Constraining Dark Energy with Clusters of Galaxies

Carlos Cunha

University of Michigan

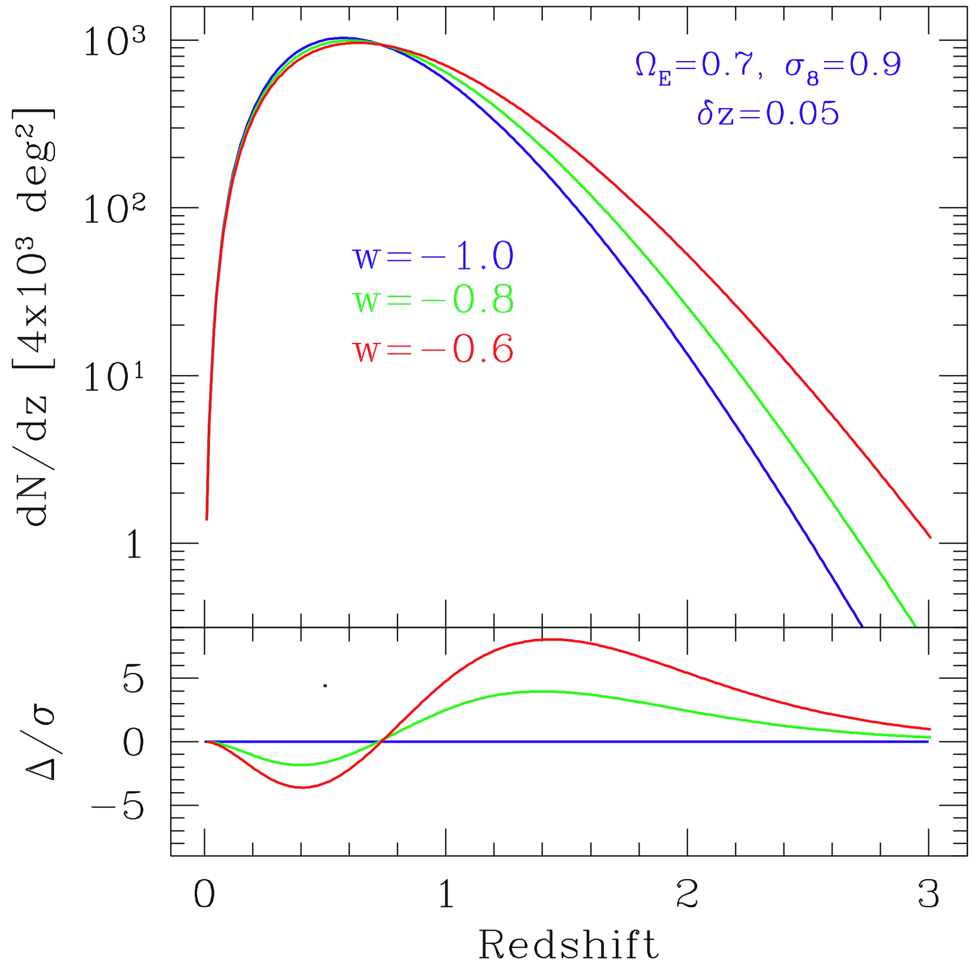
Benasque, August 10, 2010

Cluster sensitivity to cosmology

Models are normalized to produce same cluster abundance at low redshifts

For models with **larger w**:

- **less** volume --> **less** clusters at **low** redshift.
- structure grows **less** rapidly --> **more** clusters at **high** redshift.



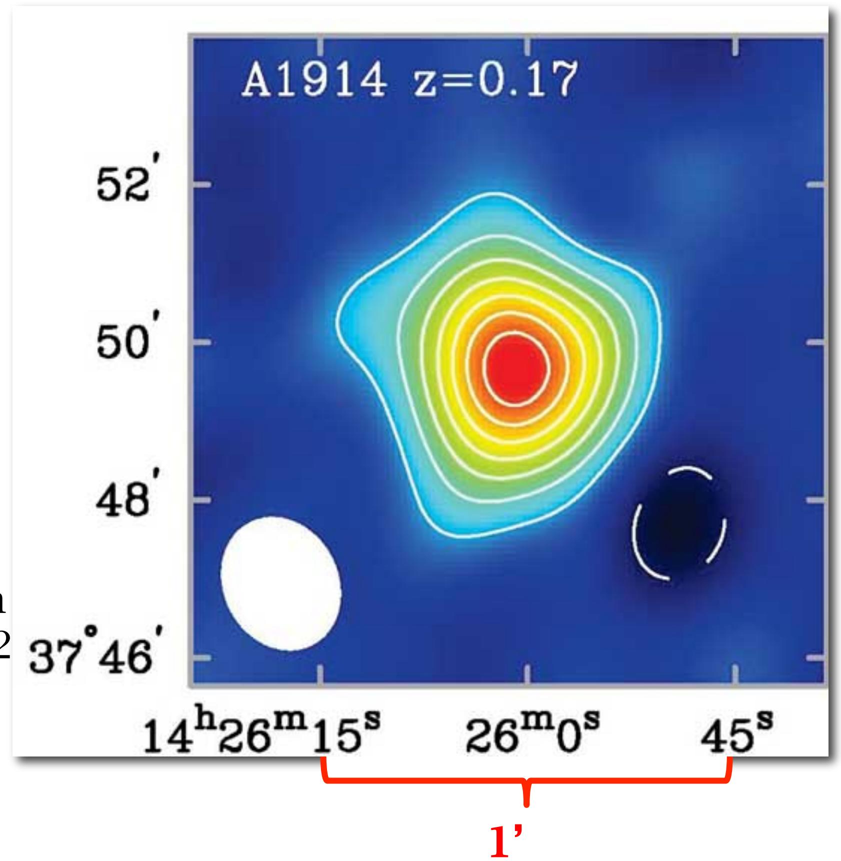
Mohr, J. 2005

Cluster counts and mass estimation

Cluster mass is not an observable. We must rely on observable proxies for mass, which have intrinsic scatter. For example:

- **Sunyaev Zel'dovich (SZ)**: Inverse Compton scatter of CMB photons off **cluster gas** (electrons)
  - *No redshift dependence, low scatter, but high* M_{th} *(~*$10^{14.2}\ h^{-1}\ M_{sun}$*)*

Carlstrom et al. 2002



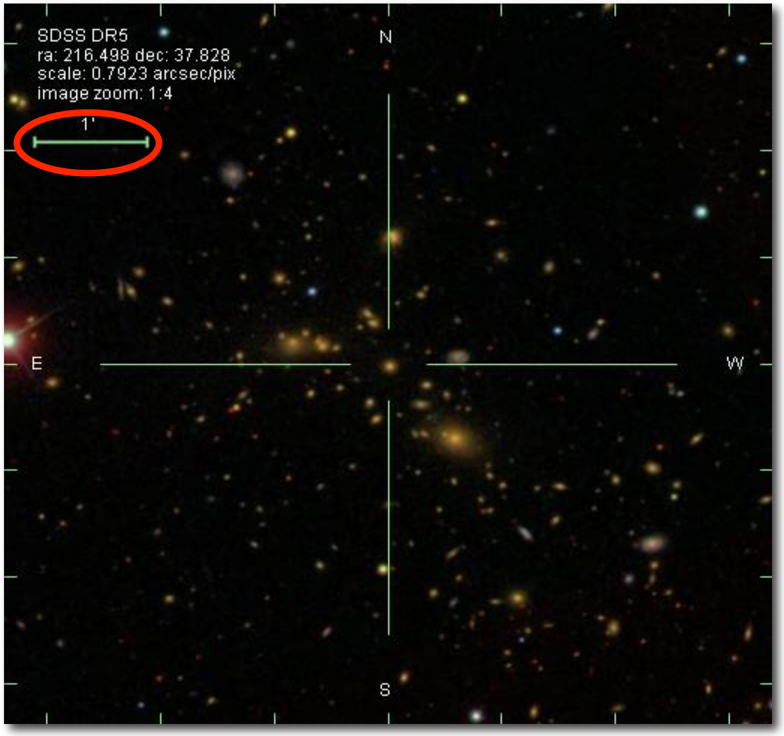
Cluster Abell 1914

Cluster counts and mass estimation

Cluster mass is not an observable. We must rely on observable proxies for mass, which have intrinsic scatter. For example:

