Large-Scale Bias, Primordial Non-Gaussianity and the Bispectrum

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Outline

- Renormalized Perturbation Theory (RPT): brief intro + update

- Primordial non-Gaussianity (PNG): scale-dependent bias in local PNG

- Local bias or Peak-Background split?

- Non-local PNG: theory, simulations, mock catalogs

- Using the bispectrum to constrain PNG

- The bispectrum of SDSS galaxies

- *M. Crocce and R.S. (2010)*
- *K.C. Chan, M. Manera, L. Hui and R.S. (2010)*
- *R.S., E. Sefusatti, C. McBride, J. Frieman, et al (2010)*

Renormalized Perturbation Theory (RPT)

- partial resummation of PT contributions

- truncation of RPT expansion accounts for all nonlinearities down to a given scale (the impact of smaller scales is highly suppressed in fluid limit).

- "Initial" conditions (density perturbations after decoupling) play a crucial role. They act as a (stochastic) source: observables (expectation values) correspond to averages over the statistics of initial conditions.

- In RPT, the linear propagator gets ``renormalized" due to nonlinearities,

Final density / velocity

↓

$$G_{ab}(k,\eta)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta\Psi_a(\mathbf{k},\eta)}{\delta\phi_b(\mathbf{k}')} \right\rangle$$

↑

Initial Conditions (act as sources)

For Gaussian initial conditions, the nonlinear propagator can be related to the cross-correlation between initial and final conditions,

$$G_{ab}(k,\eta)\,\langle\phi_b(\mathbf{k})\phi_c(\mathbf{k}')\rangle = \langle\Psi_a(\mathbf{k},\eta)\,\phi_c(\mathbf{k}')\rangle.$$

In this sense the propagator measures the memory of perturbations to their initial conditions. The asymptotics are,

$$G_{ab}(k\to 0,\eta) = g_{ab}(\eta), \qquad G_{ab}(k\to\infty,\eta) = 0$$

impossible to recover at fixed order in PT!

$$g_{ab}(\eta) = \frac{\mathrm{e}^{\eta}}{5}\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} - \frac{\mathrm{e}^{-3\eta/2}}{5}\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix},$$

growing mode $\phi_a(\mathbf{k}) \propto (1,1)$

decaying mode $\phi_a(\mathbf{k}) \propto (1,-3/2)$

$$\eta \equiv \ln D_+$$

The resummation of the propagator can be carried out exactly in the high-k limit!

The result is:

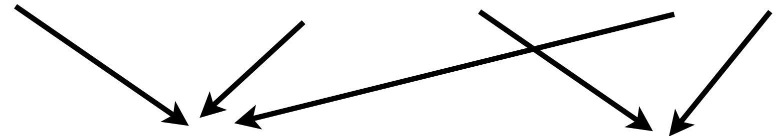
$$G_{ab}(k,\eta) \simeq g_{ab}(\eta)\ \exp\Big(-\frac{1}{2}k^2\sigma_v^2(\mathrm{e}^{\eta}-1)^2\Big) \qquad \text{(high-k limit)}$$

with:

$$\sigma_v^2 \equiv \frac{1}{3}\int d^3q\,\frac{P(q)}{q^2}$$

For the power spectrum, RPT reorganizes the PT expansion,

$$P(k,z) = D_+^2(z)\, P_0(k) + P_{1\mathrm{loop}}(k,z) + P_{2\mathrm{loop}}(k,z) + \dots$$



into,

$$P(k,z) = G^2(k,z)\, P_0(k) + P_{\mathrm{MC}}(k,z)$$

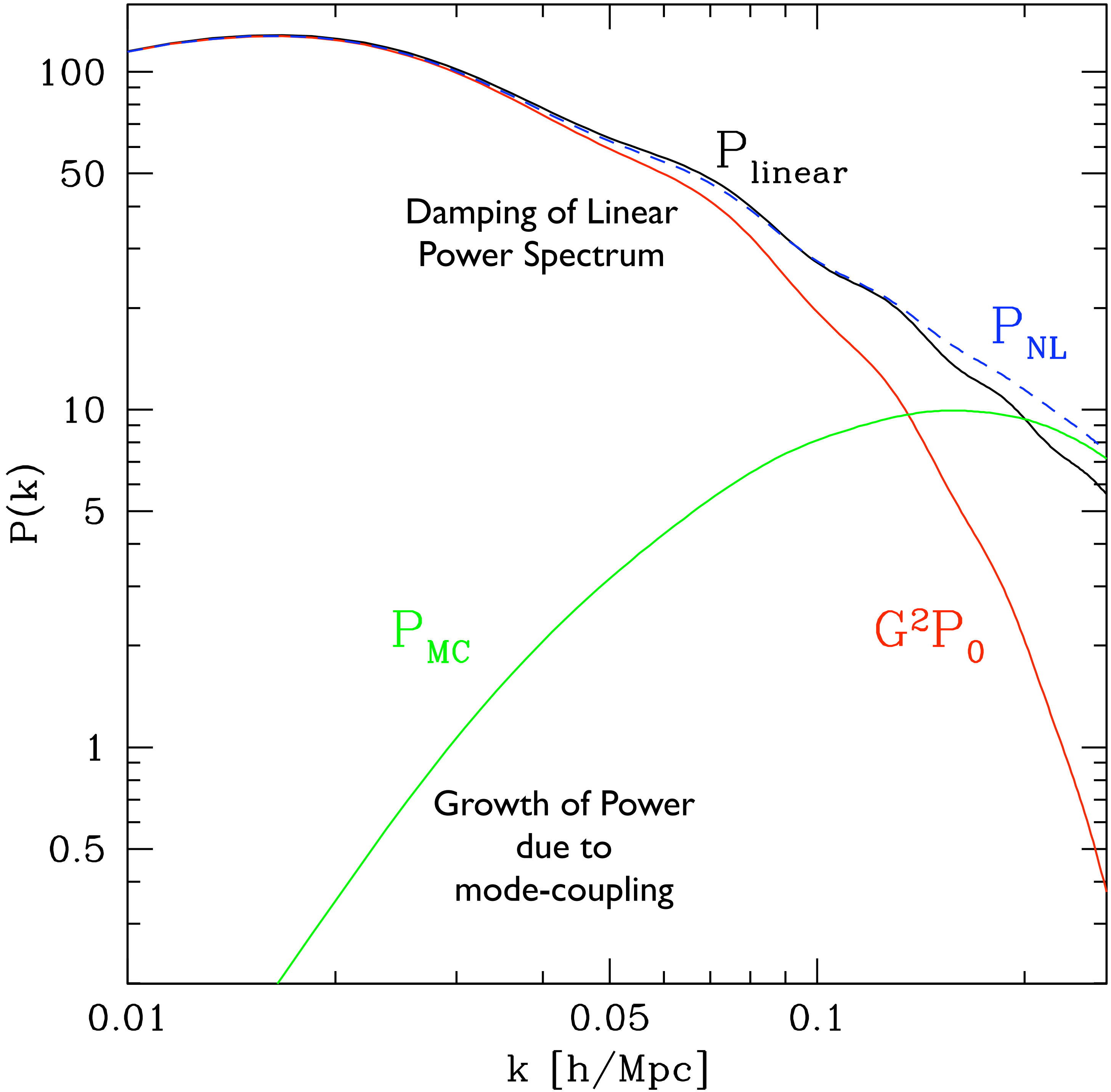
with,

$$P_{\mathrm{MC}}(k,z) = P_{\mathrm{MC}}^{1\mathrm{loop}}(k,z) + P_{\mathrm{MC}}^{2\mathrm{loop}}(k,z) + \dots$$

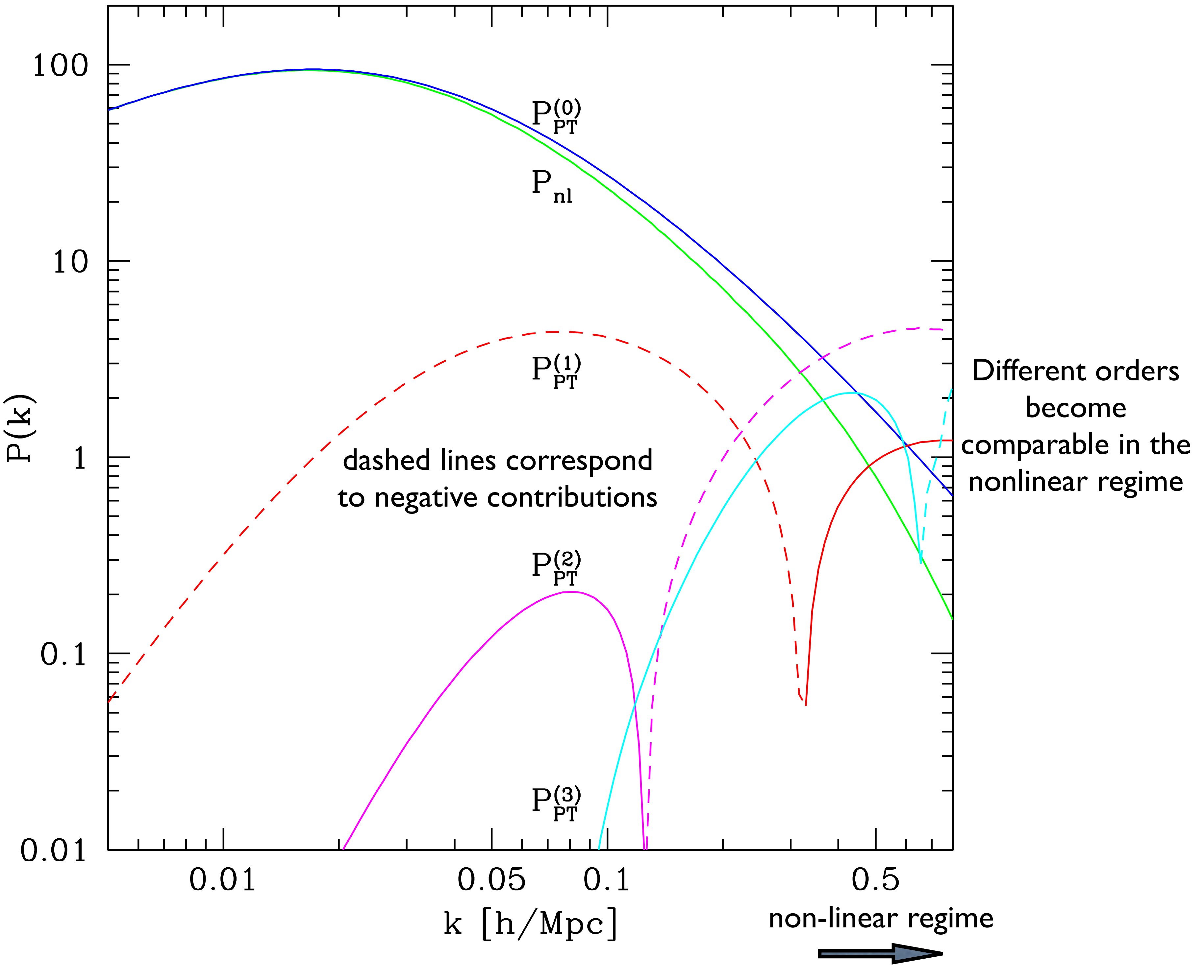
Thus, non linear effects can be divided (exactly) into two classes,

