Searching for Cosmic Superstrings in the CMB

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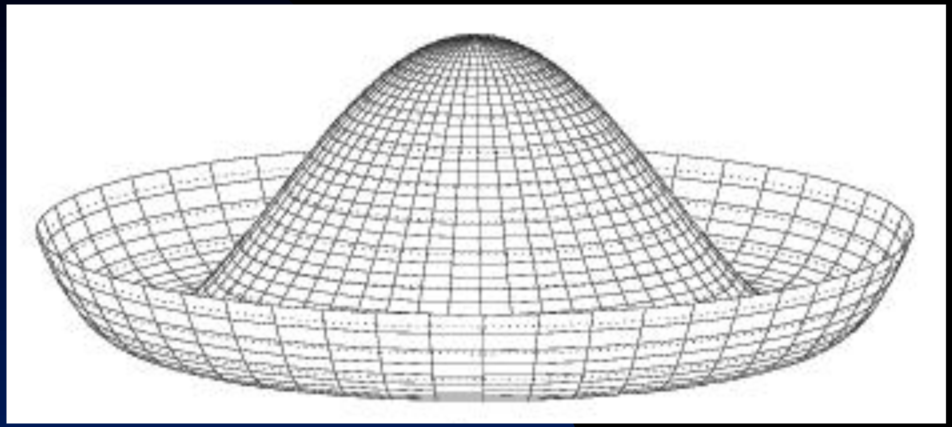
1. Brief review of cosmic strings.
2. Why cosmic superstrings.
3. Modeling strings with junctions.
4. Observational signatures and constraining g_s and μ_F.

Benasque, Aug 2nd 2010

Original cosmic strings, in gauge theory :



Spontaneously broken $U(1)$ symmetry, has magnetic flux tube solutions (Nielsen-Oleson vortices).

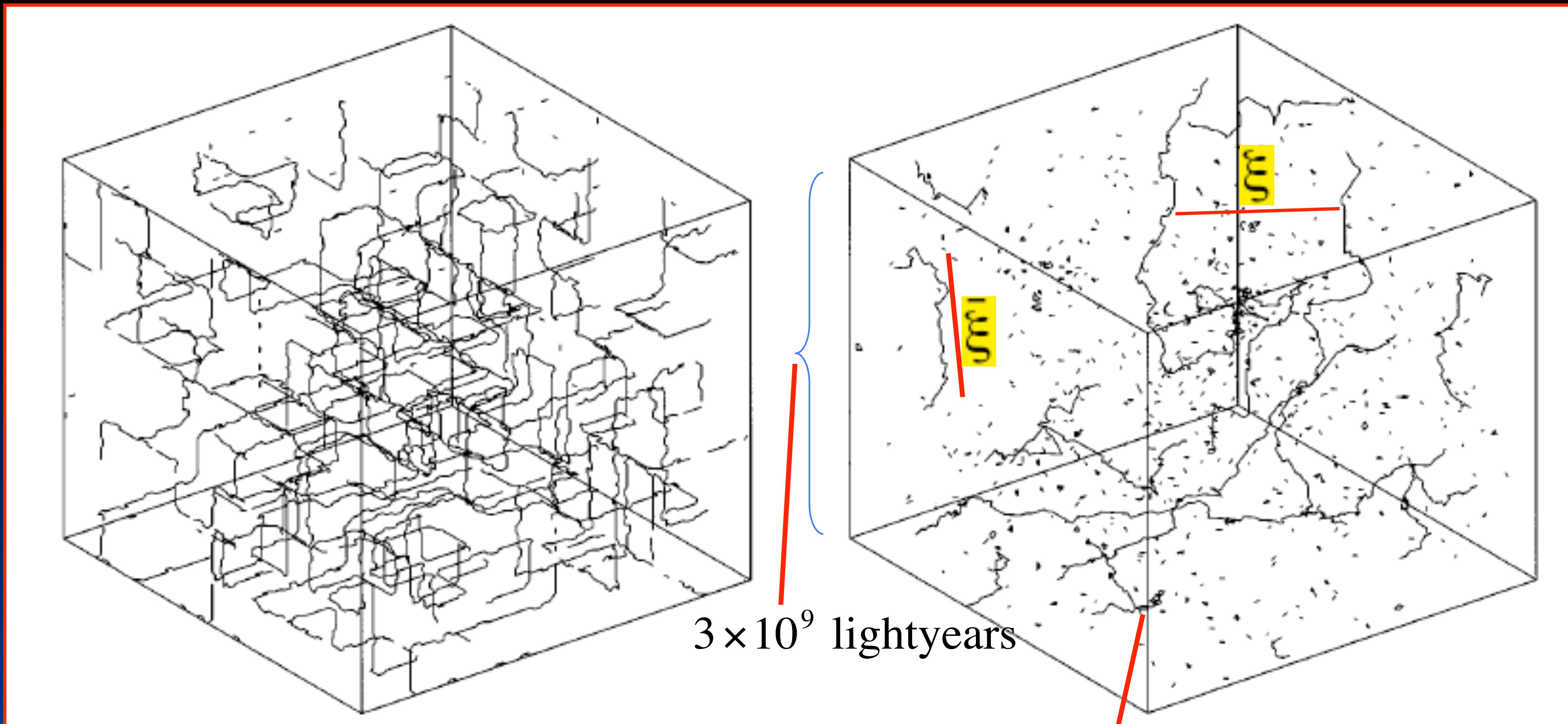
Network would form in early universe phase transitions where $U(1)$ symmetry *becomes* broken. Higgs field roles down the potential in different directions in different regions (Kibble 76).

String tension : μ Dimensionless coupling to gravity : G μ

GUT scale strings : G $\mu \sim 10^{-6}$ -- size of string induced metric perturbations.

Length scales on networks

[Vincent et al]



Initial

Scaling

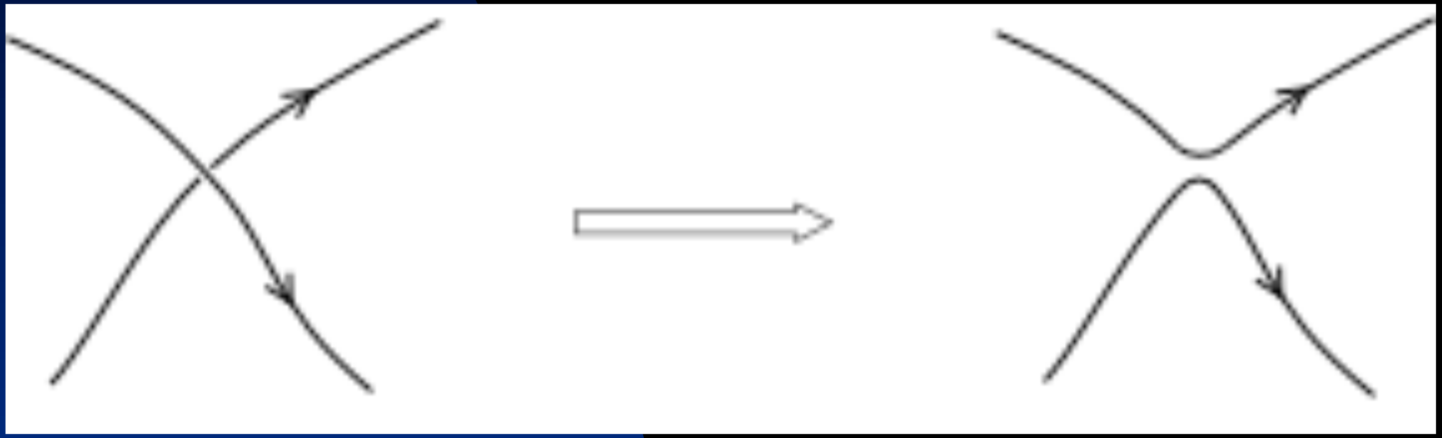
$\bar{\xi}$ - persistence length of string

ξ - interstring distance

ζ - small scale structure on network

Observational consequences : 1980's and 90's

Single string networks evolve with Nambu-Goto action, decaying primarily by forming loops through intercommutation and emitting gravitational radiation and possibly particles.



For gauge strings, reconnection probability P~1

Scaling solutions are reached where energy density in strings reaches constant fraction of background energy density:

$$\rho_{string}/\rho_{rad} \sim 400G\mu$$

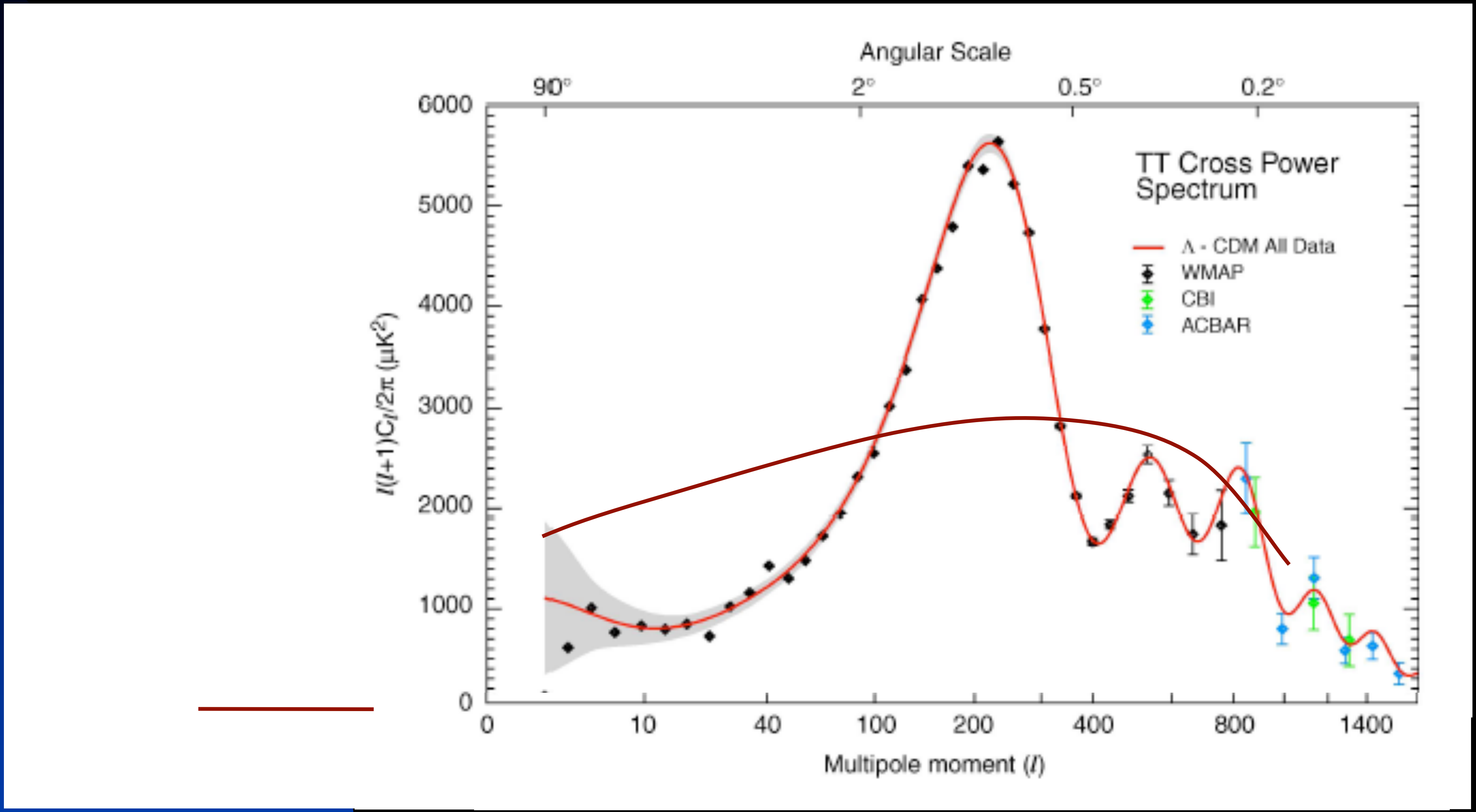
$$\rho_{string}/\rho_{mat} \sim 60G\mu$$

[Albrecht &Turok; Bennett & Bouchet; Allen & Shellard]

Density increases as P decreases because takes longer for network to lose energy to loops. Recent re-analysis of loop production mechanisms suggest two distributions of long and small loops.

Unfortunately they didn't do the full job!

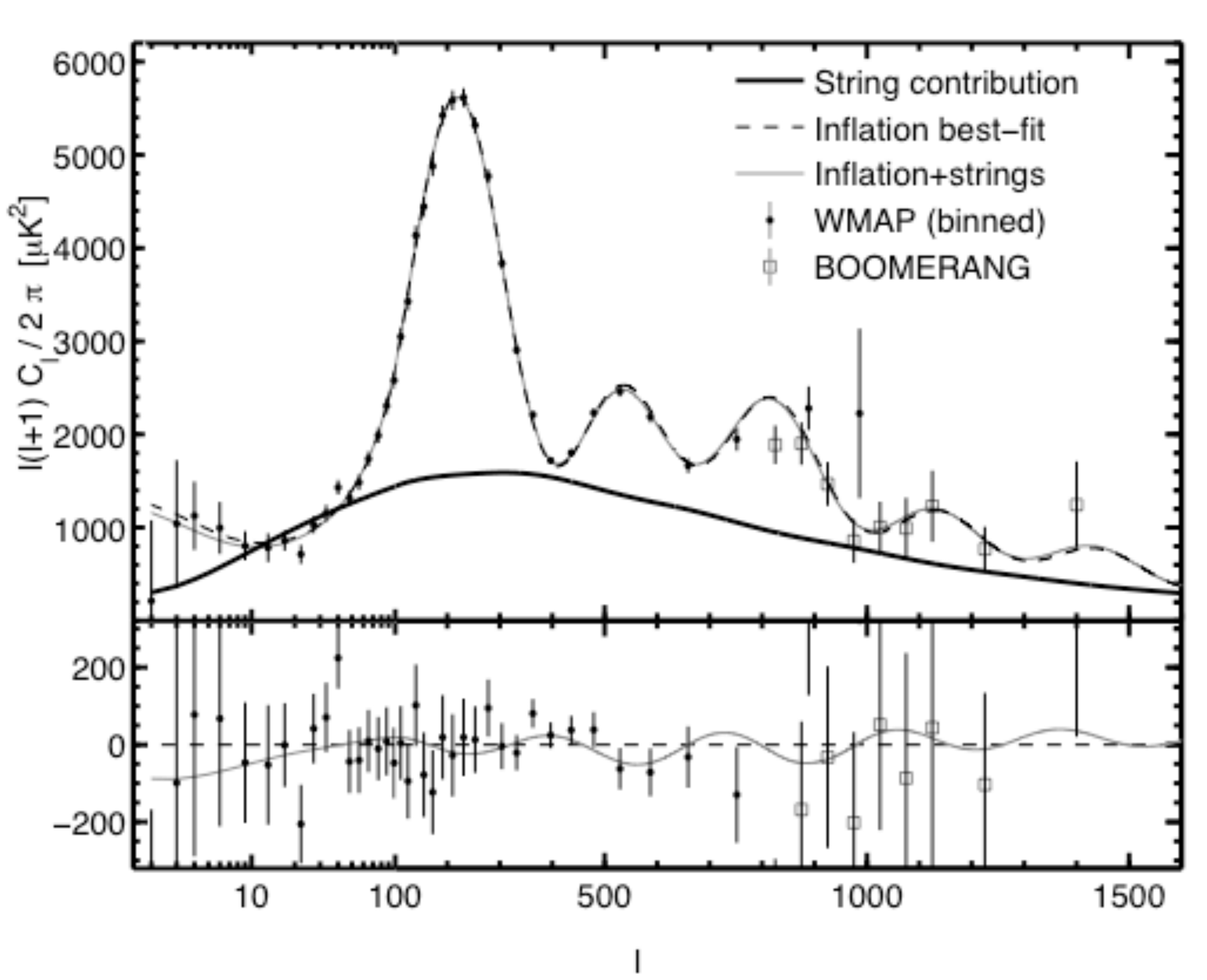
CMB power spectrum



Acoustic peaks come from temporal coherence. Inflation has it, strings don't. String contribution < 13% implies $G\mu < 10^{-6}$.

E.g. Pogosian et al 2004, Bevis et al 2004.

They may not do the full job but they can still contribute



Hybrid Inflation type models

String contribution < 11% implies $G\mu < 0.7 * 10^{-6}$.

Bevis et al 2007.

Pulsar bounds on gravitational wave emission could also be problematic for GUT scale strings:

Strings produce stochastic GW, $\Omega_{GW} \sim 10^{-1.5}\, G\mu$.
(Allen '95, Battye, Caldwell, Shellard '97)

Kaspi, Taylor, Ryba '94: $\Omega_{GW} < 1.2 \times 10^{-7}$, $G\mu < 10^{-5.5}$

Lommen, Backer '01: $\Omega_{GW} < 4 \times 10^{-9}$, $G\mu < 10^{-7}$

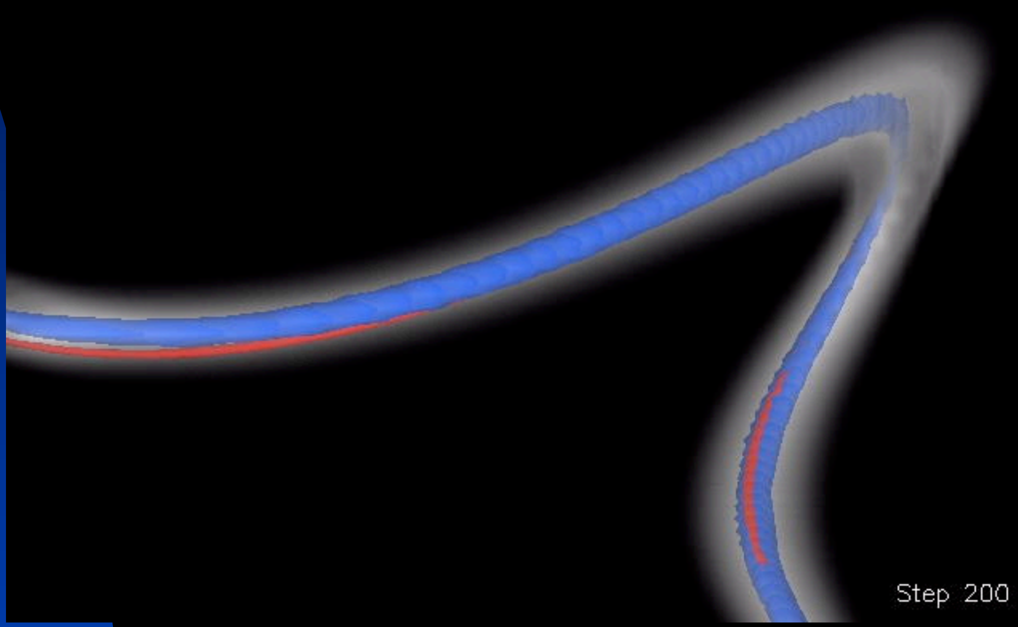
In relevant frequency range ~ 0.1 inverse year

Siemens et al 07 -- very tight constraint on strings

Need to reduce string tension although uncertainty in string calculation.

Any smoking guns?