Optical/IR: clustering of **galaxies** in physical and color space

– *High-scatter, but lowest M_{th} ($\sim 10^{13.5}$ - 10^{14} h^{-1} M_{sun}).*



SDSS image

Cluster Abell 1914

Cluster counts and mass estimation

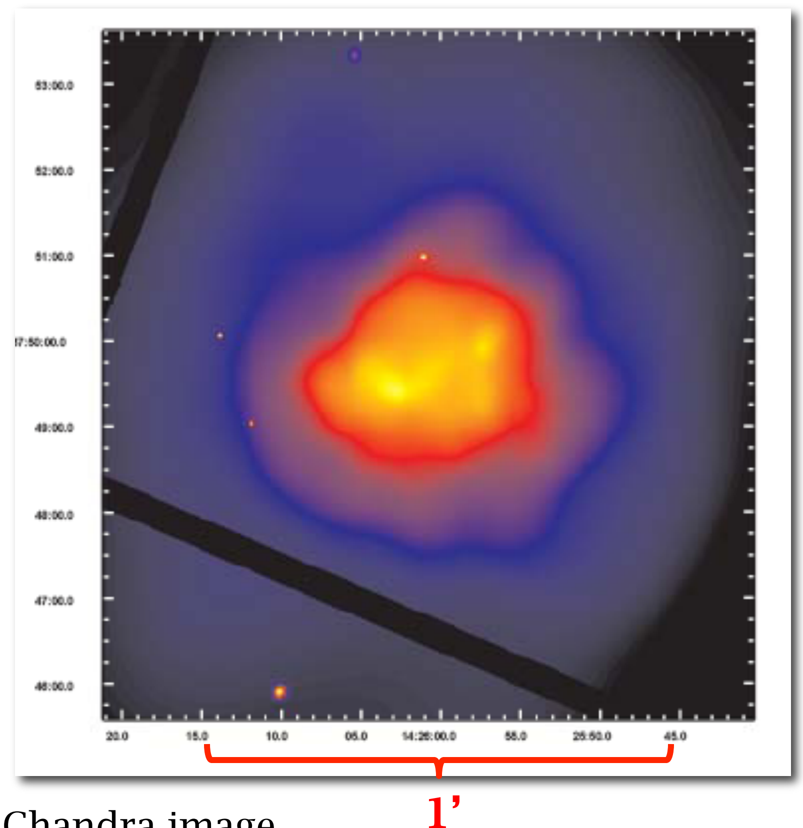
Cluster mass is not an observable. We must rely on observable proxies for mass, which have intrinsic scatter. For example:

X-ray: thermal emission from hot cluster **gas**

– *Many choices of observables. Temperature (from spectrum), luminosity, f_{gas} .*

– *Assumes hydrostatic equilibrium. Lowest scatter, but risk of bias, depending on observable.*

Cluster Abell 1914



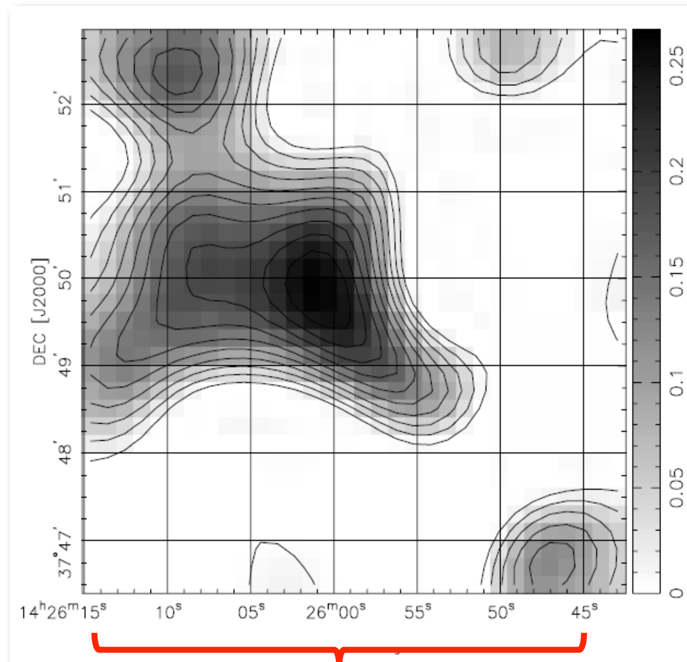
Chandra image
Jeltema et al. 2005

Cluster counts and mass estimation

Cluster mass is not an observable. We must rely on observable proxies for mass, which have intrinsic scatter. For example:

Weak Lensing: Directly probes **(projected) total mass**.

– *Low redshift. Requires stacking (does it? -> shear peaks)*



1’

Cluster Abell 1914

Shear map from Dahle et al. 2002

Predicting cluster counts

The mean number of clusters with mass $M > M_{th}$ is given by

$$\bar{m}(M > M_{th}, z) = \underbrace{\int dV}_{\text{Depends on geometry.}} \int_{M_{th}}^{\infty} d\ln M \underbrace{\frac{dn}{d\ln M}}_{\text{Mass Function}}$$

Depends on geometry.

Mass Function (eg. Jenkins).
Derived from power spectrum.
Depends on cosmology through growth of structure.

Can get more information by **binning** in redshift and mass:

$$\bar{m}(M_i, z_k) = \int d\Omega \int_{z_k}^{z_{k+1}} dz \frac{D_A^2(z)}{H(z)} \int_{M_i}^{M_{i+1}} d\ln M \frac{dn}{d\ln M}$$

Things get more complicated when we include errors in redshift and mass, selection effects, and observational errors …

Predicting cluster counts

The **mean number of clusters** in a bin of $\vec{M}_{obs}=(M^1_{obs}, M^2_{obs}, \ldots, M^n_{obs})$ and z_{phot} is

$$\overline{m}(\vec{M}_{obs}, z_{phot}) = \int dz_{phot} \int d\Omega \int d\ln M \int d\ln \vec{M}_{obs}$$

$$\frac{dn}{d\ln M} P(\vec{M}_{obs}, z_{phot} \mid M, z) P(\vec{M}_{obs}) \phi(\vec{M}_{obs}, z_{phot})$$

Theoretical uncertainties

Errors in <u>observables</u> and photometric redshifts

Observational errors (photon counts, …)

Selection function

Mass is not directly observable => must rely on indirect proxies for mass which are statistically related to the true mass.

Modeling $P(\vec{M}_{obs} \mid M)$

Simulations and observations suggest mass-observable errors are (mostly) well-represented by **log-normal** distributions:

$$P(M^i_{obs} \mid M) = \frac{1}{\sqrt{2\pi\sigma^2_{\ln M}}} \exp\left[-x^2(M^i_{obs}, M)\right]$$

where

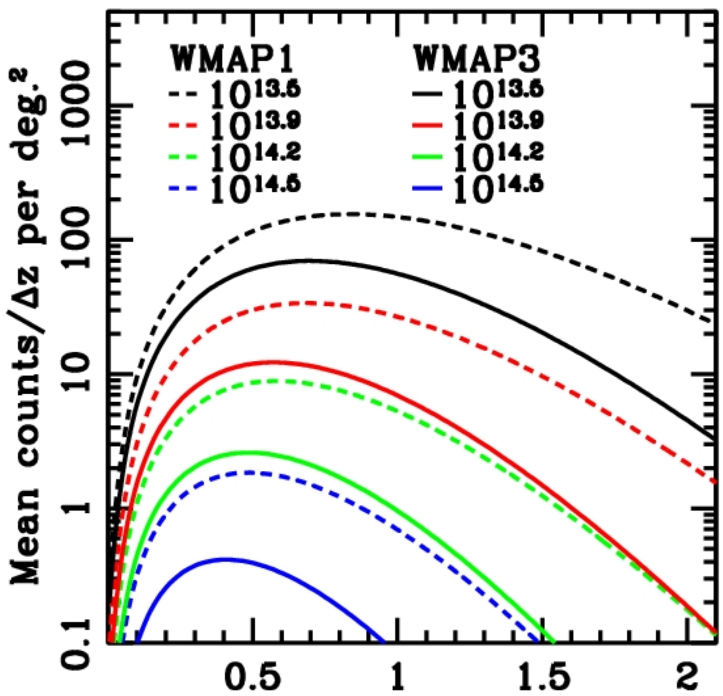
$$x(M^i_{obs}, M) = \frac{\ln M^i_{obs} - \ln M - \ln M_{bias}}{\sqrt{2\sigma^2_{\ln M}}}$$

