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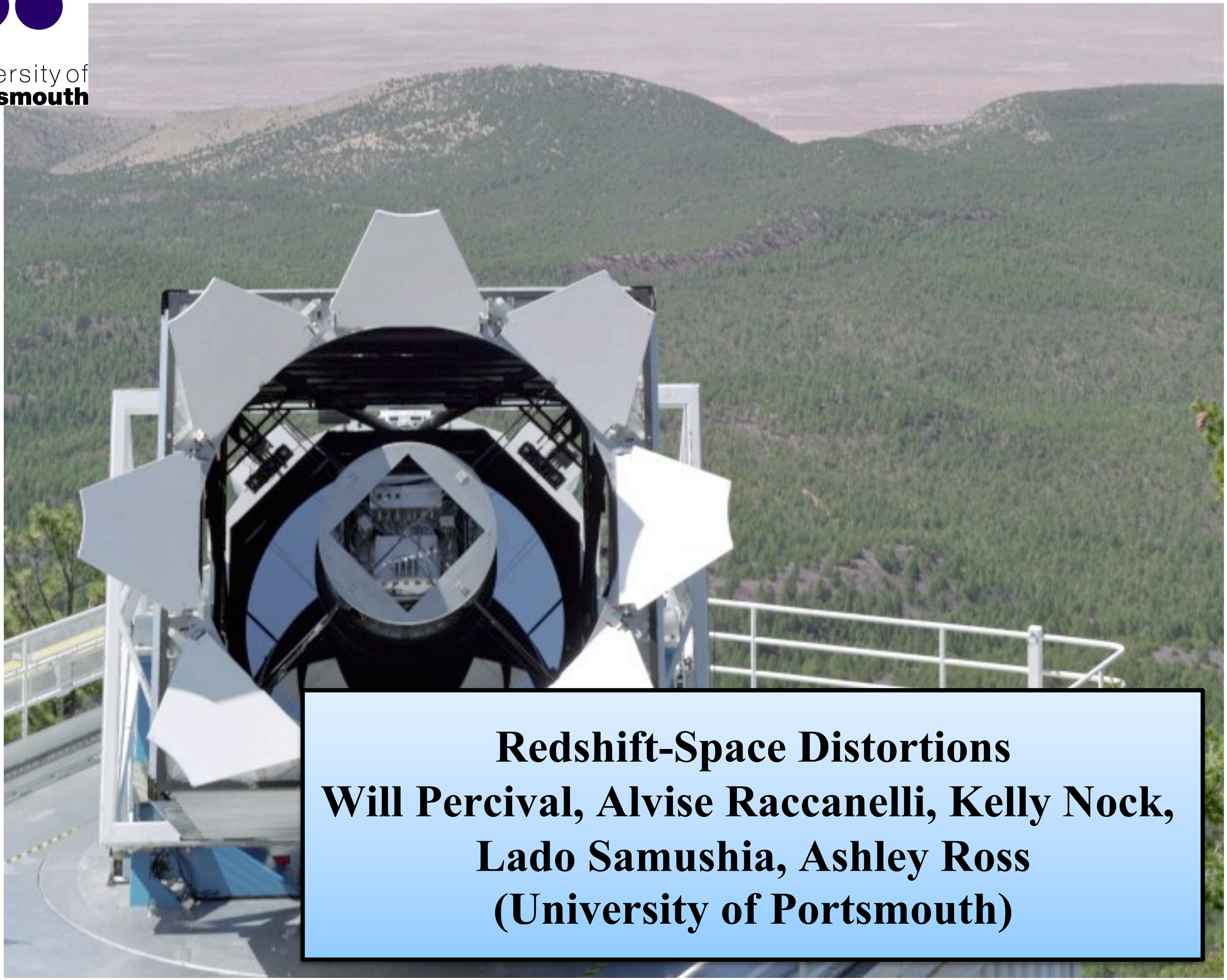

Redshift-Space Distortions

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Lado Samushia, Ashley Ross
(University of Portsmouth)

redshift-space distortions

When we measure the position of a galaxy, we measure its position in redshift-space; this differs from the one in real-space because of its peculiar velocity:

$$s(r) = r - v_r(r)\hat{r}$$

Where s and r are positions in redshift- and real-space and v_r is the peculiar velocity in the radial direction

RSD on small scales

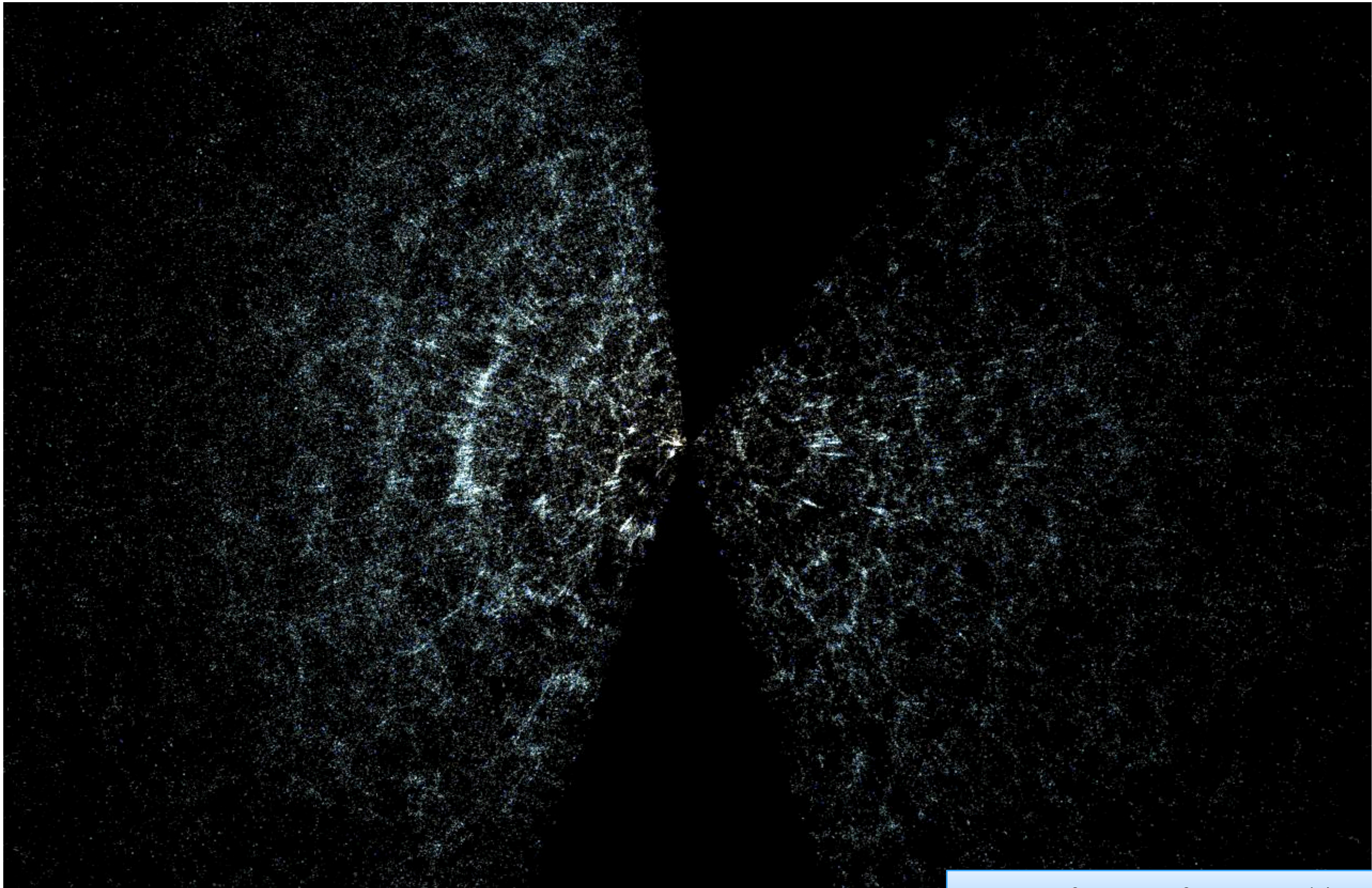
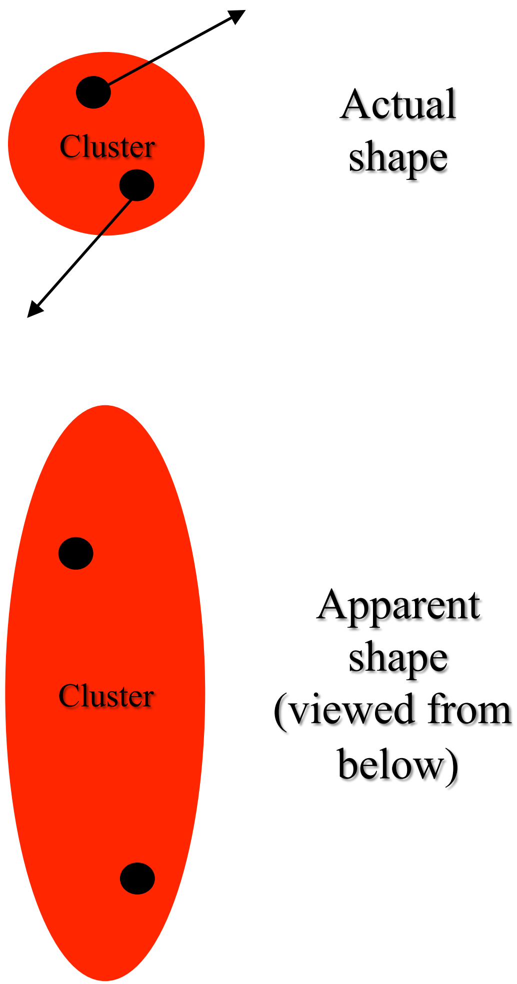