- those that are proportional to the initial power at same k.
- those that create power at k even if there was no power to begin with (mode-coupling)

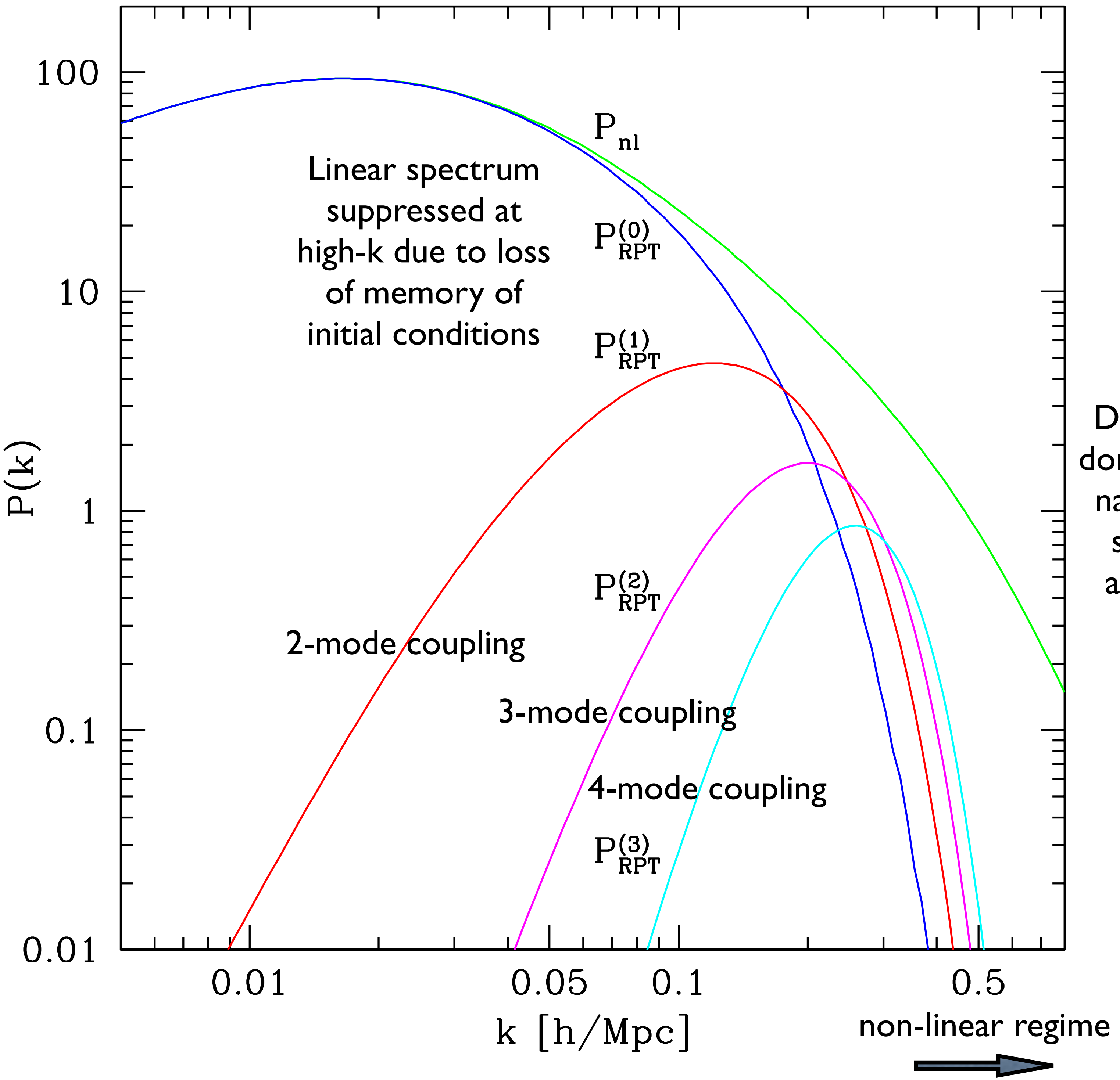
The Power Spectrum in RPT



PT expansion (schematic)

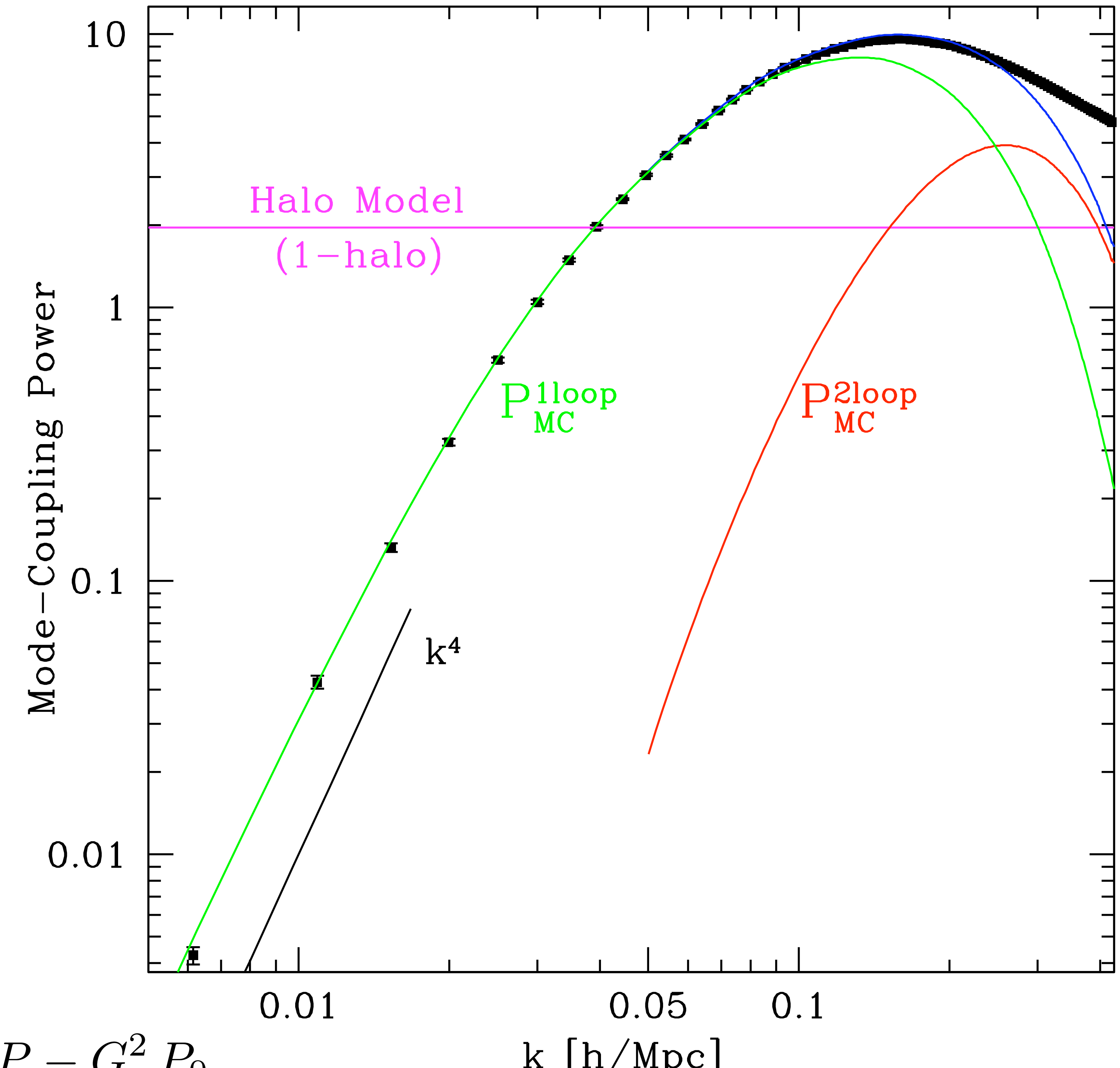


RPT expansion (schematic)



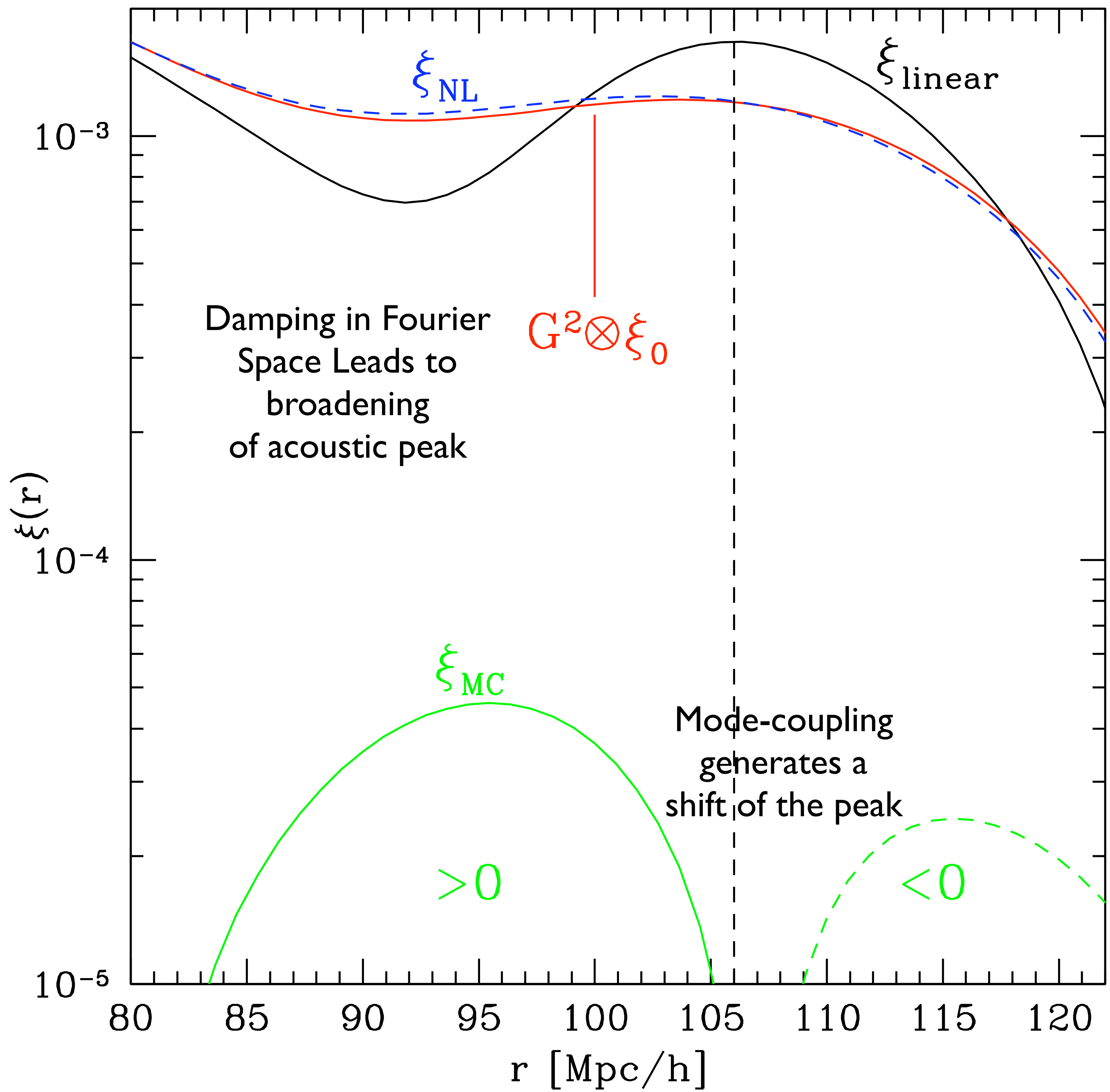
Different orders dominate only in a narrow range of scales and are always positive