$$\ln M_{bias} = \ln M_{bias}(M, z)$$

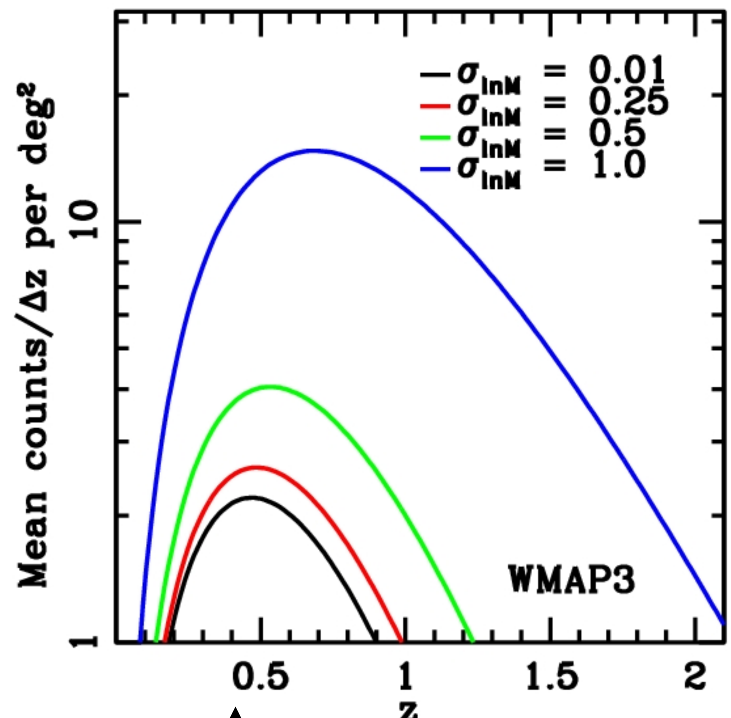
$$\sigma^2_{\ln M} = \sigma^2_{\ln M}(M, z)$$

Cluster Counts

Cosmological and **mass nuisance** parameters affect counts in similar ways:



$\sigma_{\ln M} = 0.25$



$M_{th} = 10^{14.2} h^{-1} M_{sun}$

Noise in Counts

• Counts in an angular bin will deviate from the mean because of **Poisson errors** (shot noise) and **large-scale clustering**.

• If we can predict the **noise** given the cosmology and mass-nuisance parameters, then we can use the noise to improve constraints!

Using clustering information is one of the techniques referred to as self-calibration.

Lima & Hu (2004, 2005, 2007)

Predicting the Noise

The sample covariance of the counts due to clustering is

$$S_{ij} = \left\langle (m_i - \overline{m_i})(m_j - \overline{m_j}) \right\rangle$$

Observable

$$= \frac{b_i \overline{m_i}\; b_j \overline{m_j}}{V_i\, V_j} \int \frac{d^3k}{(2\pi)^3} W_i^*(k) W_j(k) P(k)$$

Theoretical Prediction

i, j: Correspond to redshift bins.

Overbars correspond to spatial averages at fixed redshift bins.

Cross-Calibration

Parametrization of OPT + SZ

Optical:

- Lower M_{th} ($10^{13.5-13.7}$)
- Lower max. redshift (z_{max}~1)
- Larger scatter (σ_{lnM}=0.5)

10 nuisance parameters

SZ:

- Higher M_{th} ($10^{14.2-14.4}$)
- Higher max. redshift (z_{max}~2)
- Lower scatter (σ_{lnM}=0.25)

6 nuisance parameters

Describing **redshift** and **mass evolution** of **mass-observable** relation (3^{rd} order polynomials).

Based on expected sensitivities of DES (optical) and SPT (SZ) surveys.

Cross-Calibration

Old:

Method I $\longrightarrow \overline{m}(M^{1}_{obs}, z) \longrightarrow \ln L_1$

$+$

Method II $\longrightarrow \overline{m}(M^{2}_{obs}, z) \longrightarrow \ln L_2$

$$\ln L_1 + \ln L_2 \longrightarrow \ln L_{tot}$$

New:

Method I, Method II $\longrightarrow \overline{m}(M^{1}_{obs}, M^{2}_{obs}, z) \longrightarrow \ln L_{tot}$

