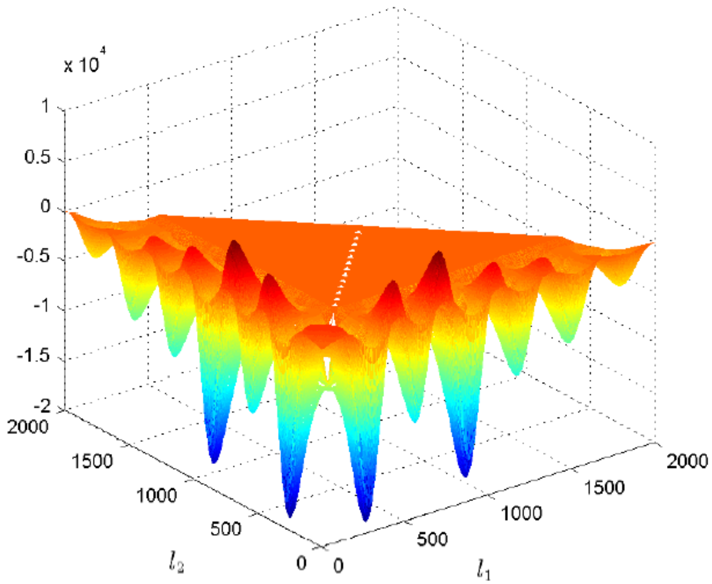
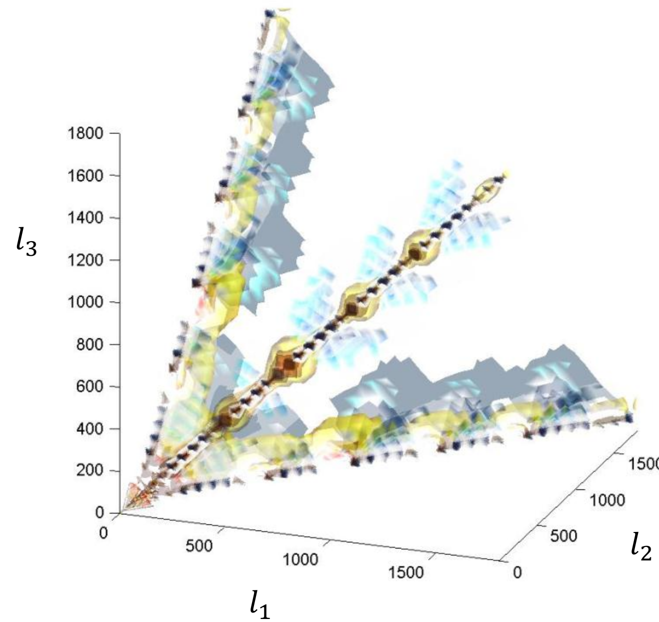
The Shape of the CMB Lensing Bispectrum





Lewis, Challinor & Hanson: *in prep*

Following Goldberg & Spergel 1998, Seljak & Zaldarriaga 1999, Hu 2001, etc.

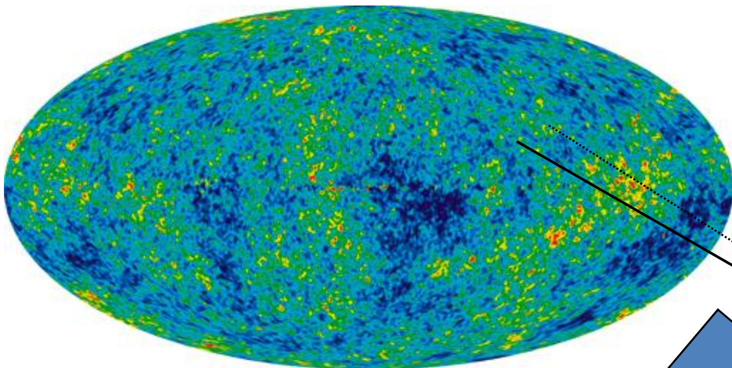
University of Sussex

Antony Lewis

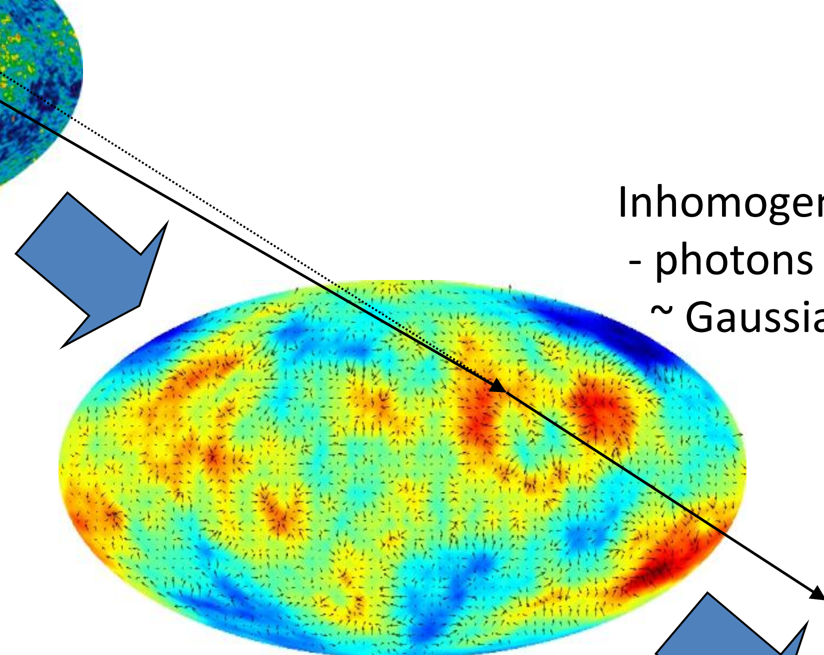
http://cosmologist.info/

CMB lensing

Last scattering surface

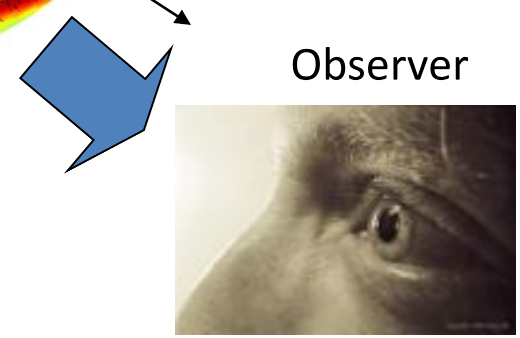


Gaussian LSS



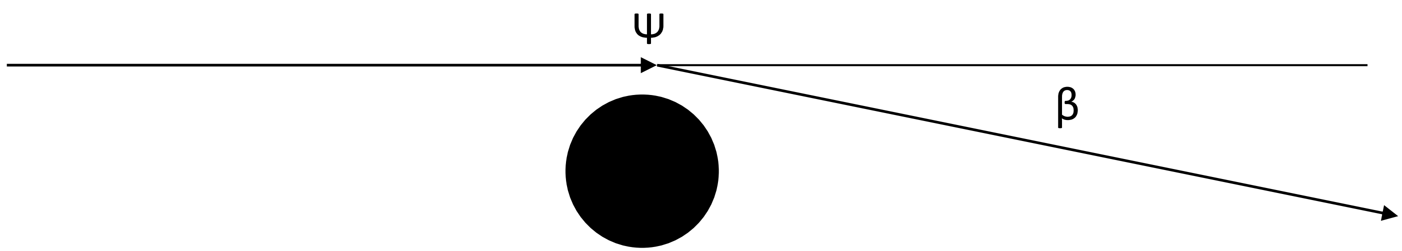
Inhomogeneous universe
- photons deflected by
~ Gaussian potentials

Observer



$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

Lensing order of magnitudes



General Relativity: $\beta = 4\,\Psi$

($\beta \ll 1$)

Potentials linear and approx Gaussian: $\Psi \sim 2 \times 10^{-5}$

$\beta \sim 10^{-4}$

Characteristic size from peak of matter power spectrum ~ 300Mpc

Comoving distance to last scattering surface ~ 14000 MPc

pass through ~50 lumps

assume uncorrelated

total deflection $\sim 50^{1/2} \times 10^{-4} = O(10^{-3})$

~ 2 arcminutes

(neglects angular factors, correlation, etc.)

So why does it matter?

- 2arcmin: ell ~ 3000

  - on small scales CMB is very smooth so lensing dominates the linear signal

- Deflection angles coherent over 300/(14000/2) ~ 2°

  - comparable to CMB scales

  - expect 2arcmin/60arcmin ~ 3% effect on main CMB acoustic peaks

Lensed temperature depends on deflection angle:

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

$$\boldsymbol{\alpha} = \delta\theta = -2\int_0^{\chi^*} \mathrm{d}\chi \frac{f_K(\chi^* - \chi)}{f_K(\chi^*)} \nabla_\perp \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi)$$