Possibly through strong non-gaussian nature of stochastic gravitational wave emission from loops which contain kinks and cusps. [Damour & Vilenkin 01 and 04]



[Blanco-Pillado and Olum]

Cusp: x'=0 for instant in an oscillation

Kink: x' discontinuous, occurs every intercommuting -- common

Both produce beams of GW, cusps much more powerful

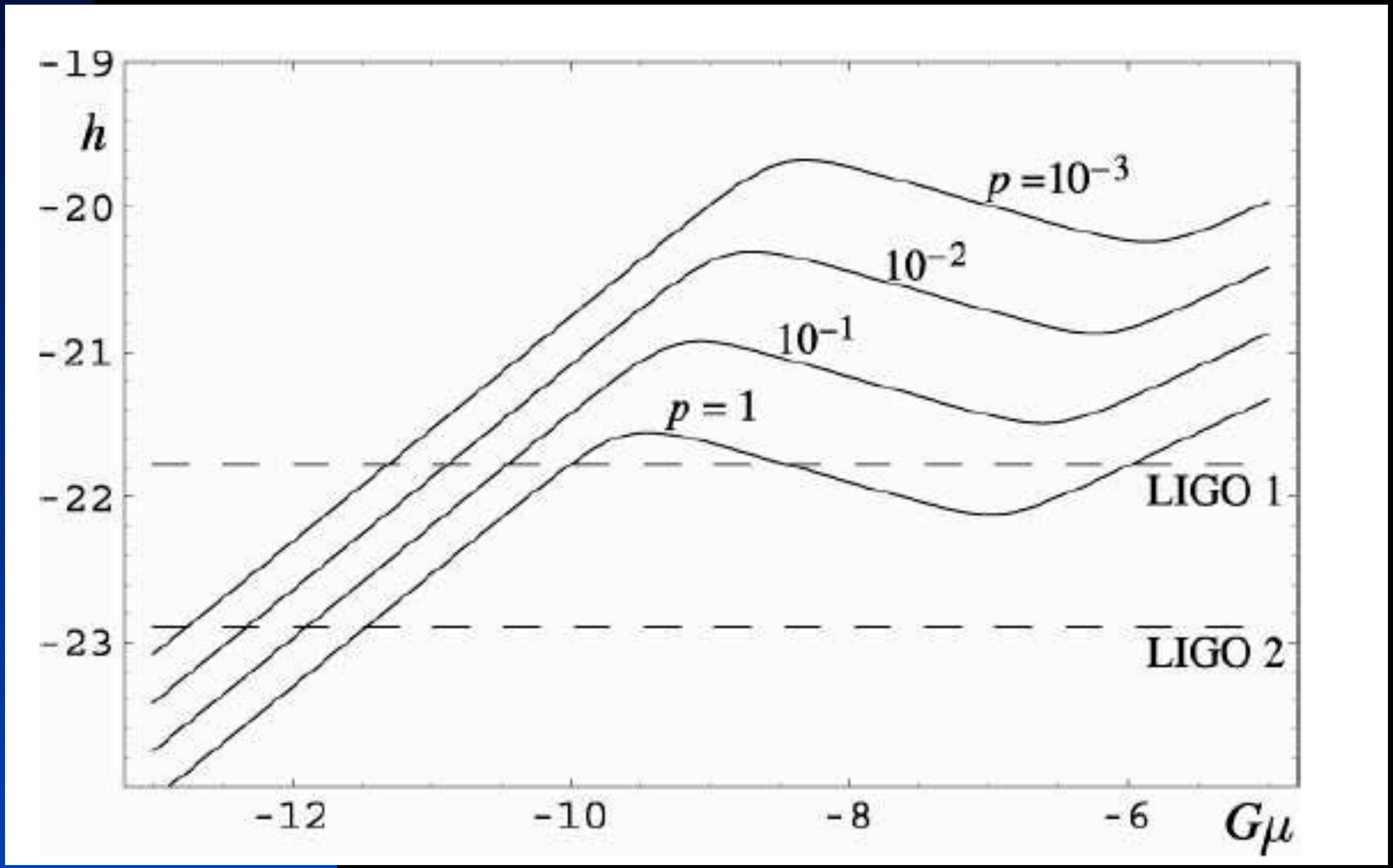
In loop network, if only 10% of loops have cusps, bursts of GW above `confusion' GW noise could be detected by LIGO and LISA for $G\mu \sim 10^{-12}$!

$\log_{10} h$

strain

LIGO I

LIGO II



[Damour & Vilenkin 04]

Noise levels

Bursts emitted by cusps in LIGO frequency range f_{ligo}=150 Hz

In 1980's Fundamental (F) strings excluded as being cosmic strings [Witten 85]:

1. F string tension close to Planck scale (e.g. Heterotic)

$$G\mu = \frac{\alpha_{GUT}}{16\pi} \geq 10^{-3}$$

Cosmic strings deflect light, hence constrained by CMB:

$$G\mu \propto \frac{\delta T}{T} \leq 10^{-6}$$

Consequently, cosmic strings had to be magnetic or electric flux tubes arising in low energy theory

2. Why no F strings of cosmic length?

a. Diluted by any period of inflation as with all defects.

b. They decay ! (Witten 85)

1990's: along came branes --> new one dimensional objects:

1. Still have F strings
2. D-strings
3. Higher dimensional D-, NS-, M- branes partly wrapped on compact cycles with only one non-compact dimension left.
4. Large compact dimensions and large warp factors allow for much lower string tensions.
5. Dualities relate strings and flux tubes, so can consider them as same object in different regions of parameter space.

What do they imply for cosmic strings?

Strings surviving inflation:

D-brane-antibrane inflation leads to formation of D1 branes in non-compact space [**Burgess et al; Majumdar & Davis; Jones, Sarangi &Tye; Stoica & Tye**]

Form strings, not domain walls or monopoles.

$$10^{-11} \leq G\mu \leq 10^{-6}$$

In general for cosmic strings to be cosmologically interesting today we require that they are not too massive (from CMB constraints), are produced after inflation (or survive inflation) and are stable enough to survive until today [**Dvali and Vilenkin (2004); EJC,Myers and Polchinski (2004)**].

What sort of strings?

Expect strings in non-compact dimensions where reheating will occur: F1-brane (fundamental IIB string) and D1 brane localised in throat. [Majumdar & Davis, Jones,Stoica & Tye, Dvali & Vilenkin]

D1 branes - defects in tachyon field describing D3-anti D3 annihilation, so produced by Kibble mechanism.

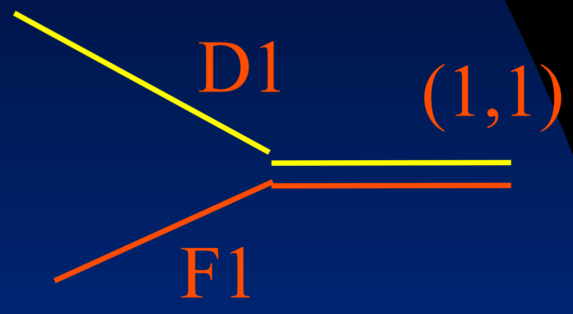
Strings created at end of inflation at bottom of inflationary throat. Remain there because of deep pot well. Eff 4d tensions can be reduced because they depend on warping and 10d tension $\bar{\mu}$

$$\mu = e^{2A(x_\perp)}\bar{\mu}$$

Depending on the model considered these strings can be metastable, with an age comparable to age of the universe

F1-branes and D1-branes --> also (p,q) strings for relatively prime integers p and q. [Harvey & Strominger; Schwarz]

Interpreted as bound states of p F1-branes and q D1-branes [Polchinski;Witten]



Tension in 10d theory:

$$\mu_i \equiv \mu_{(p_i,q_i)} = \frac{\mu_F}{g_s}\sqrt{p_i^2 g_s^2 + q_i^2}$$

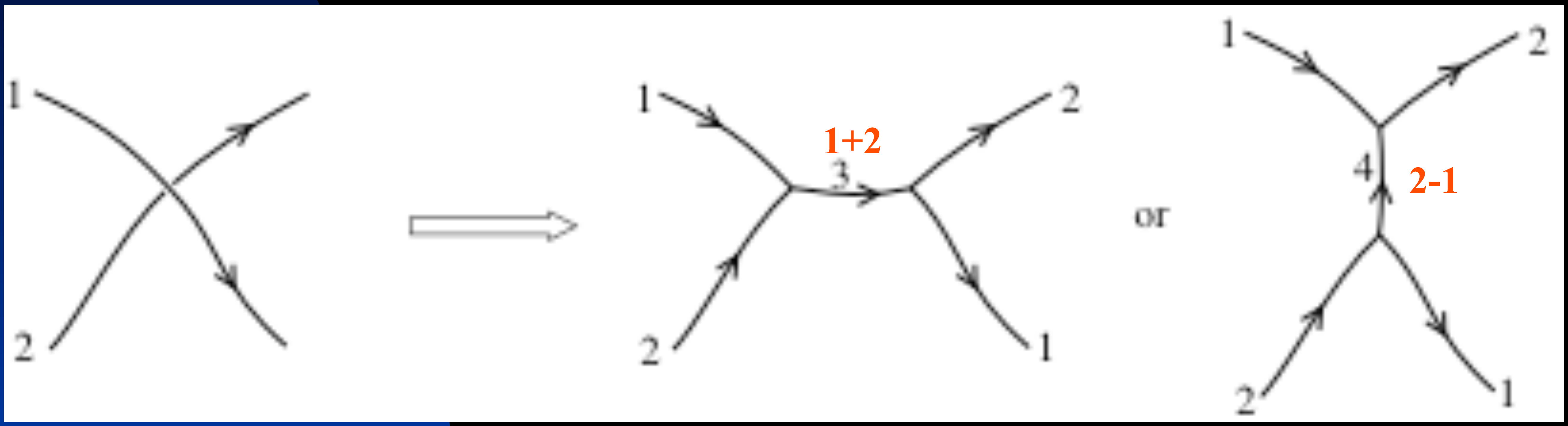
Distinguishing cosmic superstrings

1. Intercommuting probability for gauged strings P~1 always ! In other words when two pieces of string cross each other, they reconnect. Not the case for superstrings -- model dependent probability [Jackson et al 04].

2. Existence of new `defects' D-strings allows for existence of new hybrid networks of F and D strings which could have different scaling properties, and distinct observational effects.

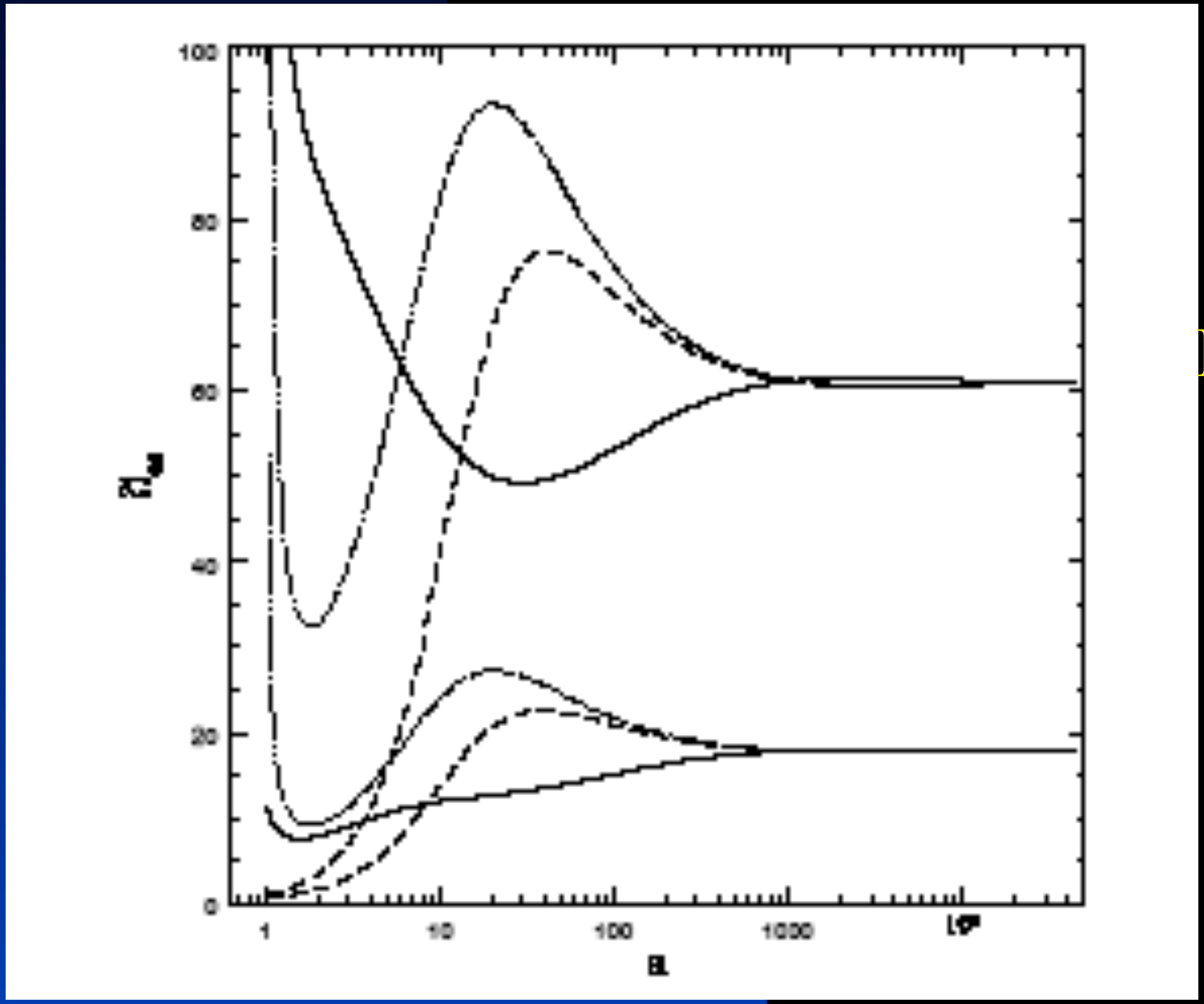
(p,q) string networks -- exciting prospect.

Two strings of different type cross, can not intercommute in general -- produce pair of trilinear vertices connected by segment of string.



What happens to such a network in an expanding background? Does it scale or freeze out in a local minimum of its PE [Sen]? Then it could lead to a frustrated network scaling as w=-1/3

Including multi-tension cosmic superstrings [Tye et al 05, Avgoustidis and Shellard 07, Urrestilla and Vilenkin 07, Avgoustidis and EJC 10].



Density of (p,q) cosmic strings.

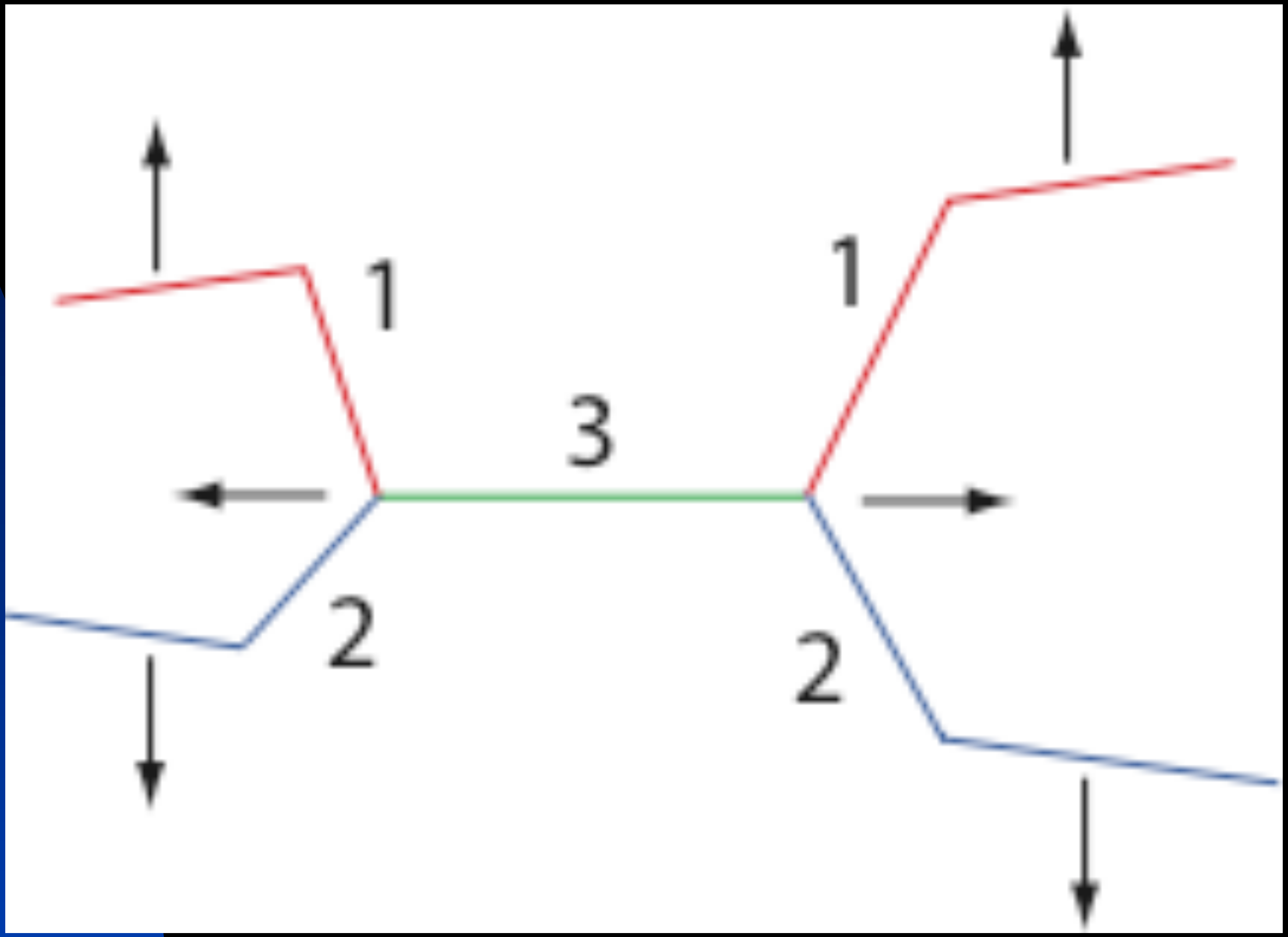
Density of D1 strings.

Scaling achieved indep of initial conditions, and indep of details of interactions.

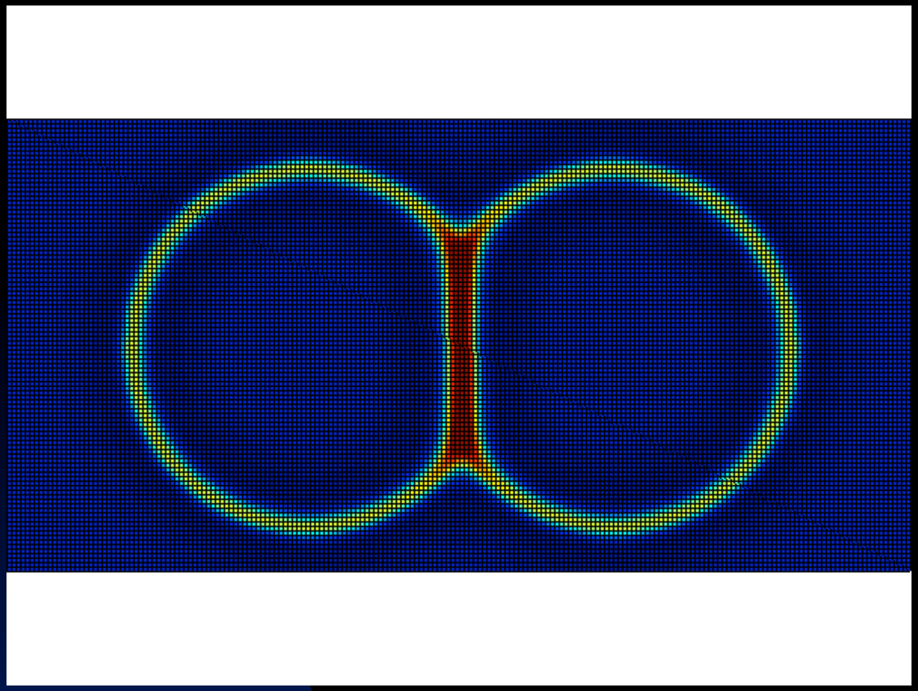
Modelling strings with junctions -- solve the modified Nambu-Goto equations

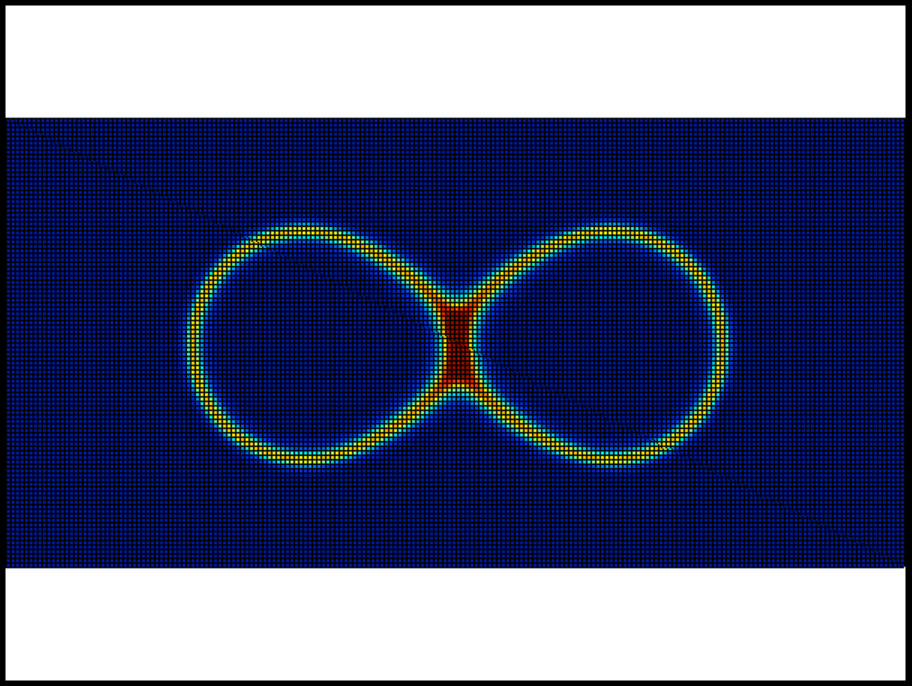
EJC, Kibble and Steer: hep-th/0601153, hep-th/0611243

EJC, Firouzjahi, Kibble and Steer: arXiv: 0712.0808

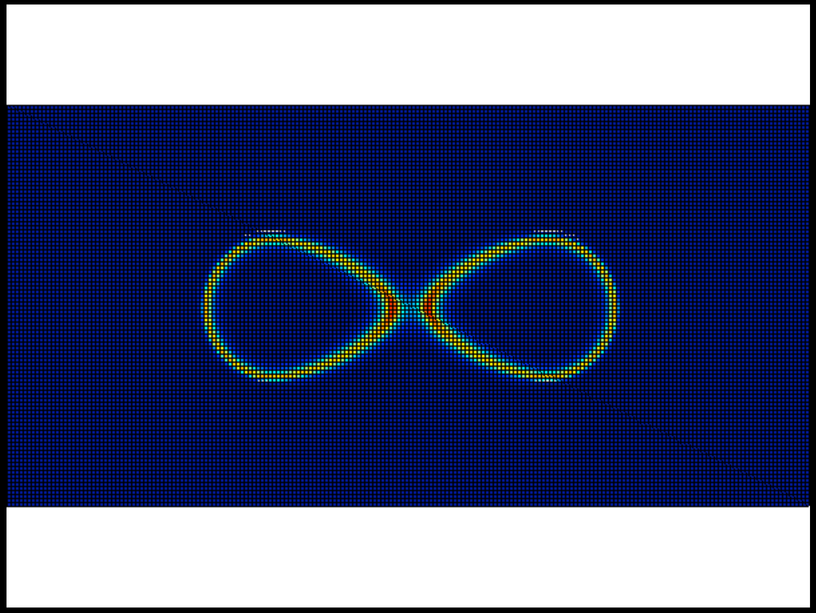


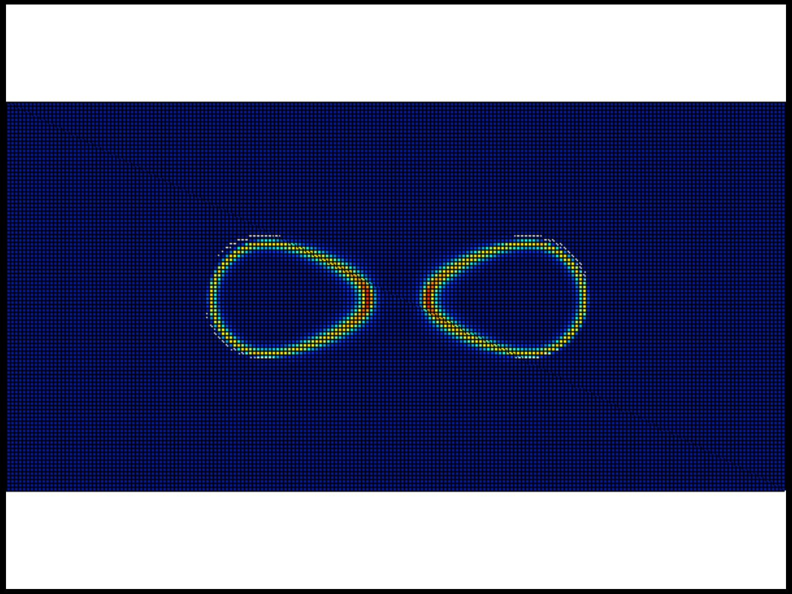
Need to account for the fact that there is a constraint -- three strings meet at a junction and evolve with that junction.