Explicit calculation of Mode-Coupling power to 2-loops in RPT

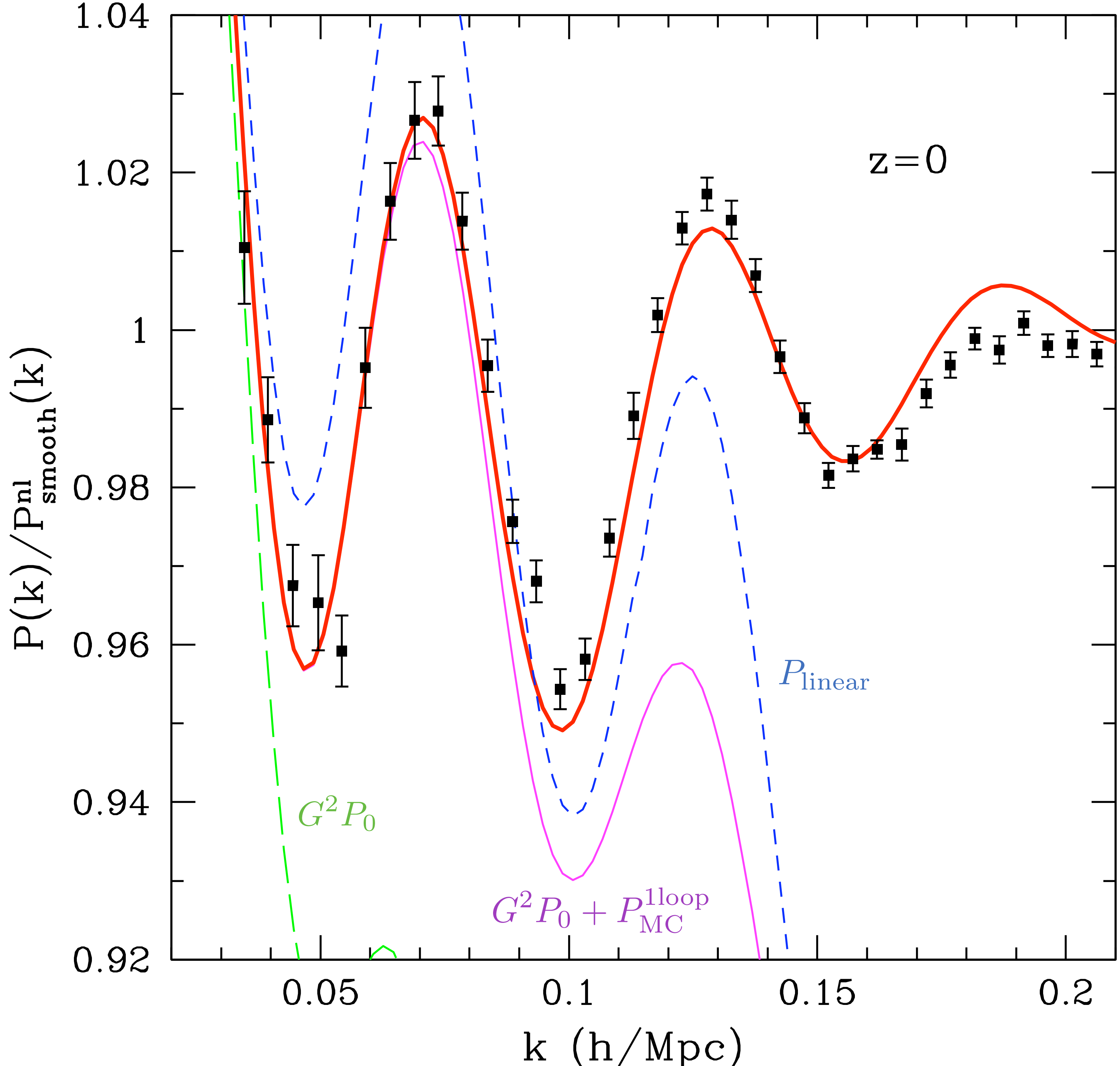


$$P_{\rm MC} \equiv P - G^2\, P_0$$

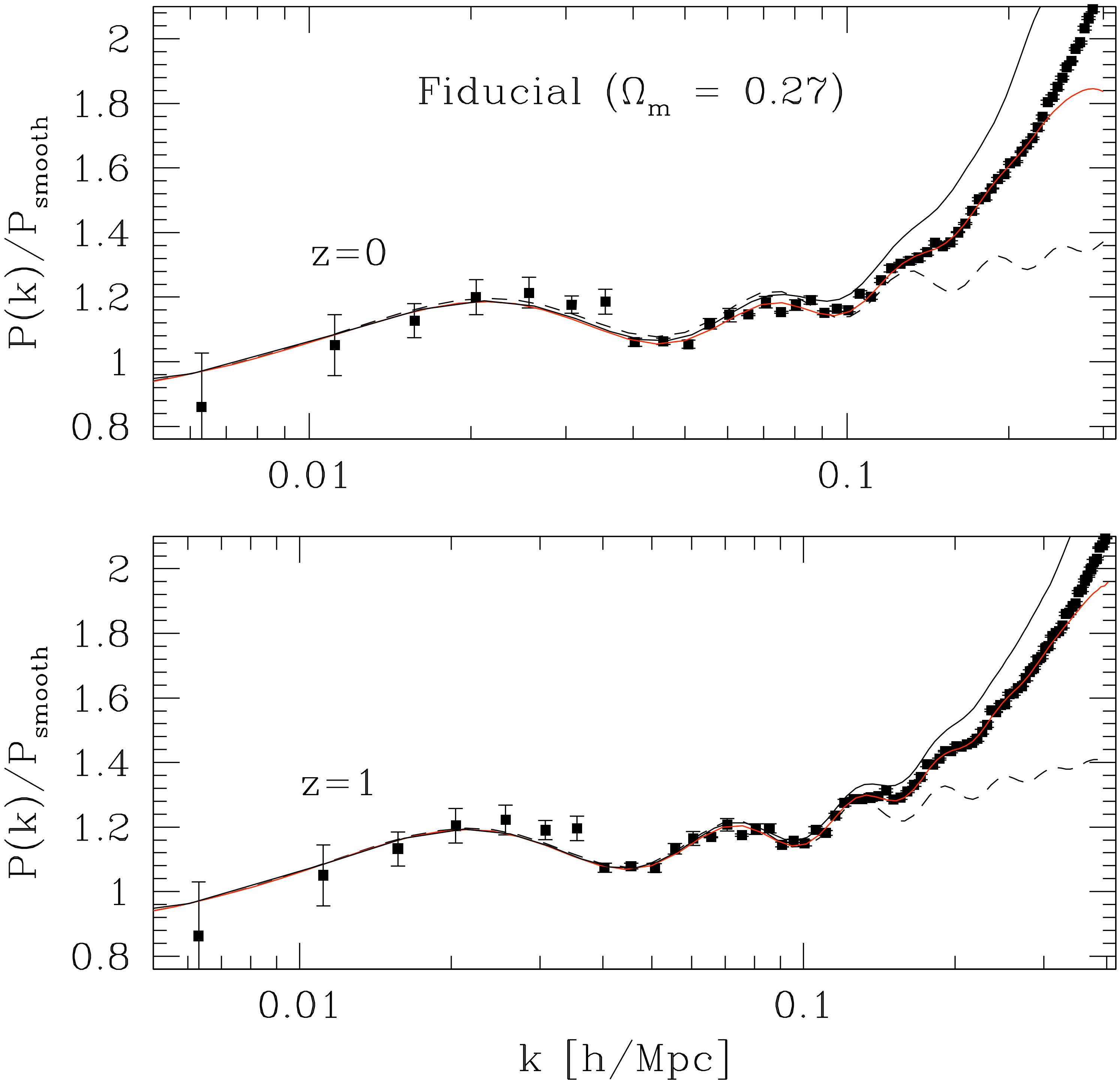
The Two-Point Function in RPT



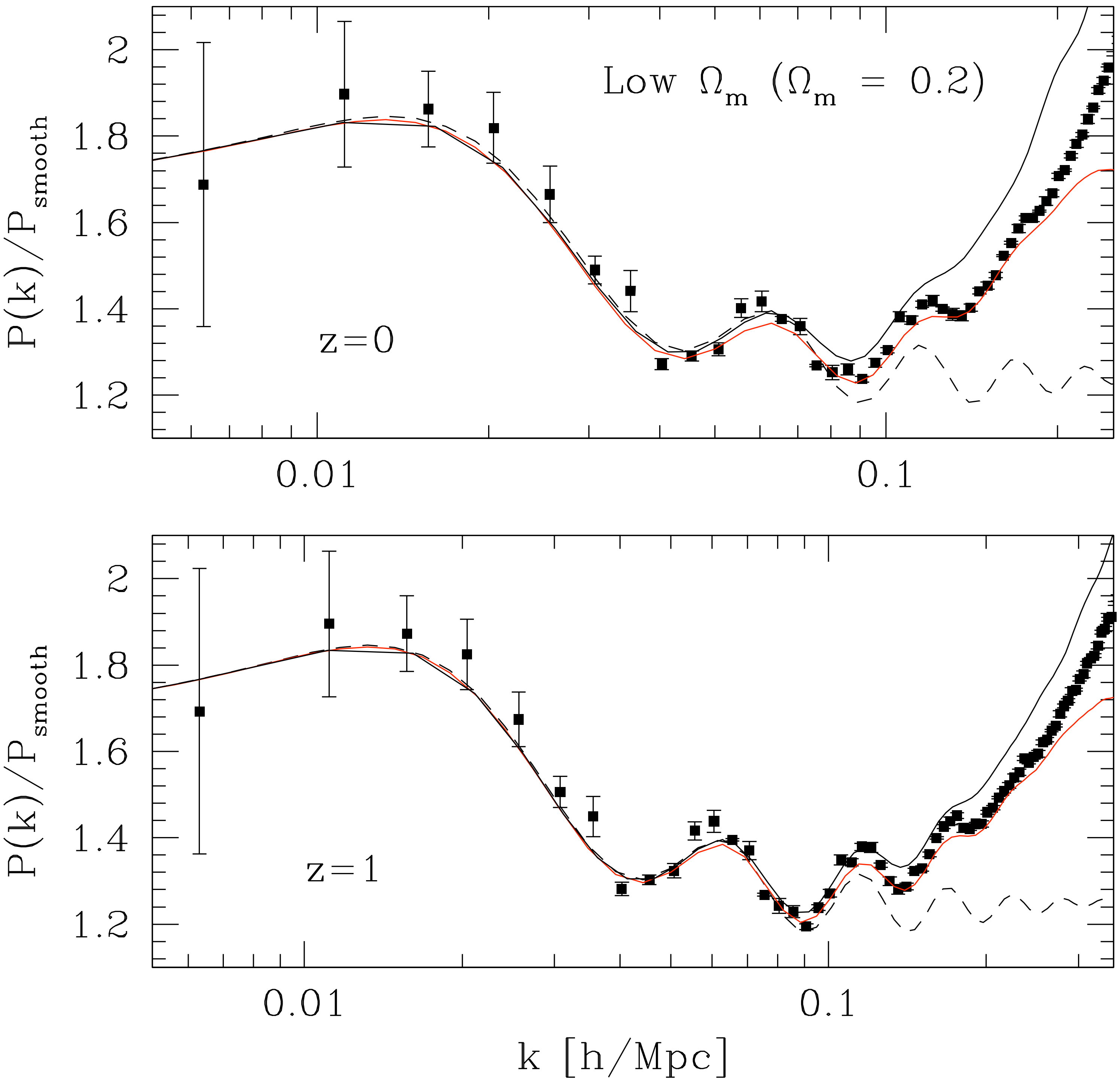
Nonlinear Evolution of Acoustic Oscillations