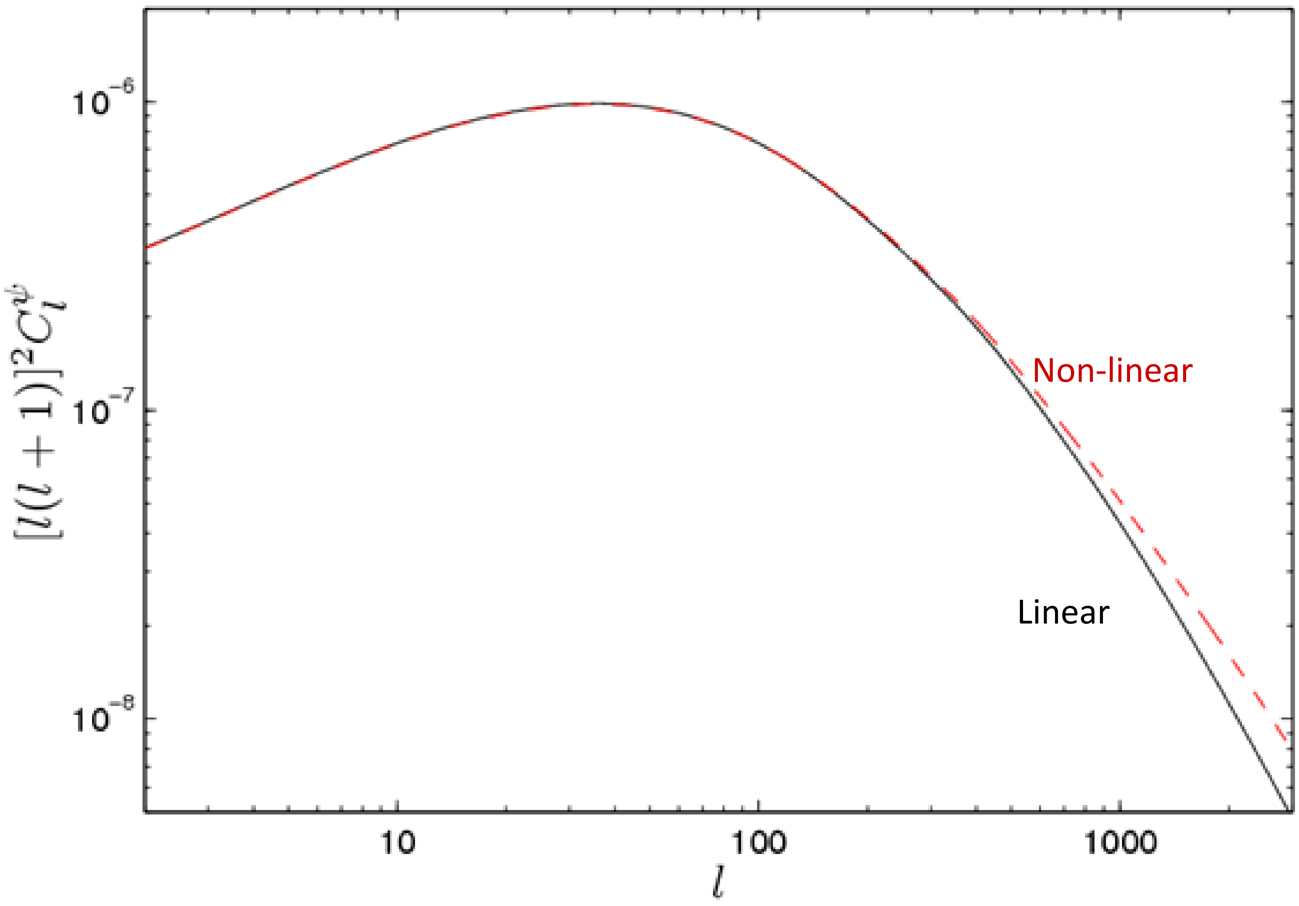
Lensing Potential

Deflection angle on sky given in terms of lensing potential $\boldsymbol{\alpha} = \nabla\psi$

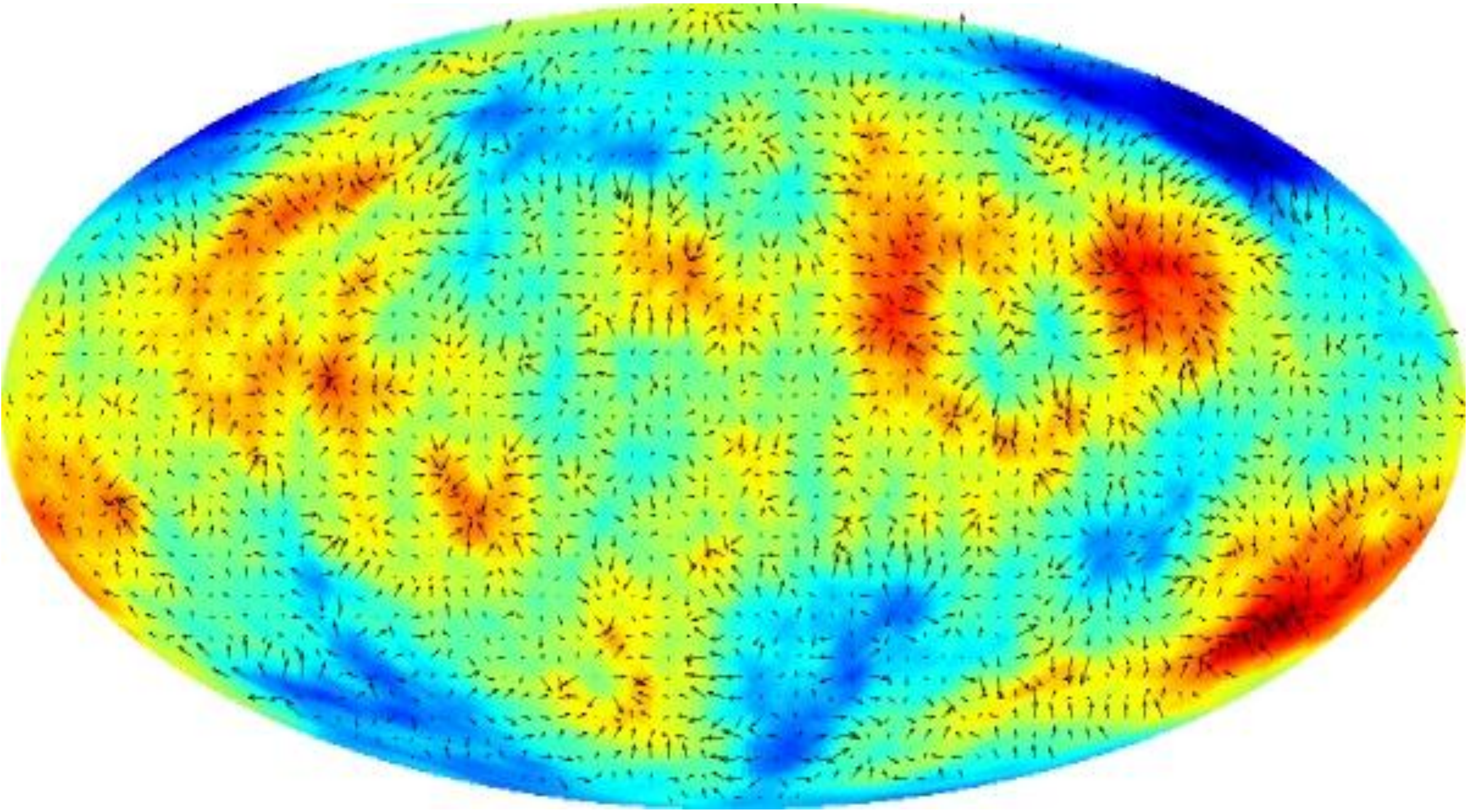
$$\psi(\hat{\mathbf{n}}) = -2\int_0^{\chi_*} \mathrm{d}\chi\, \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi) \frac{f_K(\chi^* - \chi)}{f_K(\chi^*) f_K(\chi)}$$

$$\bar{X}(\mathbf{n}) = X(\mathbf{n}') = X(\mathbf{n} + \nabla\psi(\mathbf{n}))$$

Deflection angle power spectrum



Deflections $O(10^{-3})$, but coherent on degree scales



LensPix sky simulation code:
http://cosmologist.info/lenspix
Lewis 2005, Hammimeche & Lewis 2008

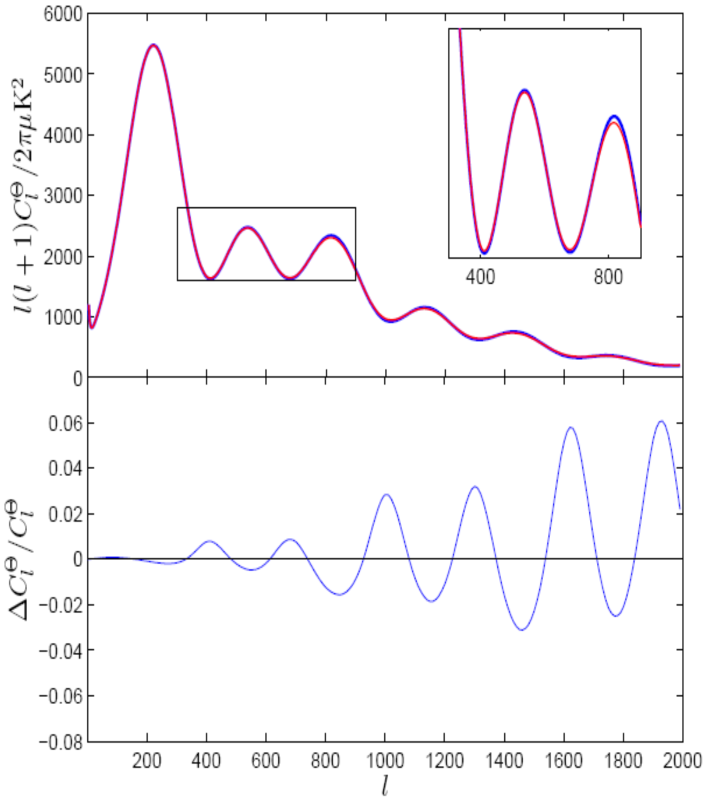
Lensed temperature power spectrum

- Good approximation: Gaussian LSS, Gaussian lensing potentials

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

Fully non-perturbative result:

$$\tilde{C}_{l'} \approx \sum_l \frac{2l+1}{2} C_l \int_{-1}^{1} \mathrm{d}\cos\beta \, d^{l'}_{00}(\beta) e^{-l(l+1)\sigma^2(\beta)/2} \sum_{n=-l}^{l} I_n \left[l(l+1) C_{\mathrm{gl},2}(\beta)/2 \right] d^l_{n-n}(\beta)$$



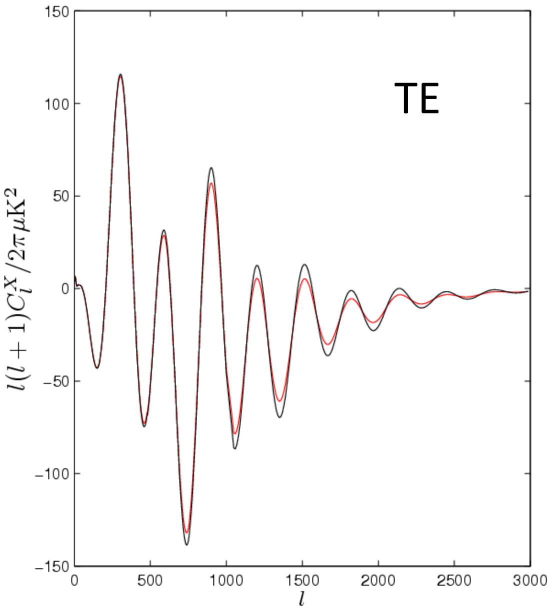
$\sim W_{l\,l'} C_{l'}$ - convolution of unlensed C_l

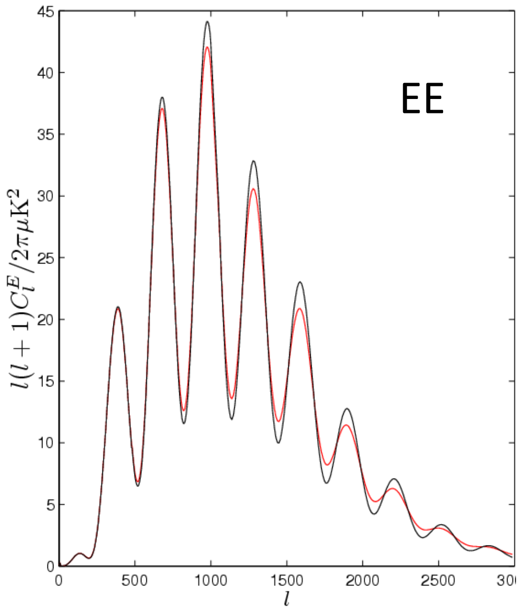
- W is non-linear in lensing potential power

