Francis Bernardeau Benasque, Monday 16th August IPhT Saclay

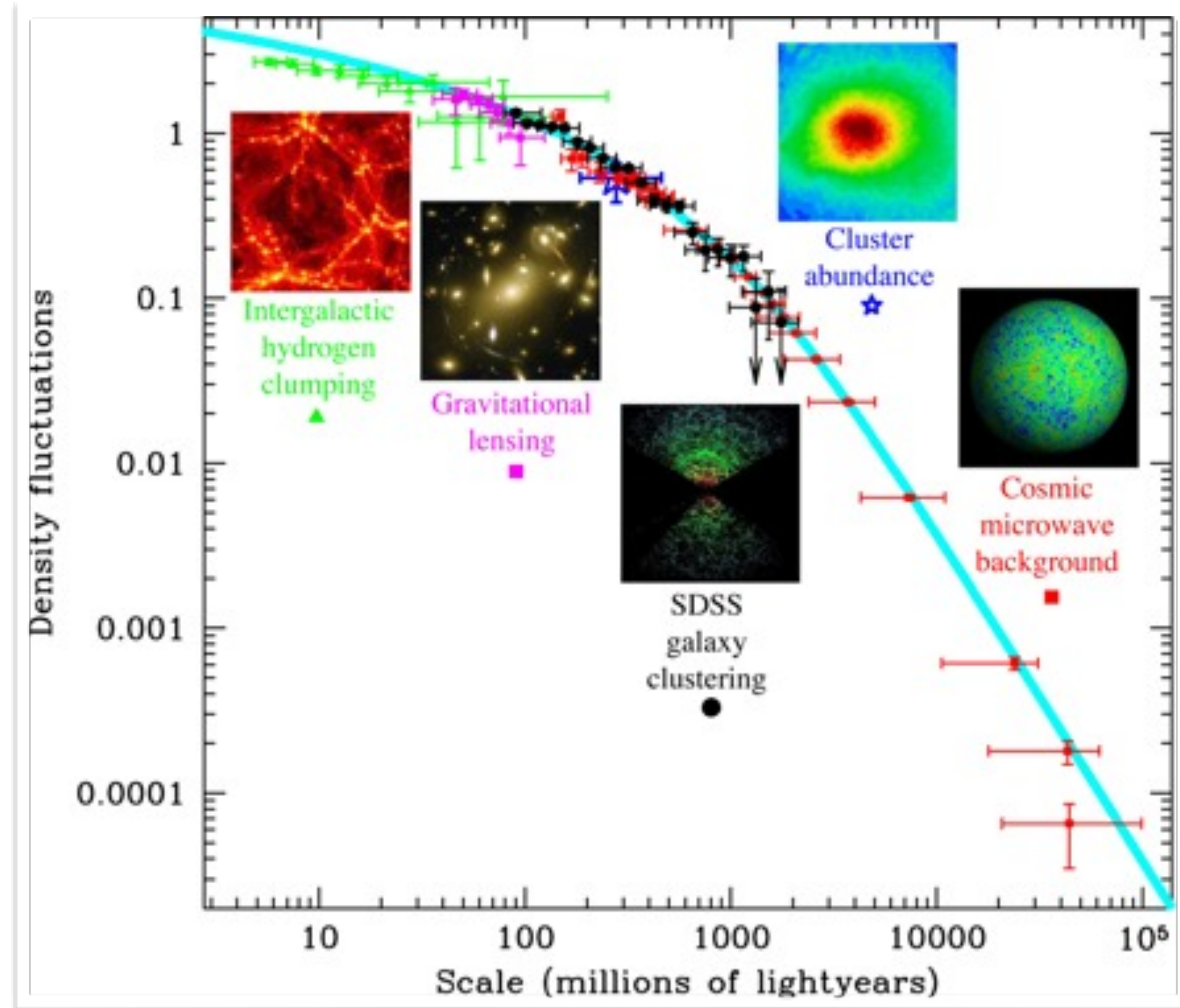
Testing gravity with large-scale structure formation

FB, Ph. Brax, in prep.

