Inhomogeneous Universe Structure

- Backreaction
  - What is it?
  - Why is the standard argument that not a counter argument? (Ruth Durrer)
  - Discuss

Backreaction
Apparent acceleration
Observational constraints

Backreaction in a nutshell

$$
\begin{aligned}
3\frac{\ddot{a}}{a} &= -4\pi G_N(\rho + 3p) \\
3\frac{\dot{a}^2}{a^2} &= 8\pi G_N \rho - 3\frac{k}{a^2} \\
\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) &= 0 \,,
\end{aligned}
$$

$$\langle A \rangle \equiv \int d^3x \sqrt{{}^{(3)}g}\, A \Big/ \int d^3x \sqrt{{}^{(3)}g}$$

Backreaction in a nutshell

$$
\begin{aligned}
3\frac{\ddot{a}}{a} &= -4\pi G_N \langle\rho\rangle + \mathcal{Q} \\
3\frac{\dot{a}^2}{a^2} &= 8\pi G_N \langle\rho\rangle - \frac{1}{2}\langle\mathcal{R}\rangle - \frac{1}{2}\mathcal{Q} \\
& \partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0 \\
\mathcal{Q} &\equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle \ ,
\end{aligned}
$$

$a(t) \propto V(t)^{1/3}$ $V(t) = \int d^3x \sqrt{^{(3)}g}$ $\langle A\rangle \equiv \int d^3x \sqrt{^{(3)}g}\, A / \int d^3x \sqrt{^{(3)}g}$

Backreaction in a nutshell

$$3\frac{\ddot{a}}{a} = -4\pi G_N\langle\rho\rangle + \mathcal{Q}$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N\langle\rho\rangle - \frac{1}{2}\langle\mathcal{R}\rangle - \frac{1}{2}\mathcal{Q}$$

