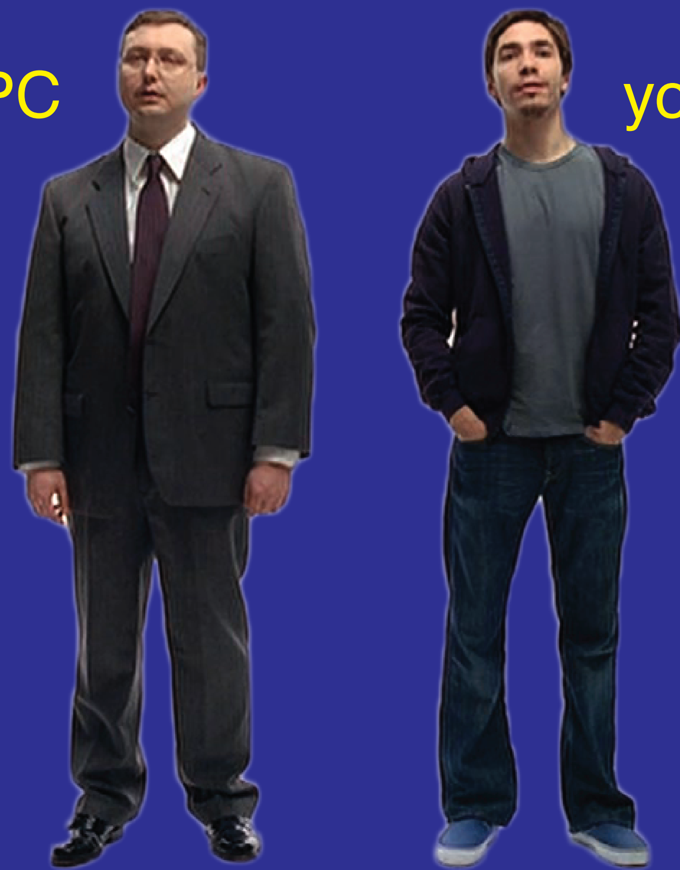
Principal Components

Hola!
Soy un PC

Y
yo soy *f*(*x*)



and Paradigm Falsification

Wayne Hu

Benasque, Spain
August 2010

Principal Components

Hola!
Soy un PC

Y
yo soy *f*(*x*)

un-PC

gm Falsifi

yne Hu

Benasque, Spain
August 2010

Outline

- Why PCs
- Ionization History $x_e(z)$
- Dark Energy Equation of state $w(z)$ Interrelated!
- Inflaton Potential $V(\phi)$

- Collaborators:
  - Cora Dvorkin
  - Dragan Huterer
  - Michael Mortonson
  - Hiranya Peiris
  - Earlier work with Gil Holder, Kenji Kadota

Why PCs

- Principal components are the eigenbasis of the projected or actual covariance matrix for a discrete representation of $f(x_i)$
- Rank ordered in observability and decorrelated linear combination

Advantages:

- Define according to Fisher projected covariance matrix – no a posteriori bias in looking for features
- Efficient – can keep only observable modes and never requires MCMC over large correlated discrete space
- Complete – can include as many modes as required to make basis observationally complete
- Paradigm testing – rapidly explore all possible observational outcome of a given paradigm
- Falsifiable predictions for other observables not yet measured

Why PCs

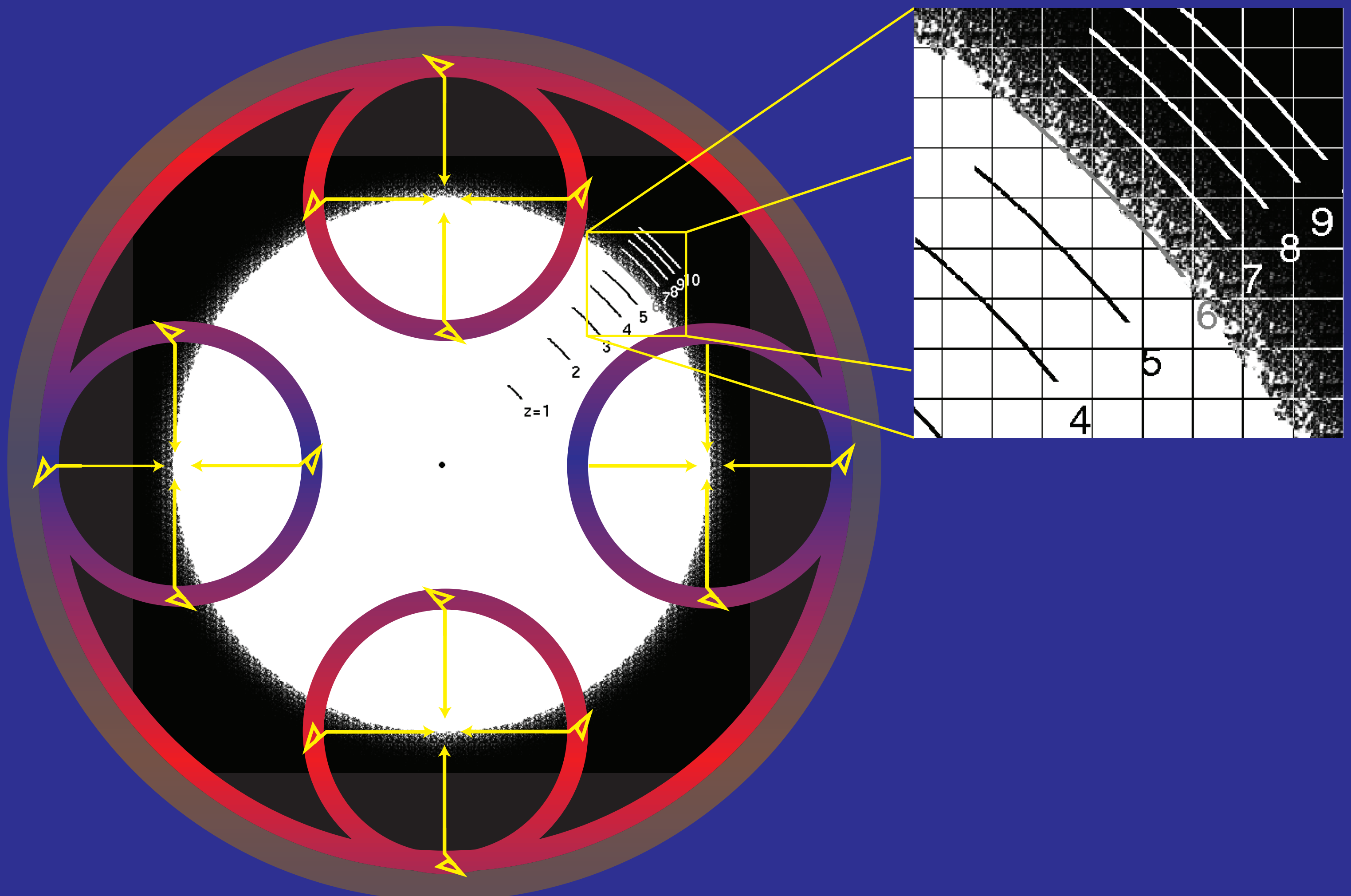
Disadvantages:

- Modes are non local
- Constraints define a heavily filtered reconstruction, e.g. sets function to zero beyond observable range
- Eigenfunctions not ranked by their importance within a certain class of models – e.g. freezing and thawing $w(z)$ – better for paradigm testing than model testing
- Completeness requires more parameters than required by model or data
- Including unconstrained modes can break orthogonality, requiring external prior or regularization (e.g. Gaussian processes, see S. Habib's talk)

Ionization History $x_{\rm e}(z)$

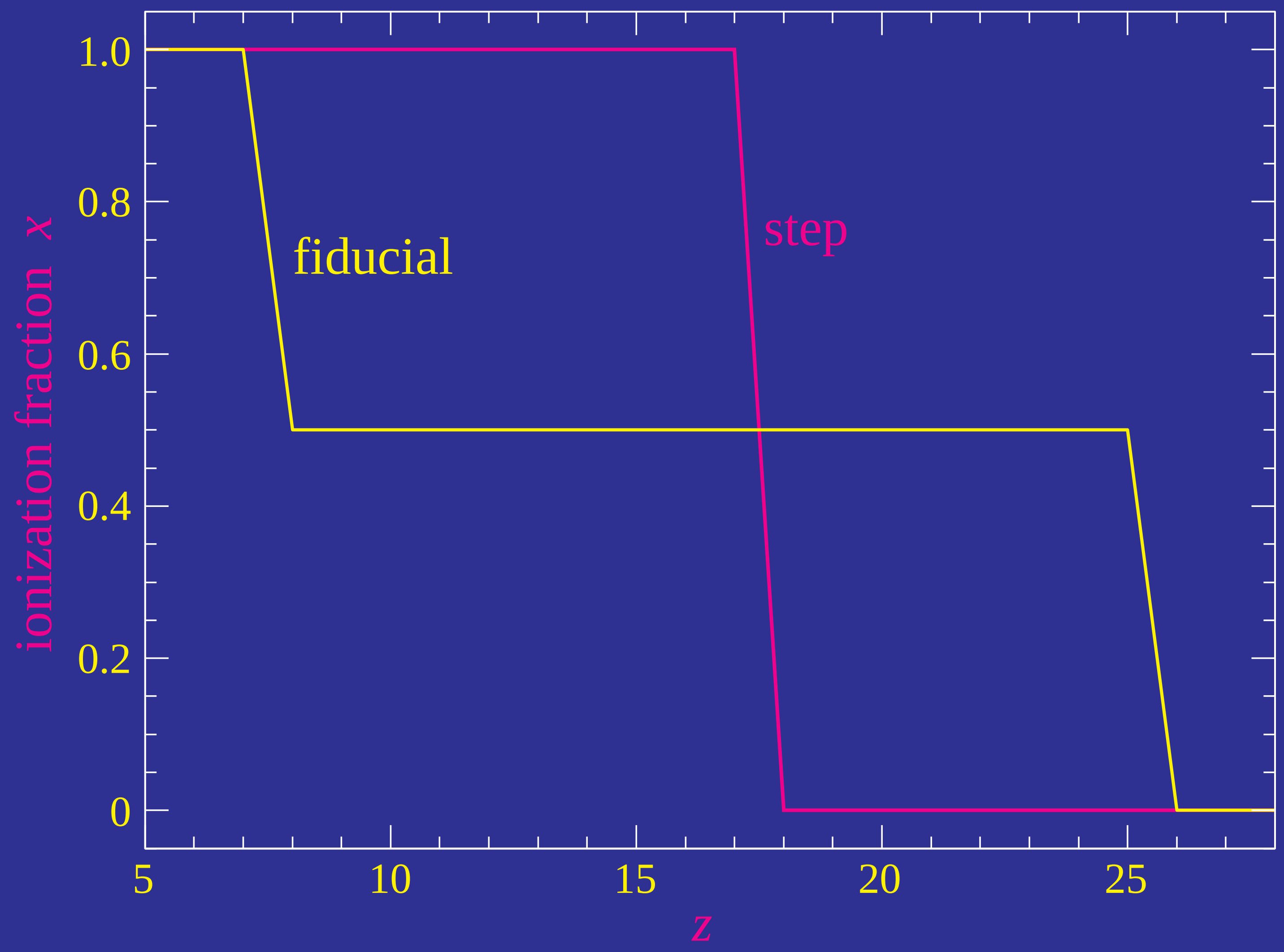
Polarization & Reionization

- Rescattering of anisotropic radiation during reionization leads to large scale polarization
- Sensitive to the average ionization fraction



Ionization History

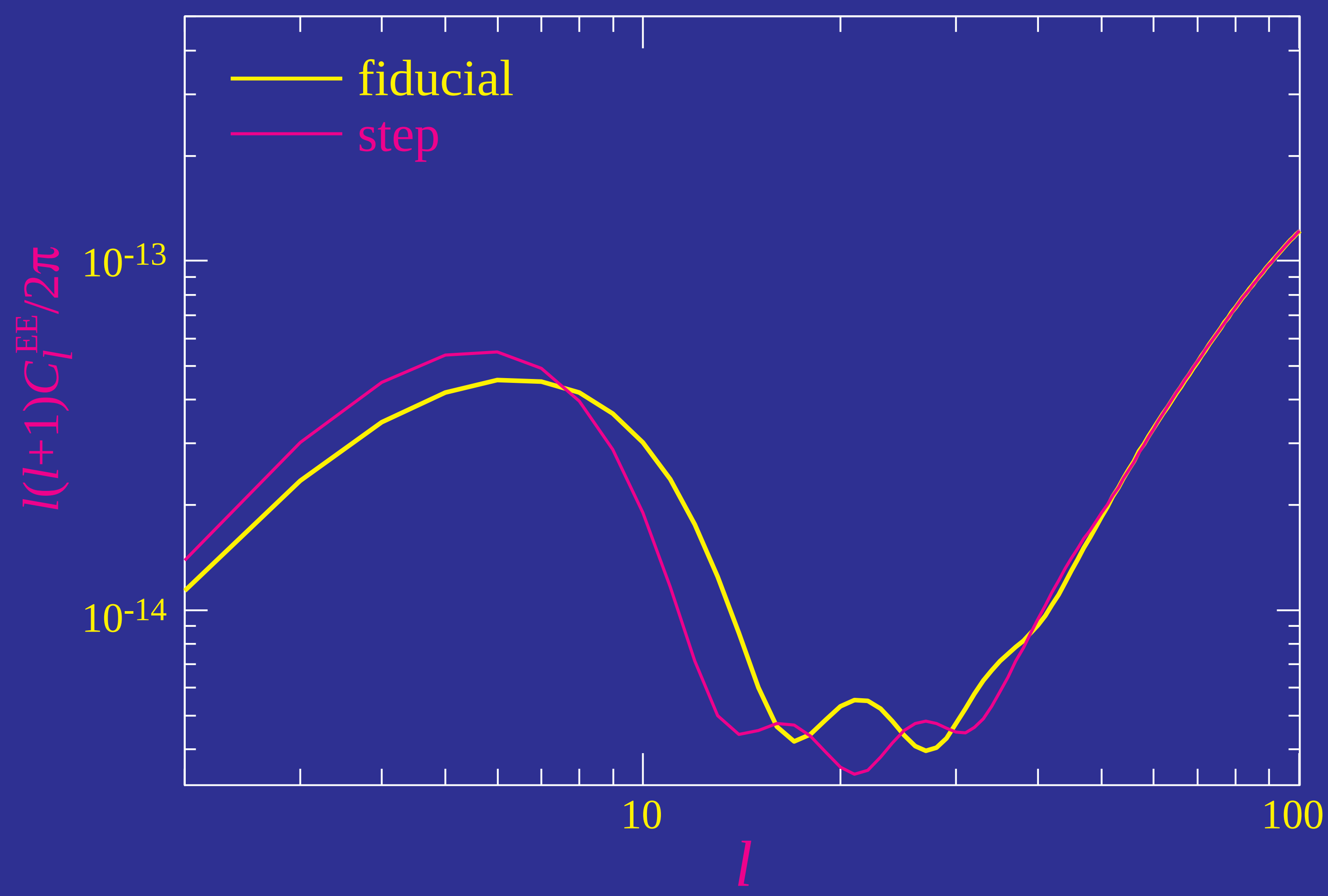
- Two models with same optical depth τ but different ionization history



