Primordial Nongaussianity
and
Large-scale Structure

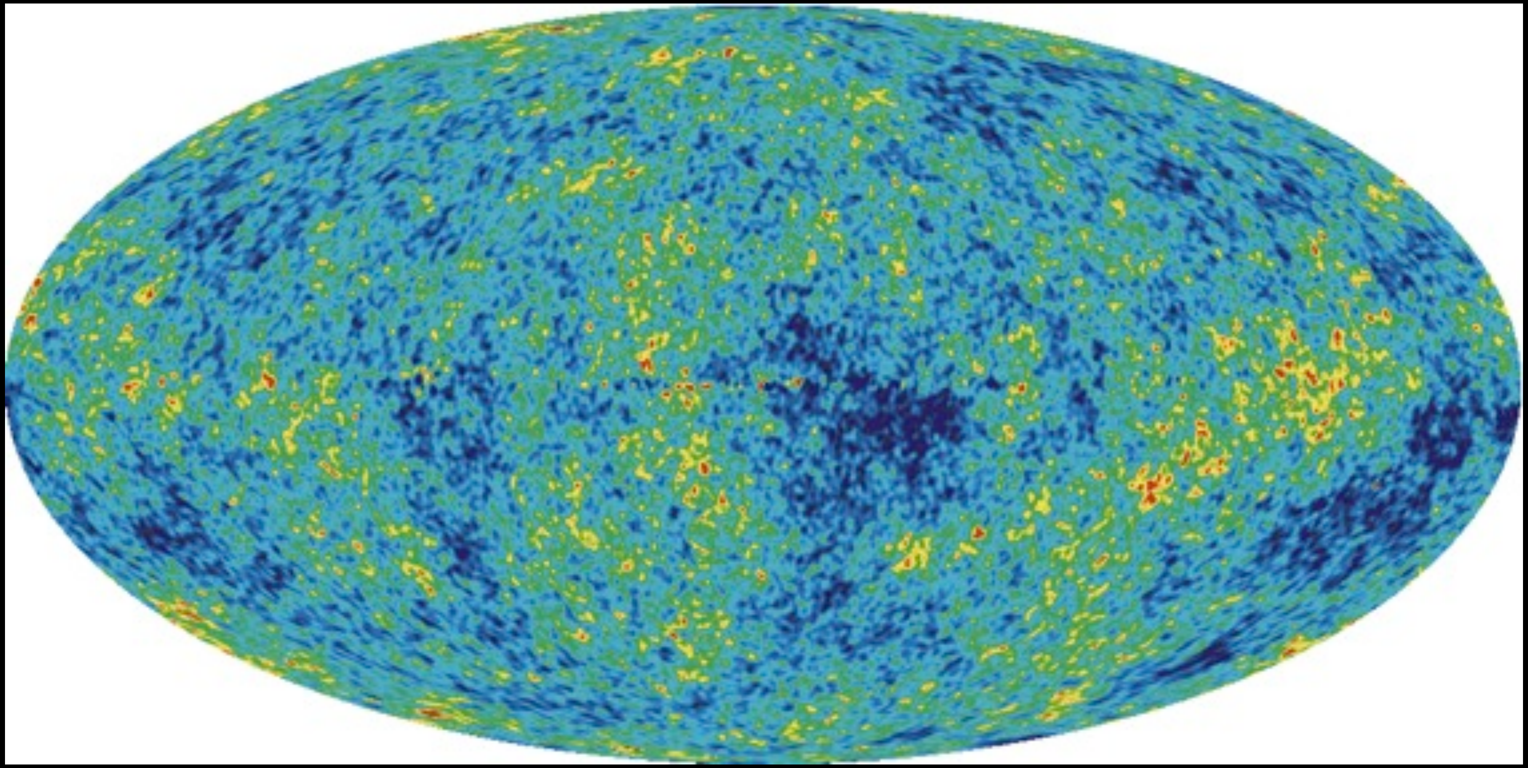
This talk: overview of field, and some ongoing work

Dragan Huterer
(University of Michigan)

Some slides courtesy of O. Doré

Initial conditions in our universe

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$

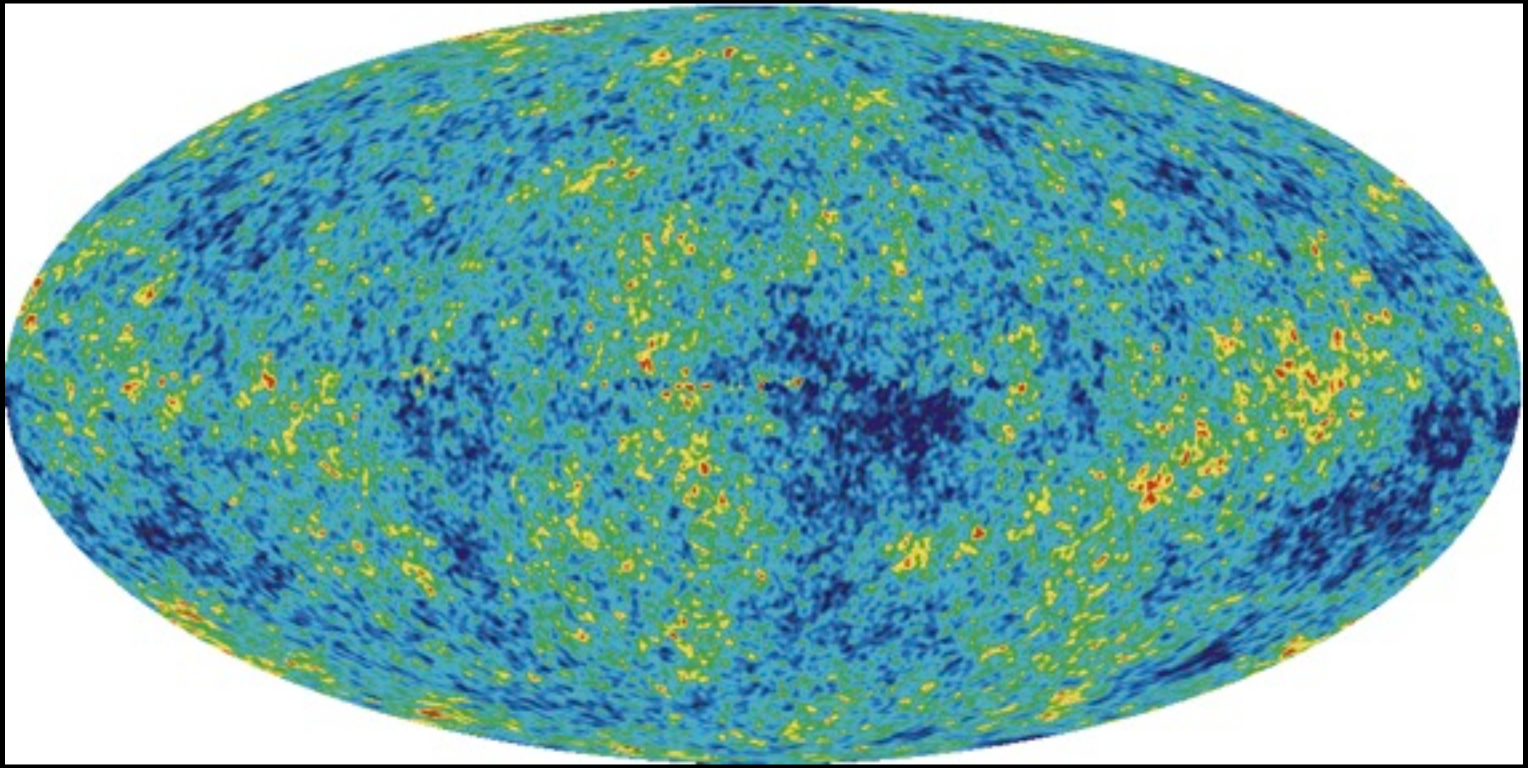


Generic inflationary predictions:

- Nearly scale-invariant spectrum of density perturbations
- Background of gravity waves
- (Very nearly) gaussian initial conditions:

Initial conditions in our universe

$$\frac{\delta T}{T}(\theta,\phi)=\sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$



Isotropy:

$$\langle a_{\ell m}\, a_{\ell' m'}\rangle \equiv C_{\ell\ell' mm'} = C_\ell \delta_{\ell\ell'}\delta_{mm'}$$

Gaussianity:

$$\langle a_{\ell m}\, a_{\ell' m'}\, a_{\ell'' m''}\rangle = 0 \quad \text{etc.}$$

Inflation generically predicts (very nearly) gaussian random fluctuations

- Nongaussianity is proportional to slow-roll parameters, V′/V and V″/V

- Reasonable and commonly used approximation: the "local" model of primordial nongaussianity

$$\Phi = \Phi_{\rm G} + f_{\rm NL}\left(\Phi_{\rm G}^2 - \langle\Phi_{\rm G}^2\rangle\right)$$

- Inflation predicts f_{NL}~O(0.1), which is basically extremely small

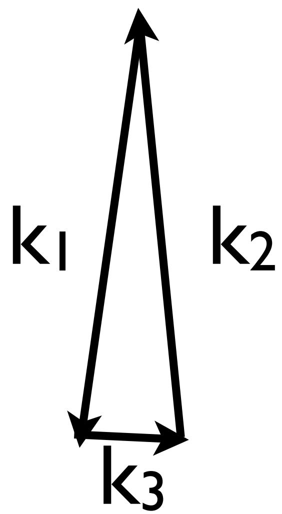
- More exotic inflationary models can produce observable NG, however

Salopek & Bond 1990; Verde et al 2000; Komatsu & Spergel 2001; Maldacena 2003

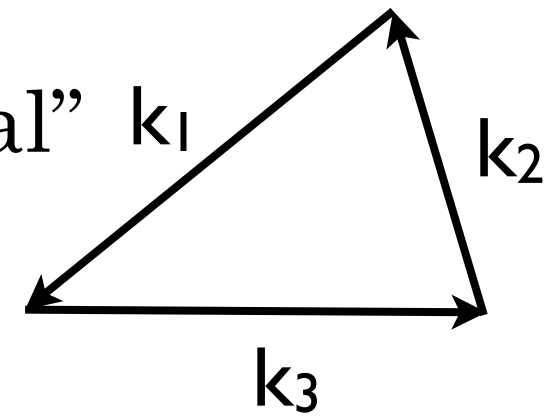
3-pt function as a measure of cosmological NonGaussianity (NG)

