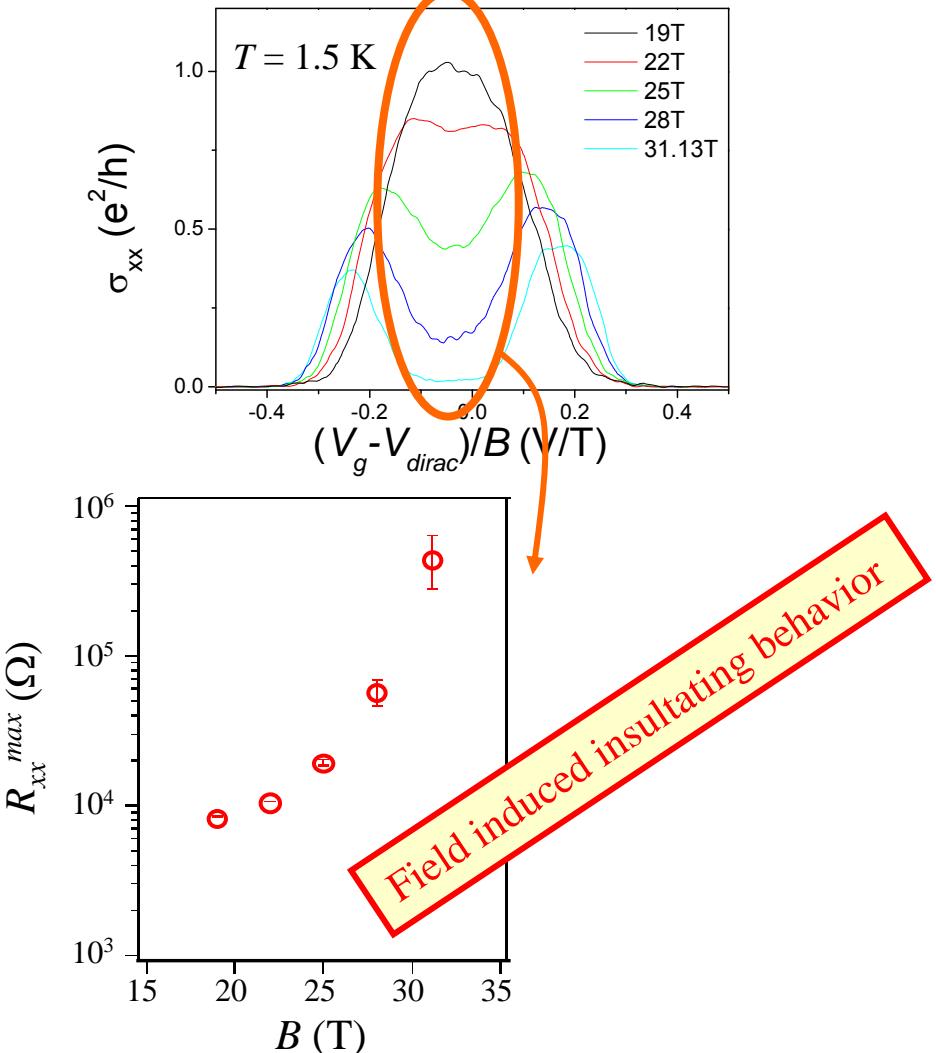


# Transport Gap in $\nu=0$ state

## Activation Energy Gap at 31T



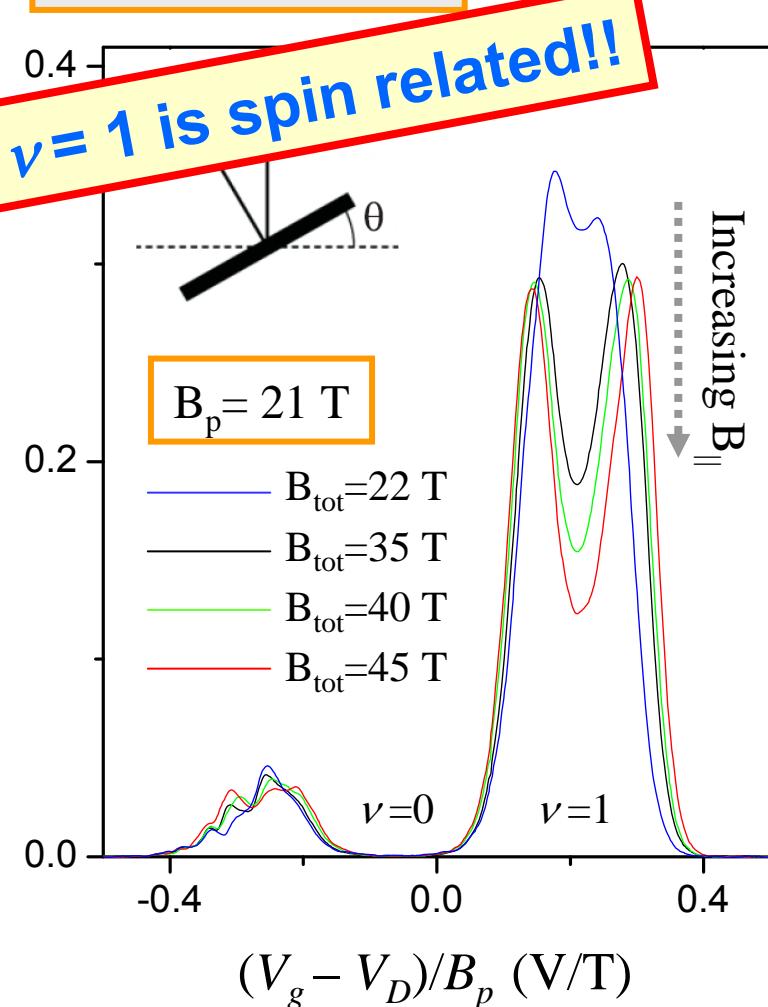
## Magnetic Field Dependence



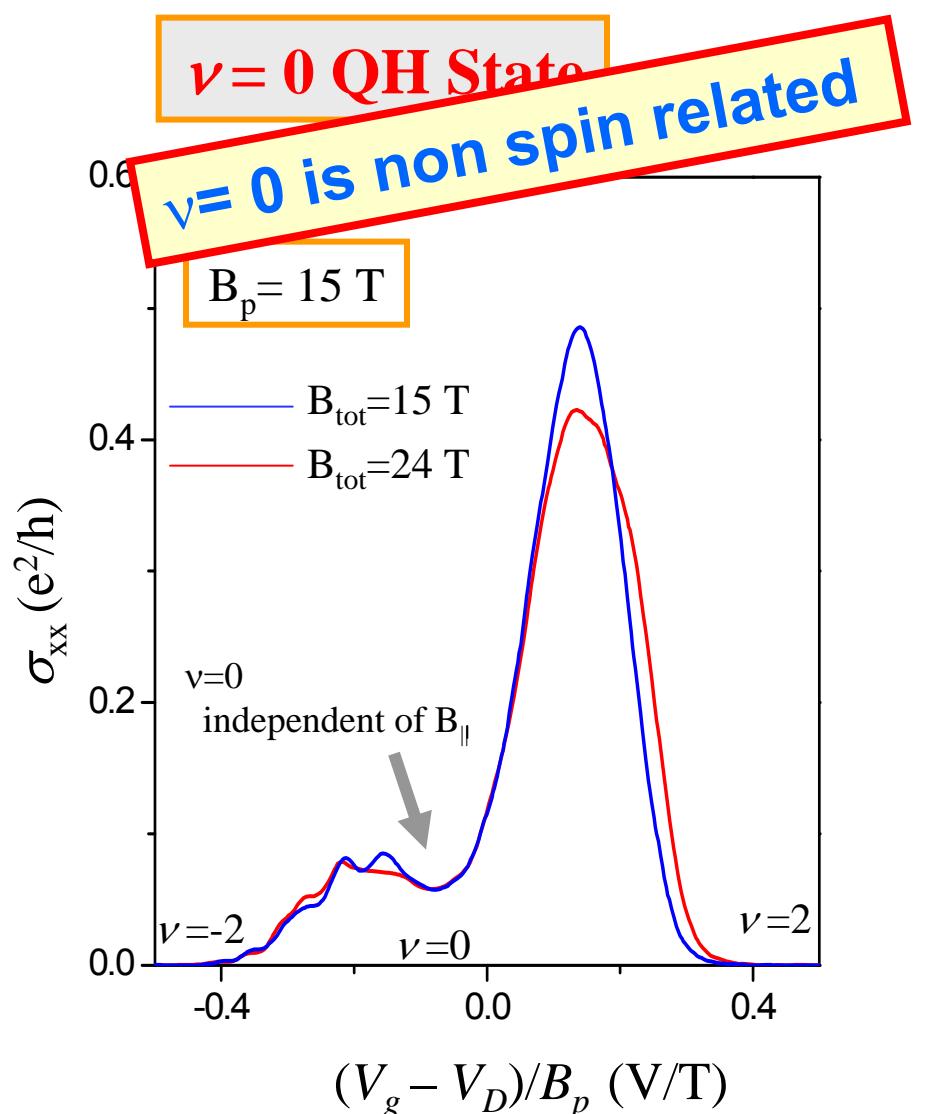
Similar to J. Chekelsky, L. Li, N. P. Ong, PRL (2007) PRB (2008)