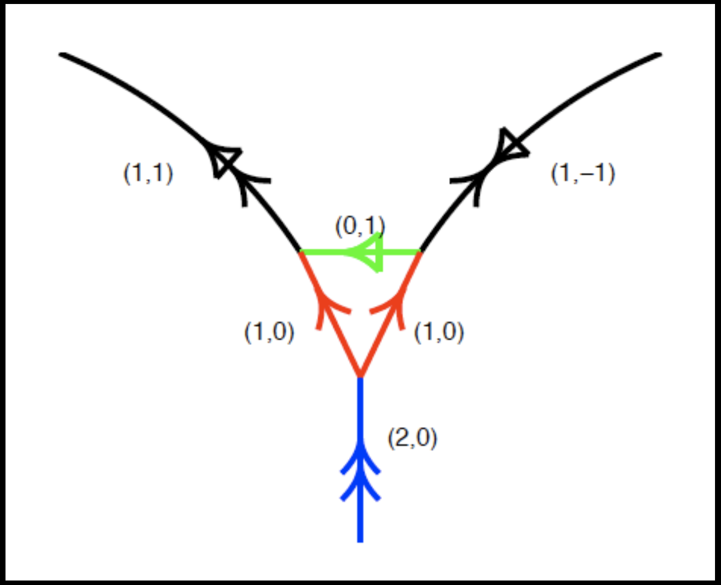


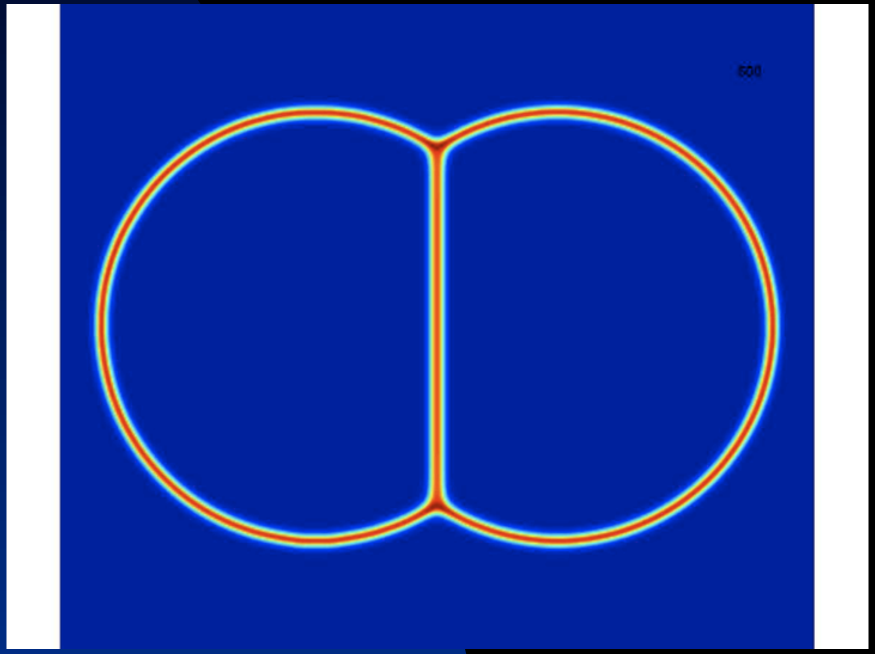
Field theory simulations of collapsing butterfly shape with two equal tensions on the wings. Bevis et al 09

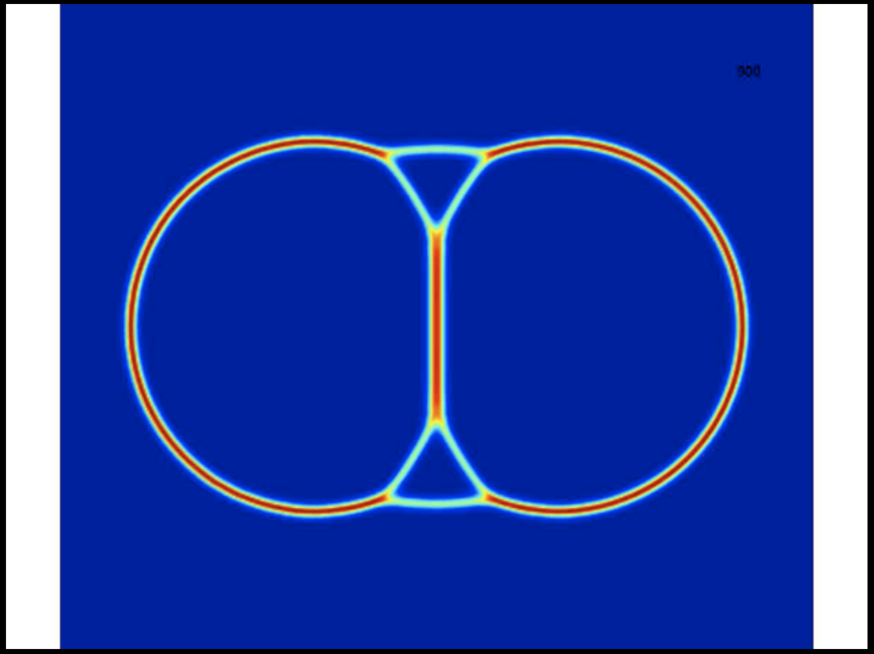


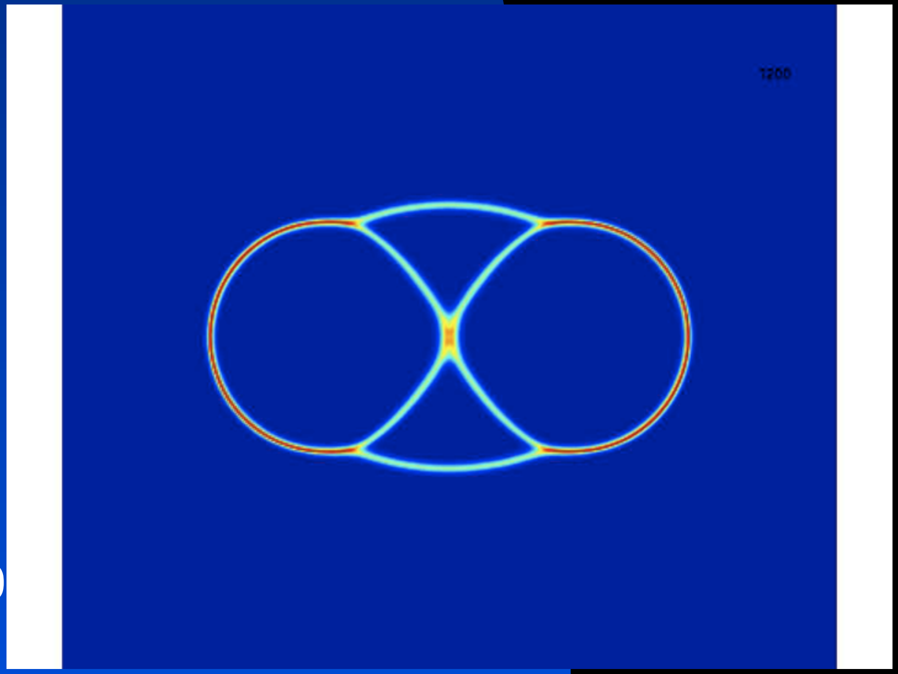


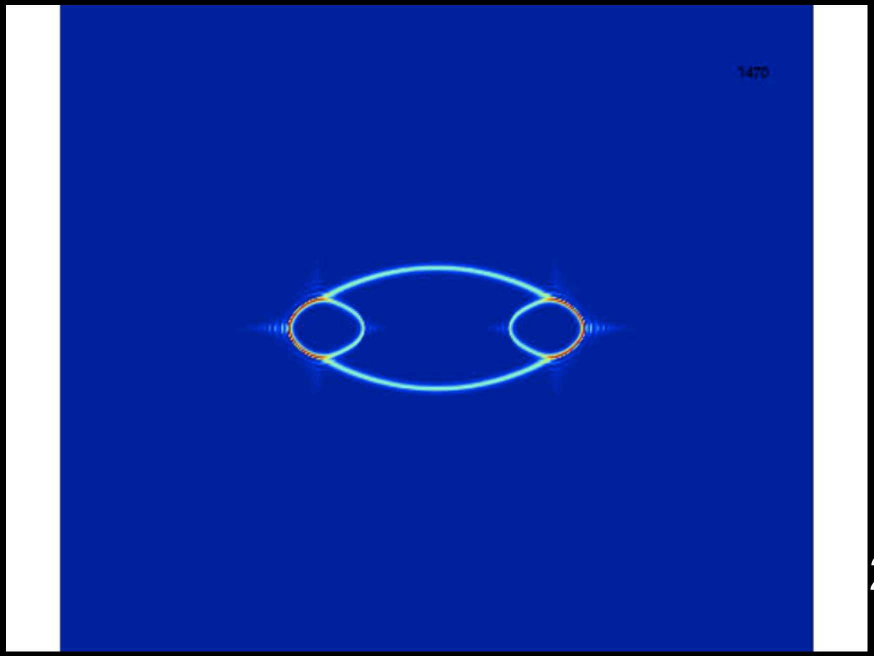
However - there exist some neat triangular instabilities -- our very own loop corrections - which we can explain with the NG equations !



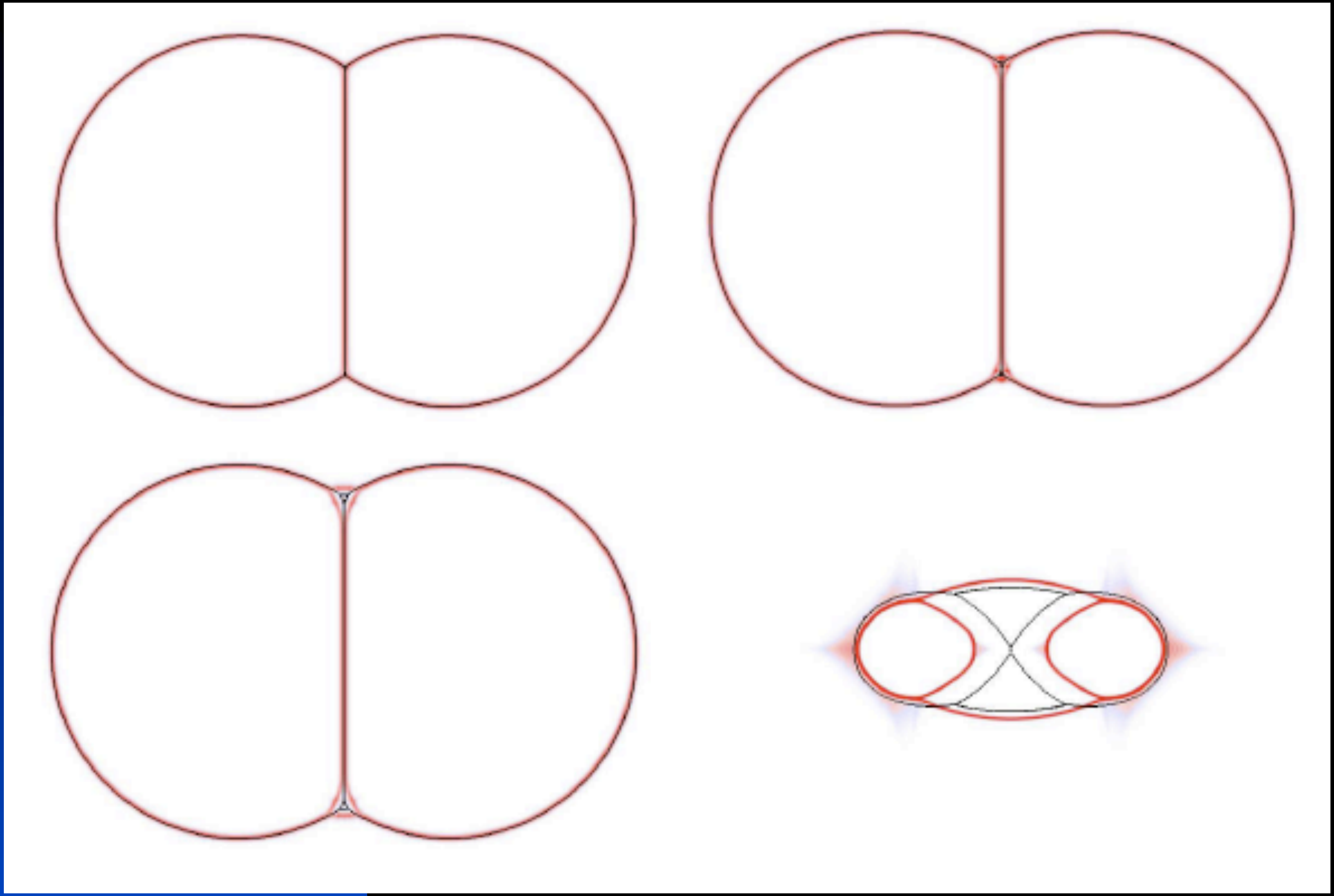




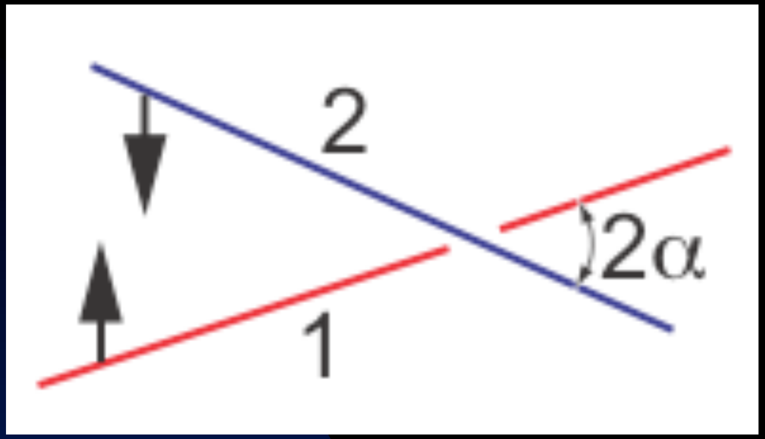




Excellent agreement between field theory (red) and NG (black)



Consider 2 strings crossing

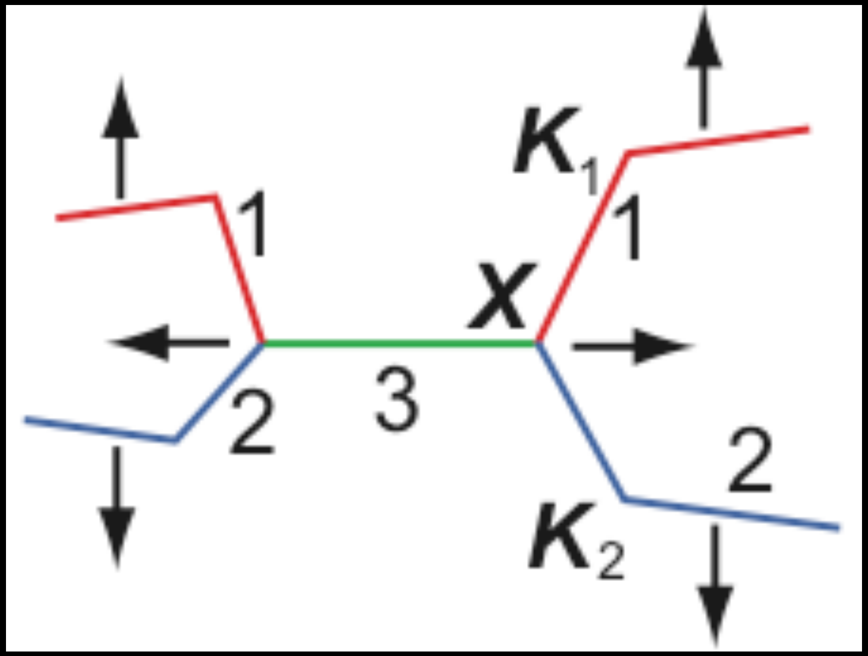


Take $\mu_1 = \mu_2$ and, for $t < 0$,

$$\mathbf{x}_{1,2}(\sigma,t) = (-\gamma^{-1}\sigma\cos\alpha, \mp\gamma^{-1}\sigma\sin\alpha, \pm vt)$$

$$\gamma^{-1} = \sqrt{1-v^2}$$

If 1,2 exchange partners, and are joined by 3, it must lie on *x* or *y* axis (for small α or large α, resp) Assume *x*-axis. Then for $t > 0$,



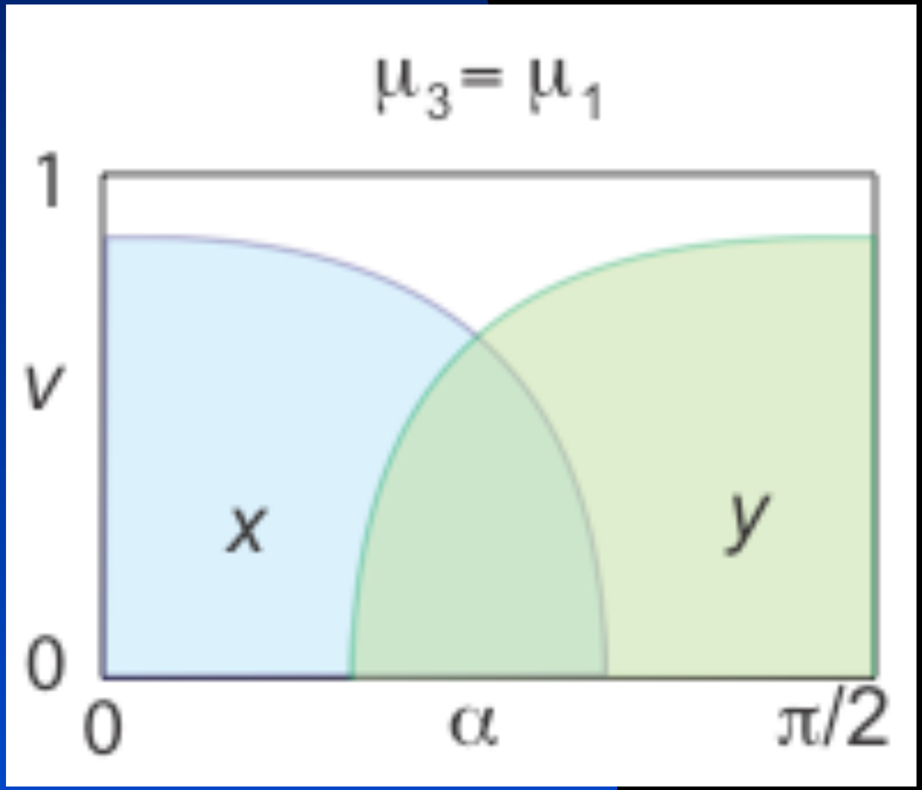
$$\mathbf{x}_3(\sigma,t) = (\sigma,0,0),$$