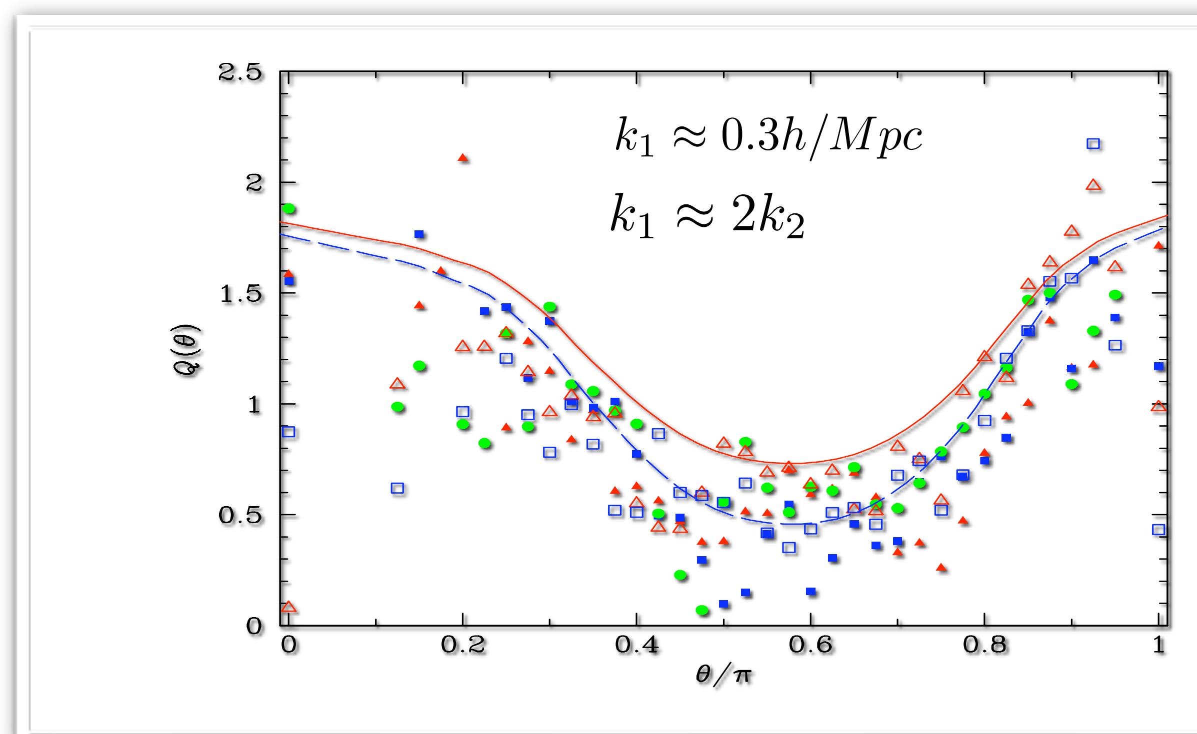
Which observables ?

The power spectrum is the obvious choice

- Dark energy equation of state
- Neutrino mass
- f_{NL} parameter
  - Theoretical uncertainties in the nonlinear evolution
  - *Not to mention biasing*



Bispectra offer richer information...



Introduction : a self-gravitating expanding dust fluid

A self-gravitating expanding dust fluid

The Vlasov equation (collisionless Boltzmann equation) - f(x,p) is the phase space density distribution - are fully nonlinear.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial}{\partial t} f(\mathbf{x},\mathbf{p},t) + \frac{\mathbf{p}}{ma^2}\frac{\partial}{\partial \mathbf{x}} f(\mathbf{x},\mathbf{p},t) - m\frac{\partial}{\partial \mathbf{x}}\Phi(\mathbf{x})\frac{\partial}{\partial \mathbf{p}} f(\mathbf{x},\mathbf{p},t) = 0$$

$$\Delta\Phi(\mathbf{x}) = \frac{4\pi G m}{a}\left(\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p} - \bar{n}\right)$$

This is what N-body codes aim at simulating...

