Cosmological density peaks and the scale-dependence of bias

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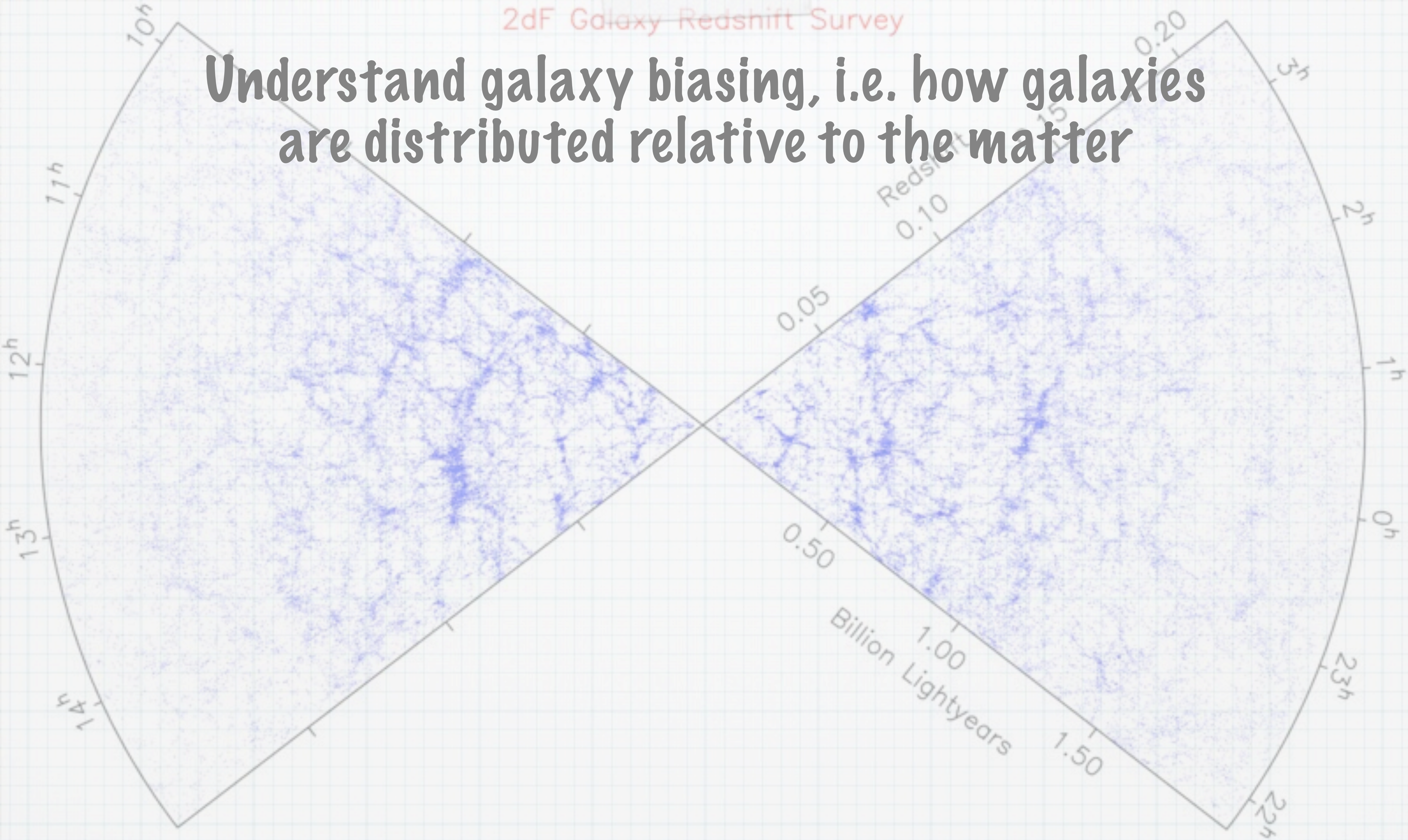
Motivation



Understand galaxy biasing, i.e. how galaxies are distributed relative to the matter

Outline

* Local bias vs. density peaks
* Clustering of density peaks in a Gaussian random field
* Bias and the baryon acoustic oscillation (BAO)

Characterizing galaxy clustering

* The full hierarchy of (connected) N-point correlations describes the spatial distribution of galaxies (Peebles 1973,...)

$$\xi_g(\mathbf{x}_1, \cdots, \mathbf{x}_N)$$

* In the large scale limit, the 2-point correlation takes the simple form (Kaiser 1984, but cf. Dragan Huterer's talk)

$$\xi_g(r) = b_I^2 \xi(r)$$

* Topological measures provide complementary information (won't be discussed here though)

Local bias model

* Essentially all models of halo biasing are based on the local bias model (Kaiser 1984; Szalay 1988; Fry & Gaztanaga 1993)

$$\delta_{hR}(M, \mathbf{x}) = \sum_N \frac{b_N(M)}{N!} [\delta_R(\mathbf{x})]^N$$

* The bias parameters are derived using the peak-background split argument (Bardeen et al. 1986; Cole & Kaiser 1989; Sheth & Tormen 1999)

$$b_N(M) = \left(-\frac{1}{\sigma_0}\right)^N \bar{n}^{-1} \frac{\partial^N [\bar{n}(\nu)]}{\partial \nu^N}, \quad \nu \equiv \delta_c(z_0)/\sigma_0(M)$$

* Halo velocities are unbiased

$$\mathbf{v}_{hR}(M, \mathbf{x}) = \mathbf{v}_R(\mathbf{x})$$

Some issues

* Eulerian or Lagrangian biasing ?
* Which value of the smoothing radius R shall we use ?
* How does discreteness affects clustering ?

Another approach: the peak model

(Peacock & Heavens 1985; Hoffman & Shaham 1985; Bardeen, Bond, Kaiser, Szalay 1986; Coles 1989; Lumsden, Heavens & Peacock 1989,...)