Image of SDSS, from U. Chicago

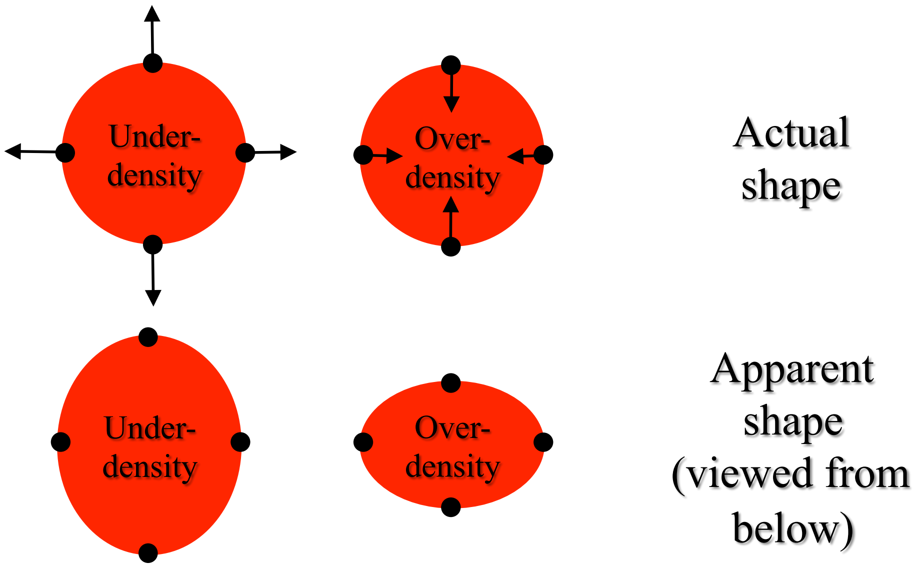
RSD on small scales

- Virial motions of galaxies in collapsed objects misinterpreted as Hubble flow
- Leads to apparent elongation of clusters along line-of-sight
- non-linear physics, so hard to extract cosmological information



RSD on large scales

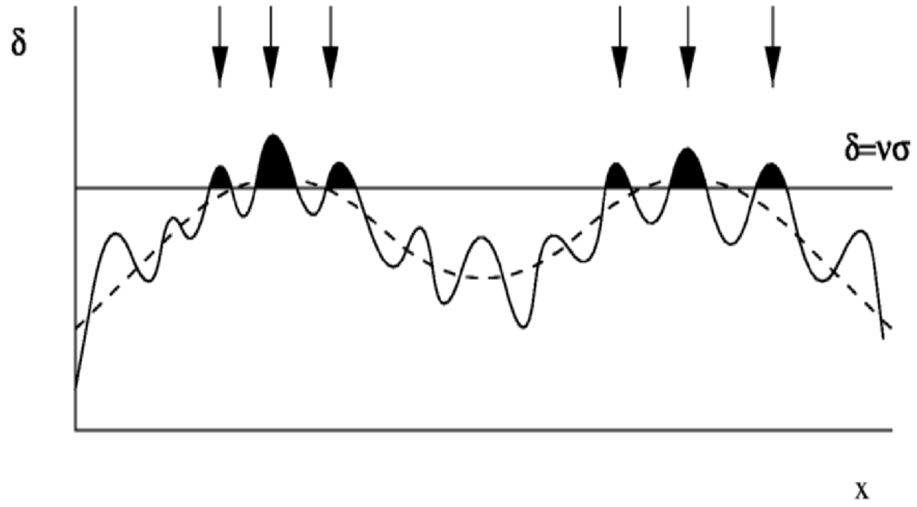
- Structure growth is
  - driven by the motion of matter
  - inhibited by the cosmological expansion
- Motion of galaxies carries an imprint of the rate of growth of large-scale structure.
- On large scales, galaxies move coherently towards the overdensities and away from underdensities



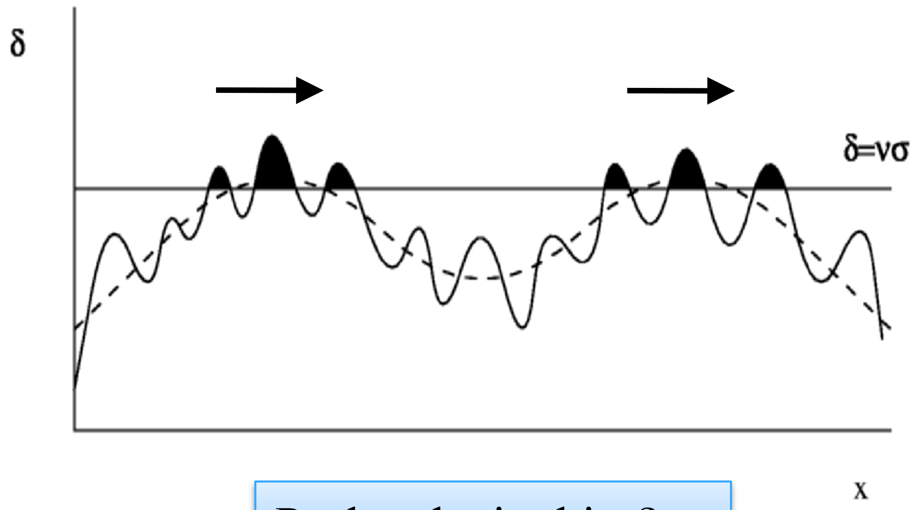
Galaxies act as test particles

Galaxies act as test particles with the flow of matter

On large-scales, the distribution of galaxy velocities is unbiased provided that the positions of galaxies fully sample the velocity field



Peak overdensity bias



Peak velocity bias?

If fact, we can expect a small peak velocity-bias due to motion in Gaussian random fields (Percival & Schafer 2008, MNRAS 385, L78)

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What parameter do RSD measure?