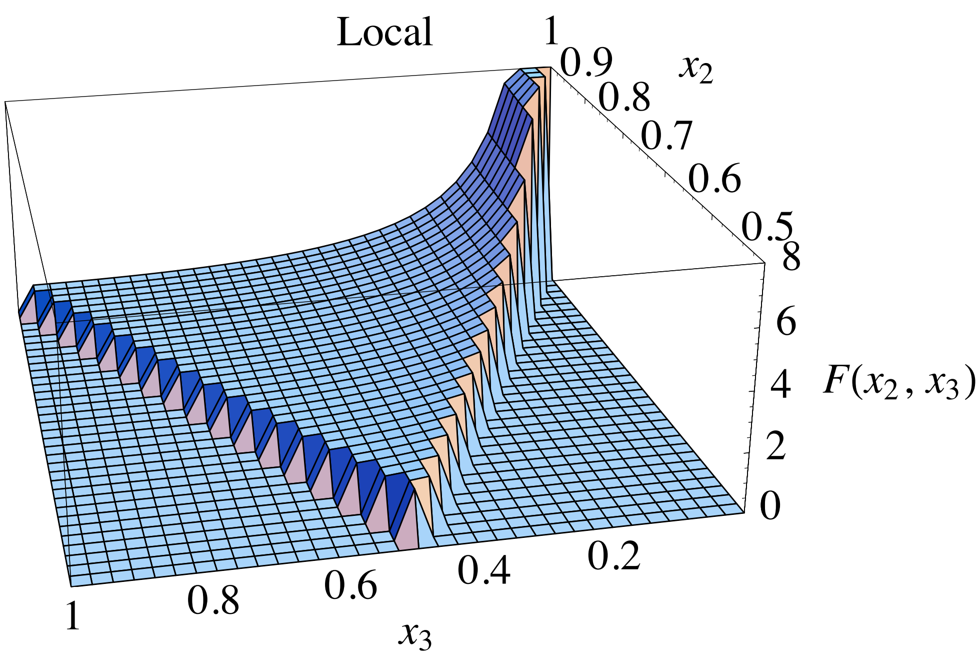
- Principal measure of NG: three-pt correlation function (e.g. Luo & Schramm 1993)

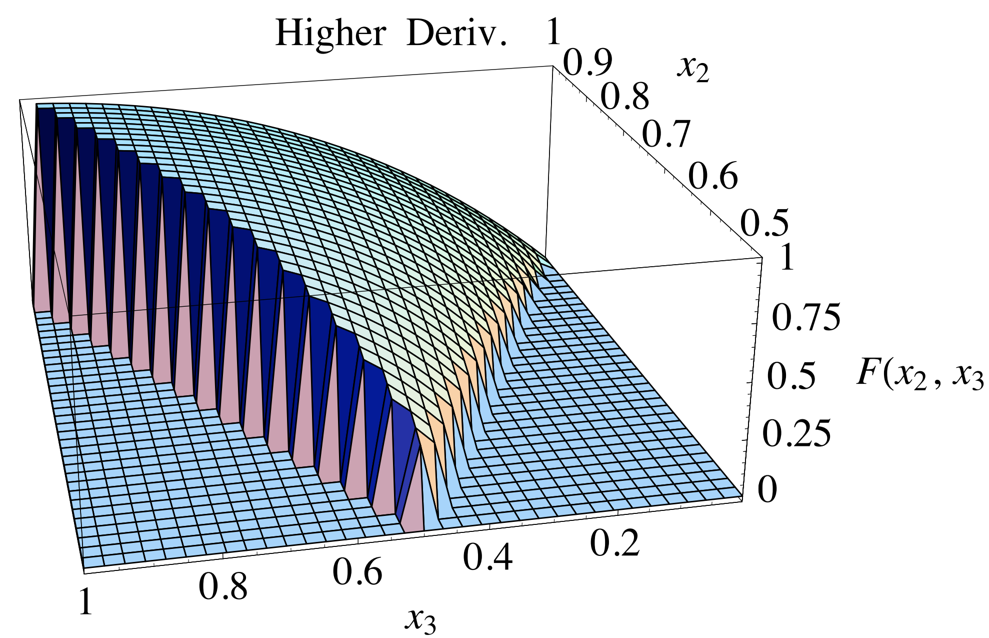
"local"



"equilateral"







Babich, Creminelli & Zaldarriaga 2004

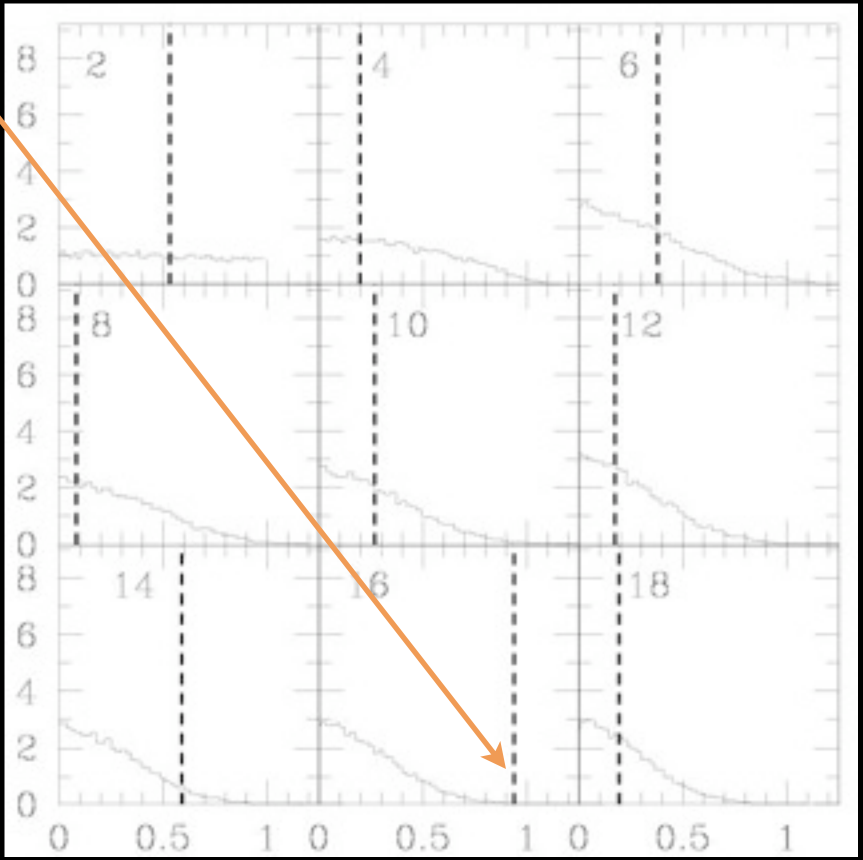
Brief history of NG measurements: 1990's

Early 1990s; COBE: Gaussian CMB sky (Kogut et al 1996)

1998; COBE: claim of NG at l=16 equilateral bispectrum
(Ferreira, Magueijo & Gorski 1998)

but explained by a known systematic effect!
(Banday, Zaroubi & Gorski 1999)

(and anyway isn't unexpected given all
bispectrum configurations you can measure;
Komatsu 2002)

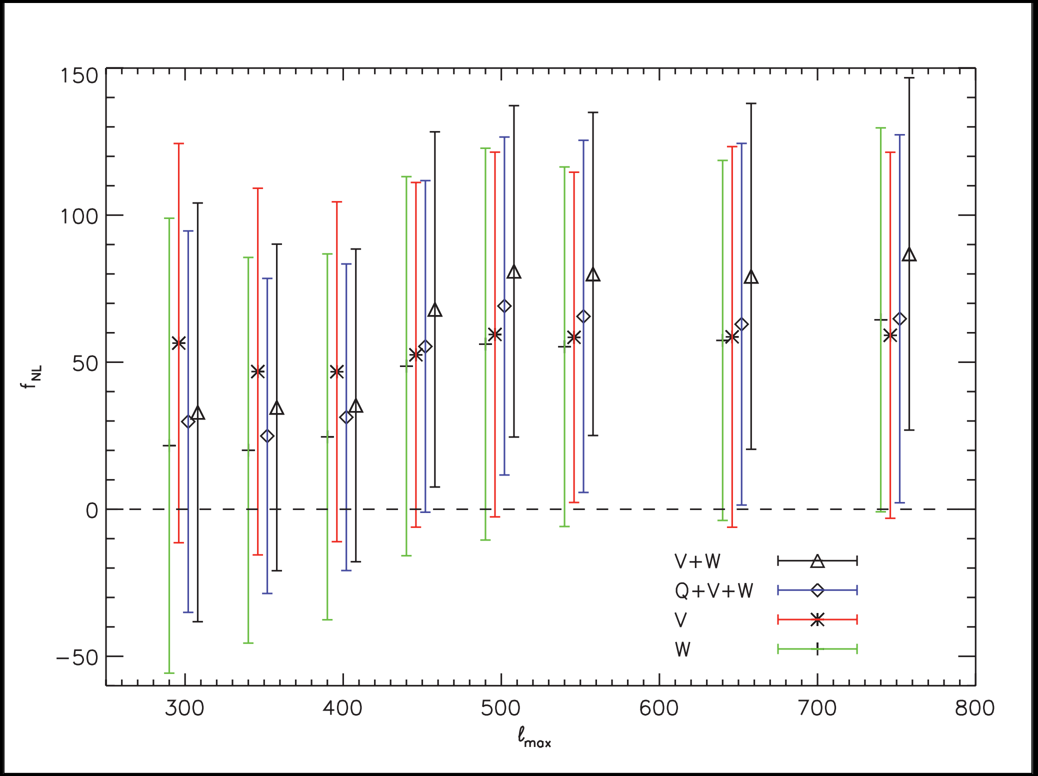


Brief history of NG measurements: 2000's

Pre-WMAP CMB: all is gaussian (e.g. MAXIMA; Wu et al 2001)

WMAP pre-2008: all is gaussian
(Komatsu et al. 2003; Creminelli, Senatore, Zaldarriaga & Tegmark 2007)

$-36 < f_{NL} < 100$ (95% CL)



Dec 2007, claim of NG in WMAP
(Yadav & Wandelt arXiv:0712.1148)

$27 < f_{NL} < 147$ (95% CL)