The rules of the game: single flow equations

Peebles '80 ;Fry '84
FB, Colombi, Gaztañaga, Scoccimarro, '02

$$\begin{aligned}
\frac{\partial}{\partial t}\delta(\mathbf{x},t) + \frac{1}{a}\nabla_i.\left[(1+\delta(\mathbf{x},t))\mathbf{u}_i(\mathbf{x},t)\right] &= 0 \\
\frac{\partial}{\partial t}\mathbf{u}_i(\mathbf{x},t) + \frac{\dot{a}}{a}\mathbf{u}_i(\mathbf{x},t) + \frac{1}{a}\mathbf{u}_j(\mathbf{x},t)\,\mathbf{u}_{i,j}(\mathbf{x},t) &= -\frac{1}{a}\nabla_i\Phi(\mathbf{x},t) \\
\nabla^2\Phi(\mathbf{x},t) - 4\pi G\overline{\rho}(t)a^2\,\delta(\mathbf{x},t) &= 0.
\end{aligned}$$

+ expansion with respect to initial density fields

$$\delta(\mathbf{x},t) = \delta^{(1)}(\mathbf{x},t) + \delta^{(2)}(\mathbf{x},t) + \ldots$$

GR corrections effects:
Yoo et al. '09 PRD
B, Bonvin, Vernizzi, '10 PRD

▸ Motion equations in Fourier space in the single flow approximation

$$\begin{aligned}\frac{1}{H}\dot{\delta}(k,t)+\theta(k,t) &= -\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\\ &\times\ \alpha(\mathbf{k}_1,\mathbf{k}_2)\delta(k_1,t)\ \theta(k_2,t)\\ \frac{1}{H}\dot{\theta}+(2+\frac{\dot{H}}{H^2})\theta+\frac{3}{2}\Omega_m\delta_{\mathrm{m}} &= -\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\\ &\times\ \beta(\mathbf{k}_1,\mathbf{k}_2)\theta(\mathbf{k}_1)\theta(\mathbf{k}_2)\end{aligned}$$

$$\boxed{\alpha(\mathbf{k}_1,\mathbf{k}_2)=\frac{\mathbf{k}_{12}.\mathbf{k}_1}{k_1^2}=1+\frac{\mathbf{k}_1.\mathbf{k}_2}{k_1^2}}$$

$$\boxed{\beta(\mathbf{k}_1,\mathbf{k}_2)=\frac{\mathbf{k}_{12}^2(\mathbf{k}_1.\mathbf{k}_2)}{2k_1^2k_2^2}=\frac{(\mathbf{k}_1.\mathbf{k}_2)^2}{k_1^2k_2^2}+\frac{\mathbf{k}_1.\mathbf{k}_2}{2k_1^2}+\frac{\mathbf{k}_1.\mathbf{k}_2}{2k_2^2}}$$

- linear order = growth rate of structure
- higher order terms = mode couplings
- equations can be solved to any arbitrary order

$$\begin{aligned}\delta^{(n)}(\mathbf{k}) &= \int \mathrm{d}^3\mathbf{k}_1\ldots\mathrm{d}^3\mathbf{k}_n\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{1\ldots n})\,\delta^{(1)}(\mathbf{k}_1)\ldots\delta^{(1)}(\mathbf{k}_n)\,F_n^{(s)}(\mathbf{k}_1,\ldots,\mathbf{k}_n)\\ \frac{\theta^{(n)}(\mathbf{k})}{f} &= \int \mathrm{d}^3\mathbf{k}_1\ldots\mathrm{d}^3\mathbf{k}_n\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{1\ldots n})\,\delta^{(1)}(\mathbf{k}_1)\ldots\delta^{(1)}(\mathbf{k}_n)\,G_n^{(s)}(\mathbf{k}_1,\ldots,\mathbf{k}_n)\end{aligned}$$

$f\equiv\frac{\mathrm{d}\log D_+}{\mathrm{d}\log a}$

... this is the reduced velocity divergence

Gravity induced mode couplings have been computed and observed!