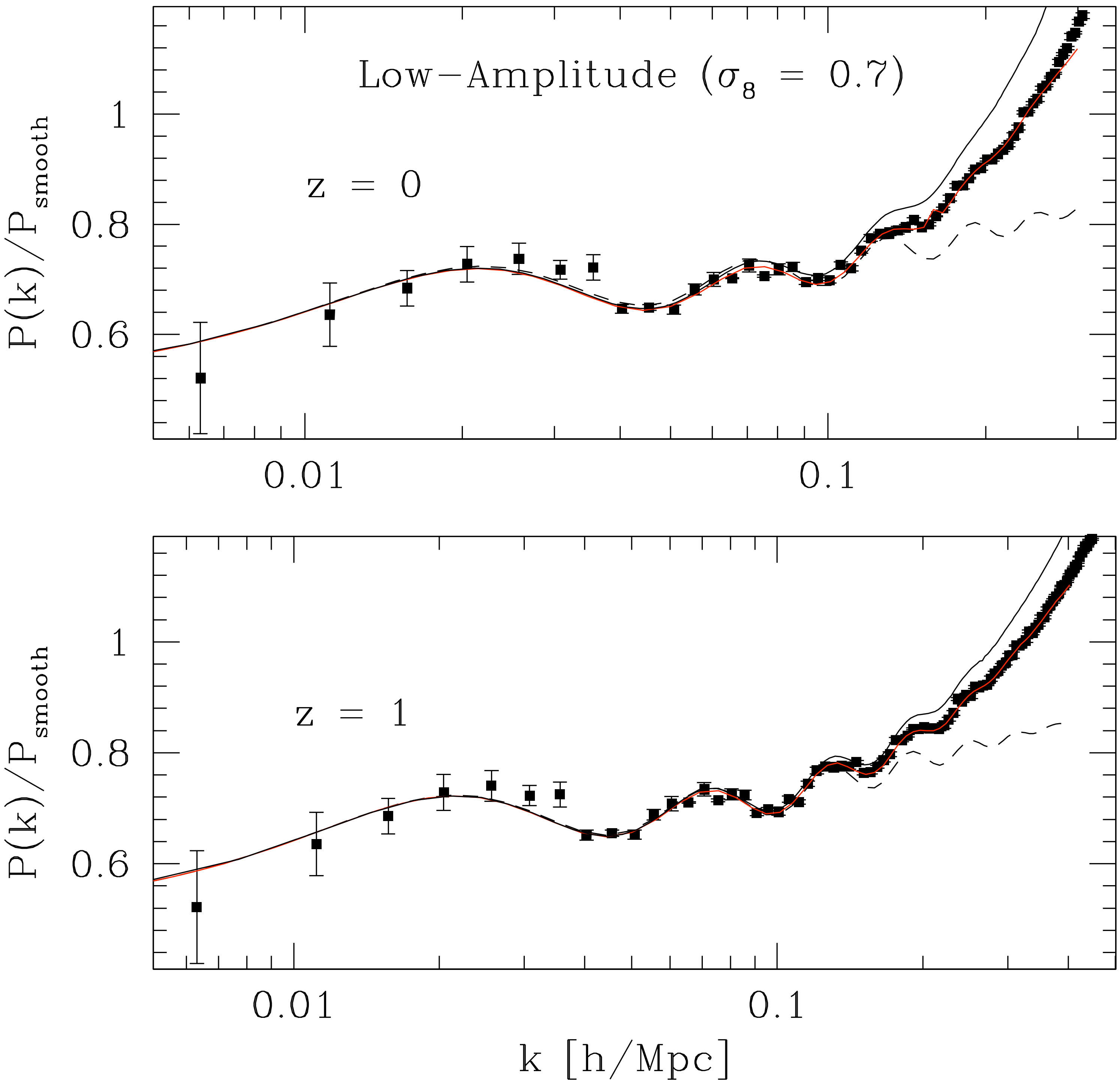
Power Spectrum: Dependence on Cosmology



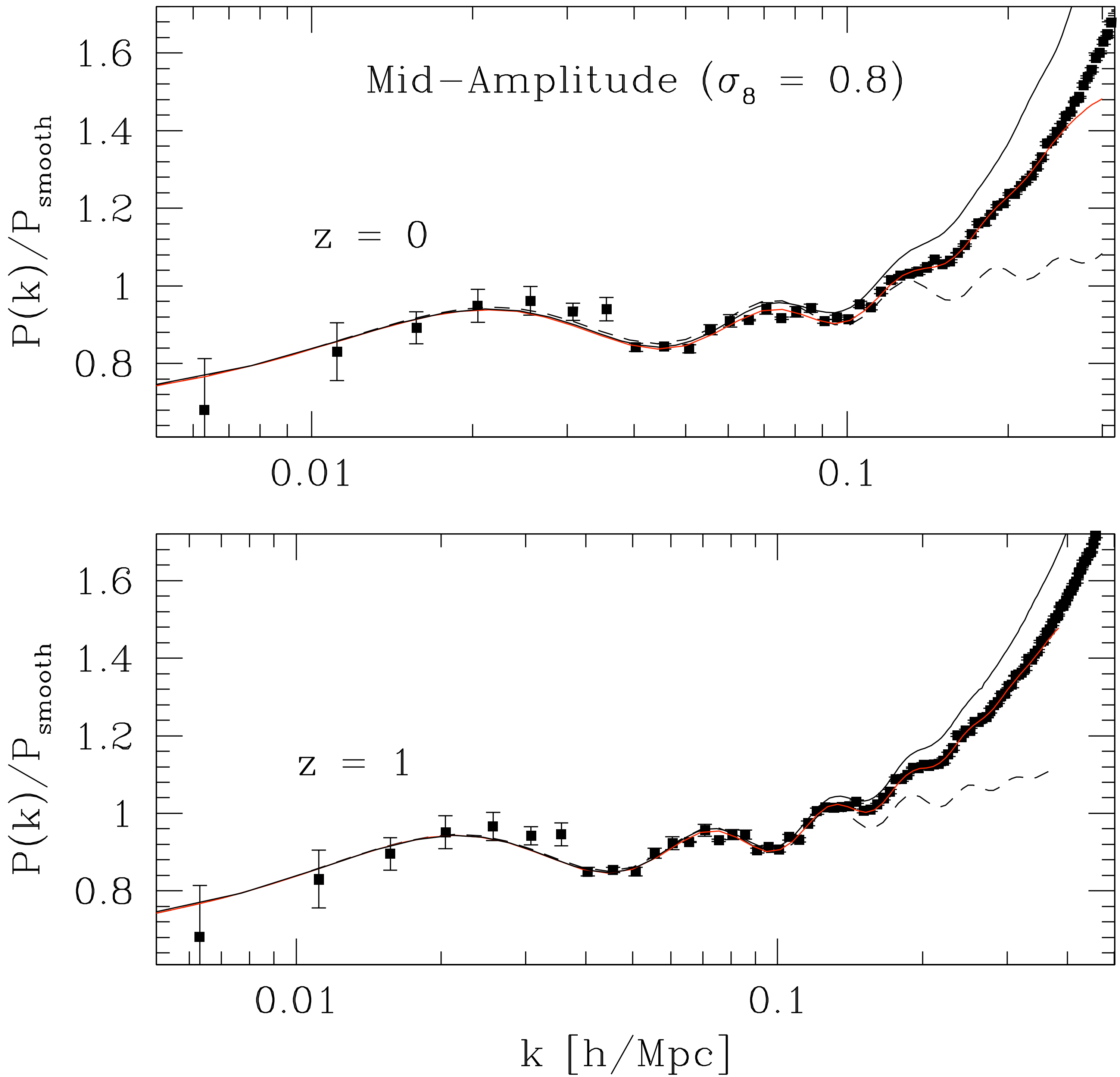
Power Spectrum: Dependence on Cosmology



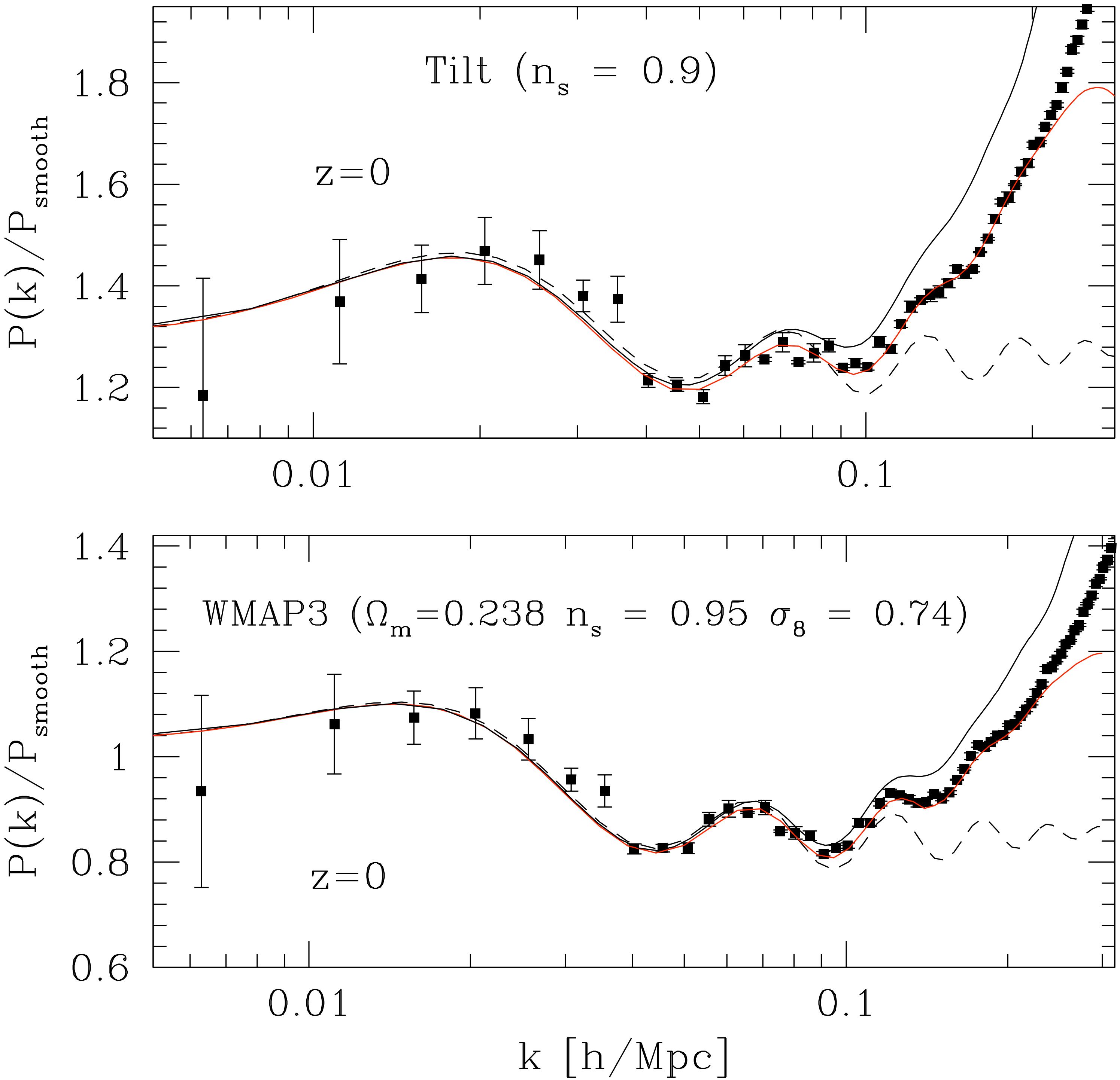
Power Spectrum: Dependence on Cosmology



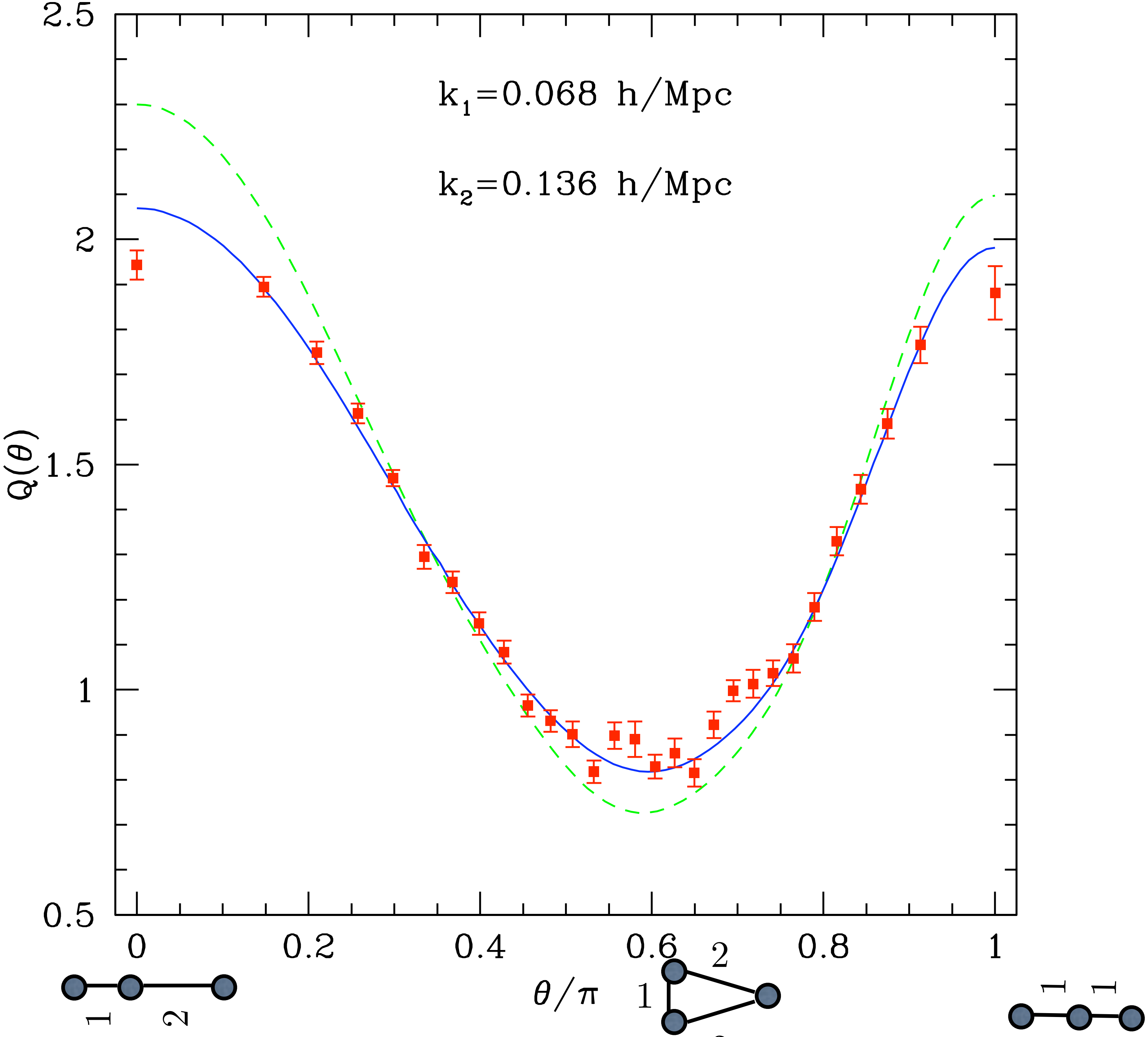
Power Spectrum: Dependence on Cosmology



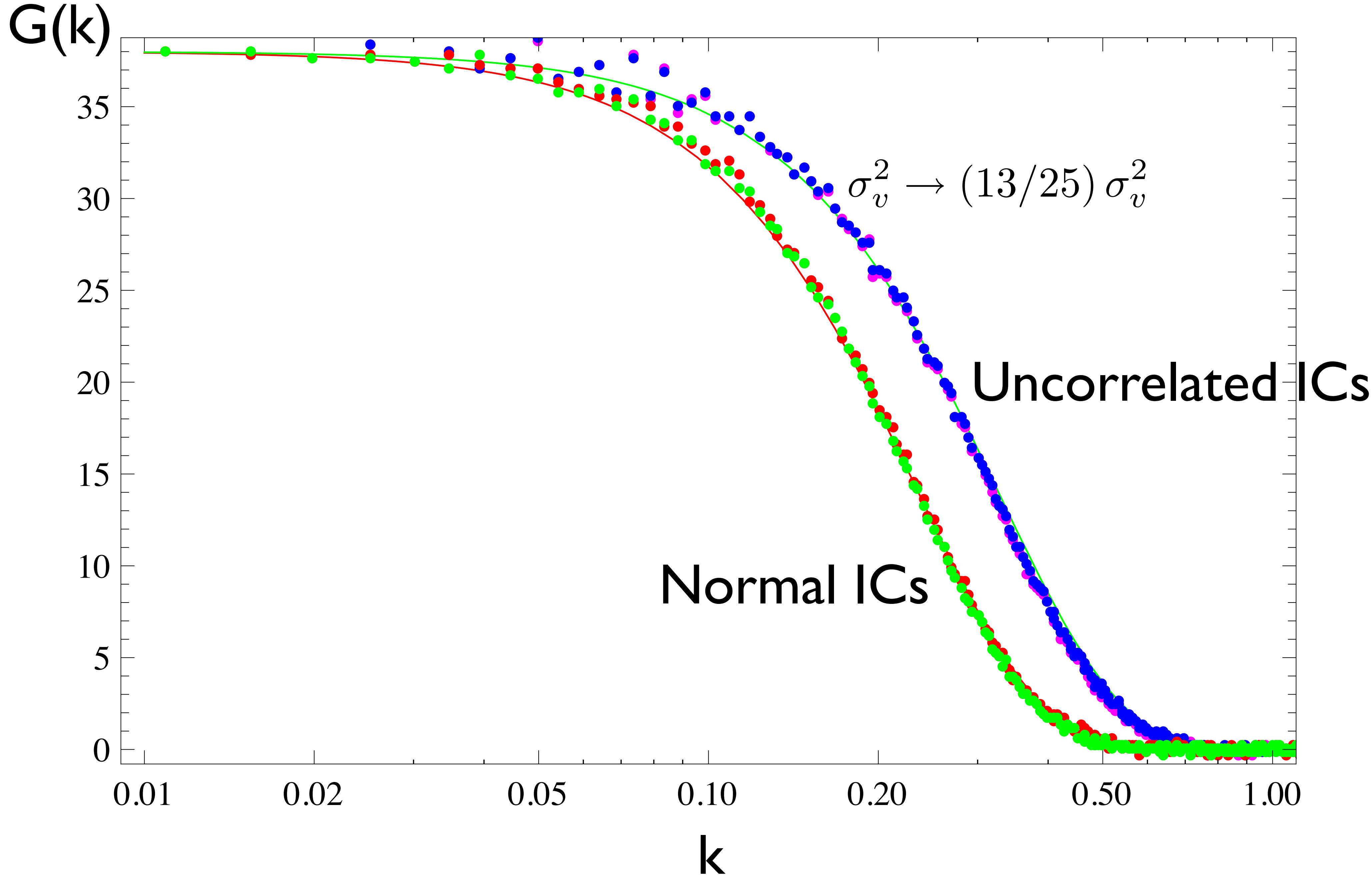
Power Spectrum: Dependence on Cosmology



Bispectrum



Testing the fundamentals of Propagator Resummation



Large-Scale Bias in local PNG

- In local models of primordial non-Gaussianity (PNG) we have for the Bardeen potential,

$$\Phi = \phi + f_{\rm NL}\phi^2$$

which implies for it a bispectrum,

$$B = 2 f_{\rm NL} P_1 P_2 + {\rm cyc.}$$

- For biased tracers (galaxies, halos), this model leads to a scale-dependent bias at large scales (Dalal et al 2008),

$$b_1(k) = b_{10} + \Delta b_1(k, f_{\rm NL})$$

where b~1/k^2 at low-k.

There are basically three derivations of this effect:

- Peak Background Split (PBS): objects correspond to $\delta_{\rm lin} > \delta_c$

$$\Delta b_1(k, f_{\rm NL}) = \frac{2 f_{\rm NL}}{M(k)}(b_{10} - 1)\delta_c$$

Slosar et al