* DM haloes are local density maxima of the evolved mass distribution -> include the peak constraint
* Since it is difficult to work out the properties of density peaks in a highly non-Gaussian field, consider instead the clustering of <u>local maxima of the initial Gaussian density field</u>
* Well-behaved point process which can account for the discrete nature of DM haloes

Peak correlation functions

* Use the peak constraint to write the peak number density as (Kac 1943; Rice 1951; BBKS)

$$n_{\rm pk}(\nu', M, \mathbf{x}) = \sum_{\mathbf{x}_{\rm pk}} \delta^{(3)}(\mathbf{x}-\mathbf{x}_{\rm pk}) = \frac{3^{3/2}}{R_1^3}|{\rm det}\zeta(\mathbf{x})|\,\delta^{(3)}[\boldsymbol{\eta}(\mathbf{x})]\,\theta(\lambda_3)\,\theta(\nu'-\nu)$$

$$\nu(\mathbf{x}) \equiv \delta_M(\mathbf{x})/\sigma_0,\ \ \eta_i(\mathbf{x}) \equiv \partial_i\delta_M(\mathbf{x})/\sigma_1,\ \ \zeta_{ij}(\mathbf{x}) \equiv \partial_i\partial_j\delta_M(\mathbf{x})/\sigma_2$$

$$\zeta \equiv -{\rm O}\Lambda{\rm O}^\top,\ \ \Lambda = {\rm diag}(\lambda_1,\lambda_2,\lambda_3),\ \ \lambda_1 \geq \lambda_2 \geq \lambda_3$$

$$R_1 \equiv \sqrt{3}\frac{\sigma_1}{\sigma_2}$$

$$\sigma_n^2(M, z_0) \equiv \frac{1}{2\pi^2}\int_0^\infty dk\, k^{2(n+1)} P_\delta(k, z_0)[W_M(k)]^2$$

* Calculate the ensemble averages (BBKS; Regos & Szalay 1995; Matsubara 1999; Matsubara 2003; Desjacques 2008; Desjacques & Sheth 2010)

$$\langle n_{\rm pk}(\nu', \mathbf{x}_1)\cdots n_{\rm pk}(\nu', \mathbf{x}_N)\rangle$$

Number density

* The number density of peaks of height ν is (BBKS)

$$\bar{n}_{\rm pk}(\nu, M) \equiv \langle n_{\rm pk}(\nu, M, \mathbf{x})\rangle = \frac{1}{(2\pi)^2 R_1^3} e^{-\nu^2/2}\, G_0^{(0)}(\gamma_1, \gamma_1\nu)$$

$$\gamma_1 \equiv \frac{\sigma_1^2}{\sigma_0\sigma_2}, \quad 0 < \gamma_1 < 1$$

$$G_n^{(\alpha)}(\gamma_1, \omega) \equiv \int_0^\infty du\, u^n f(u, \alpha) \frac{e^{-(u-\omega)^2/2(1-\gamma_1^2)}}{\sqrt{2\pi(1-\gamma_1^2)}}$$

Peaks vs. Excursion set Theory

Spherical collapse	$\sqrt{\frac{2}{\pi}}\nu\, e^{-\nu^2/2}$	Press & Schechter Bond et. al.
Ellipsoidal collapse	$A\sqrt{\frac{2}{\pi}}\sqrt{a}\nu\, e^{-a\nu^2/2}\left[1+(a\nu^2)^q\right]$	Sheth & Tormen
Non-Markovian (+stochastic barrier)	$\sqrt{\frac{2}{\pi}}\left[(1-a\kappa)\sqrt{a}\nu\, e^{-a\nu^2/2}+a^{3/2}\kappa\frac{\nu}{2}\Gamma\left(0,\frac{a\nu^2}{2}\right)\right]$	Maggiore & Riotto
Peaks (ignoring cloud-in-cloud)	$G_0^{(0)}(\gamma_1,\gamma_1\nu)\, e^{-\nu^2/2}$	BBKS

(cf. Michele Maggiore's talk)

Peak biasing

* At the first order (i.e. large scale), the 2-point correlation of peaks can be thought of as arising from the biasing relation (Desjacques 2008)

$$\delta n_{\rm pk}(\nu, M, \mathbf{x}) = (\hat{b}_I \delta_M)(\mathbf{x}) \equiv b_\nu \delta_M(\mathbf{x}) - b_\zeta \partial^2 \delta_M(\mathbf{x})$$

$$b_\nu(\nu, M) = \frac{1}{\sigma_0}\left(\frac{\nu - \gamma_1 \bar{u}}{1-\gamma_1^2}\right), \quad b_\zeta(\nu, M) = \frac{1}{\sigma_2}\left(\frac{\bar{u} - \gamma_1 \nu}{1-\gamma_1^2}\right)$$

* In Fourier space,

$$\delta n_{\rm pk}(\nu, M, \mathbf{k}) = \hat{b}_I(k)\delta_M(\mathbf{k}), \quad \hat{b}_I(k) \equiv \left(b_\nu + b_\zeta k^2\right)$$

* Compare with local $f_{\rm NL}$ primordial NG (Dalal et al. 2008):

$$\hat{b}_{\rm NG}(k) = \left(b_\nu + f_{\rm NL}\frac{b_\phi}{k^2}\right)$$

2-point peak correlation

Work in Progress with Martin Crocce, Roman Scoccimarro, Ravi Sheth

* Up to second order, this is

$$
\begin{aligned}
\xi_{\rm pk}(\nu, M, r) = & \, (\hat{b}_I^2 \xi_0^{(0)})(r) + \frac{1}{2}(\xi_0^{(0)} \hat{b}_{II}^2 \xi_0^{(0)})(r) \\
& - \frac{3}{\sigma_1^2}(\xi_1^{(1/2)} \hat{b}_{II} \xi_1^{(1/2)})(r) - \frac{5}{\sigma_2^2}(\xi_2^{(1)} \hat{b}_{II} \xi_2^{(1)})(r) \left(1 + \frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1, \gamma_1 \nu)\Big|_{\alpha=1}\right) \\
& + \frac{5}{2\sigma_2^4}\left[(\xi_0^{(0)})^2 + \frac{10}{7}(\xi_2^{(2)})^2 + \frac{18}{7}(\xi_4^{(2)})^2\right]\left(1 + \frac{2}{5}\partial_\alpha \ln G_0^{(\alpha)}(\gamma_1, \gamma_1 \nu)\Big|_{\alpha=1}\right)^2 \\
& + \frac{3}{2\sigma_1^4}\left[(\xi_0^{(1)})^2 + 2(\xi_2^{(1)})^2\right] + \frac{3}{\sigma_1^2 \sigma_2^2}\left[3(\xi_3^{(3/2)})^2 + 2(\xi_1^{(3/2)})^2\right]
\end{aligned}
$$

where $$\xi_\ell^{(n)}(r) \equiv \frac{1}{2\pi^2}\int_0^\infty dk\, k^{2(n+1)} P_\delta(k, z_0) j_\ell(kr) [W_M(k)]^2$$

* In Fourier space, the 2nd order bias is

$$\hat{b}_{II}(q_1, q_2) = b_{\nu\nu} + b_{\nu\zeta}(q_1^2 + q_2^2) + b_{\zeta\zeta} q_1^2 q_2^2$$

Peak-background split

* b_ν and $b_{\nu\nu}$ are exactly the same as the first- and second-order biases returned by a peak-background split

