Toward a model for the galaxy bispectrum with generic initial conditions

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Primordial non-Gaussianity and the **Large-Scale Structure**

We expect effects

- on the **matter higher-order correlation functions**

  $\langle\phi\phi\phi\rangle \Rightarrow \langle\delta\delta\delta\rangle$

- on the **cluster abundance**

  $\langle\phi\phi\phi\rangle \Rightarrow s_3 \sim \langle\delta^3\rangle \Rightarrow n(M)$

- on the **halo and galaxy bias relation**

  $\langle\phi\phi\phi\rangle \Rightarrow [...] \Rightarrow b_{eff}(k, f_{NL})$, for *local* NG

The **galaxy bispectrum** is affected by all these effects (this is good and bad)

The problem is to **separate these effects from other sources of non-Gaussianity** ...

The **matter** bispectrum: Perturbation Theory

A solution for the matter density in Fourier space

$$\left.\begin{array}{l} \textit{Continutity eq.} \\ \textit{Euler eq.} \\ \textit{Poisson eq.} \end{array}\right\} \Rightarrow \quad \delta(\mathbf{k}) \stackrel{\rm PT}{\simeq} \delta^{(1)}(\mathbf{k}) + \delta^{(2)}(\mathbf{k}) + \delta^{(3)}(\mathbf{k}) + \ldots$$

with

$$\begin{aligned} \delta^{(1)}(\mathbf{k}) &= \delta_L(\mathbf{k}) \quad \textit{linear} \text{ matter density} \\ \delta^{(2)}(\mathbf{k}) &= \int dq^3 F_2(\mathbf{k}-\mathbf{q},\mathbf{q})\delta_L(\mathbf{k}-\mathbf{q})\delta_L(\mathbf{q}) \end{aligned}$$

$+$ the initial conditions, *i.e.* the *initial correlators*

$$\begin{aligned} \langle \delta^{(1)}_{\mathbf{k}_1}\delta^{(1)}_{\mathbf{k}_2}\rangle &\equiv \delta_D(\mathbf{k}_1+\mathbf{k}_2)P_L(k_1) \\ \langle \delta^{(1)}_{\mathbf{k}_1}\delta^{(1)}_{\mathbf{k}_2}\delta^{(1)}_{\mathbf{k}_3}\rangle &\equiv \delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_I(k_1,k_2,k_3) \quad \Leftarrow \quad N.G.\ \ I.C. \\ \langle \delta^{(1)}_{\mathbf{k}_1}\delta^{(1)}_{\mathbf{k}_2}\delta^{(1)}_{\mathbf{k}_3}\delta^{(1)}_{\mathbf{k}_4}\rangle_c &\equiv \delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T_I(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) \quad \Leftarrow \quad N.G.\ \ I.C. \end{aligned}$$

$\Rightarrow$ a *perturbative* expression for the 3-point function

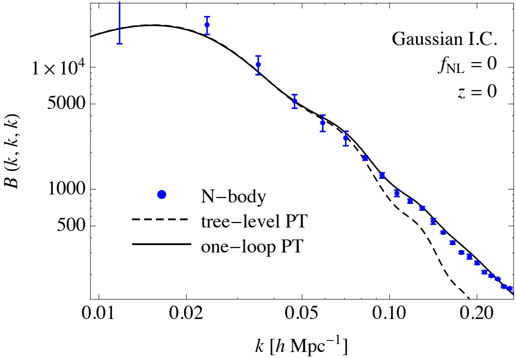
$$\begin{array}{ccccccc} \langle\delta\delta\delta\rangle & \stackrel{\rm PT}{\simeq} & \langle\delta^{(1)}\delta^{(1)}\delta^{(1)}\rangle & + & \langle\delta^{(1)}\delta^{(1)}\delta^{(2)}\rangle & + & \langle\delta^{(1)}\delta^{(2)}\delta^{(2)}\rangle + \langle\delta^{(1)}\delta^{(1)}\delta^{(3)}\rangle + \ldots \\ B(k_1,k_2,k_3) & \simeq & B_I(k_1,k_2,k_3) & + & B_G^{tree}(k_1,k_2,k_3) & + & \text{loop corrections} \end{array}$$

The *component* **induced by gravity**,

$$B_G^{tree}(k_1,k_2,k_3)=2F_2(\mathbf{k}_1,\mathbf{k}_2)P_L(k_1)P_L(k_2)+\mathrm{perm.}$$

is present even at large scales

... with a well defined **dependence on scale** $\Rightarrow$



[ES, Crocce, Desjacques (2010)]