$$\partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0$$

$$\mathcal{Q} \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle\ ,$$

Expansion of local volume element

shear of local volume element

$a(t) \propto V(t)^{1/3}$

$V(t) = \int d^3x\sqrt{^{(3)}g}$

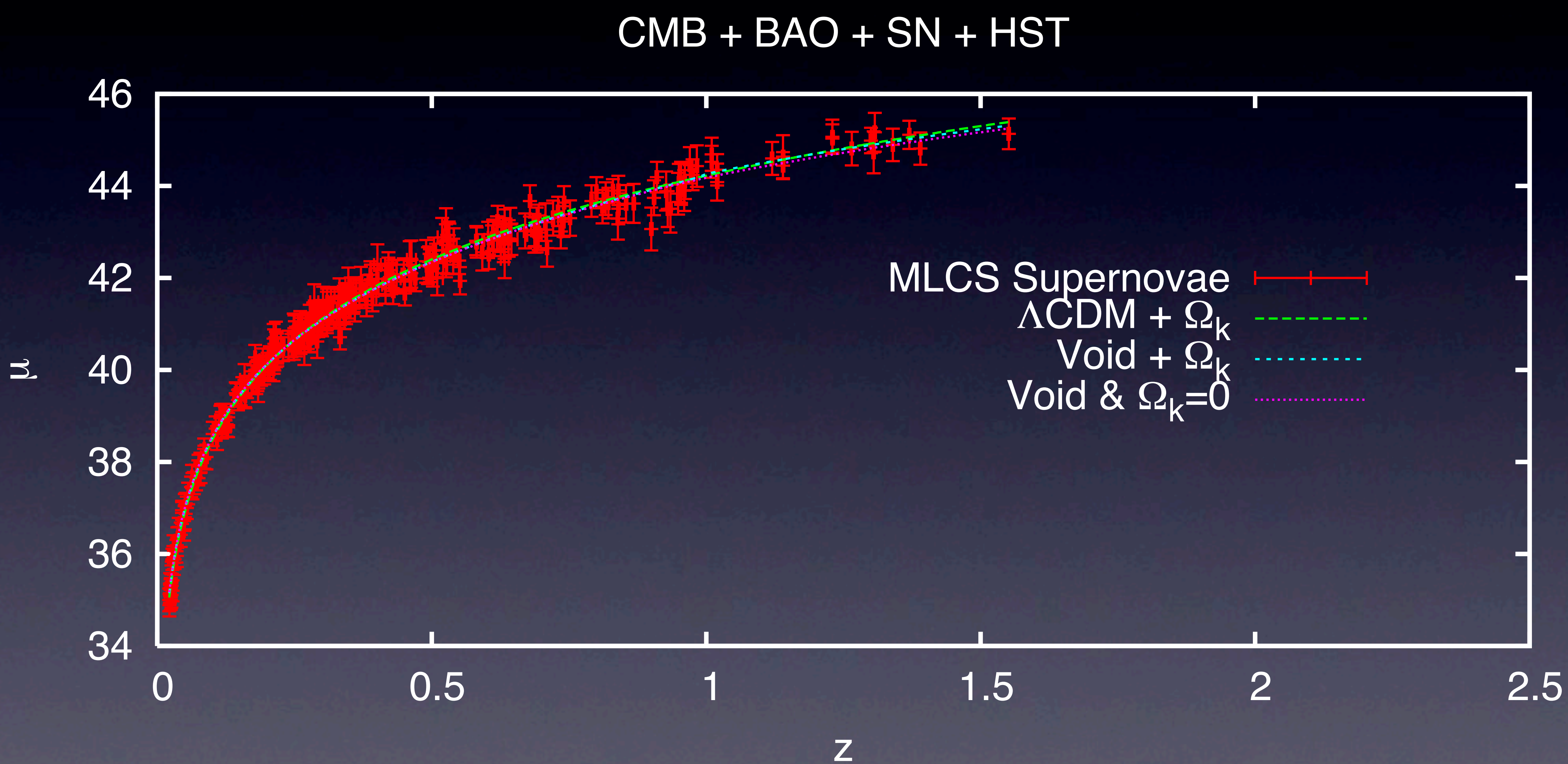
$\langle A\rangle \equiv \int d^3x\sqrt{^{(3)}g}\,A/\int d^3x\sqrt{^{(3)}g}$

Inhomogeneous Universe Structure

- Constraining power of CMB (Ruth Durrer)
  - Discuss
- Inhomogeneities and LCDM (Romano)
- Apparent acceleration:
  - How does it (usually) work?
  - Observational constraints
  - Discuss