- Gaussian Field Peaks in high-threshold limit $(\nu \gg 1)$

$$\Delta b_1(k, f_{\rm NL}) = \frac{2 f_{\rm NL}}{M(k)}\nu^2$$

Matarrese & Verde

- Local Eulerian bias model $(\delta_g = b_1 \delta + \frac{b_2}{2}\delta^2 + \ldots)$

$$\Delta b_1(k, f_{\rm NL}) = \frac{2 f_{\rm NL}}{M(k)} b_2\, \sigma^2$$

Taruya et al

where M relates the density to the Bardeen potential through the Poisson eqn

$$M(k) = \frac{2c^2 k^2 T(k) D(z)}{3\Omega_m H_0^2} \sim k^2 \qquad (k \to 0)$$

In local Eulerian models and peaks there is a generic formula (for any type primordial non-Gaussianity) for the low-k power change

$$\Delta P(k) \sim \int B(-\mathbf{k}, \mathbf{q}, \mathbf{k}-\mathbf{q}) d^3q$$

Grinstein & Wise 86
Matarrese, Lucchin & Bonometto 86

$$= \int M(k)M(q)M(|\mathbf{k}-\mathbf{q}|)B_\Phi(-\mathbf{k}, \mathbf{q}, \mathbf{k}-\mathbf{q}) d^3q$$

In PBS one splits long wavelength from small scale fluctuations,

$$\phi = \phi_\ell + \phi_s$$

$$\delta \sim \delta_\ell + \nabla^2\phi_s + f_{\rm NL}\nabla^2(\phi_\ell + \phi_s)^2 \equiv \delta_\ell + \delta_s$$

thus small scales perturbations (objects) will be sensitive to phi and its derivatives at long wavelength.