The **matter** bispectrum: **Gaussian** initial conditions

The **shape dependence** of gravity-induced non-Gaussianity

The *component* **induced by gravity**,

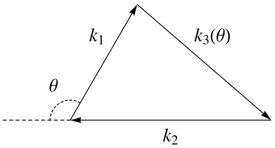
$$B_G^{tree}(k_1,k_2,k_3) = 2F_2(\mathbf{k}_1,\mathbf{k}_2)P_L(k_1)P_L(k_2) + \mathrm{perm.}$$

is present even at large scales

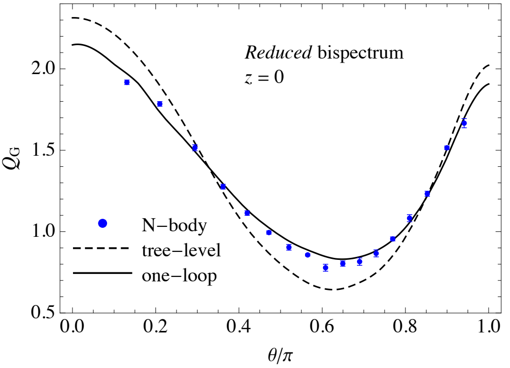
... and with a well defined **dependence on shape** $\Rightarrow$

Reduced bispectrum:

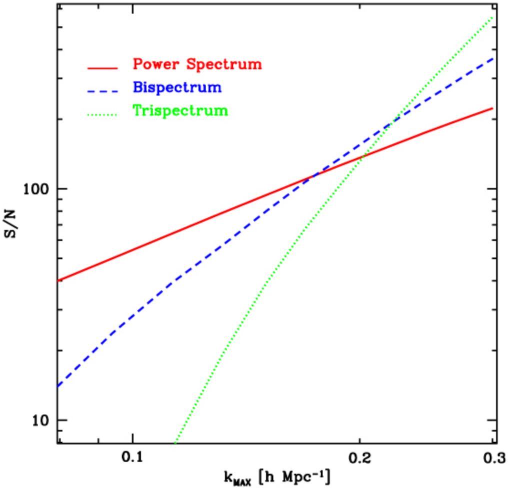
$$Q \equiv \frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_3)}$$



$k_1 = 0.1\,h\,\mathrm{Mpc}^{-1}$ and $k_2 = 1.5k_1$



[ES, Crocce, Desjacques (2010)]



Ideal Geometry
$V = 0.3\, h^{-3}\, \mathrm{Gpc}^3$
$\bar{n} = 0.003(\, h^{-1}\, \mathrm{Mpc})^{-3}$

$$\left(\frac{S}{N}\right)^2_P = \sum_k^{k_{\rm max}} \frac{P^2}{\Delta P^2}$$

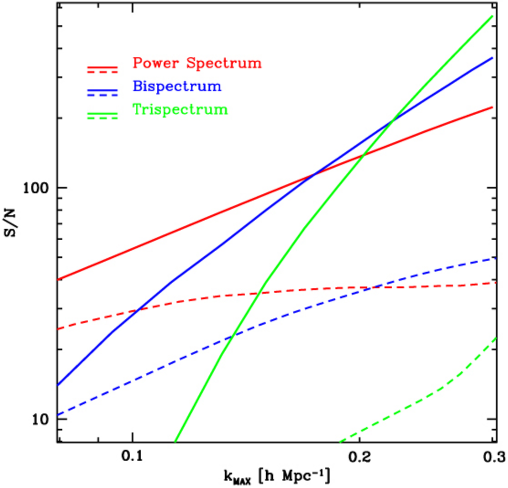
$$\left(\frac{S}{N}\right)^2_B = \sum_{\rm triangles}^{k_{\rm max}} \frac{B^2}{\Delta B^2}$$

$$\left(\frac{S}{N}\right)^2_{\widetilde{T}} = \sum_{\rm quads}^{k_{\rm max}} \frac{\widetilde{T}^2}{\Delta \widetilde{T}^2}$$

Sums over *all configurations* up to $k_{\rm max}$

[ES & Scoccimarro (2005)]

The **matter** bispectrum: Cumulative Signal-to-Noise



Ideal Geometry *vs.* SDSS

$$\left(\frac{S}{N}\right)^2_P = \sum_{k_i,k_j}^{k_{\rm max}} P_i (C^P_{ij})^{-1} P_j$$