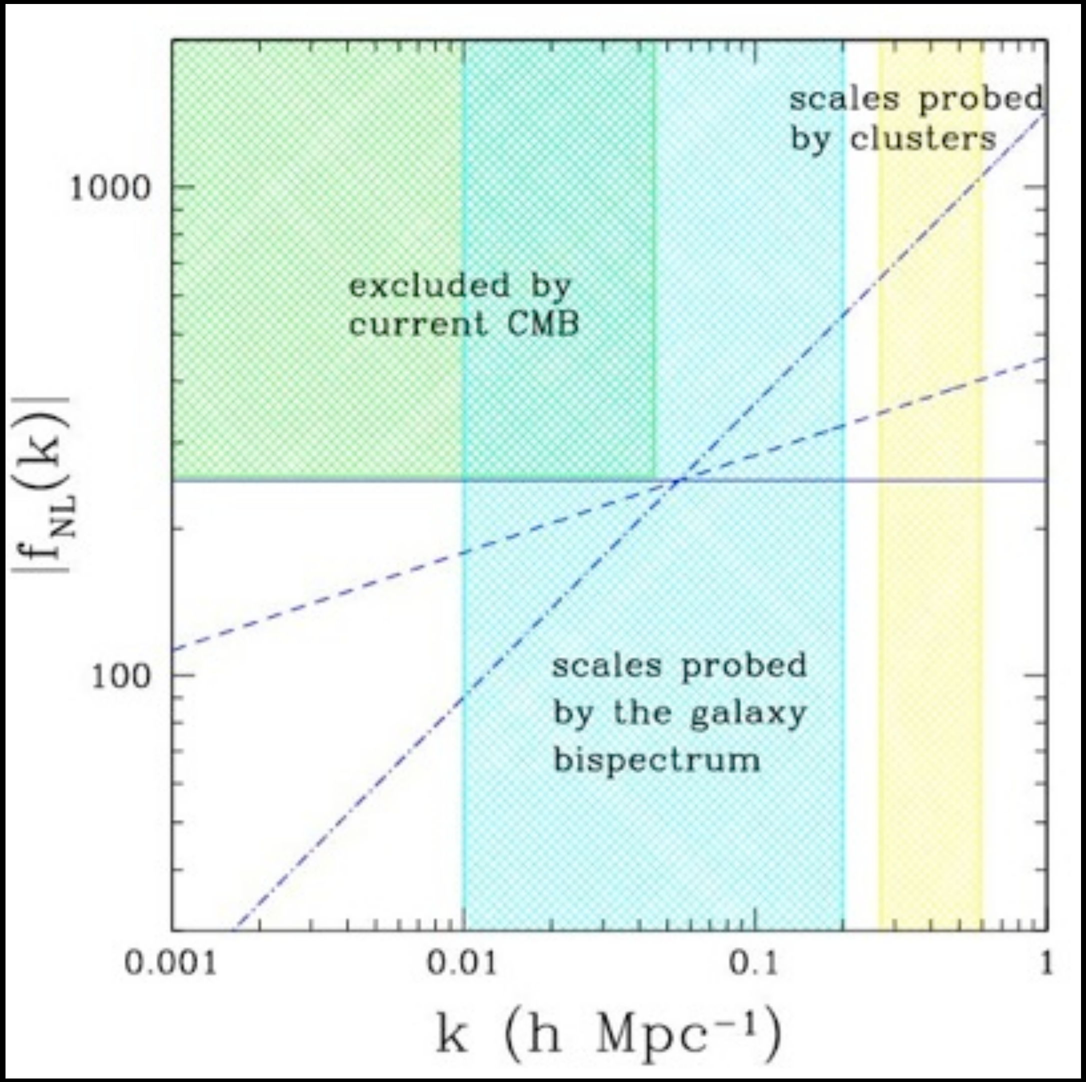
Current constraints from WMAP

Band	Foreground[b]	f_{NL}^{local}	f_{NL}^{equil}	f_{NL}^{orthog}	b_{src}
V+W	Raw	59 ± 21	33 ± 140	-199 ± 104	N/A
V+W	Clean	42 ± 21	29 ± 140	-198 ± 104	N/A
V+W	Marg.[c]	$\boxed{32 \pm 21}$	26 ± 140	-202 ± 104	-0.08 ± 0.12
V	Marg.	43 ± 24	64 ± 150	-98 ± 115	0.32 ± 0.23
W	Marg.	39 ± 24	36 ± 154	-257 ± 117	-0.13 ± 0.19

Komatsu et al. 2010

Future: much better constraints, $\sigma(f_{NL})<O(10)$ with Planck

Constraints from future LSS surveys



LoVerde, Miller, Shandera & Verde, 2008

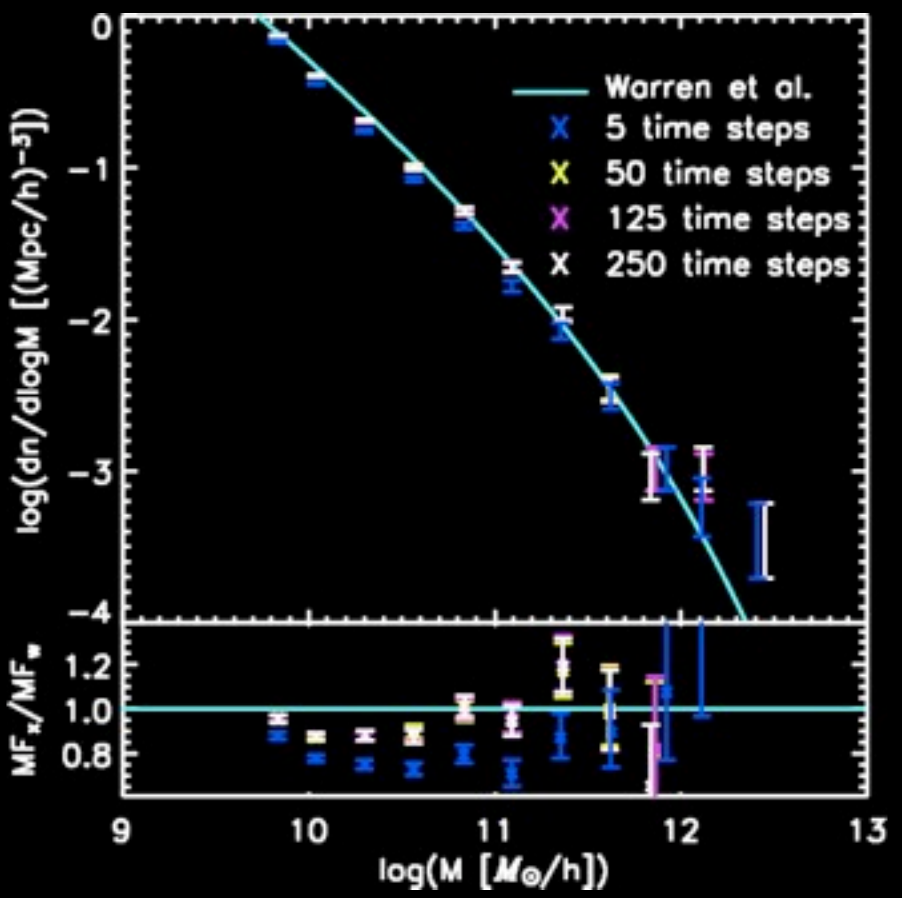
Abundance of halos: the mass function

Lots of interest in using halo counts as a cosmological probe.

- Mass function can be computed precisely (~5%) and robustly for standard cosmology (Jenkins et al. 01, Warren et al. 03)

- dN/dM appears universal — i.e. f(σ) — for standard cosmologies

$$\sigma^2(M,z) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k) W^2(k,M)\,dk$$



Mass function, usual analytic approach

Press & Schechter 1974:

$$\frac{dn}{dM}dM = \frac{\rho_M}{M}\left|\frac{dF}{dM}\right|dM \qquad F(>M) = 2\int_{\delta_c/\sigma(M)}^{\infty} P_G(\nu)d\nu$$

therefore
$$\left(\frac{dn}{d\ln M}\right)_{\mathrm{PS}} = 2\,\frac{\rho_M}{M}\frac{\delta_c}{\sigma}\left|\frac{d\ln\sigma}{d\ln M}\right| P_{\mathrm{G}}(\delta/\sigma)$$

"Extended Press-Schechter" (EPS): $P_G(\nu) \rightarrow P_{NG}(\nu)$

Matarrese, Verde & Jimenez (2000; MVJ):
follow EPS, then expand P_{NG} in terms of skewness, do the integral
(also LoVerde, Miller, Shandera & Verde 2008)

However, no convincing reason why either should work!
Need to check these formulae with simulations

Simulations with nongaussianity (f_{NL})