Kaplinghat et al. (2002); Hu & Holder (2003)

Distinguishable History

- Same optical depth, but different coherence - horizon scale during scattering epoch

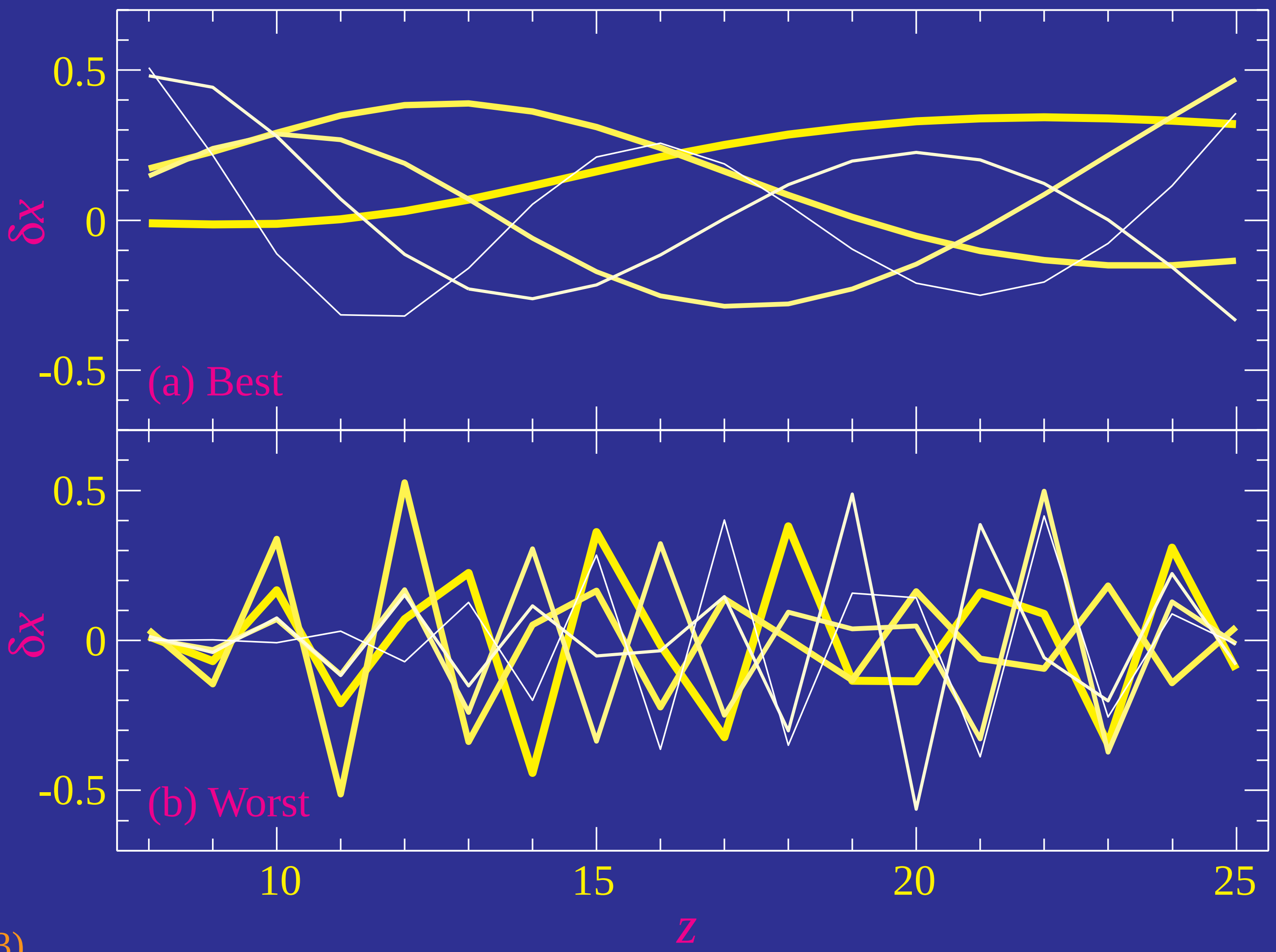


Kaplinghat et al. (2002); Hu & Holder (2003)

Principal Components

- Eigenvectors of the Fisher Matrix

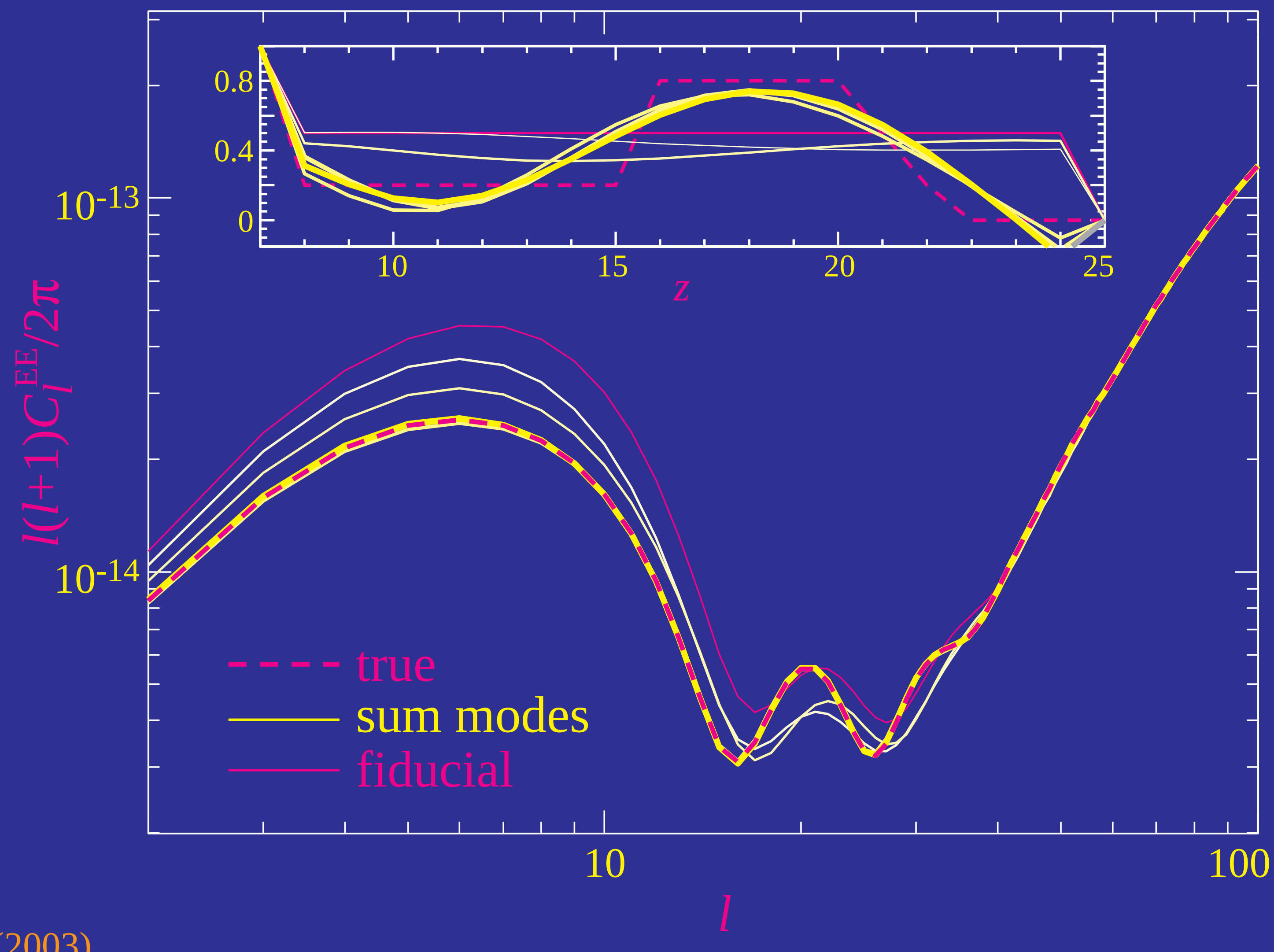
$$F_{ij} \equiv \sum_{\ell} (\ell + 1/2) T_{\ell i} T_{\ell j} = \sum_{\mu} S_{i\mu} \sigma_{\mu}^{-2} S_{j\mu}$$



Hu & Holder (2003)

Representation in Modes

- Reproduces the power spectrum with sum over >3 modes
  more generally 5 modes suffices: e.g. total τ=0.1375 vs 0.1377

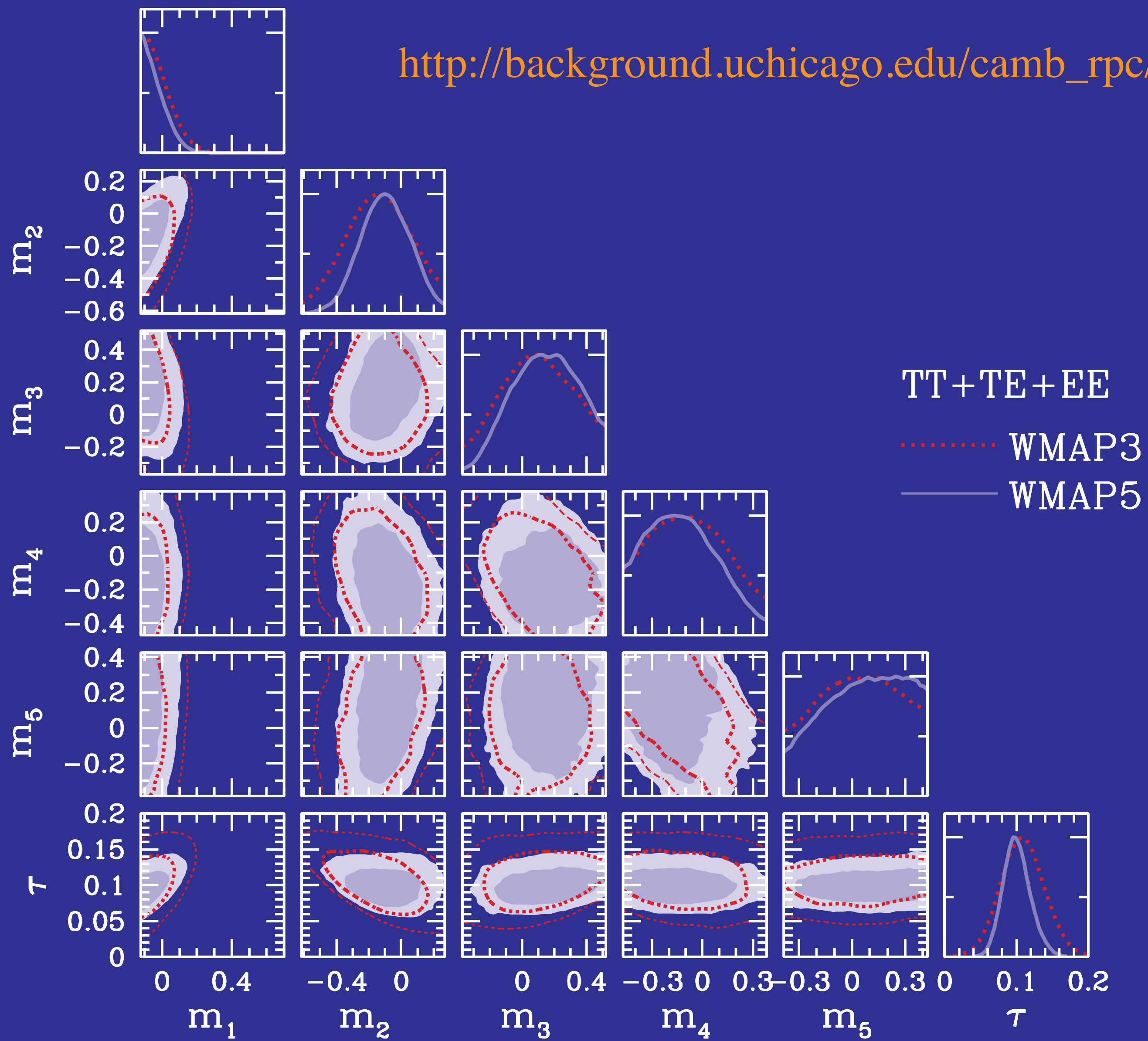


Hu & Holder (2003)

WMAP5 Ionization PCs

- Only first two modes constrained, τ=0.101±0.017

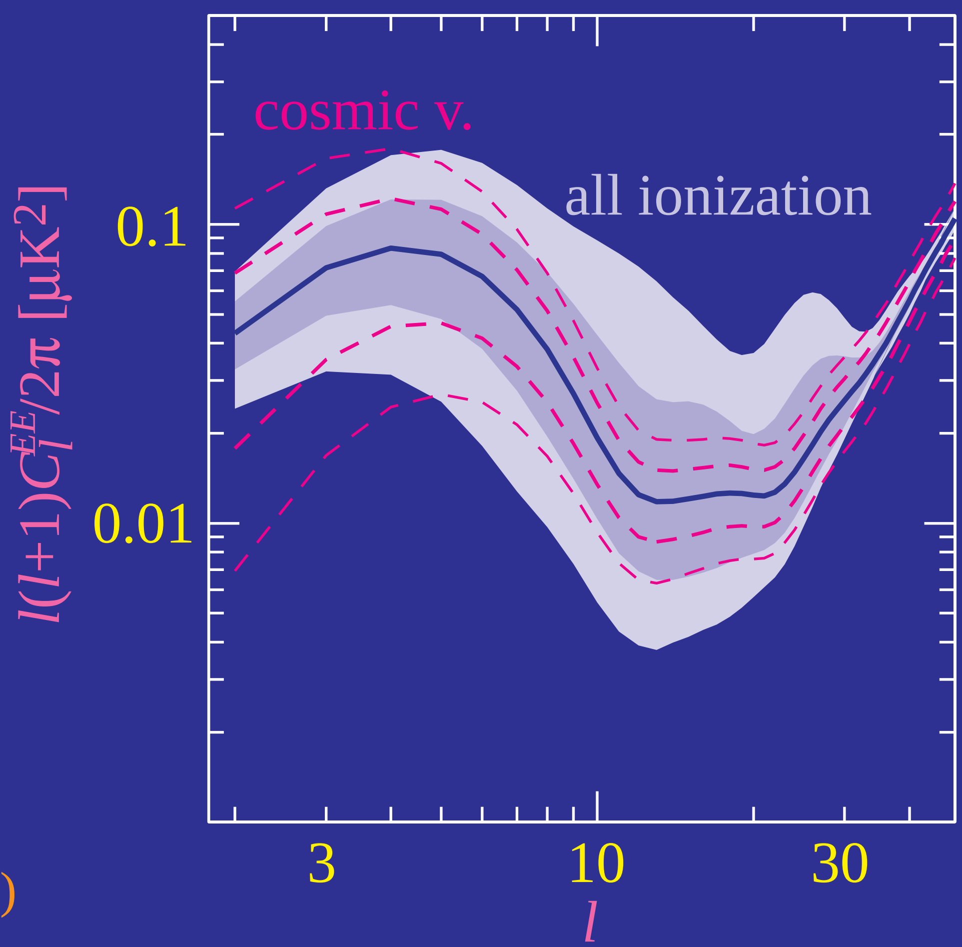
http://background.uchicago.edu/camb_rpc/