$$F_2^{(s)} = \left(\frac{3\nu_2}{4} - \frac{1}{2}\right) + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_2^2} + \left(\frac{3}{2} - \frac{3\nu_2}{4}\right)\frac{(\mathbf{k}_1.\mathbf{k}_2)^2}{k_1^2\,k_2^2}$$

This shape is expected (for CDM) irrespectively of background evolution, neutrino mass, etc...

$$\nu_2(\Omega_m, \Omega_\Lambda, \omega, \dots) = 34/21 + \dots$$

Einstein-de Sitter case

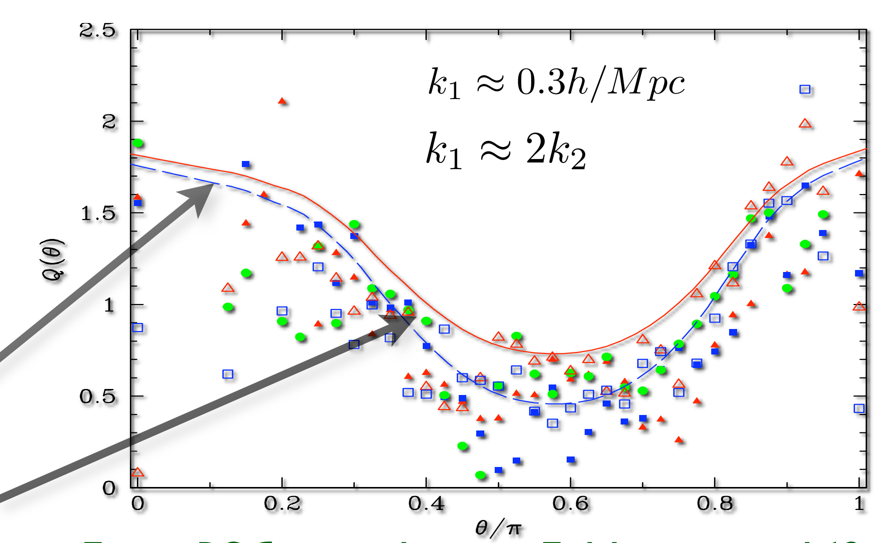
Related observables (cosmic shear, redshift galaxy gatalogues)

Observations are closely related (through projections, shape integration) to the density and the reduced velocity divergence power spectra

$$\begin{aligned} B_\delta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_2) &= F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2)\,P(k_1)\,P(k_2) + \mathrm{sym.} \\ B_{\tilde\theta}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_2) &= G_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2)\,P(k_1)\,P(k_2) + \mathrm{sym.} \end{aligned}$$

Flattened configurations

Equilateral configurations



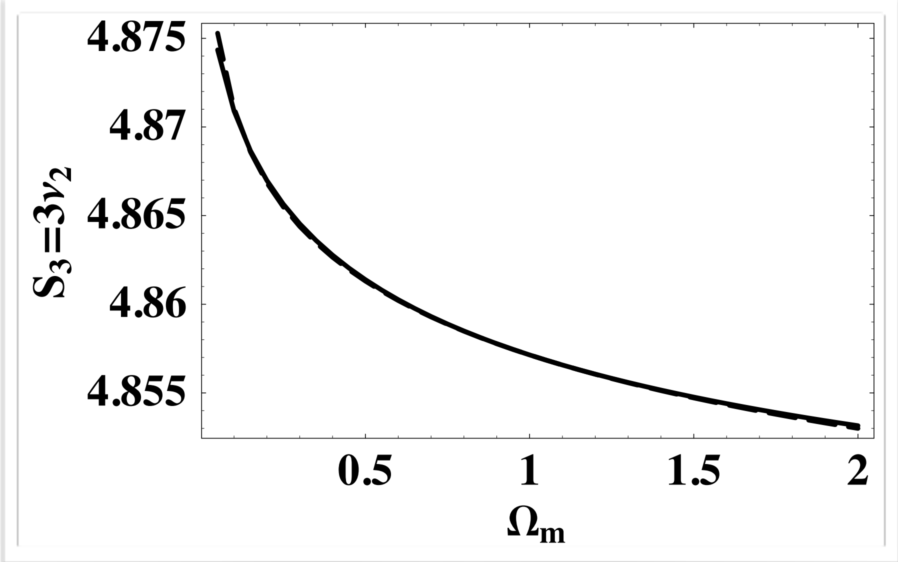
From PSCz catalogue, Feldman et al.'01

Message 1: the bispectrum amplitude (as measured by ν_2) is very weakly dependent (compared to the current precision level) on the energy content of the universe.