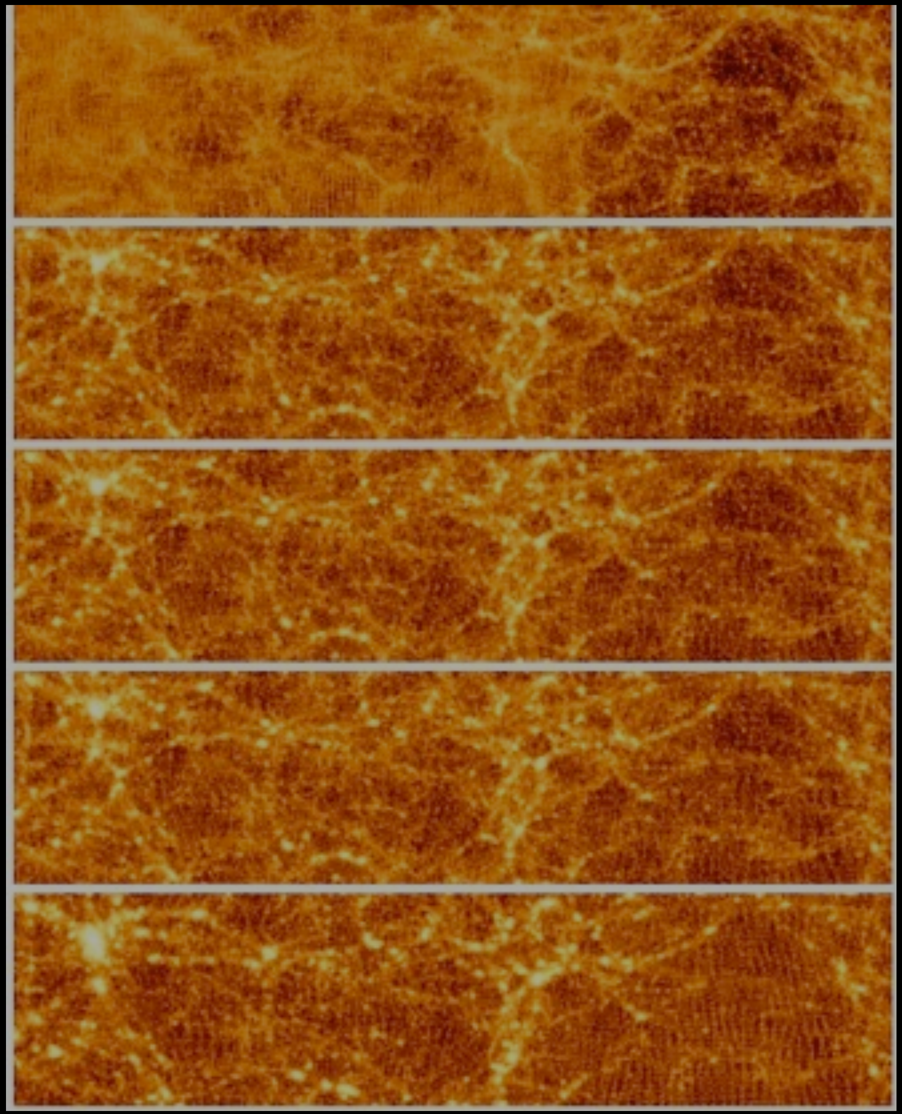
f_{NL}=-5000

f_{NL}=-500

f_{NL}=0

f_{NL}=+500

f_{NL}=+5000



80 Mpc/h

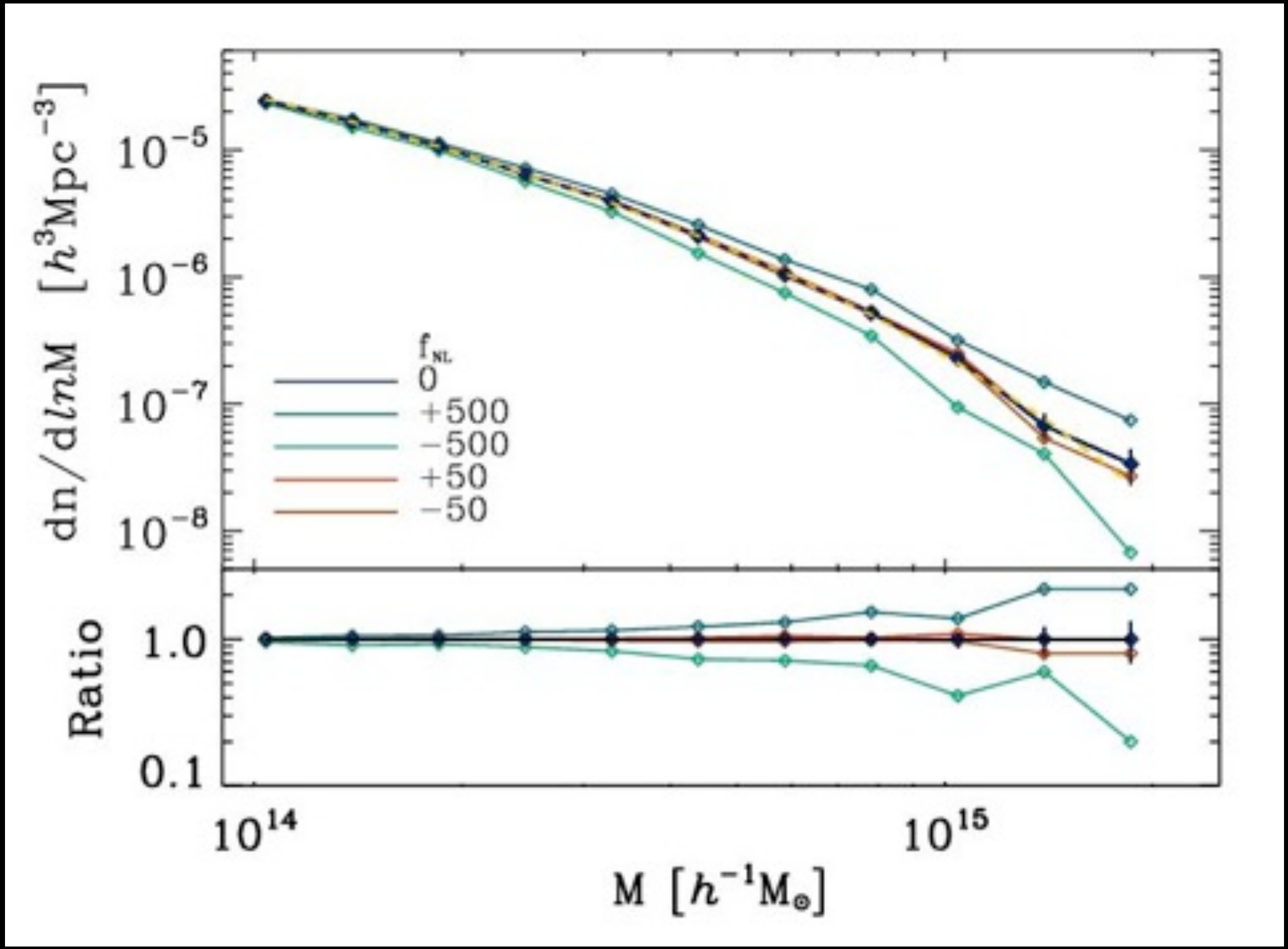
375 Mpc/h

- Under-dense region evolution decrease with f_{NL}
- Over-dense region evolution increase with f_{NL}

- Same initial conditions, different f_{NL}
- Slice through a box in a simulation $N_{part}=512^3$, L=800 Mpc/h

Dalal, Doré, Huterer & Shirokov, arXiv:0710.4560, PRD 2008

The measured halo mass function

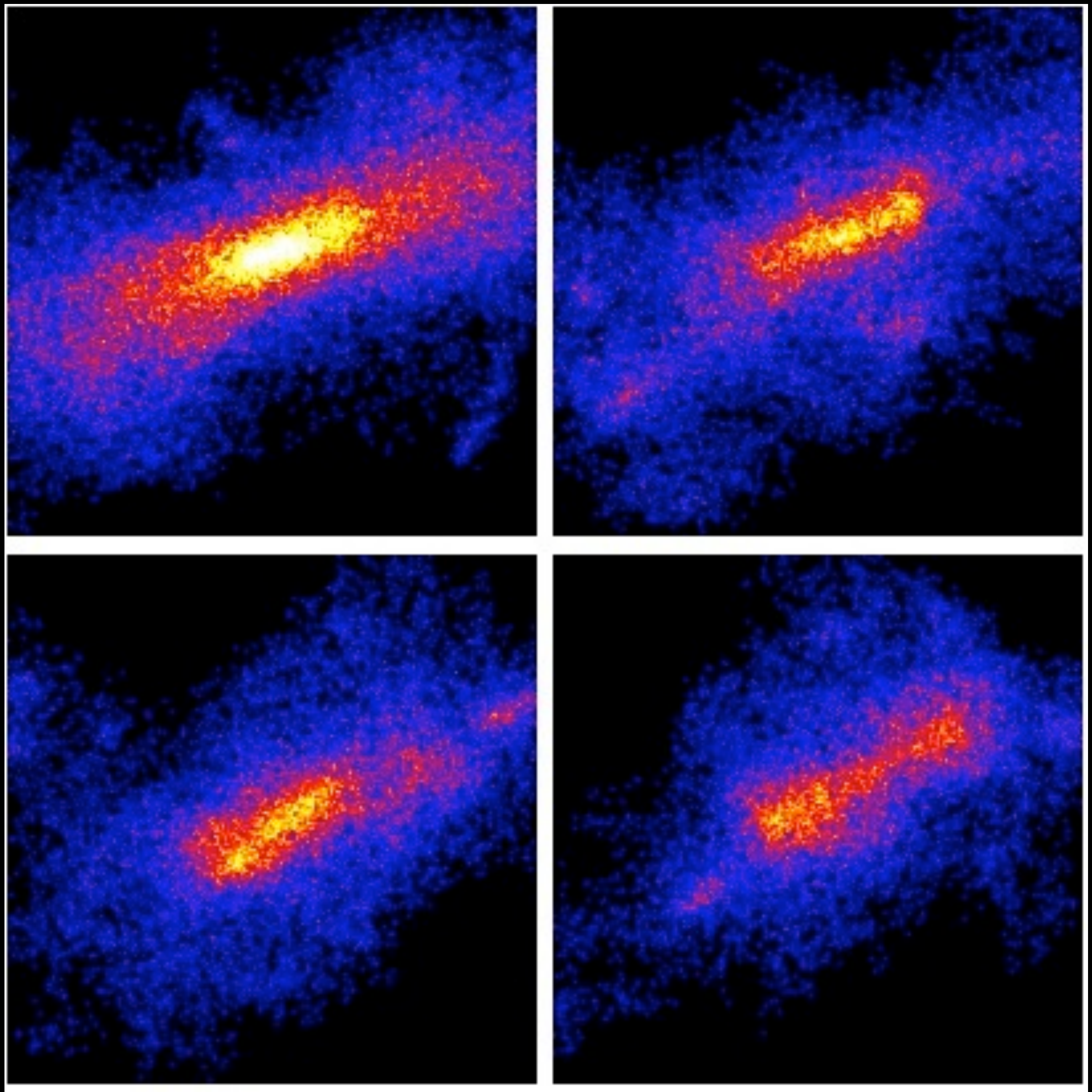


- 512^3 (1024^3) particle simulations with box size 800 (1600) Mpc/h
- Gracos code (www.gracos.com); add quadratic Phi term in real space; apply transfer function in Fourier space

Looking at one individual cluster

f_{NL}=+5000
M=1.2 10^{16} $M_\odot$

f_{NL}=+500
M=5.9 10^{15} $M_\odot$

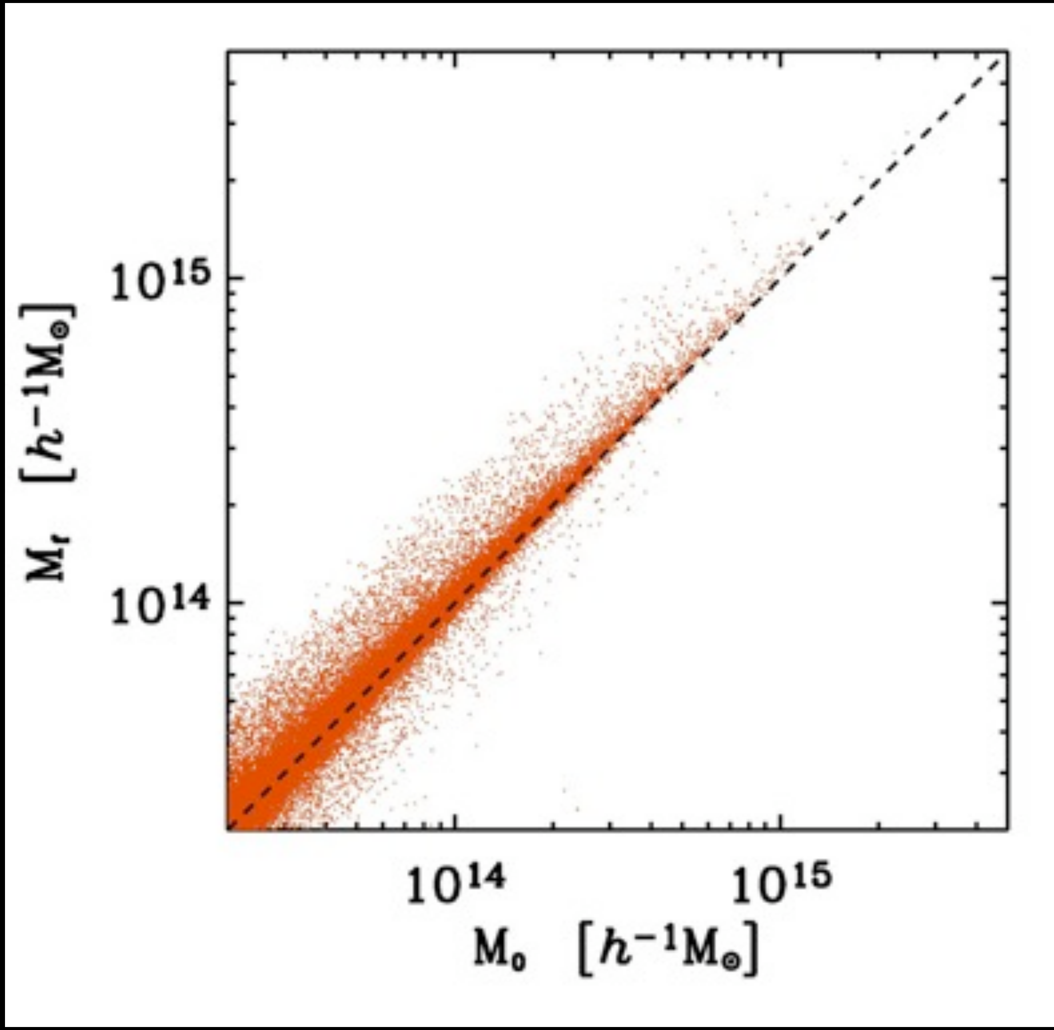


f_{NL}=0
M=5.1 10^{15} $M_\odot$

f_{NL}=-500
M=4.3 10^{15} $M_\odot$

- Most massive cluster in our simulation
- For small enough f_{NL}, same peaks arise, with different heights (implying different masses)
- Can we extend to any cluster?