Results

SZ

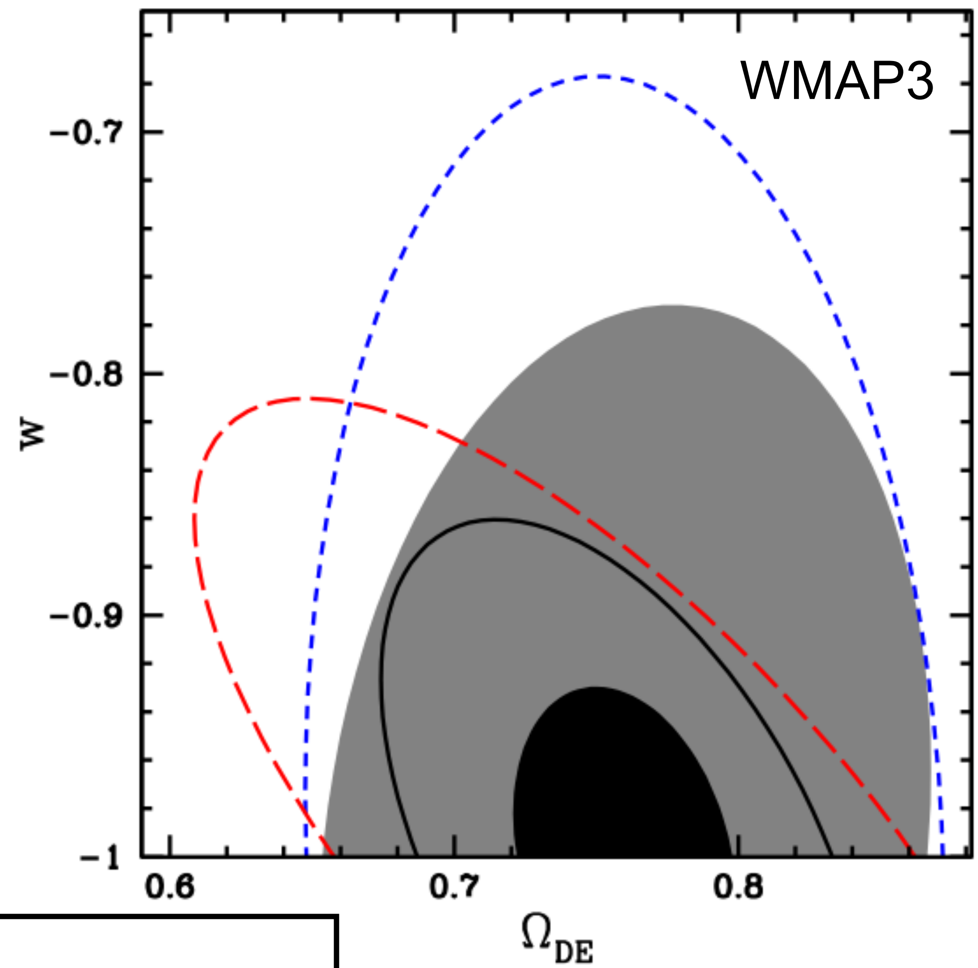
OPT

SZ + OPT

Cross. cal. using only SZ ∩ OPT

Full cross-calibration

No priors on 16 nuisance parameters



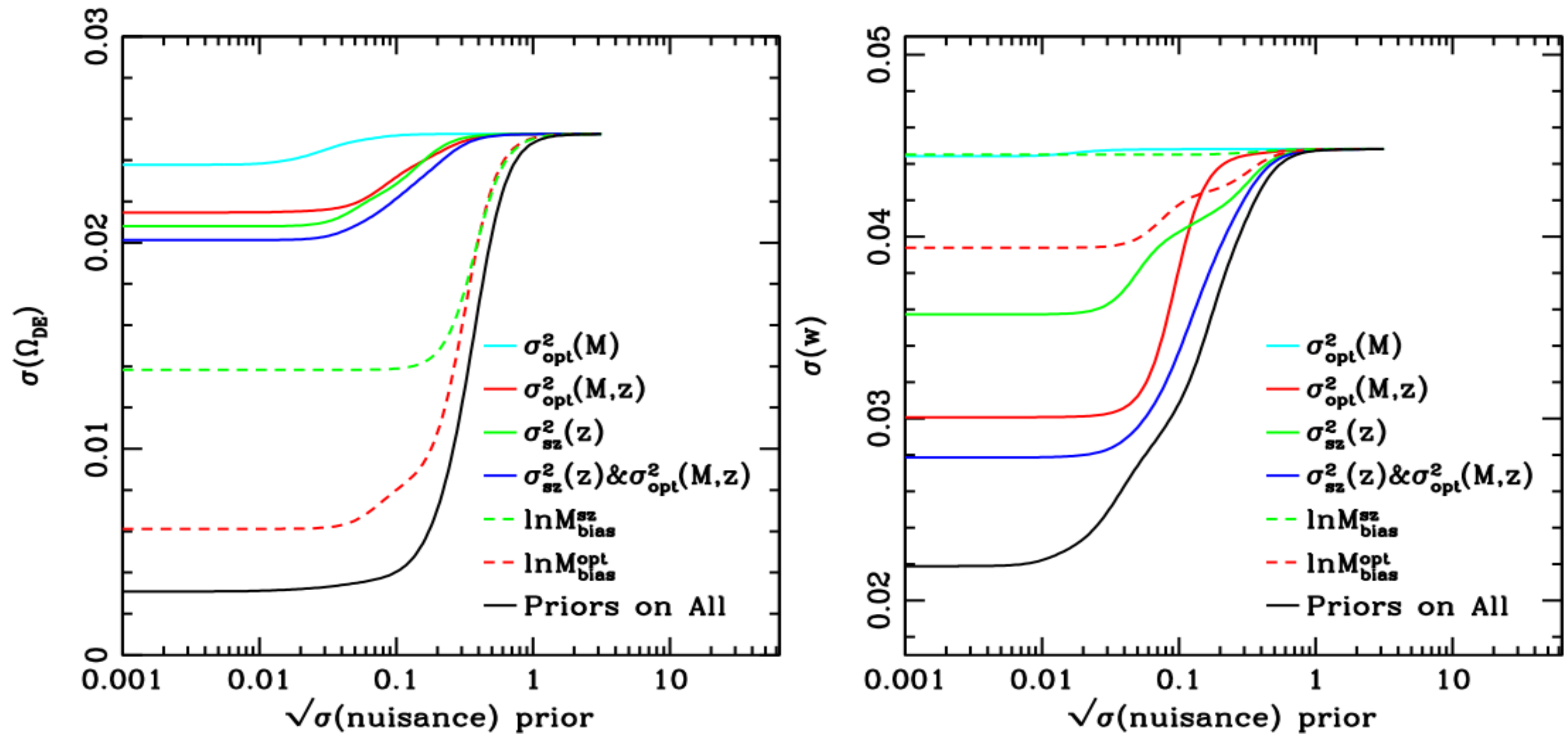
Correlation: ρ=0 (fixed)

1% priors on: $\Omega_m h^2$, $\Omega_b h^2$, n (tilt), ln A_s

Cunha (2009)

Results

Priors on nuisance parameters will further improve constraints



A sample of ~100 clusters with detailed mass modeling is needed to achieve $\sigma(\sigma^2_{\ln M}) \sim (0.1)^2$ at z~1.

Cunha (2009)

Degeneracies

- A model for the cluster counts is a function of the **cosmological** and the **mass nuisance** parameters, which can be **degenerate**.
- Choice of parametrization affects results only up to a point.
- Dark energy effects on growth and geometry are very smooth, so only low-order evolution of systematics matters.
- If we can trust simulations and physical models for errors, much stronger constraints are possible.
- Simulations do not need to estimate parameters correctly, the cross-calibration can do that.

Uncertainties in the mass function

The mass function

Recall:

$$\bar{m}(M > M_{th}, z) = \int dV \int_{M_{th}}^{\infty} d\ln M \, \frac{dn}{d\ln M}$$

$$\frac{dn(M,z)}{dM} = f(\sigma)\frac{\bar{\rho}_m}{M}\frac{d\ln\sigma^{-1}}{dM}$$

Choice: Tinker mass function (Tinker et al. 2008)

$$f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right]e^{-c/\sigma^2}$$

Parameters:

$$A(z) = A_0(1+z)^{A_x}$$

$$a(z) = a_0(1+z)^{a_x}$$

$$b(z) = b_0(1+z)^{-\alpha}$$

$A_0 = 0.186$	$A_x = -0.14$
$a_0 = 1.47$	$a_x = -0.06$
$b_0 = 2.57$	$\alpha = 0.0107$
$c = 1.19$	

The galaxy bias

$$b(M;z) = 1 + \frac{a_c\delta_c^2/\sigma^2 - 1}{\delta_c} + \frac{2p_c}{\delta_c\left[1 + (a_c\delta_c^2/\sigma^2)^{p_c}\right]}$$

$$a_c = 0.75$$
$$\delta_c = 1.69$$
$$p_c = 0.30$$

Sheth & Tormen (1999)

Bias is only needed to calculate clustering of clusters (self-calibration).

In total, we have **7+3** nuisance parameters for the mass-function and linear bias, and **6** nuisance parameters describing the mass-observable relation, $P(M_{obs}|M)$.

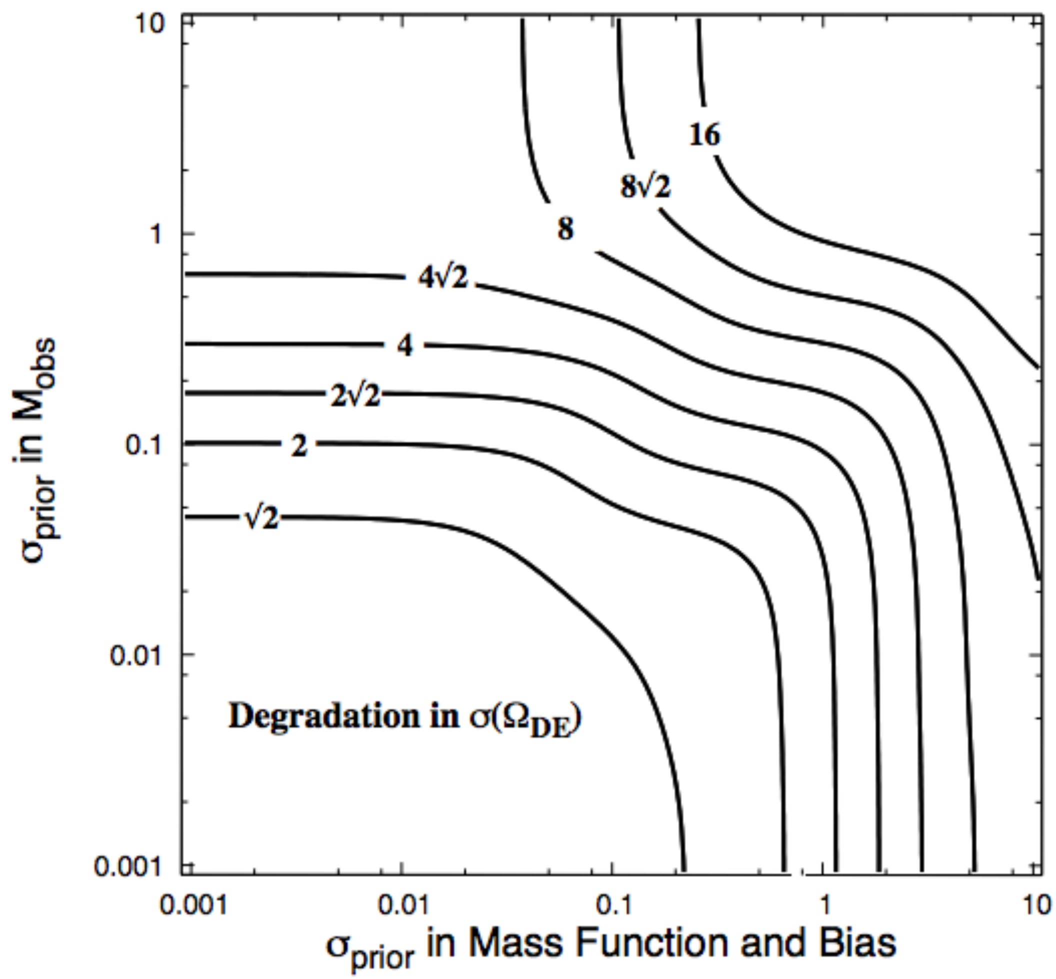
Degradation in $\sigma(\Omega_{DE})$

SZ Survey

Baseline constraints (infinitely sharp priors):