Two ways of writing the over-density in linear limit

$$\begin{aligned} \delta_{\rm gal}(k,\mu) &= b\delta_{\rm mass}(1+\mu^2\beta) \\ \delta_{\rm gal}(k,\mu) &= b\delta_{\rm mass}+\mu^2 f\delta_{\rm mass} \end{aligned}$$

Two ways of writing the power spectrum

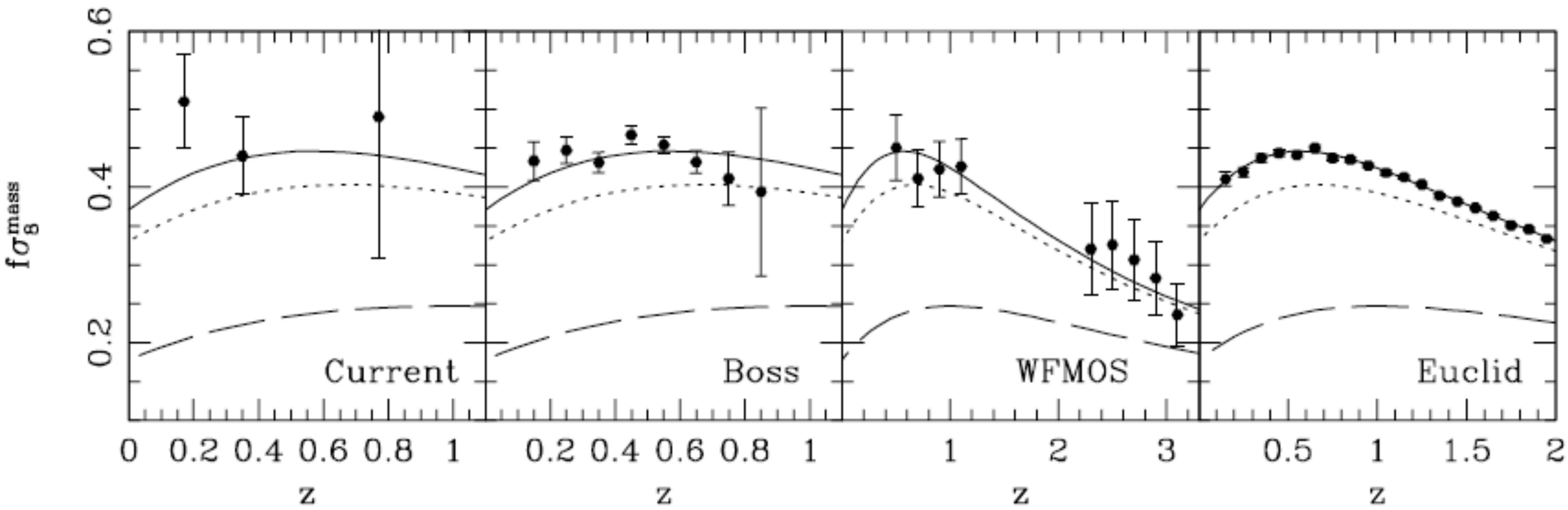
$$\begin{aligned} P_{\rm gal}(k,\mu) &= b^2P_{\rm mass}(1+\beta\mu^2)^2 \\ P_{\rm gal}(k,\mu) &= P_{\rm mass}(b^2+2\mu^2 bf+\mu^4 f^2) \end{aligned}$$

We measure the normalizations of the galaxy over-density field ($b_\delta\sigma_8$), and the galaxy velocity field ($fb_v\sigma_8$, with $b_v=1$). You can obviously measure any combinations of these (e.g. β), or other combinations.

Do we need to know galaxy bias?

RSD constrain $f\sigma_8$, which is as good a test of GR as f. A constraint on $b\delta$ or σ_8 allows us to normalise the amplitude of the velocity field

$$f \equiv \frac{d\log D}{d\log a} \qquad f\sigma_8 \propto \frac{dD}{d\log a}$$



Song, Percival, 2008, astro-ph/0807.0810

Direct estimator for the velocity power spectrum

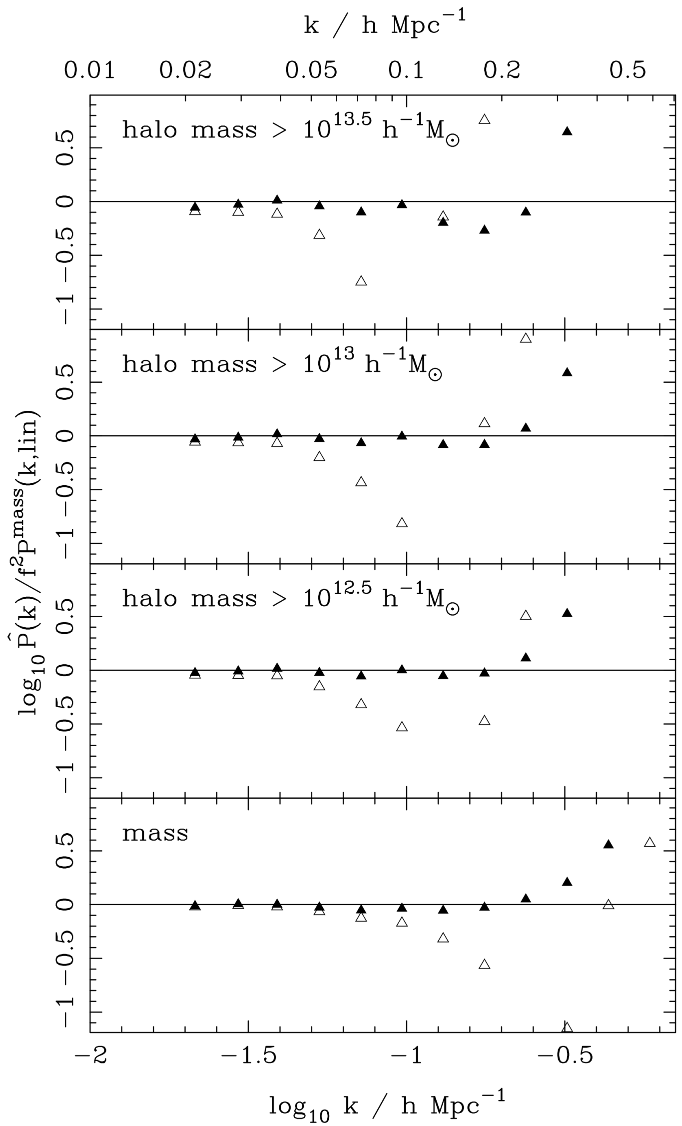
Can construct an estimator for the linear **mass velocity** power spectrum (mass power spectrum multiplied by f^2)

$$\hat{P}(k) = \frac{7}{48}\left[5(7P_0 + P_2) - \sqrt{35}(35P_0^2 + 10P_0P_2 - 7P_2^2)^{1/2}\right]$$

Where P_0 and P_2 are the standard expansions of the power in Legendre polynomials

On large-scales, the primary systematic is a possible velocity-bias.