Building the P($M_f|M_0$) distribution



$f_{NL} = 500$

- Idea: identify the *same* cluster for different f_{NL}, keep track how its mass changed!
- Significantly saves computational expense (relative to brute-force fitting n(M, f_{NL}))

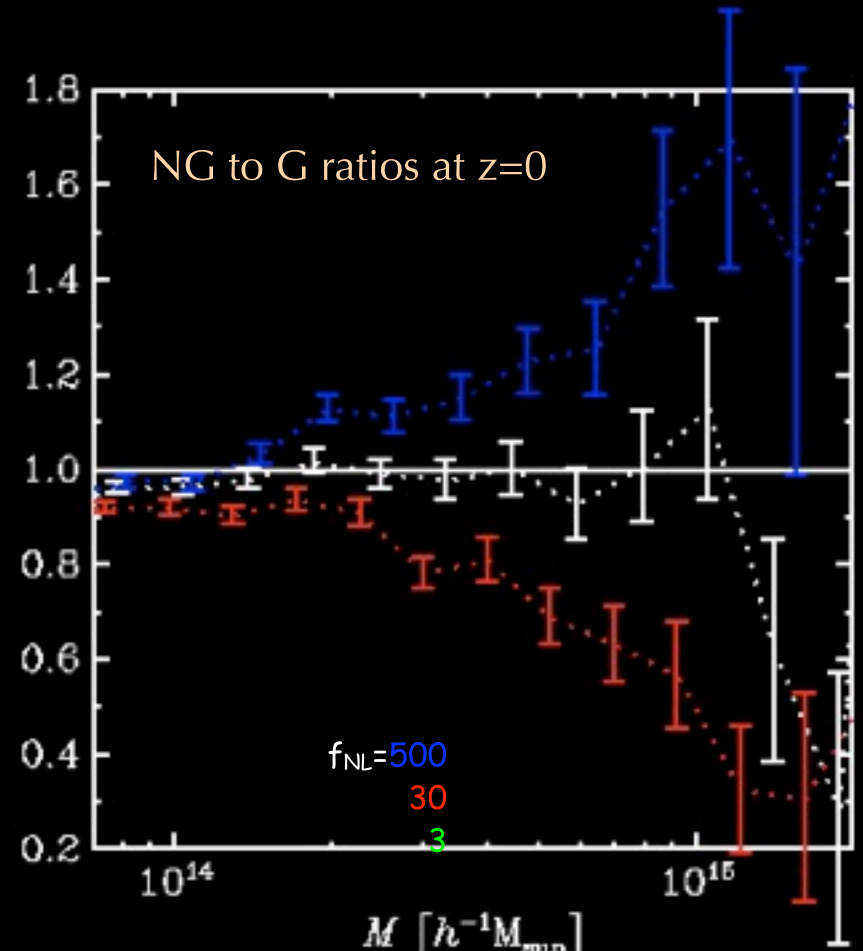
Dalal et al. NG mass function

- If the mapping $M_0 \to M_f$ is described by a PDF $dP/dM_f(M_0)$, then the non-gaussian mass function is a convolution over the (known) gaussian mass function

$$\boxed{\frac{dN}{dM} = \int \frac{dP(M_f|M_0)}{dM_f} \frac{dN}{dM_0} dM_0}$$

usual Gaussian mass function (e.g. Jenkins et al)

non-Gaussian mass function

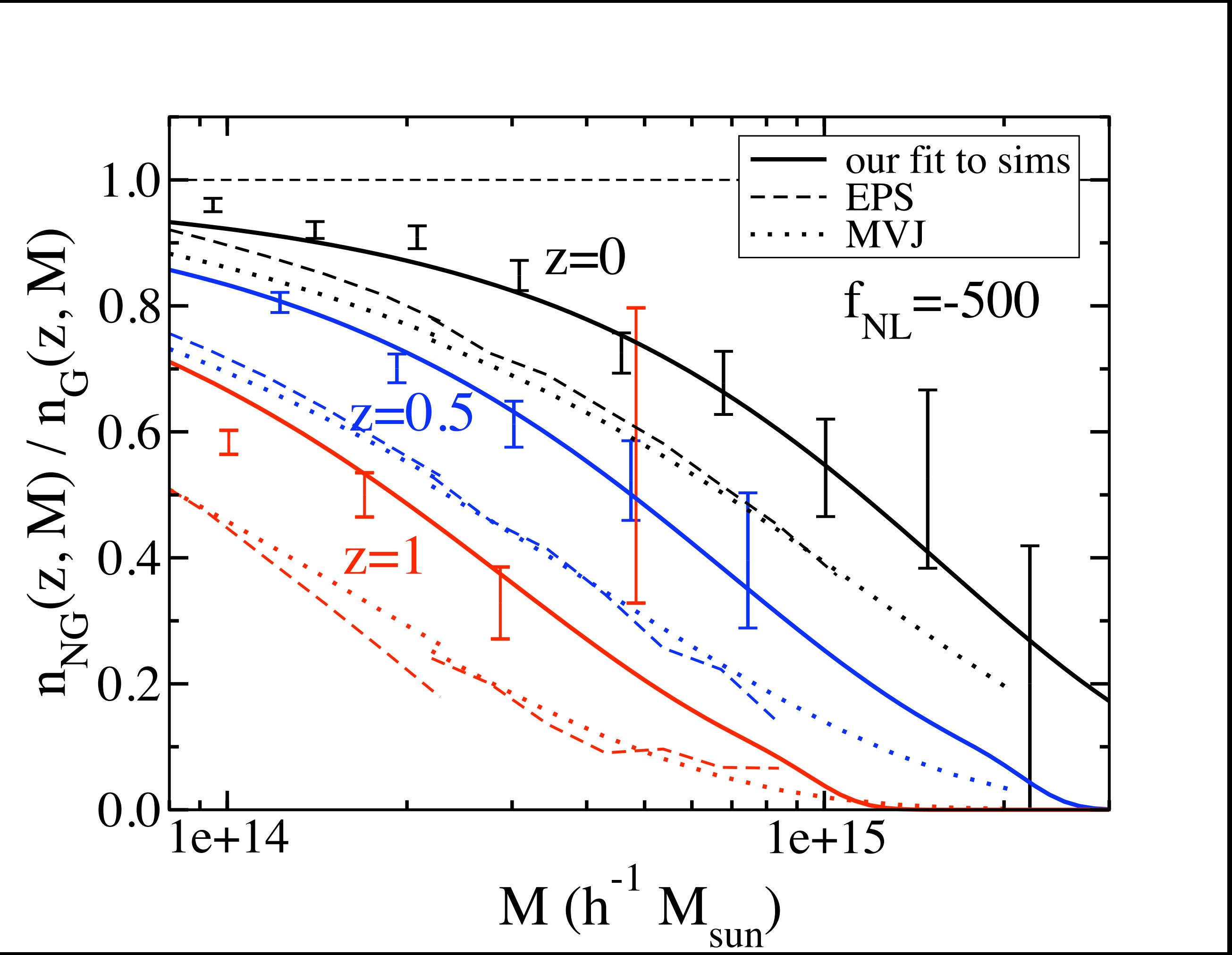


Mean and variance of $P(M_f/M_0)$ are well fit by:

$$\left[\frac{\bar{M}_f}{M_0}\right] - 1 = 6.\ 10^{-5} f_{NL} \sigma_8\ \sigma(M_0, z)^{-2}$$

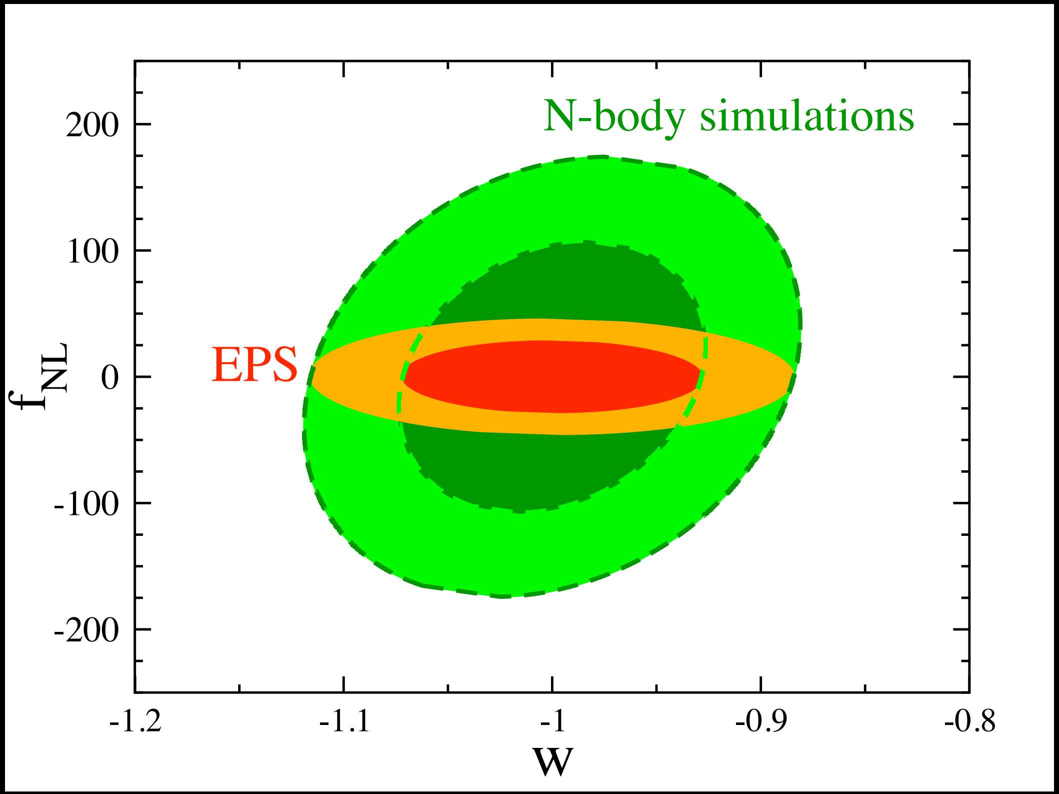
$$\sigma\left(\left[\frac{\bar{M}_f}{M_0}\right] - 1\right) = 0.012\ \left(f_{NL}\sigma_8\right)^{0.4}\ \sigma(M_0, z)^{-0.5}$$

Old fitting functions are discrepant; off by O(100%) wrt truth



Moreover, it is not much harder to run a simulation than evaluate Extended Press-Schechter n(M)

Cosmological constraints - dark energy and NG



Cluster **counts alone**;
SPT-type survey, ~7,000 clusters, 4000 sq.deg., 0.1<z<1.5
+ Planck cosmological parameter prior

N.B. Planck bispectrum will provide stronger constraints

Effects of primordial NG
on the bias of virialized objects

Does galaxy/halo bias depend on NG?