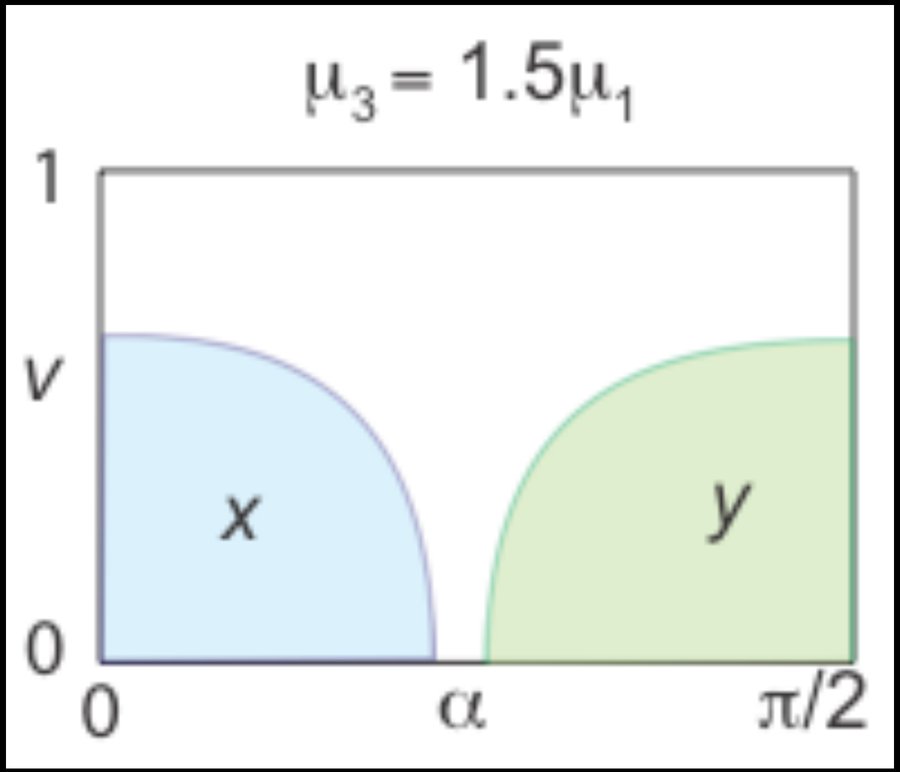
Consider vertex ***X*** on right. Require it moves to right: $\dot{s}_3 > 0,$

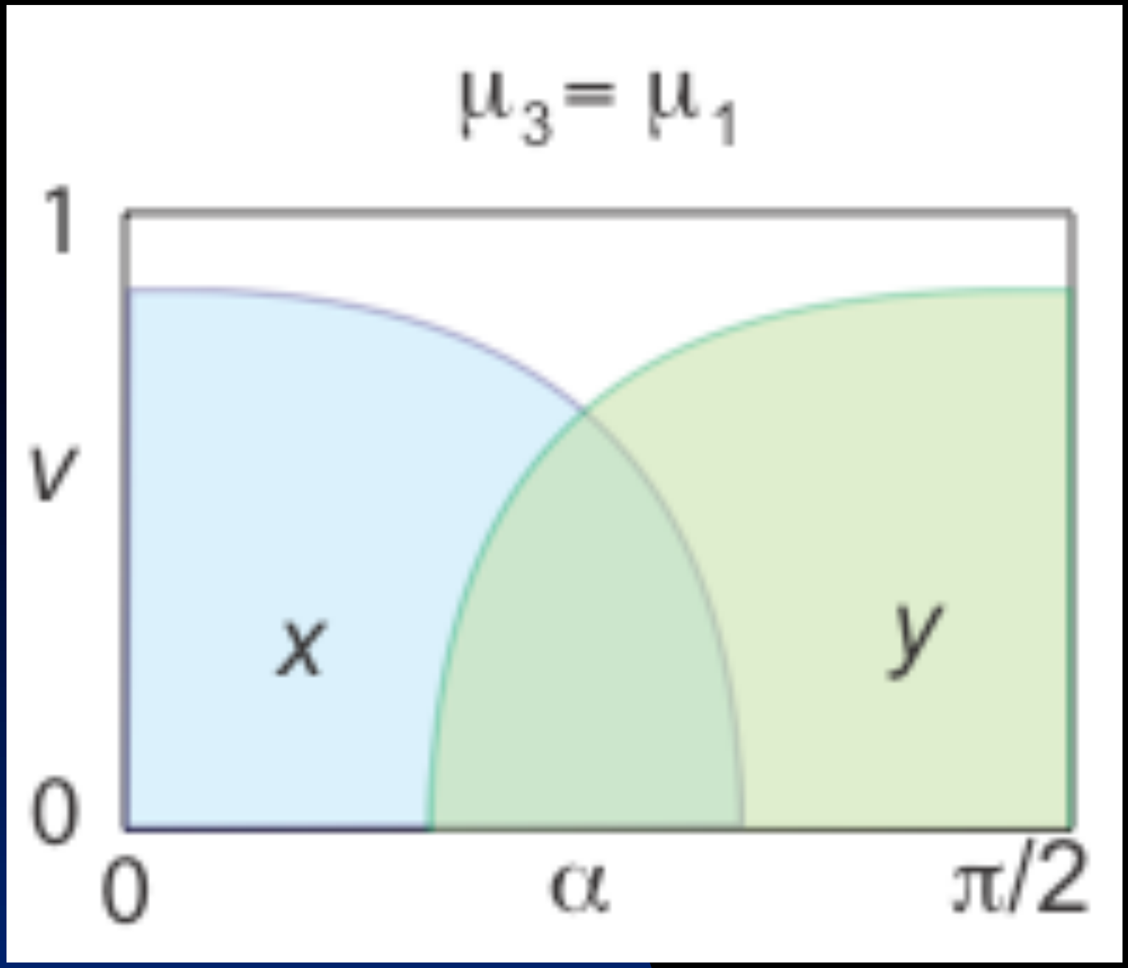
$$\dot{s}_3 = \frac{2\mu_1\gamma^{-1}\cos\alpha - \mu_3}{2\mu_1 - \mu_3\gamma^{-1}\cos\alpha}$$ with $$\mu_3 < 2\mu_1$$

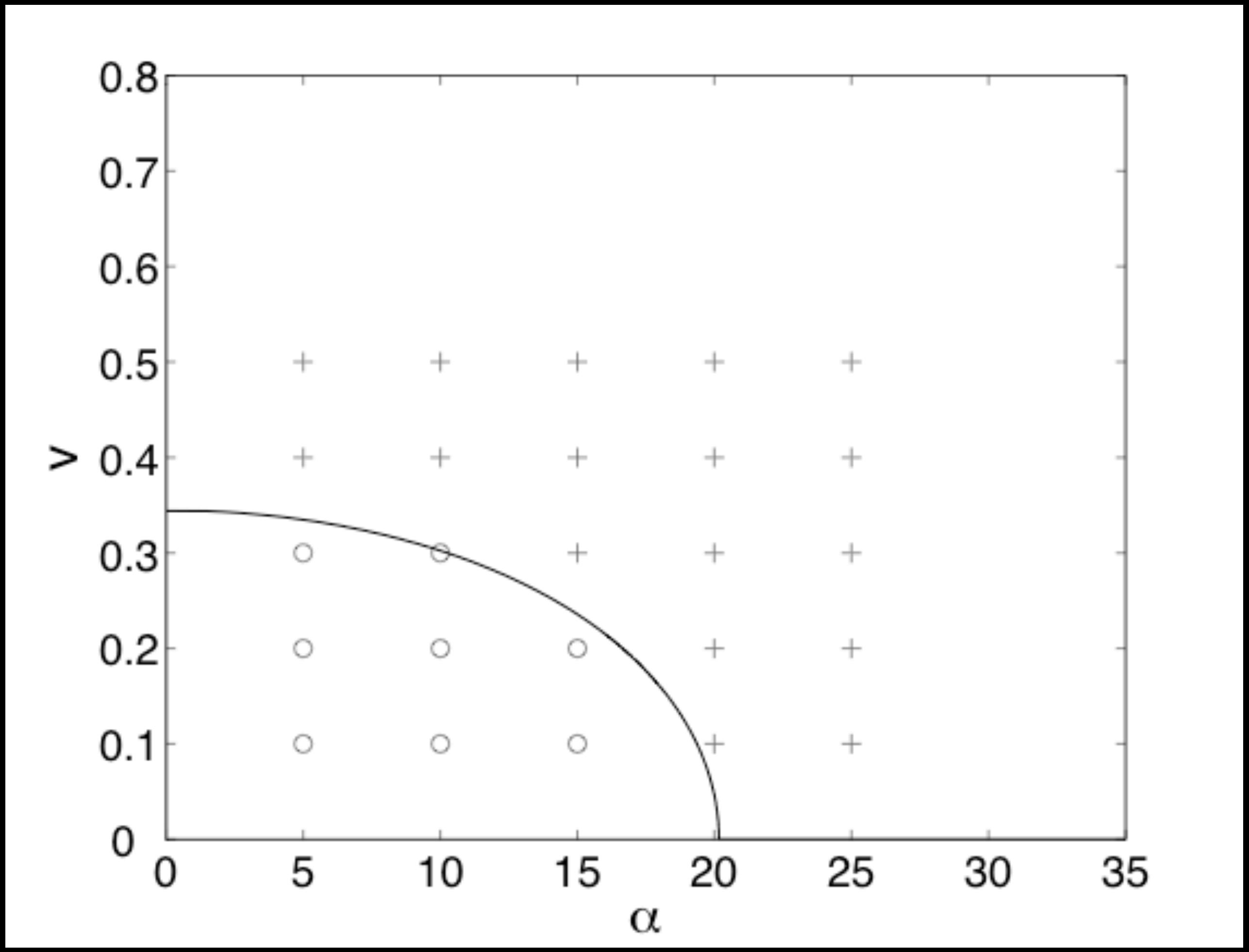
But $\dot{s}_3 > 0,$ implying $$\alpha < \arccos\left(\frac{\mu_3\gamma}{2\mu_1}\right)$$

Kinematically allowed regions are:









Note: **neither** is possible unless

$$\gamma < \frac{2\mu_1}{\mu_3}$$

e.g., if $\mu_3 = \mu_1$,

we require $v < \frac{\sqrt{3}}{2}$

Type I Abelian strings which have stable n=2 string solutions show similar features. Circles form junctions, crosses have reconnections. Solid line is prediction based on junctions-- Salmi et al 07

Recap single one-scale model: (Kibble + many...)

Infinite string density $\rho = \frac{\mu}{L^2}$

$$\dot{\rho} = -2\frac{\dot{a}}{a}\rho - \frac{\rho}{L}$$

Expansion — Loss to loops

Correlation length $L(t) = \xi(t)t, \quad a(t) \sim t^\beta$ Scale factor

$$\frac{\dot{\xi}}{\xi} = \frac{1}{2t}\left(2(\beta - 1) + \frac{1}{\xi}\right)$$

Scaling solution $\xi = [2(1-\beta)]^{-1}.$

Need this to understand the behaviour with the CMB.

Velocity dependent model: (Shellard and Martin)

$$\dot{\rho} = -2\frac{\dot{a}}{a}(1+v^2)\rho - \frac{\tilde{c}\,v\rho}{L},$$

RMS vel of segments

$$\dot{v} = (1-v^2)\left(\frac{k}{L} - 2\frac{\dot{a}}{a}v\right)$$

Curvature type term encoding small scale structure

$$k = \frac{2\sqrt{2}}{\pi}\left(\frac{1-8v^6}{1+8v^6}\right)$$

$$\xi^2 = \frac{k(k+\tilde{c})}{4\beta(1-\beta)}, \qquad v^2 = \frac{k(1-\beta)}{\beta(k+\tilde{c})}$$

Both correlation length and velocity scale

Multi tension string network: (Avgoustidis & Shellard 08, Avgoustidis & EJC 10)

$$\dot{\rho}_i = -2\frac{\dot{a}}{a}(1+v_i^2)\rho_i - \frac{c_i v_i \rho_i}{L_i} - \sum_{a,k} \frac{d^k_{ia}\bar{v}_{ia}\mu_i \ell^k_{ia}(t)}{L_a^2 L_i^2} + \sum_{b,\, a\leq b} \frac{d^i_{ab}\bar{v}_{ab}\mu_i \ell^i_{ab}(t)}{L_a^2 L_b^2}$$

Expansion

Loop of `i' string

Segment of `i' collides with`a' to form segment `k' -- removes energy

Segment of `i' forms from collision of `a' and `b' -- adds energy

$$\dot{v}_i = (1-v_i^2)\left[\frac{k_i}{L_i} - 2\frac{\dot{a}}{a}v_i + \sum_{b,\, a\leq b} b^i_{ab}\frac{\bar{v}_{ab}}{v_i}\frac{(\mu_a+\mu_b-\mu_i)}{\mu_i}\frac{\ell^i_{ab}(t)L_i^2}{L_a^2 L_b^2}\right]$$

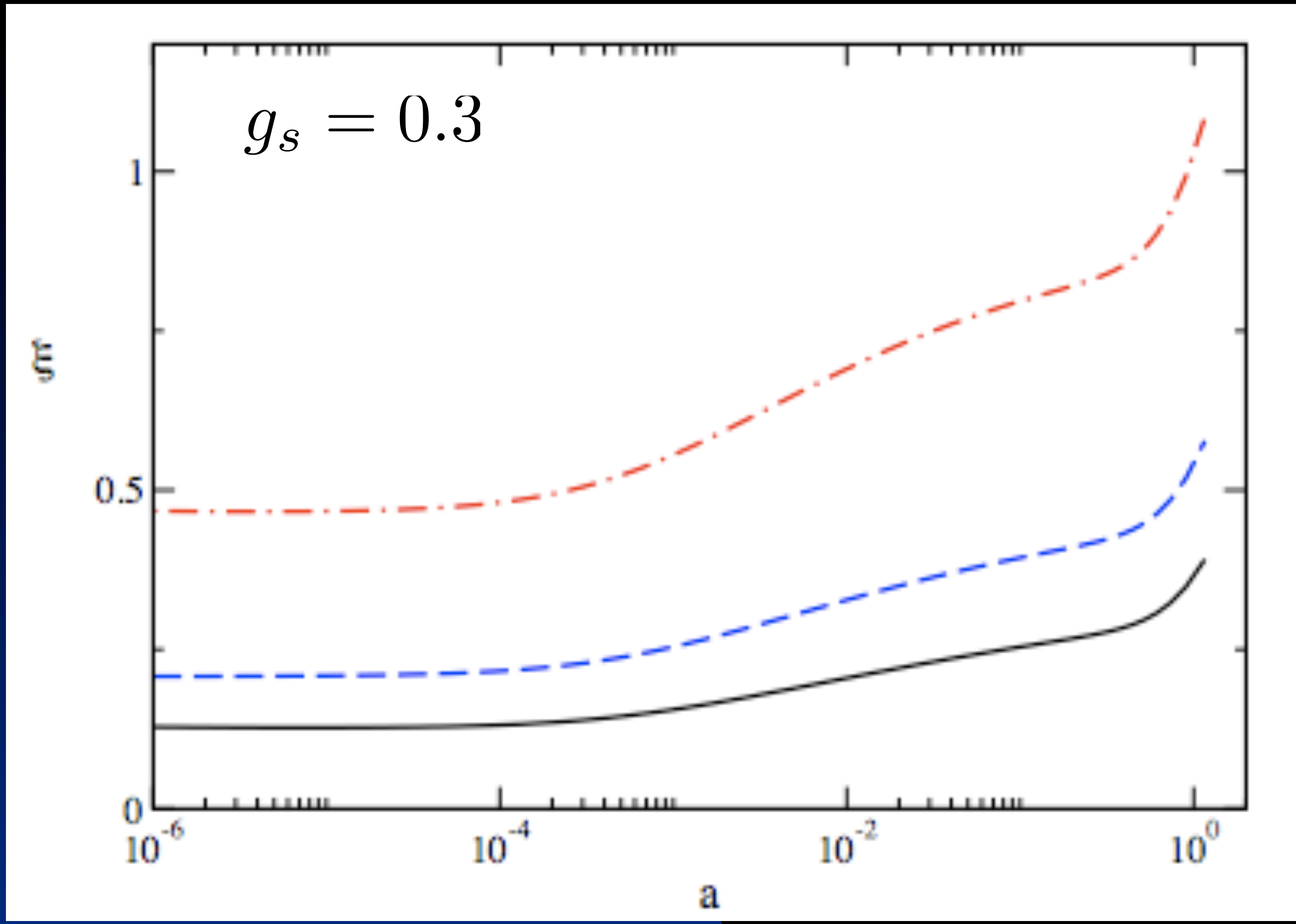
$$v_{ab} = \sqrt{v_a^2 + v_b^2} \qquad \mu_i \equiv \mu_{(p_i,q_i)} = \frac{\mu_F}{g_s}\sqrt{p_i^2 g_s^2 + q_i^2} \qquad \rho_i = \frac{\mu_i}{L_i^2}$$

`k' segment length $\quad \ell^k_{ij} = \dfrac{L_i L_j}{L_i + L_j}$

d^k_{ia} incorporate the probabilities of intercommuting and the kinetic constraints. They have a strong dependence on the string coupling g_s and we are still getting to the bottom of that dependence -- not easy !

$$\{(p,q)_i\} = \{(1,0),(0,1),(1,1),(1,2),(2,1),(1,3),(3,1)\}\ , \quad (i = 1, ..., 7)$$

preliminary results from work in progress with Pourtsidou, Avgoustidis, Pogosian and Steer



Example - 7 types of (p,q) string. Only first three lightest shown - scaling rapidly reached in rad and matter.

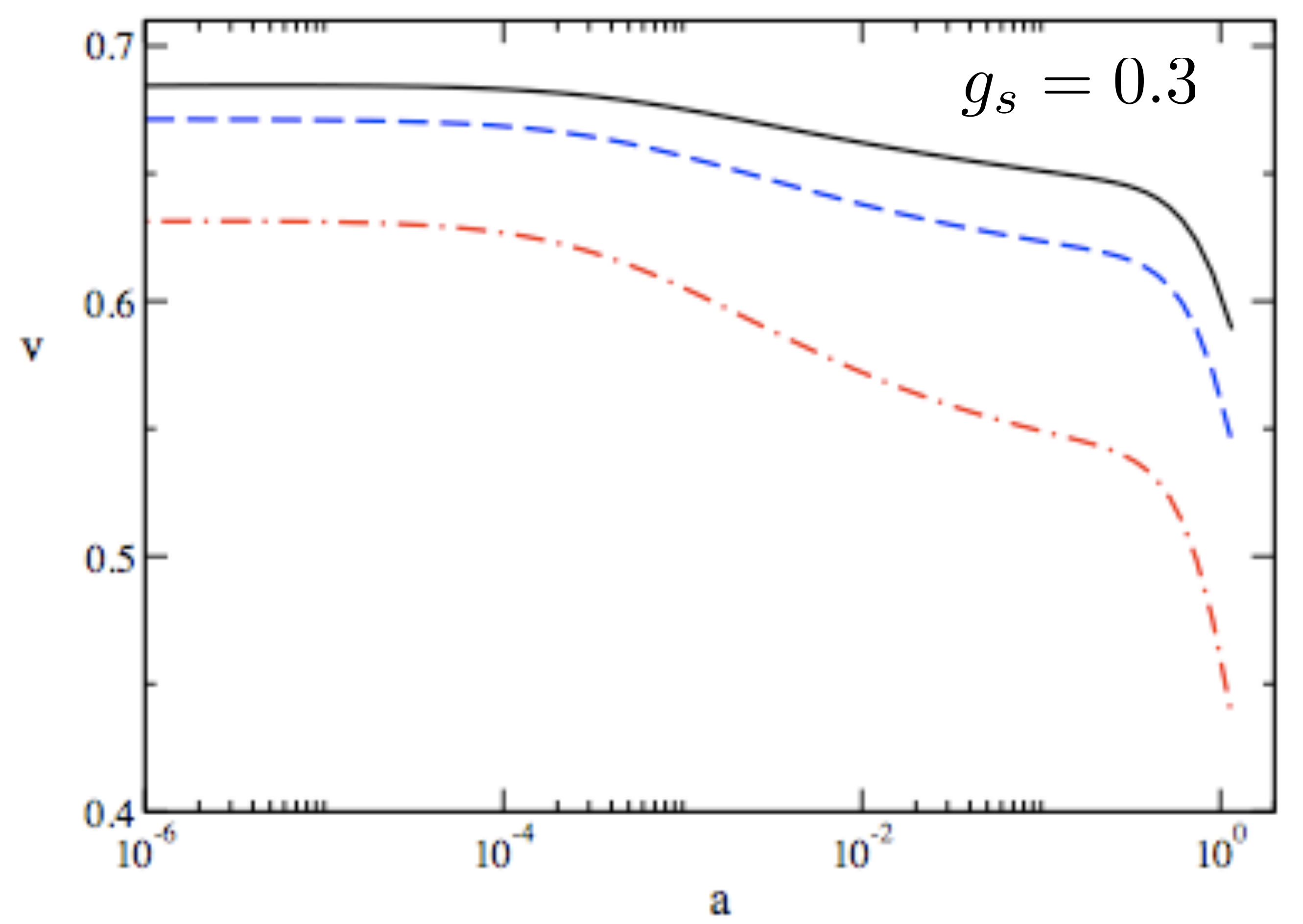
Densities of rest suppressed.