Mortonson & Hu (2008)

Model-Independent Reionization

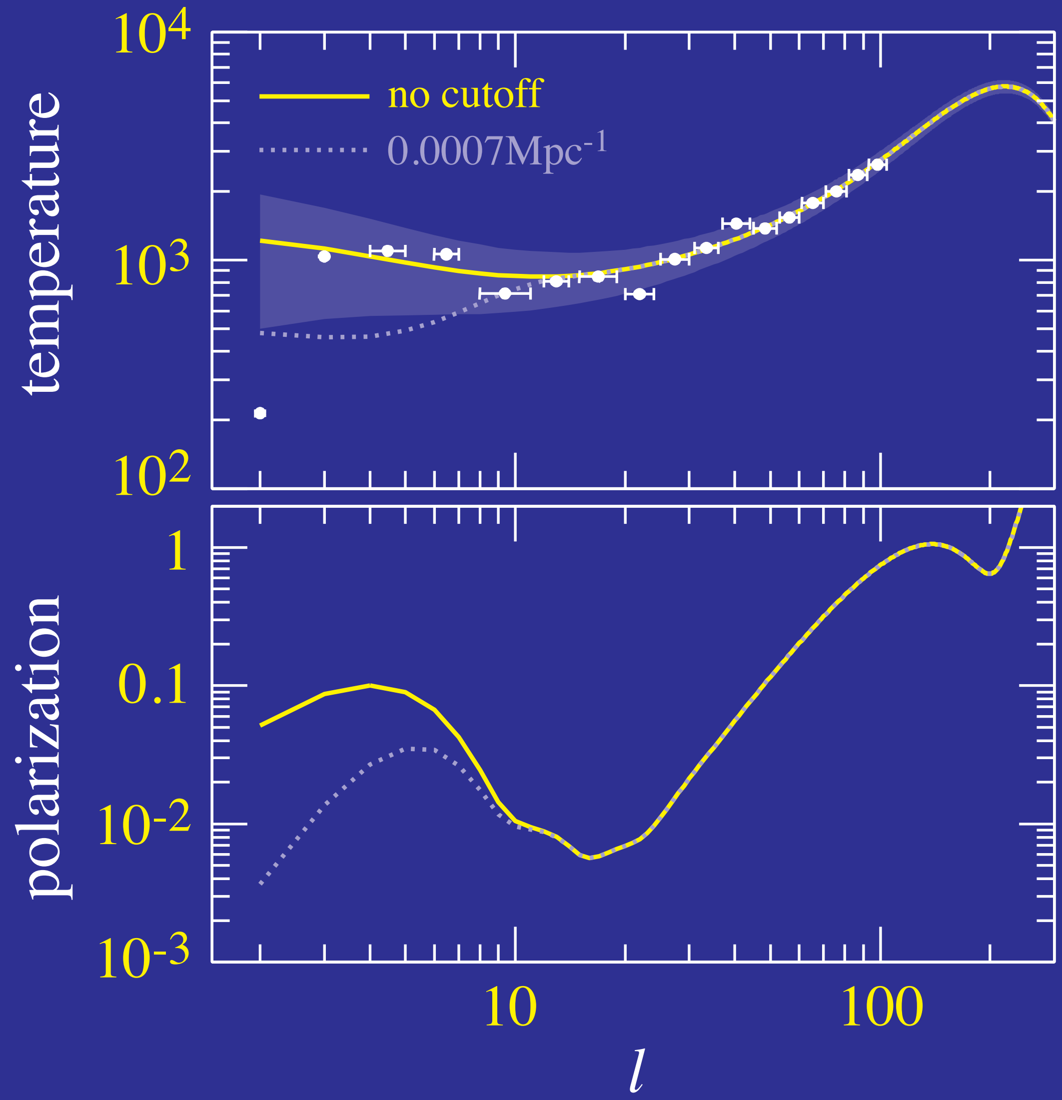
- All possible ionization histories at $z<30$
- Detections at $20<l<30$ required to further constrain general ionization which widens the τ-n_s degeneracy allowing $n_s=1$
- Quadrupole & octopole predicted to better than cosmic variance test ΛCDM for anomalies



Mortonson & Hu (2008)

Horizon-Scale Power

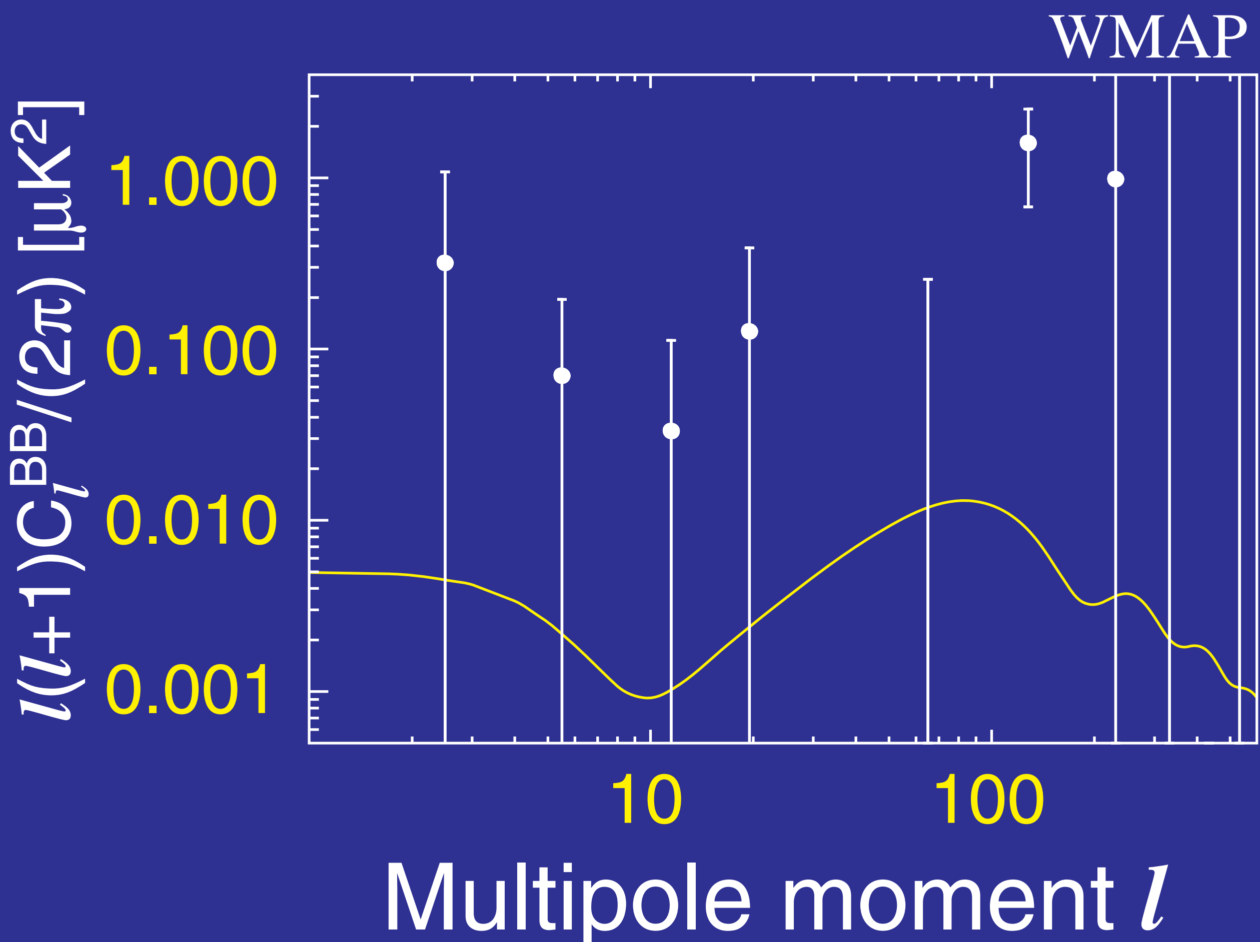
- Polarization is a robust indicator of horizon scale power and disfavors suppression as explanation of low quadrupole independently of ionization or acceleration model



Mortonson & Hu (2009)

Tensor Slope

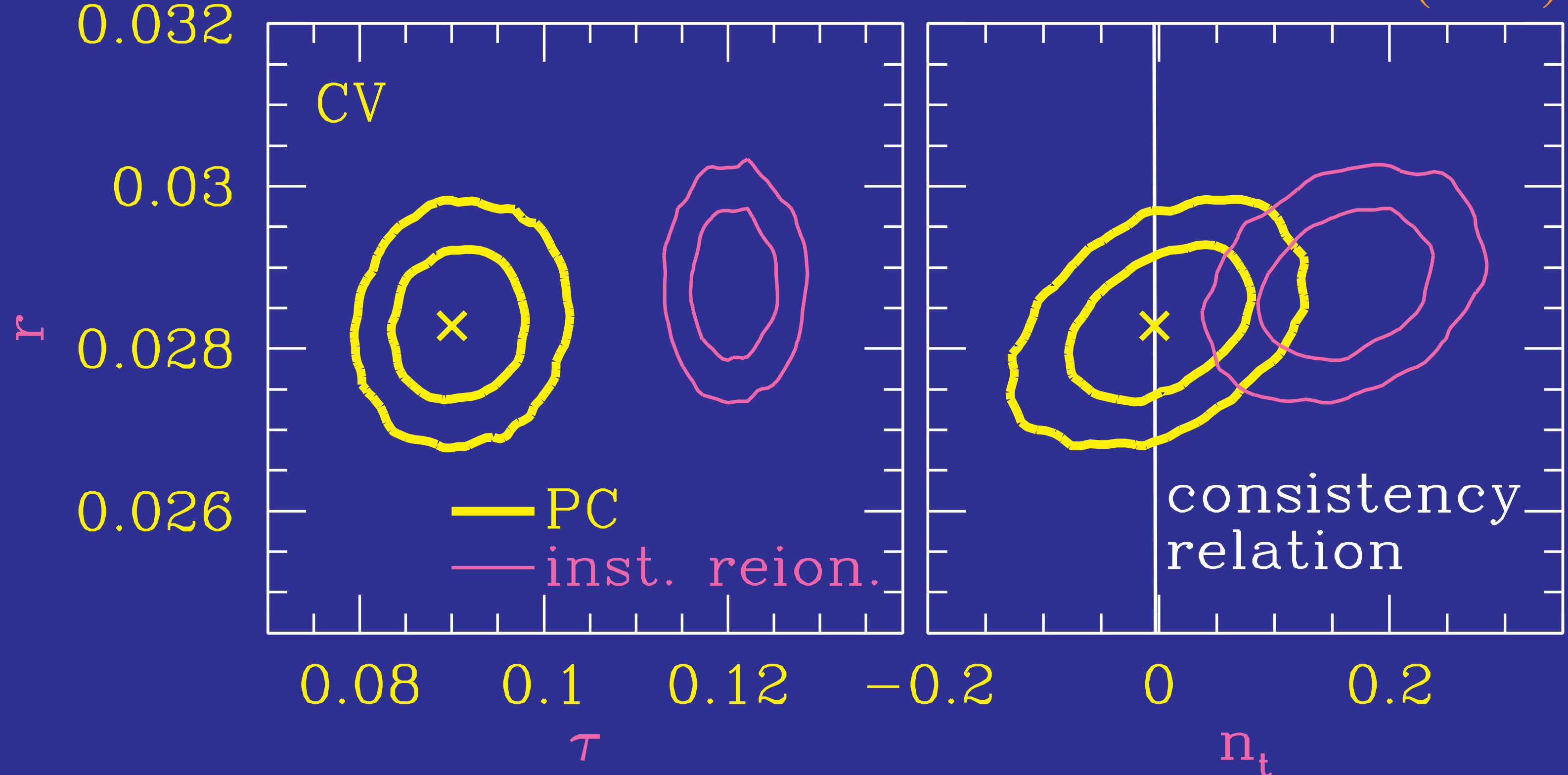
- If degree scale tensors are observed, reionization enables test of slow roll infation through consistency between n_T-r