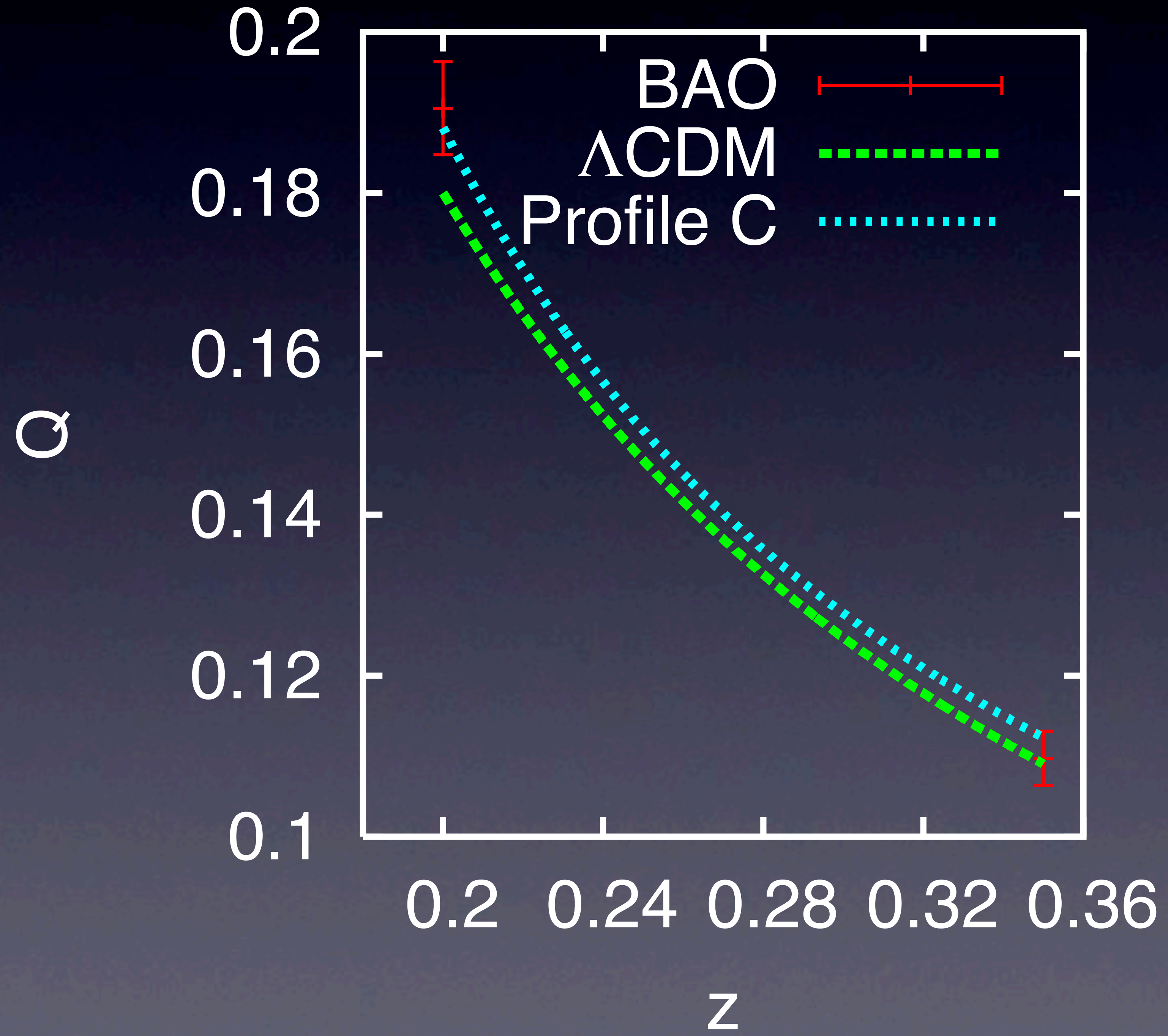
Backreaction
Apparent acceleration
Observational constraints