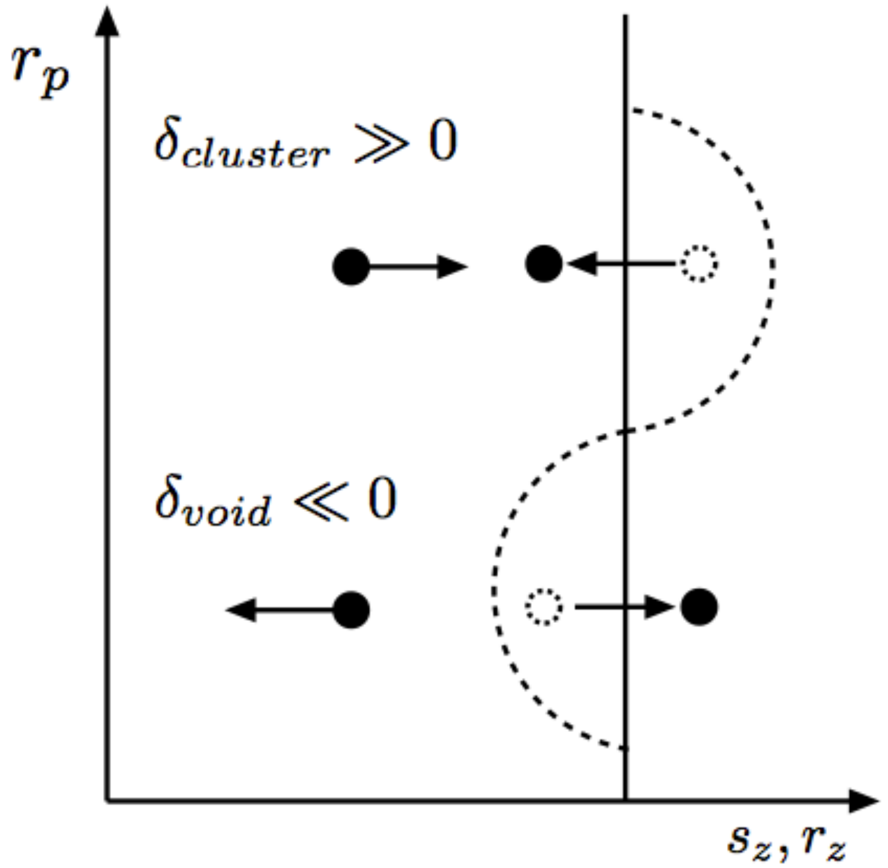
This estimator follows the plane-parallel, distant observer limit



Percival, White, 2008, astro-ph/0808.0003

Projected clustering measurements

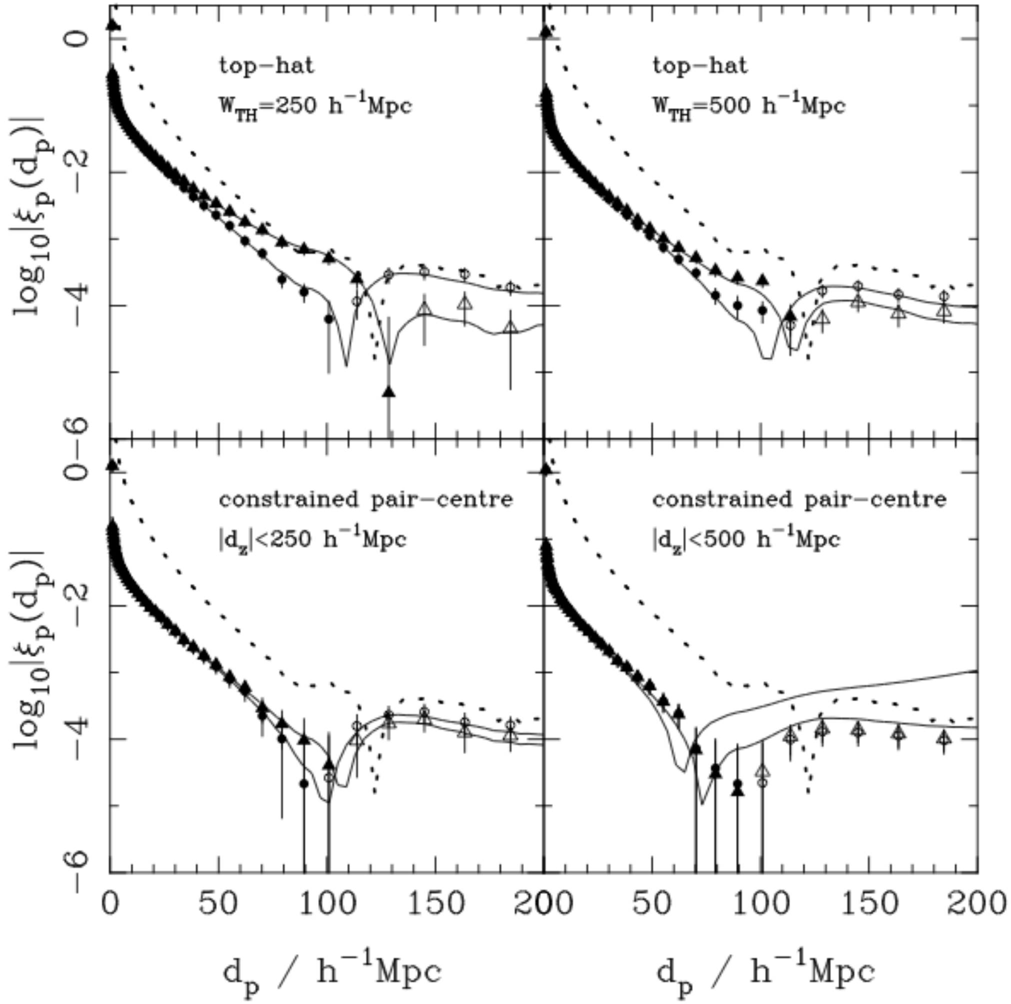
We even have to include RSD when modeling projected measurements (even though RSD do not change angular positions)



Nock et al 2010, arXiv:1003.0896

Projected clustering measurements

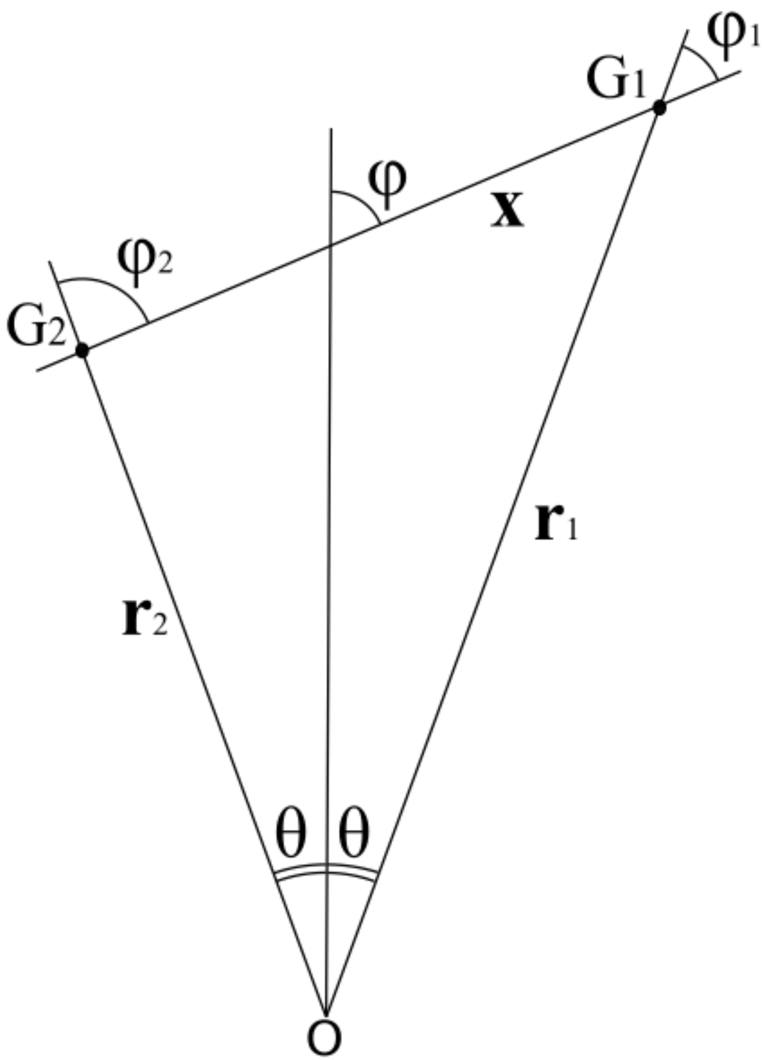
Although there are ways to mitigate RSD effects – for example by binning based on pair centers, rather than galaxy redshifts