# Degeneracy Lifting: Spin or Pseudo Spin?

$\nu=1$  QH State



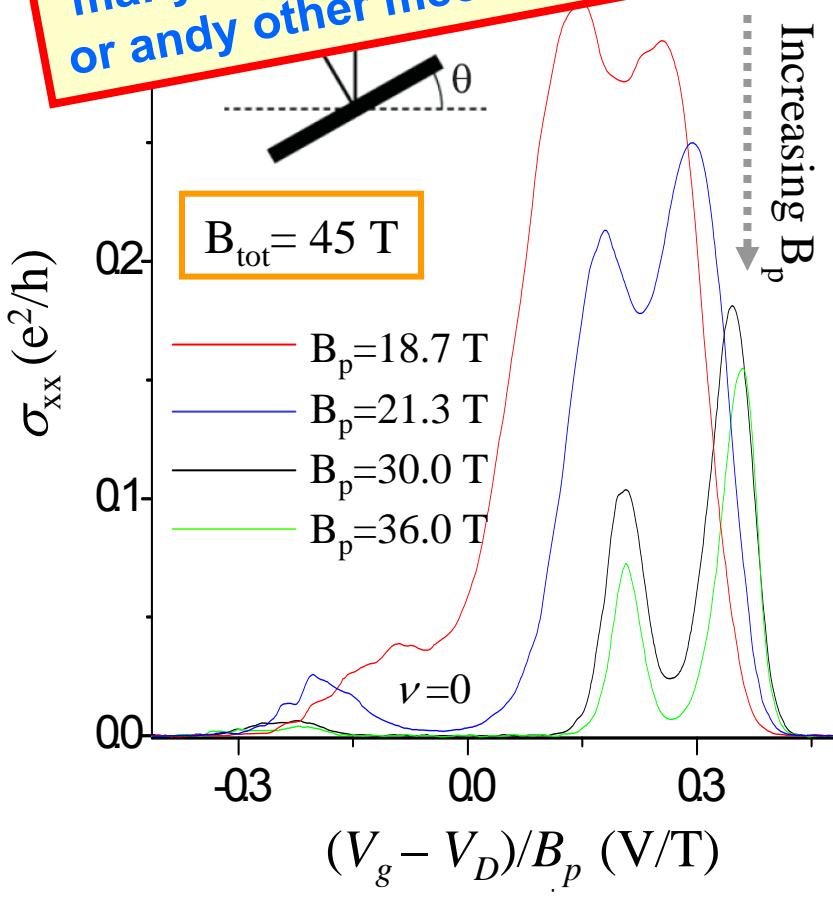
$\nu=0$  QH State



# Caveats

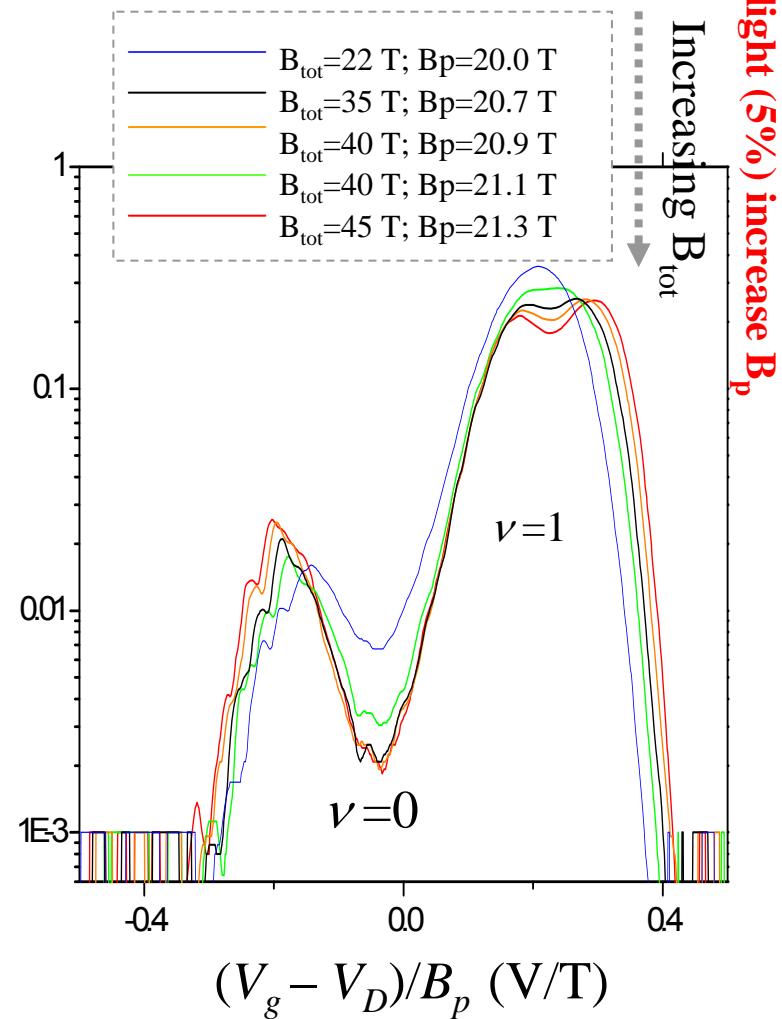
$\nu = 1$  QH State

$\nu = 1$  is not bare spin related:  
many-body enhanced spin effect?  
or andy other mechanism?