Consistency Relation & Reionization

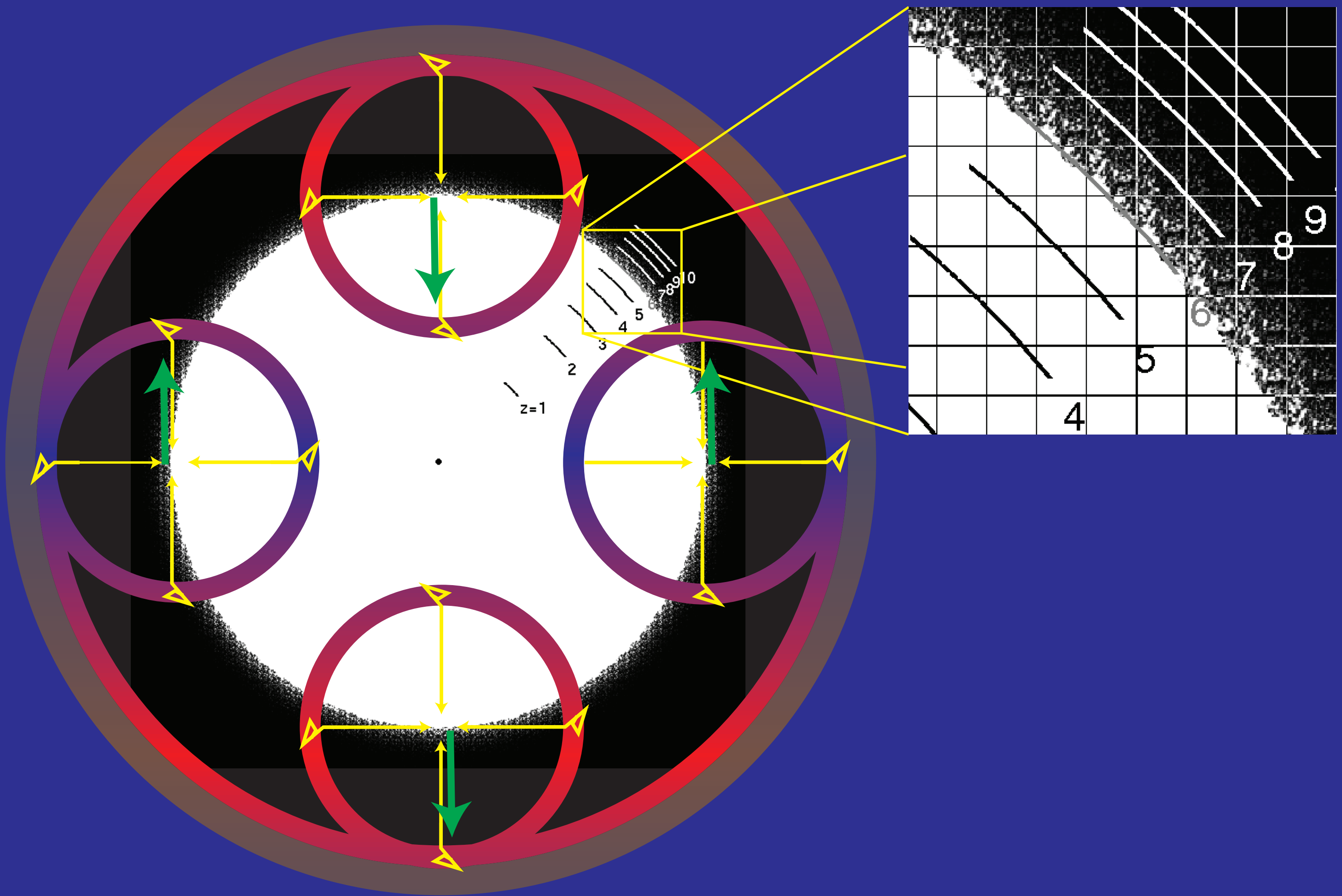
- By assuming the wrong ionization history can falsely rule out consistency relation
- Principal components eliminate possible biases

Mortonson & Hu (2007)



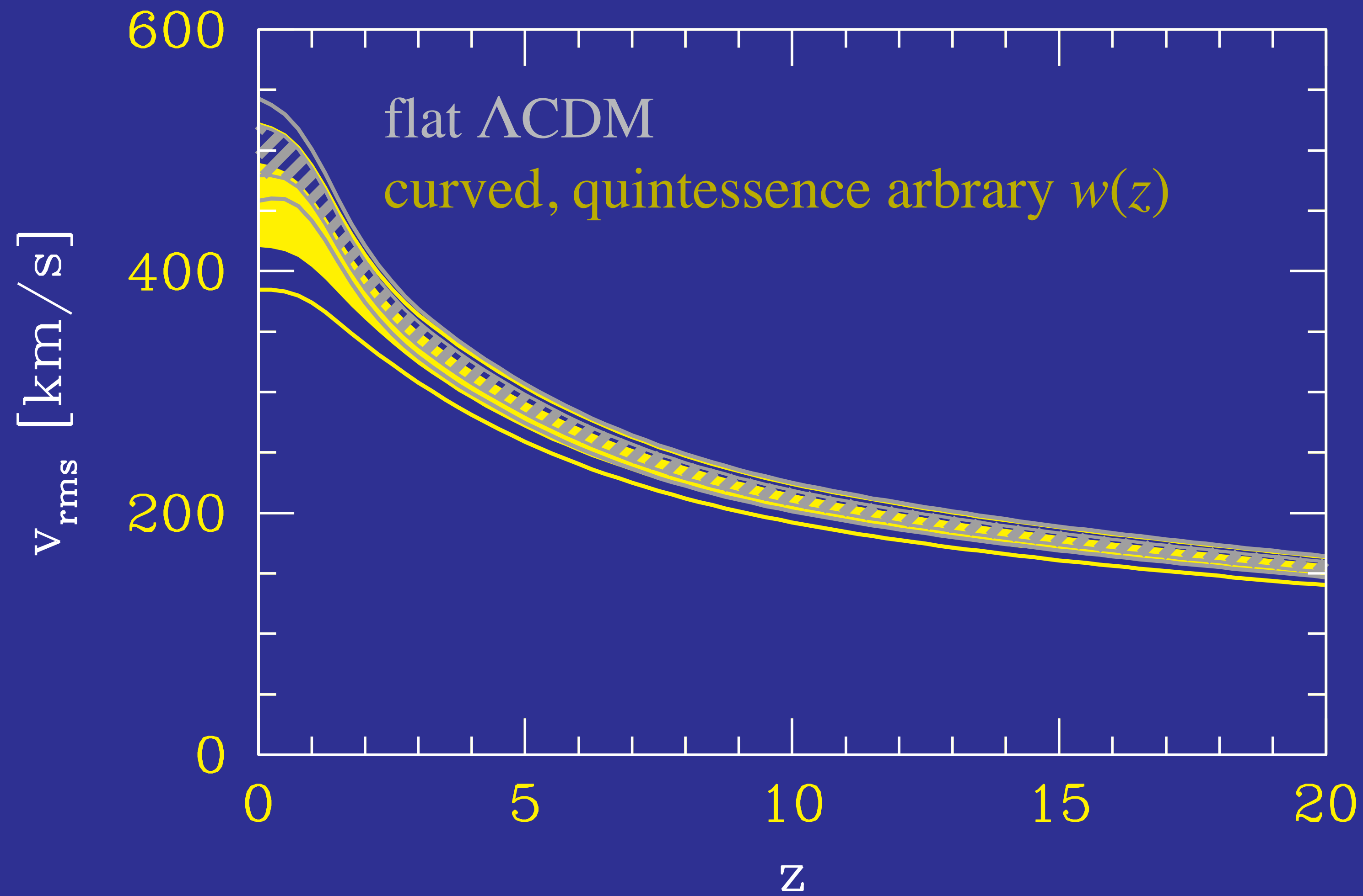
Inhomogeneous Ionization

- Provides a source for modulated Doppler effect that appears on the scale of the ionization region



Linear Velocity Field

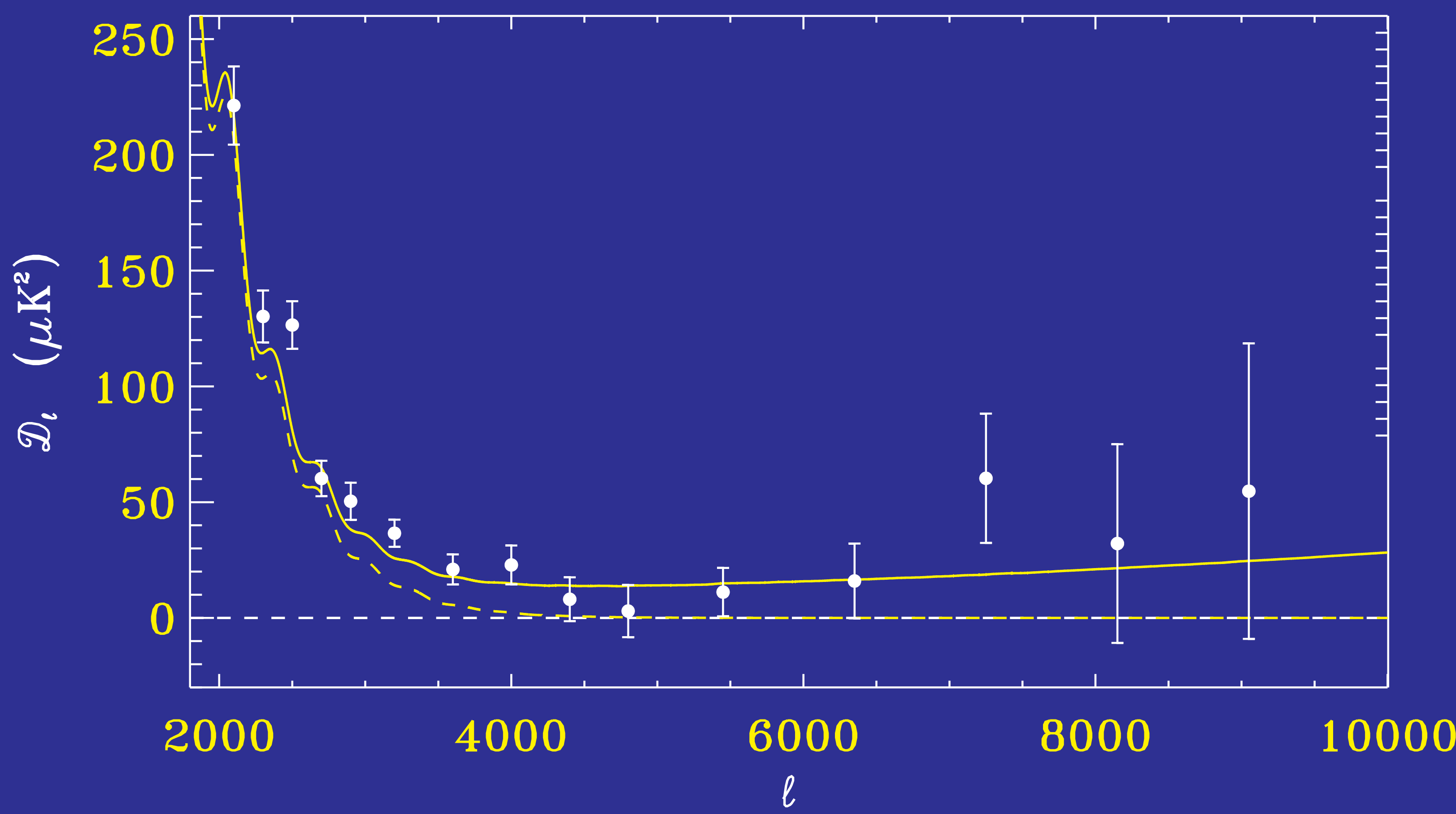
- Even given dark energy, curvature uncertainties, rms linear velocity well determined at z~10



Mortonson, Hu, Huterer (2009); Mortonson & Hu (2010)

Observational Constraints

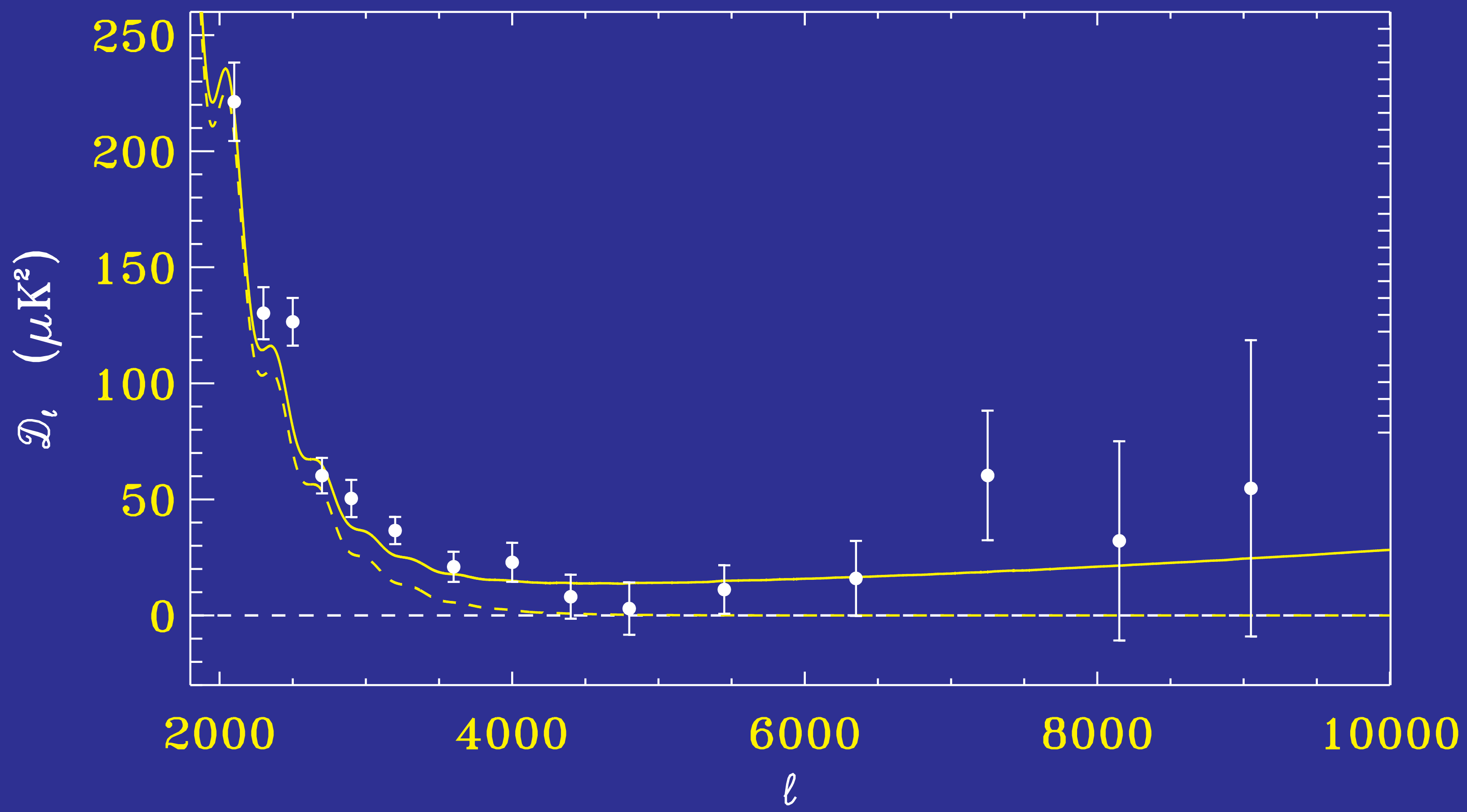
- SPT detection of secondary anisotropy (likely SZ dominated, low level) sets upper limit on modulated Doppler contributions



SPT Hall et al - Leuker et al (2010)