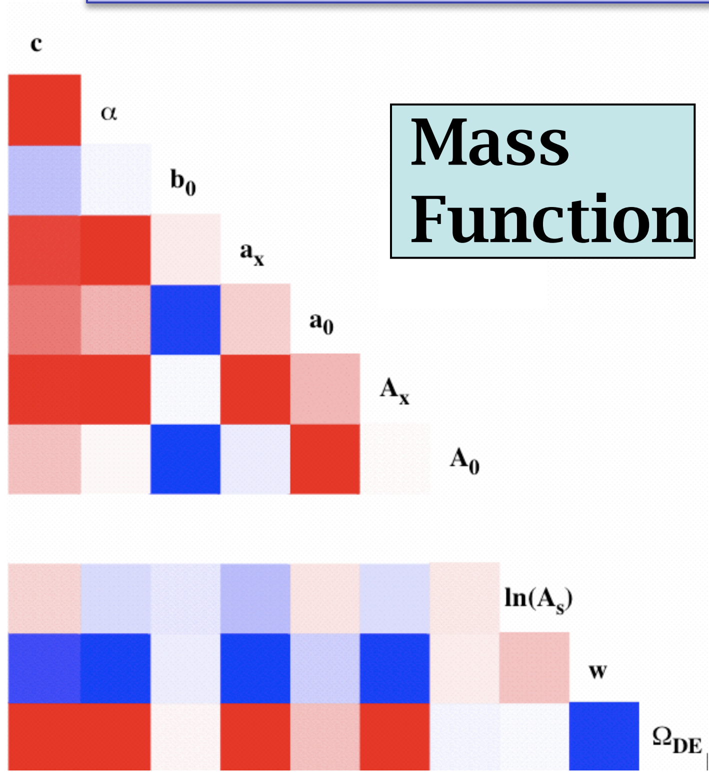
$$\sigma(\Omega_{DE}) = 0.01$$

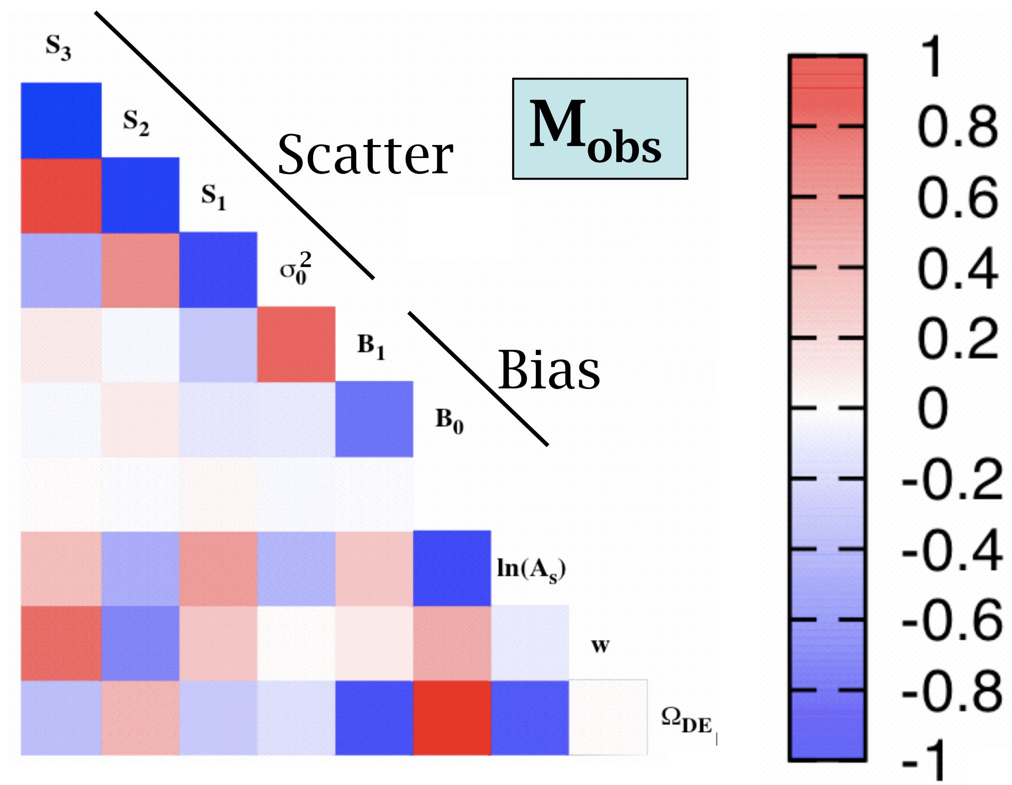
$$\sigma(w) = 0.05$$



Cunha & Evrard (2010)

Correlations (are complicated)





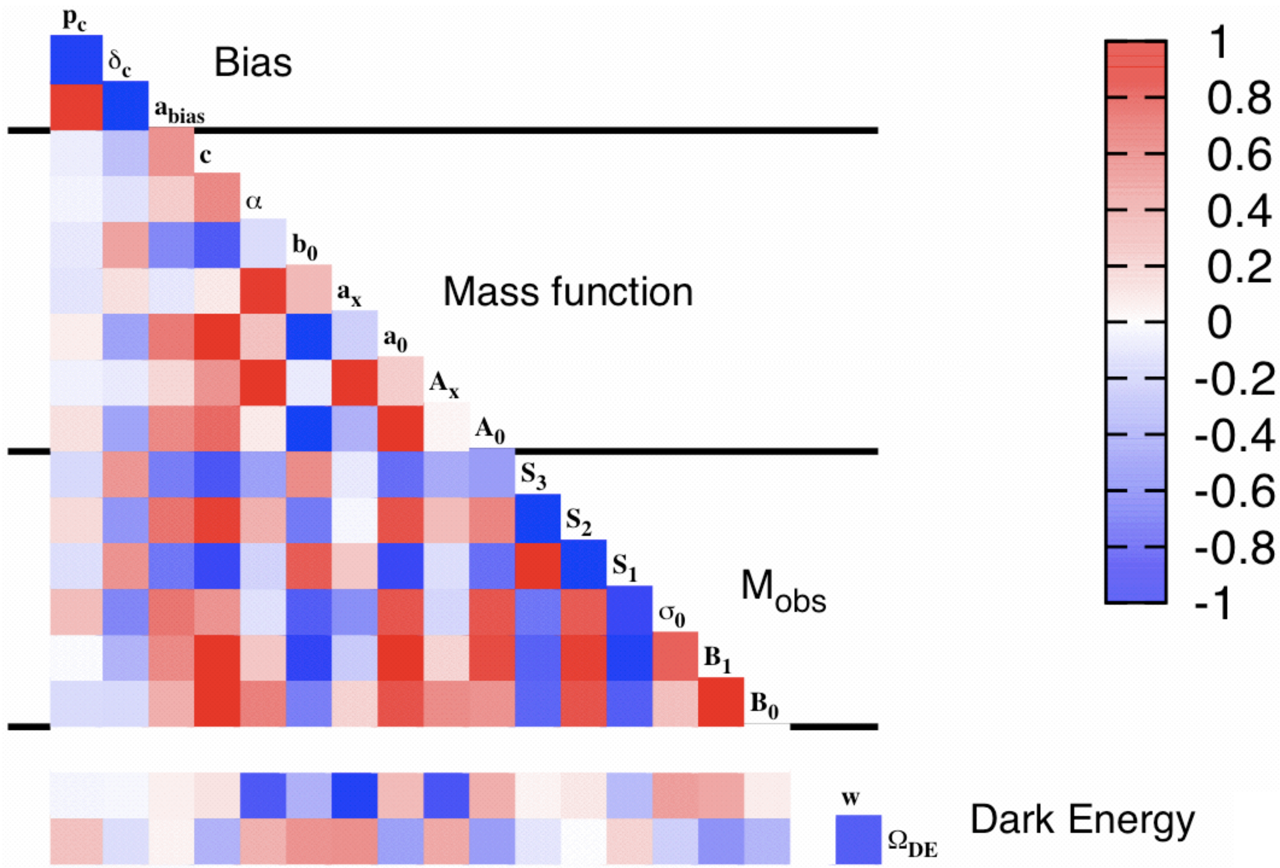
$$f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right]e^{-c/\sigma^2}$$