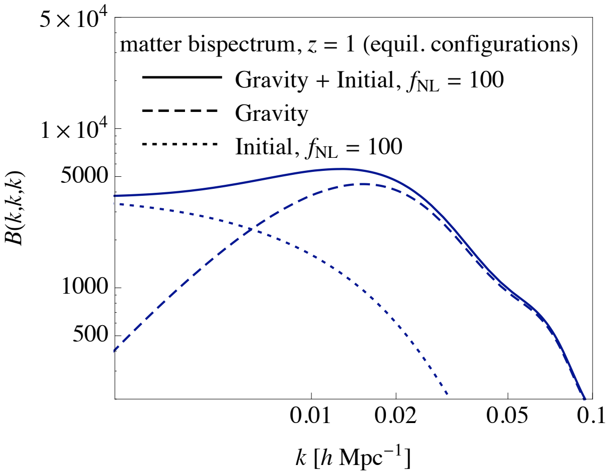
$$\left(\frac{S}{N}\right)^2_B = \sum_{\rm triangles\ i,j}^{k_{\rm max}} B_i (C^B_{ij})^{-1} B_j$$

$$\left(\frac{S}{N}\right)^2_{\widetilde{T}} = \sum_{\rm quads\ i,j}^{k_{\rm max}} \widetilde{T}_i (C^T_{ij})^{-1} \widetilde{T}_j$$

Sums over *all configurations* up to $k_{\rm max}$

[ES & Scoccimarro (2005)]

At large scales: Gravity + PNG $\Rightarrow$ $B \simeq B_I + B_G^{tree}$



$\Leftarrow$ Equilateral triangles

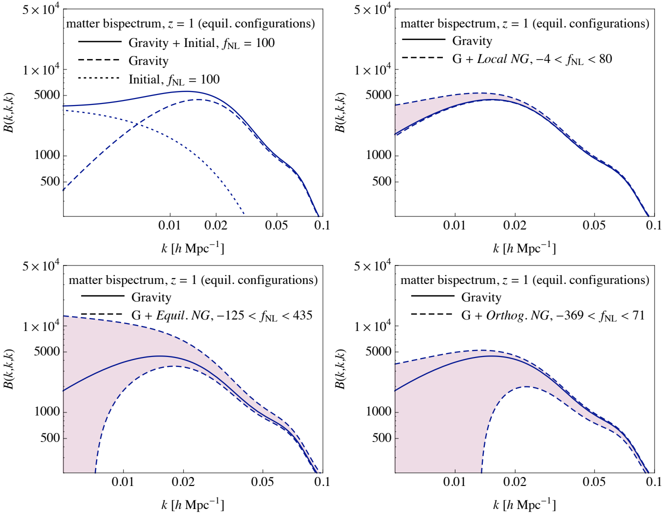
$B(k,k,k)$ *vs.* k

$$\frac{B_I}{B_G} \overset{k\rightarrow 0}{\sim} \frac{f_{NL}}{D(z)k^2}$$

for a broad range of models

The **matter bispectrum** and **primordial non-Gaussianity**

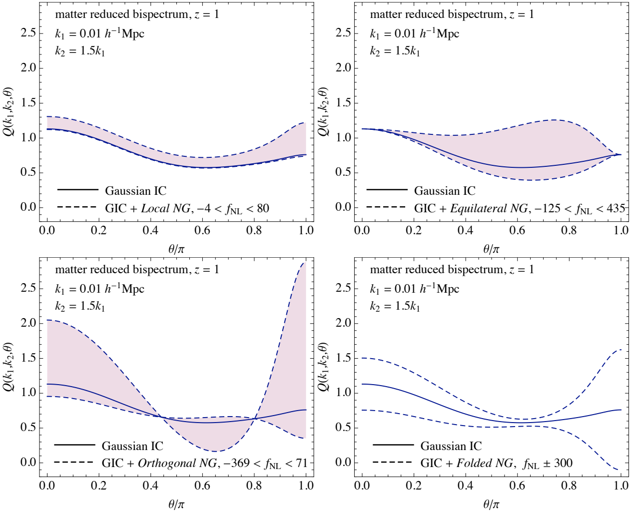
The **scale-dependence** of primordial non-Gaussiani*ties*



[Liguori, ES, Fergusson & Shellard (review, 2010)]