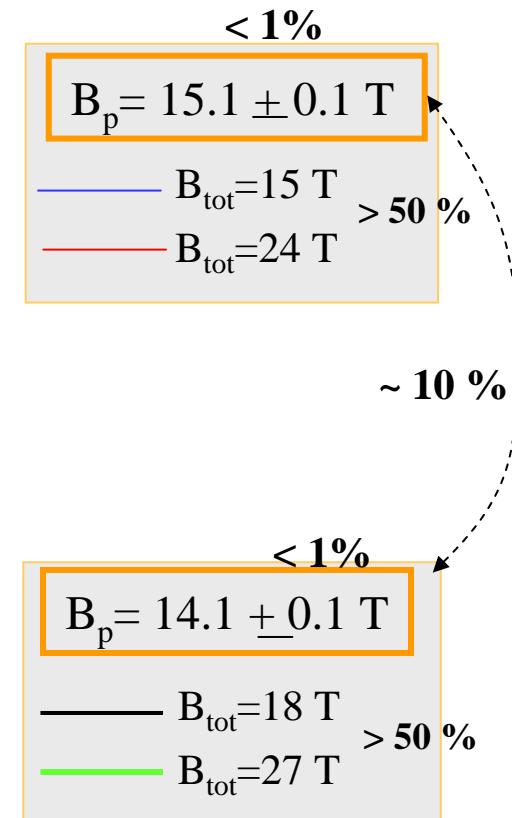
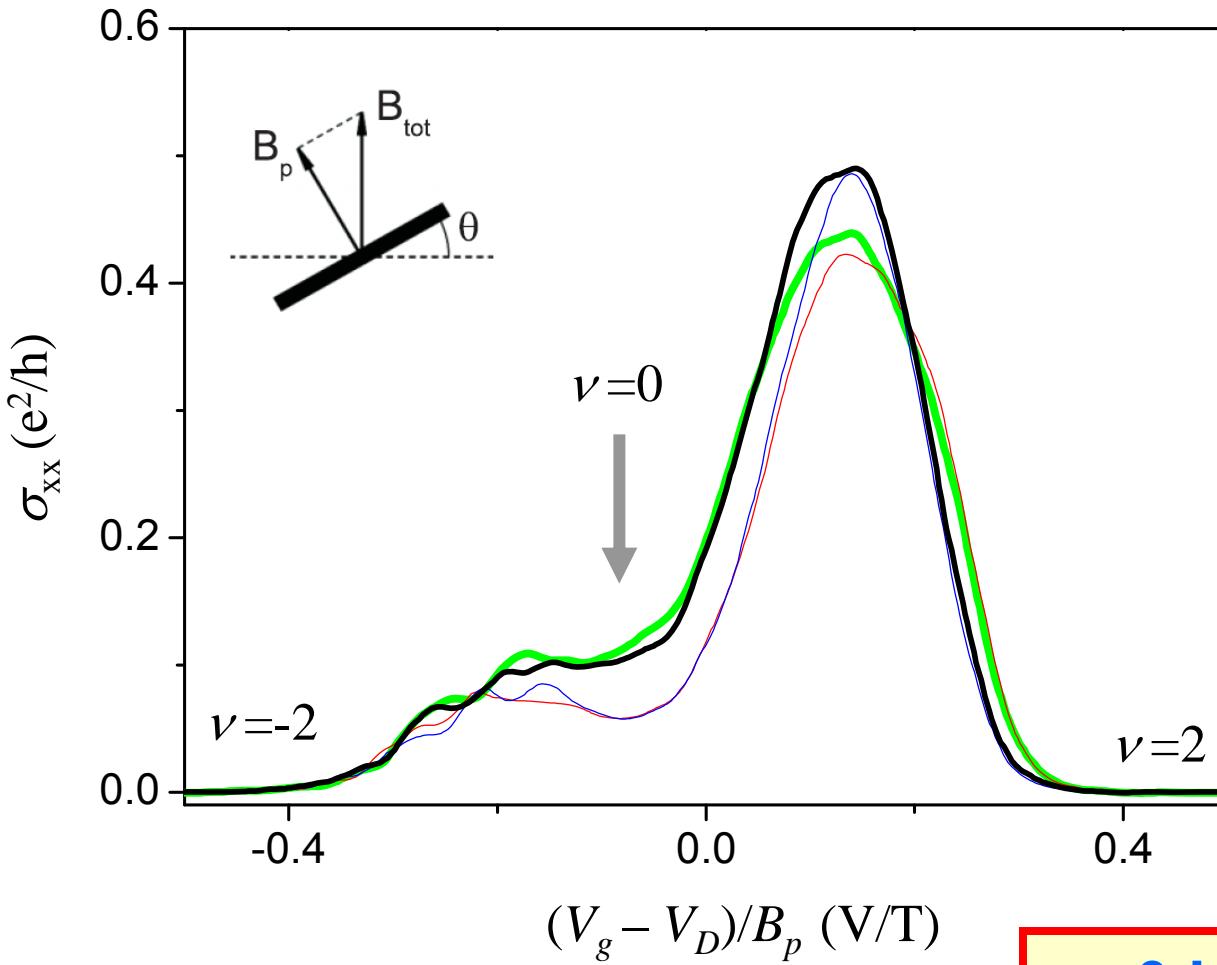


Require < 5% experimental accuracy  
in tilting angle adjustment...

$\nu = 0$  QH State



# $\nu = 0$ Quantum Hall Splitting: Tilting Angle Adjustment



$\nu = 0$  is non spin related

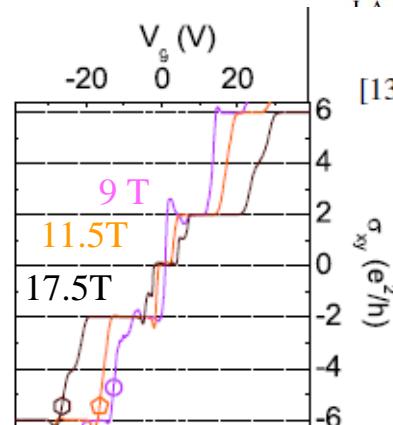
# Brief History of LL Symmetry Breaking Hierarchy

PRL 96, 136806 (2006)

PHYSICAL REVIEW LETTERS

## Landau-Level Splitting in Graphene in High Magnetic Fields

Y. Zhang,<sup>1</sup> Z. Jiang,<sup>1,3</sup> J. P. Small,<sup>1</sup> M. S. Purewal,<sup>1</sup> Y.-W. Tan,<sup>1</sup> M. Fazlollahi,<sup>1</sup> J. D. Chu,<sup>1</sup> A. Jaszczak,<sup>4</sup> H. L. Stormer,<sup>1,2</sup> and P. Kim<sup>1</sup>



[13] The fact that the  $\nu = \pm 1$  and  $\nu = \pm 4$  solved at similar magnetic fields suggests splitting responsible for these QH states same origin.

PRL 99, 106802 (2007)

## 2009 New experimental findings in bulk (Corbino) measurements:

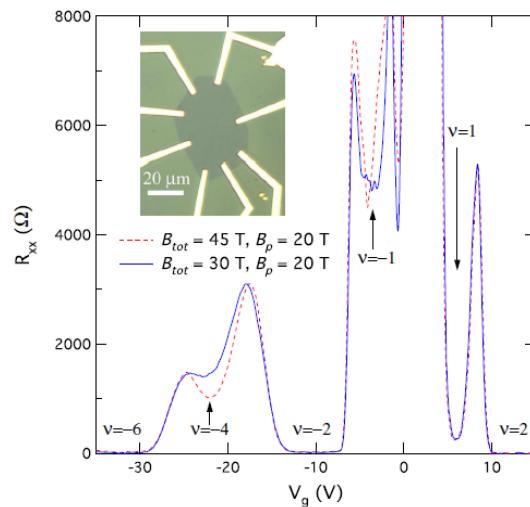
- $\nu=1$  QH state  
-> spin (but manybody enhanced) related
- $\nu=0$  QH state  
-> pseudo spin related

PHYSICAL REVIEW LETTERS

week ending  
7 SEPTEMBER 2007

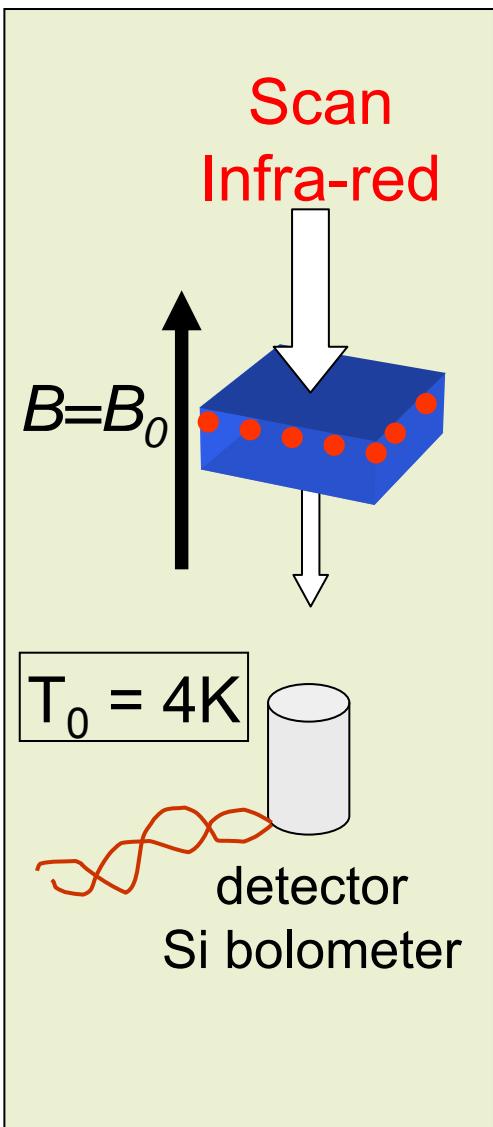
## Quantum Hall States near the Charge-Neutral Dirac Point in Graphene

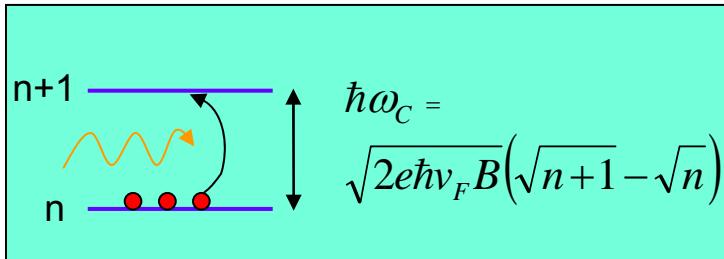
Z. Jiang,<sup>1,2,\*</sup> Y. Zhang,<sup>1,†</sup> H. L. Stormer,<sup>1,3,4</sup> and P. Kim<sup>1</sup>