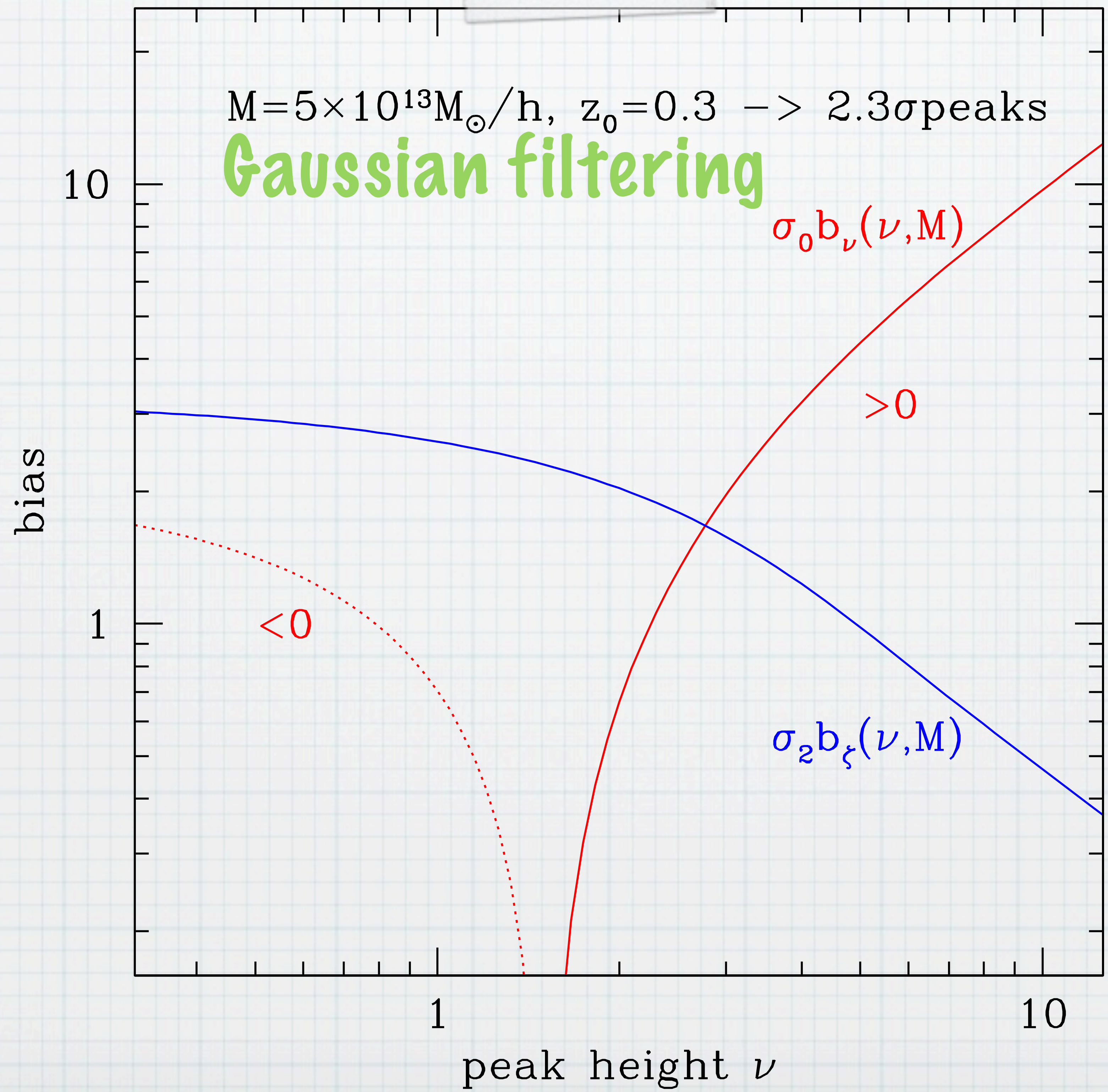
$$b_\nu(\nu, M) = -\frac{1}{\sigma_0}\frac{\partial \ln \bar{n}_{\rm pk}(\nu, M)}{\partial \nu} \equiv b_I(\nu, M)$$

$$b_{\nu\nu}(\nu, M) = \frac{1}{\sigma_0^2}\bar{n}_{\rm pk}^{-1}\frac{\partial^2 \bar{n}_{\rm pk}(\nu, M)}{\partial \nu^2} \equiv b_{II}(\nu, M)$$

* So, the peak correlation can also be written

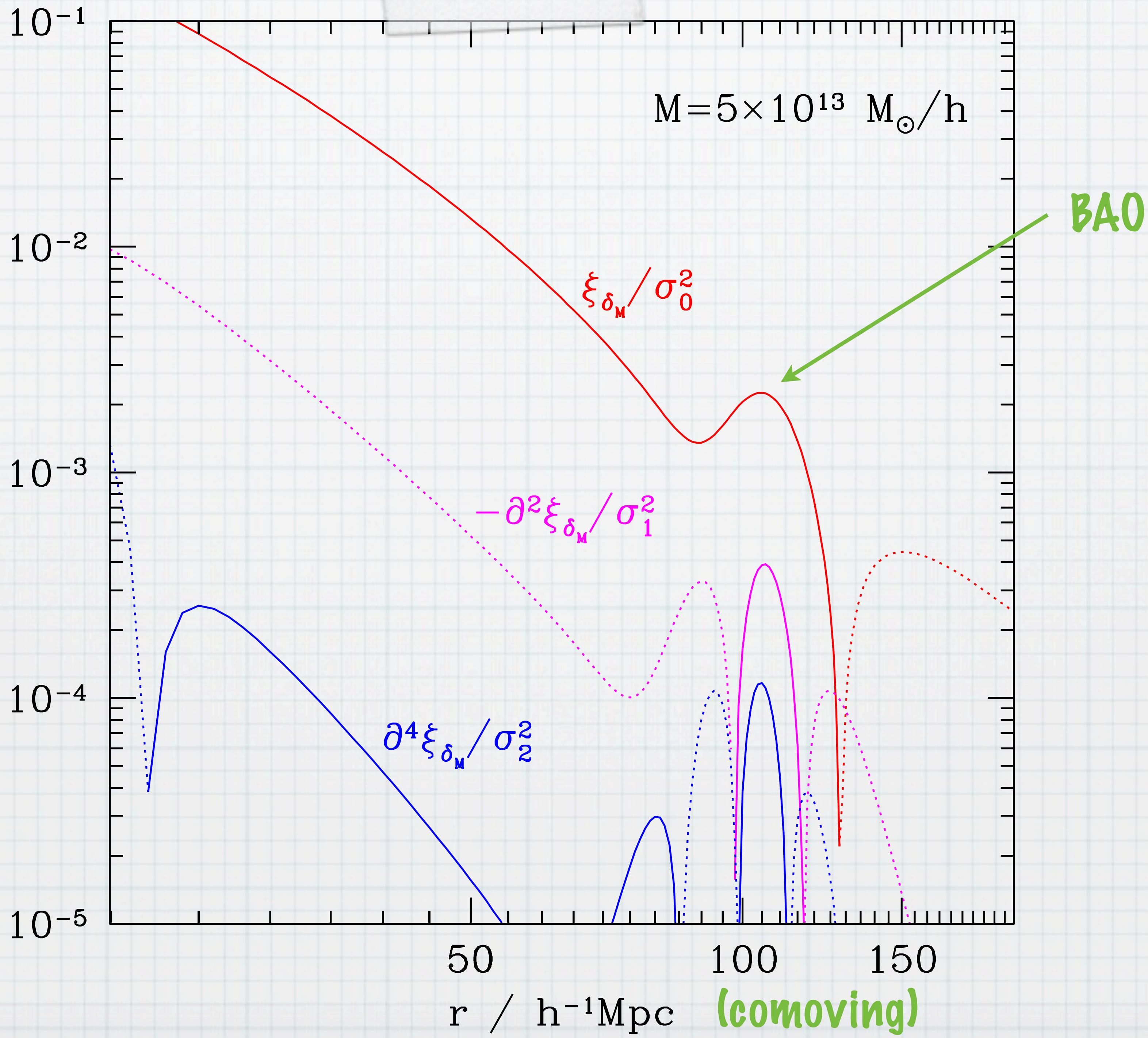
$$\xi_{\rm pk}(\nu, r) = b_I^2 \xi_0^{(0)}(r) + \frac{1}{2} b_{II}^2 \left[\xi_0^{(0)}(r)\right]^2 + \text{other terms}$$