The **matter bispectrum** and **primordial non-Gaussianity**

The **shape-dependence** of primordial non-Gaussian*ities*



[Liguori, ES, Fergusson & Shellard (review, 2010)]

$$B_m(k_1,k_2,k_3) \stackrel{PT}{=} B_I(k_1,k_2,k_3) + B_G^{tree}(k_1,k_2,k_3) + B_m^{1-loop}(k_1,k_2,k_3) + \ldots$$

The 1-loop corrections are *several*

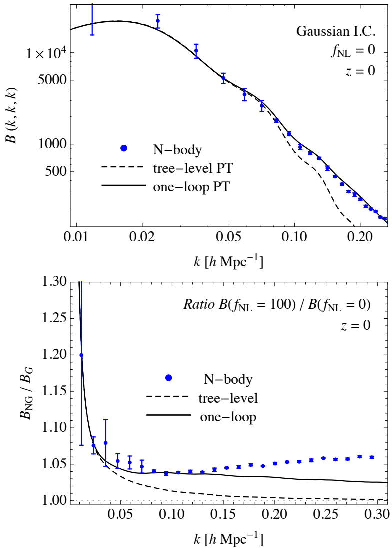
$$B_m^{1-loop} = B_{112}^{II} + B_{122}^{I} + B_{122}^{II} + B_{113}^{I} + B_{113}^{II} + B_{222}^{I} + B_{123}^{I} + B_{123}^{II} + B_{114}^{I},$$

for instance

$$\begin{aligned} B_{112}^{II} &= \int d^3q\ F_2(\mathbf{q},\mathbf{k}_3-\mathbf{q})\ T_0(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{k}_3-\mathbf{q}), \\ B_{122}^{I} &= 2\ P_0(k_1)\ F_2(\mathbf{k}_1,\mathbf{k}_3)\int d^3q\ F_2(\mathbf{q},\mathbf{k}_3-\mathbf{q})\ B_0(k_3,q,|\mathbf{k}_3-\mathbf{q}|) + \mathrm{perm.} \\ &\ldots \\ B_{222}^{I} &= 8\int d^3q F_2(-\mathbf{q},\mathbf{q}+\mathbf{k}_1)F_2(-\mathbf{q}-\mathbf{k}_1,\mathbf{q}-\mathbf{k}_2)F_2(\mathbf{k}_2-\mathbf{q},\mathbf{q})P_0(q)P_0(|\mathbf{k}_1+\mathbf{q}|)P_0(|\mathbf{k}_2-\mathbf{q}|), \\ &\ldots \end{aligned}$$

$\Rightarrow$ Extra sensitivity to B_0 (and a mild one to T_0)

[Scoccimarro (1997); ES (2009)]



⇐ Gaussian I.C.

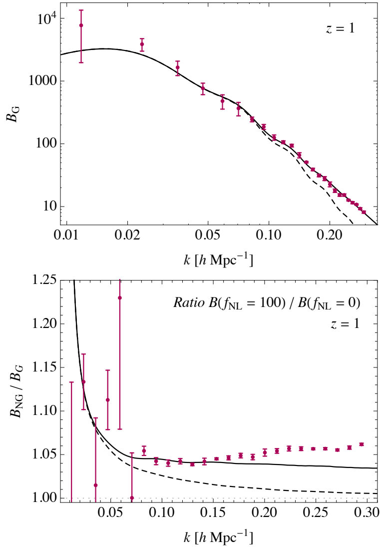
Equilateral configurations:
$B(k,k,k)$ *vs.* k
$z = 0$

⇐ *Local* non-Gaussian I.C.
relative effect

[ES, Crocce & Desjacques (2010)]

The **matter bispectrum** and **primordial non-Gaussianity**

Small scales: *N-body simulations vs. PT*

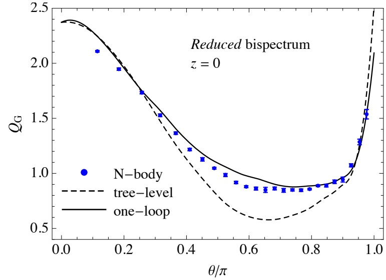


$\Leftarrow$ Gaussian I.C.