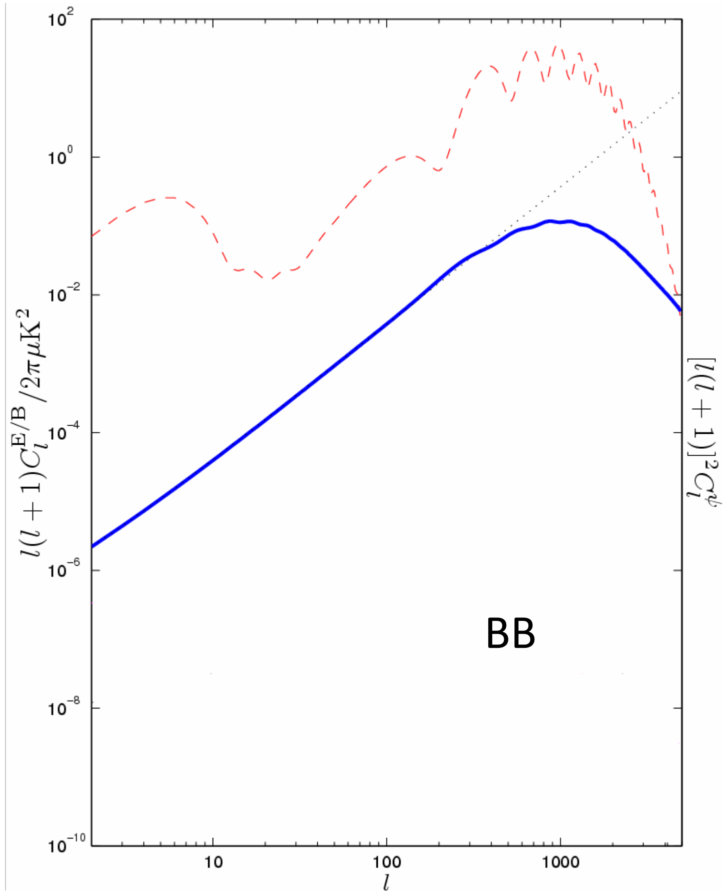
Full-sky calculation accurate to 0.1% in CAMB

Seljak astro-ph/9505109 (flat sky)
Challinor & Lewis, astro-ph/0502425
Lewis & Challinor Phys Rept, astro-ph/0601594

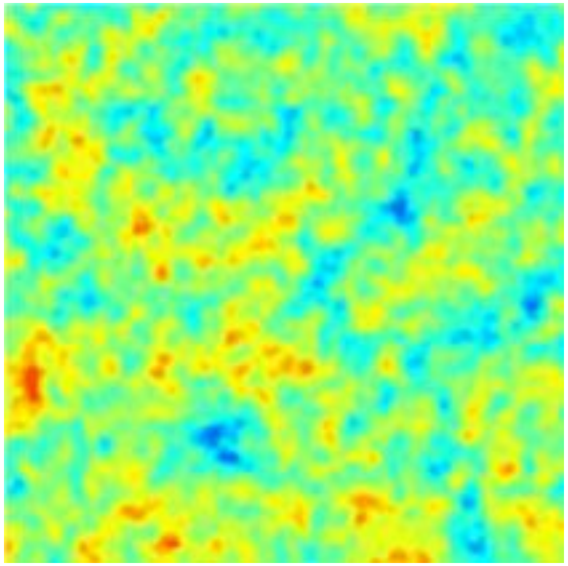
Lensed polarization power spectra

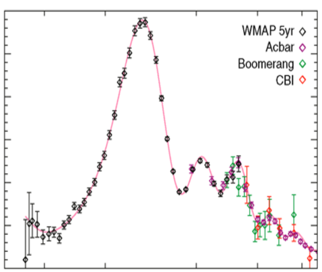




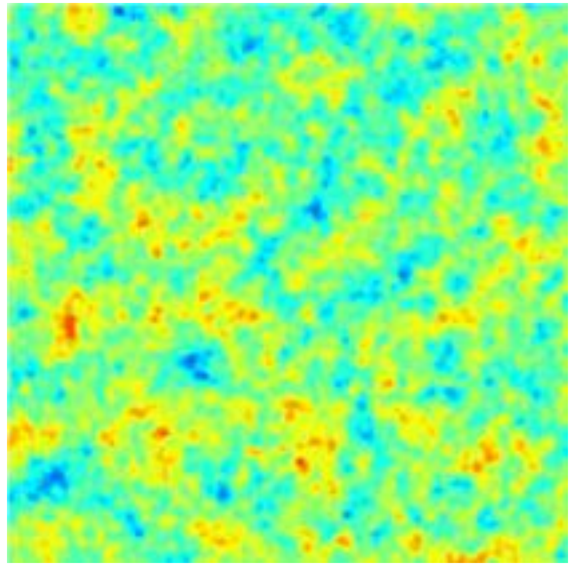


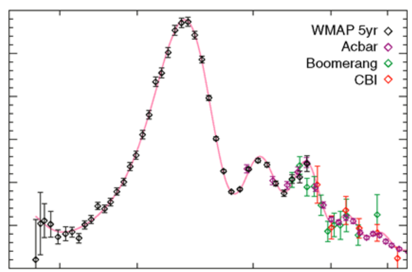
Magnified



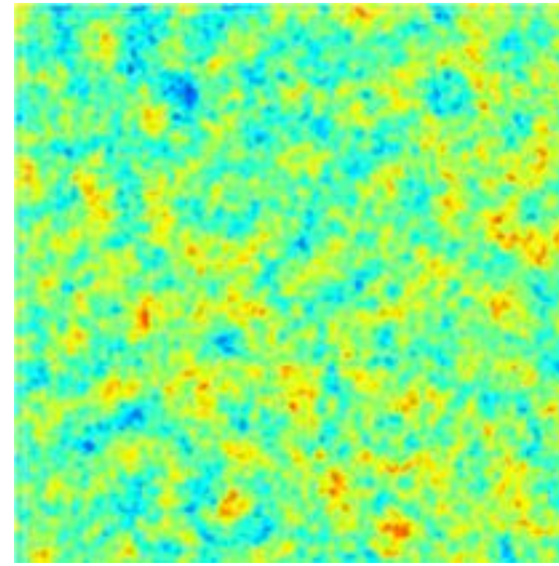


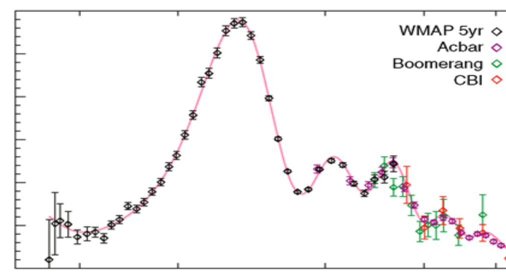
Unlensed





Demagnified





Bispectrum in ultra-squeezed limit

Large scale lensing convergence κ, for all vectors parallel and $l_1 \ll l_2 \sim l_3$

$$\langle T(l_1)\tilde{T}(l_2)\tilde{T}(l_3)\rangle = C_{l_1}^{T\kappa} \langle \frac{d}{d\kappa}[\tilde{T}(l_2)\tilde{T}(l_3)]\rangle$$
$$\approx \delta(l_1 + l_2 + l_3) C_{l_1}^{T\kappa} \frac{d}{d\kappa} \tilde{\mathrm{C}}_{l_2}$$

Magnification $\approx 2\kappa \Rightarrow \frac{\mathrm{d}}{\mathrm{d}\kappa} \approx -2\frac{d}{d\ln l}$ $\qquad$ $\kappa = -\nabla^2\psi/2$

Reduced bispectrum $\qquad b_{l_1 l_2 l_3} \approx l_1^2 C_{l_1}^{T\psi} \frac{1}{l_2^2} \frac{d(l_2^2 \tilde{C}_{l_2})}{d\ln l_2}$

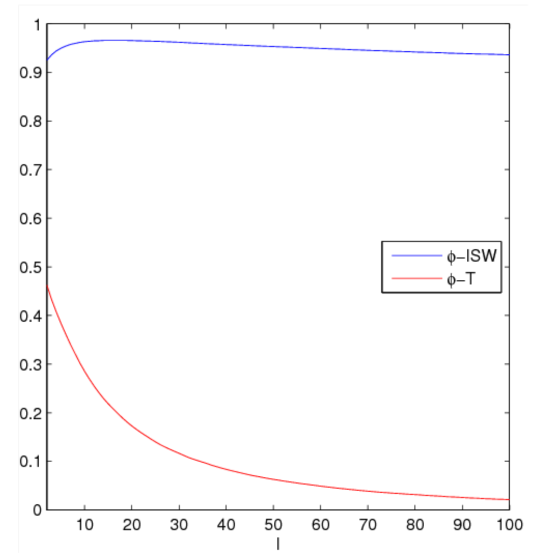
Lensing potential-temperature correlation

Slope of the *lensed* temperature power spectrum

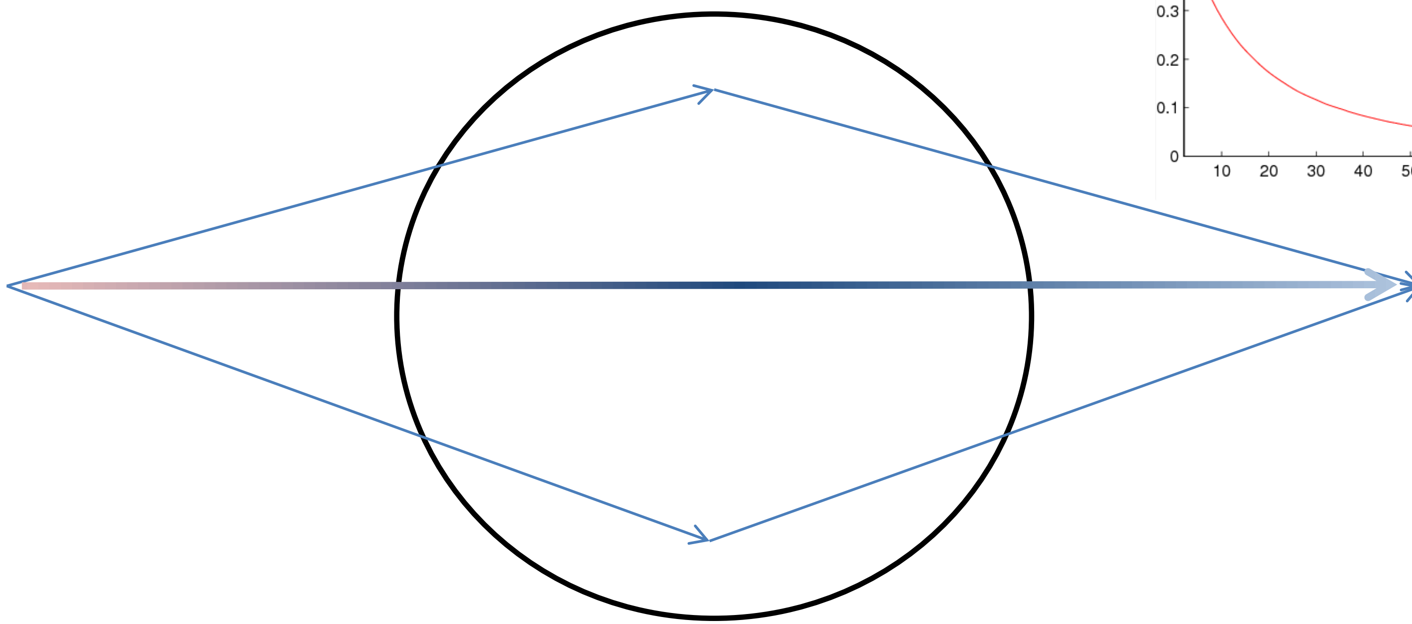
Creminelli & Zaldarriagaa 2004; c.f. Maldacena 2003, Creminelli & Zaldarriaga 2004 for primordial bispectrum

Why is there a correlation between large-scale lenses and the temperature?