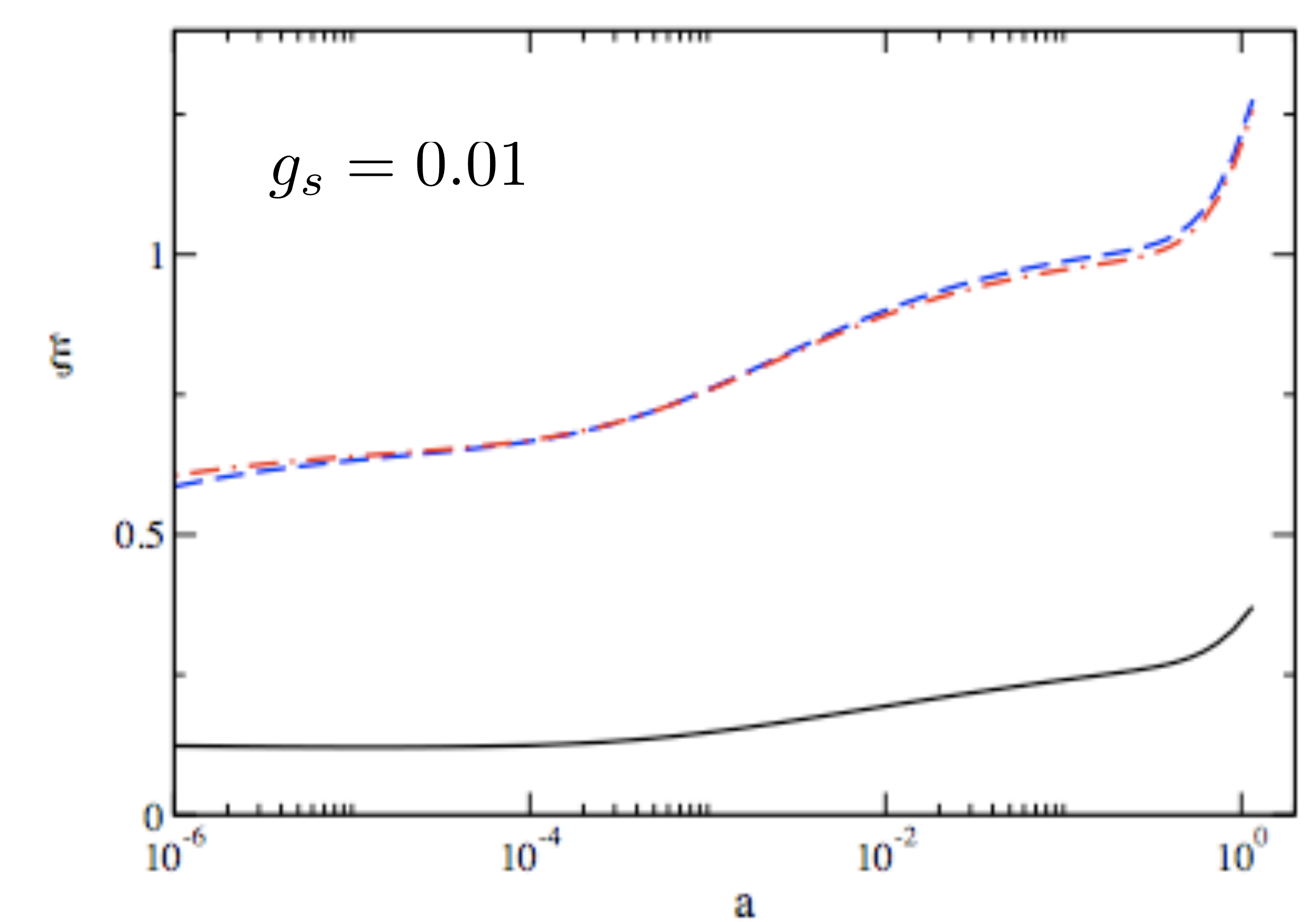
Black -- (1,0) -- Most populous
Blue dash -- (0,1)
Red dot dash -- (1,1)

Deviation from scaling at end as move into Λ domination.

Velocities of first three most populous strings:

F and D strings dominate both the number density and the energy density for larger values of g_s=0.3 - 1





As before for correlation lengths but now with g_s=0.01

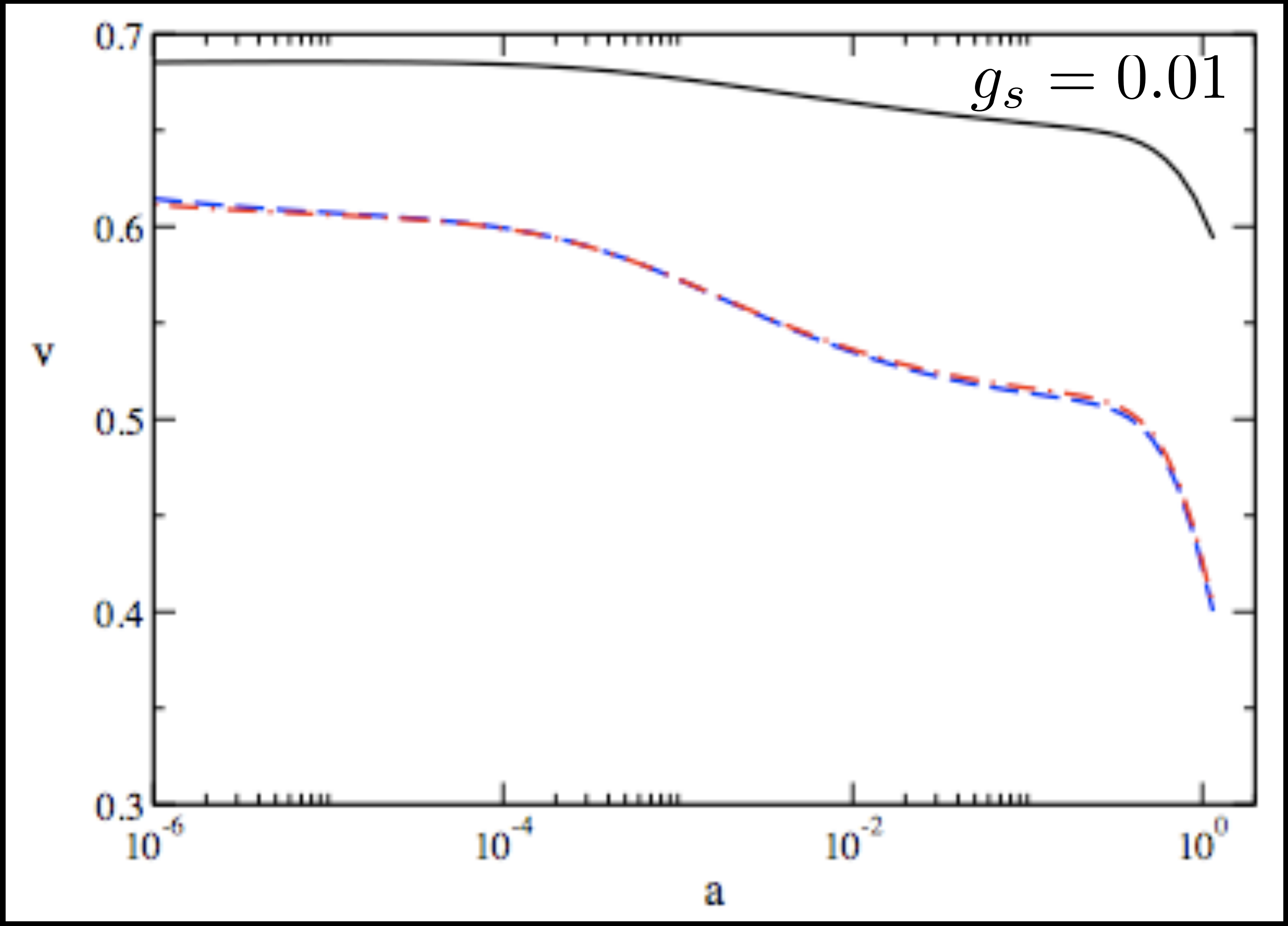
Black -- (1,0) -- Most populous
Blue dash -- (0,1)
Red dot dash -- (1,1)

Note (0,1) and (1,1) almost identical because tensions so similar. Note also F string has much larger number density, where as heavier D string (100 times here) is less common. Same is true for (F,D) string, so now have two heavy and one light string.

As before for velocities but now with g_s=0.01

Now have situation where energy density of network is dominated by the heavier and rare D and (F,D) strings even though the light F string is more populous. This is in contrast to previous case.

Will see this impacts on position of B-mode peak in CMB.



Strings and the CMB

Modified CMBACT (Pogosian) to allow for multi-tension strings.
Shapes of string induced CMB spectra mainly obtained form large scale properties of string such as correlation length and rms velocity given from the earlier evolution eqns.
Normalisation of spectrum depends on:

$$C_l^{strings} \propto \sum_{i=1}^{N} \left(\frac{G\mu_i}{\xi_i} \right)^2$$

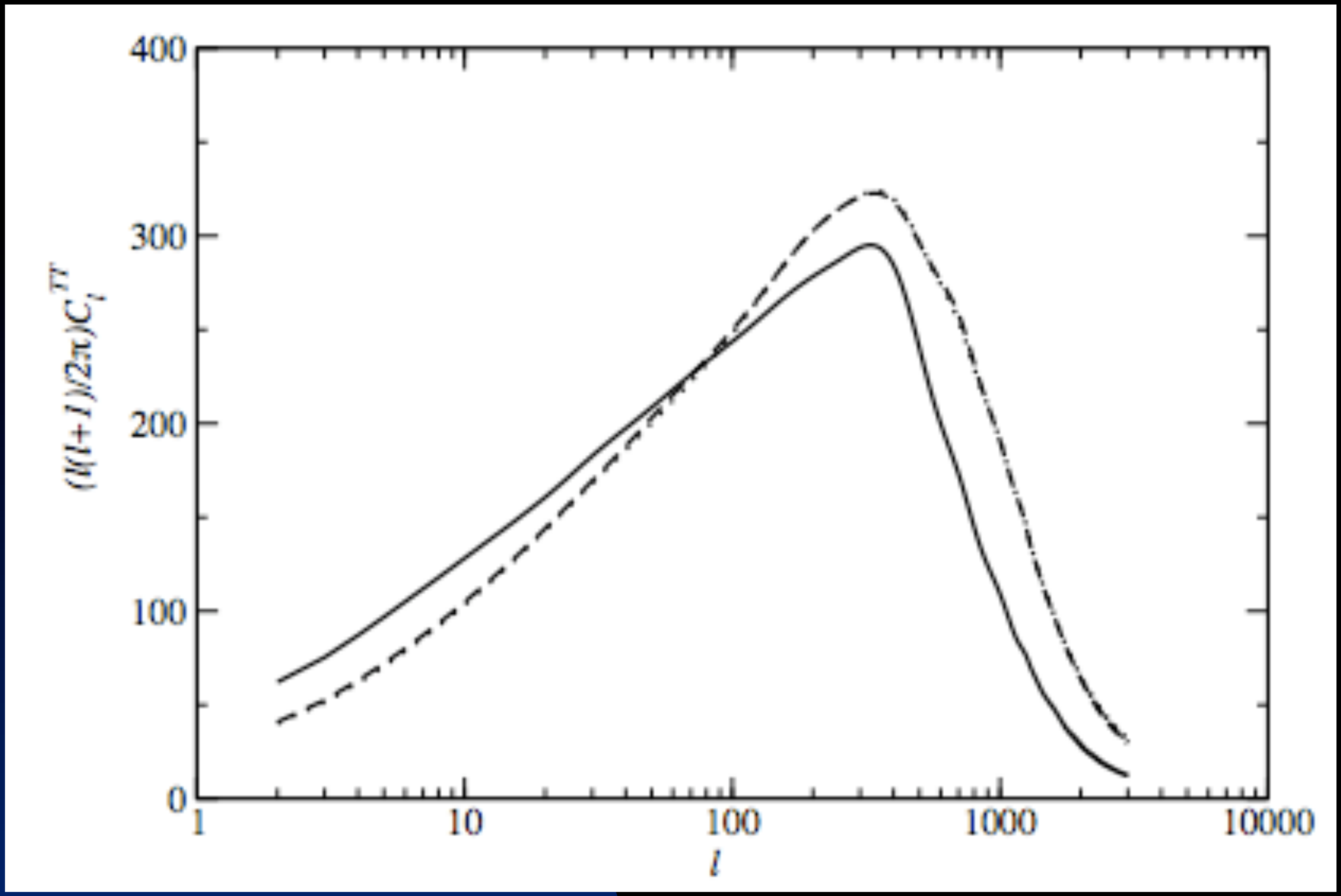
i.e. on tension and correlation lengths of each string

Since strings can not source more than 10% of total CMB anisotropy, we use that to determine the fundamental F string tension which is otherwise a free parameter. So μ_F chosen to be such that:

$$f_s = C^{TT}_{strings}/C^{TT}_{total} = 0.1$$

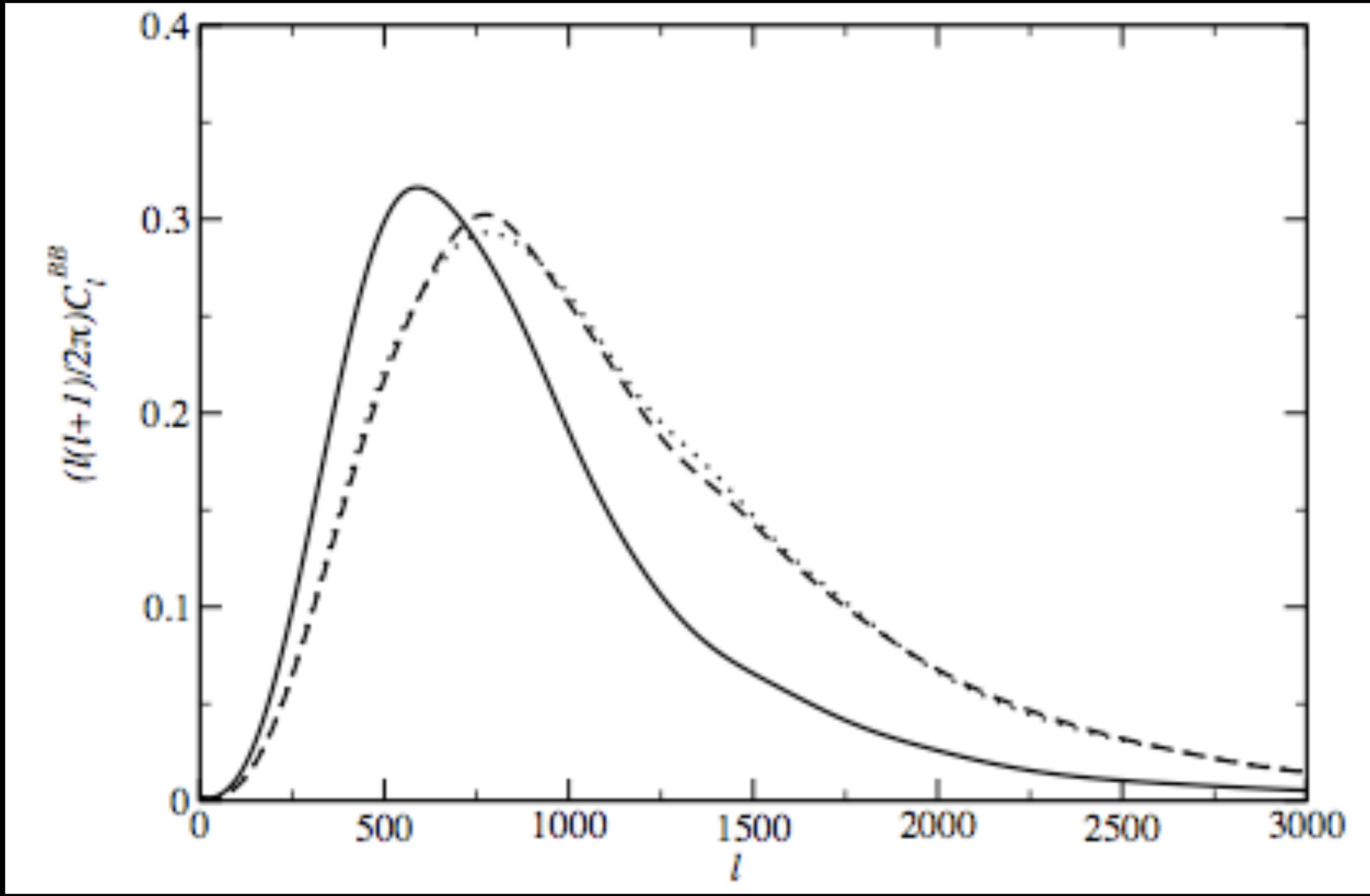
where

$$C^{TT} \equiv \sum_{\ell=2}^{2000} (2\ell+1) C_\ell^{TT}$$



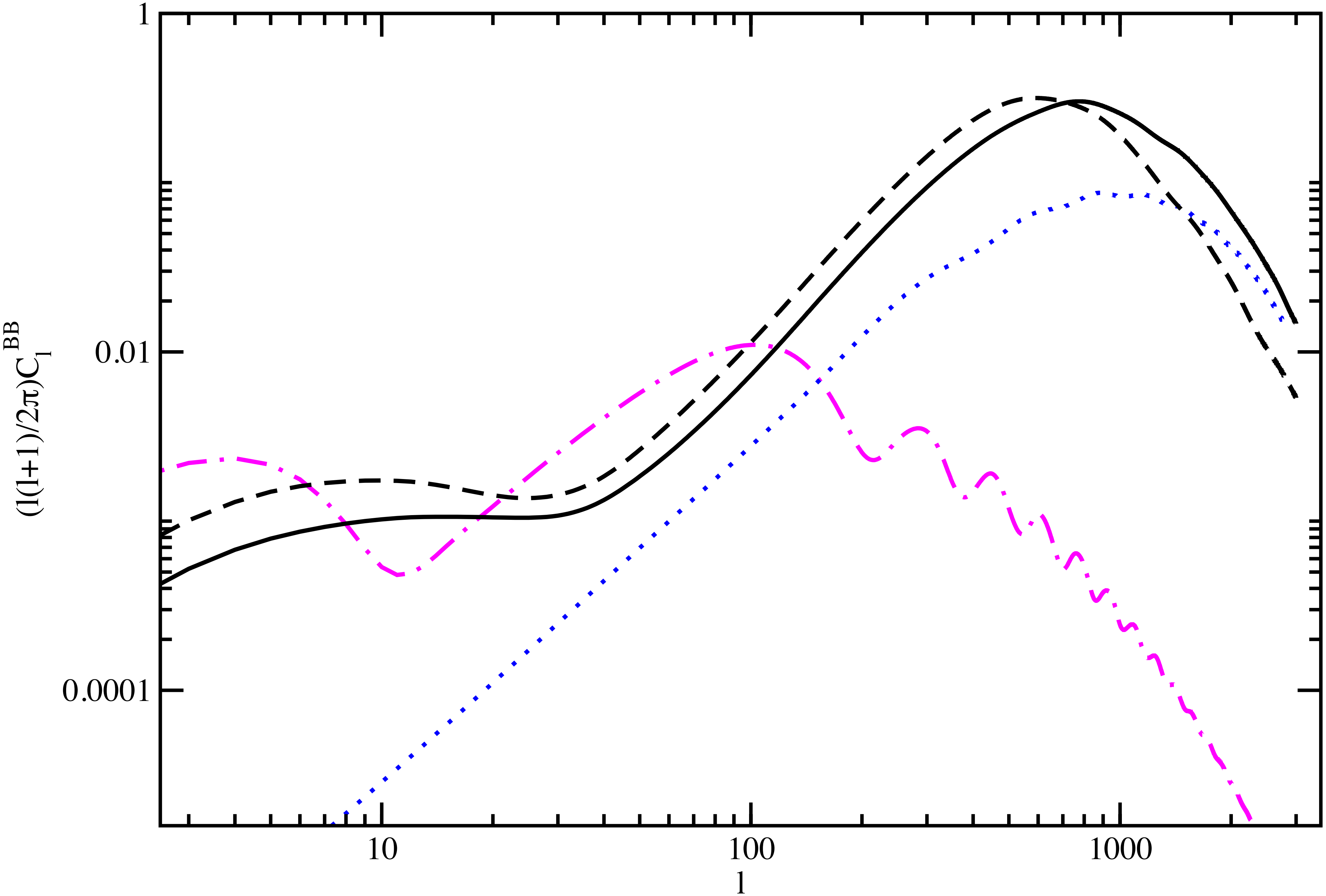
Left:
Normalised TT power spectra for 3 different string couplings.
Solid black is g_s=0.01
Dotted line is g_s=0.3
Dashed line is g_s=1

Note degeneracy in g_s=0.3 and 1.