Flat universe

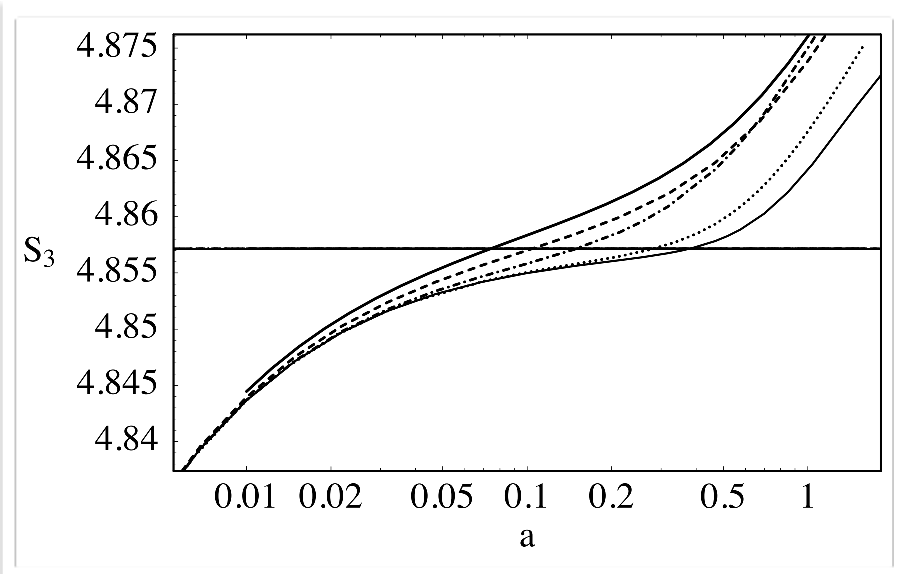
$$\nu_2 = \frac{4}{3} + \frac{2}{7}\Omega_m^{-1/143}$$

The PT review + refs in it



Different quintessence models (Ratra Peebles and extensions)

Benabed, FB, PRD, '01



What is the sensitivity of F_2 to the laws of gravity?

Can its measurement be used to test gravity ?