Observational Constraints

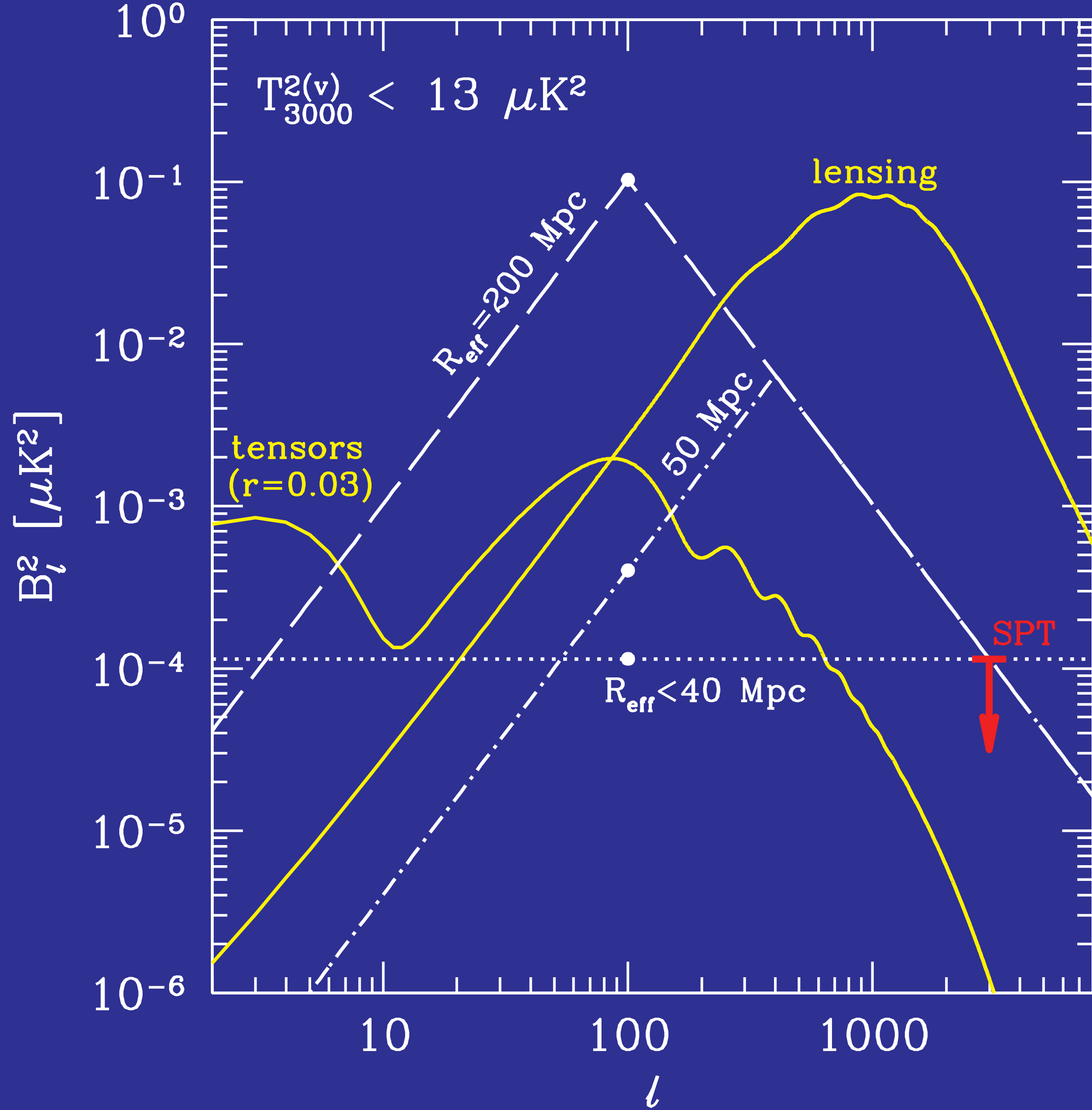
- Combined with well-determined velocity, rms optical depth fluctuation at arcmin scale $\delta\tau<0.0036$ (conservative 95% CL)



SPT Hall et al - Leuker et al (2010); Mortonson & Hu (2010)

Inferred B-Mode Limits

- With SPT optical depth constraint, arcminute B-modes highly contrained; degree scale depends on ionization bubble size



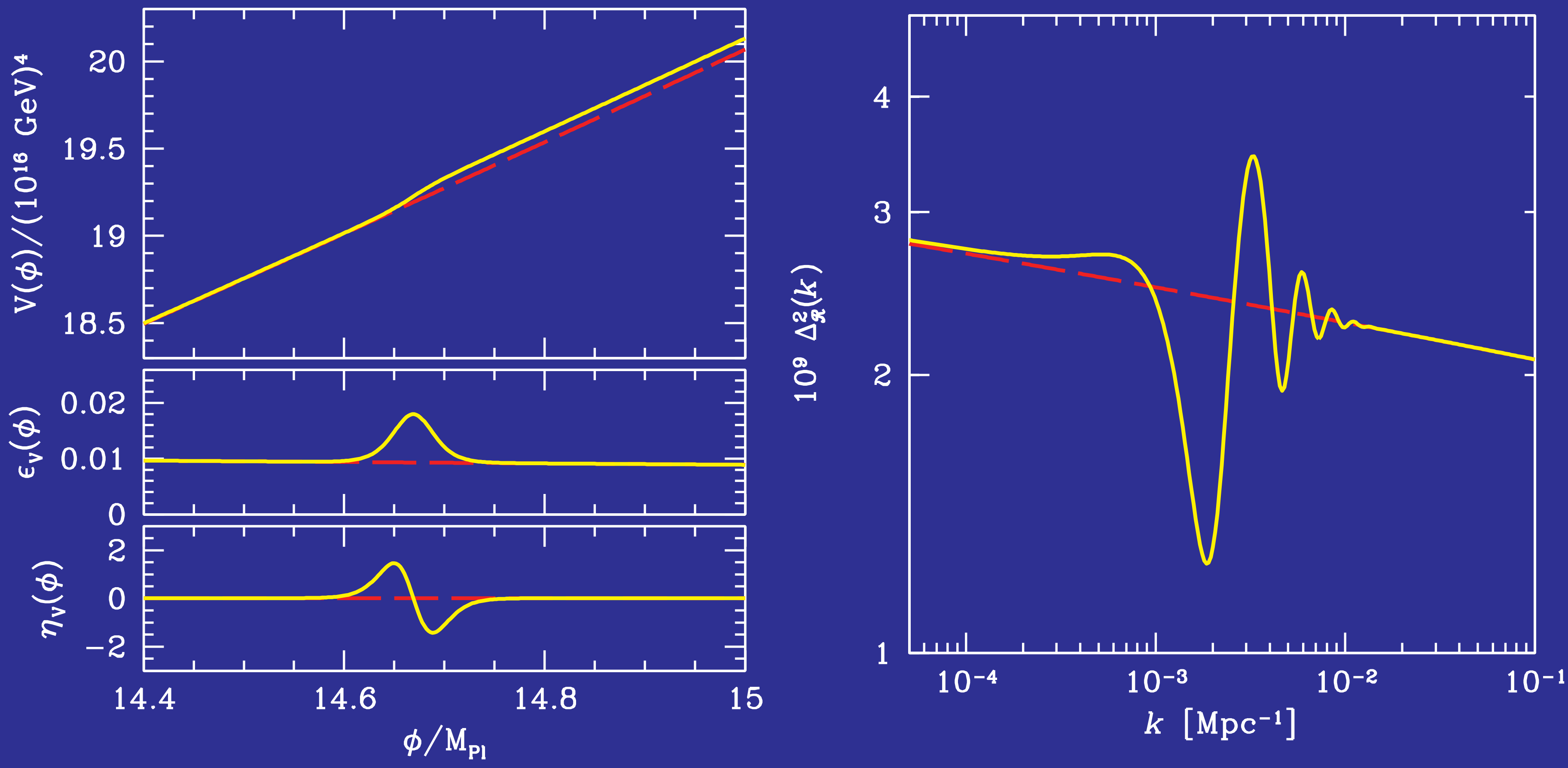
$$R_{\rm eff} = \bar{R} e^{4\sigma^2_{\ln R}}$$

SPT Hall et al - Leuker et al (2010); Mortonson & Hu (2010)

Inflaton Potential $V(\phi)$

Features in Potential

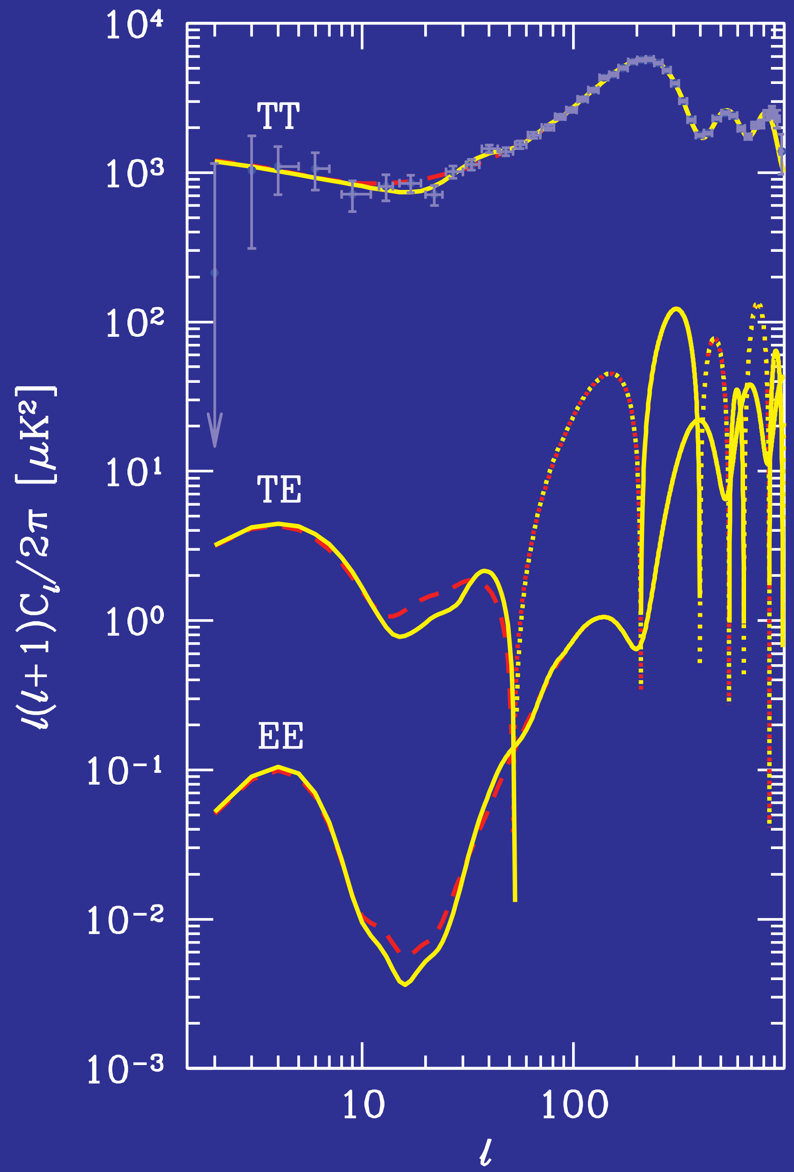
- Rolling of inflaton across a sharp feature causes ringing



Mortonson, Dvorkin, Peiris, Hu (2008) [Covi et al 2006; Hamann et al 2007]

Features in Potential

- Possible expanation of glitches
- Predicts matching glitches in polarization
- Falsifiable independent of ionization history through PC analysis
- Planck 2.5-3σ
- Cosmic variance 5-8σ



Mortonson, Dvorkin, Peiris, Hu (2008)

Inflaton Fluctuations

- Single field inflaton fluctuations obey the linearized Klein-Gordon equation for $u = a\delta\phi$

$$\ddot{u} + \left[k^2 - \frac{\ddot{z}}{z}\right] u = 0$$

where

$$z(\eta) = \dot{\phi}/H$$

- Oscillatory response to rapid slow down or speed up of roll $\dot{\phi}$ due to features in the potential
- Single function $z(\eta)$ controls curvature fluctuations but
  - direct PC or other functional constraints cumbersome
  - link to $V(\phi)$ obscured

Generalized Slow Roll

- Green function approach allowing slow roll parameters to be strongly time varying (Stewart 2002)
- Generalized for large features by promoting second order to non-linear in controlled fashion (Dvorkin & Hu 2009)
- Functional constraints on the source function of deviations from scale invariance

$$G'(\ln\eta) = \frac{2}{3}\left[\frac{f''}{f} - 3\frac{f'}{f} - \left(\frac{f'}{f}\right)^2\right], \qquad f = 2\pi\eta z(\eta)$$