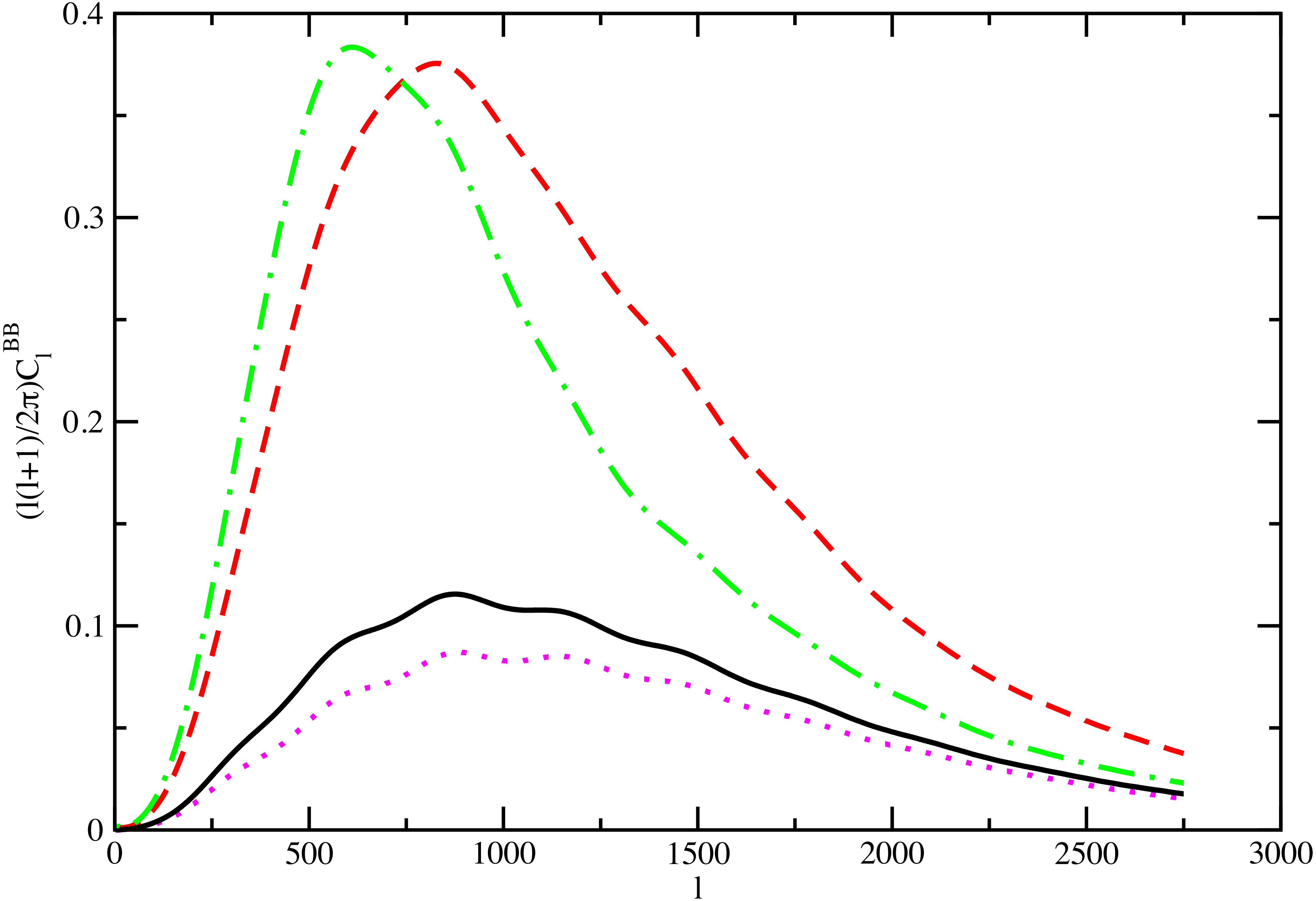


Right:
Normalised BB power spectra for 3 different string couplings.
Solid black is g_s=0.01
Dotted line is g_s=0.3
Dashed line is g_s=1

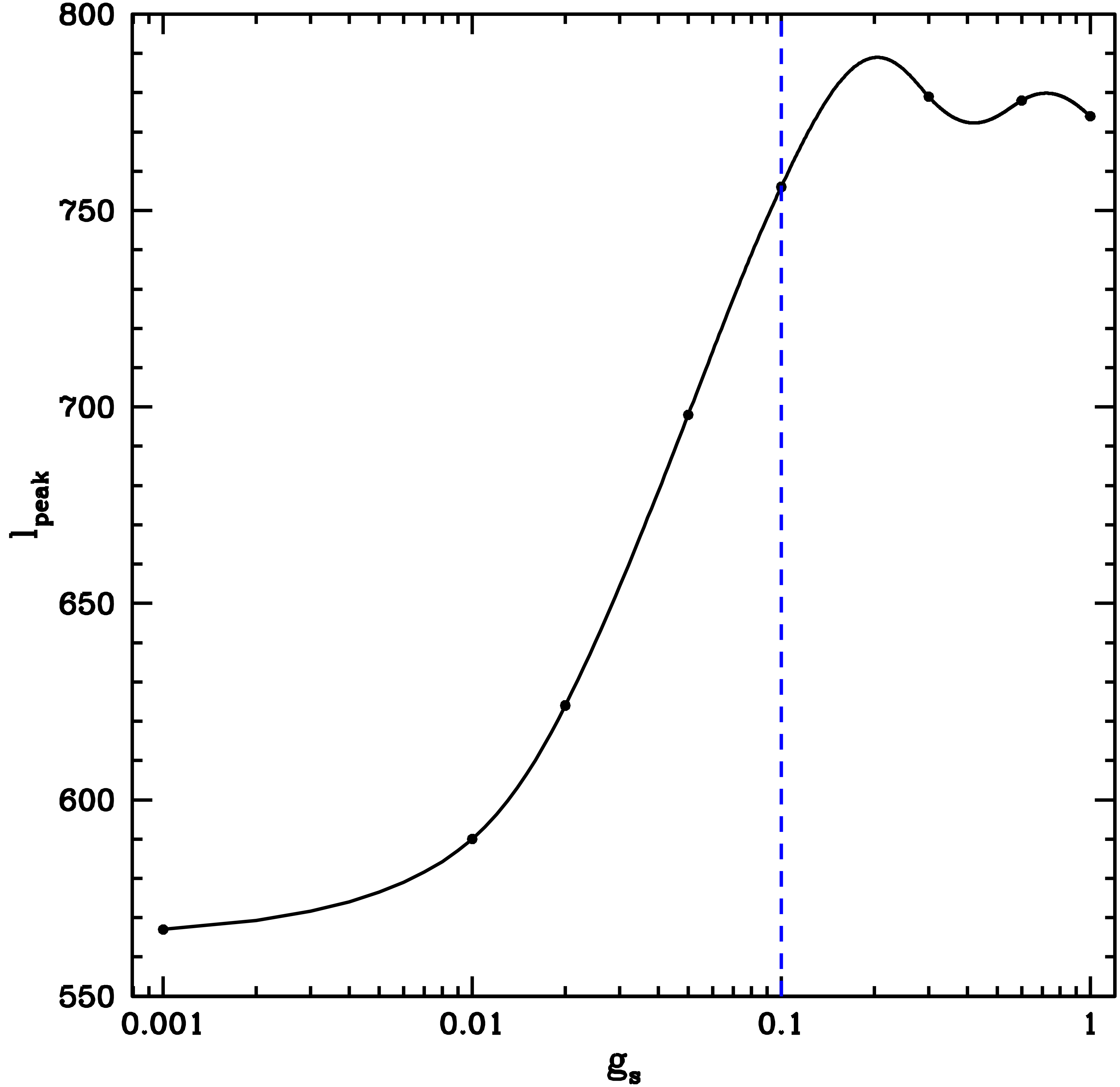
Note small string coupling leads to discernible move in the peak of the BB spectra to small l -- showing impact of changing scaling solutions wrt light and heavy strings.



B type polarisation spectra due to cosmic superstrings assuming 10% string contribution. Solid black (g_s=0.3) and dashed black line (g_s=0.01). Expected spectra for E to B lensing (blue dot) and primordial grav waves assuming r=0.1 (magenta-dot-dash) also shown.



Lensing prediction (magenta dot). Sum of strings and lens sourced B-mode power for g_s=0.3 and f_s=0.001 (Black). Strings show up as excess power at high l over lensing prediction. Also shown is sum of strings and lensing contributions for g_s=0.3 and f_s=0.01 (red-dash) and g_s=0.01 and f_s=0.01 (green-dash).

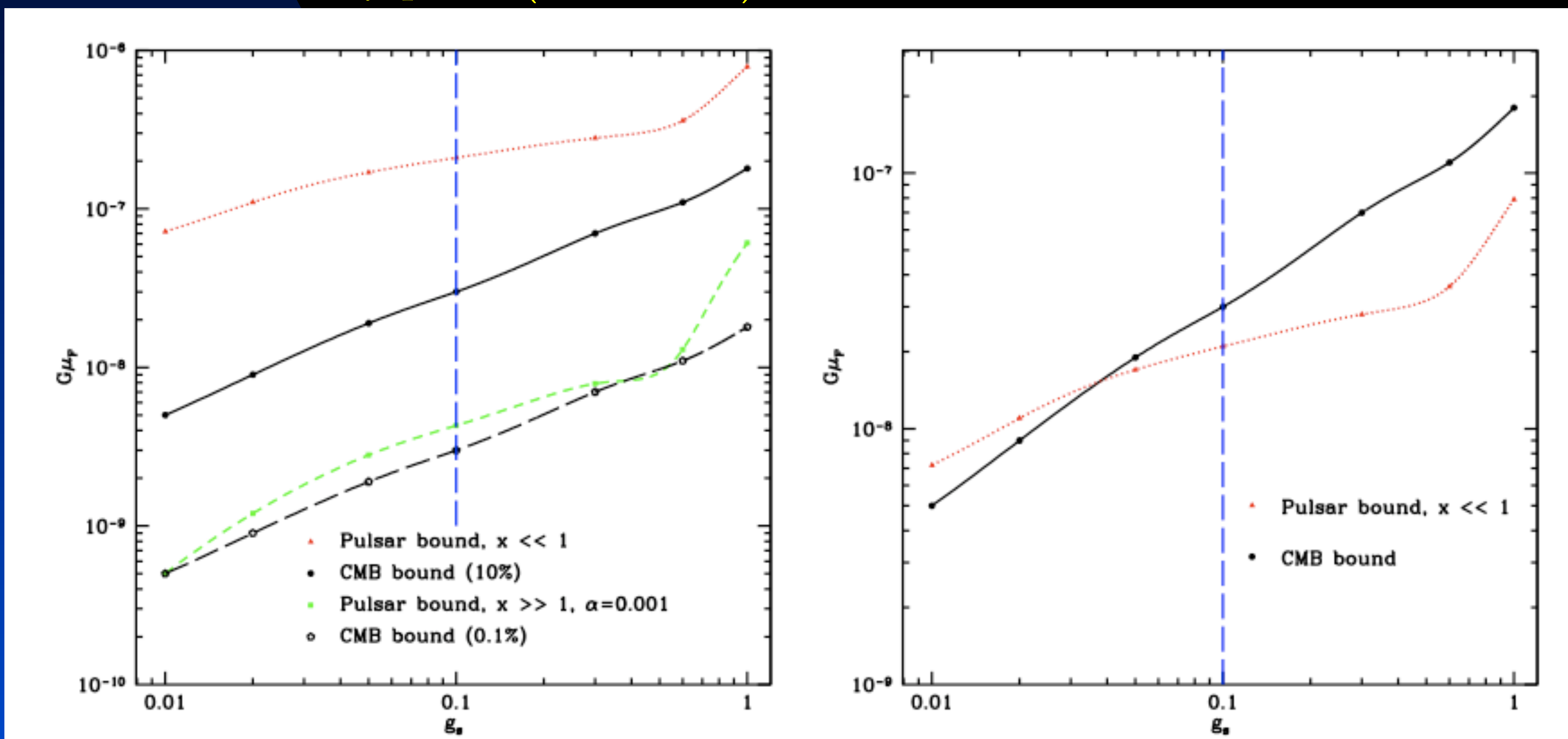


Position of the peak of the BB spectrum as a function of the string coupling g_s. The transition from high l values to lower values occurs when the density of string becomes dominated by the heavy rarer strings.

Using cosmology to constrain μ_F and g_s

Aim use a combination of measurements to constrain the allowed parameter space making use of the fact they ahve different dependencies on the parameters. For example combining CMB and pulsar timing (Battye and Moss 10)

$$\Omega_{\rm g}h^2 = 1.17\times 10^{-4}\sum_{i=1}^{3} G\mu_i\left(\frac{1-\langle v^2_{\rm rad,i}\rangle}{\xi^2_{\rm rad,i}\Omega_{\rm m}}\right)\frac{(1+1.4x_i)^{3/2}-1}{x_i} \qquad x_i=\alpha/(\Gamma G\mu_i)$$



Conclusions

If we are lucky with inflation in string models, we may form metastable F and D strings which will survive long enough to be of interest. To really understand their impact we need to know their dynamical properties.

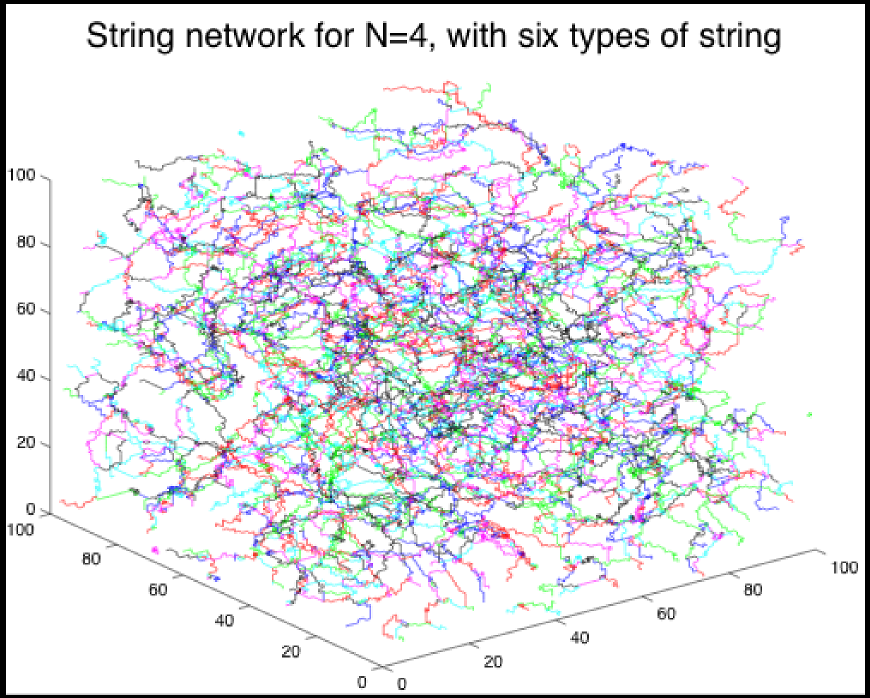
1. What does a network of strings with junctions look like? Will need to incorporate kinematic constraints.

2. What are their distinctive observational signatures, either through Gravitational waves, lensing or cmb?

3. We are beginning to address some of these questions thanks to a combination of analytic and numerical approaches and are finding some interesting results.

Lots still to do though !

(p,q) string networks -- mimic with field theory. Under sym breaking G -->K (non-Abelian) find defects that do not intercommute.



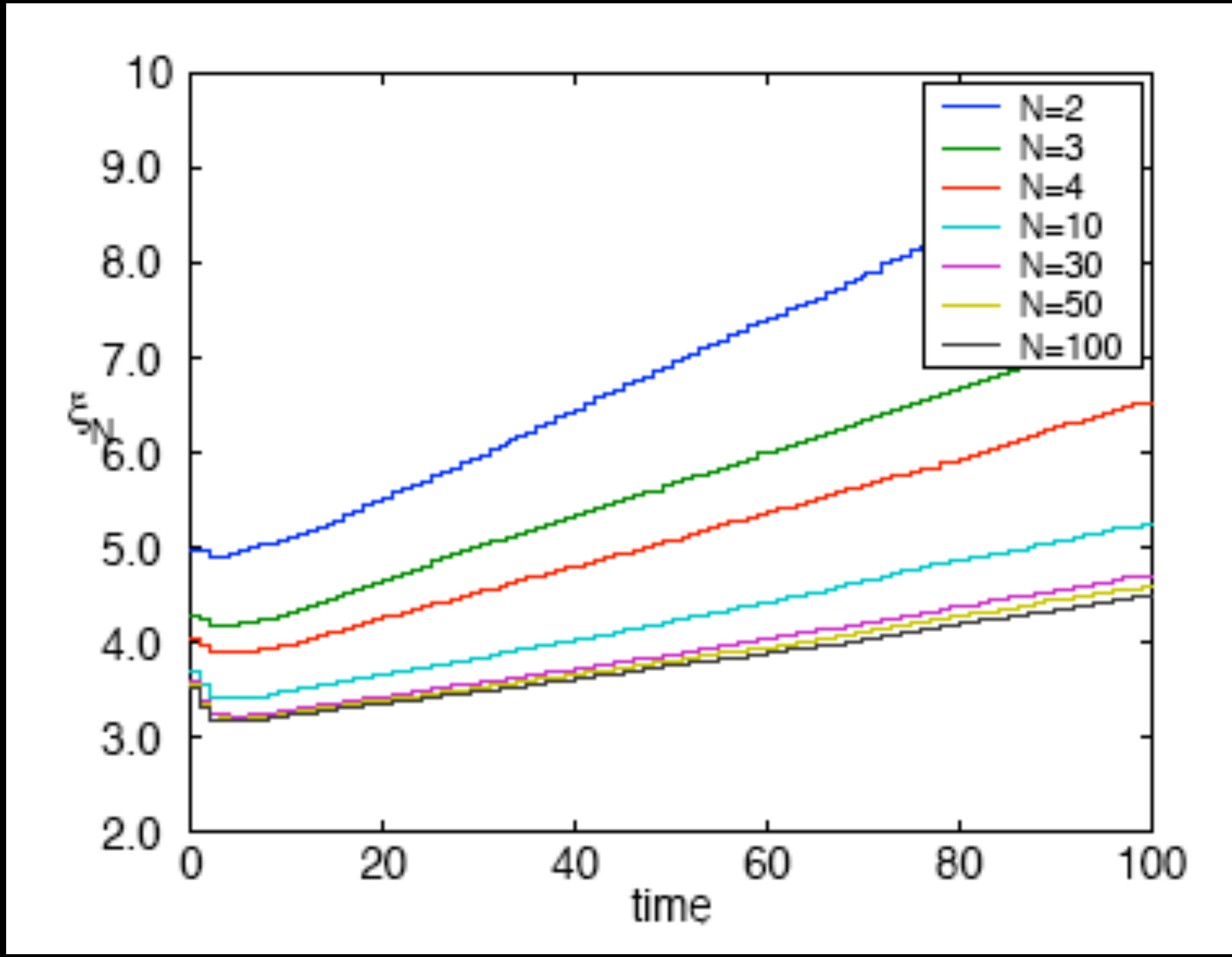
$K = S_3$ and S_8 - [Spergel & Pen 96]

Modelling the case $K = S_N$

Numerically: Scaling solutions seem to exist for all N :

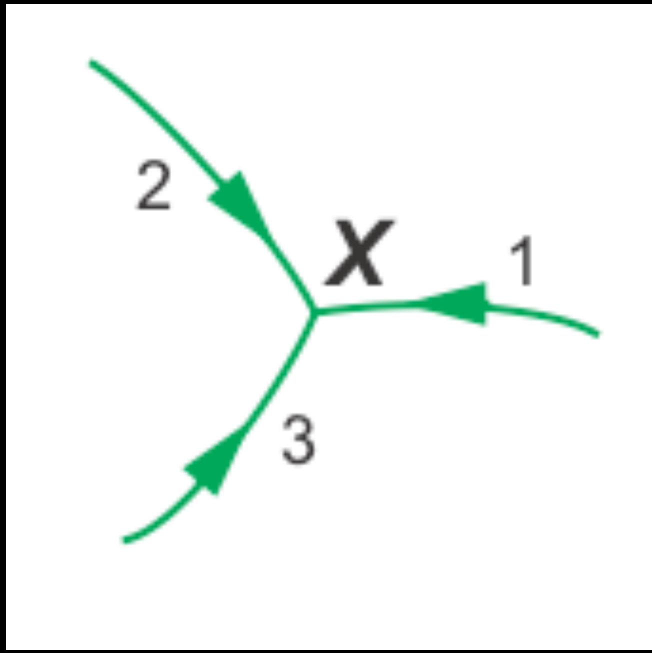
$$\rho \sim \mu \xi^{-2}$$

$$\xi_N(t) = \xi_0(N) + \alpha_N t$$



[Vachaspati and Vilenkin 87] [EC and Saffin 05] [Saffin 06; Hindmarsh and Saffin 06]