1. Changing gravity

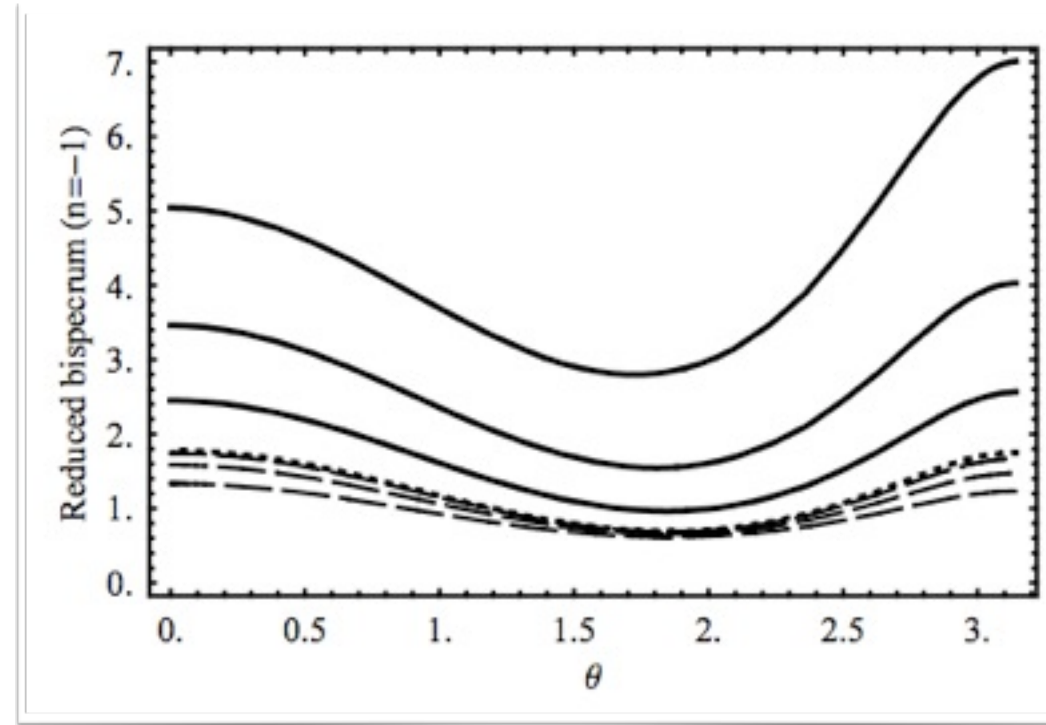
Jain, Zhang PRD '08

There are many ways of doing so...

work here based on Brax et al. astro-ph/1005.3735

$$\frac{1}{H}\dot{\theta}(k) + (2 + \frac{\dot{H}}{H^2})\theta(k) + \frac{3}{2}\Omega_m\,\xi(k,t)\,\delta_{\rm m}(k) = \ldots$$

If the change is $\xi(k,a) \approx \frac{ak/k_c}{1+ak/k_c}$ (large scale effective 5D gravity)



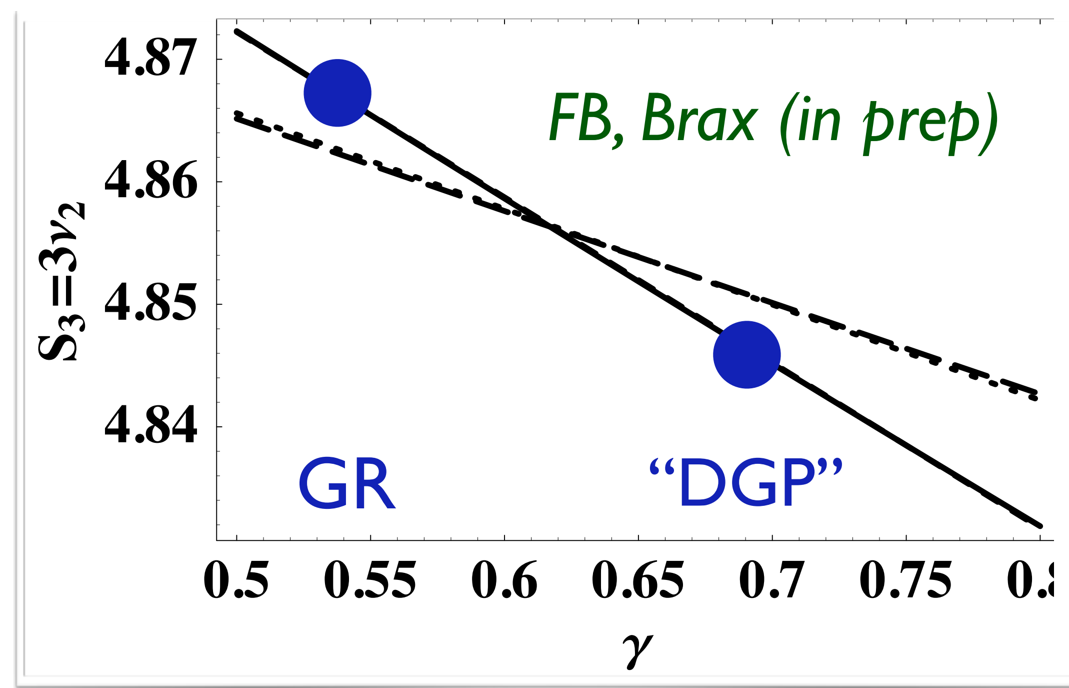
J-Ph Uzan, FB. PRD '01; FB, '04 (astro-ph)