Supernovae

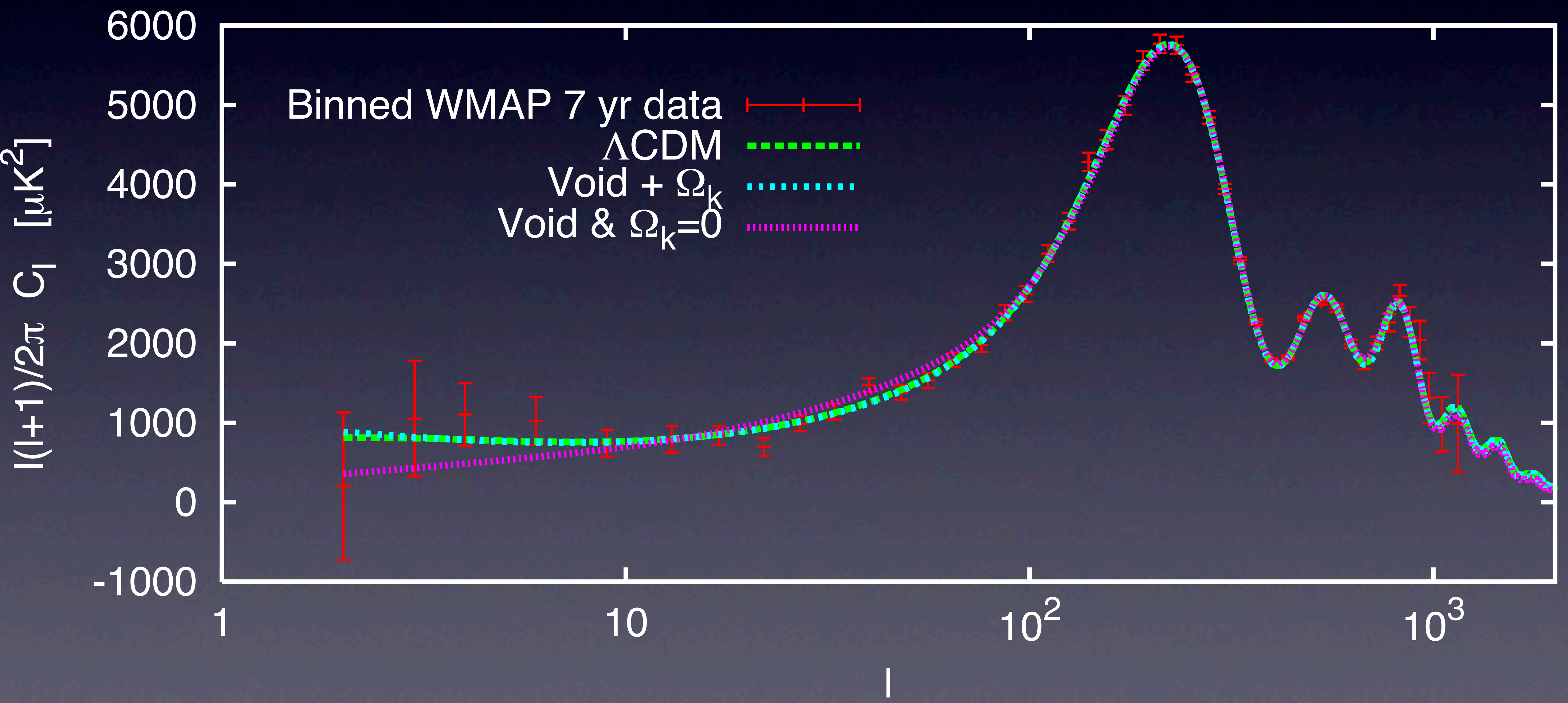


LCDM Void in EdS Void in Curved FLRW

BAO



CMB



Binned WMAP 7 yr data
ΛCDM
Void + Ω_k
Void & Ω_k=0
$l(l+1)/2\pi \ C_l \ [\mu K^2]$
6000
5000
4000
3000
2000
1000
0
-1000
1
10
10^2
10^3
l

How good is it?

Model	CMB	BAO	SN	$\mathrm{HST}_{74\pm4}$	total χ^2
ΛCDM	3372.7	1.8	239.7	2.1	3616.3
Profile C	3389.8	0.3	235.4	27.8	3653.3
Profile E	3373.3	3.0	242.7	15.3	3634.3

$H_0 = 74 \pm 4$ from [Riess et al., 2009]

How good is it?

Datasets		#dof	ΛCDM + Ω_k	EdS Void	Curved Void
CMB+BAO+SN	+HST$_{62\pm6}$	3406	(3614.1)	+50.5	+14.6
CMB	+SN+LRG+HST$_{62\pm6}$	3426	(3639.1)	+30.3	+12.7

$H_0 = 62 \pm 6$ from [Sandage et al., 2006]

How crazy is it?

Profile	($\Delta\chi^2$ vs ΛCDM)	$\Omega_{k,out}$	$\Omega_{k,in}$	δ_0	t_0 [Gyr]
A	(13.7)	-0.20	0.76	-0.67	17.6
B	(3.2)	-0.18	0.80	-0.71	17.3
C	(1.5)	-0.19	0.83	-0.75	16.8
D	(1.6)	-0.15	0.98	-0.98	17.7
E	(3.1)	+0.40	0.94	-0.73	15.4

$$\Omega_k > 0 \equiv \text{closed}$$

Inhomogeneous Universe Discussion points:

- Copernican Principle
- Observations
    - distance measures $\left\{ \right.$
        - CMB
        - BAO
        - SN
        - HST
    - CMB: parameters at last scattering, ISW, Caldwell-Stebbins
    - Clusters: $\frac{\Omega_b}{\Omega_m}$, $\frac{\delta\rho}{\rho}$
    - Lensing: $\frac{\delta\rho}{\rho}$
    - kSZ: inhomogeneous expansion
    - LSS: ???
    - Does anyone know $\Omega_{m,\mathrm{local}}$?
- Distinguish from DE: realtime cosmology, ...?
- Considerations for FLRW: Swiss Cheese, Meat Balls, ...?