First order bias parameters



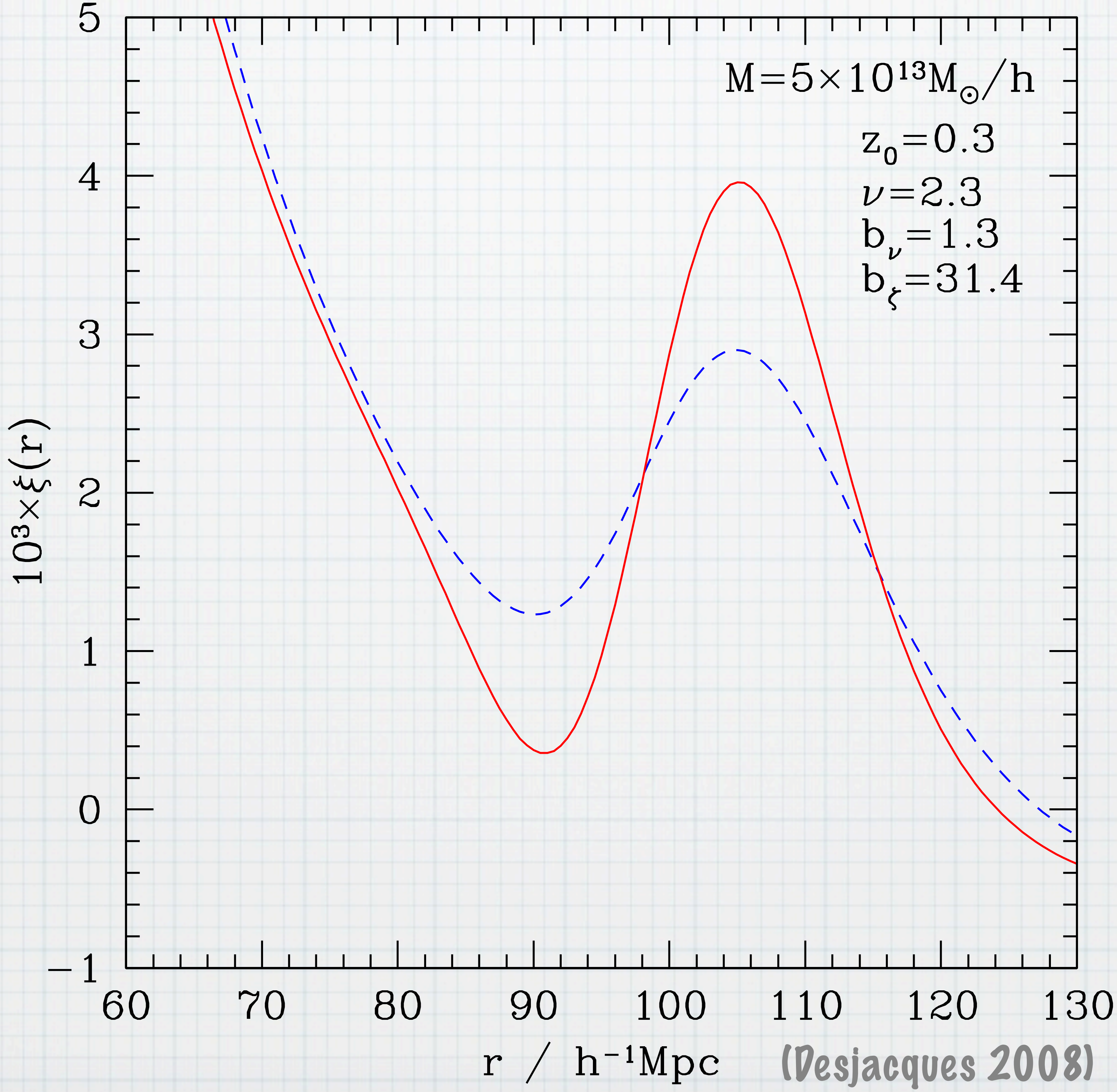
Large scale correlation in CDM models

$$\xi_{\rm pk}(\nu, r) = b_\nu^2 \xi_{\delta_M}(r) - 2b_\nu b_\zeta \partial^2 \xi_{\delta_M}(r) + b_\zeta^2 \partial^4 \xi_{\delta_M}(r), \quad \xi_{\delta_M} \equiv \xi_0^{(0)}$$