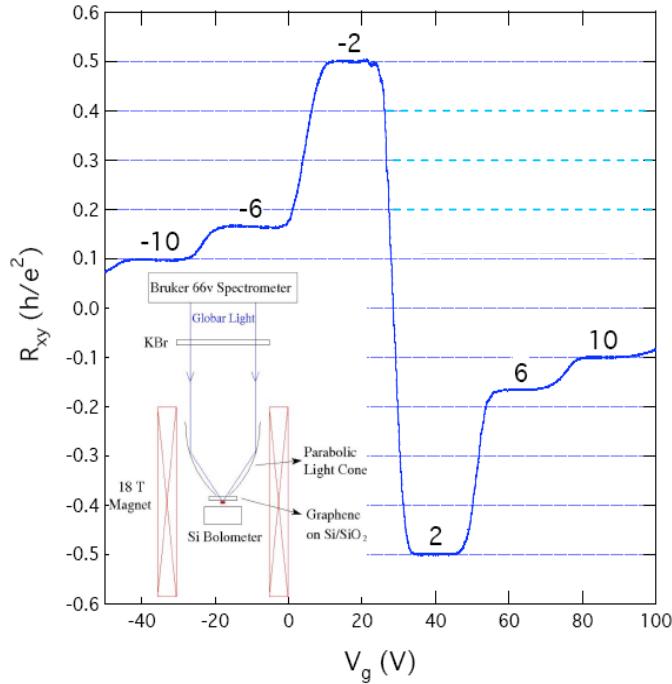
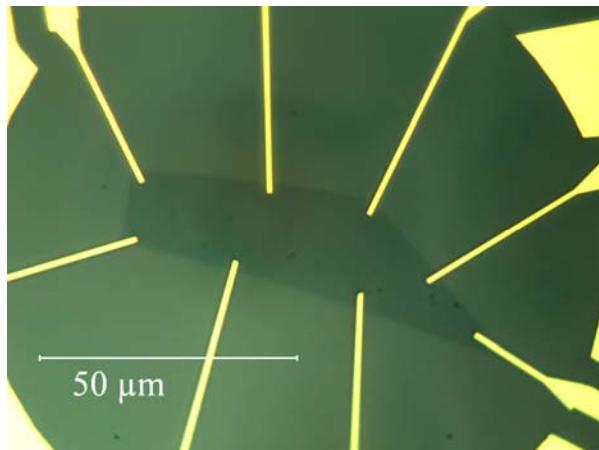


We investigate the quantum Hall (QH) states near the charge-neutral Dirac point of a high mobility graphene sample in high magnetic fields. We find that the QH states at filling factors  $\nu = \pm 1$  depend only on the perpendicular component of the field with respect to the graphene plane, indicating that they are not spin related. A nonlinear magnetic field dependence of the activation energy gap at filling factor  $\nu = 1$  suggests a many-body origin. We therefore propose that the  $\nu = 0$  and  $\pm 1$  states arise from the lifting of the spin and sublattice degeneracy of the  $n = 0$  Landau level, respectively.

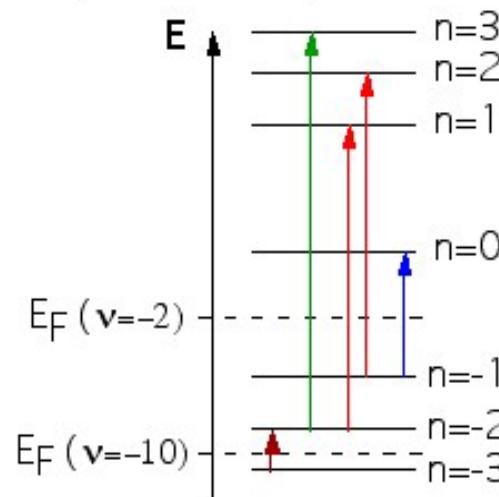
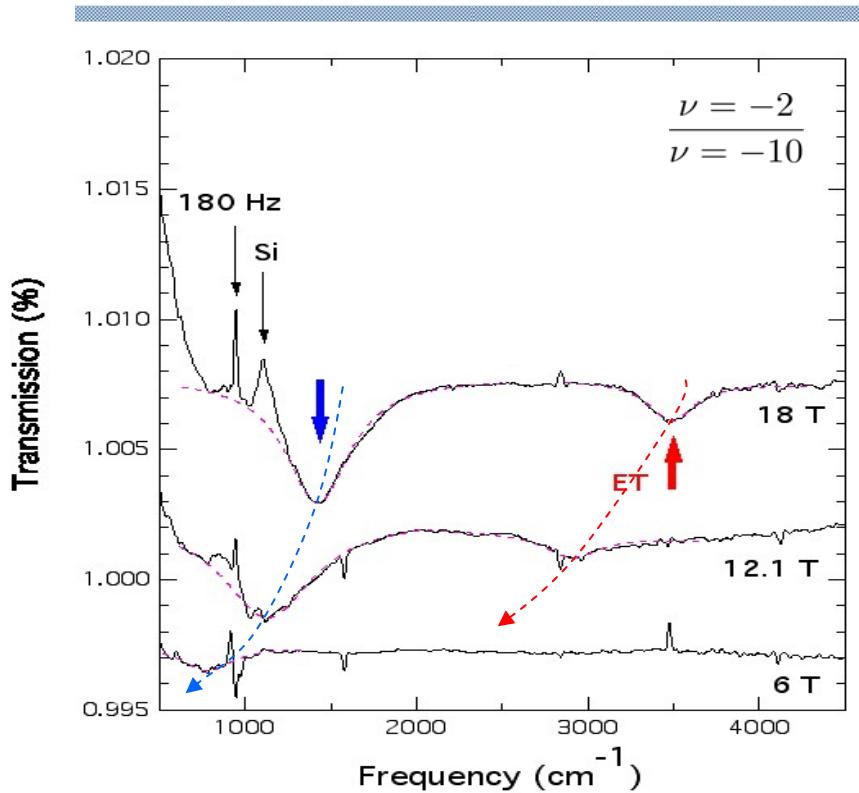
# Energy Gap Measurement: Cyclotron Resonance