Equilateral configurations:
$B(k, k, k)$ *vs.* k
$z = 1$

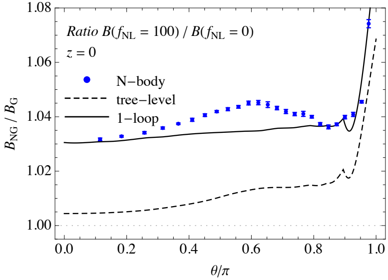
$\Leftarrow$ *Local* non-Gaussian I.C.
relative effect

[ES, Crocce & Desjacques (2010)]



⇐ Gaussian I.C.

Generic configurations:
$Q(\mathbf{k}_1, \mathbf{k}_2, \theta)$ *vs.* k
w/ $k_1 = 0.14\, h\, \mathrm{Mpc}^{-1}$
and $k_2 = 0.15\, h\, \mathrm{Mpc}^{-1}$
$z = 0$



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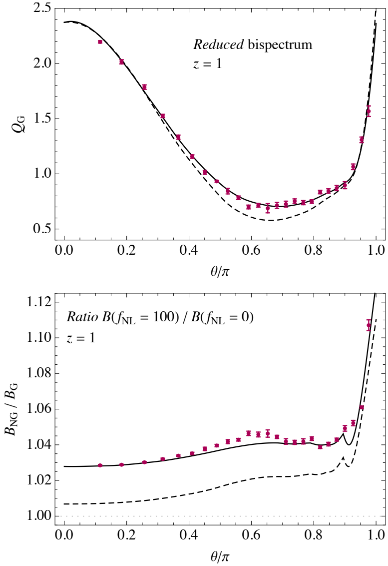
[ES, Crocce & Desjacques (2010)]



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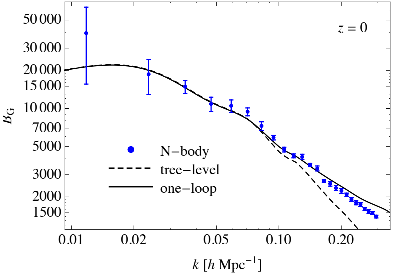
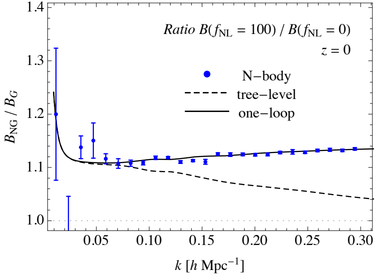


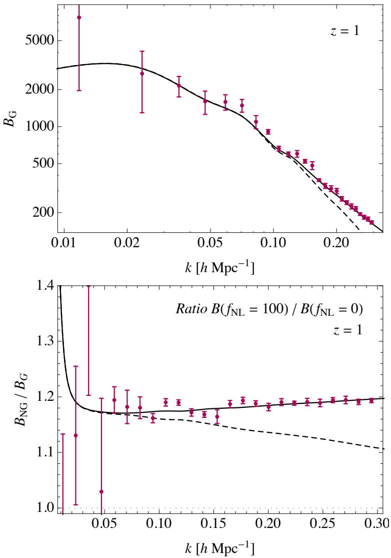
⇐ Gaussian I.C.

Squeezed configurations:
$B(\Delta k, k, k)$ *vs.* k
w/ $\Delta k = 0.01\, h\,\mathrm{Mpc}^{-1}$
$z = 0$



⇐ *Local* non-Gaussian I.C.
relative effect

[ES, Crocce & Desjacques (2010)]



$\Leftarrow$ Gaussian I.C.

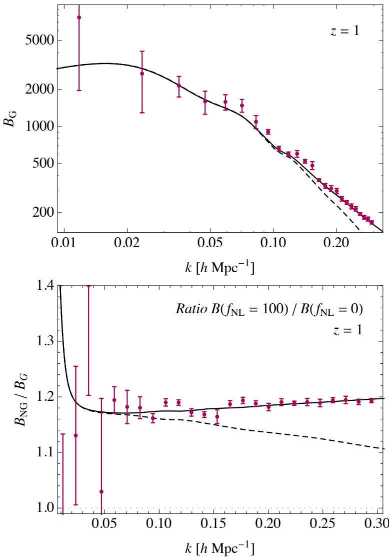
Squeezed configurations:
$B(\Delta k, k, k)$ *vs.* k
w/ $\Delta k = 0.01\, h\,\mathrm{Mpc}^{-1}$
$z = 1$

$\Leftarrow$ *Local* non-Gaussian I.C.
relative effect

[ES, Crocce & Desjacques (2010)]

The **matter bispectrum** and **primordial non-Gaussianity**

Small scales: *N-body simulations vs. PT*



$\Leftarrow$ Gaussian I.C.