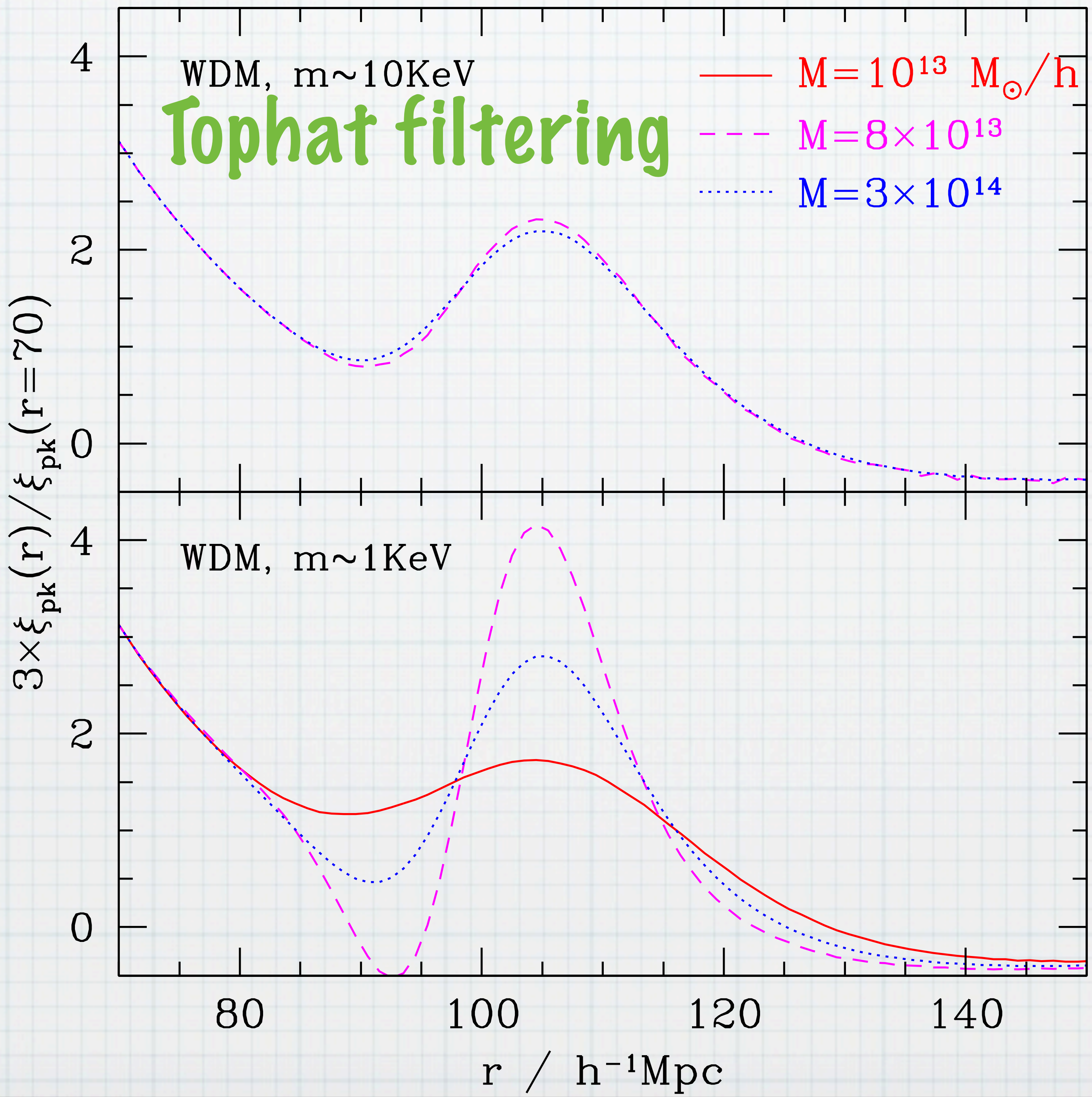


Local bias : $b_\nu^2 \xi_\delta$

Peaks : $b_\nu^2 \xi_{\delta_M} - 2 b_\nu b_\zeta \partial^2 \xi_{\delta_M} + b_\zeta^2 \partial^4 \xi_{\delta_M}$



WDM transfer function



Gravitational evolution

* In a first approximation, the initial density peaks move along straight lines (Zel'dovich 1970)

$$\mathbf{x}_{\rm pk}(z) = \mathbf{q}_{\rm pk} - D(z)\nabla\Phi(\mathbf{q}_{\rm pk})$$

* The peak correlation function can be formally written (Bharadwaj 1996)

$$\begin{aligned}\bar{n}_{\rm pk}^2\left[1+\xi_{\rm pk}(\nu,M,r,z)\right] &= \int d^3\mathbf{v}_1 d^3\mathbf{v}_2\, P_2(\mathbf{v}_1,\mathbf{v}_2;r,z|{\rm pk}) \\ &= \int d^3\mathbf{r}'\int d^3\mathbf{v}_1 d^3\mathbf{v}_2\,\delta^{(3)}[\mathbf{r}'-\mathbf{r}+\Delta\mathbf{v}_{12}]\; P_2(\mathbf{v}_1,\mathbf{v}_2;\mathbf{r}',z_i|{\rm pk})\end{aligned}$$

Gravitational evolution (II)

* The peak power spectrum as a function of redshift is

$$P_{\rm pk}(\nu, M, k, z) = G^2(k,z)\left[\hat{b}_{\rm vel}(k) + \frac{D(z_0)}{D(z)}\hat{b}_I(k,z_0)\right]^2 P_{\delta_M}(k,z_0) + P_{\rm MC}(\nu, M, k, z)$$

$$P_{\delta_M}(k,z_0) \equiv P_\delta(k,z_0)[W_M(k)]^2$$

$$G^2(k,z) \equiv \left(\frac{D(z)}{D(z_0)}\right)^2 \exp\left(-\frac{1}{3}k^2\sigma^2_{\rm vpk}(z)\right)$$

* Note the similarity with Renormalized Perturbation Theory (RPT, Crocce & Scoccimarro 2006)

$$P_\delta(k,z) = G^2_\delta(k,z)P_\delta(k,z_0) + P_{\rm MC}(k,z)$$

$$G^2_\delta(k,z) \equiv \left(\frac{D(z)}{D(z_0)}\right)^2 \exp\left(-\frac{1}{3}k^2\sigma^2_v(z)\right)$$

Velocity bias

Assumption: DM haloes locally move with the dark matter flows. This implies

- Local bias: $\sigma_h^2 = \sigma_v^2$

  $$\theta_h(\mathbf{k}) = \theta(\mathbf{k}), \quad (\theta \equiv \nabla \cdot \mathbf{v})$$

- Peaks: $\sigma_{\rm pk}^2 = \sigma_v^2 \left(1 - \gamma_0^2\right), \quad 0 < \gamma_0 < 1$ (BBKS)

  $$\theta_{\rm pk}(\mathbf{k}) = \left(1 - \frac{\sigma_0^2}{\sigma_1^2} k^2\right) \theta(\mathbf{k}) \equiv \hat{b}_{\rm vel}(k)\theta(\mathbf{k})$$ (Desjacques & Sheth)

This velocity bias is <u>statistical</u> (as opposed to <u>physical</u>)

Redshift distortions

* For a local bias model with <u>unbiased</u> velocities (Kaiser 1987)

$$P_h^s(k,\mu) = \left(b_I + f\mu^2\right)^2 P_\delta(k,\mu)$$

* In the peak model, the Kaiser expression becomes (Desjacques & Sheth 2010)

$$P_{\rm pk}^s(k,\mu) = \left[\hat{b}_I(k) + f\hat{b}_{\rm vel}(k)\mu^2\right]^2 P_\delta(k,\mu)$$

* Standard manipulations applied to the peak model would lead to k-dependent estimates of the growth rate f