If the change is such that

$$f_k \equiv \frac{\mathrm{d}\log D_+}{\mathrm{d}\log a} = \Omega_{\rm m}{}^{\gamma} \quad (\gamma^{\rm GR} \approx 0.55)$$

standard parameterization (Amendola & Quercellini, '04, Linder '05, Reyes et al. Nature, etc.),

$$\nu_2(\gamma) = \nu_2^{\rm GR} - .075\,(\gamma - \gamma^{\rm GR})\,(\Omega_m - 1)^{1.5}$$



2. In presence of a dilaton field

Brax et al. astro-ph/1005.3735

$$S=\int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\mathcal{R}-M_{\rm Pl}^2g^{\mu\nu}k^2(\phi)\partial_\mu\phi\partial_\nu\phi-V(\phi)\right\}+\int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m(\psi_m^{(i)},A^2(\phi)g_{\mu\nu})\,,$$

This extra field φ that is responsible of massive gravity effects. Its effect are suppressed in dense regions through the Chameleon mechanism.

$$V(\phi)=A^4(\phi)V_0\exp(-\phi)$$

$$A(\phi)=1+\frac{A_2}{2}(\phi-\phi_0)^2+\dots$$

$$k^2(\phi)=3\left(\frac{\mathrm{d}\log A}{\mathrm{d}\phi}\right)^2+\frac{1}{\lambda^2}$$

A new force term: $\mathrm{F}_i=-\frac{1}{a(t)}\left(\Phi(\mathbf{x},t)_{,i}+\frac{\mathrm{d}\log A}{\mathrm{d}\phi}(\bar{\phi}+\delta\phi)\phi(\mathbf{x},t)_{,i}\right)$

Newton potentials, $\Phi=\Psi$ with standard Poisson equation

An effective potential for the dilaton field

$$V_{\rm eff.}(\phi)=A^4(\phi)V_0\exp(-\phi)+A(\phi)\rho_m$$

$$\frac{m_\varphi^2}{H^2}\approx\frac{3A_2}{2}(\Omega_{\rm m}+4\Omega_\Lambda)\left[\lambda^{-2}+3\left(\frac{\Omega_{\rm m}}{\Omega_\Lambda}+4\right)^{-2}\right]^{-1}$$