$$\hbar\omega_c = \sqrt{2e\hbar v_F B} (\sqrt{n+1} - \sqrt{n})$$



# Landau Level Spectroscopy with IR Measurement

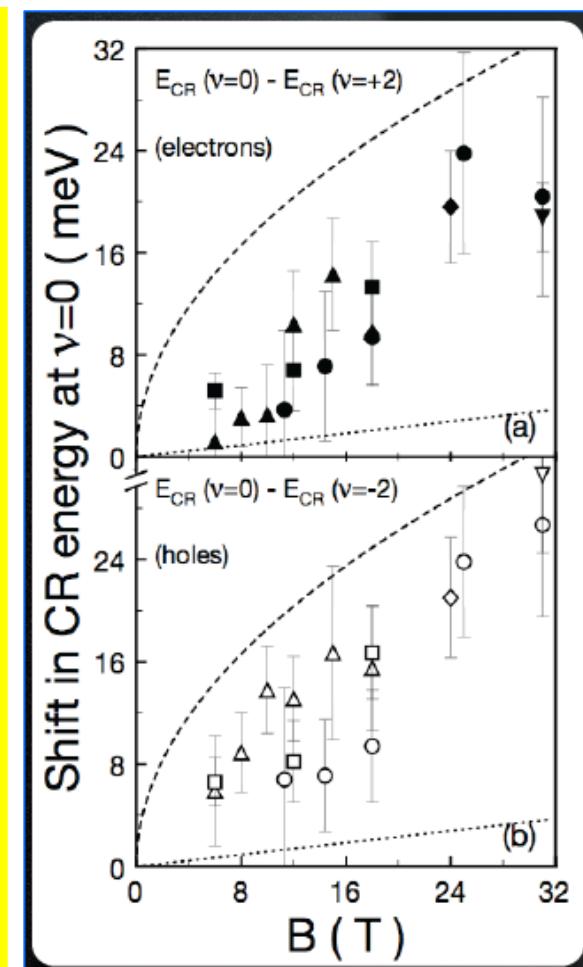
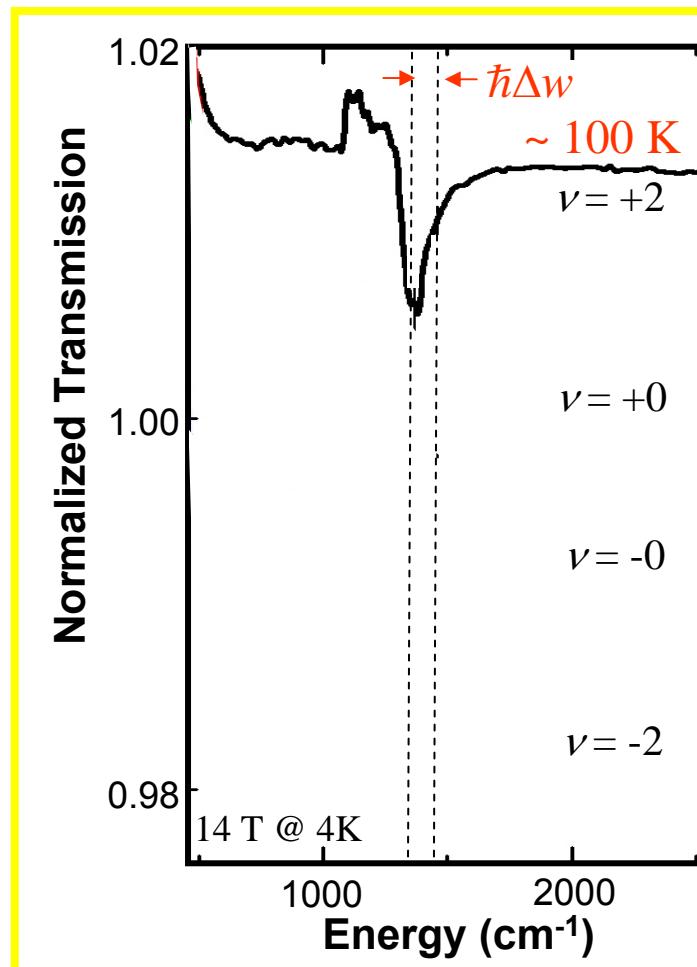
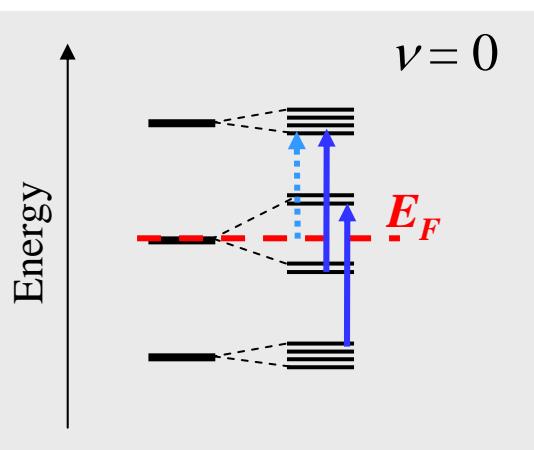
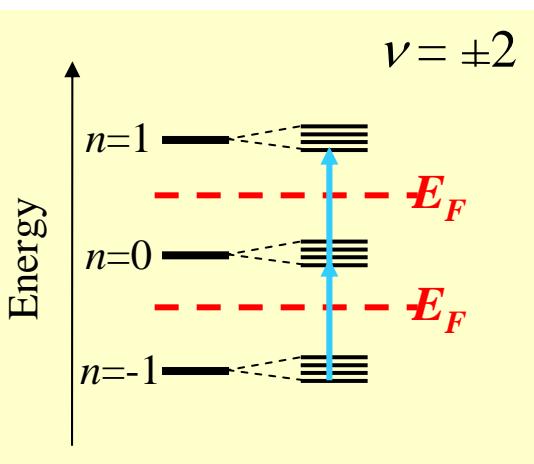


Measuring energy between  
LL centers in bulk

- R. S. Deacon, K.-C. Chuang, R. J. Nicholas, K. S. Novoselov, and A. K. Geim, Phys. Rev. B **76**, 081406(R) (2007).  
Z. Jiang, E. A. Henriksen, L.-C. Tung, Y.-J. Wang, M. E. Schwartz, M. Y. Han, P. Kim, and H. L. Stormer, Phys. Rev. Lett. **98**, 197403 (2007).  
E. A. Henriksen, Z. Jiang, L.-C. Tung, M. E. Schwartz, M. Takita, Y.-J. Wang, P. Kim, and H. L. Stormer, Phys. Rev. Lett **100**, 087403 (2008).  
M. L. Sadowski, G. Martinez, M. Potemski, C. Berger, and W. A. de Heer, Phys. Rev. Lett. **97**, 266401 (2006).

# $\nu = 0$ Gap Measurement by IR Spectroscopy

## Landau Level Hierarchy

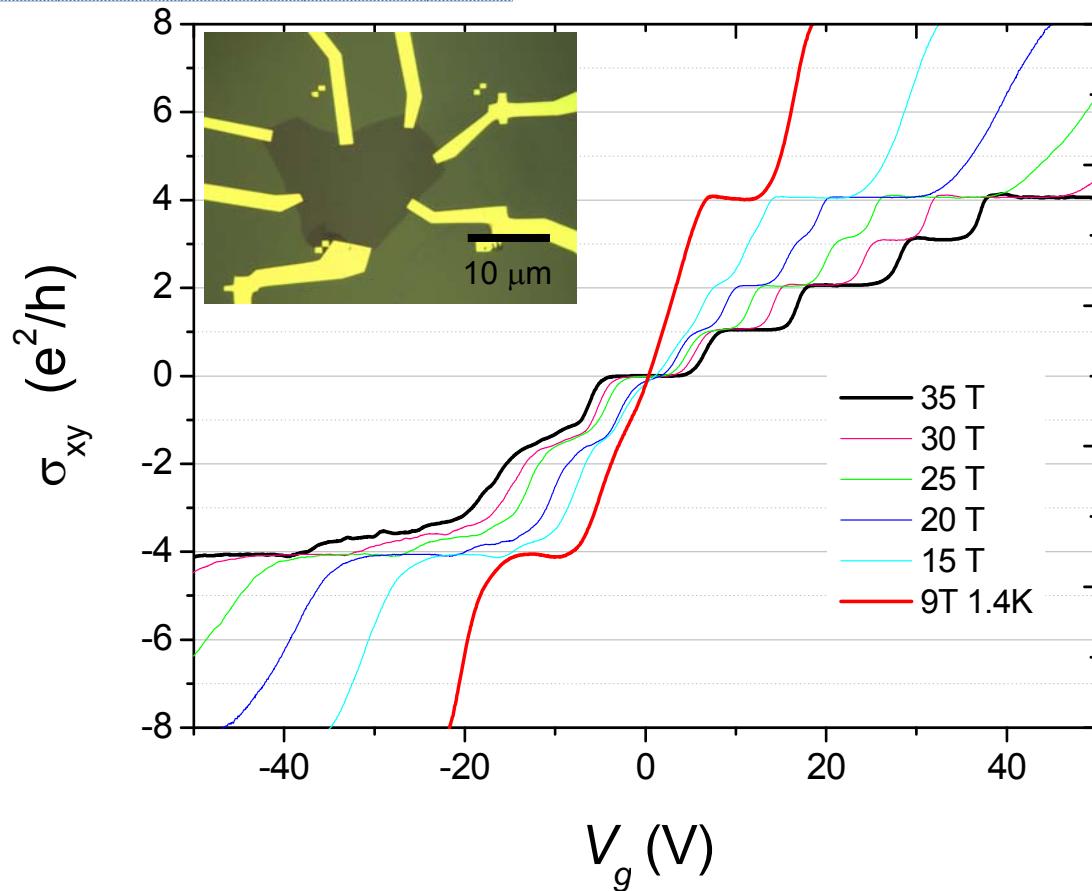
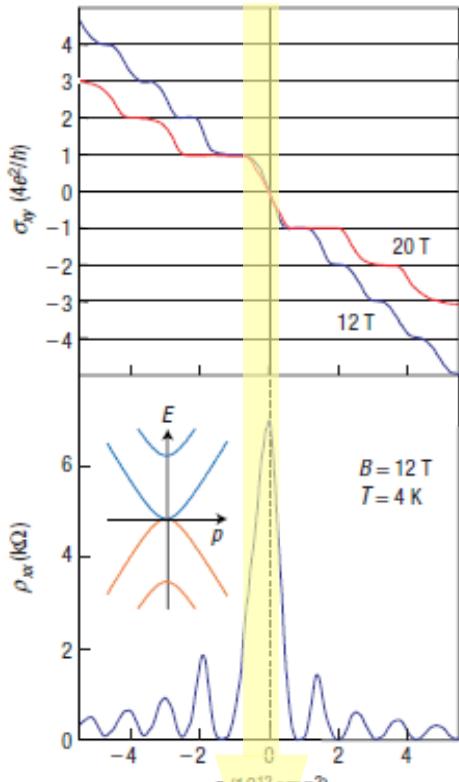