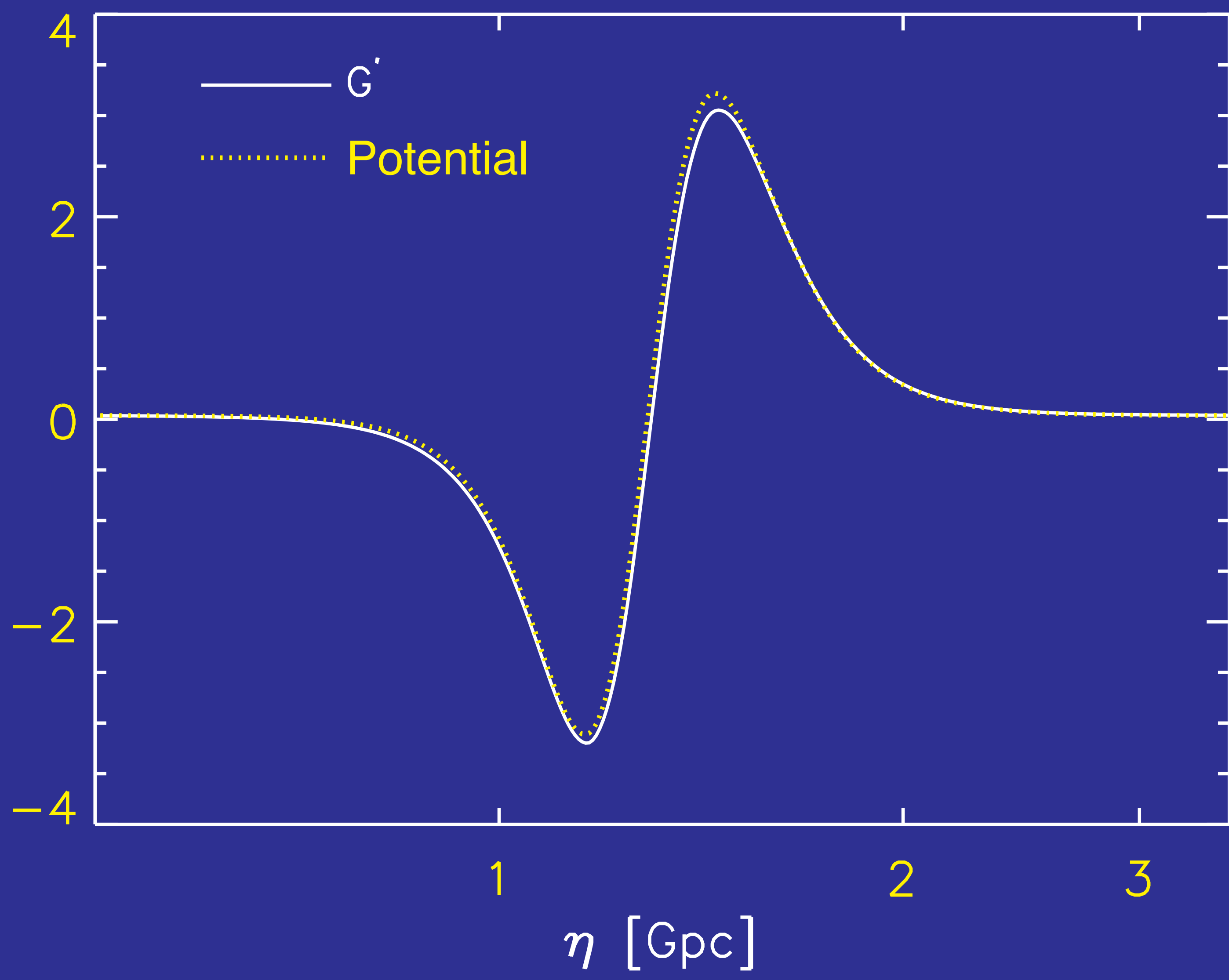
- As long as large features are crossed on order an e-fold or less

$$G' \approx 3\left(\frac{V'}{V}\right)^2 - 2\frac{V''}{V}$$

same combination that enters into tilt n_s in slow roll

GSR and the Potential

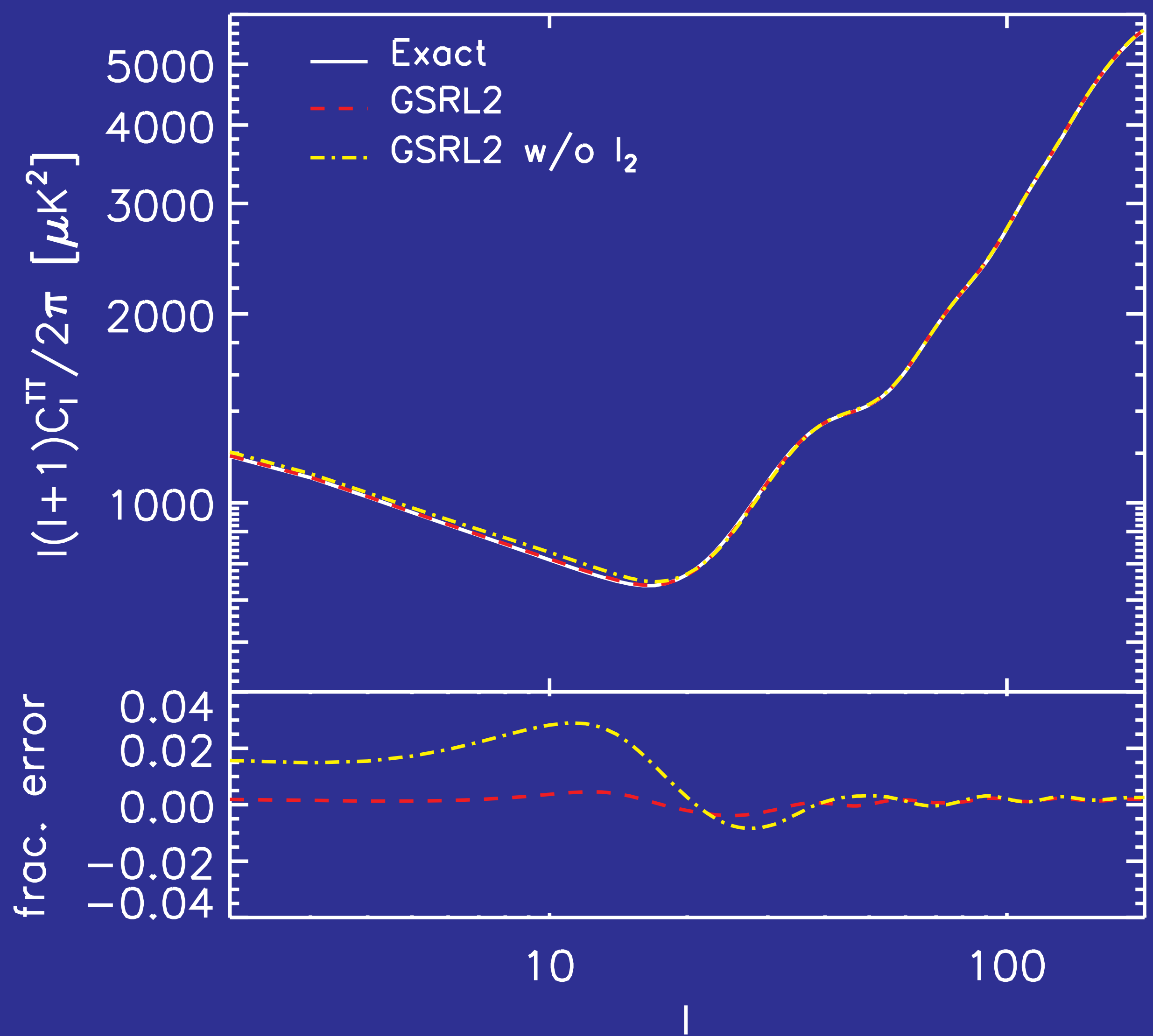
- GSR source function G' vs potential combination $3(V'/V)^2 - 2V''/V$



Dvorkin & Hu (2009)

GSR Accuracy

- ~2% for order unity features (can be improved to <0.5% with iteration)



Dvorkin & Hu (2009)

Generalized Slow Roll

- Heuristically, a non-linear mapping or transfer function

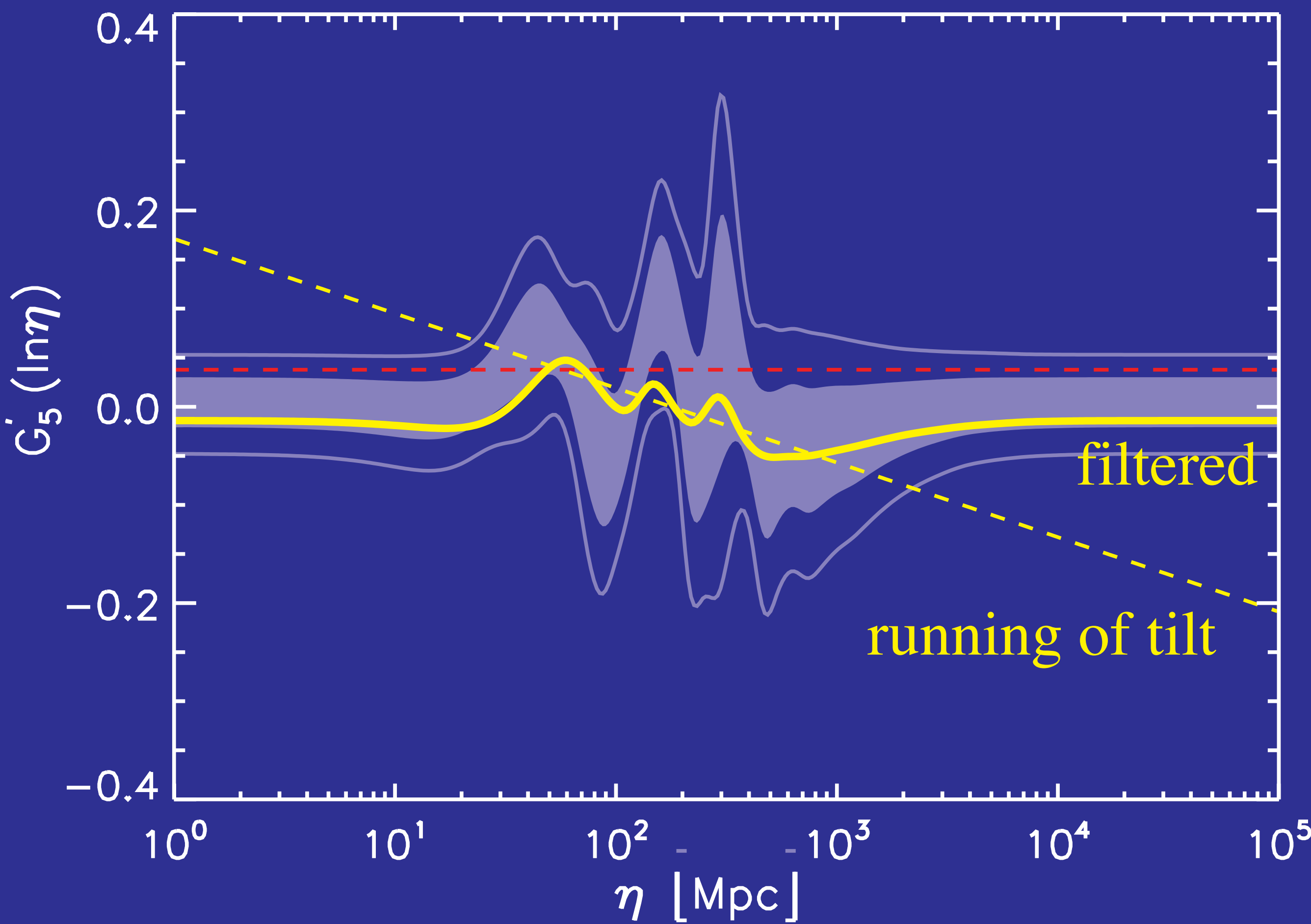
$$\Delta^2_{\mathcal{R}}(k) = A_s T[G'(\ln \eta)]$$

- Allows only initial curvature spectra that are compatible with single field inflation
- Disallowed behavior falsifies single field inflation
- PC decomposition of G' allows efficient computation - precompute responses and combine non-linearly
- Changes in initial power spectrum do not require recomputing radiation transfer in CMB – fast parameters in CAMB
- Bottleneck is WMAP likelihood evaluation. Fast OMP parallelized code ($\sim 5N_{\rm core}$ speedup)

http://background.uchicago.edu/wmap_fast

Functional Constraints on Source

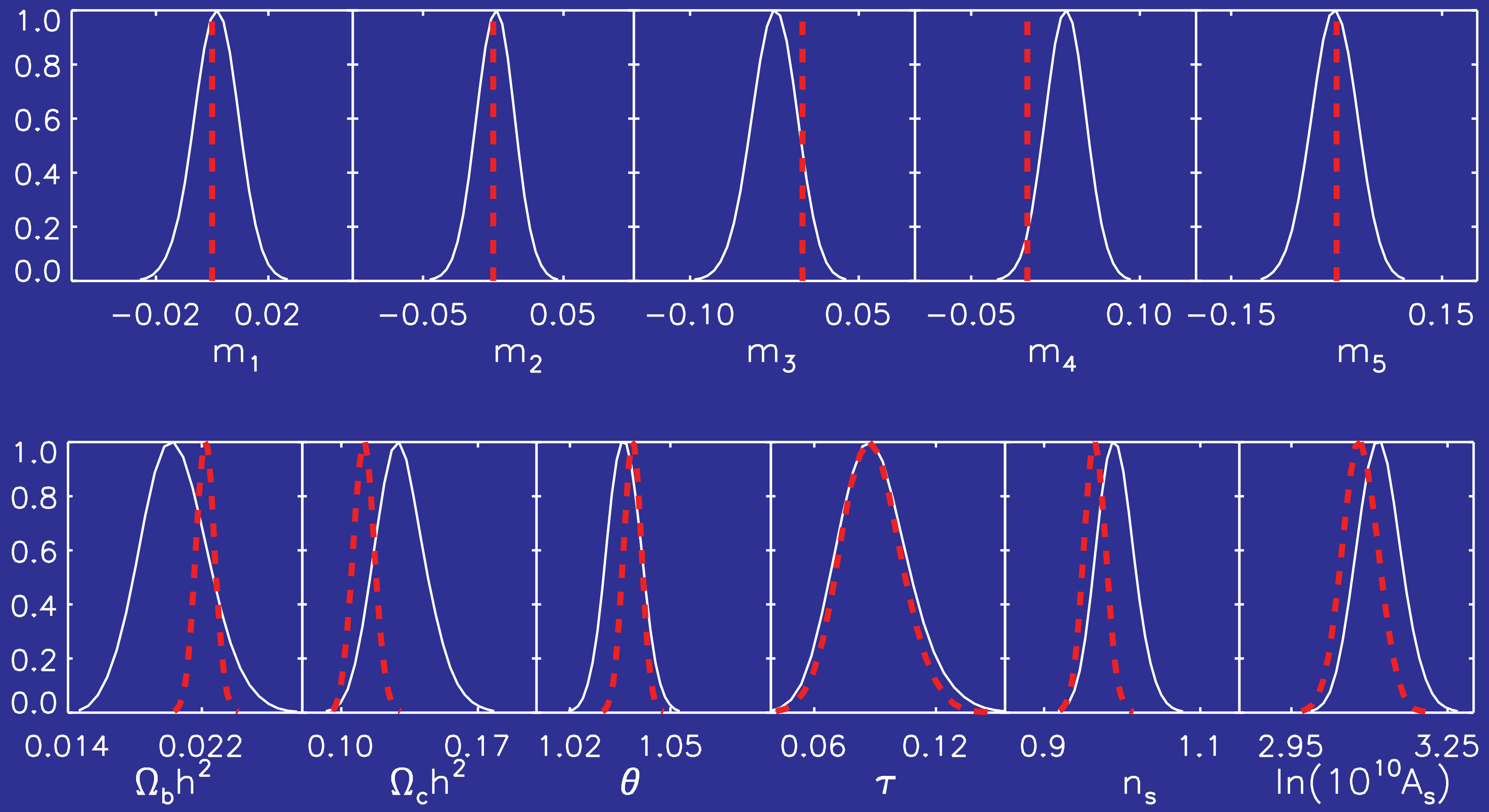
- 5 nearly Gaussian independent constraints on deviations from scale invariance for model testing
- Not a reconstruction due to truncation



Dvorkin & Hu (2010)