Fake dark energy as a consequence of ignoring inhomogeneities

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Outline

1. Consequences of ignoring large scale inhomogeneities
2. Geodesic equations and luminosity distance $D_L(z)$
3. Apparent cosmological observables and “fake” dark energy

Outline

1. Consequences of ignoring large scale inhomogeneities
2. Geodesic equations and luminosity distance $D_L(z)$
3. Apparent cosmological observables and "fake" dark energy

- If we try to fit cosmological data with a homogeneous and isotropic model we can miss important effects from large scale inhomogeneities
- Local gravitational red-shift due to large scale inhomogeneities can in fact be mistaken for evolving dark energy
- The effects can be important even for relatively small inhomogeneities compatible with inflation theory
- While in the past attention has been focused on trying to explain experimental data with different type of inhomogeneous solutions without dark energy (Moffat, Dabrowski, Celerier, Moon, Sasaki, Romano etc.) here we consider the effects of inhomogeneities in presence of a cosmological constant.

FLRW case

- $$\left(\frac{\dot{a}_F}{a_F}\right)^2 = -\frac{k}{a_F^2} + \frac{\rho_0}{3a_F^3} + \frac{\Lambda}{3} \,.$$

  Introducing the conformal time η , such that $d\eta = dt/a_F$,

  $$a_F(\eta) = \frac{\rho_0 L^2}{3\phi(\frac{\eta}{2L}; g_2, g_3) + kL^2} \,; \quad g_2 = \frac{4}{3}k^2 L^4 \,,$$

  $$g_3 = \frac{4}{27}\left(2k^3 - \Lambda\rho_0^2\right) L^6 \,,$$

  where $\phi(x; g_2, g_3)$ is the Weierstrass elliptic function satisfying the differential equation,

  $$\left(\frac{d\phi}{dx}\right)^2 = 4\phi^3 - g_2\phi - g_3 \,.$$

  and we have explicitly introduced the length scale L.

LTB extension

- Inspired by the FLRW case we get

$$g_2 = \frac{4}{3}k(r)^2 L^4 \,, \quad g_3 = \frac{4}{27}\left(2k(r)^3 - \Lambda\rho_0(r)^2\right)L^6 \,.$$

$$a(\eta, r) = \frac{\rho_0(r)L^2}{3\phi\left(\frac{\eta}{2L}; g_2(r), g_3(r)\right) + k(r)L^2} \,.$$

where we have introduced the length L for dimensional consistency. In the numerical calculations we will set $L = H_0^{-1} = 1$.

Outline

- The luminosity distance in a LTB space-time is

  $D_L(z) = (1+z)^2 R\left(t(z), r(z)\right) = (1+z)^2 r(z) a\left(\eta(z), r(z)\right)$,

  where $\Big(t(z), r(z)\Big)$ or $\Big((\eta(z), r(z)\Big)$ is the solution of the radial geodesic equation as a function of z.
- The past-directed radial null geodesics is given by

$$\frac{dt}{dr} = -\frac{R_{,r}(t,r)}{\sqrt{1+2E(r)}} .$$

from which we can get:

$$\begin{aligned} \frac{dr}{dz} &= \frac{\sqrt{1+2E(r(z))}}{(1+z)\dot{R}_{,r}[r(z),t(z)]} , \\ \frac{dt}{dz} &= -\frac{R_{,r}[r(z),t(z)]}{(1+z)\dot{R}_{,r}[r(z),t(z)]} . \end{aligned}$$