We can compare the three models by using PS and peak theory, in which case,

$$(b_{10} - 1)\delta_c = \nu^2 - 1$$

$$b_2^{\rm peaks}\sigma^2 = 2(1-\nu_2)\frac{\nu^2-1}{\nu^2}\delta_c + (\nu^2 - 3)$$

gravity

so all three formulae agree in the high-peak limit!

However, the nature of bias in these models is quite different...

In PBS, to linear order we have:

$$\delta_g = 2f_{\rm NL}(b_1 - 1)\delta_c\phi + b_1\,\delta$$

which looks different from a local model where corrections are quadratic. We should be able to distinguish which one is correct...

Or are they different ways of writing effectively the same biasing process?
(McDonald 2009)

In RPT language, the 1/k^2 bias in local models arises through mode-coupling, in PBS through the propagator. We can then distinguish between linear non-local and non-linear local bias by computing the "galaxy propagator" G(k)

$$G(k)\ \delta_D(\mathbf{k}-\mathbf{k}') \equiv \langle\ \frac{\partial\delta_g(\mathbf{k})}{\partial\delta_I(\mathbf{k}')}\ \rangle$$

for PBS we get:

$$G(k) = \frac{2f_{\rm NL}}{M(k)}(b_1-1)\delta_c + b_1 G_{\rm dm}(k) \to 1/k^2 \qquad (k\to 0)$$

while for local models, if

$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + \frac{b_3}{6}\delta^3 + \ldots$$

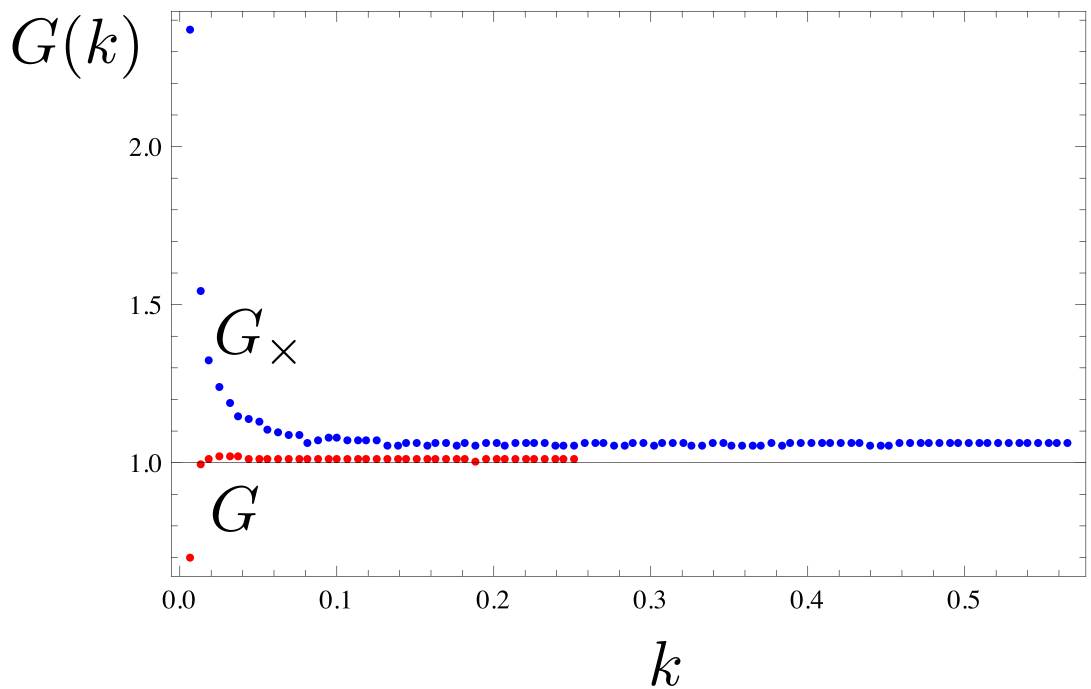
then

$$G(k) = G_{dm}(k)(b_1 + \frac{b_3}{2}\sigma^2 + \ldots)$$

Computing the propagator for non-Gaussian ICs is tricky, but can be done.
Bernardeau, Crocce and Sefusatti (2010) showed that to leading order in PNG,

$$G(k) = G_\times - \frac{1}{2P(k)} \int \Gamma_2(k,-q) B_0(k-q,q) d^3q$$

essentially b_2!



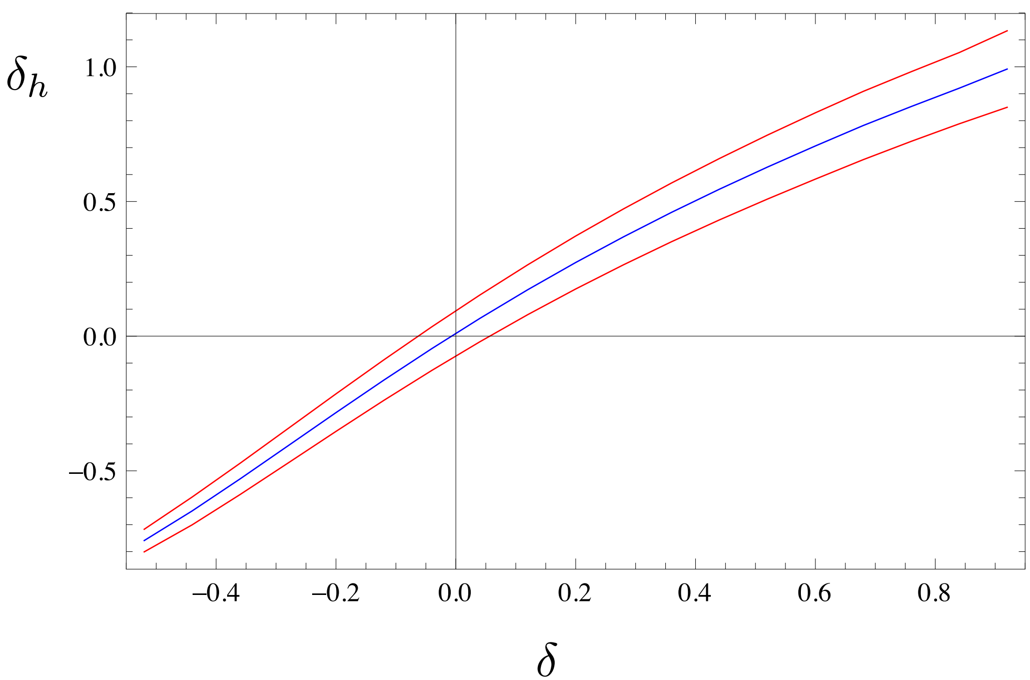
test on numerical realization of a local model with b1=1 and b2=0.5

Take halos in mass bin of 1e13-1e14 Msun/h at z=0:

they have b1>1 but b2<0! (There are no cancellations in G-formula if PBS right)

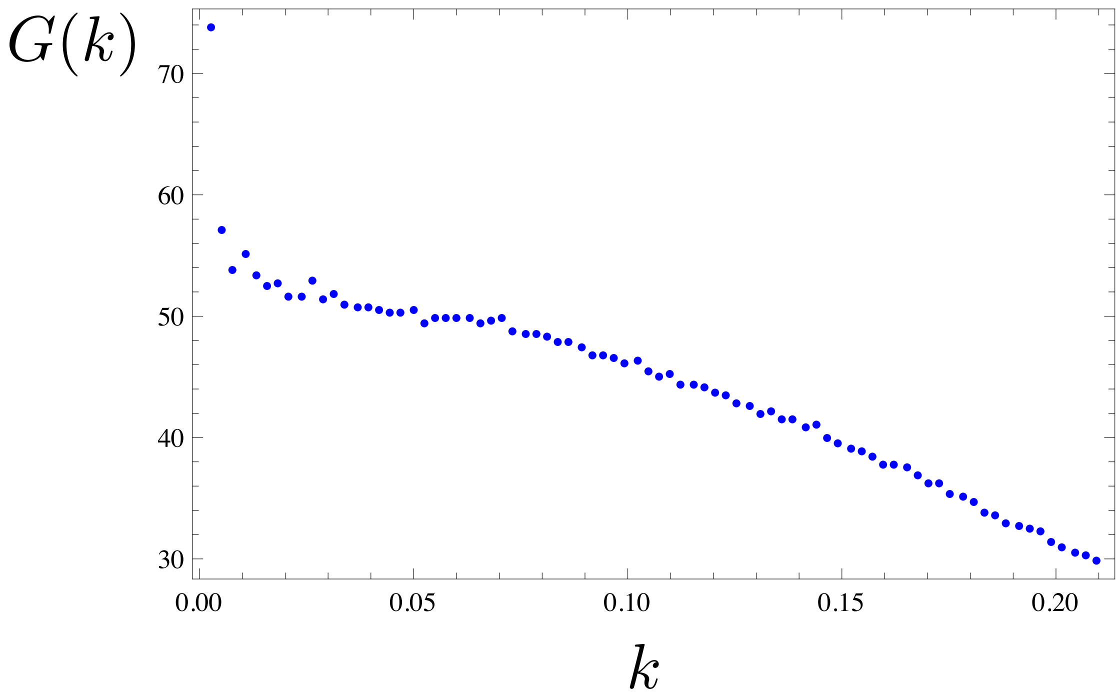
- PBS says P(k=0) and G(k=0) should be enhanced as 1/k^2