Nock et al 2010, arXiv:1003.0896

Wide-angle RSD

- Geometry is actually a different triangle for each pair of galaxies
- In plane-parallel limit,
  - $\theta=0$
  - $\phi_1=\phi_2=\phi_3$
  - $r_1=r_2$



Raccanelli 2010, arXiv:1006:1652

The RSD operator

In the linear regime, we can write a linear RSD operator between real-space and redshift-space overdensities

$$\delta^s = \mathbf{S}\delta^r$$

The Jacobian of the real to redshift-space mapping can be written

$$d^3s = \left(1 + \frac{v_r}{r}\right)^2 \left(1 + \frac{\partial v_r}{\partial r}\right) d^3r$$

Giving an operator – on matter over-density field

$$\mathbf{S} = b + f\left(\frac{\partial^2}{\partial r^2} + \frac{\alpha(r)\partial}{r\partial r}\right)\nabla^{-2}$$

Where

$$\alpha(r) = \frac{\partial \ln r^2 \bar{N}^s(r)}{\partial \ln r}$$

“Mode-coupling” for RSD

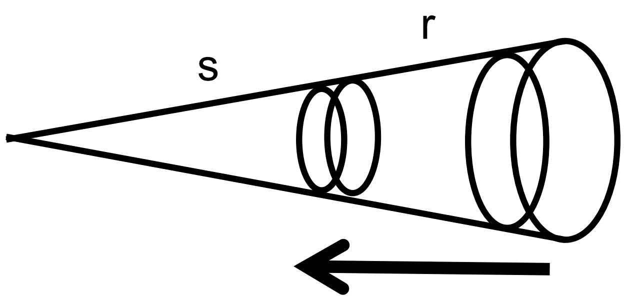
Applying the full linear operator gives

$$\delta^s(\mathbf{s}) = \delta^r(\mathbf{r}) - \left(\frac{\partial v_r}{\partial r} + \frac{\alpha(\mathbf{r})v_r}{r}\right)$$

The second term vanishes in the distance observer approximation r>>$\mathbf{v_r}$

Called the “mode-coupling” term, but perhaps “mode-confusion” would be better

Corrects for the extra increase in correlations when the volume is different between real and redshift-space



Increase in amplitude of over-density because of volume change

The plane-parallel limit

RSD operator in plane-parallel limit r>>$\mathbf{v_r}$

$$\mathbf{S} = b + f\left(\frac{\partial^2}{\partial r^2} + \cancel{\frac{\alpha(r)\partial}{r\partial r}}\right)\nabla^{-2}$$

Remaining operator commutes with the translation operator $-i\nabla$

So
$$\frac{\partial^2}{\partial z^2}\nabla^{-2} = \frac{k_z^2}{k^2} = \mu_k^2, \quad \mu_k \equiv \hat{z}\cdot\hat{k}$$

Giving the standard Kaiser operator

$$\mathbf{S} = b + f\mu_k^2$$

Each Fourier mode is simply amplified, but relative amplitude is unaffected

Kaiser 1987, review by Hamilton 1998

The full power spectrum

Following Papai & Szapudi (2008) we can write the full power spectrum

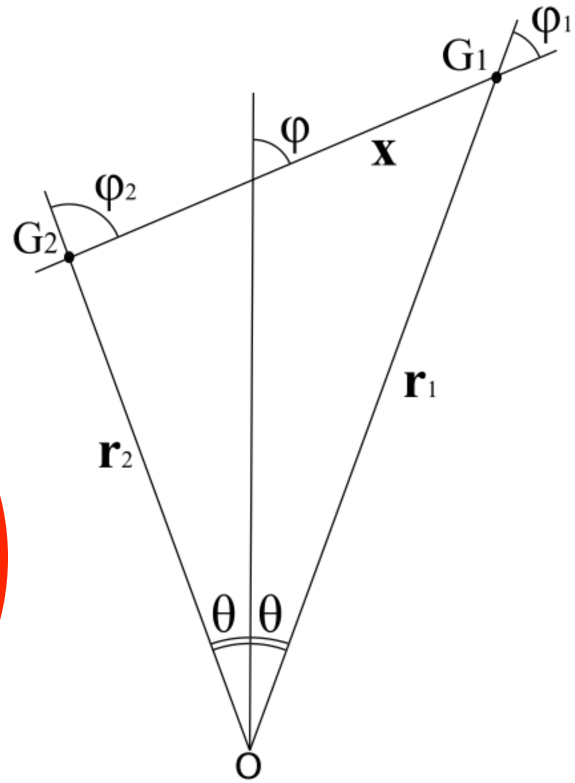
$$\langle \delta^s(\mathbf{r}_1)\delta^{s*}(\mathbf{r}_2)\rangle = \int \frac{d^3k}{(2\pi)^3} P(k) e^{ik(r_1 - r_2)}$$

$$\left[1 + \frac{f}{3} + \frac{2f}{3} L_2(\mu_1) - \frac{i\alpha f}{r_1 k} L_1(\mu_1)\right]$$

$$\left[1 + \frac{f}{3} + \frac{2f}{3} L_2(\mu_2) - \frac{i\alpha f}{r_2 k} L_1(\mu_2)\right]$$

where

$$\mu_1 = \cos(\phi_1), \quad \mu_2 = \cos(\phi_2)$$



Wide-angle RSD terms

Mode-coupling RSD terms

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Tripolar Spherical Harmonics Expansion

Szalay et al. (1998), Szapudi (2004) and Papai & Szapudi (2008) proposed and devloped a method to express the angular dependence of the correlation function using tripolar spherical harmonics:

$$S_{l_1 l_2 l}(\hat{x}, \hat{r}_1, \hat{r}_2)$$
$$= \sum_{m_1, m_2, m} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & m \end{pmatrix} C_{l_1 m_1}(\hat{r}_1) C_{l_2 m_2}(\hat{r}_2) C_{lm}(\hat{x})$$

where $\begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & m \end{pmatrix}$ are 3-j Wigner symbols

and $C_{lm} \equiv \sqrt{\frac{4\pi}{2l+1}} Y_{lm}$ are standard normalized spherical harmonics

They then give coefficients for the expansion of the redshift-space correlation function

$$\xi^s(x, \phi, \theta) = \sum_{l_1, l_2, l} B^{l_1 l_2 l}(x, \phi_1, \phi_2) S_{l_1 l_2 l}(\hat{x}, \hat{r}_1, \hat{r}_2)$$

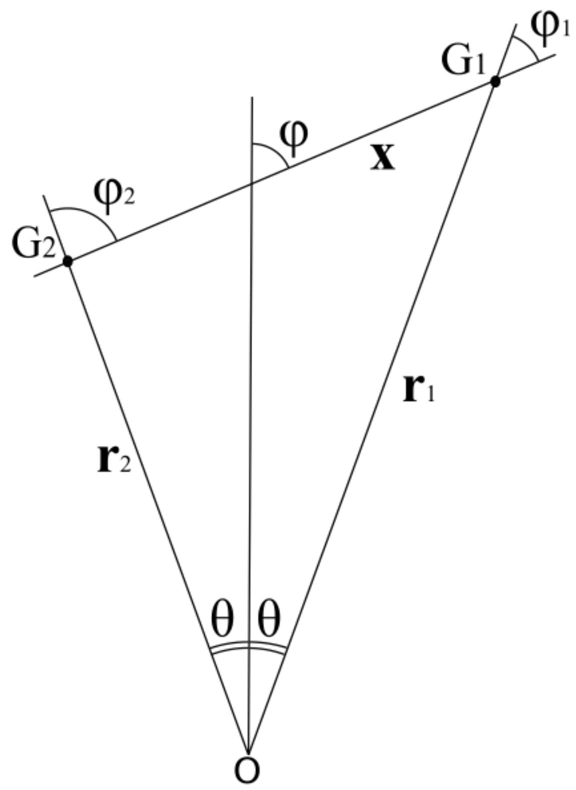
Testing wide angle RSD: method

- A standard approach to analysing wide-angle RSD would be to create and analyse a mock sample as you would with real data:
  - – locate an observer within the simulation output
  - – translate all galaxies from real into redshift-space based on that observer
  - – sample from these galaxies based on desired radial distribution
  - – split pairs into bins in φ, θ, x and count pairs
  - – estimate the correlation function
- However, this is time-consuming as not all pairs will be of the required separation
- We can do better …