- Taking advantage of the analytical solution we can write the geodesics equations

$$\frac{d\eta}{dz} = -\frac{\partial_r t(\eta,r) + F(\eta,r)}{(1+z)\partial_\eta F(\eta,r)} \equiv p(\eta,r)\,,$$
$$\frac{dr}{dz} = \frac{a(\eta,r)}{(1+z)\partial_\eta F(\eta,r)} \equiv q(\eta,r)\,,$$

where

$$F(\eta,r) \equiv \frac{R_{,r}}{\sqrt{1+2E(r)}} =$$
$$= \frac{\partial_r(a(\eta,r)r) - a^{-1}\partial_\eta(a(\eta,r)r)\,\partial_r t(\eta,r)}{\sqrt{1-k(r)r^2}}\,.$$

It is important that the functions p, q, F have explicit analytical forms.

Outline

- In a flat FLRW model for a given observed $D_L(z)$ we have the relations,

$$H^{app}(z) = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1} ,$$

$$Q^{app}(z) = \frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right) = (H^{app}(z))^{-1} ,$$

$$q^{app}(z) = -1 - \frac{d\ln(Q^{app}(z))}{d\ln(1+z)} = q^{app}(D_L(z)) ,$$

$$w_{DE}^{app}(z) = \frac{(2(1+z)/3)\, d\ln H^{app}/dz - 1}{1 - (H_0/H^{app})^2\Omega_{0m}(1+z)^3} .$$

- These are valid under the assumption of flatness and homogeneity. Apparent observables are those given above with the luminosity distance $D_L(z)$ obtained for a central observer in a LTB model.

- In order to make a connection between the LTB model and a universe with primordial curvature perturbations from inflation, we introduce the following metric which describes a spherically symmetric space-time after inflation at scales much exceeding the Hubble one:

$$ds^2 = -dt^2 + a_F^2(t)e^{2\zeta(r)}(dr^2 + r^2 d\Omega^2)\,.$$

-

$$ds^2 = -dt^2 + \frac{(R_{,r})^2\, dr^2}{1 + 2\,E(r)} + R^2 d\Omega^2\,, \tag{1}$$

$R = a_F(t)e^{\zeta} r$, we find the exact relation:

$$1 + 2E(r) = [1 + r\zeta'(r)]^2.$$

- In the linear approximation, this reduces to

$$k(r) = -2\frac{\zeta'(r)}{r}\,.$$

- Motivated by observations we consider the curvature perturbation $\zeta(r)$ of the amplitude $\sim 5\times 10^{-5}$. Specifically we study the four different types of inhomogeneities,

$$\text{Type I}^- : k(r) = \frac{A}{r_0^2}[l(r) - l'(0)re^{-r/r_0}]; \qquad H_0 r_0 = 0.1\,,\ A = 10^{-4}\,,\ \Delta = 0.02\,,$$

$$\text{Type I}^+ : k(r) = \frac{A}{r_0^2}[l(r) - l'(0)re^{-r/r_0}]; \qquad H_0 r_0 = 0.1\,,\ A = -10^{-4}\,,\ \Delta = 0.02\,,$$

$$\text{Type II}^- : \zeta(r) = A[l(r) - l'(0)re^{-r/r_0}]; \qquad H_0 r_0 = 0.2\,,\ A = 5\times 10^{-5}\,,\ \Delta = 0.05\,,$$

$$\text{Type II}^+ : \zeta(r) = A[l(r) - l'(0)re^{-r/r_0}]; \qquad H_0 r_0 = 0.2\,,\ A = -5\times 10^{-5}\,,\ \Delta = 0.05\,,$$

where the function $l(r)$ is defined as

$$l(r) = \left[\tanh\left(\frac{H_0(r-r_0)}{\Delta}\right) - 1\right]\,.$$

- In all cases, the cosmological constant is assumed to be the same as the one implied by the best fit ΛCDM model corresponding to $\Omega_\Lambda = 0.7$, and $H_0 = H^{app}(z=0)$ is adjusted to the observed Hubble constant.