$$\sigma_{\ln M}^2 = \sigma_0^2 + \sum_{i=1}^{3} S_i z^i$$

$$\ln M_{bias} = B_0 + B_1 \ln(1+z)$$

Cunha & Evrard (2010)

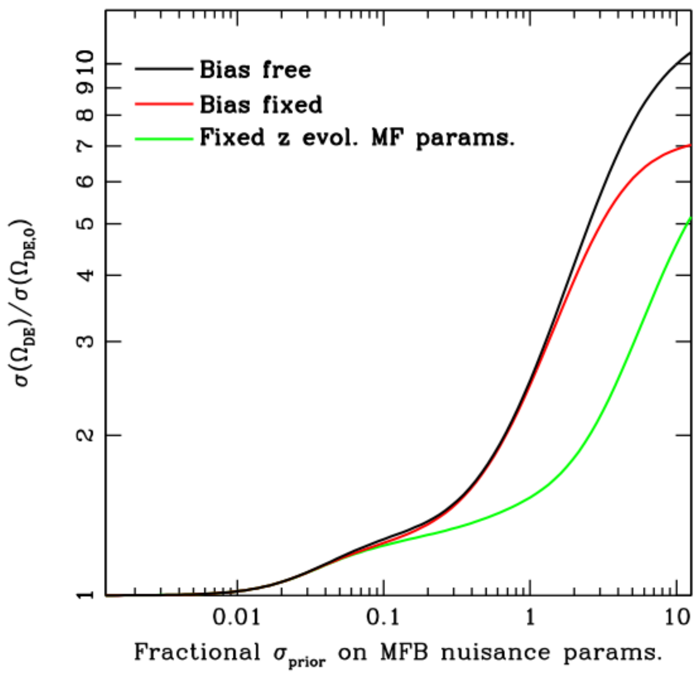
Very Complicated

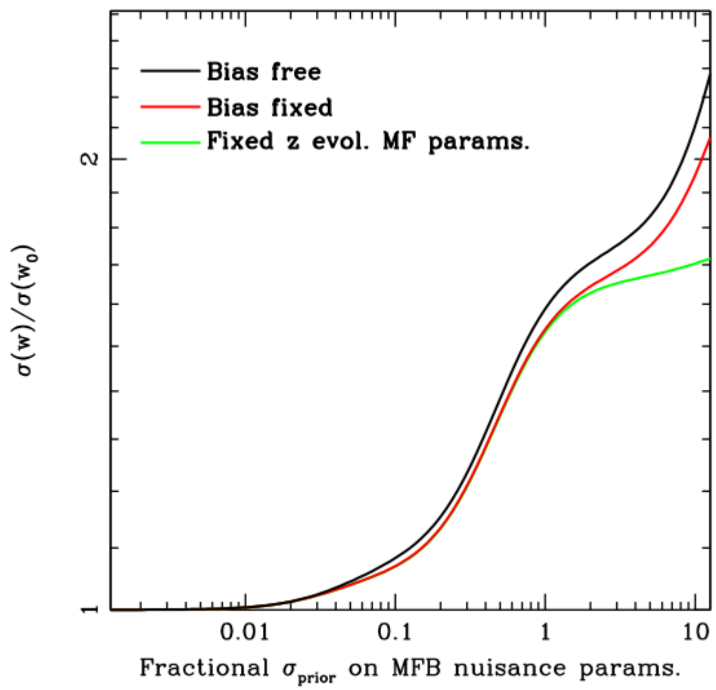


$$f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right]e^{-c/\sigma^2}$$

Cunha & Evrard (2010)

Mass function nuisance parameters





Nuisance parameters describing redshift evolution of mass function only dominate when prior uncertainty is large (nearly flat priors).

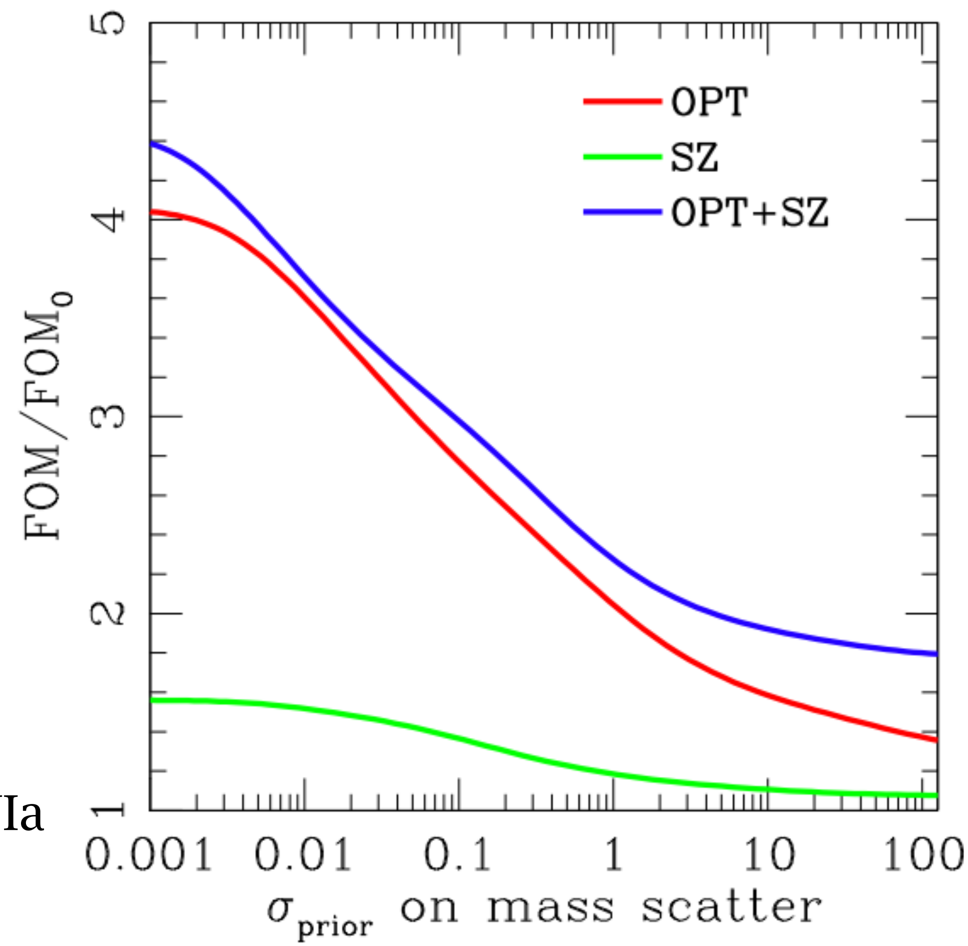
Cunha & Evrard (2010)