- The tree-level approximation breaks-down at relatively large scales
- There is a 5 - 15% effect of non-Gaussian Initial Conditions *for all triangles*, at small scales and at any redshift, for $f_{NL} = 100$

$\Leftarrow$ *Local* non-Gaussian I.C.
relative effect

[ES, Crocce & Desjacques (2010)]

We can do better:

- The **resummation** of infinite sub-sets of perturbative contributions in **RPT**, can be **extended to non-Gaussian initial conditions**, for arbitrary non-Gaussian models
- Renormalized, **"non-linear" kernels** can be obtained as a function of the initial correlators, so that

$$\begin{aligned} B(k_1,k_2,k_3) \quad &= \quad \Gamma^{(1)}(k_1)\Gamma^{(1)}(k_2)\Gamma^{(1)}(k_3)B_0(k_1,k_2,k_3)) \\ &\quad +2\Gamma^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\Gamma^{(1)}(k_1)\Gamma^{(1)}(k_2)P_0(k_1)P_0(k_2) + \mathrm{perm.} \end{aligned}$$

and they can be **computed in the high-k limit**

$$\Gamma^{(n)}(\mathbf{k}_1,\ldots,\mathbf{k}_n) \to f(k)F_n(\mathbf{k}_1,\ldots,\mathbf{k}_n), \quad \mathbf{k} = \sum_i \mathbf{k}_i$$

with a **damping function**

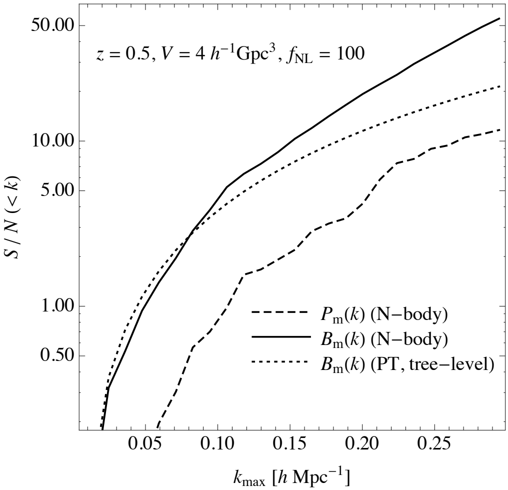
$$\log f(k) = \sum_{p=2}^{\infty} \frac{\langle(\mathbf{d}\cdot\mathbf{k})^p\rangle}{p!}(D_+ - 1)^p\,, \quad \mathbf{d} = \int d^3\mathbf{q}\,\frac{\mathbf{q}}{q^2}\delta_0(\mathbf{q})$$

that does not depende on B_0, very close to the Gaussian case,

$$\langle(\mathbf{d}\cdot\mathbf{k})^2\rangle = -k^2\sigma_d^2, \quad \langle(\mathbf{d}\cdot\mathbf{k})^3\rangle = 0, \quad \langle(\mathbf{d}\cdot\mathbf{k})^4\rangle \sim \int T_0$$

[Bernardeau, Crocce & ES (2010), see also Bartolo *et al.* (2010)]

Cumulative, "**non-Gaussian**", signal-to-noise



$$\left(\frac{S}{N}\right)^2_P = \sum_k^{k_{\mathrm{max}}} \frac{(P_{NG}-P_G)^2}{\Delta P^2}$$
$$\left(\frac{S}{N}\right)^2_B = \sum_{\mathrm{triangles}}^{k_{\mathrm{max}}} \frac{(B_{NG}-B_G)^2}{\Delta B^2} \tag{1}$$

Sums over *all configurations* up to k_{max}

[ES, Crocce & Desjacques (*in preparation*)]

Non-linear bias is a source of **additional non-Gaussianity**

If we assume **local bias**:

$$\delta_g(x) = f[\delta(x)] \simeq b_1\delta(x) + \frac{b_2}{2}\delta^2(x)$$

And the **tree-level** prediction for the *galaxy* 3-point function

$$\langle\delta_g(x_1)\delta_g(x_2)\delta_g(x_3)\rangle = b_1^3\langle\delta(x_1)\delta(x_2)\delta(x_3)\rangle + b_1^2 b_2\langle\delta(x_1)\delta(x_2)\delta^2(x_3)\rangle + \mathrm{perm.}$$