WMAP Constraints on 5PCs

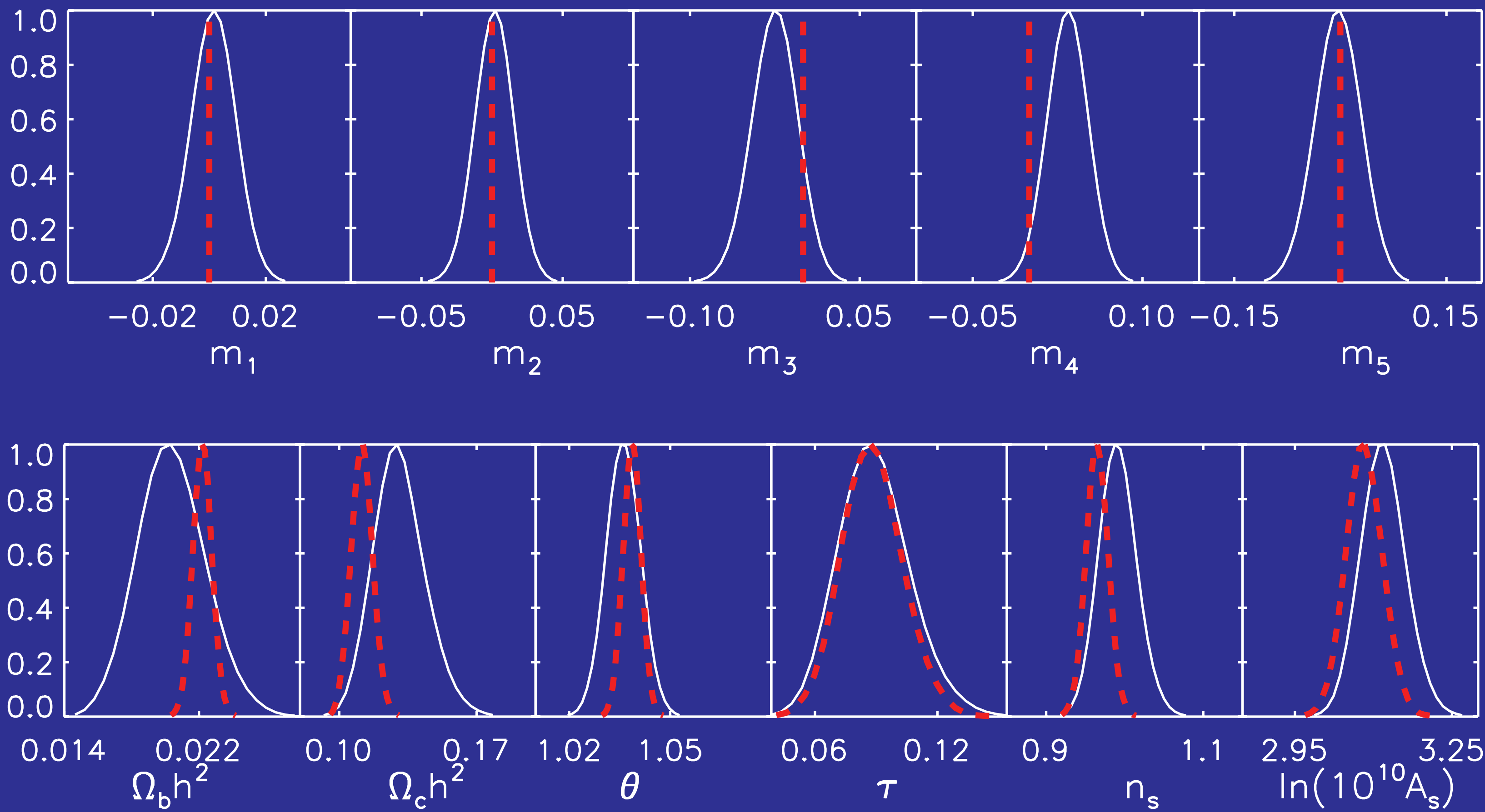
- 1 out of 5 shows a 95% preference for non-zero values though only if CDM density is high



Dvorkin & Hu (2010)

WMAP Constraints on 5PCs

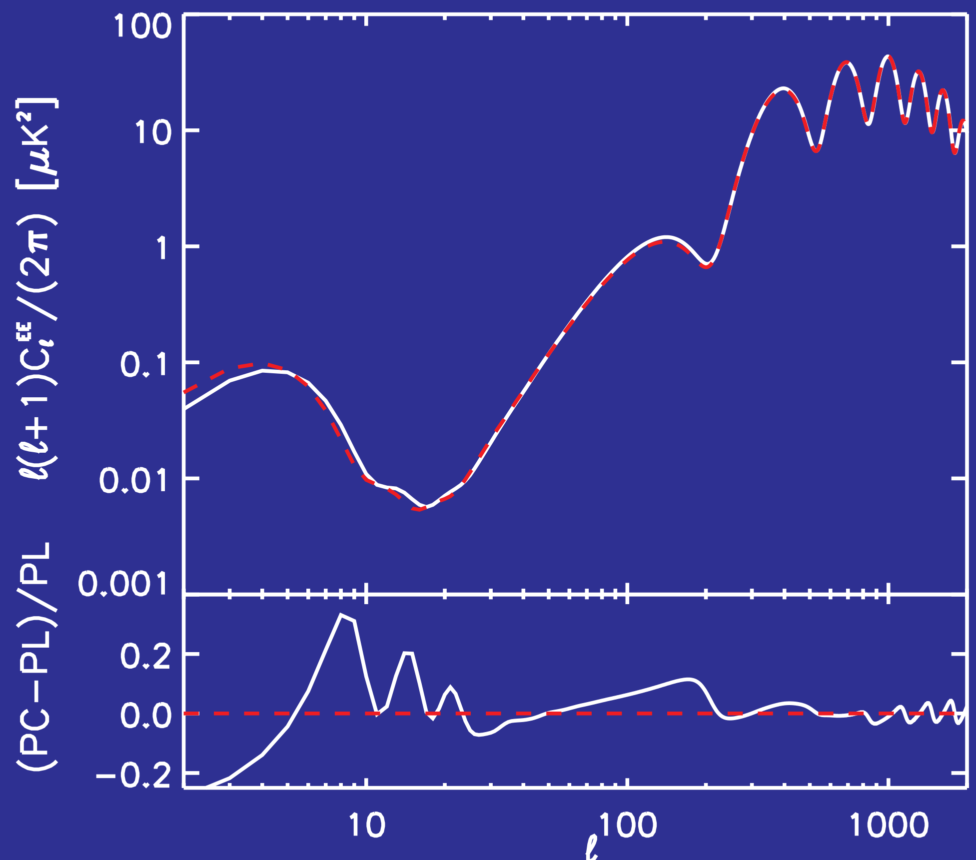
- Interestingly 4th component carries most of the information about running of tilt
- But outside of the PC range data does not prefer a constant running of that size - local preference around few 100Mpc



Dvorkin & Hu (2010)

Predictive Power

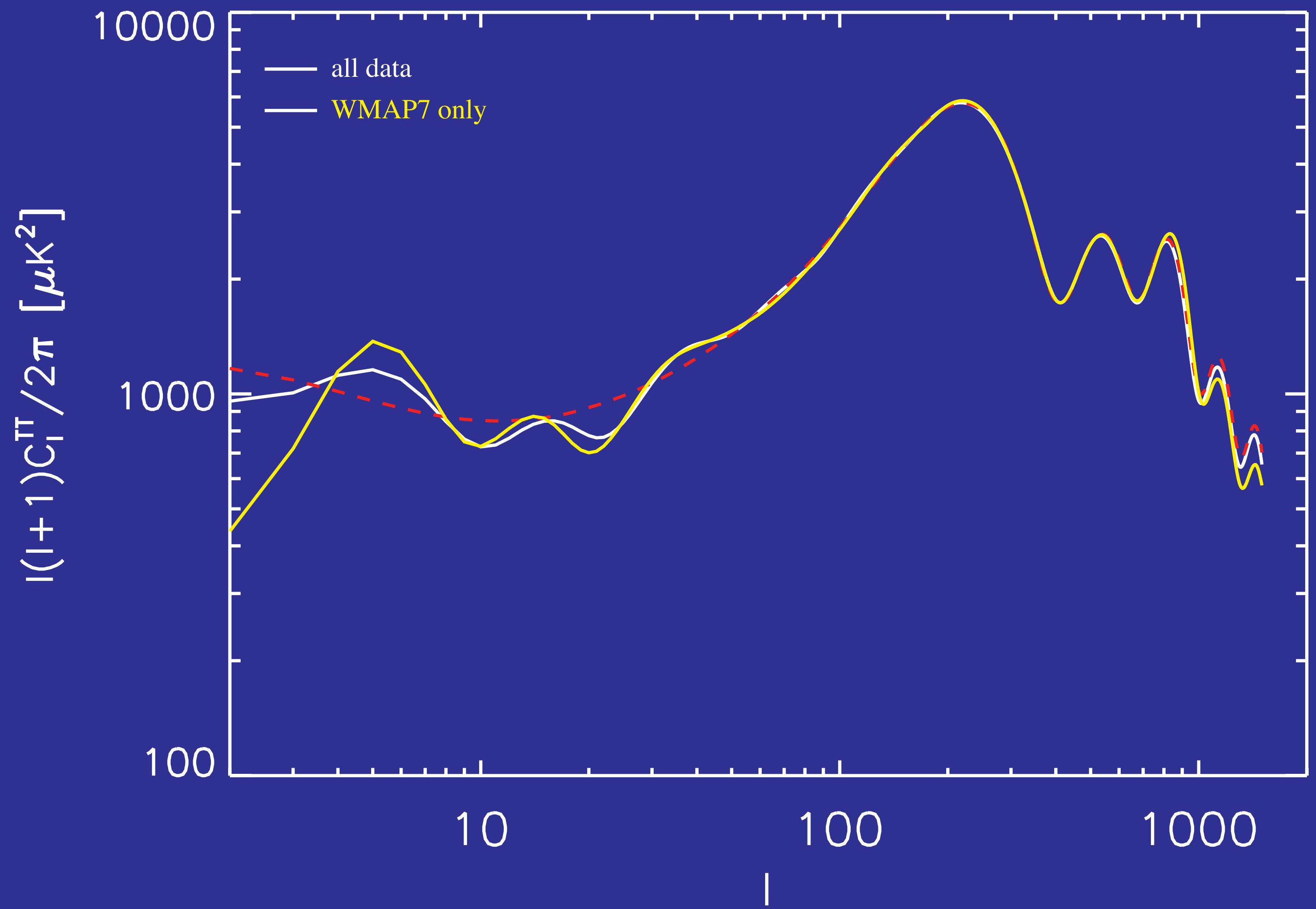
- Models make a prediction for corresponding features or lack thereof in polarization
- Falsification would imply features are not inflationary and potentially even rule out single field inflation



Dvorkin & Hu (2010)

Complete Basis

- 20 PCs are required for a complete basis that includes large features in poorly constrained region of data



Dvorkin & Hu (in prep)

Dark Energy $w(z)$

Smooth Dark Energy

- Physical model of cosmic acceleration must specify 2 scalar closure relations + energy-momentum conservation [Hu (1998) - see also Martin Kunz talk]
- Density and anisotropic stress (or Newton G, slip)

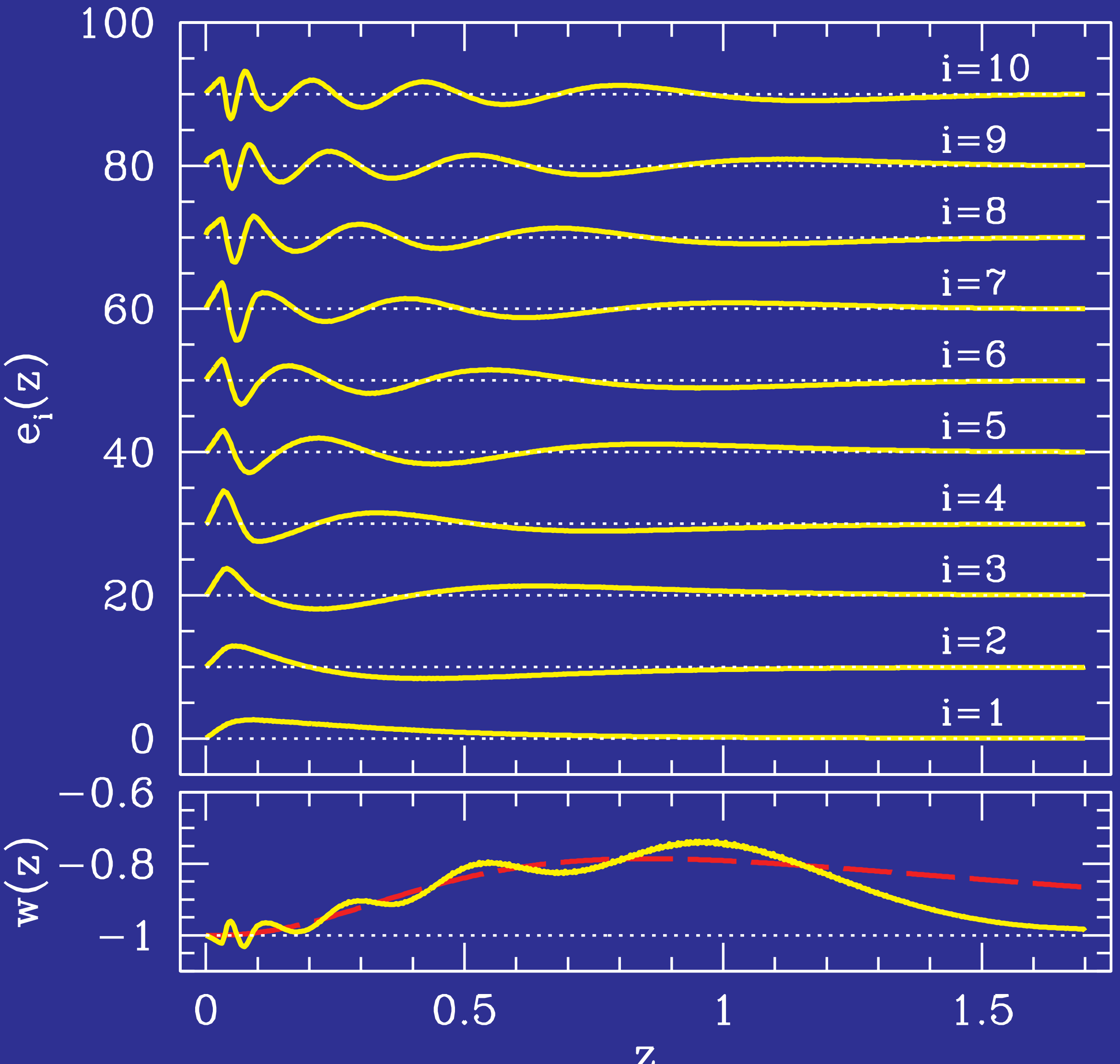
  http://camb.info/ppf Fang, Hu, Lewis (2009)
- Quintessence: no linear anisotropic stress, sound speed c_s=1
- K-essence: variable sound speed
- Below sound horizon dark energy density fluctuations negligible compared with dark matter
  [caution! not true for momentum fluctuation in all gauges]
- Impact on structure formation comes purely from effect on background expansion
- Smooth dark energy hypothesis highly falsifiable

Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate
- Hence geometric measurements of the expansion rate predict the growth of structure
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)

Equation of State PCs

- 10 PCs defined for StageIV (SNAP+Planck) define an observationally complete basis out to z=1.7

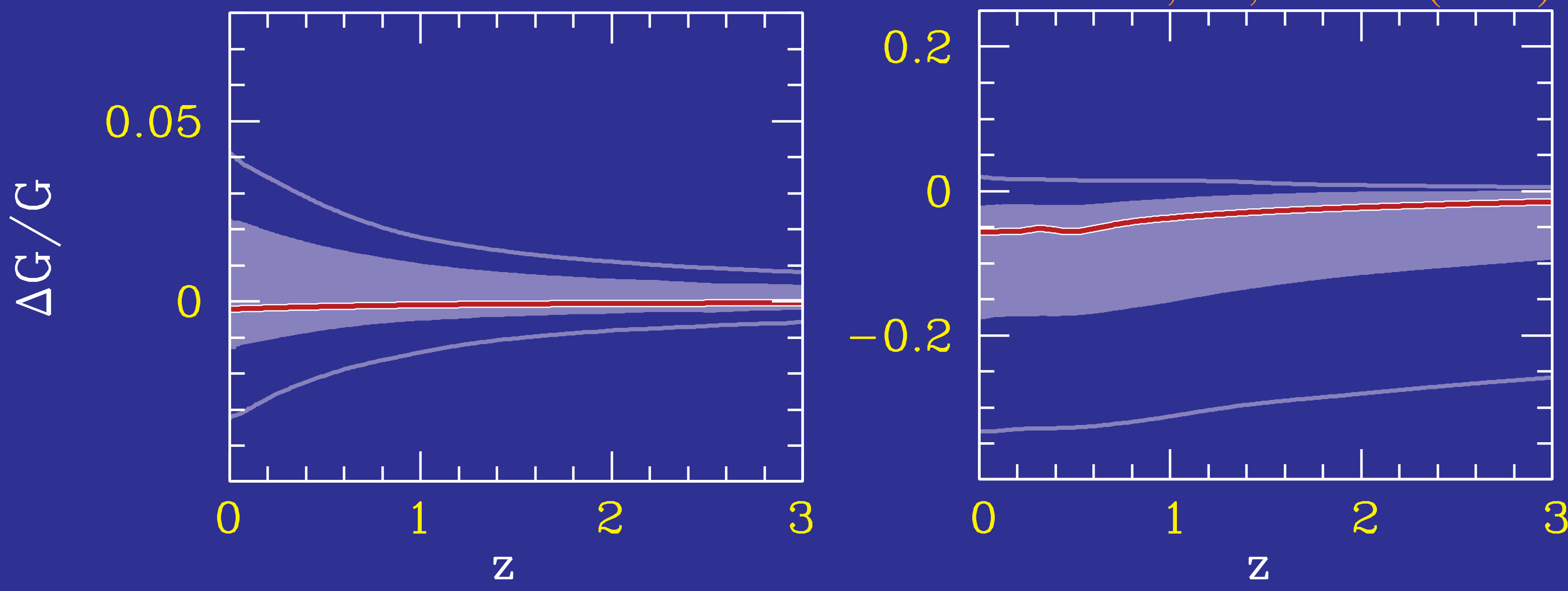


Mortonson, Huterer, Hu (2010)

Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)

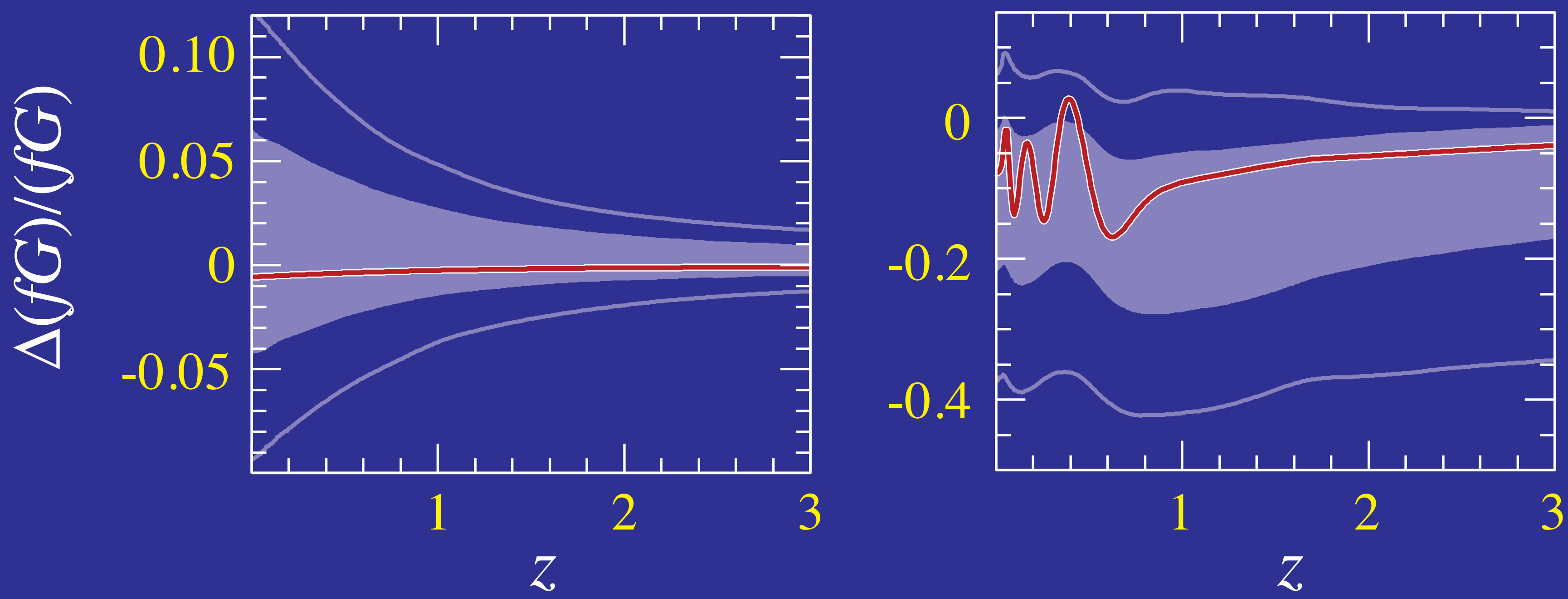


Cosmological Constant Quintessence

- Deviation significantly >2% rules out Λ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in H_0]

Redshift Space Distortion

- Redshift space distortions measure fG or $f\sigma_8$
- Measurements in excess of ~5% of ΛCDM would rule out quintessence

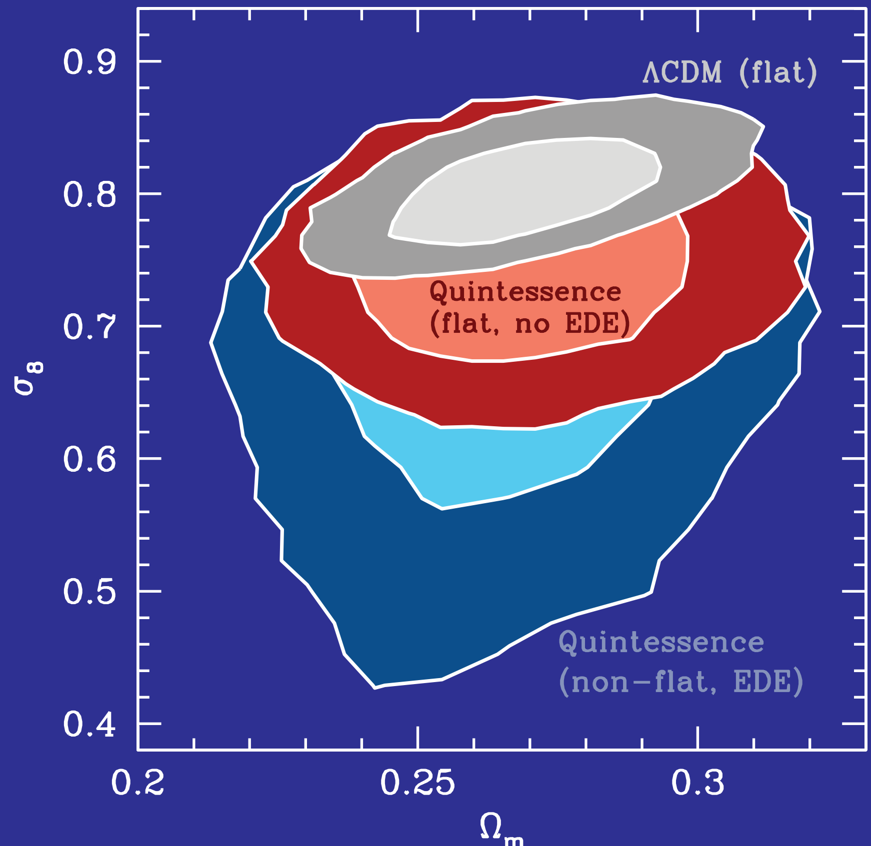


Cosmological Constant Quintessence

Mortonson, Hu, Huterer (2009)

Quintessence Falsified?

- No excess numbers of massive z>1 X-ray or SZ clusters with Gaussian initial conditions (Jee et al 2009, Brodwin et al 2010)
- No excess power in gravitational lensing at high z relative to low z (Bean 0909.3853)

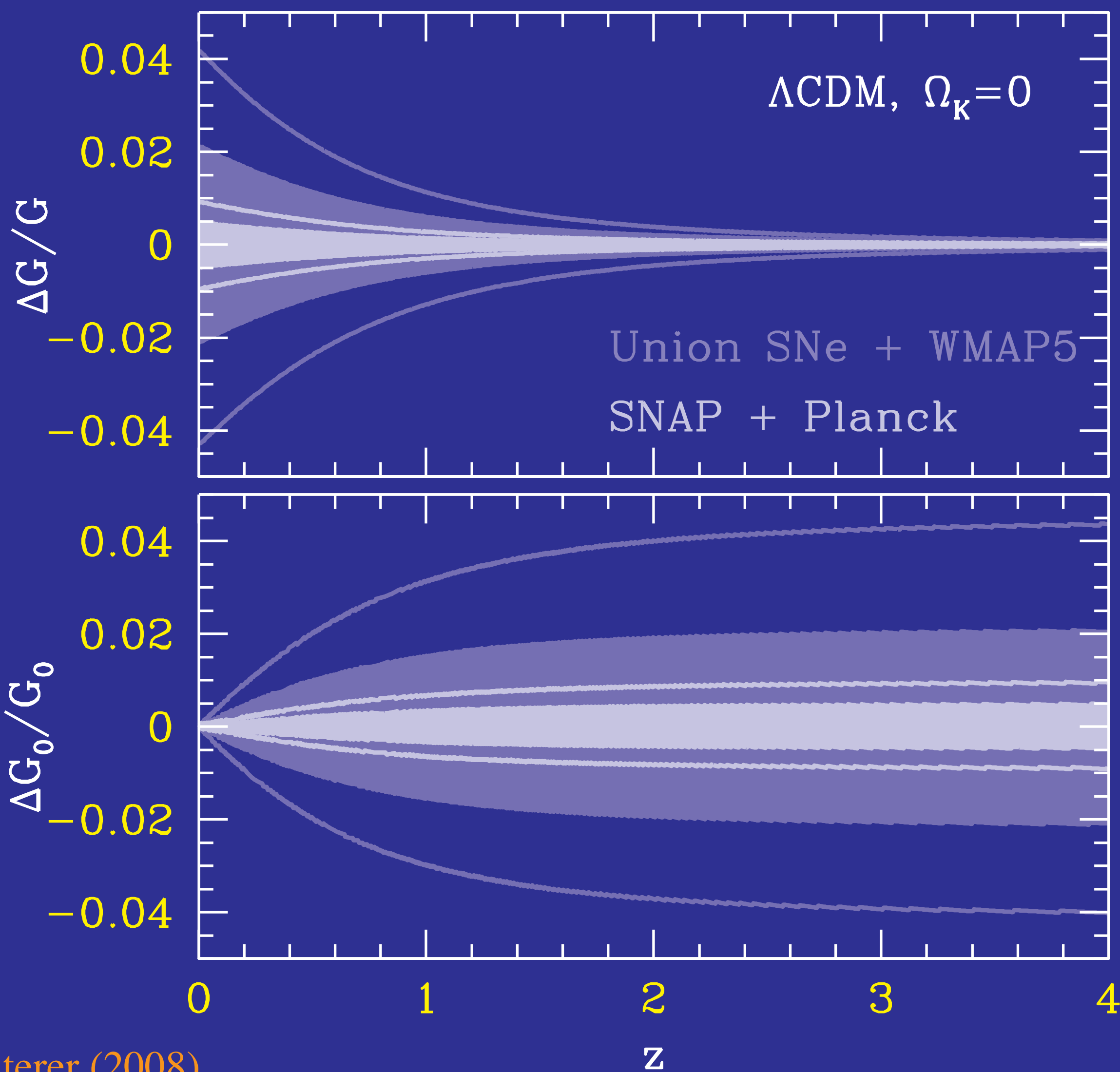


Mortonson, Hu, Huterer (2009 and in prep)

- Given astrophysical systematics, expect purported 2σ violations of smooth dark energy predictions will be common in coming years!

Future Improvements

- Future Stage IV (SNAP+Planck) predictions sharpened by 2-3 and more importantly provide control of systematic errors



Mortonson, Hu, Huterer (2008)

Summary

- PC analysis is a useful, general technique for imposing functional constraints in cosmology

  Efficient, observationally complete

- Explore observational consequences within the whole paradigm rather than a specific functional form for $f(x)$

  Ionization history $x_e(z)$

  Inflaton potential $V(\phi)$

  Dark energy equation of state $w(z)$

- Make falsifiable predictions for new observables

  Polarization predictions for low ℓ anomalies

  Polarization predictions for single field inflation beyond slow roll

  Growth of structure predictions given distance measures