Take σ on each leg j to increase towards the vertex, position $\boldsymbol{X}(t)$



$$S = -\sum_j \mu_j \int dt\, d\sigma\, \theta(s_j(t) - \sigma)\sqrt{\boldsymbol{x}_j'^2(1 - \dot{\boldsymbol{x}}_j^2)}$$

$$+\sum_j \int dt\, \boldsymbol{f}_j(t) \cdot [\boldsymbol{x}_j(s_j(t), t) - \boldsymbol{X}(t)]$$

Varying $\boldsymbol{x}_j \Rightarrow \ddot{\boldsymbol{x}}_j - \boldsymbol{x}_j'' = \boldsymbol{0},$

boundary terms $\Rightarrow \mu_j(\boldsymbol{x}_j' + \dot{s}_j \dot{\boldsymbol{x}}_j) = \boldsymbol{f}_j$ at $(s_j(t), t)$

Varying $\boldsymbol{X} \Rightarrow \sum_j \boldsymbol{f}_j = \boldsymbol{0}$ Varying $\boldsymbol{f}_j \Rightarrow \boldsymbol{x}_j(s_j(t), t) = \boldsymbol{X}(t)$

Varying $s_j \Rightarrow \boldsymbol{f}_j \cdot \boldsymbol{x}_j' = \boldsymbol{x}_j'^2$ (not independent of other eqns)

$$\boldsymbol{x}_j(\sigma,t) = \tfrac{1}{2}[\boldsymbol{a}_j(\sigma+t) + \boldsymbol{b}_j(\sigma-t)]$$ with $$\boldsymbol{a}_j'^2 = \boldsymbol{b}_j'^2 = 1$$

$$\boldsymbol{x}_j(s_j(t),t) = \boldsymbol{X}(t) \;\Rightarrow\; \boldsymbol{a}_j(s_j+t) + \boldsymbol{b}_j(s_j-t) = 2\boldsymbol{X}(t)$$

$$\sum_j \boldsymbol{f}_j = \boldsymbol{0} \;\Rightarrow\; \sum_j \mu_j[(1+\dot{s}_j)\boldsymbol{a}_j' + (1-\dot{s}_j)\boldsymbol{b}_j'] = \boldsymbol{0}$$

Initial conditions at $t = 0 \Rightarrow$ values of $\boldsymbol{a}_j'(\sigma)$ and $\boldsymbol{b}_j'(\sigma)$ for $\sigma < s_j(0)$

So for $t > 0$, values of $\boldsymbol{b}_j'(s_j(t)-t)$ (ingoing wave) are known, but not those of $\boldsymbol{a}_j'(s_j(t)+t)$ (outgoing wave)

So use $(1+\dot{s}_j)\boldsymbol{a}_j' - (1-\dot{s}_j)\boldsymbol{b}_j' = 2\dot{\boldsymbol{X}}$ to eliminate $\boldsymbol{a}_j'$

$$\Rightarrow \sum_j \mu_j(1-\dot{s}_j)\boldsymbol{b}_j' = -(\mu_1+\mu_2+\mu_3)\dot{\boldsymbol{X}}$$

Need $\dot{s}_j$ Find: $(\sum_j \mu_j)^2 \dot{s}_1 = -(\sum_j \mu_j)(1-\dot{s}_1)\sum_k \mu_k (1-\dot{s}_k) c_{1k} + \sum_{j,k} \mu_j \mu_k (1-\dot{s}_j)(1-\dot{s}_k) c_{jk}$

where: $c_{ij} = \boldsymbol{b}_i'(s_i - t) \cdot \boldsymbol{b}_j'(s_j - t)$

As a check, summing 3 eqs $\Rightarrow \mu_1 \dot{s}_1 + \mu_2 \dot{s}_2 + \mu_3 \dot{s}_3 = 0$ (gives energy conservation.)

Hence eliminate $\dot{s}_3$ and solve for $\dot{s}_1$, $\dot{s}_2$

Final solution

$$\frac{\mu_1(1-\dot{s}_1)}{\mu_1+\mu_2+\mu_3} = \frac{M_1(1-c_{23})}{M_1(1-c_{23})+M_2(1-c_{31})+M_3(1-c_{12})}$$

$M_1 = \mu_1^2 - (\mu_2 - \mu_3)^2$ etc.

Note: because $c_{ij} = \mathbf{b}_i'(s_i - t)\cdot \mathbf{b}_j'(s_j - t)$

these are **differential equations** for $s_j(t)$

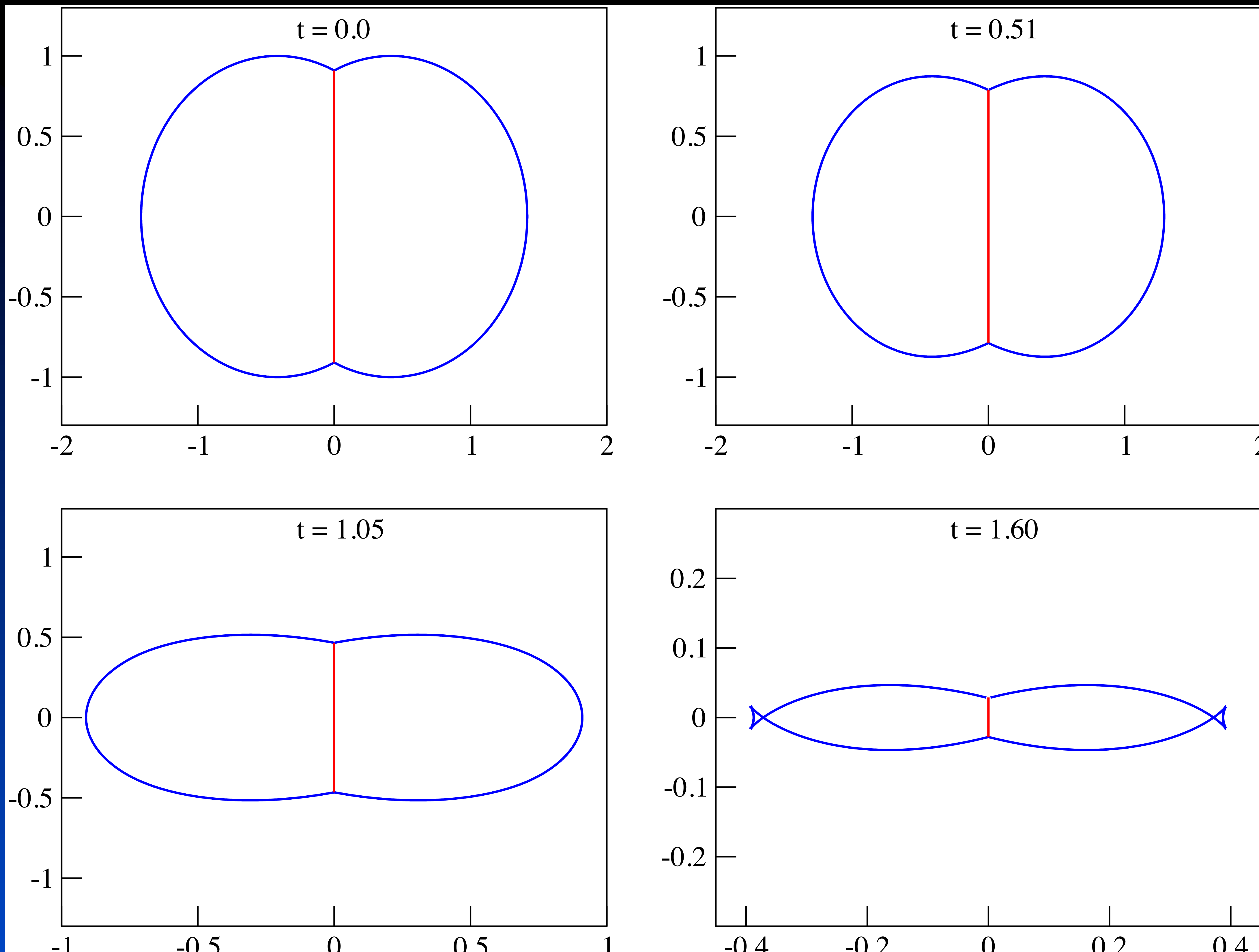
Also since $\dot{s}_j < 1$ and $c_{ij} < 1$ all $M_j > 0$

i.e. μ_j satisfy **triangle inequalities** (obvious if $\dot{\mathbf{X}} = \mathbf{0}$)

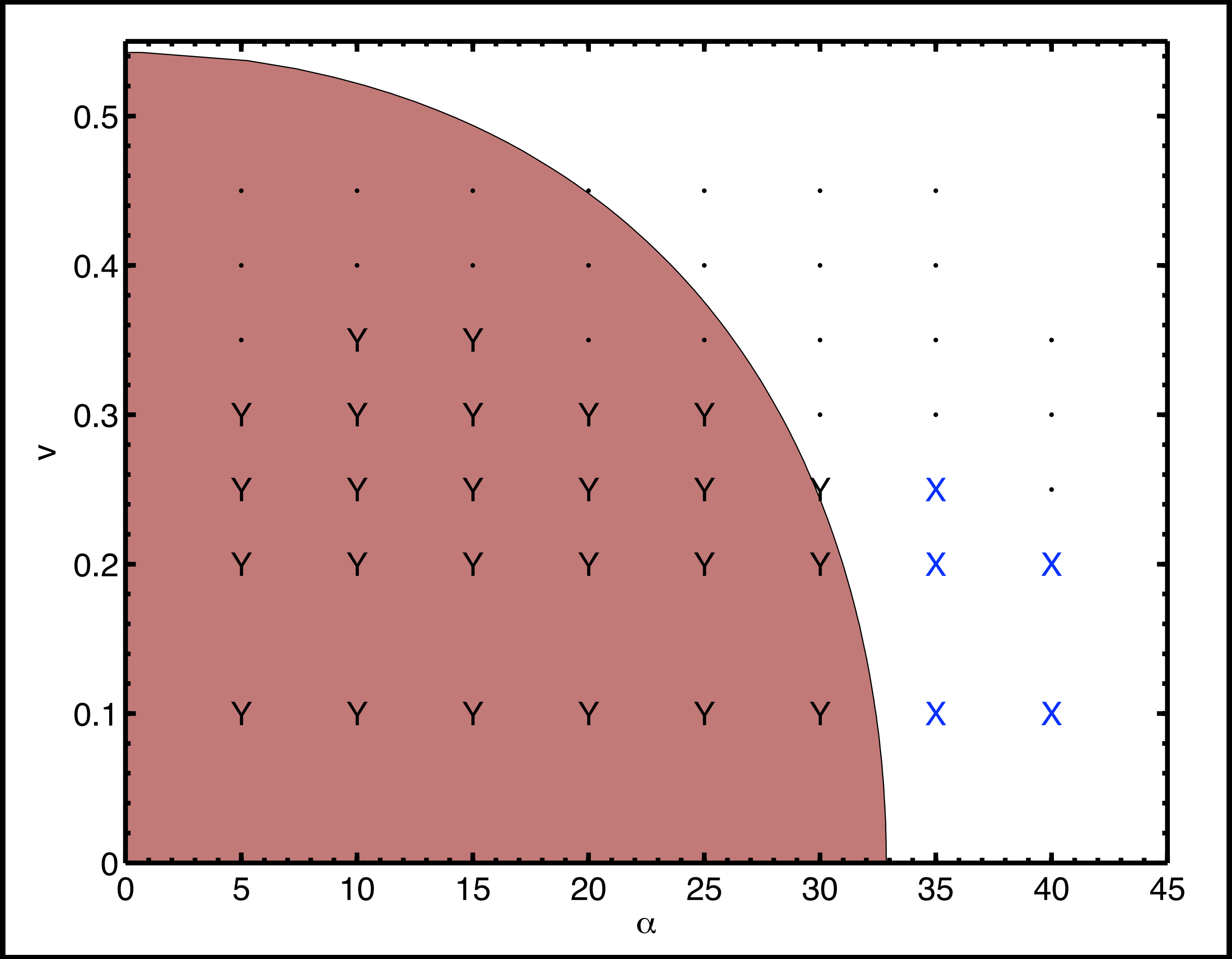
— e.g. if $\mu_3 > \mu_1 + \mu_2$ string 3 is unstable

Butterfly configuration -- in the plane

Bevis et al 09



Typically, how many kinks, cusps on loops made of junctions ?



Two different Type I Abelian strings which combine to form stable (p,q) type strings. Again show similar features, but there is a difference from Nambu-Goto prediction (solid line). Bevis and Saffin 08

Condition $\dot{s}_3 > 0$

provides kinematical constraints

Abelian strings, in white or *z* region, must pass through one another.

Non-abelian-strings, in *z* region, may be linked along the *z* axis; in white region, they will be locked.

06/23/2008

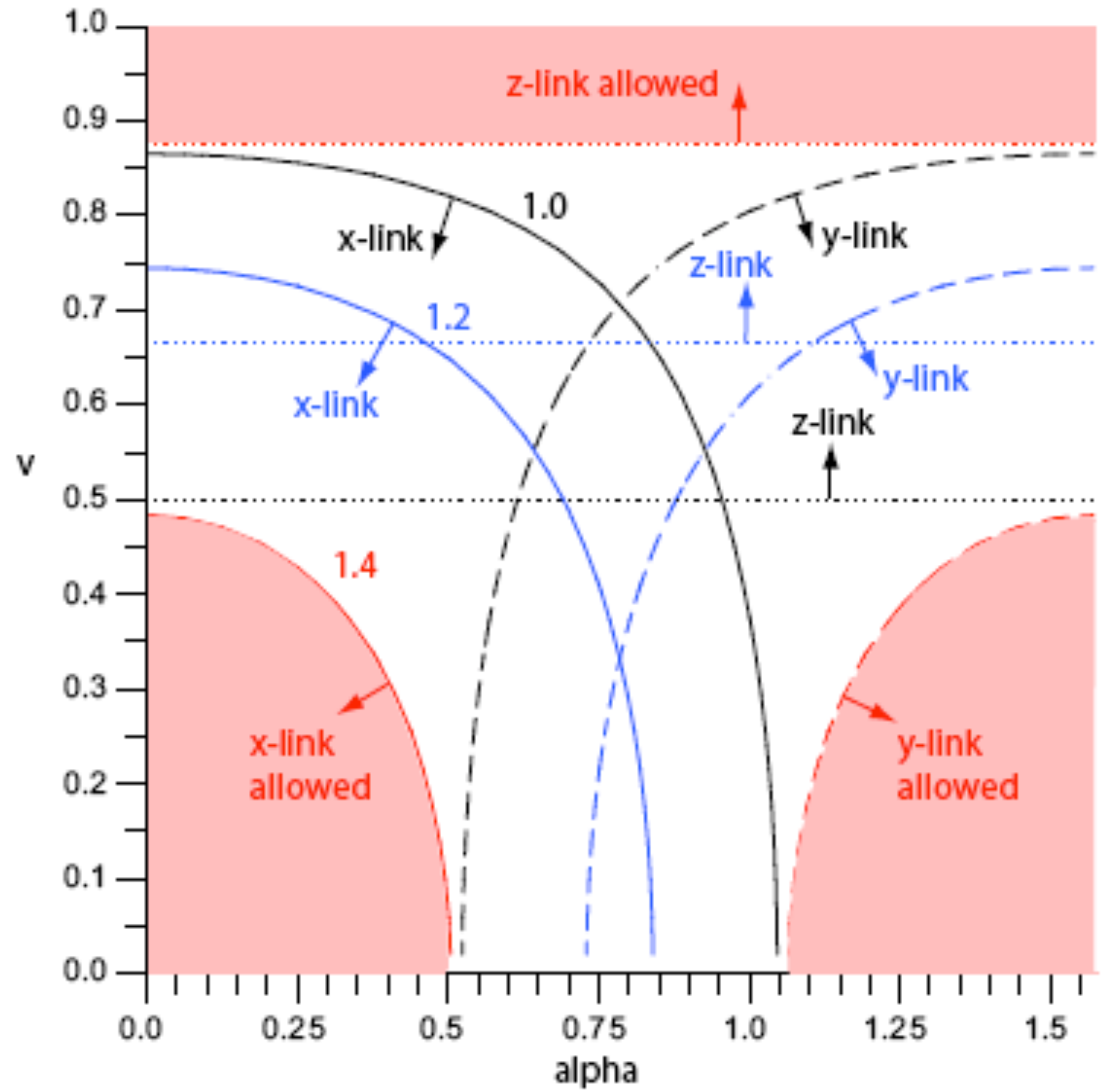


FIG. 2: Kinematic constraints for $\mu_- = 0$. Allowed regions for x-links are to the left of the full curves; for y-links to the right of the dashed curves; and, for z-links in the non-abelian case, above the horizontal dotted lines. The values of μ_3 are 1.4 (red), 1.2 (blue), 1.0 (black). Allowed regions are shaded for the $\mu_3 = 1.4$ case.

Problem because now lack symmetry. If 1,2 exchange partners, and are joined by 3, all we know is it must be parallel to xy-plane. Consider x-link:

New string at angle θ to x-axis and moving in z-direction with velocity u

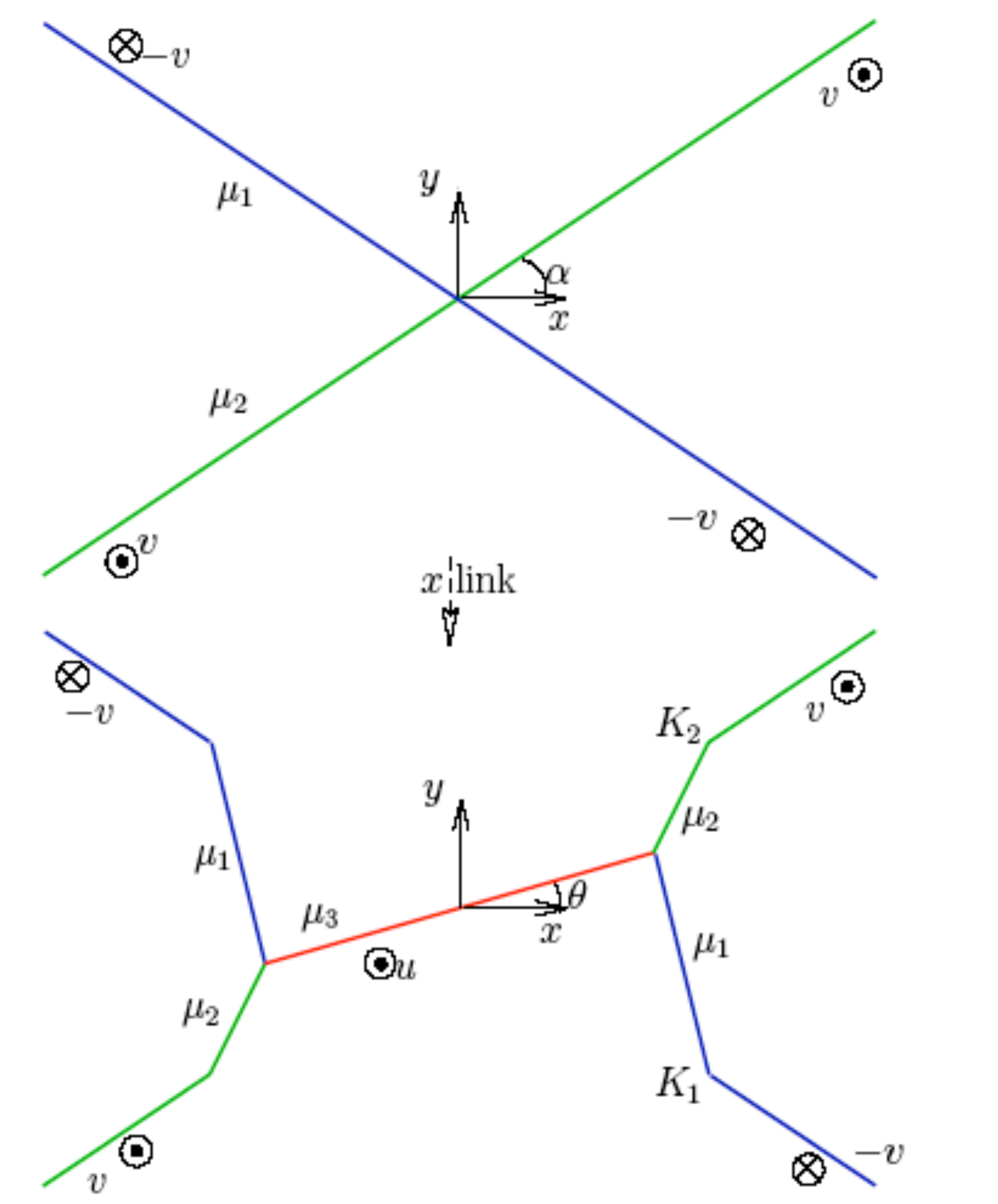


FIG. 1: Two colliding strings, of unequal tension, joined by a third string (an x-link).