Raccanelli 2010, arXiv:1006:1652

Testing wide angle RSD: method

- We can allow the origin (observer) to move to match each pair of galaxies to exactly the required angular separation
  - – decide on the value of θ for which we wish to analyse pairs
  - – take each galaxy pair from the simulation with real-space separation < Rmax
  - – for each pair randomly choose ϕ
  - – choose the location of the origin giving ϕ and θ
  - – move galaxies according to their expected RSD
  - – weight the pair by a function of ϕ, x to match desired distribution
  - – split pairs into bins in ϕ, x and counts pairs
  - – estimate the correlation function
- Weighting also allow multiple galaxy distributions to be analysed using a single simulation run



Raccanelli 2010, arXiv:1006:1652

Testing wide angle RSD: results

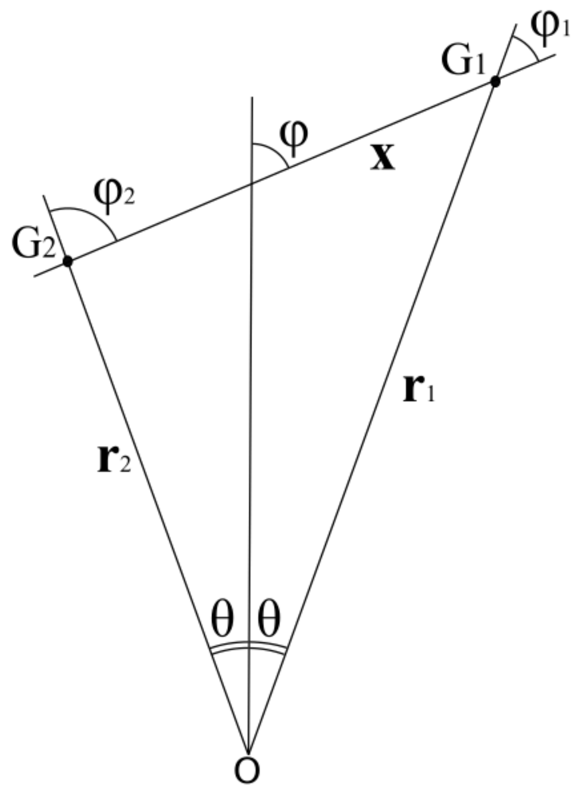
- Use data from the Hubble Volume Simulation, analysing 8 different galaxy configurations
- Two different theta: 0.355, 0.71 radians, (20, 40 degrees)
- 4 different values of α