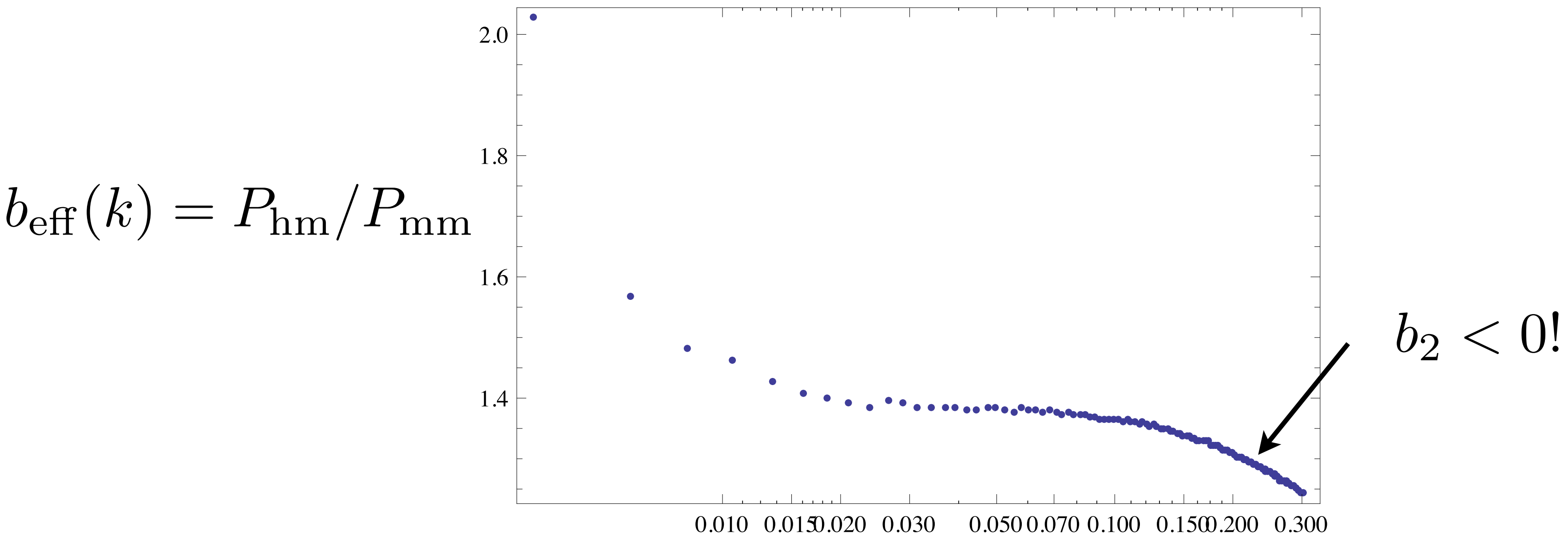
- local bias says that P(k=0) should be suppressed as -1/k^2 and G(k=0) constant at low-k.



R=50 Mpc/h

The local model expectations do not match results:





Large-Scale Bias in non-local PNG

- In single-field inflationary models, we are instead interested in models that correspond to non-local PNG. For example, the equilateral model has a Bardeen potential bispectrum,

$$(6f_{\rm NL})^{-1}B_{\rm equil} = -P_1P_2 - 2(P_1P_2P_3)^{2/3} + P_1^{1/3}P_2^{2/3}P_3$$

(permutations are understood), whereas the orthogonal model reads

$$(6f_{\rm NL})^{-1}B_{\rm ortho} = -3P_1P_2 - 8(P_1P_2P_3)^{2/3} + 3P_1^{1/3}P_2^{2/3}P_3$$

We are interested in generating such bispectra from quadratic (non-local) models, i.e.

$$\Phi = \phi + f_{\rm NL}\ K[\phi,\phi]$$

where K is the appropriate non-local quadratic kernel that generates the desired bispectrum. For simplicity we assume scale-invariance.

For the local model, to leading order one can find a general solution

$$K_{12}^{\mathrm{loc}} = (1-u) + u\,\frac{(k_1^3+k_2^3)}{2|k_1+k_2|^3},$$

where u is a free parameter that leaves invariant the bispectrum. However, if one has an admixture of second term (nonzero u), it does contribute a significant low-k (k^-6) component to the power spectrum through loops (which are UV sensitive). Then one must then set u=0.

For more complicated models these regularity constraints similarly restrict the free parameters that leave the bispectrum invariant.

In the EQ and ORT case, there is k^-6 and k^-4 contributions that one must take care of.

IF, however, one does leave e.g. the k^-4 in, the PBS formula leads to a different large-scale bias scaling than GW/MLB to leading order in fnl.

What's the predicted low-k power for non-local PNG? Local models predict

$$\Delta P(k) \sim \int M(k)M(q)M(|\mathbf{k}-\mathbf{q}|)B_\Phi(-\mathbf{k},\mathbf{q},\mathbf{k}-\mathbf{q})d^3q \propto P(k) \qquad (k \to 0)$$

Using PBS, one gets generically

$$\Delta P(k) \sim \int M(k)M(q)M(|k-q|)B_{\phi\Phi\Phi}(-k,q,k-q)d^3q$$

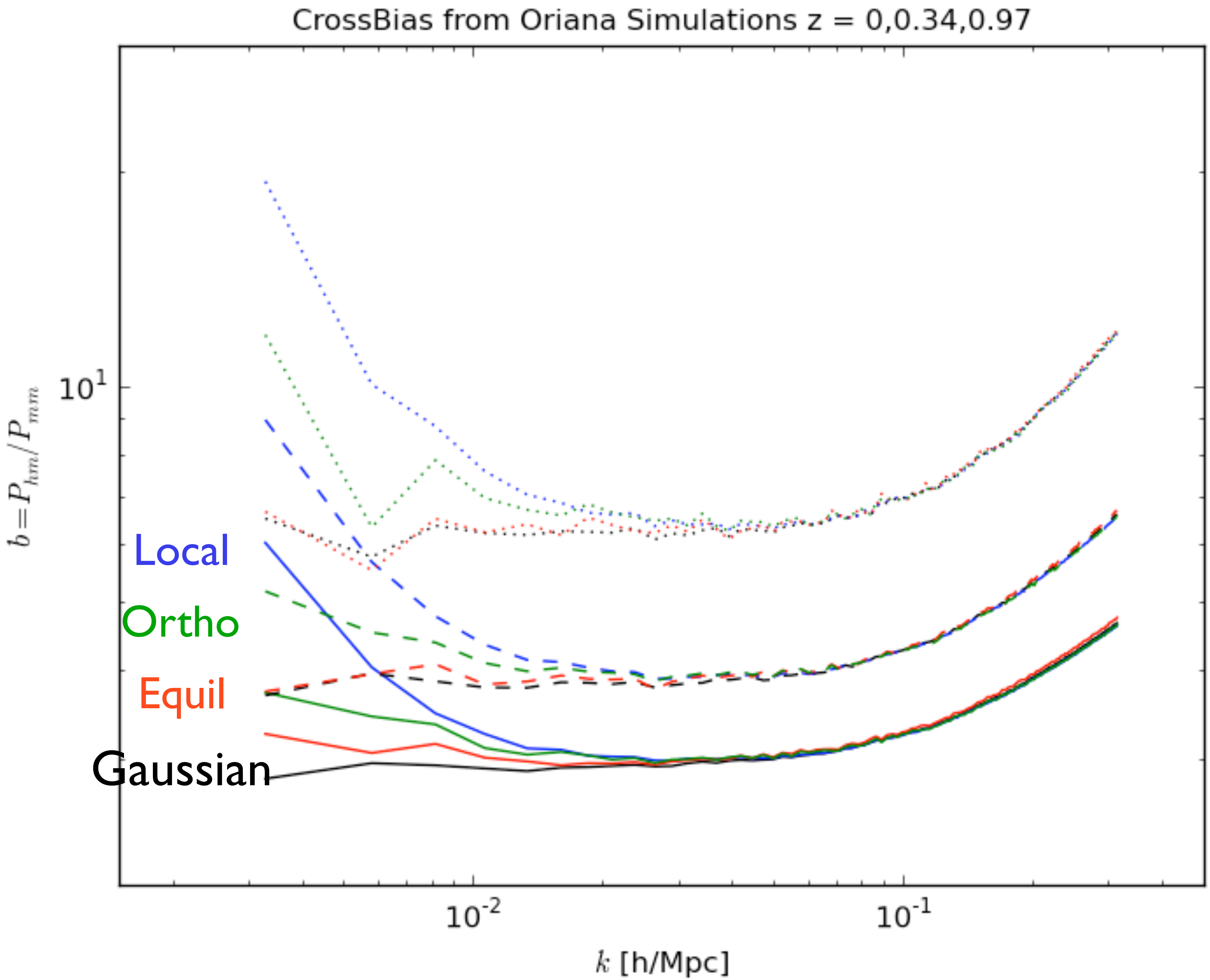
Which differs slightly (apart from the usual prefactors in nu) in one term absent in the bispectrum. Again this difference is due to MC (local) vs Propagator (PBS, mediated by a Gaussian Field).

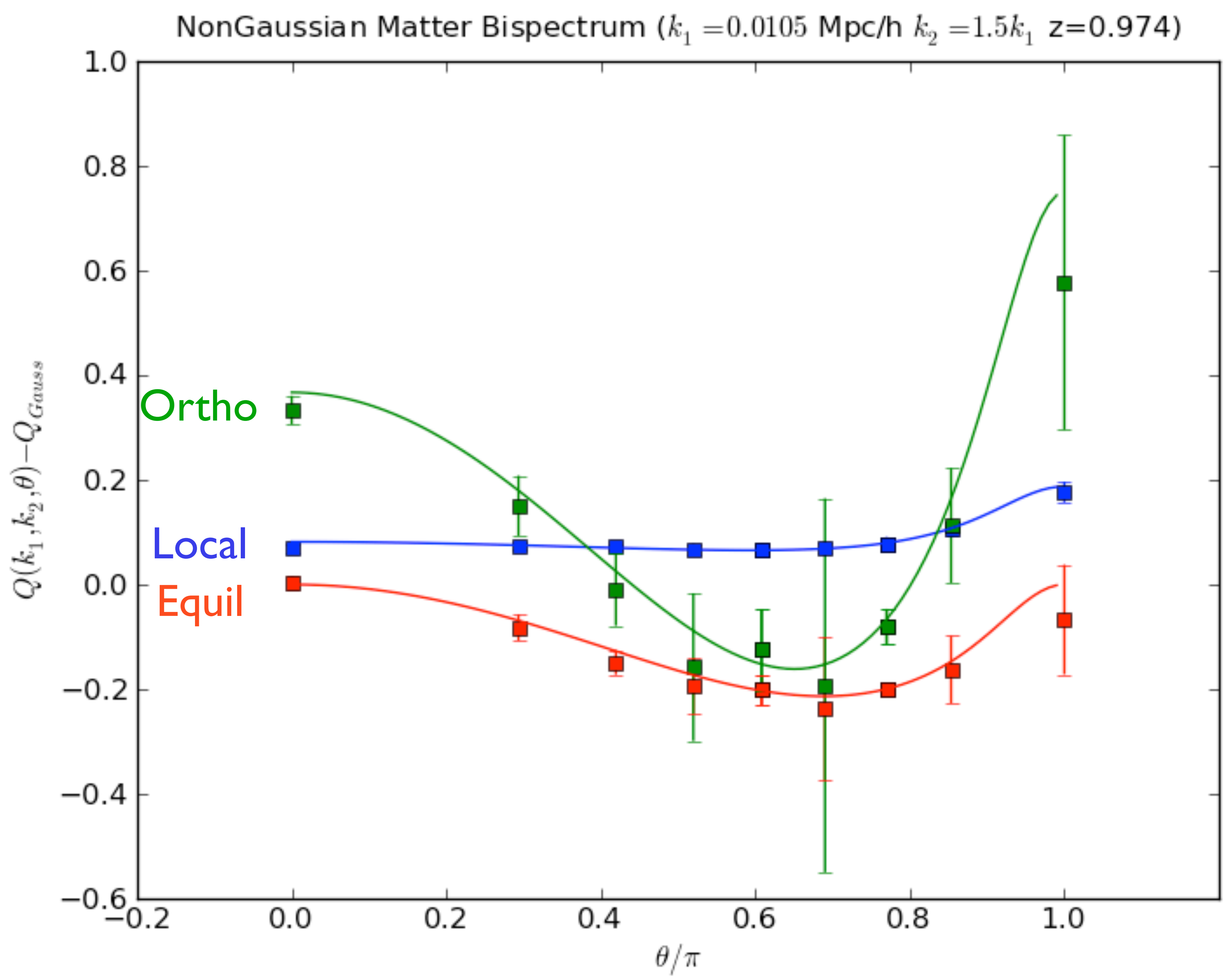
If the kernel K is not too singular as k->0 (1-loop power constraint), the low-k bias scalings agree.