Rate of change of string lengths

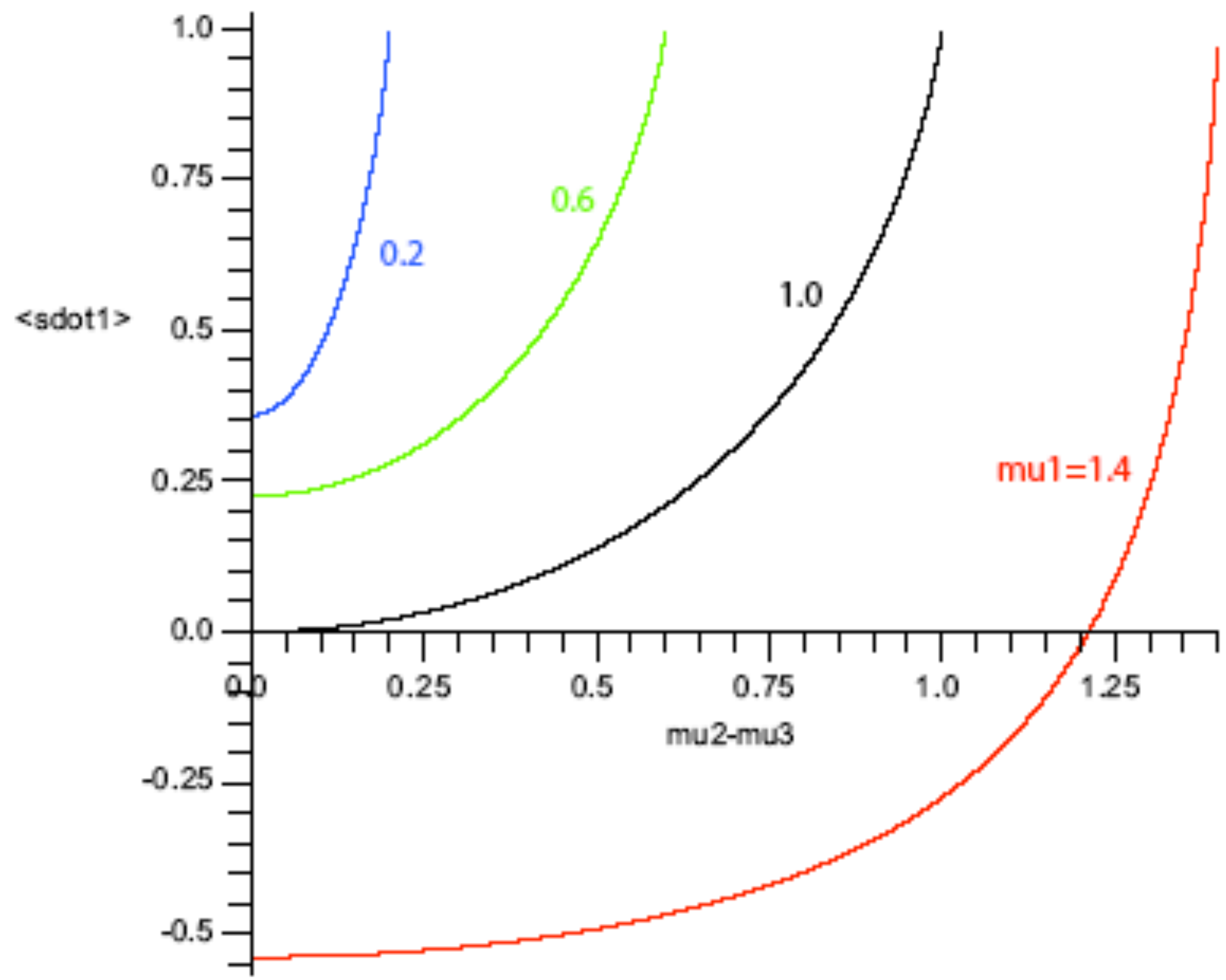
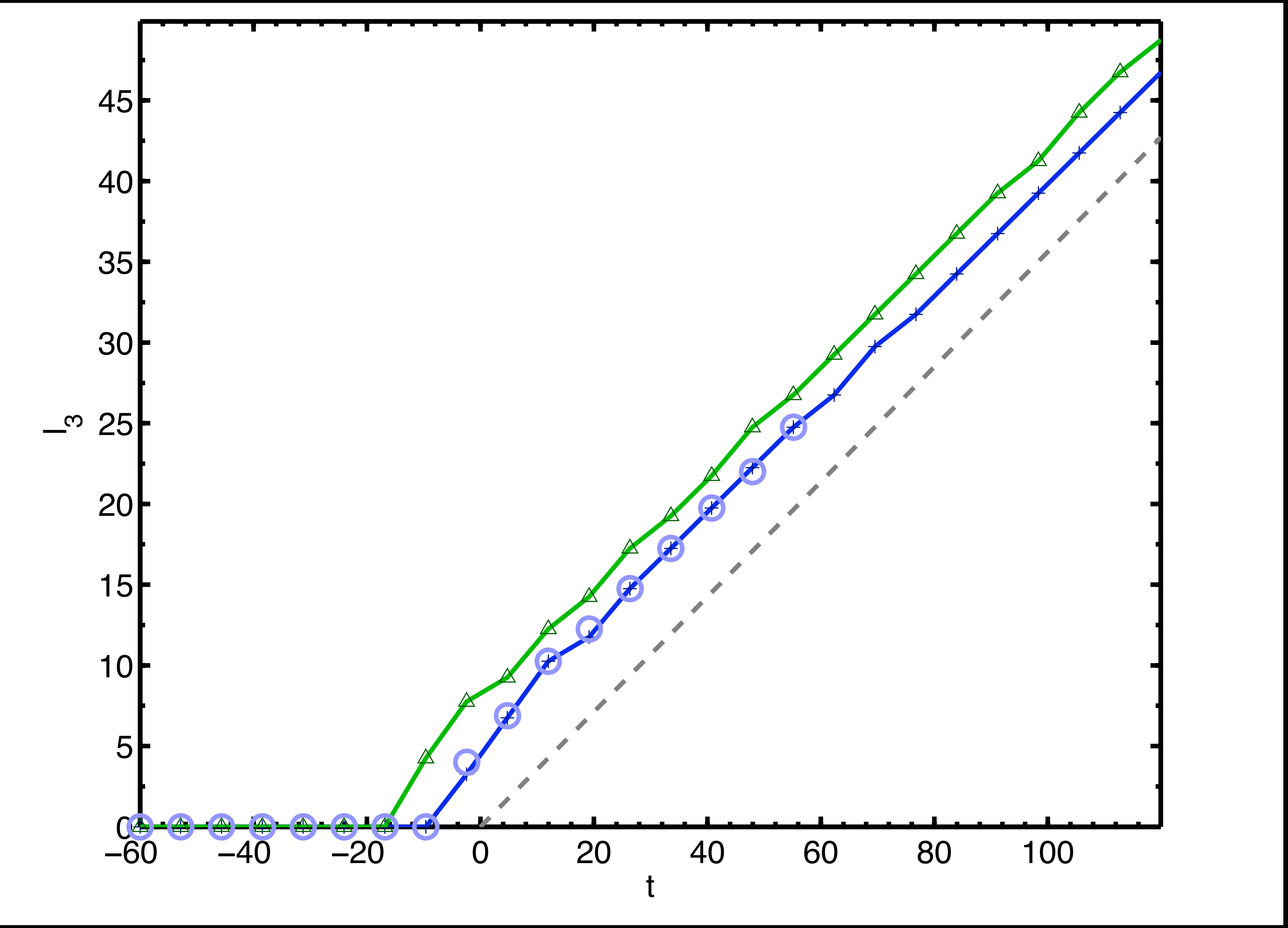


FIG. 5: Average value of $\dot{s}_1$ plotted against $|\mu_2 - \mu_3|$ for $\mu_1 = 0.2$ (blue), 0.6 (green), 1.0 (black) and 1.4 (red).

Note light strings have positive velocity and so seem to grow at the expense of heavy strings

Physical bridge length



Field theory simulation matches Nambu-Goto prediction (dashed line) very well. Bevis and Saffin 08

Average string velocity

$$
\begin{aligned}
\langle \dot{x}_1^2 \rangle &= \langle (1 - x_1'^2) \rangle \\
&= \frac{\mu_1^2 - 13\bar{\mu}^2}{15(\mu_1^2 - \bar{\mu}^2)} + \frac{(4\mu_1 - \bar{\mu})(\mu_1 + \bar{\mu})^2}{15\mu_1(\mu_1 - \bar{\mu})^2} \ln \frac{2\mu_1}{\mu_1 + \bar{\mu}} \\
&+ \frac{(4\mu_1 + \bar{\mu})(\mu_1 - \bar{\mu})^2}{15\mu_1(\mu_1 + \bar{\mu})^2} \ln \frac{2\mu_1}{\mu_1 - \bar{\mu}},
\end{aligned}
\qquad (64)
$$

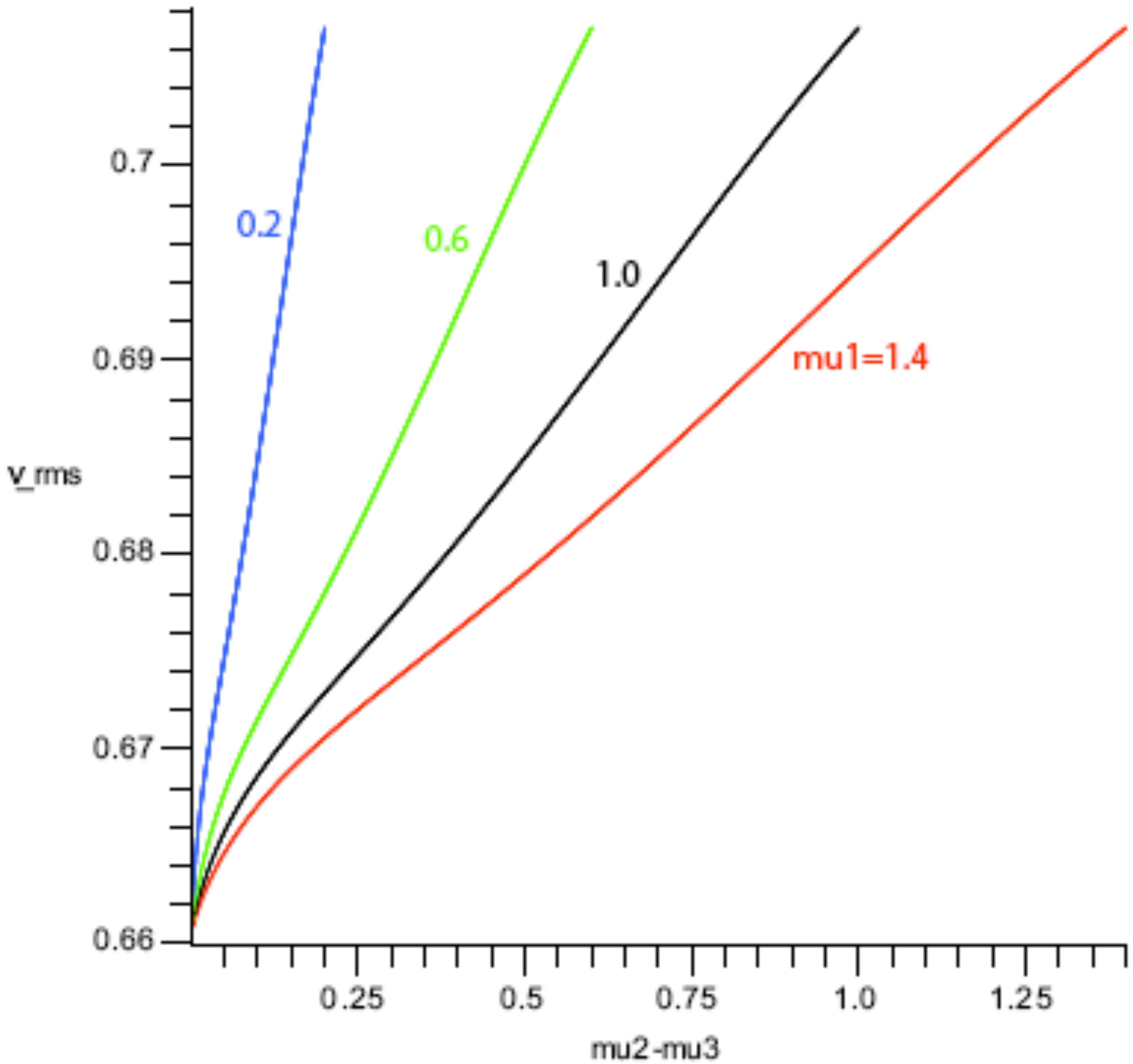


FIG. 6: R.m.s. value of the string velocity v plotted against $|\mu_2 - \mu_3|$ for several values of μ_1.

Note even for case of equal tensions, around the junctions

$$v_{\rm rms} < \frac{1}{\sqrt{2}}$$

the result for Nambu-Goto strings

Collision of Cosmic Superstrings

[EJC, Firouzjahi,Kibble,Steer --arXiv [0712.0808] hep-th

Consider forming junctions between (p,q) strings. Presence of fluxes implies need to generalise DBI action:

$$S = -\sum_i \mu_i \int d\tau d\sigma \sqrt{-X_i'^2 \dot{X}_i^2 - \lambda^2 {F^i_{\tau\sigma}}^2}\,\theta\left(s_i(\tau) - \sigma\right)$$
$$+\sum_i \int d\tau \left(f_i \cdot \left[X_i(s_i(\tau),\tau) - \bar{X}(\tau)\right] + g_i \left[A^i_\tau(s_i(\tau),\tau) + \dot{s}_i A^i_\sigma(s_i(\tau),\tau) - \bar{A}(\tau)\right]\right)$$

where $\mu_i = \frac{|q_i|}{g_s \lambda} \quad \lambda = 2\pi\alpha'$

Constraints as before with original tension replaced by

$$\bar{\mu}_i = \sqrt{p_i^2 + \frac{q_i^2}{g_s^2}}$$

plus two new constraints:

$$\sum_i p_i = 0 \qquad \sum_i q_i = 0$$

Conservation of electric flux and charge at the junctions

Constraints on (p,q) string junction formation

Strings with charges (p_1,q_1) and (p_2,q_2) collide and become linked by a string with charges $(p_3,q_3) = -(p_1 + p_2,\ q_1 + q_2)$

t<0: $\mathbf{x_{1,2}}(\sigma, \mathbf{t}) = (-\gamma^{-1}\sigma\cos\alpha, \mp\gamma^{-1}\sigma\sin\alpha, \pm\mathbf{vt})$

t>0: $\mathbf{x_3}(\sigma, \mathbf{t}) = (\gamma_{\mathbf{u}}^{-1}\sigma\cos\theta, \gamma_{\mathbf{u}}^{-1}\sigma\sin\theta, \mathbf{ut})$

Ex: Collision of F-string (1,0) with a D-string (0,1). The basic building blocks for (p,q) strings.Third string is (1,1) string and forms for $0<v<v_c$.

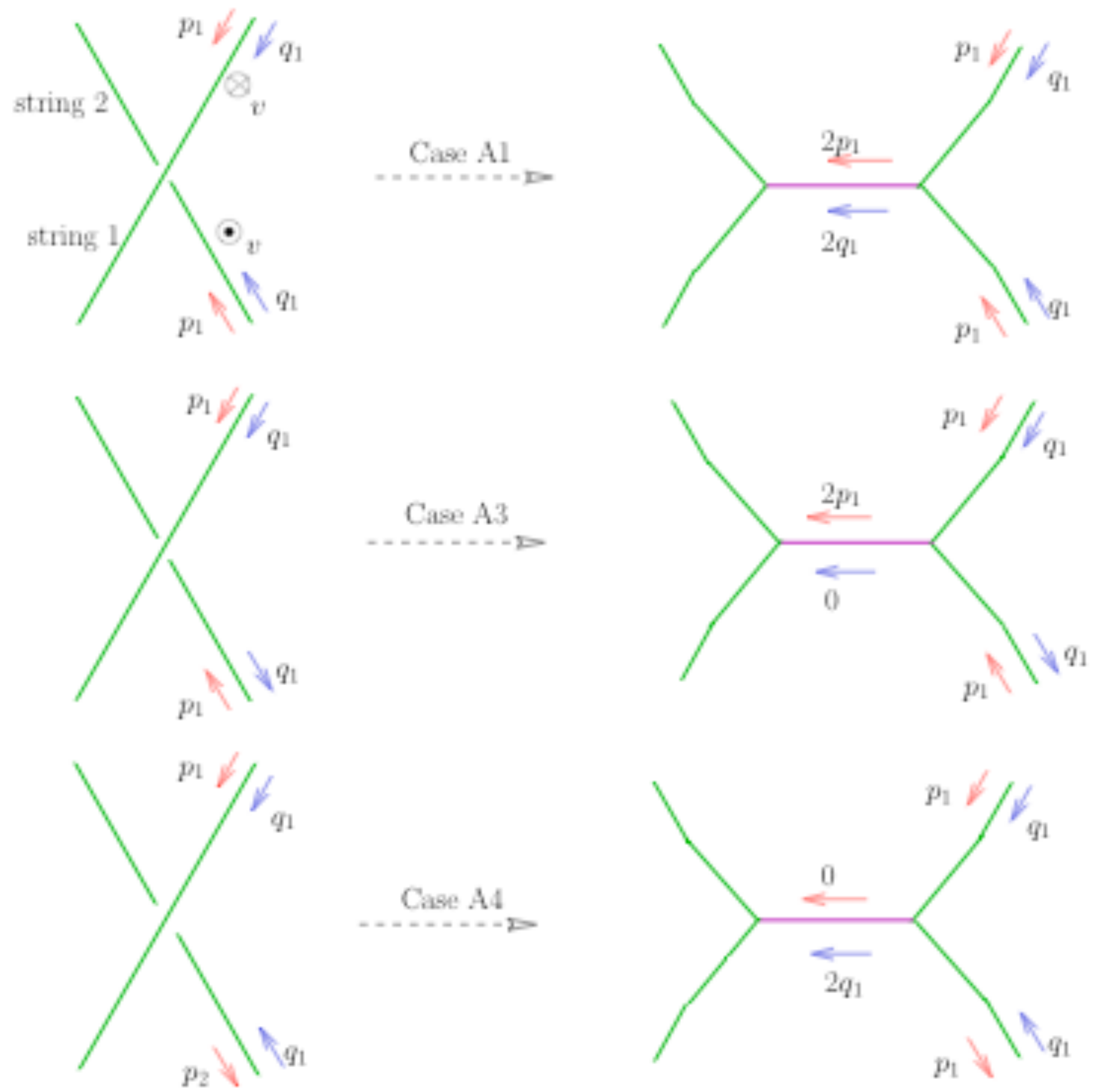
$$v_c^2 = \frac{(1+g_s^2) - 4\cos^2\alpha\sin^2\alpha(1+g_s)^2 + \sqrt{(1+g_s^2)^2 - 4\cos^2\alpha\sin^2\alpha(1-g_s^2)^2}}{2\cos^2\alpha(1+g_s)^2(2\cos^2\alpha - 1)}$$

$v_c=0$ indep of g_s for $\alpha=\pi/4$. For $\alpha>\pi/4$ no x-link forms. For $g_s \to 0$, $v_c=1$, so half the (α,v) plane allowed. Implies very heavy D-string can always exchange partners with light F-string

Equal tension:

$$\bar{\mu}_1 = \bar{\mu}_2$$

x-link formation:



(p_2, q_2)	(p_3, q_3)	$\bar{\mu}_3$	**Kinematic constraint**	**Case**
$(p_2, q_2) = (+p_1, +q_1)$	$(-2p_1, -2q_2)$	$\bar{\mu}_3 = 2\sqrt{p_1^2 + \left(\frac{q_1}{g_s}\right)^2}$	intercommutation allowed with no link	A1
$(p_2, q_2) = (-p_1, -q_1)$	$(0, 0)$	$\bar{\mu}_3 = 0$	intercommutation allowed with no link	A2
$(p_2, q_2) = (+p_1, -q_1)$	$(-2p_1, 0)$	$\bar{\mu}_3 = 2p_1$	$\gamma^{-1} \cos\alpha \geq \frac{1}{\sqrt{1+q_1^2/(g_s^2 p_1^2)}}$	A3
$(p_2, q_2) = (-p_1, +q_1)$	$(0, -2q_1)$	$\bar{\mu}_3 = 2\frac{q_1}{g_s}$	$\gamma^{-1} \cos\alpha \geq \frac{1}{\sqrt{1+(p_1^2 g_s^2)/q_1^2}}$	A4

Collisions in a warped background

Can extend analysis beyond collision in flat space-time to collisions in warped throat such as Klebanov-Strassler:

$$ds^2 = h^2\eta_{\mu\nu}dx^\mu dx^\nu + g_s M\alpha'(d\psi^2 + \sin^2\psi\, d\Omega_2^2)$$

h- warp factor, M- number of RR $F_{(3)}$ fluxes turned on inside S^3 where internal geometry ends. Find:

$$\bar{\mu} = h^2\sqrt{\frac{q^2}{g_s^2} + \frac{M^2}{\pi^2}\sin^2\left(\frac{\pi p}{M}\right)}.$$

Can reanalyse and understand collision of an F-string and D-string in the throat.

Same basic properties as before but in terms of redefined parameters. Very useful when considering more realistic scenarios.