(small-scales: also SZ , Rees-Sciama..)



$$\Delta T_{\rm ISW}(\hat{\mathbf{n}}) = 2\int_0^{\chi_*} \mathrm{d}\chi\, \dot{\Psi}(\chi\hat{\mathbf{n}};\eta_0 - \chi)$$



Overdensity: magnification correlated with positive Integrated Sachs-Wolfe (net blueshift)

Underdensity: demagnification correlated with negative Integrated Sachs-Wolfe (net redshift)

Bispectrum as statistical anisotropy correlation

Lensing by fixed ψ field introduced statistical anisotropy

Construct quadratic estimator for ψ (Hu and Okomoto 2003)

$$\langle \tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l_1} - \mathbf{l_2})\rangle_T \propto \psi(\mathbf{l}_1)$$

Bispectrum measures cross-correlation of quadratic estimator for ψ with the large-scale temperature

For squeezed triangles, $l_1 \ll l_2, l_3$,

$$\tilde{T}(\mathbf{l_1}) \sim T(\mathbf{l_1}) \text{ and } \langle \tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle_T \propto \psi(\mathbf{l}_1)$$

$$\Rightarrow \quad \langle \tilde{T}(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l_3})\rangle \sim \langle T(\mathbf{l}_1)\psi(\mathbf{l_1})\rangle \sim C_{l_1}^{\psi T}$$

See Hanson & Lewis 0908.0963 for general optimal anisotropy estimator formalism

Accurate bispectrum calculation

Assume Gaussian fields. Non-perturbative result:

$$\langle T(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle = C_{l_1}^{T\psi}\left\langle \frac{\delta}{\delta\psi(\mathbf{l}_1)^*}\left(\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\right)\right\rangle$$

Use $\tilde{T}(\boldsymbol{x}) = T(\boldsymbol{x} + \boldsymbol{\nabla}\psi)$ $\Longrightarrow$ $$\frac{\delta}{\delta\psi(\mathbf{l}_1)^*}\tilde{T}(\mathbf{l}) = -\frac{i}{2\pi}\mathbf{l}_1\cdot\widetilde{\boldsymbol{\nabla}T}(\mathbf{l}+\mathbf{l}_1),$$

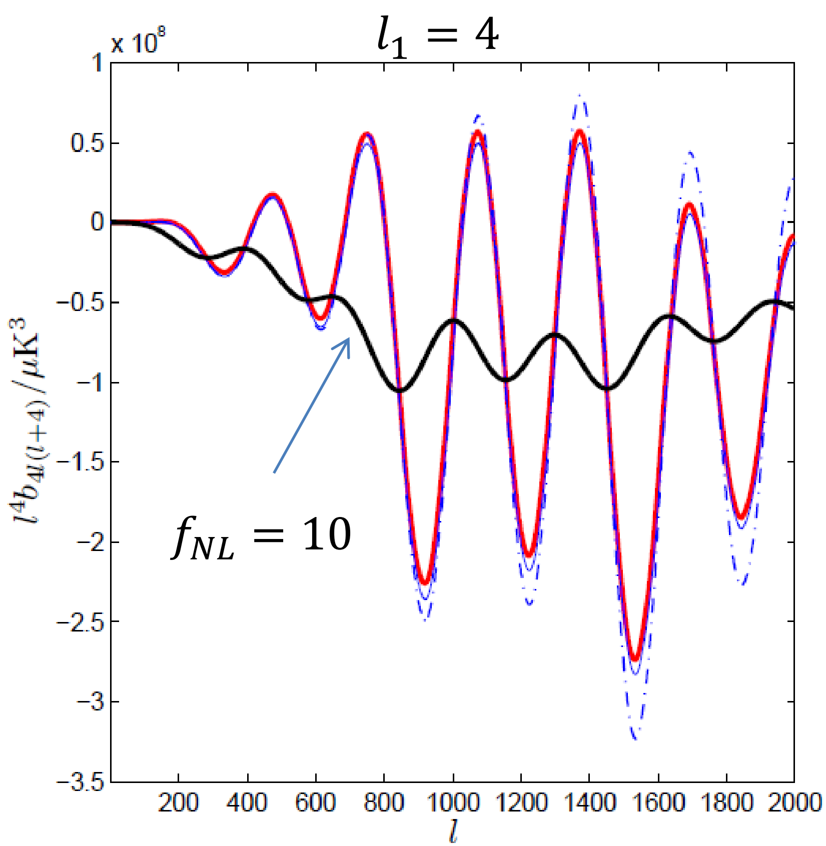
$$\begin{aligned}
\langle T(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle &= -\frac{i}{2\pi}C_{l_1}^{T\psi}\,\mathbf{l}_1\cdot\left\langle\widetilde{\boldsymbol{\nabla}T}(\mathbf{l}_1+\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\right\rangle + (\mathbf{l}_2\leftrightarrow\mathbf{l}_3)\\
&= -\frac{1}{2\pi}\delta(\mathbf{l}_1+\mathbf{l}_2+\mathbf{l}_3)C_{l_1}^{T\psi}\left[(\mathbf{l}_1\cdot\mathbf{l}_2)\tilde{C}_{l_2}^{T\nabla T} + (\mathbf{l}_1\cdot\mathbf{l}_3)\tilde{C}_{l_3}^{T\nabla T}\right]
\end{aligned}$$

~ Lensed temperature power spectrum

Lensing bispectrum depends on *changes* in the small-scale *lensed* power

$$
\begin{aligned}
b_{l_1 l_2 l_3} &\approx -C_{l_1}^{T\psi}\left[(\mathbf{l}_1\cdot\mathbf{l}_2)\tilde{C}_{l_2}^{TT} + (\mathbf{l}_1\cdot\mathbf{l}_3)\tilde{C}_{l_3}^{TT}\right] \\
&\approx l_1^2 C_{l_1}^{T\psi}\left[\frac{(\mathbf{l}_1\cdot\mathbf{l}_2)^2}{l_1^2 l_2^2}\left.\frac{\mathrm{d}\tilde{C}_l^{TT}}{\mathrm{d}\ln l}\right|_{l_2} + \tilde{C}_{l_2}^{TT}\right].
\end{aligned}
$$

$$(\mathbf{l_1} + \mathbf{l_2} + \mathbf{l_3} = 0)$$



- Quite large signal. Expect $\sim 5\sigma$ with Planck. Cosmic variance $\sim 7\sigma$.

- Using lensed power spectra important at 5-20% level: leading-order result (using unlensed spectra) not accurate enough

If lensing is neglected get bias $\Delta f_{NL} \sim 9$ on primordial local models with Planck
(see e.g. Hanson et al 0905.4732, Mangilli 0906.2317)

BUT:

- Lensing bispectrum depends on power difference: has phase shift compared to any adiabatic primordial bispectrum (and different scale dependence)

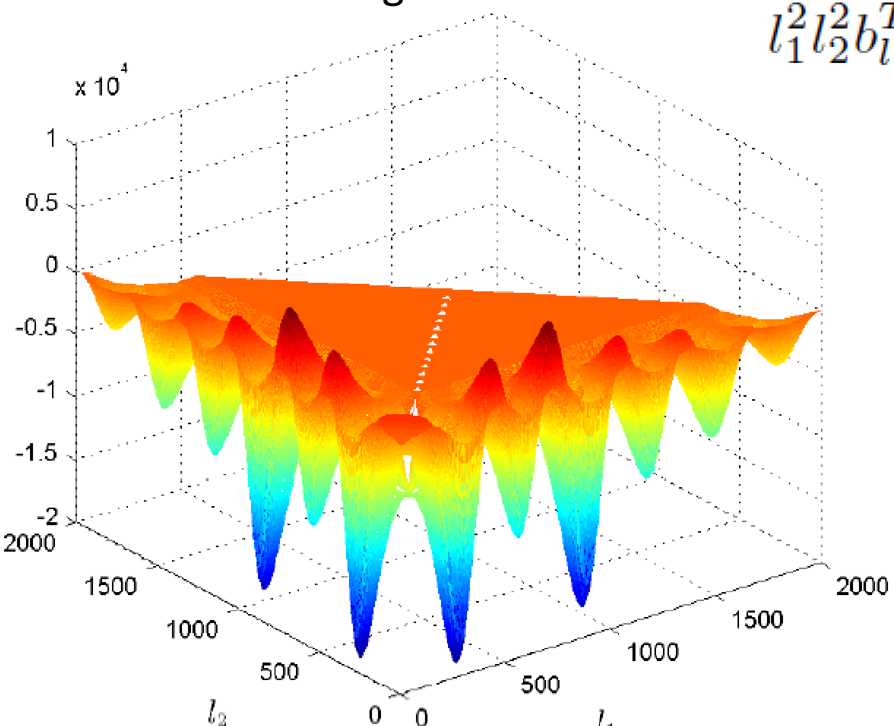
- Lensing bispectrum is strongly scale dependent (small ISW for larger l_1)