corresponding to the *reduced galaxy bispectrum*

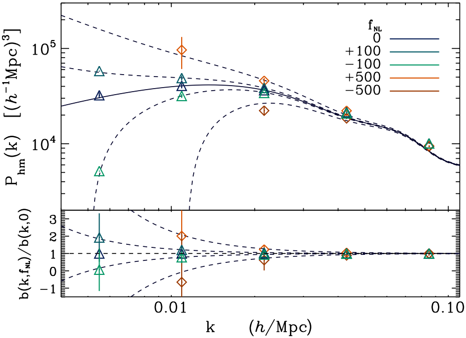
$$Q_B^{(g)} = \frac{1}{b_1}\left[Q_G(k_1,k_2,k_3) + Q_I(k_1,k_2,k_3)\right] + \frac{b_2}{b_1^2}$$

Then we can expect very good constraints on f_{NL}, after marginalizing over bias, from the initial component alone

	$\Delta f_{NL}^{\mathrm{loc.}}$	$\Delta f_{NL}^{\mathrm{eq.}}$
Euclid, $0 < z < 2$	1.5	7
Planck (CMB)	6	30

[ES & Komatsu (2007); ES, Yadav, Liguori, Pajer & Jackson (2009)]

Dalal *et al.*, (2008): **the bias of halos receives a large correction** (at large scales!) for **local primordial non-Gaussianity**



⇒ a **non-local bias** relation (cf. Porciani's talk, Giannantonio & Porciani, 2010):

$$\delta_g(x) = f[\delta(x), \phi(x)] = b_1\delta(x) + c_1(f_{NL})\phi(x) + \frac{b_2}{2}\delta^2(x) + c_2(f_{NL})\delta(x)\phi(x) + \ldots$$

with

$$b_1 = \frac{1}{n_{NG}}\frac{\partial n_{NG}}{\partial \delta} = b_{1,G} + \Delta b_{1,NG}(f_{NL}), \qquad c_1 = 2f_{NL}\delta_c(b_1 - 1), \qquad \ldots$$

The **galaxy bispectrum** and **primordial non-Gaussianity**

The simplest model for the galaxy bispectrum is then

$$\begin{aligned} B_g(k_1, k_2, k_3) &= b_1^3 B(k_1, k_2, k_3) + b_1^2 c_1 B_{\delta\delta\phi}(k_1, k_2, k_3) \\ &\quad + b_1^2 b_2 P(k_1) P(k_2) + \mathrm{perm.} + b_1^2 c_2 P(k_1) P_{\delta\phi}(k_2) + \mathrm{perm.} \end{aligned}$$

a "tree-level" expression
(but we keep the matter and matter-potential correlators at 1-loop)

We study its validity (at large scales) with two halo populations:

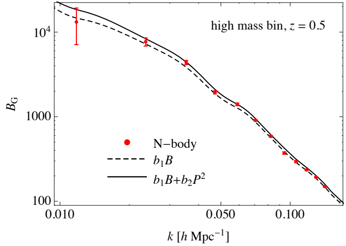
Low mass: $8.8 \times 10^{12}\, h^{-1}\, \mathrm{M}_\odot < M < 1.6 \times 10^{13}\, h^{-1}\, \mathrm{M}_\odot$
High mass: $M > 1.6 \times 10^{13}\, h^{-1}\, \mathrm{M}_\odot$

So far we considered only the matter-matter-halo bispectrum, $B_{\delta\delta h}$

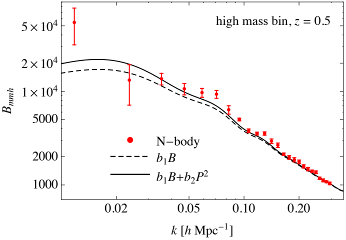
The **galaxy bispectrum** and **primordial non-Gaussianity**

First we determine the Gaussian bias parameters $b_{1,G}$ and $b_{2,G}$ from *all configurations*, up to $k_{max} = 0.07\, h\,\mathrm{Mpc}^{-1}$

Isosceles triangles
$B(2k, 2k, k)$ *vs* k
(Gaussian IC)



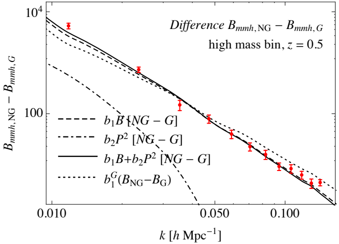
Squeezed triangles
$B(k, k, \Delta k)$ *vs* k
(Gaussian IC)