Complementarity to other DE probes

The DETF figure of merit

$$w(z) = w_0 + (1-a)w_a$$
$$= w_p + (a_p - a)w_a$$

$$\boxed{FOM = \frac{1}{\sigma(w_p)\times\sigma(w_a)}}$$



Fiducial: FOM_0=116 WL+SNIa +Planck+BAO

w/ Clusters: FOM=206 - 512 WL+SNIa +Planck+BAO+Clusters

Cunha, Huterer & Frieman (2009)

Sensitivity to modifications of gravity

Fit to growth equation:

$$\ln g(a) = \int_0^a d\ln a\left[\Omega_m^{\gamma} - 1\right]$$

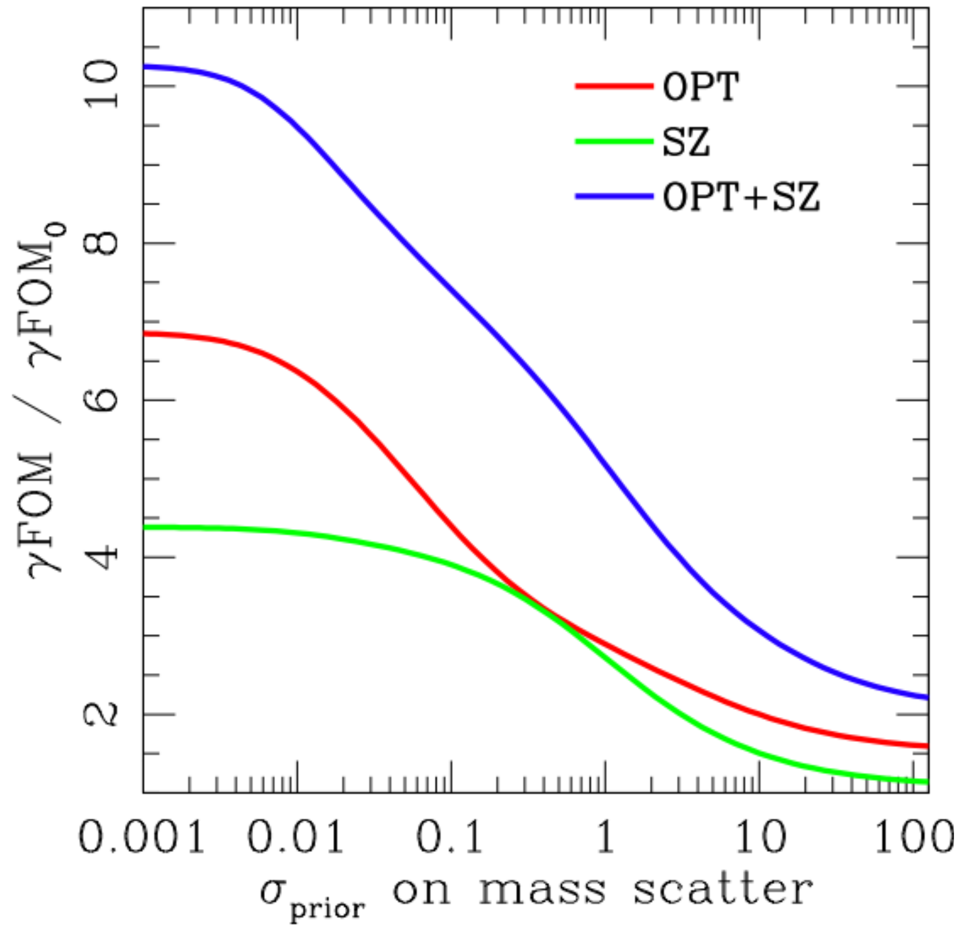
For General Relativity:

$$\boxed{\gamma = 0.55}$$

$$\gamma FOM = \frac{1}{\sigma(\gamma)}$$

Fiducial: **γFOM$_0$=4.8**
WL+SNIa+Planck+BAO

w/ Clusters: **γFOM=10 - 50**
WL+SNIa+Planck+BAO+Clusters



Cunha, Huterer & Frieman (2009)

Primordial non-Gaussianity from Clusters

Counts + Clustering of Clusters is relatively insensitive to systematics – see Dragan Huterer's talk, Cunha et al (2010), Sartoris et al (2010), Oguri (2009).

Clusters vs. Galaxies

- Galaxy catalogs will be much bigger but,
- It's the large halo-halo separations that have the signal
- Clusters are more directly related to the haloes.

Conclusions

- Cross-cal:
  - constrain mass-observable relations
  - understand selection

  Applications

  - test with simulations, real data

  Future

- Better/more simulations to constrain mass function are **very desirable**. Simulations to understand mass-observable relation are **essential** (and feasible).

- Interesting constraints are possible even with large uncertainties in sources of errors.

- Very interesting constraints if priors are available.

Cluster Research in Michigan

- Gus Evrard
- Tim McKay
- Dragan Huterer
- Elena Rasia
- David Gerdes
- Chris Miller
- Jeff McMahon
- Oleg Gnedin

- Brian Nord
- Rebecca Stanek
- Anbo Chen
- Brandon Smith

- CC
- Jorg Dietrich

- Optical
- SZ
- X-Ray
- WL
- Simulations
- Observations
- Selection
- Contamination
- Mass-observable relation
- Substructure
- Photometric redshifts
- Cross-calibration
- Cosmological constraints

Modeling $P(\vec{M}_{obs} | M)$

If all the $P(M | M)$ are Gaussians, so is their product. For two mass-observables the total probability $P(\vec{M}_{obs} | M)$ can expressed as a **bivariate lognormal** distribution.

$$P(\vec{M}_{obs} \mid M) = \frac{1}{\sqrt{2\pi|C|}} \exp\left[-x^T C^{-1} x\right]$$