cosmologists
measure

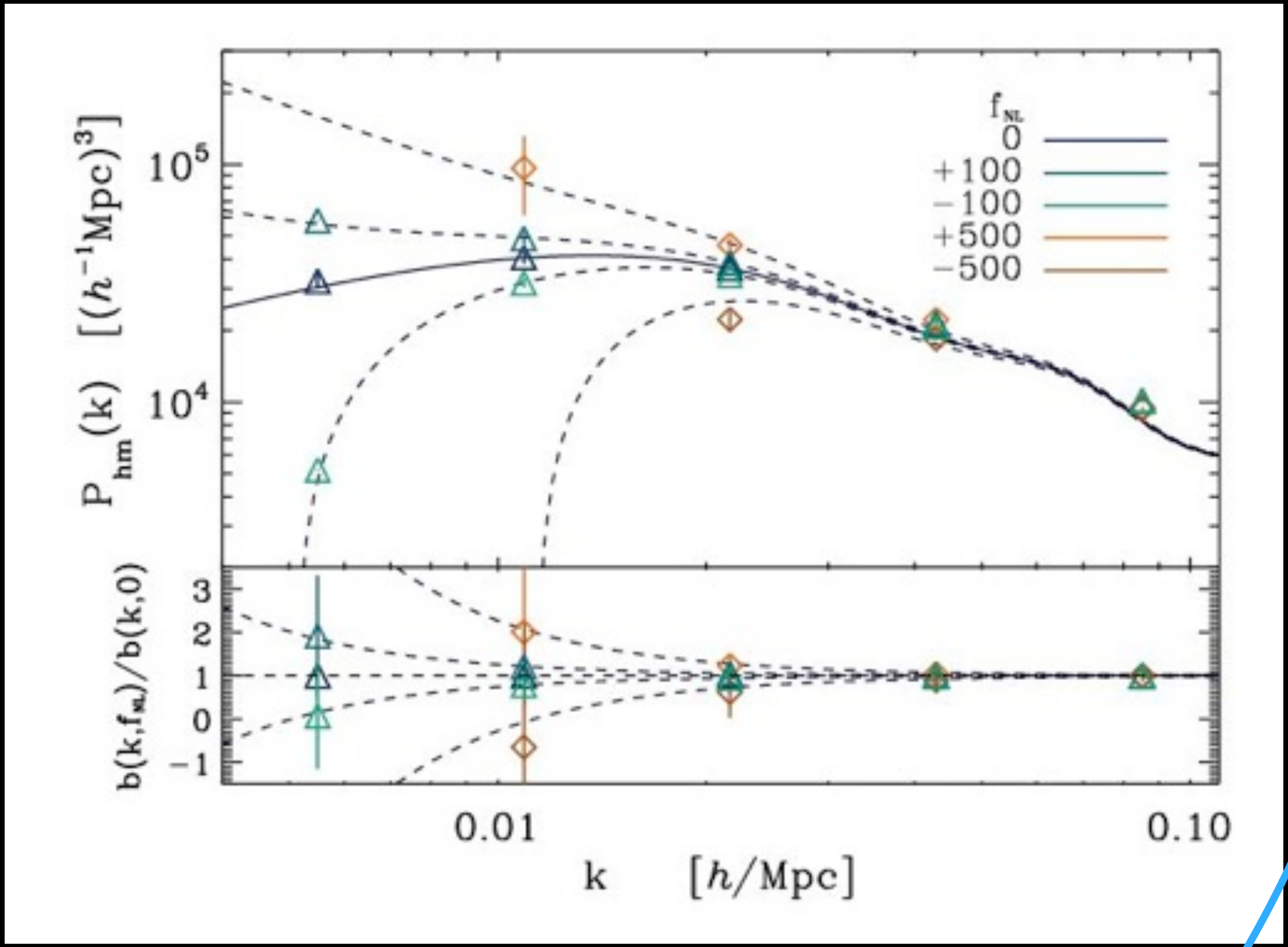
$$\text{bias} \equiv \frac{\text{clustering of galaxies}}{\text{clustering of dark matter}} = \frac{\left(\frac{\delta\rho}{\rho}\right)_{\text{halos}}}{\left(\frac{\delta\rho}{\rho}\right)_{\text{DM}}}$$

usually nuisance
parameter(s)

theory predicts

Simulations and theory both say:
large-scale bias is scale-independent

Scale dependence of NG halo bias!



$$b(k) = b_{\rm G} + f_{\rm NL}\,\frac{\rm const}{k^2}$$

Dalal, Doré, Huterer & Shirokov, arXiv:0710.4560, PRD 2008

Halo clustering with NG: Analytic confirmation

Rigorous derivations exist, but here's back-of-envelope:

$$\Phi_{\rm NG} = \phi + f_{\rm NL}(\phi^2 - \langle\phi^2\rangle)$$

Then, near the peaks of the potential

$$\begin{aligned}
\nabla^2\Phi_{\rm NG} &= \nabla^2\phi + 2f_{\rm NL}\left(\phi\nabla^2\phi + |\nabla\phi|^2\right) \\
&\approx \nabla^2\phi\,(1 + 2f_{\rm NL}\phi)
\end{aligned}$$

And in particular

$$\delta_{\rm NG} = \delta(1 + 2f_{\rm NL}\phi)$$

Halo clustering with NG: Analytic confirmation

Definition of bias: $$\delta_h = b_L\,\delta$$

With NG, for peaks: $$\delta \longrightarrow \delta + 2 f_{\rm NL}\phi\,\delta$$

Reinterpreting as change in bias $$\delta_h = b_L\,(\delta + 2 f_{\rm NL}\phi\,\delta) = (b_L + \Delta b)\,\delta$$

Using Poisson Eq. (to replace ϕ with δ) and including late-time perturbation evolution, you get

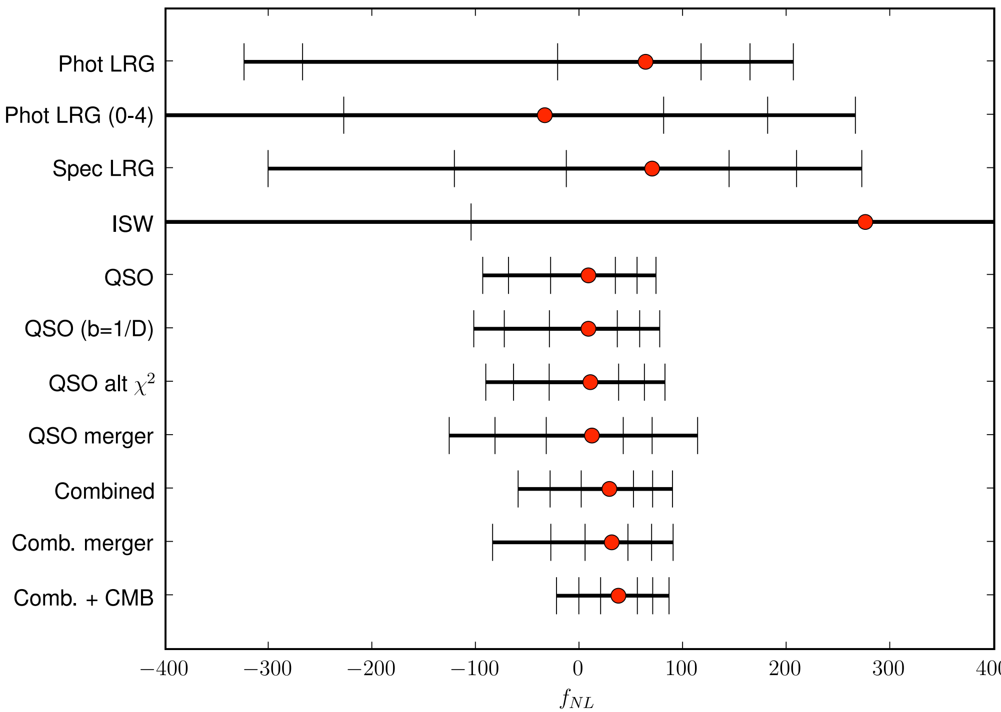
$$\Delta b(k) = f_{\rm NL}(b_G - 1)\,\delta_c\,\frac{3\,\Omega_M H_0^2}{T(k)D(a)k^2}$$

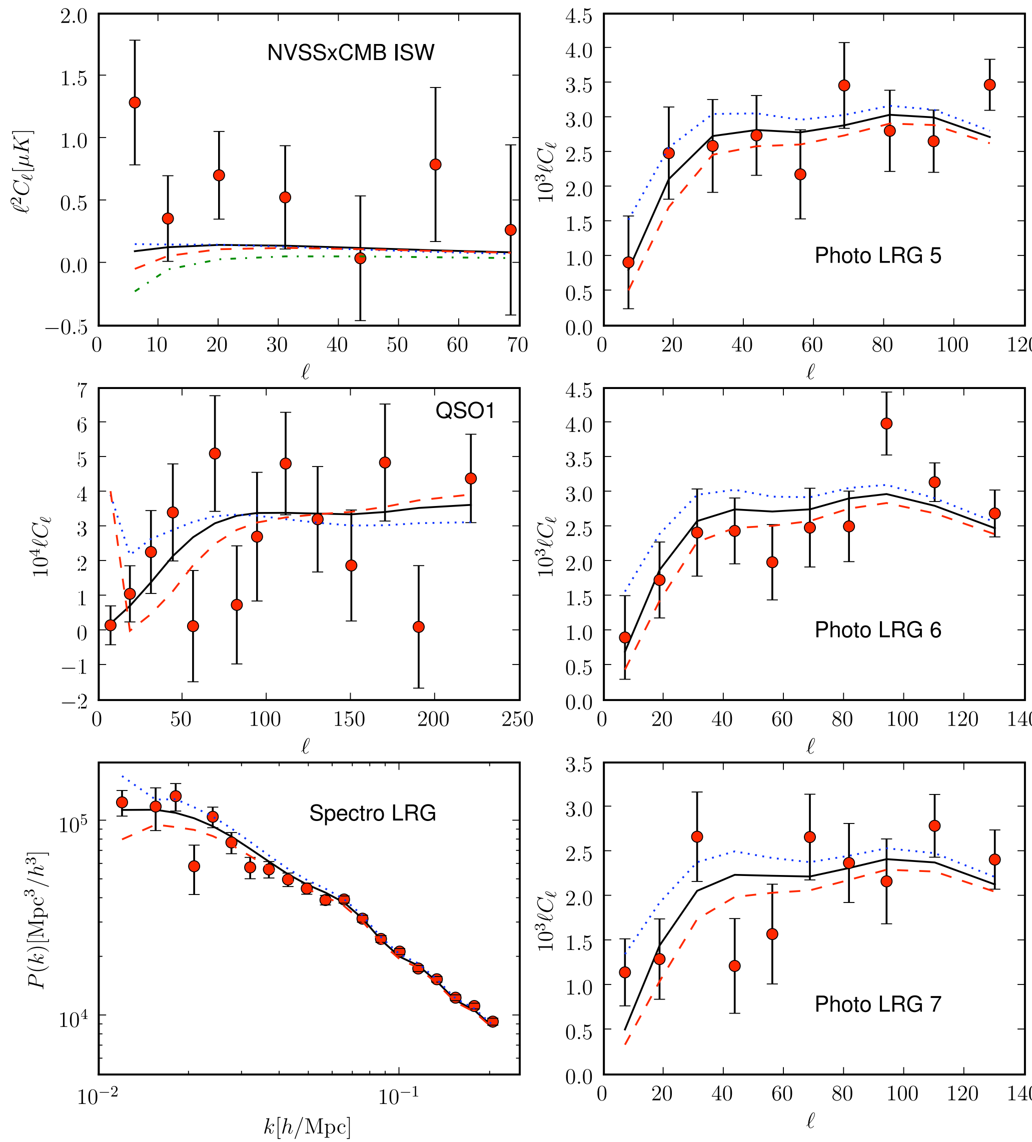
Dalal, Doré, Huterer & Shirokov, PRD 2008

Many complementary confirmations : Matarrese & Verde; Slosar et al; Afshordi & Tolley; Desjacques et al; Giannantonio & Porciani; Grossi et al; McDonald; ....)