LasDamas Simulations

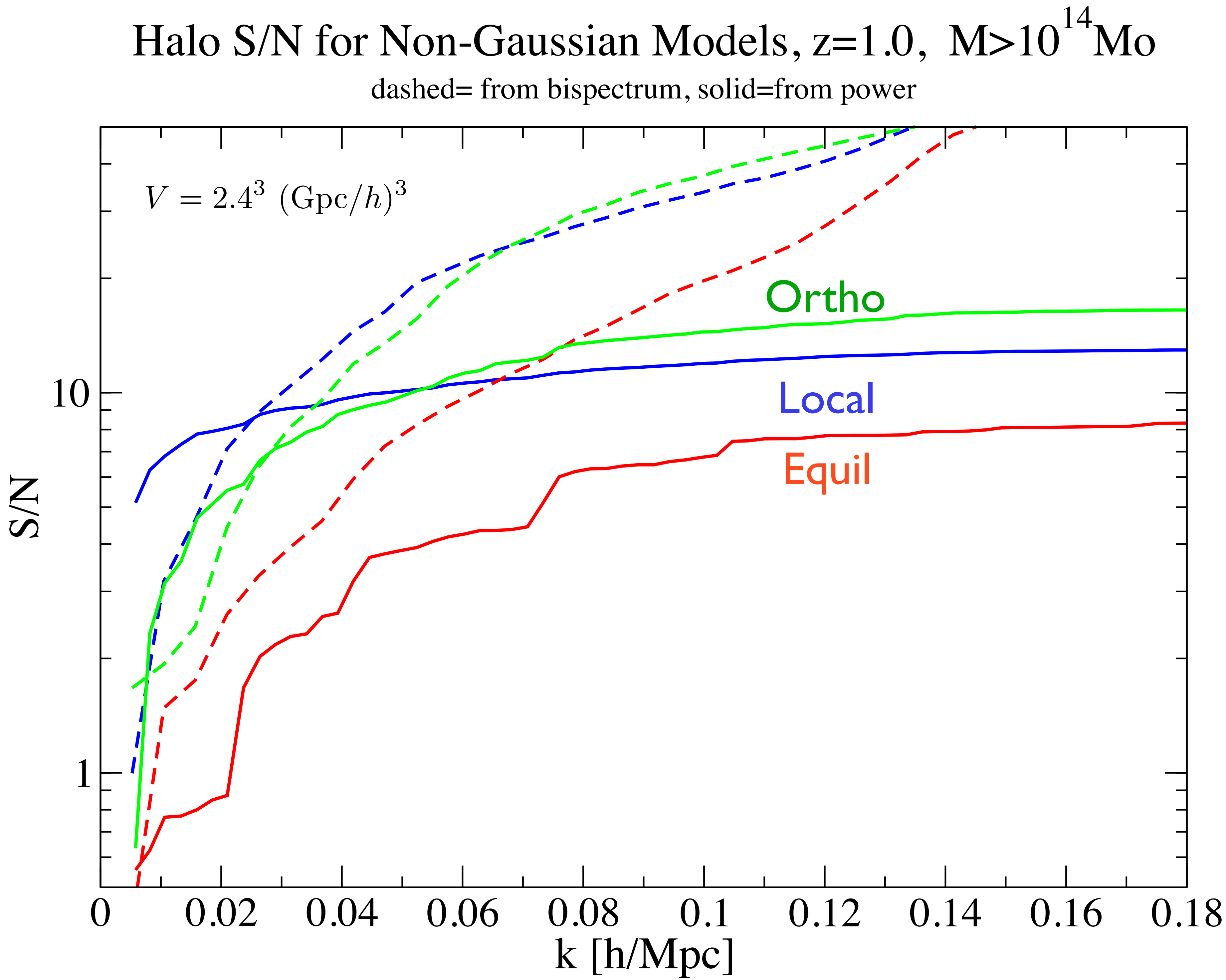
Name	Sample	Lbox	Npar	mpar	Nrealiz
Oriana (G)	LRG +Main -22	2400	1280^3	4.57E+11	42
Oriana fnl_local=+100	LRG +Main -22	2400	1280^3	4.57E+11	12
Oriana fnl_equi=-400	LRG +Main -22	2400	1280^3	4.57E+11	12
Oriana fnl_orto=-400	LRG +Main -22	2400	1280^3	4.57E+11	12
Carmen	Main -21	1000	1120^3	4.98E+10	42
Esmeralda	Main -20	640	1250^3	9.31E+09	50
Consuelo	Main -19-18	420	1400^3	1.87E+09	50

Nmocks=4 x Nrealiz, 2LPT ICs, Gaussian Mocks available at http://lss.phy.vanderbilt.edu/lasdamas/


CrossBias from Oriana Simulations z = 0,0.34,0.97
$b = P_{hm}/P_{mm}$
10^1
10^{-2}
10^{-1}
k [h/Mpc]
Local
Ortho
Equil
Gaussian

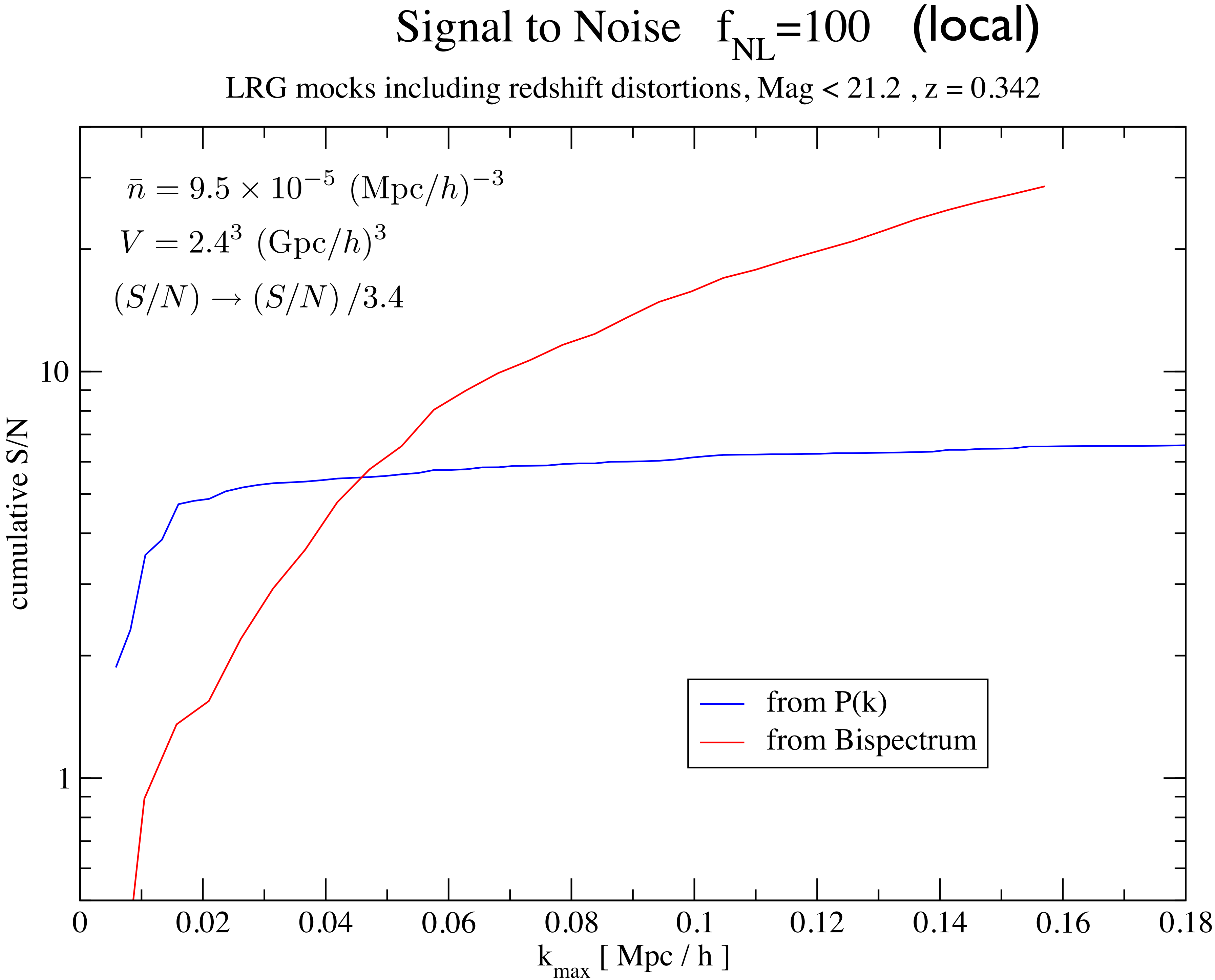


NonGaussian Matter Bispectrum (k_1 =0.0105 Mpc/h k_2 =1.5k_1 z=0.974)
Ortho
Local
Equil
$Q(k_1, k_2, \theta) - Q_{Gauss}$
θ/π
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.2
0.0
0.2
0.4
0.6
0.8
1.0



Halo S/N for Non-Gaussian Models, z=1.0, M>10^14 Mo
dashed= from bispectrum, solid=from power
$V = 2.4^3\ (\mathrm{Gpc}/h)^3$
Ortho
Local
Equil
S/N
10
1
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
k [h/Mpc]

Adding Bispectrum information helps a lot...



The Bispectrum of SDSS Galaxies

Some features of the analysis:

- Blind analysis: clustering data scrambled by random numbers, method of analysis decided on mock catalogs (including challenge cosmologies+NGICs) and then fixed *before* unscrambling.

- Gaussian and inflationary-motivated non-Gaussian initial conditions

- 2-loop RPT for the mass (Crocce & Scoccimarro; Bernardeau at al)
- Redshift distortions beyond PT (extension of Scoccimarro 04)
- Galaxy bias beyond local approximation (even for Gaussian ICs)

- Full (non-Gaussian) power+bispectrum covariance matrix determined from analytic (beat-coupling approx) and numerical approaches (PTHalos, and N-body simulations + HOD from LasDamas). Includes luminosity-dependent bias.

- Bispectrum Eigenmodes + non-Gaussian likelihood (Scoccimarro 00; Gaztañaga & Scoccimarro 05)

Conclusions

- RPT provides adequate description of DM nonlinearities at the level needed in current observations.

- Local bias does not explain the low-k behavior of the bias of mid-size halos.

- PBS calculations have been generalized to non-local PNG models. Currently testing these in detail.

- Bispectrum adds significant (S/N) to constrain PNG.

- SDSS bispectrum shows expected configuration dependence generated by gravity, multiple scales/configurations detected at high signal-to-noise.

- Stay tuned for quantitative constraints on cosmology, bias, gravity and PNG!