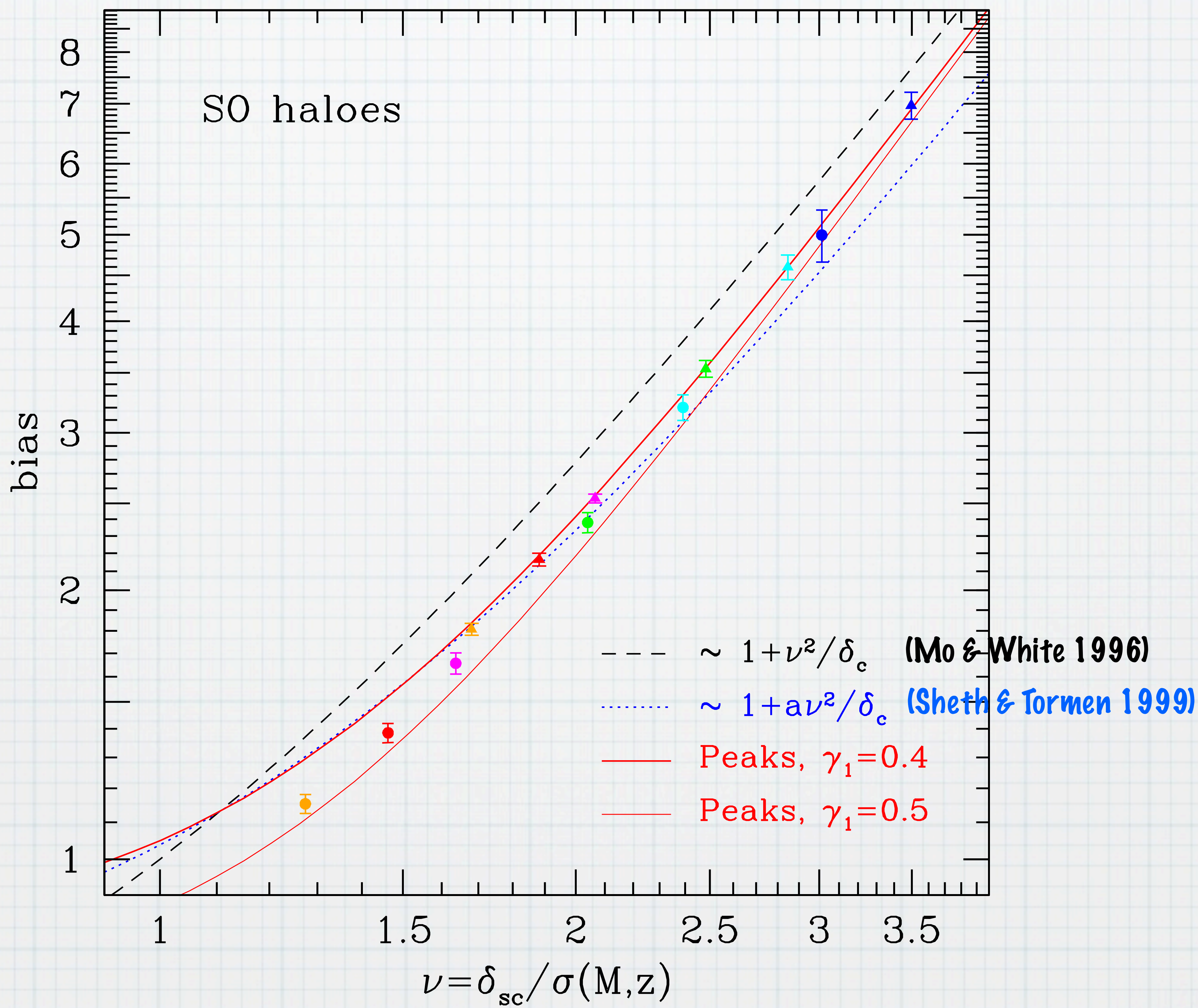
Lagrangian vs. Eulerian bias

* The Eulerian and Lagrangian first order bias parameters are related according to

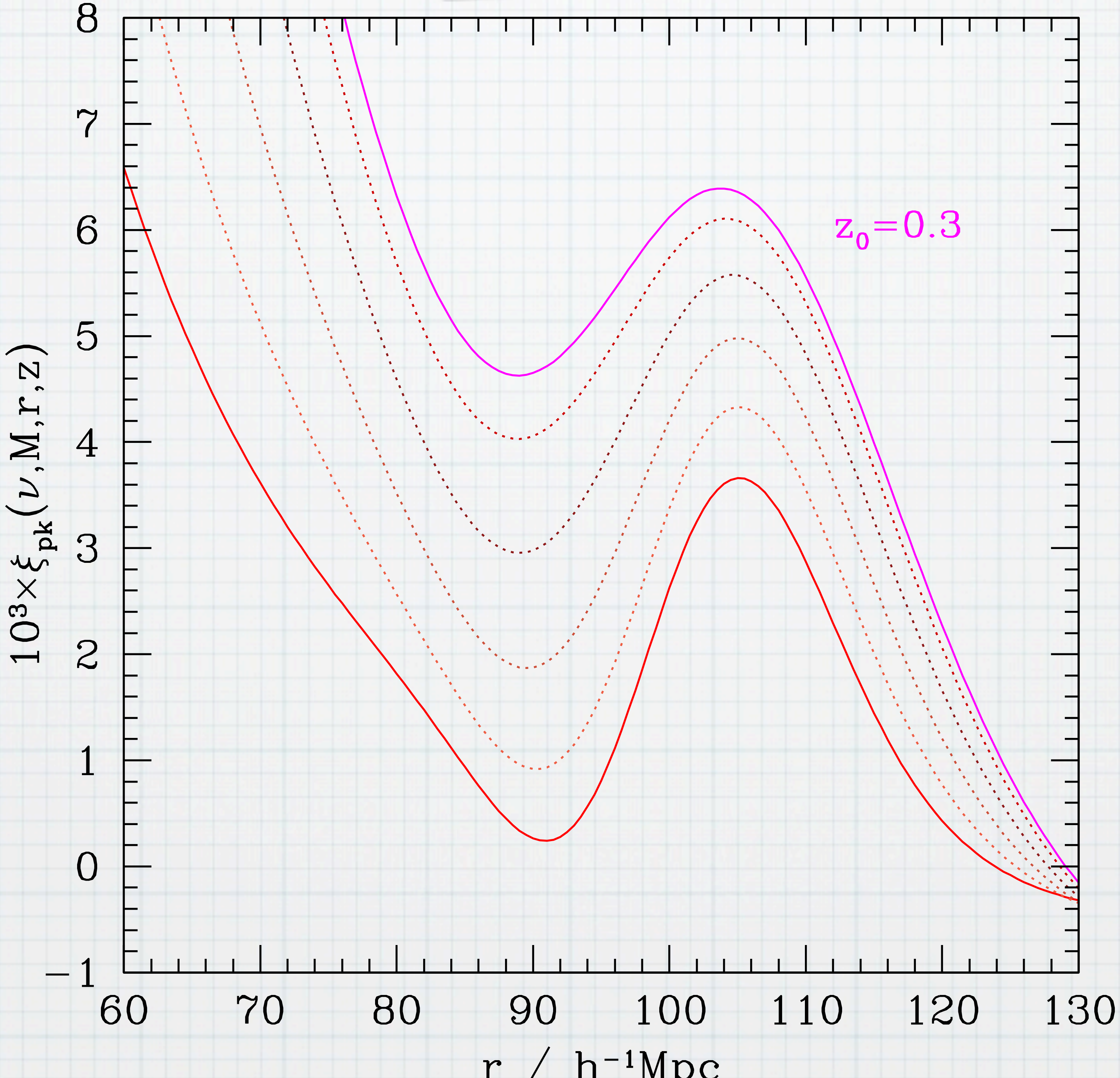
$$\hat{b}_I^{\mathrm{E}}(k,z) \equiv \hat{b}_{\mathrm{vel}}(k) + \frac{D(z_0)}{D(z)}\hat{b}_I(k,z_0)$$