Analytical approximation

- For this purpose we expand the relevant functions as

$$
\begin{aligned}
t(\eta, r) &= A_0(\eta) + A_1(\eta) r + \frac{1}{2} A_2(\eta) r^2 + \cdots \, , \\
\eta(z) &= \eta_0 + \eta_1 z + \eta_2 z^2 + \cdots \, , \\
r(z) &= r_1 z + r_2 z^2 + \cdots \, ,
\end{aligned}
$$

to get

$$
\begin{aligned}
q_0^{app} &= -\frac{2\,(r_1 a_{,r} + \eta_1 a_{,\eta})}{a} - \frac{2 r_2}{r_1} - 3 \, , \\
w_0^{app} &= -\frac{4 r_1\,(r_1 a_{,r} + \eta_1 a_{,\eta}) + (7 r_1 + 4 r_2) a}{3 r_1 a (1 - H_0^2 \Omega_M (r_1 a)^2)} \, .
\end{aligned}
$$

- We can finally get the analytical results :

$$
\begin{aligned}
q_0^{app} &= \frac{3}{2}\Omega_M - 1 + 2\,\zeta_{ss}(0), \\
w_0^{app} &= -1 + \frac{4}{3(1-\Omega_M)}\zeta_{ss}(0)\,, \\
\zeta_{ss} &= \frac{1}{(a_0 H_0)^2}\zeta_{rr}\,,
\end{aligned}
$$

- As expected the above formulae reduce to the ΛCDM case in the central flat limit,

$$
\begin{aligned}
k_0 &= -2\zeta_{rr}(0) &&= 0\,, \\
q_0^{app} &= q_0^{\Lambda CDM} &&= \frac{3}{2}\Omega_M - 1\,, \\
w_0^{app} &= w_0^{\Lambda CDM} &&= -1\,.
\end{aligned}
$$

- We have used

$$
\begin{aligned}
a_0 &= \frac{L^2\rho_0}{\zeta_{rr}(0)L^2+3\phi}\,,\\
H_0 &= -\frac{3\phi'}{2L^3\rho_0}\,,\\
\phi &= \phi\left(\frac{\eta_0}{2L};\frac{16}{3}\zeta_{rr}(0)^2L^4,-\frac{4}{27}\left(16\zeta_{rr}(0)^3+\Lambda\rho_0^2\right)L^6\right)\,,\\
\phi' &= \partial_x\phi\left(x;\frac{16}{3}\zeta_{rr}(0)^2L^4,-\frac{4}{27}\left(16\zeta_{rr}(0)^3+\Lambda\rho_0^2\right)L^6\right)\Bigg|_{x=\frac{\eta_0}{2L}}\,,\\
\Lambda &= 3(1-\Omega_M)H_0^2\,,\\
\zeta_{ss} &= \frac{1}{(a_0H_0)^2}\zeta_{rr}\,,\\
\rho_0 &= 3a_0^3\Omega_M H_0^2\,.
\end{aligned}
$$

- As a confirmation that large scale inhomogeneities look like fake dark energy we can also verify that the relation between q_0^{app} and w_0^{app} is the same as in the case of an FLRW model with dark energy:

$$
\begin{aligned}
q_0^{FLRW} &= \frac{3}{2}\Omega_m - 1 + \frac{3}{2}(1 + w_0^{DE})(1 - \Omega_M)\,, \\
q_0^{app} &= \frac{3}{2}\Omega_m - 1 + \frac{3}{2}(1 + w_0^{app})(1 - \Omega_M)\,.
\end{aligned}
$$

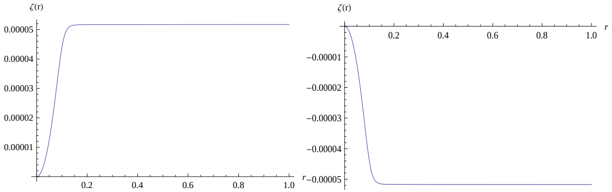

Figure: $\zeta(r)$ is plotted for inhomogeneity of type I^- and I^+.