Where C is the covariance matrix, defined as

$$C = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_1^2 \end{pmatrix}$$

And ρ is the correlation coefficient (-1 < ρ < 1*).*

OPT+SZ parameter space

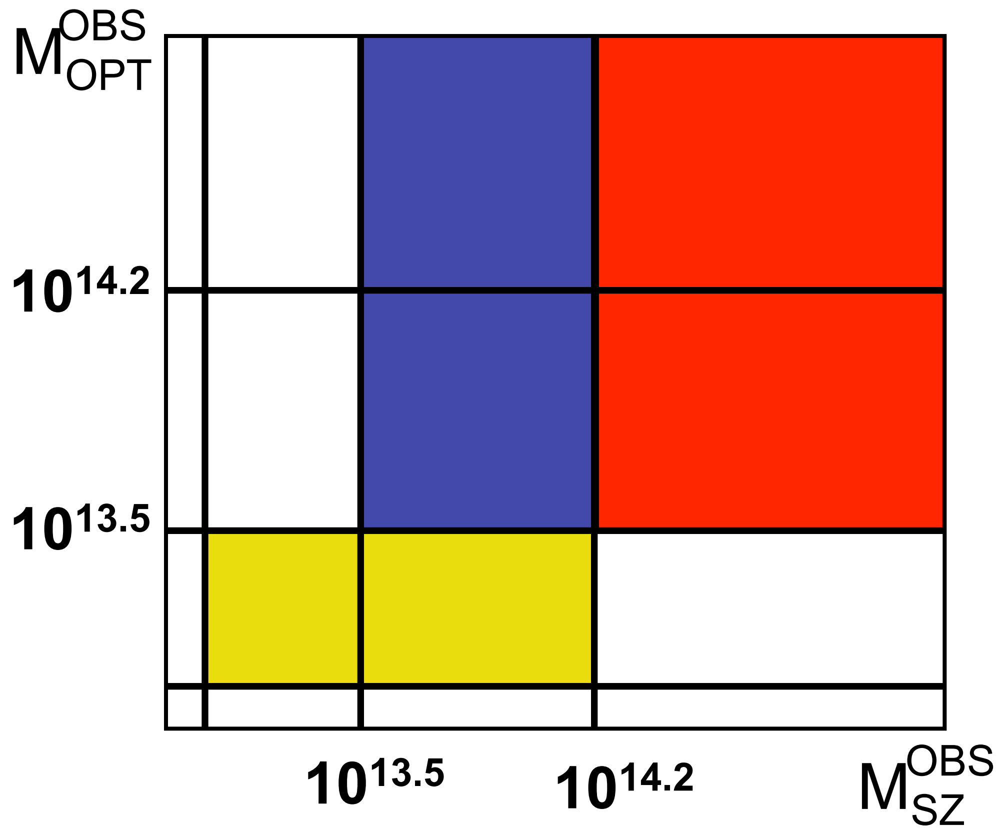
Selection

SZ+OPT

OPT

TRULY EMPTY

NEARLY EMPTY



0 < z < 1

OPT+SZ parameter space

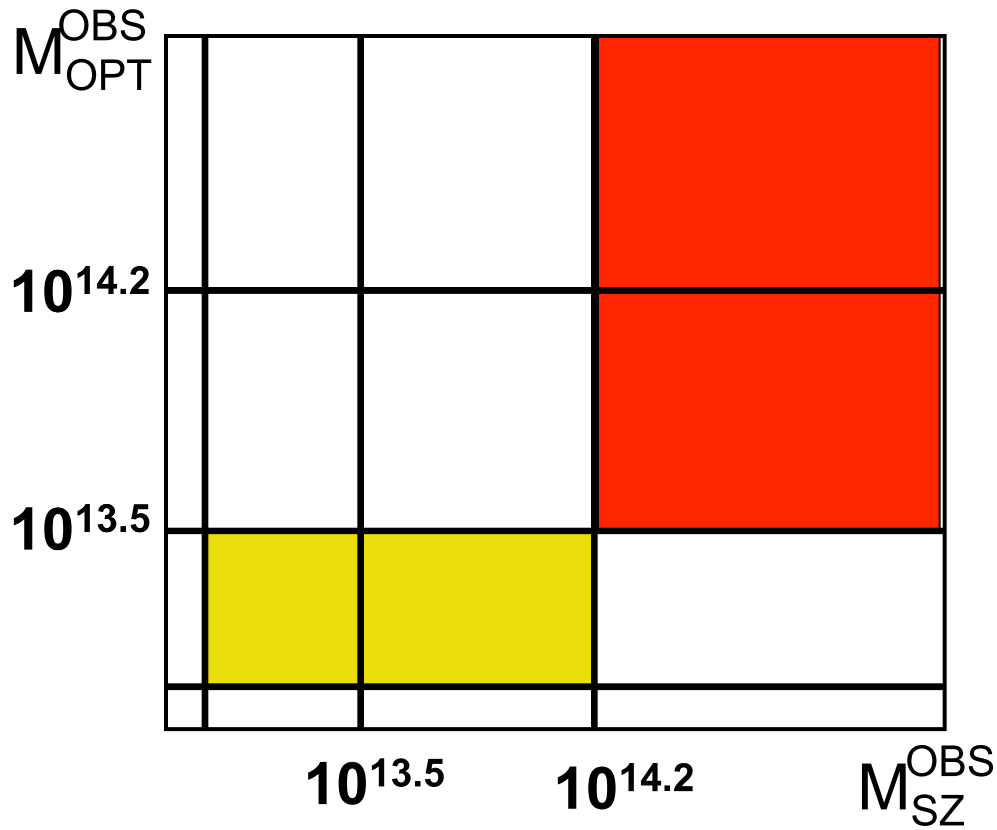
Selection (0<z<2)

SZ

EMPTY

TRULY EMPTY

TRULY EMPTY



1 < z < 2

Predicting cluster counts

Simplifying assumptions

- Work in range of M_{obs} and z_{phot} where selection is nearly complete:

$$\boxed{\phi(\vec{M}_{obs}, z_{phot}) = 1}$$

- Ignore observational errors:

$$\boxed{P(\vec{M}_{obs}) = 1}$$

- M_{obs} errors are separable from z_{phot} errors:

$$\boxed{P(\vec{M}_{obs}, z_{phot} | M, z) = P(\vec{M}_{obs} | M) P(z_{phot} | z)}$$

- Fixed $\sigma(z_{phot})=0.02$ errors for this talk.

The importance of being clustered

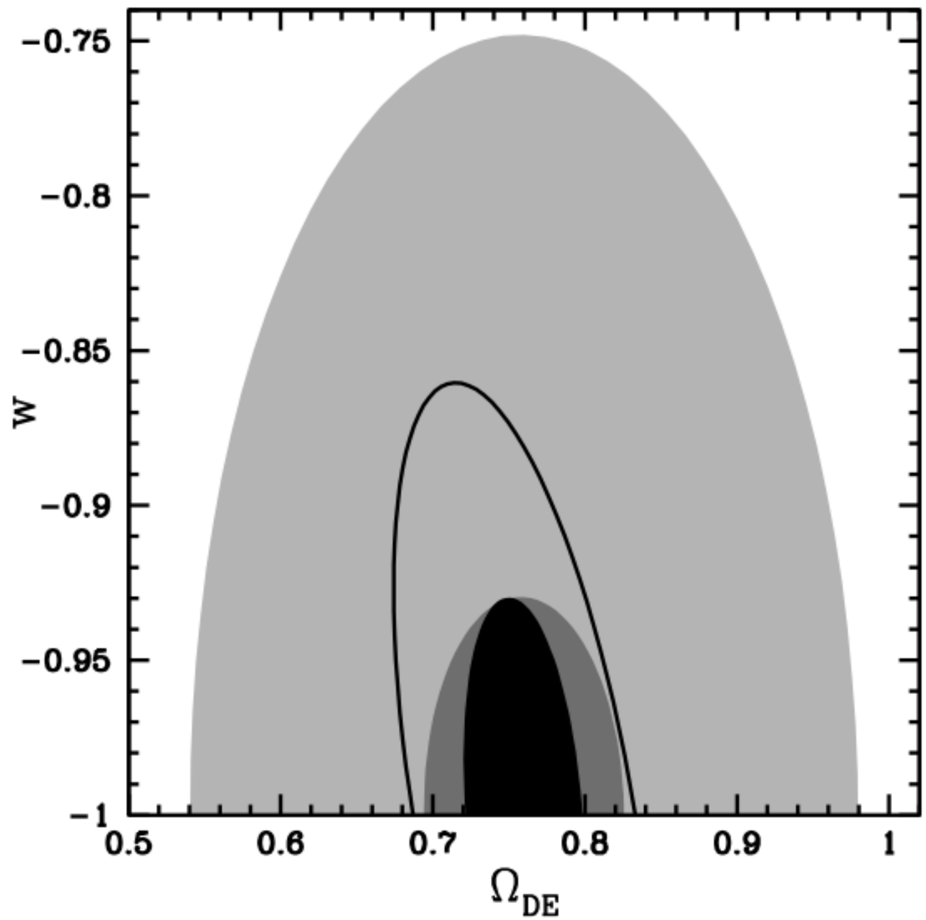
SZ + OPT (no clustering)

SZ + OPT

Full cross-cal. no clustering

Full cross-calibration

With full cross-calibration, clustering information only relevant for Ω_{DE} constraints.



Correlation: ρ=0 (fixed)

Cunha (2009)

Parametrization of OPT + SZ

Optical:

$$\ln M_{bias} = \ln M^0_{bias} + a_1(1+z) + \boxed{a_2(\ln M_{obs} - \ln M_{pivot})}$$

$$\sigma^2_{\ln M} = \sigma^2_0 + \sum_{i=1}^{3} b_i z^i \boxed{+ \sum_{i=1}^{3} c_i (\ln M_{obs} - \ln M_{pivot})^i}$$

$M_{th}=10^{13.5}\ h^{-1}\ M_{sun}$
$0 < z < 1$

$$\sigma_0^{fiducial} = 0.5$$

$$\ln M_{bias}^{fiducial} = 0$$

SZ:

$$\ln M_{bias} = \ln M^0_{bias} + a_1(1+z)$$

$$\sigma^2_{\ln M} = \sigma^2_0 + \sum_{i=1}^{3} b_i z^i$$

Based on expected sensitivities of DES (optical) and SPT (SZ) surveys.

$M_{th}=10^{14.2}\ h^{-1}\ M_{sun}$
$0 < z < 2$

$$\sigma_0^{fiducial} = 0.25$$

$$\ln M_{bias}^{fiducial} = 0$$