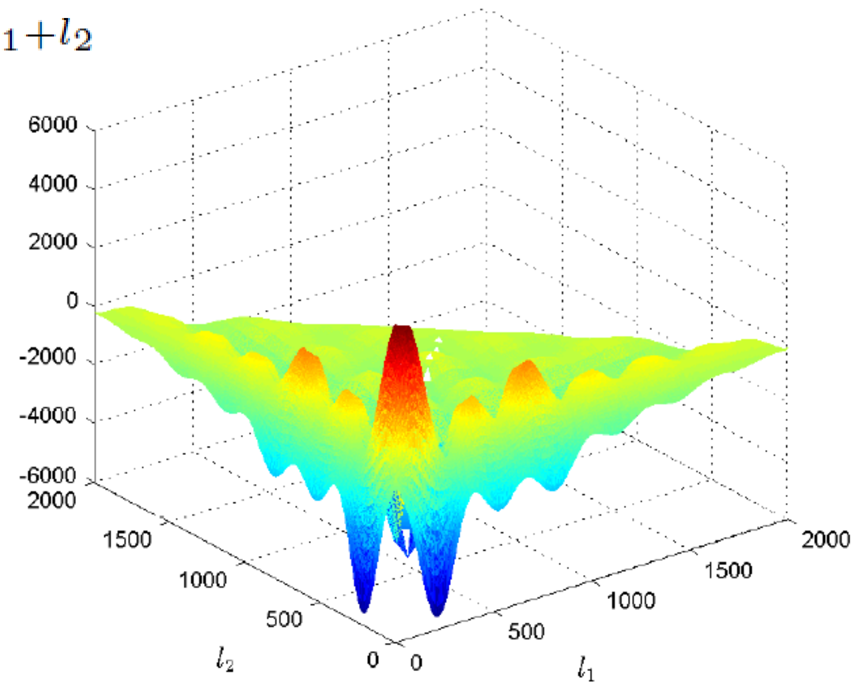
- Lensing bispectrum depends on shape of squeezed triangle ($l_1 \cdot l_2$ factor)

Lensing

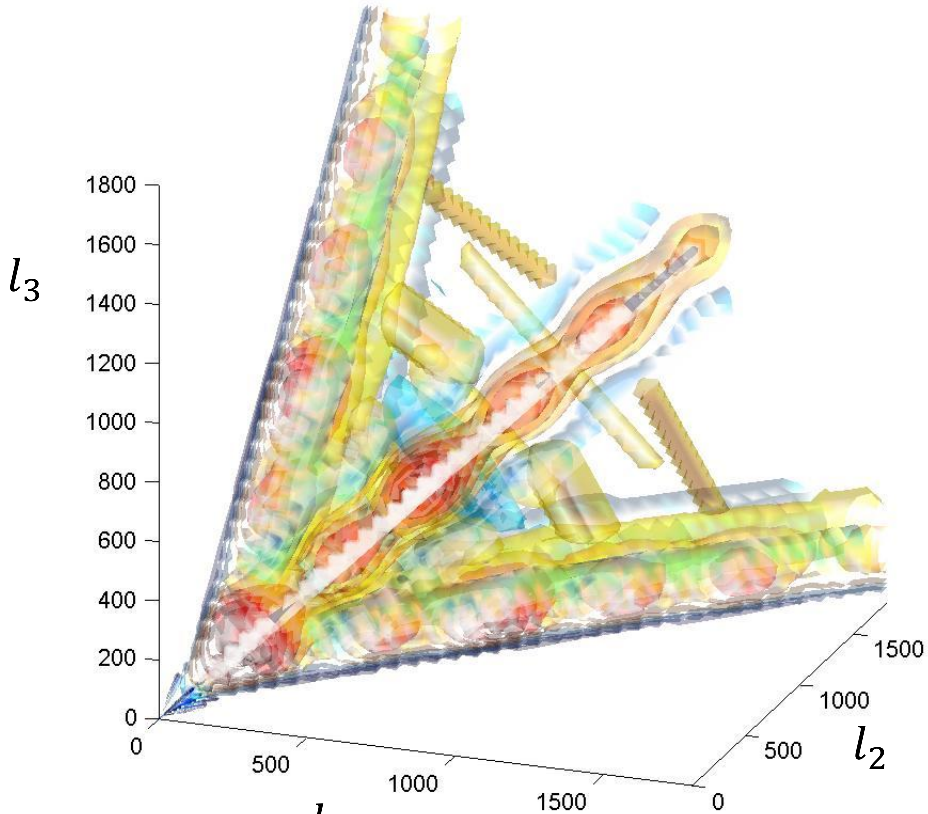
$$l_1^2 l_2^2 b^{TTT}_{l_1 l_2 l_1+l_2}$$

Local f_{NL}

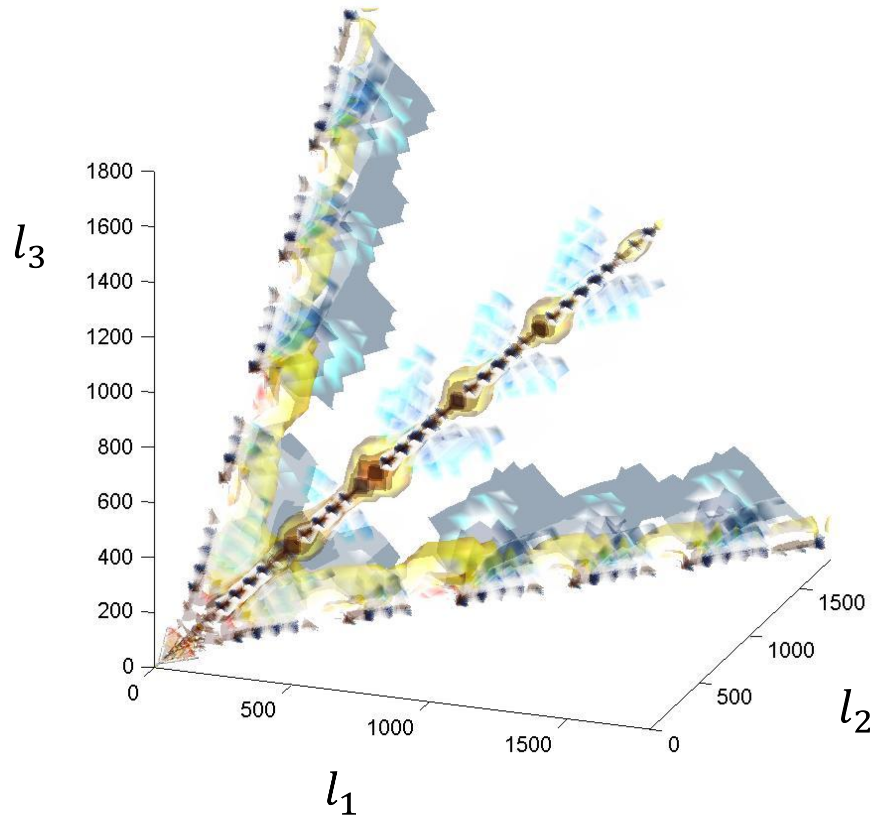




$b_{l_1 l_2 l_3}$



Local f_{NL}



CMB temperature lensing

Lensing bispectrum also squeezed triangles but quite distinctive

Temperature bispectrum correlation with local $f_{NL} \sim 30\%$: in null hypothesis can measure amplitude using optimized estimator and accurately subtract from f_{NL} estimator

CMB polarization

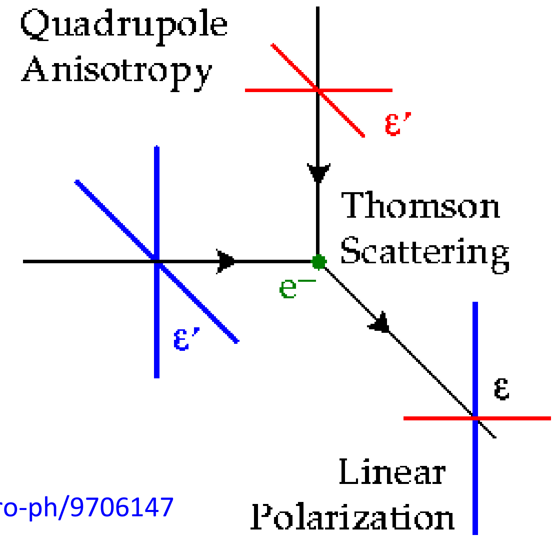
General full-sky bispectrum: $\mathbf{a}_{lm} = (T_{lm}, E_{lm}, B_{lm})^T$

$$B^{ijk}_{l_1 l_2 l_3} = \sum_{m_1 m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \langle a^i_{l_1 m_1} a^j_{l_2 m_2} a^k_{l_3 m_3} \rangle$$

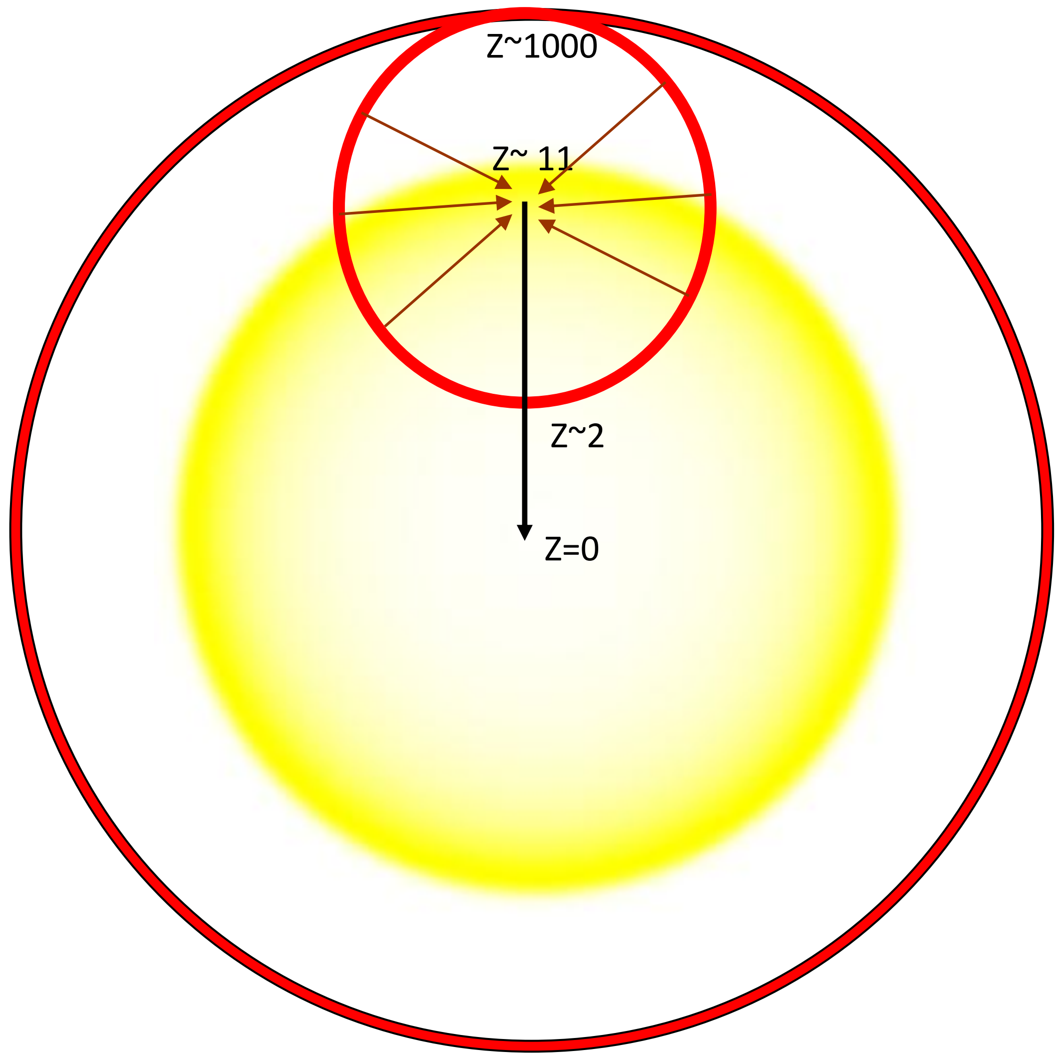
$$\approx F^{s_k}_{l_3 l_1 l_2} C^{a^i \psi}_{l_1} \tilde{C}^{a^j a^k}_{l_2} + i F^{-s_k}_{l_3 l_1 l_2} C^{a^i \psi}_{l_1} \tilde{C}^{a^j \bar{a}^k}_{l_2}$$ + perms