$$\hbar\omega_{\nu=0} - \hbar\omega_{\nu=2} = \frac{1}{2} \Delta E_{n=0}$$

Energy Gap at Dirac Point  $\sim 300\text{K} @ 31\text{ T}$

# Symmetry Breaking of $\nu=0$ QH state in Bilayer graphene

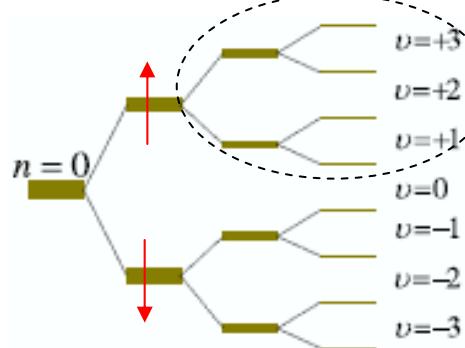
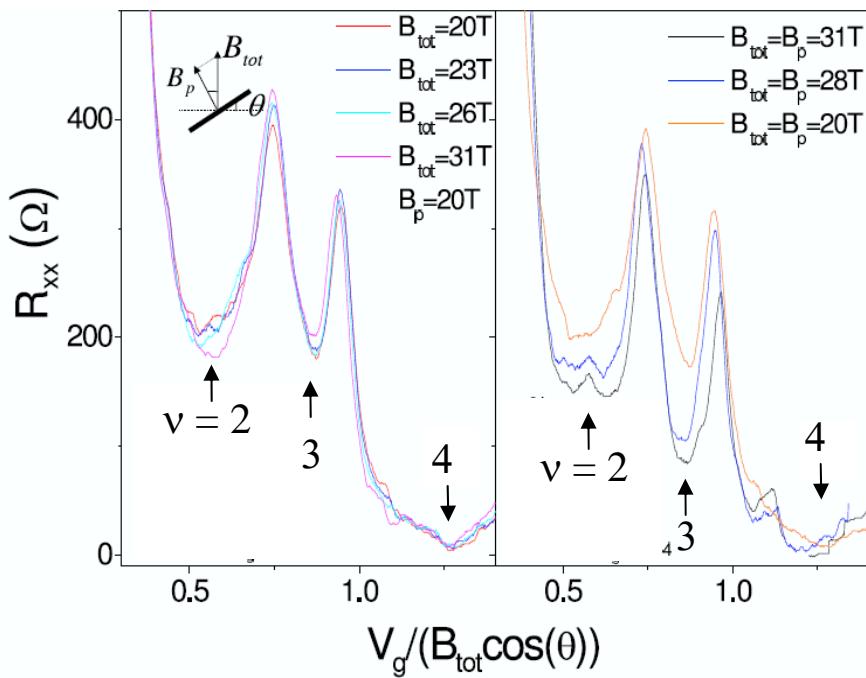
Bilayer QHE: Novoselov *et al.*,  
*Nature Physics* 2, 177-180 (2006).



- New QH states at higher field:  
 $\nu = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 8, \dots$
- All 8 degeneracy of  $n=0$  LL lifted at high magnetic fields

# Nature of Symmetry Breaking

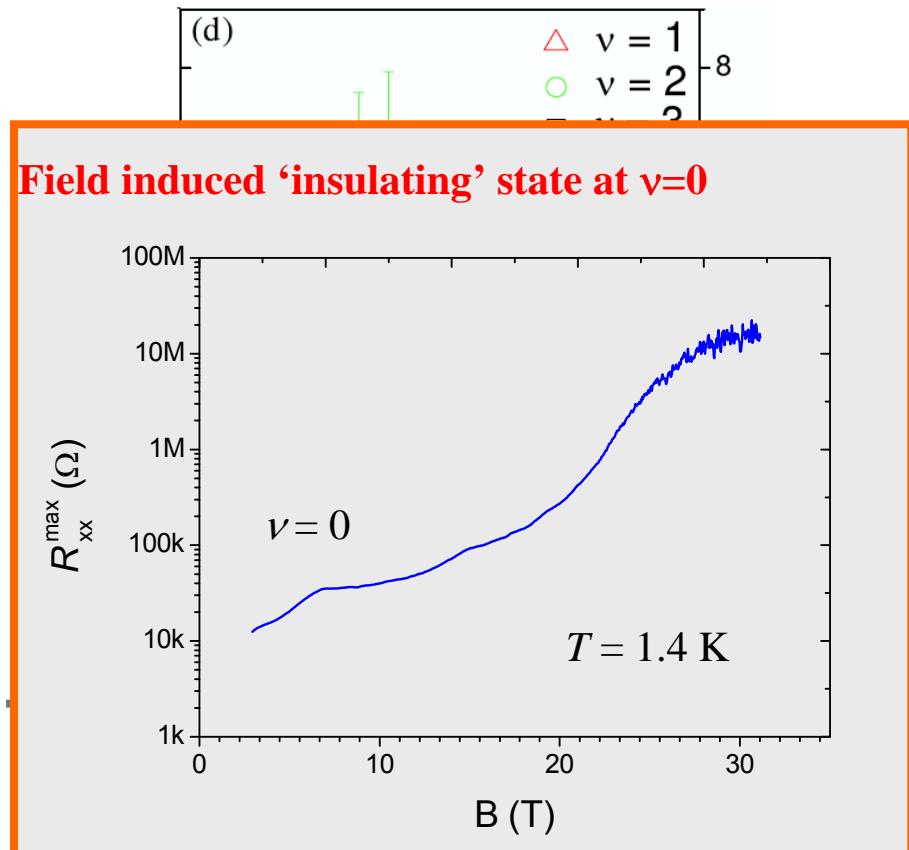
## Tilted Field Measurement



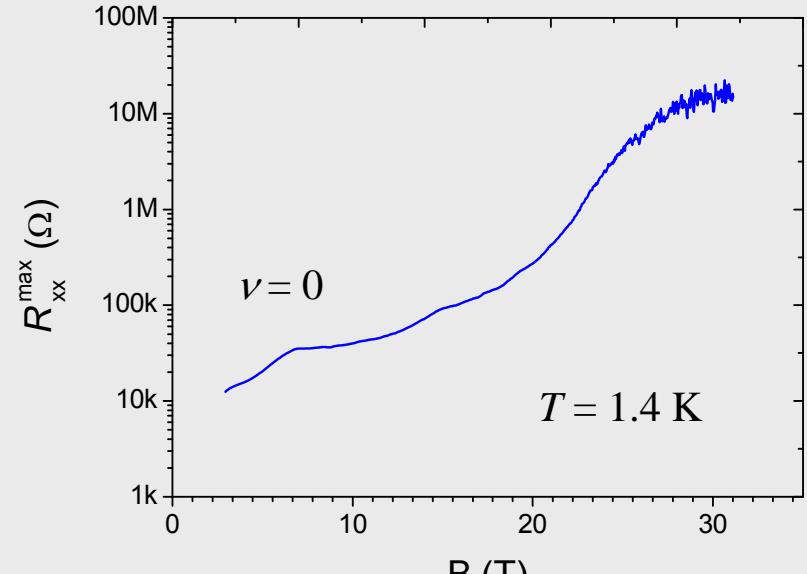
Theory

Yafis Barlas, R. Côté, K. Nomura, and A. H. MacDonald,  
Phys. Rev. Lett. 101, 097601 (2008).