or

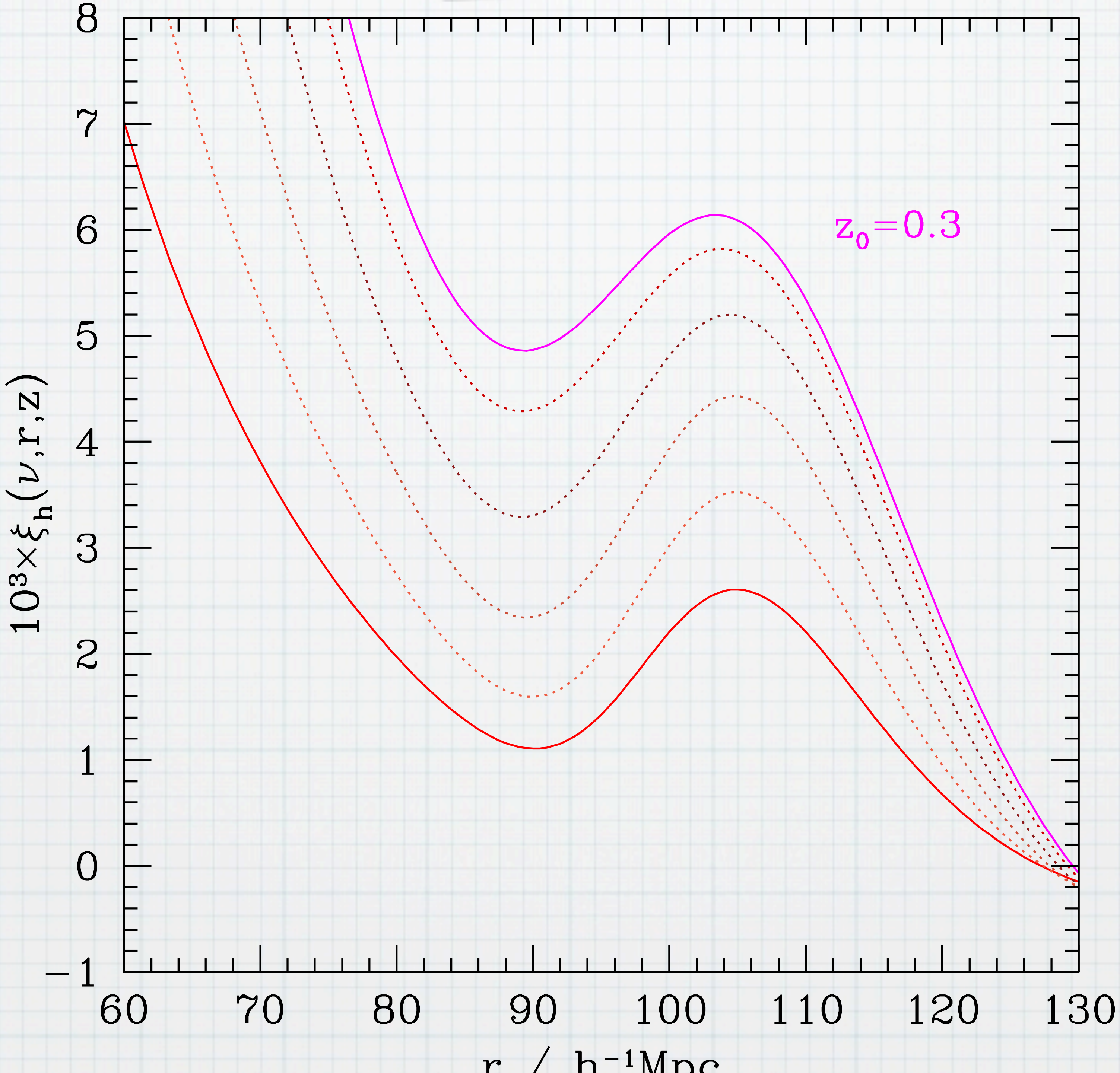
$$b_\nu^{\mathrm{E}}(z) \equiv 1 + \frac{D(z_0)}{D(z)}b_\nu(z_0), \quad b_\zeta^{\mathrm{E}}(z) \equiv \frac{D(z_0)}{D(z)}b_\zeta(z_0) - \frac{\sigma_0^2}{\sigma_1^2}$$


SO haloes
bias
8
7
6
5
4
3
2
1
1
1.5
2
2.5
3
3.5
$\nu=\delta_{sc}/\sigma(M,z)$
$\sim 1+\nu^2/\delta_c$ (Mo & White 1996)
$\sim 1+a\nu^2/\delta_c$ (Sheth & Tormen 1999)
Peaks, $\gamma_1=0.4$
Peaks, $\gamma_1=0.5$