Constraints from the bias of DM halos

Constraints from *current* data - SDSS





$f_{NL} = 8$ +/- 30 (68%, QSO)

$f_{NL} = 23$ +/- 23 (68%, all)

Slosar, Hirata, Seljak, Ho & Padmanabhan 2008

Future NG from measurements of b(k)

- Numerous cosmological probes, such as the baryon acoustic oscillations (BAO) or probes of Integrated Sachs-Wolfe effect (galaxy-CMB cross-corr) can be used to measure b(k)

- The effect (going as k^{-2}) provides a fairly unique signature and a clear target; almost no degeneracy with other cosmological parameters

- Expect accuracy of order sigma(f_{NL})<10 or even ~1 in the future

TABLE 1
Galaxy Surveys considered

survey	z range	sq deg	mean galaxy density $(h/Mpc)^3$	$\Delta f_{\rm NL}/q'$ LSS
SDSS LRG's	$0.16 < z < 0.47$	7.6×10^3	1.36×10^{-4}	40
BOSS	$0 < z < 0.7$	10^4	2.66×10^{-4}	18
WFMOS low z	$0.5 < z < 1.3$	2×10^3	4.88×10^{-4}	15
WFMOS high z	$2.3 < z < 3.3$	3×10^2	4.55×10^{-4}	17
ADEPT	$1 < z < 2$	2.8×10^4	9.37×10^{-4}	1.5
EUCLID	$0 < z < 2$	2×10^4	1.56×10^{-3}	1.7
DES	$0.2 < z < 1.3$	5×10^3	1.85×10^{-3}	8
PanSTARRS	$0 < z < 1.2$	3×10^4	1.72×10^{-3}	3.5
LSST	$0.3 < z < 3.6$	3×10^4	2.77×10^{-3}	0.7

!

Carbone, Verde & Matarrese 2008; Afshordi & Tolley 2008

Nongaussianity form clustering of galaxy **clusters**

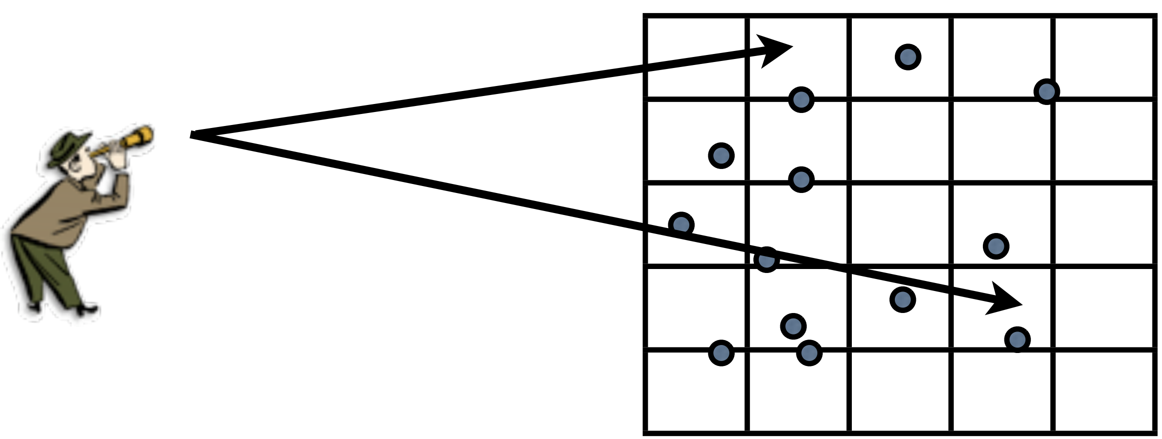
Cunha, Huterer & Doré, arXiv:1003.2416

- **Covariance** (i.e. clustering) between very distant clusters of galaxies is especially sensitive to primordial nongaussianity

- Improvement relative to counts alone: **2-3 orders of magnitude** in accuracy

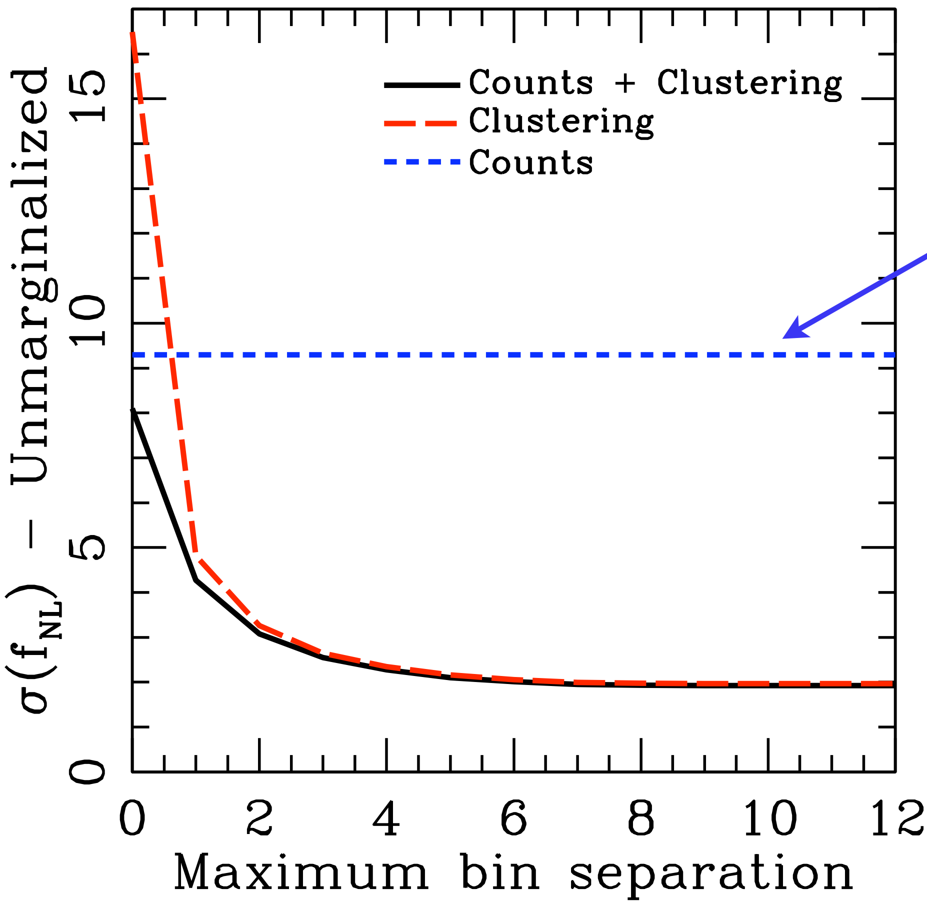
- Improvement relative to *variance* of counts: >1 order of magnitude in accuracy

- In other words:
  - **Good**: Counts ($d^2N/dzd\Omega = r^2(z)/H(z)$)
  - **Better**: Variance (of counts in cells)
  - **Best: Covariance** (of counts in cells)



N.B. calculation is numerically demanding even at the Fisher matrix level!

Nongaussianity form clustering of galaxy clusters



more "covariances"

(still much worse if marginalized over other pars.)

Marginalized errors - Full Covariance				
Nuisance parameters		Counts+Covariance		
Halo bias	$M_{\rm obs}$	$\sigma(\Omega_{\rm DE})$	$\sigma(w)$	$\sigma(f_{\rm NL})$
Marginalized	Marginalized	0.069	0.23	**6.0**
Known	Marginalized	0.065	0.22	**5.4**
Marginalized	Known	0.0036	0.014	**3.8**
Known	Known	0.0036	0.014	**1.8**

Cunha, Huterer & Doré, arXiv:1003.2416

Scale-dependent nongaussianity?
Generalized local ansatz

Becker, Huterer & Kadota, in preparation

- Motivated by inflationary models with self-interactions etc
- In general, even if you are considering standard single-field inflation, interactions may lead to scale-dependence of f_{NL}
- See Chris Byrnes talk next week

(Usual) local model...

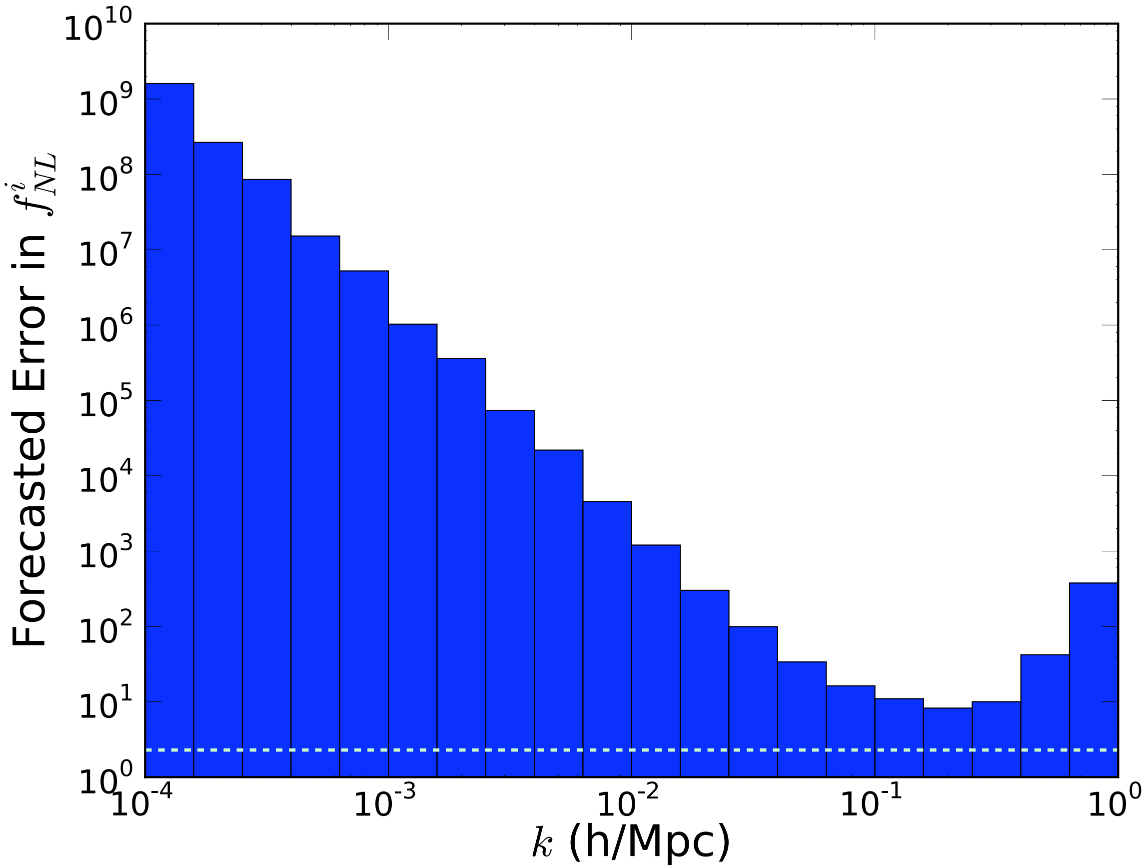
$$\Phi(x) = \phi_G(x) + f_{\rm NL}\left[\phi_G^2(x) - \langle\phi_G^2\rangle\right]$$