## Activation Energy Gap



Field induced ‘insulating’ state at  $v=0$

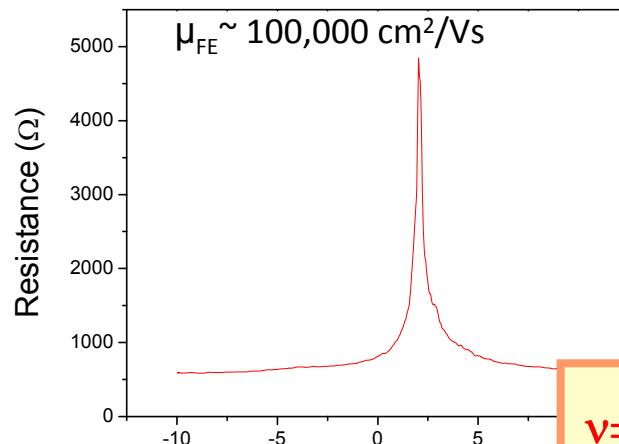


Non spin related symmetry breaking

# Quantum Hall Effect in Suspended Graphene



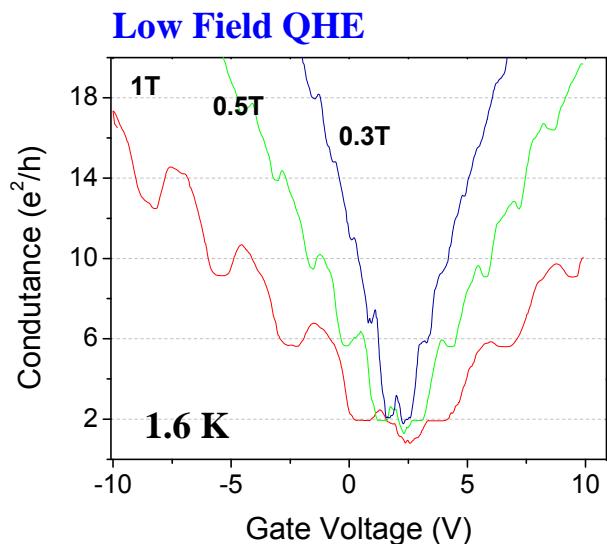
Bolotin et al., (2008); Du et al., (2008)



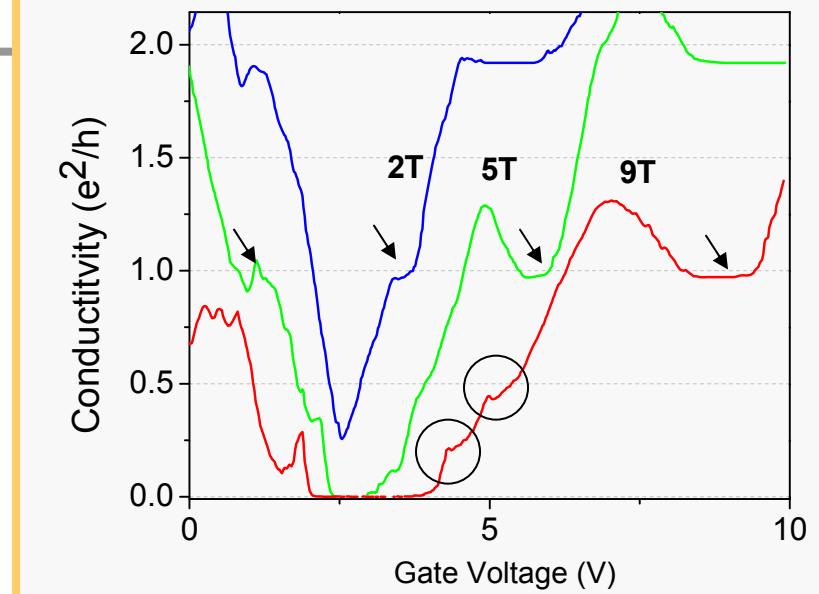
- Cleaning: current annealing  
Bachtold et al. (2007)
- Mechanical stability  
J. Lau et al. (2009)

New features for  $\nu < 1$

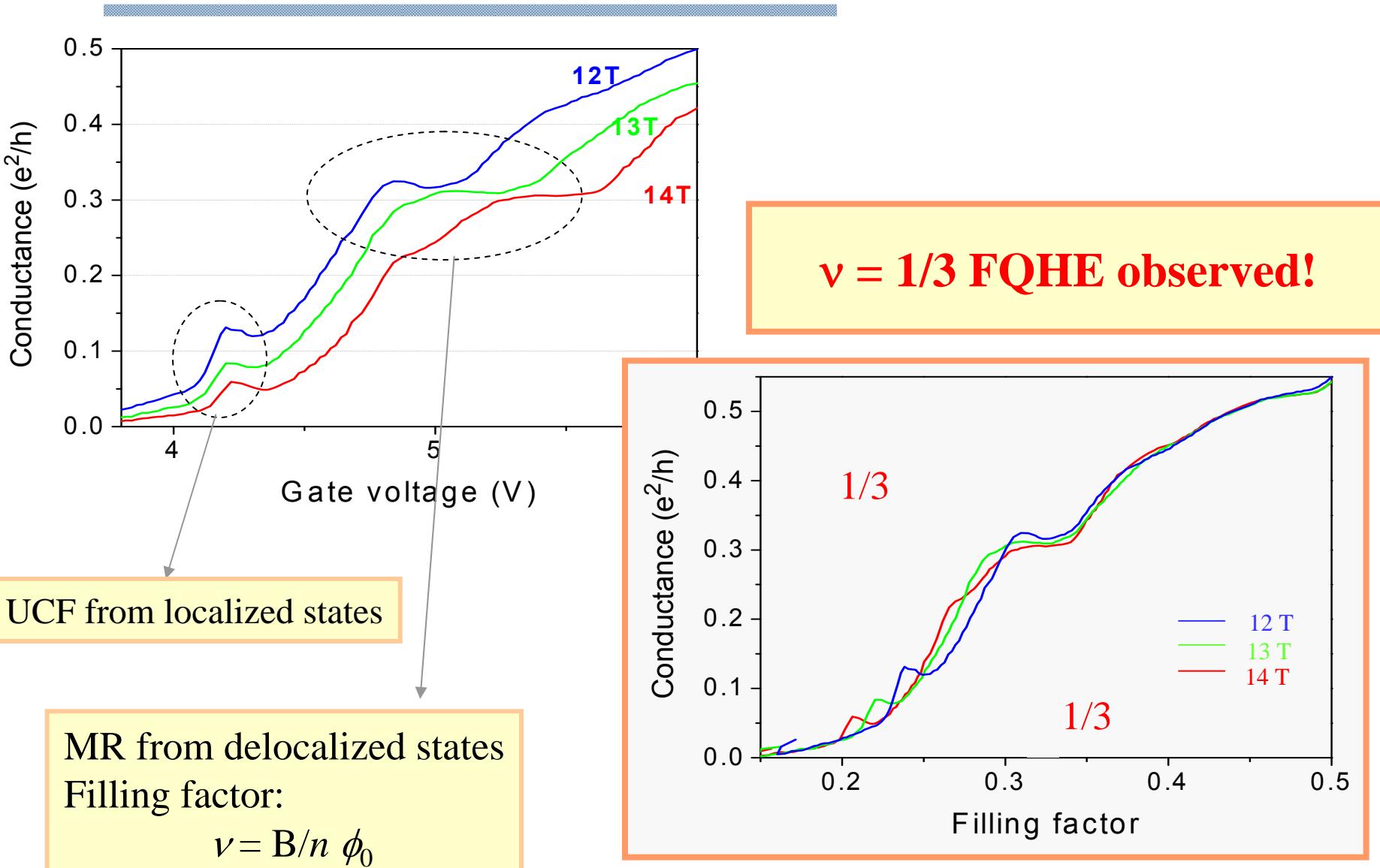
$\nu=1$  states appeared at 2 T !!