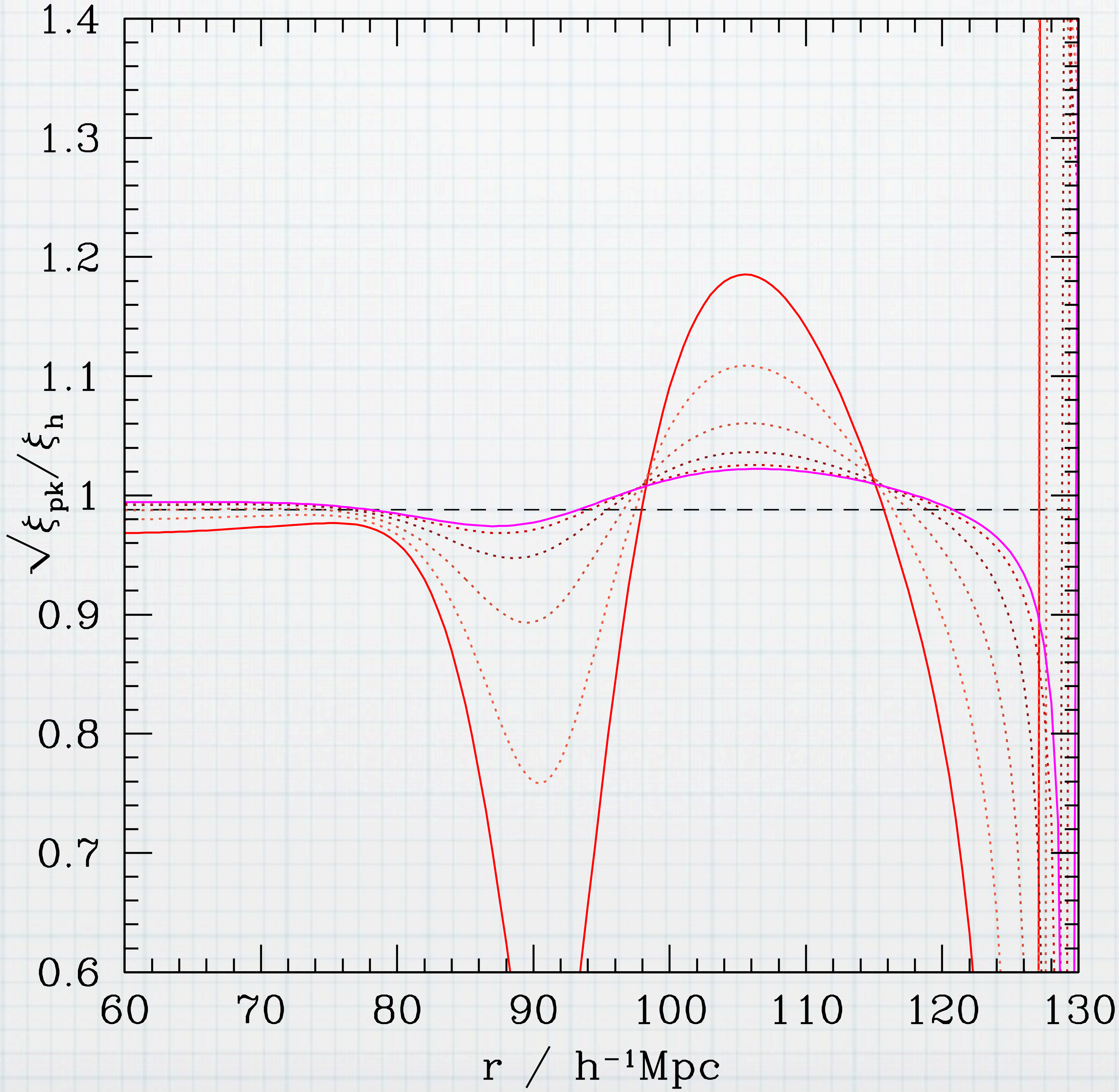
BAO feature: peaks



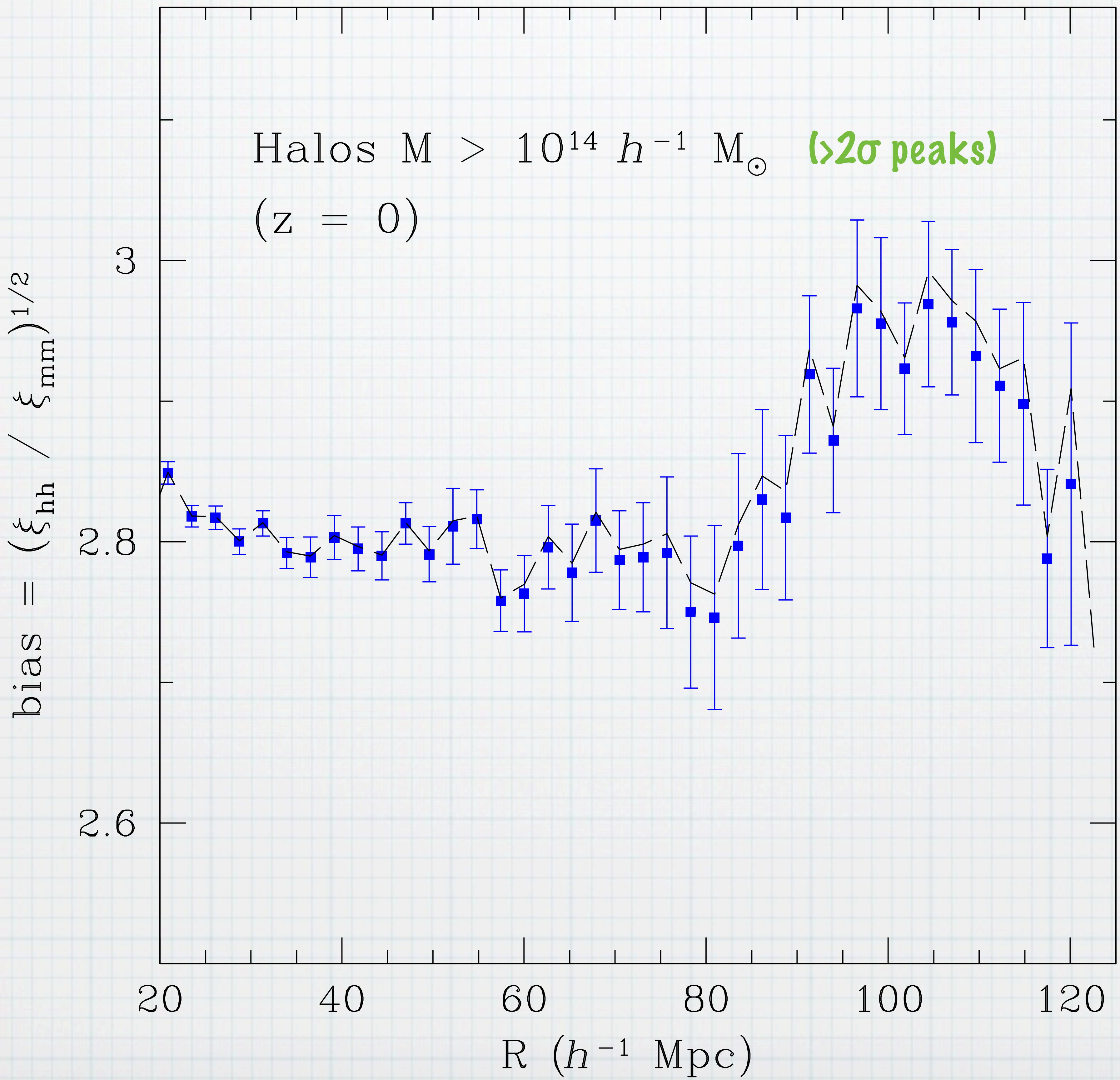
BAO feature: local bias



Scale-dependence across BAO



MICE project, 450 (Gpc/h)^3 simulation



In summary

* The peak model is an extension of the local bias model
* The spatial bias parameters are k-dependent; This generates a few percent residual scale-dependence across the BAO feature
* The peak velocities are statistically biased; The Kaiser formula acquires a velocity bias factor
* Large numerical simulations should be able to test the predictions of this model
* Are massive haloes related to local maxima of the initial density field ? cf. Cris Porciani's talk...