...we generalize to a scale dependent (non-local) model

$$\Phi(x) = \phi_G(x) + f_{\rm NL}(x) * \left[\phi_G^2(x) - \langle\phi_G^2\rangle\right]$$

$$\Phi(k) = \phi_G(k) + f_{\rm NL}(k) \int \frac{d^3k'}{(2\pi)^3} \phi_G(k')\phi_G(k-k')$$

A complete basis for $f_{NL}(k)$: piecewise-constant bins



Measurement errors from DES-type survey

Projection onto any theoretical $f_{NL}(k)$ model is now trivial:

(F = Fisher matrix; quantifies measurability of $f_{NL}(k)$ from bias of LSS)

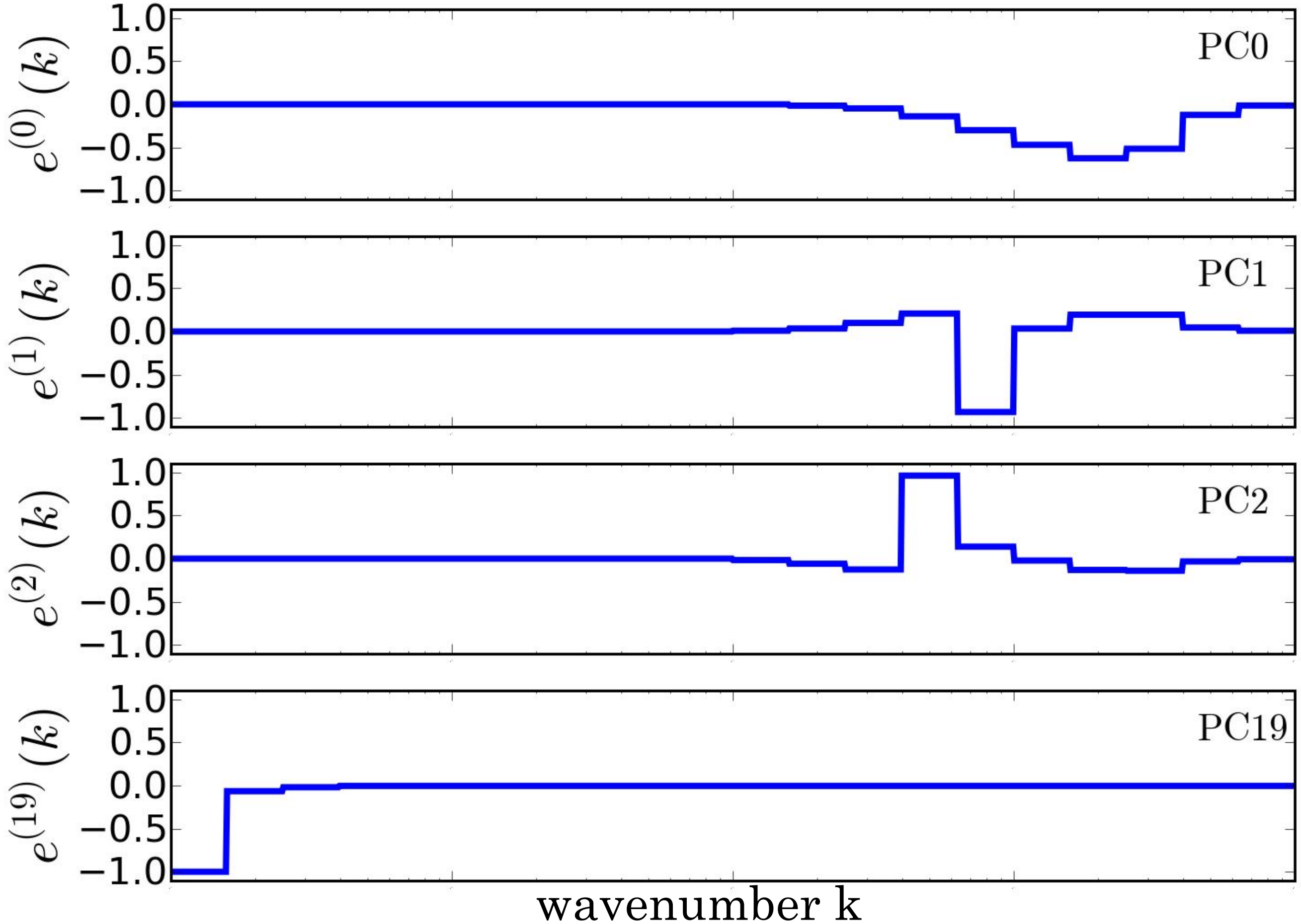
$$F_{ij}^{\text{new}} = \sum_{k,l=1}^{N} \frac{\partial p^k}{\partial q^i} \frac{\partial p^l}{\partial q^j} F_{kl}$$

Original (basis) parameters, $f_{NL}{}^i$

New parameters (e.g. $f_{NL}(k_{pivot})$, n)

Becker, Huterer & Kadota, in preparation

Principal Components of $f_{NL}(k)$



Best-measured $f_{NL}(k)$ mode

2nd-best measured...

3rd-best measured...

worst-measured mode

Overlap with local and equil. NG models:

	Local cosine	Equilateral cosine
PC 0	0.708	0.074
PC 1	0.263	0.005
PC 2	0.158	0.063

$$\cos(B_1, B_2) = \frac{B_1 \cdot B_2}{\sqrt{(B_1 \cdot B_1)\,(B_2 \cdot B_2)}}$$

$$B_1 \cdot B_2 = \sum_{k_1,k_2,k_3} \frac{B_1(k_1,k_2,k_3) B_2(k_1,k_2,k_3)}{\Delta^2 B(k_1,k_2,k_3)}$$

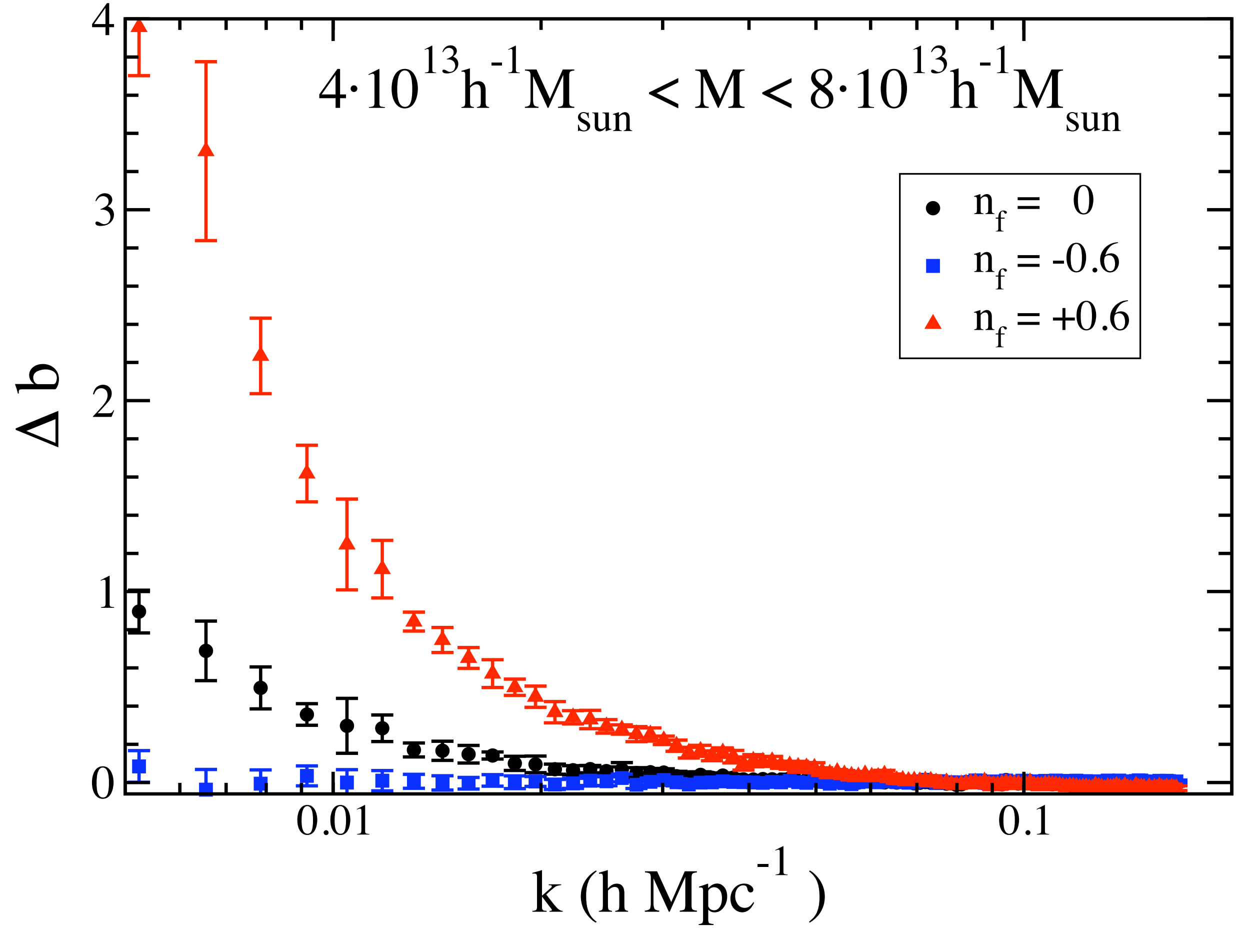
Becker, Huterer & Kadota, in preparation

Scale-dependent non-Gaussianity: comparison with simulations

S. Shandera et al, in preparation

$$f_{\rm NL}(k) = f_{\rm NL}(k_p)\left(\frac{k}{k_p}\right)^{n_f}$$

Preliminary



Conclusions

- Searching for primordial nongaussianity is one of the most fundamental tests of the early universe cosmology

- CMB bispectrum traditionally most promising tool; current results favor $f_{NL}>0$ but only at 1-2 sigma

- Mass function of cluster counts is in principle sensitive to NG, but not competitive with the CMB

- Cosmological models with (local) primordial NG lead to significant scale dependence of halo bias; theory and simulations are in remarkable agreement on this

- Therefore, LSS probes (baryon oscillations, galaxy-CMB cross-correlations, etc) are likely to lead to constraints on NG nearly 2 orders of magnitude stronger than previously thought

- sigma(f_{NL})~few expected from future LSS surveys (DES, PanStarrs, LSST, JDEM/EUCLID etc)

- See upcoming talks on NG/bias from C. Byrnes, V. Desjacques, M. Maggiore, M. Manera, C. Porciani, R. Scoccimarro and R. Smith