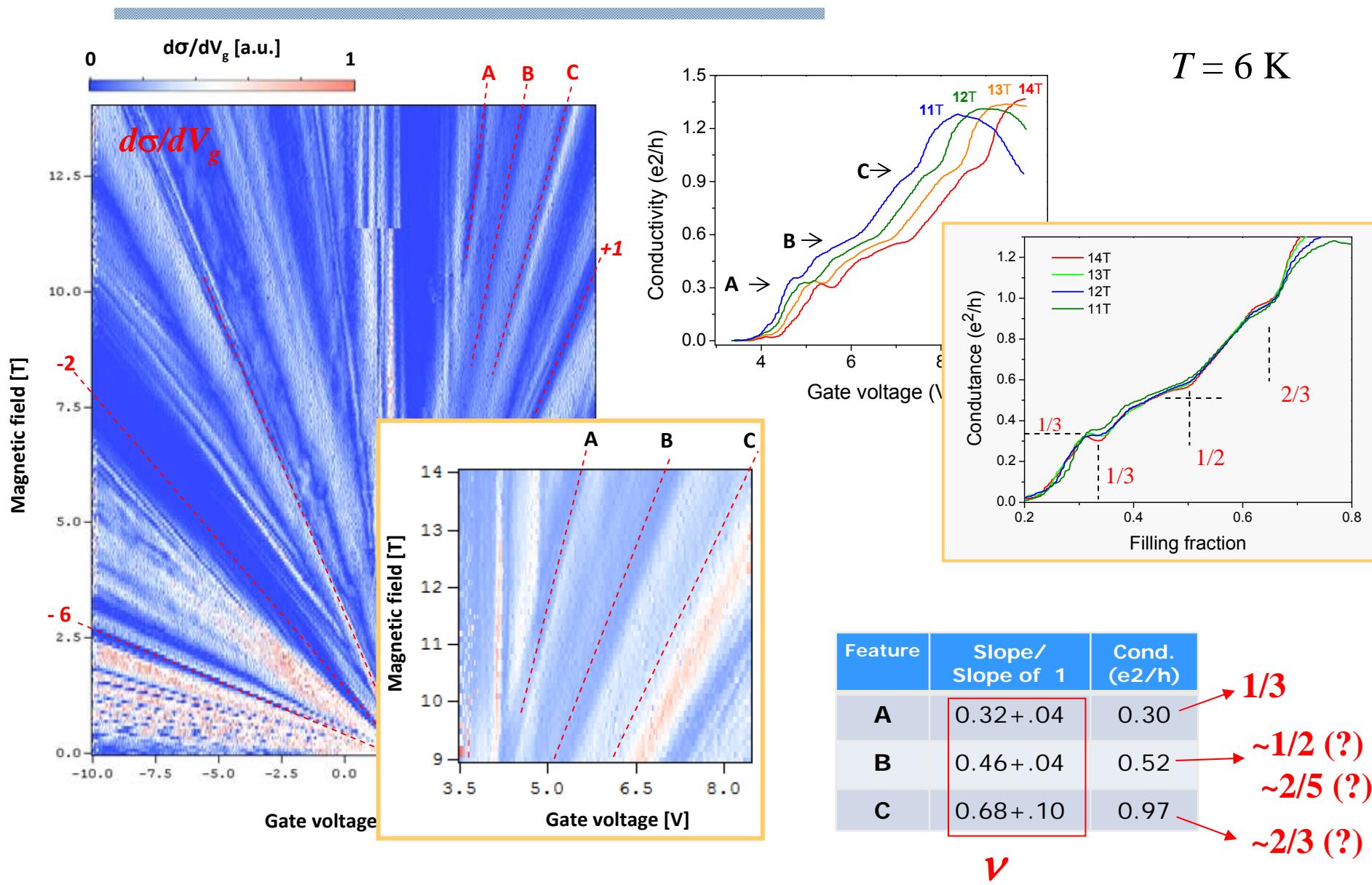
Increasing B



# Fractional Quantum Hall State in graphene

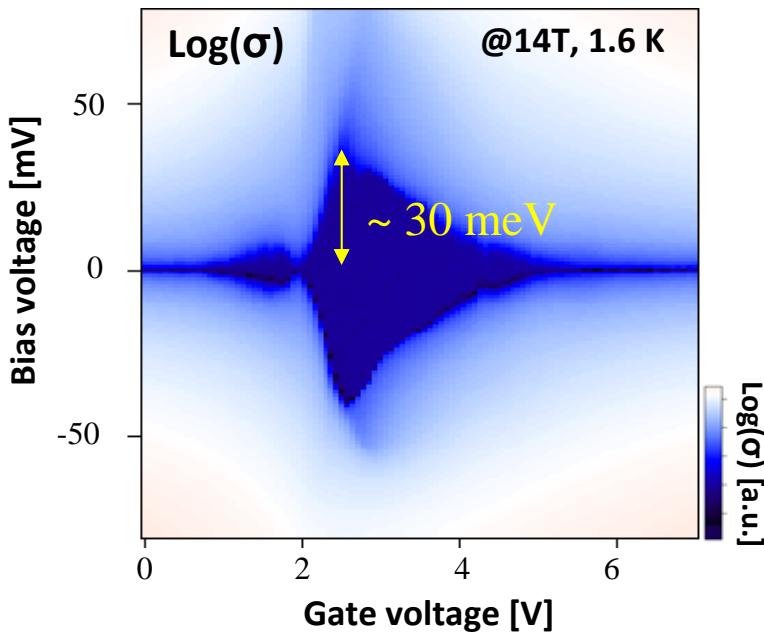
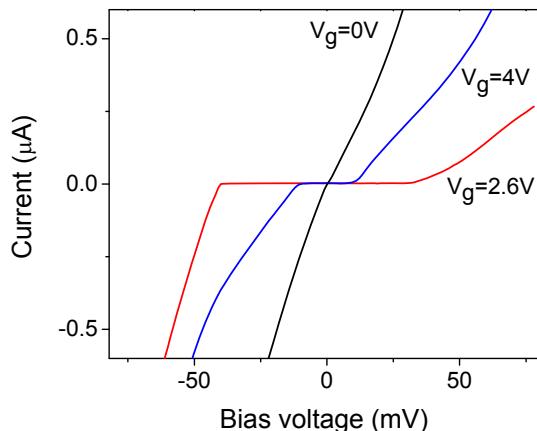


# Landau Fan Diagram : additional FQH states (?)

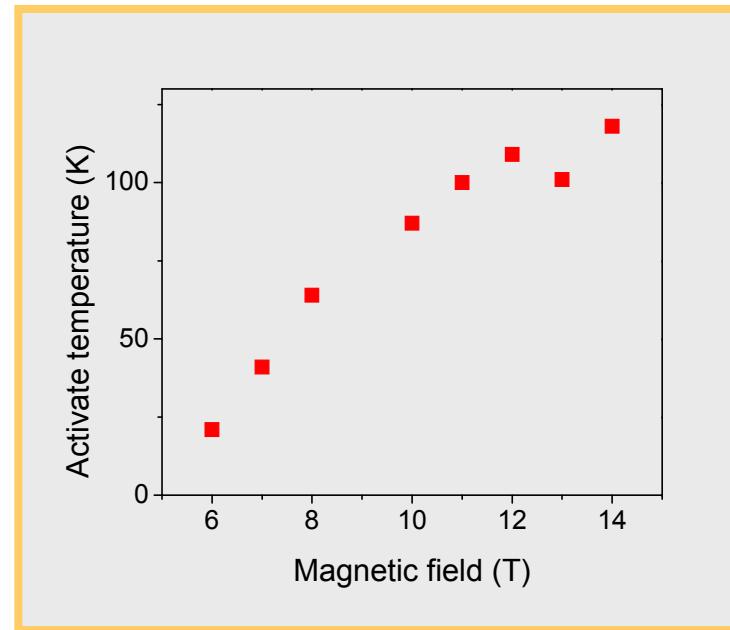
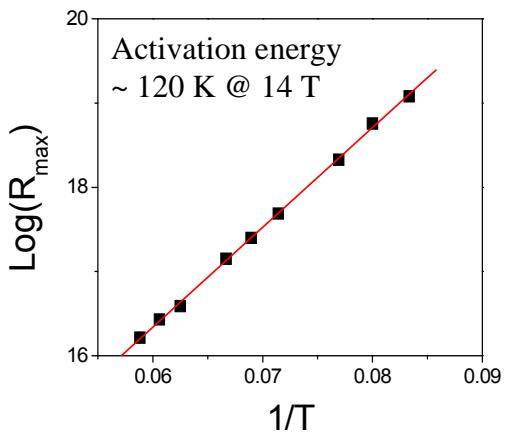


# Insulating State at $\nu = 0$ : Size of Gap

## Non-linear transport



## Activation Energy



Factor of  $\sim 10$  larger than unsuspended sample, comparable to IR meas.

# Summary

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Symmetry breaking of zero energy LL in ‘bulk’ graphene

SU(4) symmetry breaking hierarchy

Pseudo Spin ( $\nu = 0$ ); Spin –manybody enhanced ( $\nu = +/-1$ )

Insulating bulk state at high magnetic field

IR gap Measurement

Magnetic field dependent ‘Bulk Gap’  $\nu = 0$  QH state

SU(8) Symmetry breaking in bilayer graphene sample

Spin degeneracy lifting at the charge neutrality point

Fractional Quantum Hall Effect in Suspended Graphene

$\nu = 1/3$  FQH state observed

Potential other FQH states  $1/3 < \nu < 1$

Large gap in the insulating state at  $\nu = 0$

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