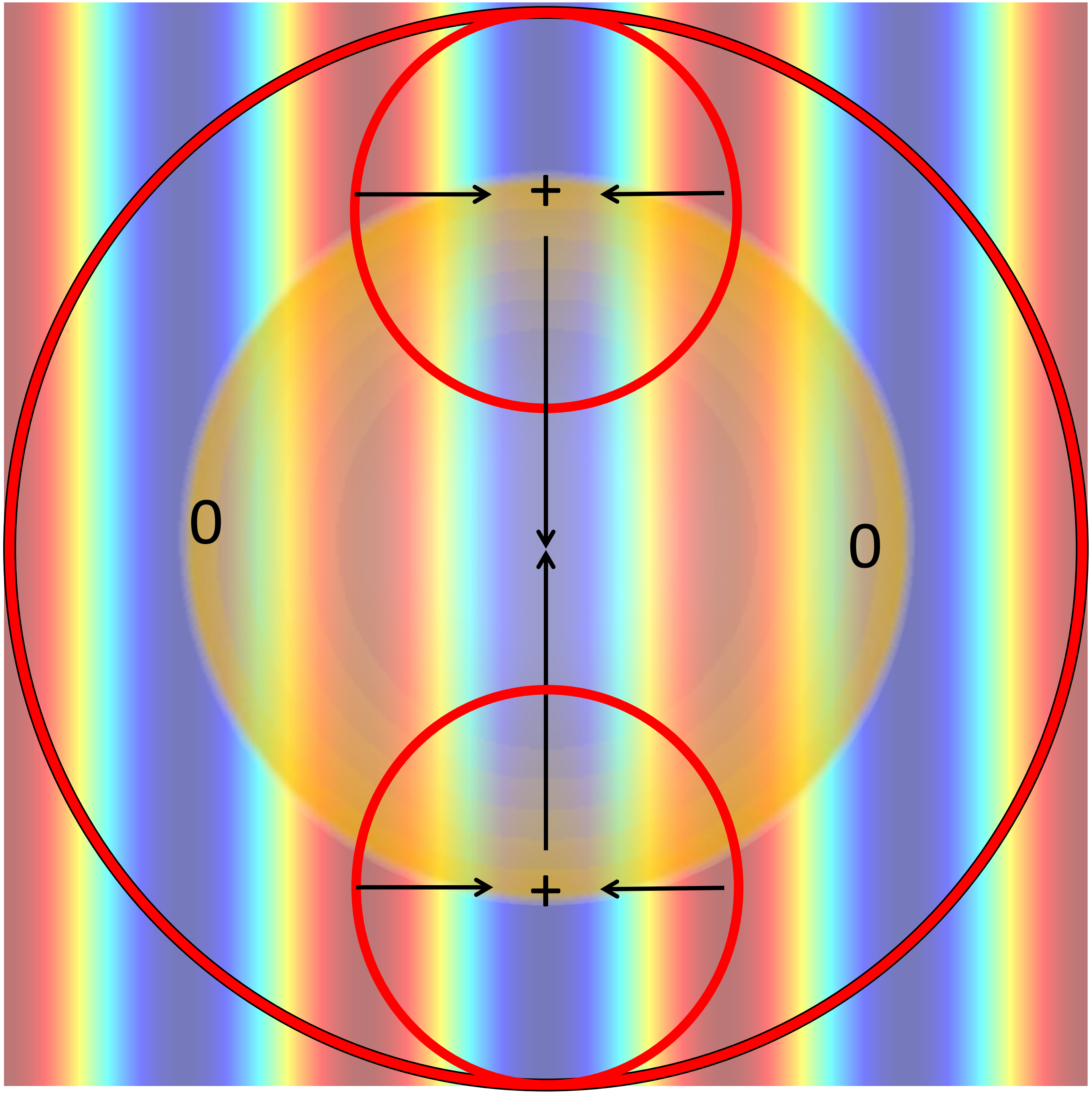
Is the polarization correlated? $C_l^{E\psi} = ?$