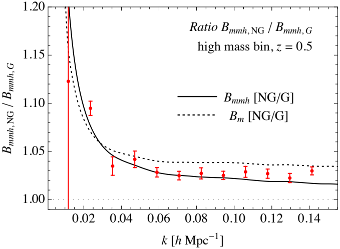


$$b_{1,G} \simeq 2.1, \quad b_{2,G} \simeq 1$$

Then we (*try to*) predict the non-Gaussian correction:

Isosceles triangles
$B(2k, 2k, k)$ *vs* k
(Non-Gaussian IC)





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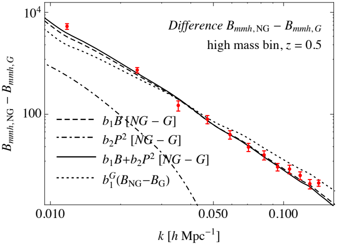

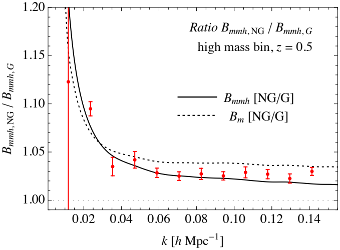

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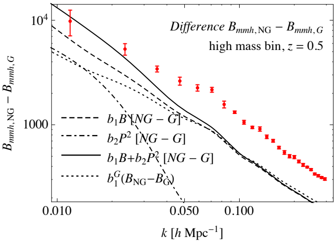
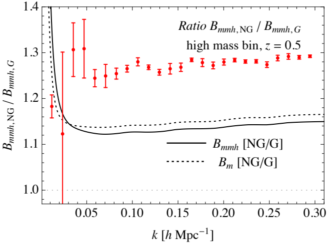
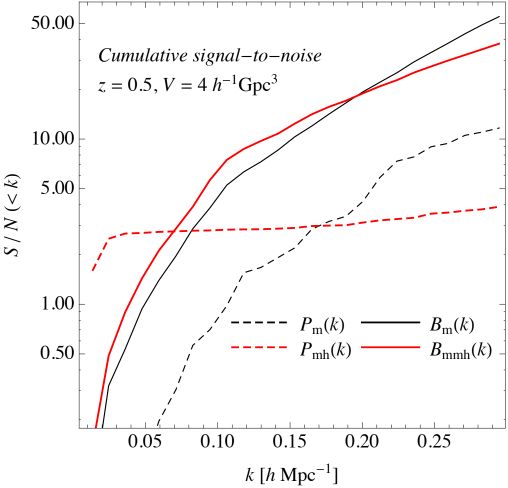




The **galaxy bispectrum** and **primordial non-Gaussianity**

Cumulative, "**non-Gaussian**", signal-to-noise (but see Scoccimarro's talk next week)



$$\left(\frac{S}{N}\right)^2_P = \sum_k^{k_{\max}} \frac{(P_{NG} - P_G)^2}{\Delta P^2}$$
$$\left(\frac{S}{N}\right)^2_B = \sum_{\text{triangles}}^{k_{\max}} \frac{(B_{NG} - B_G)^2}{\Delta B^2} \tag{2}$$

Sums over *all configurations* up to $k_{\max}$

[ES, Crocce & Desjacques (*in preparation*)]

	$\Delta f_{NL}^{\mathrm{loc.}}$	$\Delta f_{NL}^{\mathrm{eq.}}$	
CMB Bispectrum			
WMAP7 (current)	21	140	Komatsu *et al.* (2009)
Planck	6	30	
CMBPol	3	18	
LSS Power Spectrum			
SDSS QSOs (current)	25	—	Slosar *et al.* (2008)
NVSS AGNs (current)	27	—	Xia *et al.* (2010)
Euclid	$3 \sim 9$	—	Carbone *et al.* (2010)
LSST	$2 \sim 5$	—	Carbone *et al.* (2010)
LSS Bispectrum			
Euclid	< 2	< 7	ES & Komatsu (2007)
	work in progress!		
LSS Cluster abundance (w/self-calibration)			
WFXT	12	—	Sartoris *et al.* (2010)
DES	6	—	Cunha *et al.* (2010)

In principle, the constraints on primordial non-Gaussianity from the galaxy bispectrum are expected to be quantitatively (smaller errors on f_{NL}'s) and qualitatively (larger sensitivity to the shape of non-Gaussianities) better than those from other LSS probes. **In practice**, more work is needed ...