  α=0 – wide angle only, as galaxy numbers are the same for bins of equal radius

  α=0.5 – matches distribution of pairs in 3D volume

  α=2 – equal galaxy numbers in equal volumes

  α=4 – more galaxies at higher redshifts



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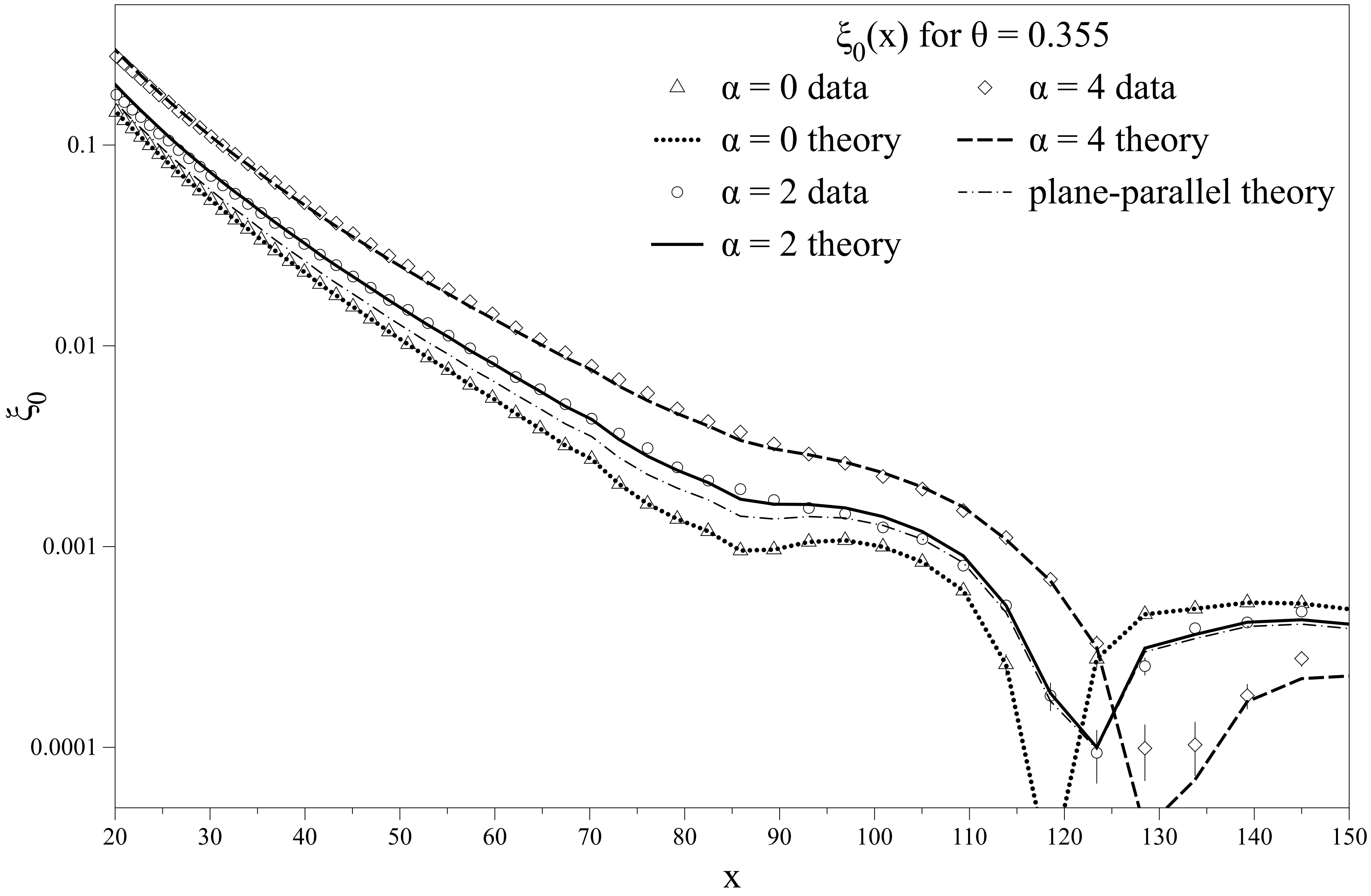
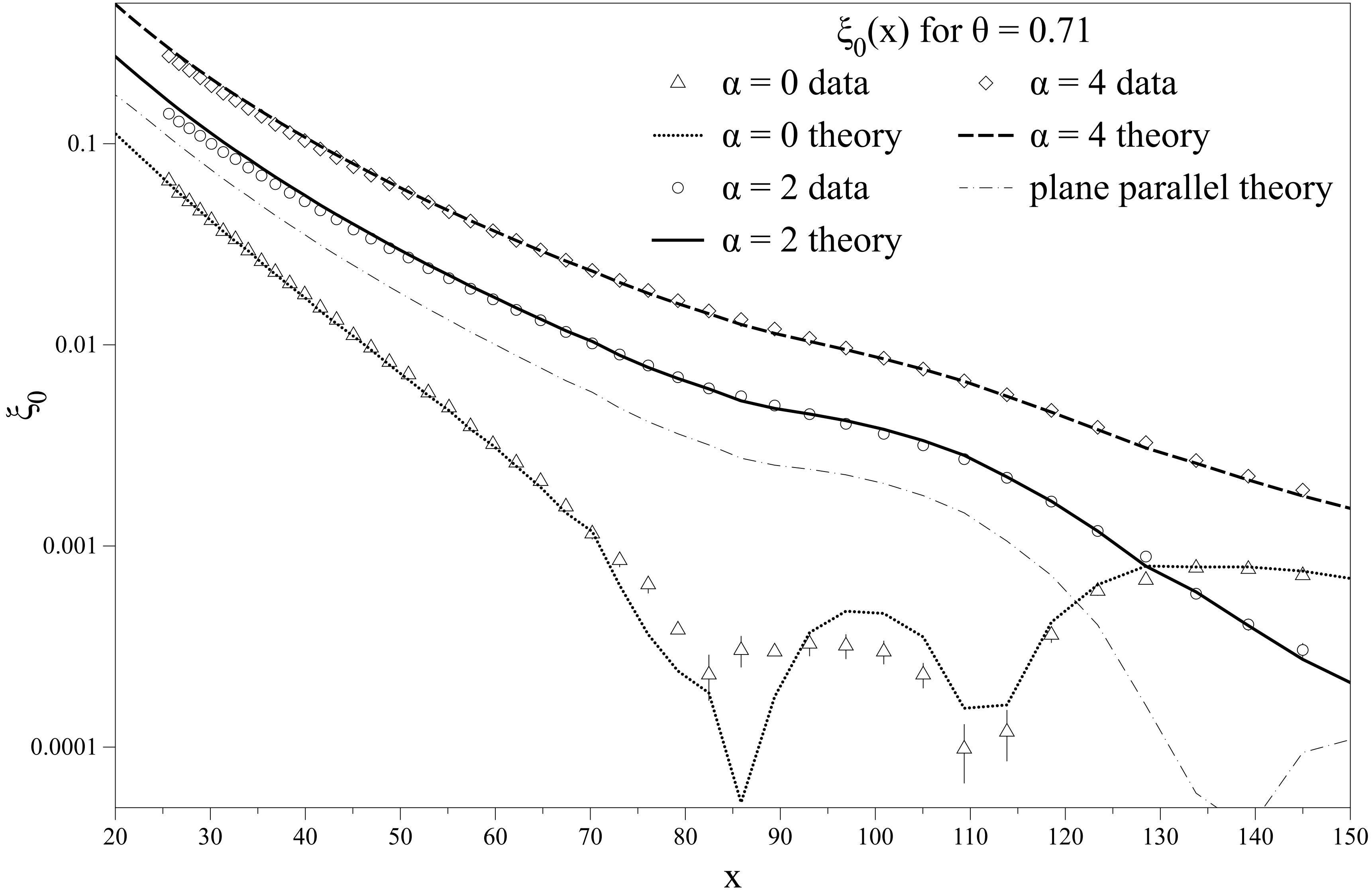
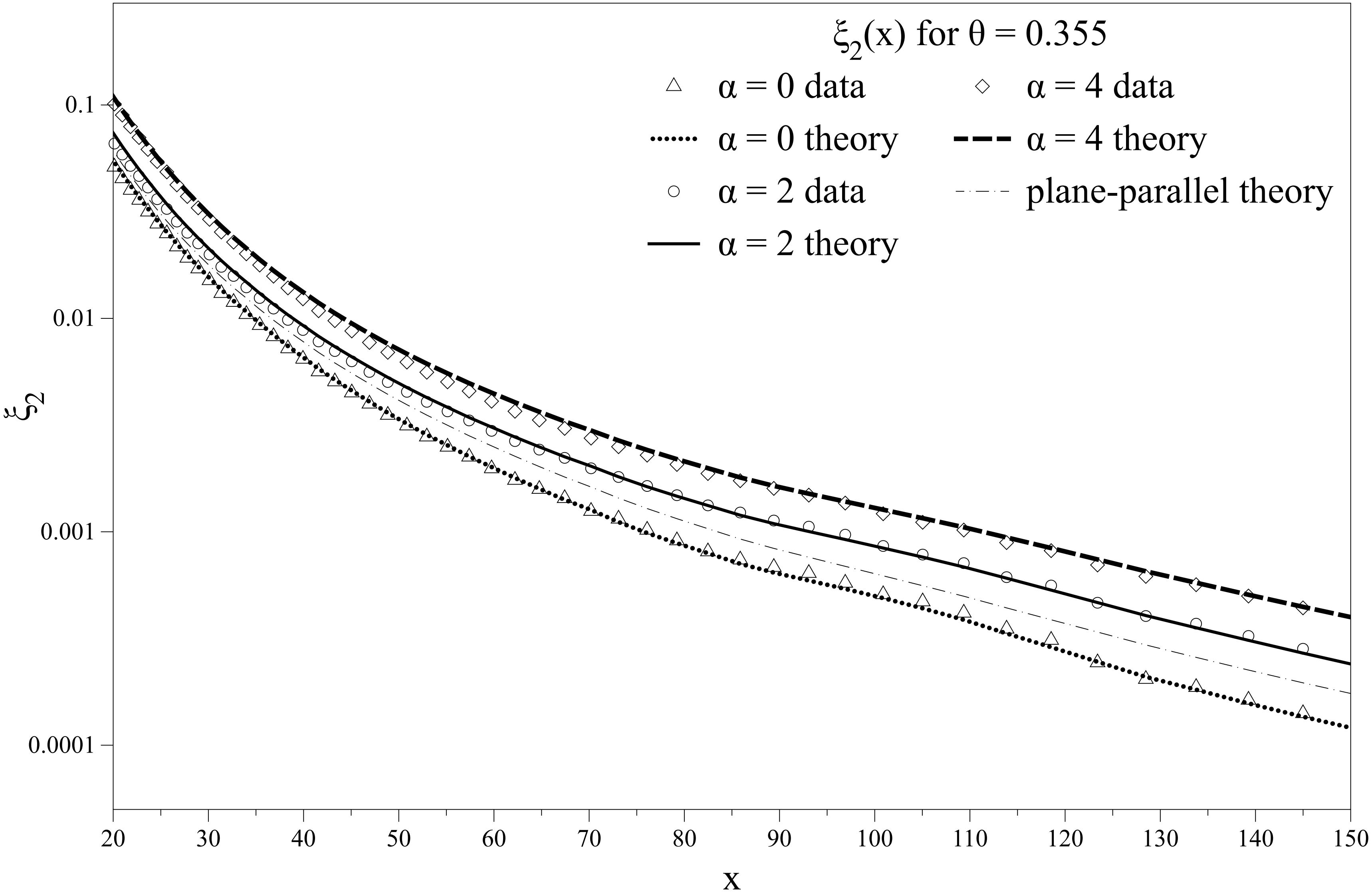
Testing wide angle RSD: results