Yes! Significant large-scale correlation due to reionization



Hu astro-ph/9706147

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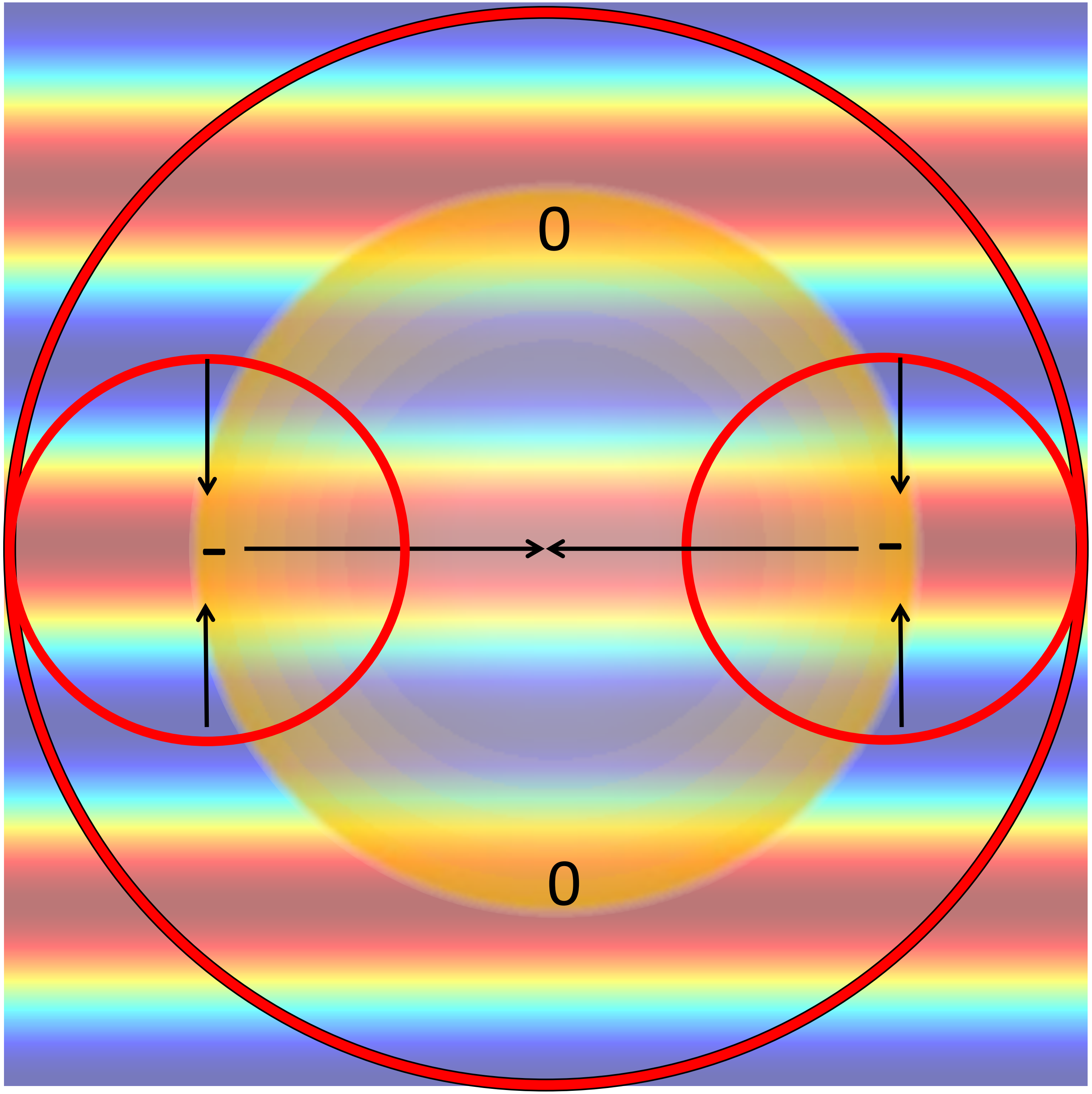
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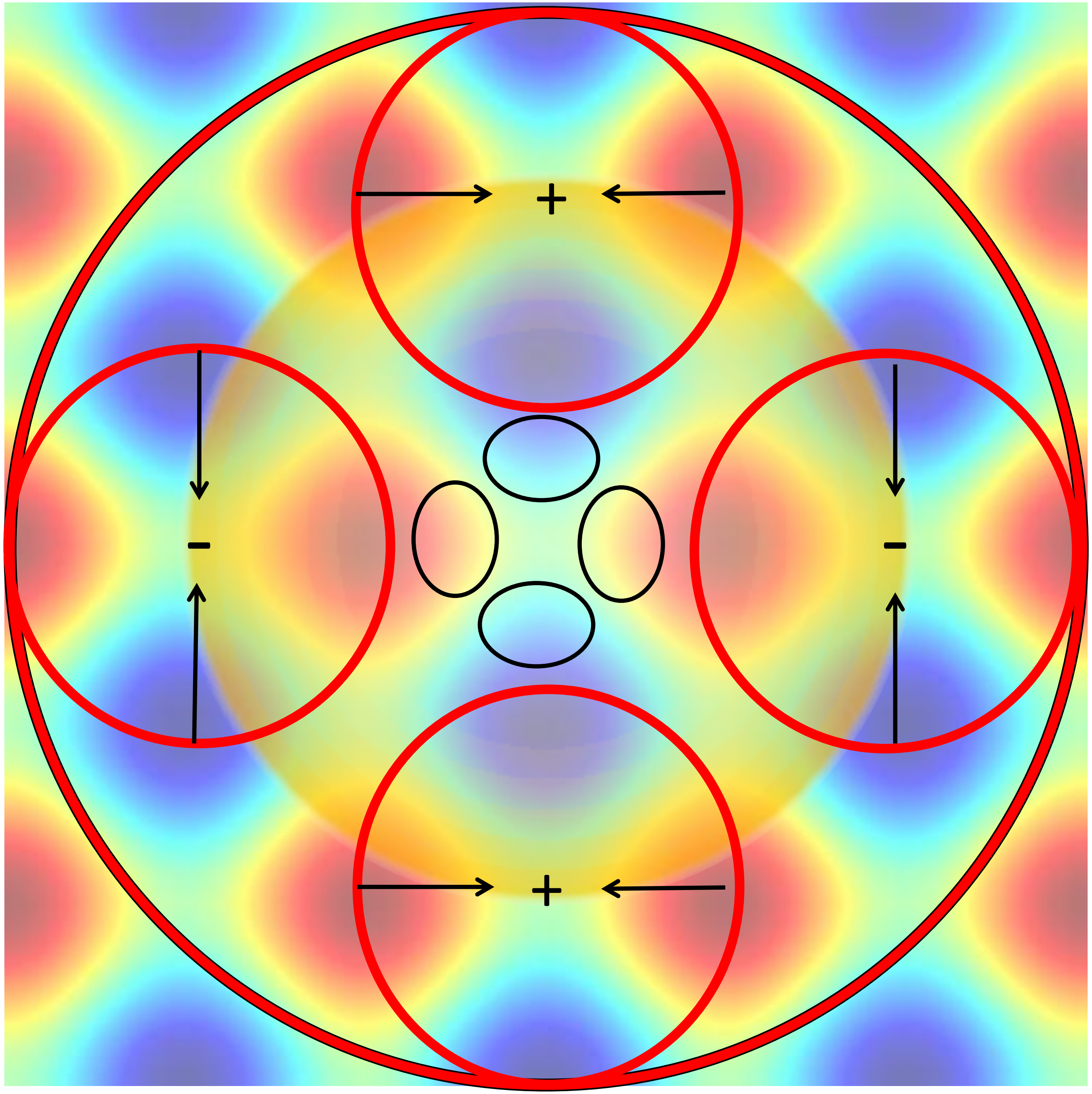
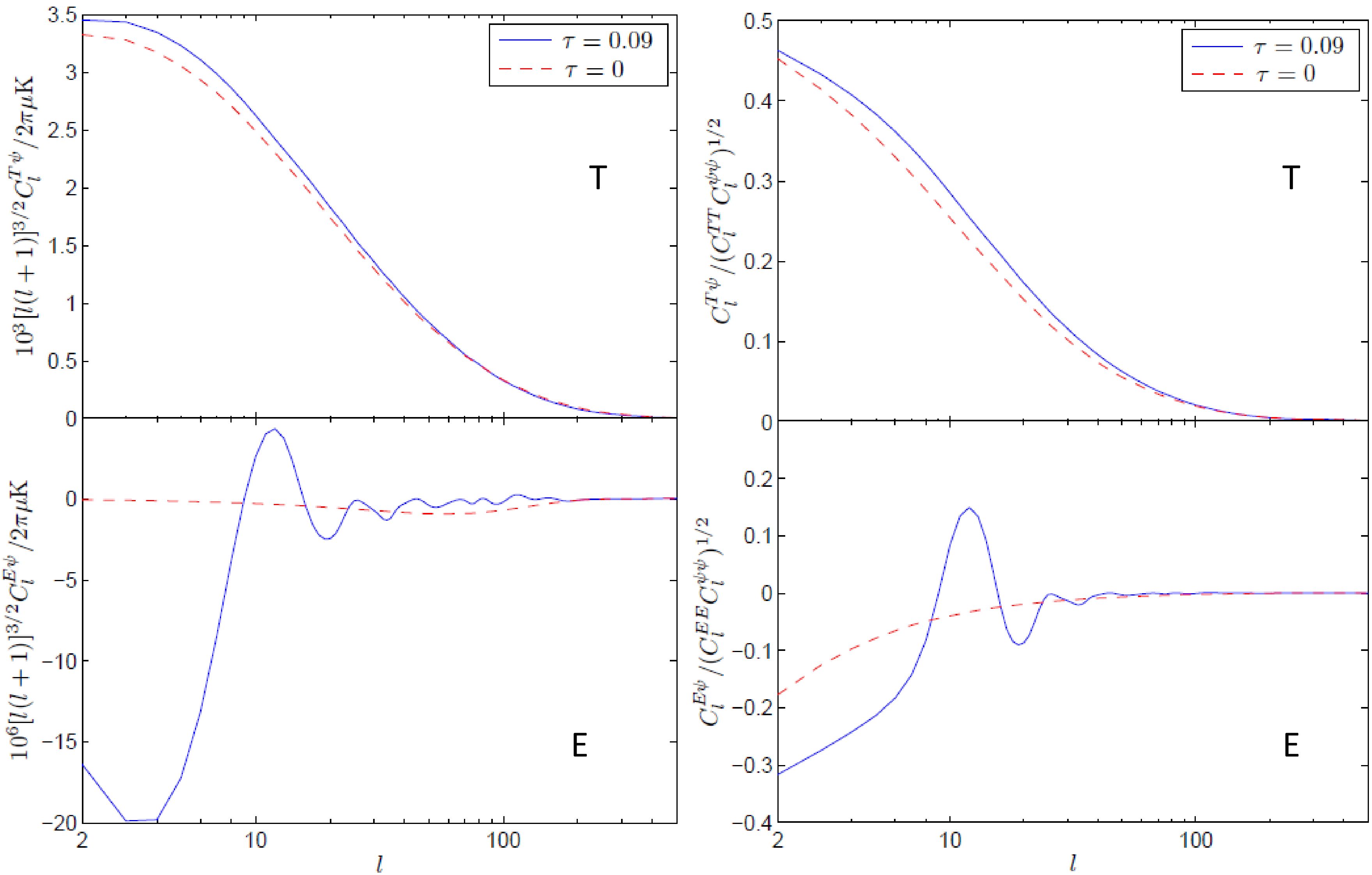
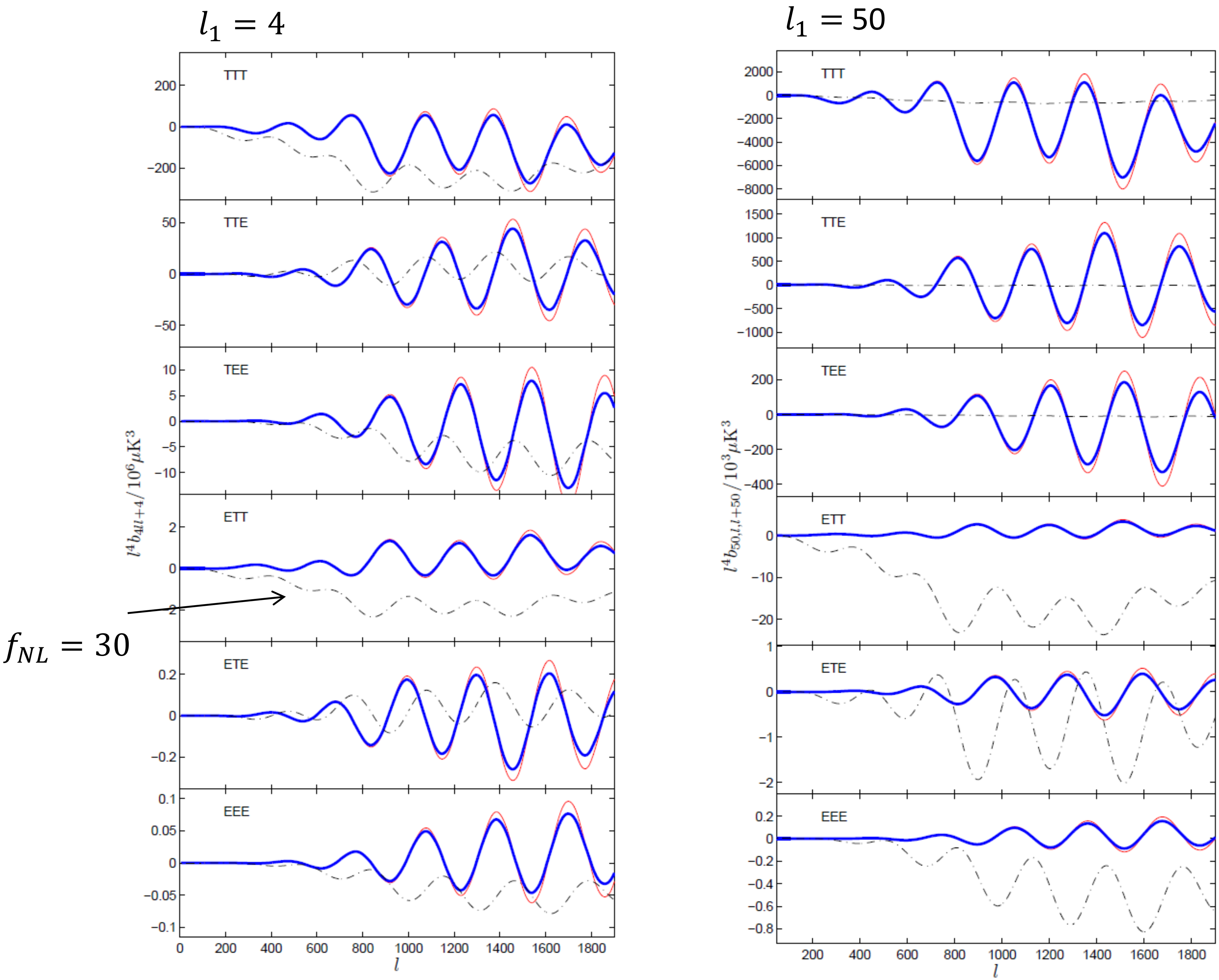