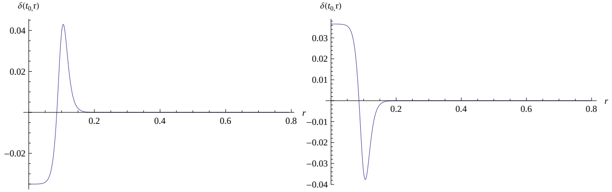

Figure: $\delta(t_0, r)$ is plotted as a function of r for inhomogeneity of types I^- and I^+.

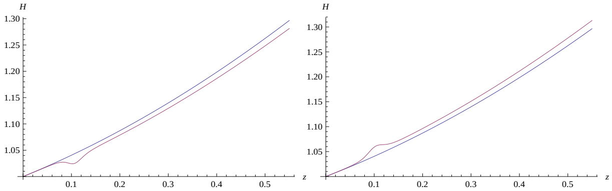

Figure: $H^{app}(z)$ is plotted for inhomogeneity of types I^- and I^+.

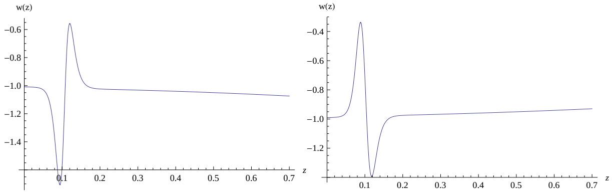

Figure: $w_{DE}^{app}(z)$ is plotted for inhomogeneity of types I^- and I^+.

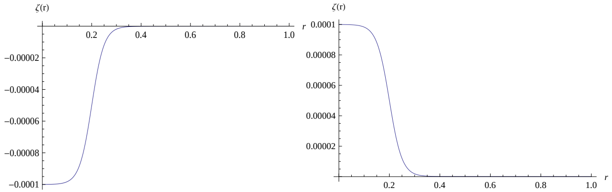

Figure: $\zeta(r)$ is plotted for inhomogeneity of type II^- and II^+.

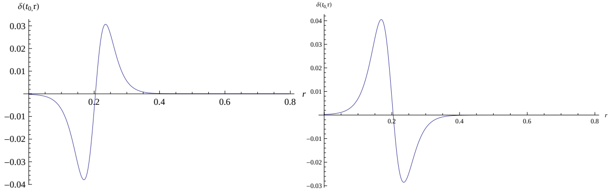

Figure: $\delta(t_0, r)$ is plotted as a function of r for inhomogeneity of types II^- and II^+.

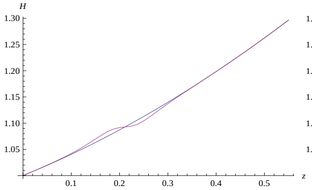

Figure: $H^{app}(z)$ is plotted for inhomogeneity of types II$^-$ and II$^+$.

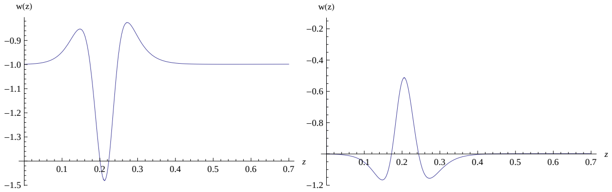

Figure: $w_{DE}^{app}(z)$ is plotted for inhomogeneity of types II$^-$ and II$^+$.

Conclusions

- We have investigated how the presence of a local inhomogeneity could affect the apparent equation of state of dark energy under the “wrong” assumption of a homogeneous FLRW background, which is commonly used in interpreting astrophysical observations in ΛCDM models.
- The presence of a local underdensity gives rise to apparent phantom behavior, while that of a local overdense region to apparent quintessence behavior.
- Our results give a semi-realistic example of inhomogeneities with the amplitude compatible with inflationary predictions which, if interpreted in the framework of a flat and homogeneous spacetime, can lead to the wrong conclusion of the presence of dark energy with an evolving equation of state.

- In general, a local inhomogeneity can lead to a confusion between local gravitational redshift and cosmological redshift due to the expansion of the Universe.
- Recent analysis of observational data could support the existence of a local underdense region, but which may not be of compensated type as the one we considered here. We will investigate in a future work what could be the constraints on the size and density contrast of such a void based on observational data.