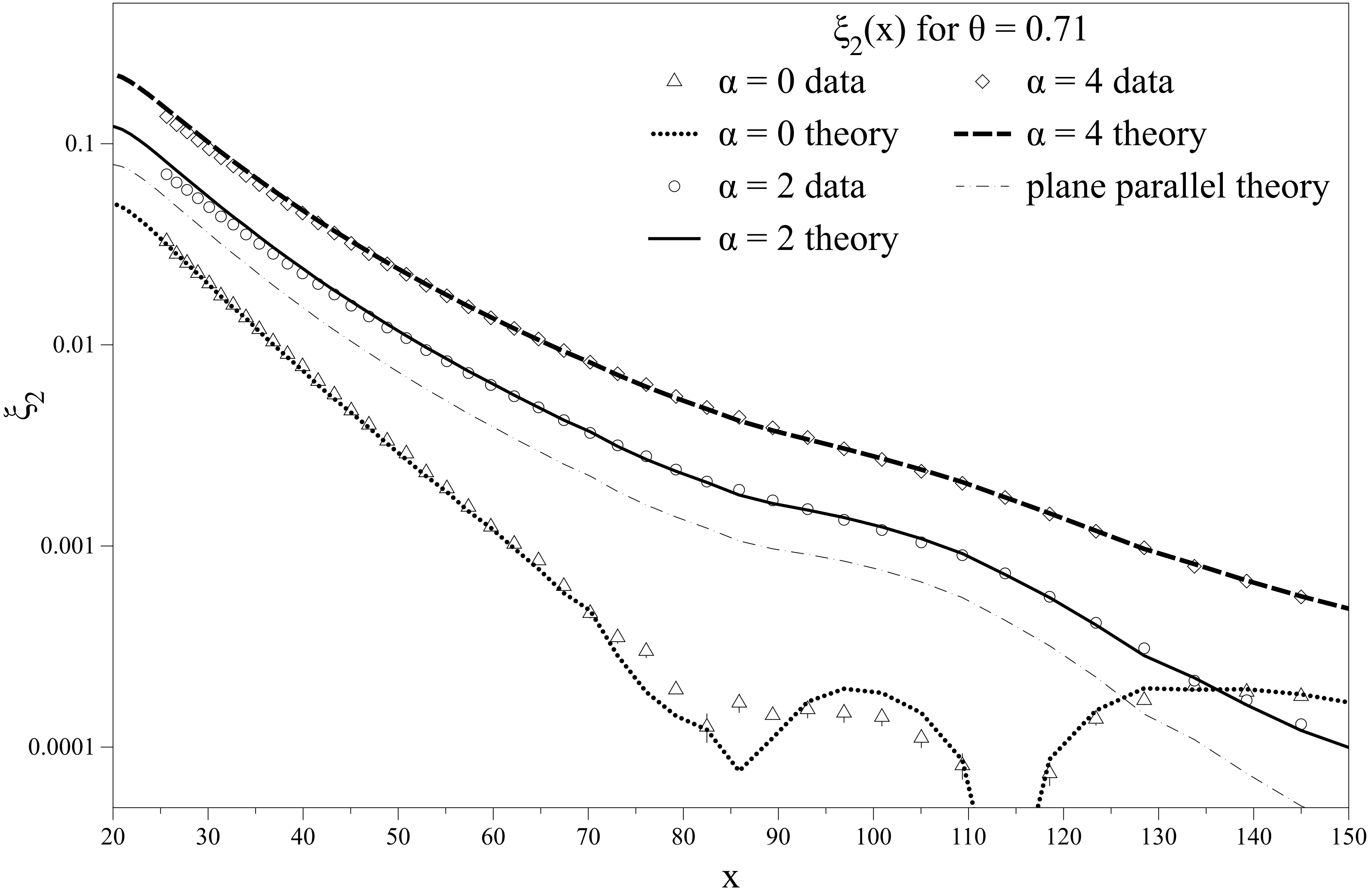
Testing wide angle RSD: results



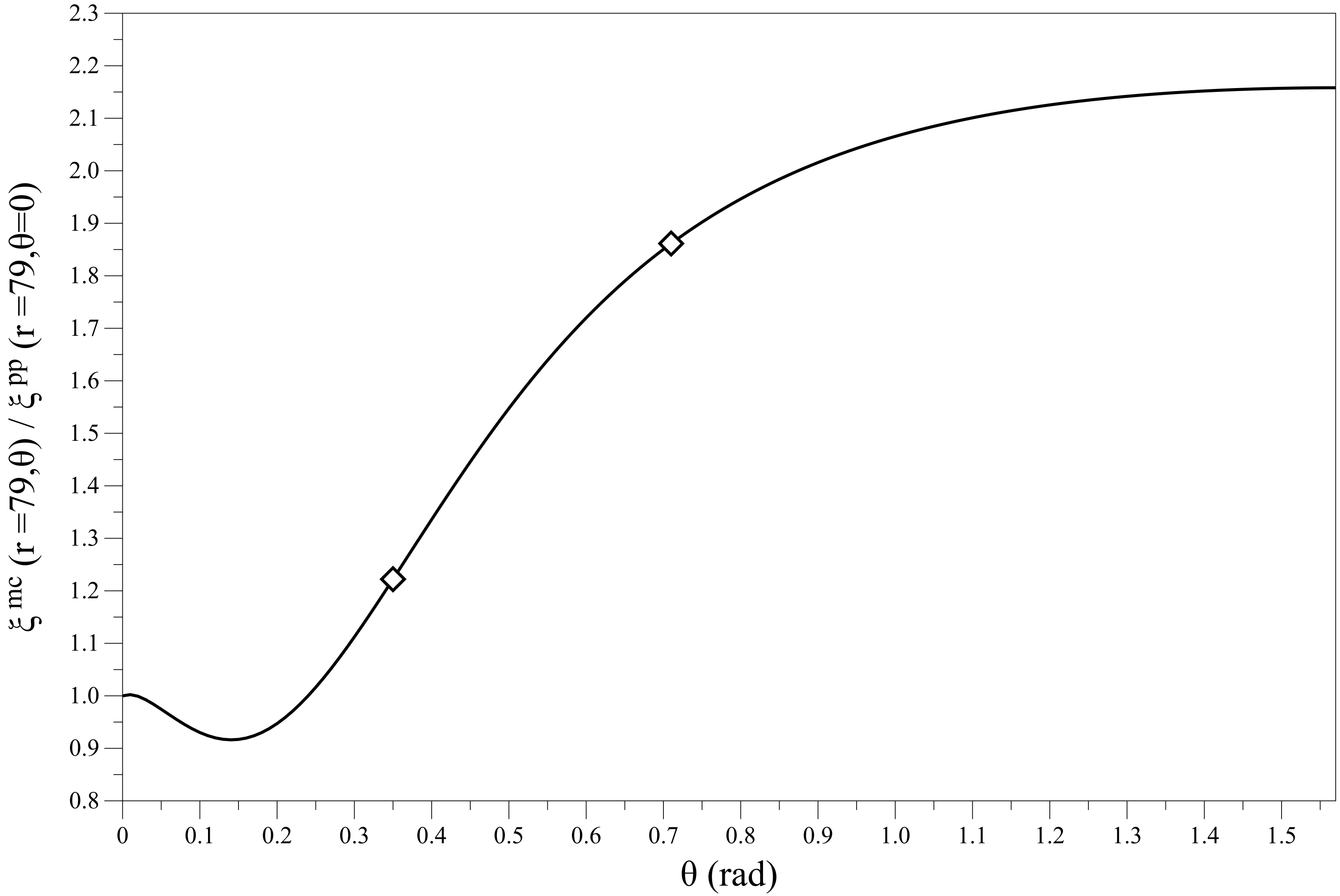
Testing wide angle RSD: results



Testing wide angle RSD: results



The importance of wide-angle RSD



Conclusions

- Linear RSD are now well understood and can easily be modeled
- They measure structure growth in a way that is fundamentally independent of galaxy overdensity bias
- For wide-angles we need to include geometrical and effects from the mode-coupling term in the Jacobian
- Current methods using tripolar spherical harmonics expansion of RSD correctly model both effects
- These effects are important for galaxies with relatively narrow separations