Lensing potential correlation power spectra

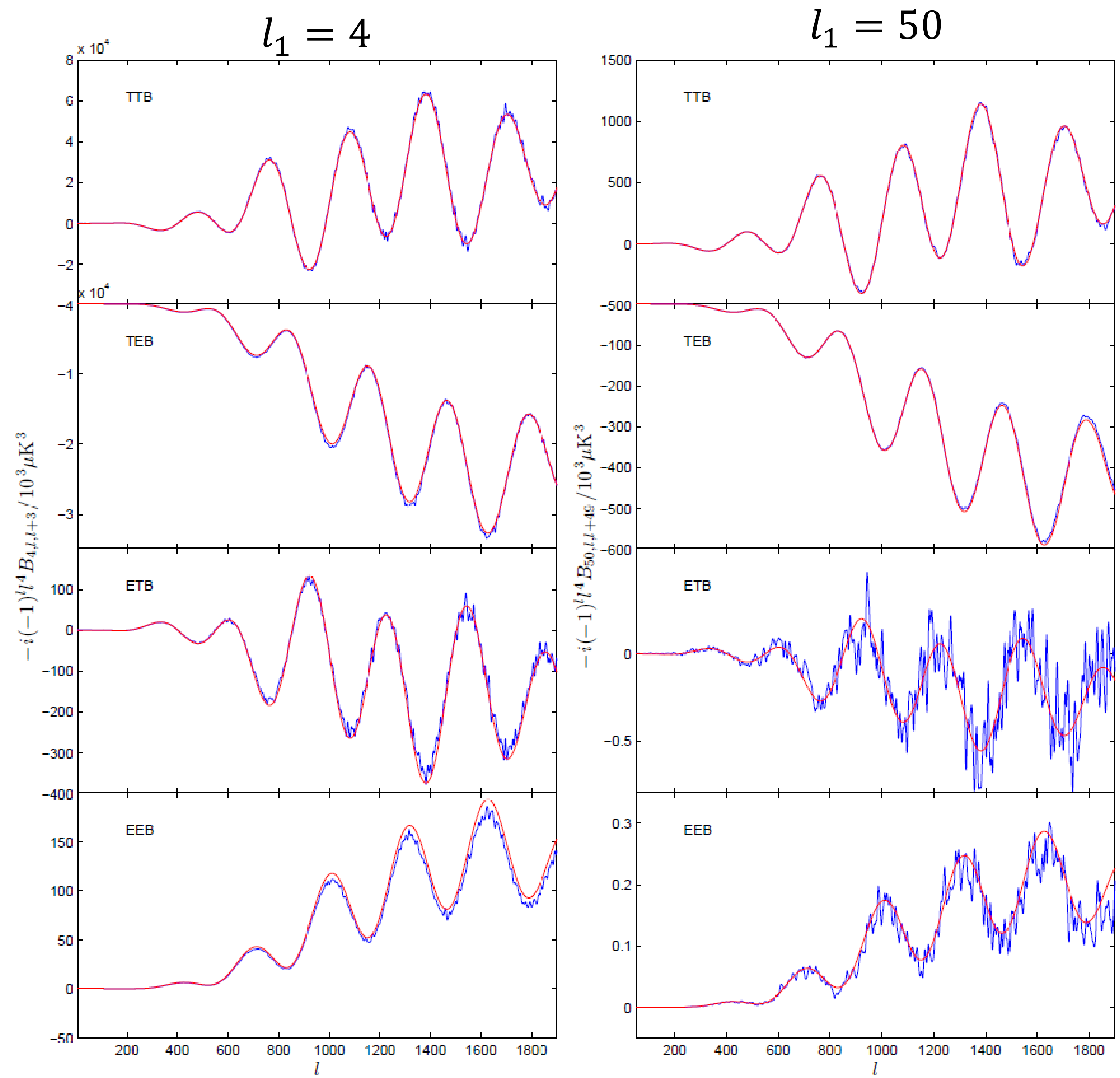


Cosmic variance: $C^{T\psi}$: $\sim 7\sigma$, $C^{E\psi}$: $\sim 2.5\sigma$



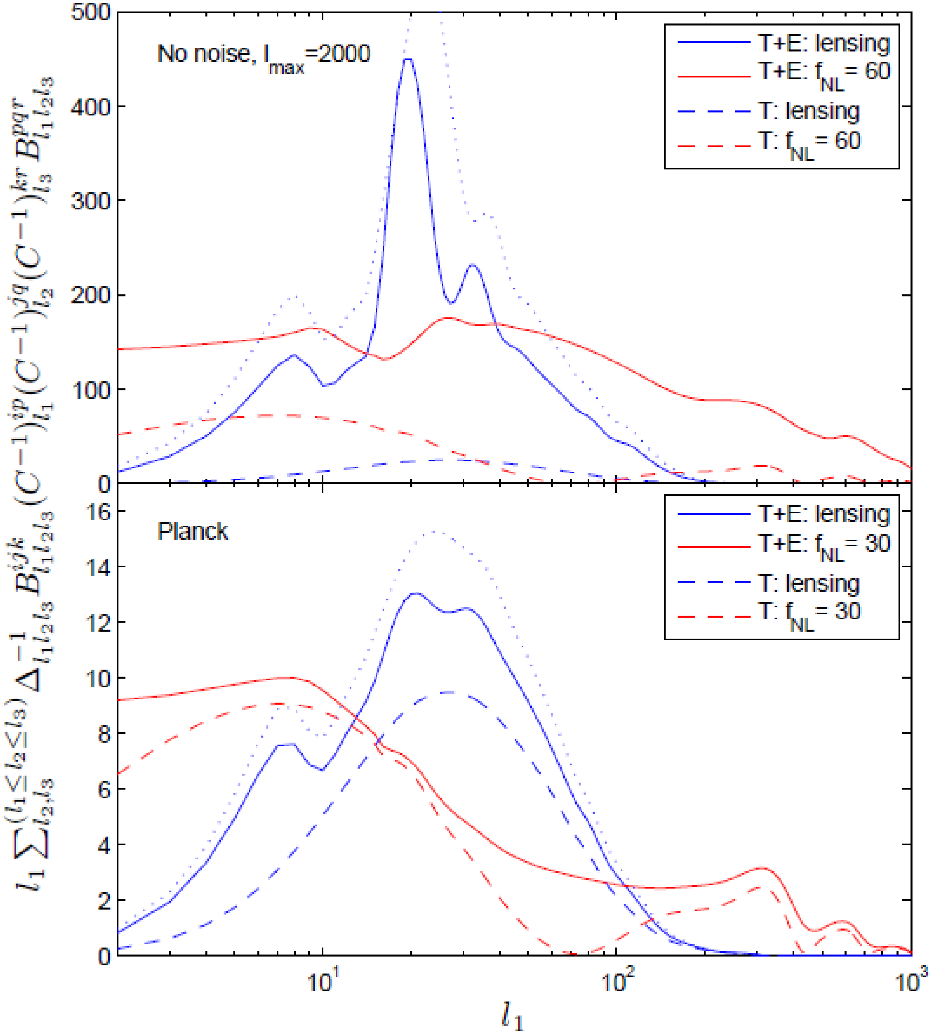
$l_1 = 4$
$l_1 = 50$
TTT
TTE
TEE
ETT
ETE
EEE
$l^4 b_{4ll+4}/10^6 \mu K^3$
$l^4 b_{50,l,l+50}/10^3 \mu K^3$
$f_{NL} = 30$
l

Also parity odd bispectra, TEB etc.



Signal to noise

Contributions to Fisher inverse variance for $b_{l_1 l_2 l_3} = 0$



Lensing signal peaks around $l_1 \sim 30$
- trade-off between size of signal and number of modes

For low noise Fisher error not correct
- signal saturates when large-scale lensing potential is reconstructed perfectly ($b_{l_1 l_2 l_3} \neq 0$).

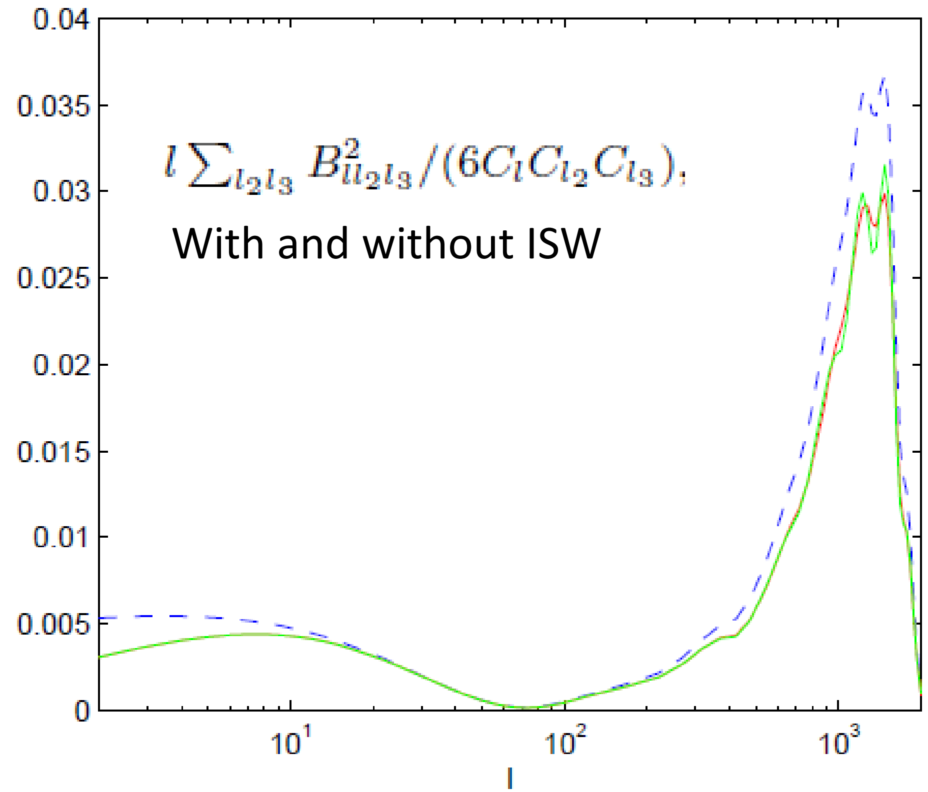
- Cosmic variance limits simply determined by cosmic variance detection limits on $C_l^{T\psi}$ and $C_l^{E\psi}$

Planck $\sim 5\sigma$; Cosmic Variance $\sim 9\sigma$

ISW–cleaning using CMB lensing ψ or other tracer?

- zero the temperature lensing bispectrum

- reduce cosmic variance on f_{NL} by ~ 10%
(Mead, Lewis & King, in prep).



c.f. Francis & Peacock 0909.2495

Conclusions

- CMB lensing bispectrum is significant

  - Temperature bispectrum from ISW-ψ correlation

  - Also E- ψ correlation ($\sim 2.5\sigma$ cosmic variance limit)

  - Distinctive phase and scale-dependence

  - Also parity-odd bispectra (TEB)

  - Equivalent to correlating a quadratic lensing reconstruction with the large-scale temperature and polarization

  - As with the power spectrum non-perturbative methods useful for accurate results

- Should be detected by Planck

  - Potential confusion with local f_{NL} but contribution easily distinguished/subtracted

  - Also SZ correlation on smaller scales, but frequency dependent; other terms includes Rees Sciama ($f_{NL} \sim 1$)

- Public codes available:

  $C_l^{E\psi}$, $C_l^{T\psi}$, $C_l^{\psi\psi}$, Local f_{NL} and lensing bispectrum in CAMB update: http://camb.info

  Lensed CMB simulation: LensPix http://cosmologist.info/lenspix