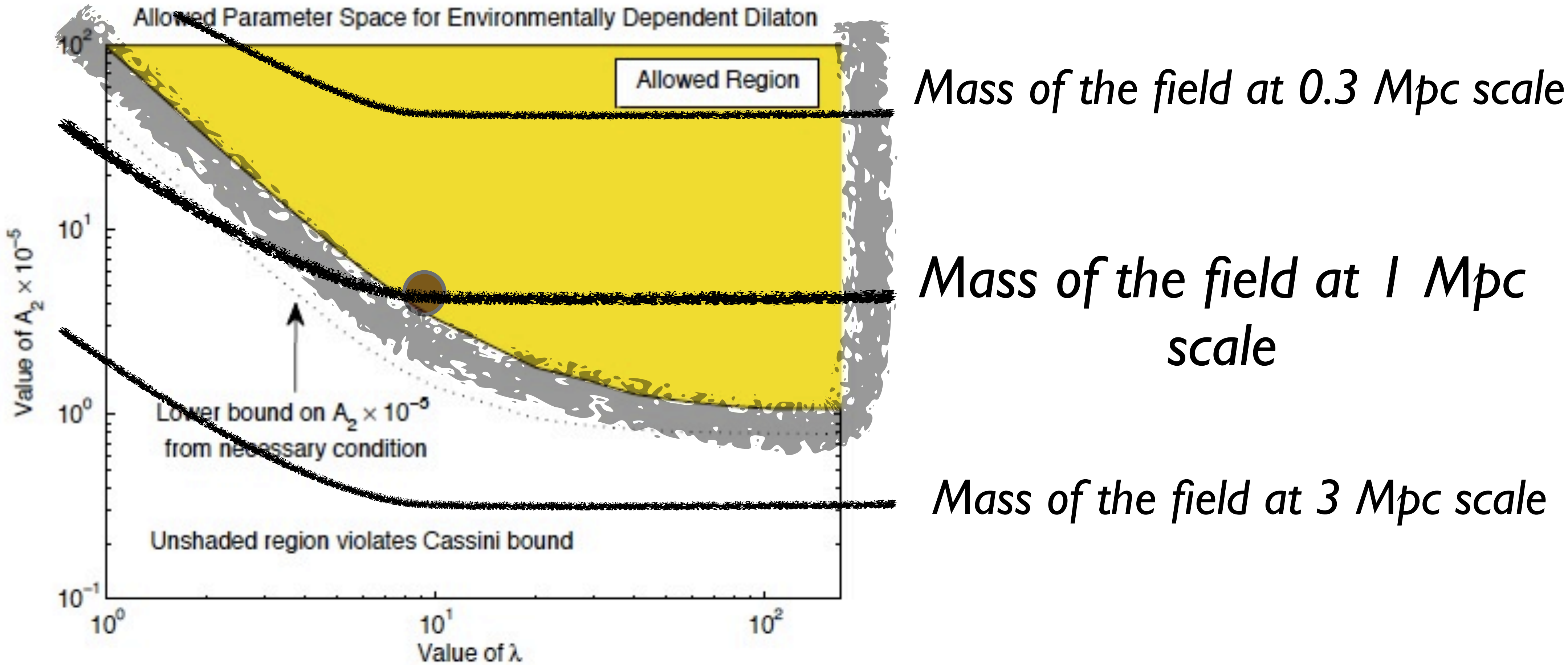


FIG. 1: Allowed parameter space for the environmentally dependent dilaton model. The shaded region is that where the presence of our galaxy is sufficient to ensure that the local value of the fifth force coupling, α, is smaller than the Cassini probe upperbound of 10^{-5}. We have modelled the galaxy as a spherical dark matter halo with NFW profile. We have taken typical values for the NFW model parameters for our galaxy: $r_{\rm vir} = 267\,{\rm kpc}$, $c = 12.0$, $M_{\rm v} = 0.91 \times 10^{12} M_\odot$. We take the galactocentric radius of the solar system, $r_\odot$ to be $r_\odot \approx 8.3\,{\rm kpc}$. These choices correspond to $\Phi(r_\odot) = 1.02\times10^{-6}$ and $\rho(r_\odot) = 0.22{\rm GeV\,cm}^{-3}$. This value for $\rho(r_\odot)$ limits $\lambda < 170$, and we have plotted the constraints on A_2 for $\lambda \in [1, 170]$. Very similar bounds on A_2 result for different realistic models of the galactic halo

Simplified case: dilaton mass and coupling parameters are determined by the background evolution

$$k^2\varphi(\mathbf{x},t) + m^2(\bar{\phi})\varphi = -4\pi G \ \frac{\mathrm{d}\log A}{k(\bar{\phi})\mathrm{d}\phi} \ \overline{\rho}(t) \ \delta_{\mathrm{m}}(\mathbf{x},t)$$

Eliminating φ leads to a new set of equation for the cosmic matter fluid,

$$\begin{aligned} \dot{\delta}(k,t) + H\theta(k,t) &= -H\,\alpha(\mathbf{k}_1,\mathbf{k}_2)\delta(k_1,t)\ \theta(k_2,t) \\ \dot{\theta}(k,t) + \left(2H + \frac{\dot{H}}{H}\right)\theta(k,t) + \frac{3}{2}H^2\Omega_m(1+\epsilon(k,t))\delta(k,t) &= -H\,\beta(\mathbf{k}_1,\mathbf{k}_2)\theta(k_1,t)\theta(k_2,t) \end{aligned}$$

$$\epsilon(k,t) = \frac{1}{1+m^2a^2/k^2}\left(\frac{\mathrm{d}\log A}{k(\bar{\phi})\ \mathrm{d}\phi}\right)^2$$: scale dependent amplification of gravity

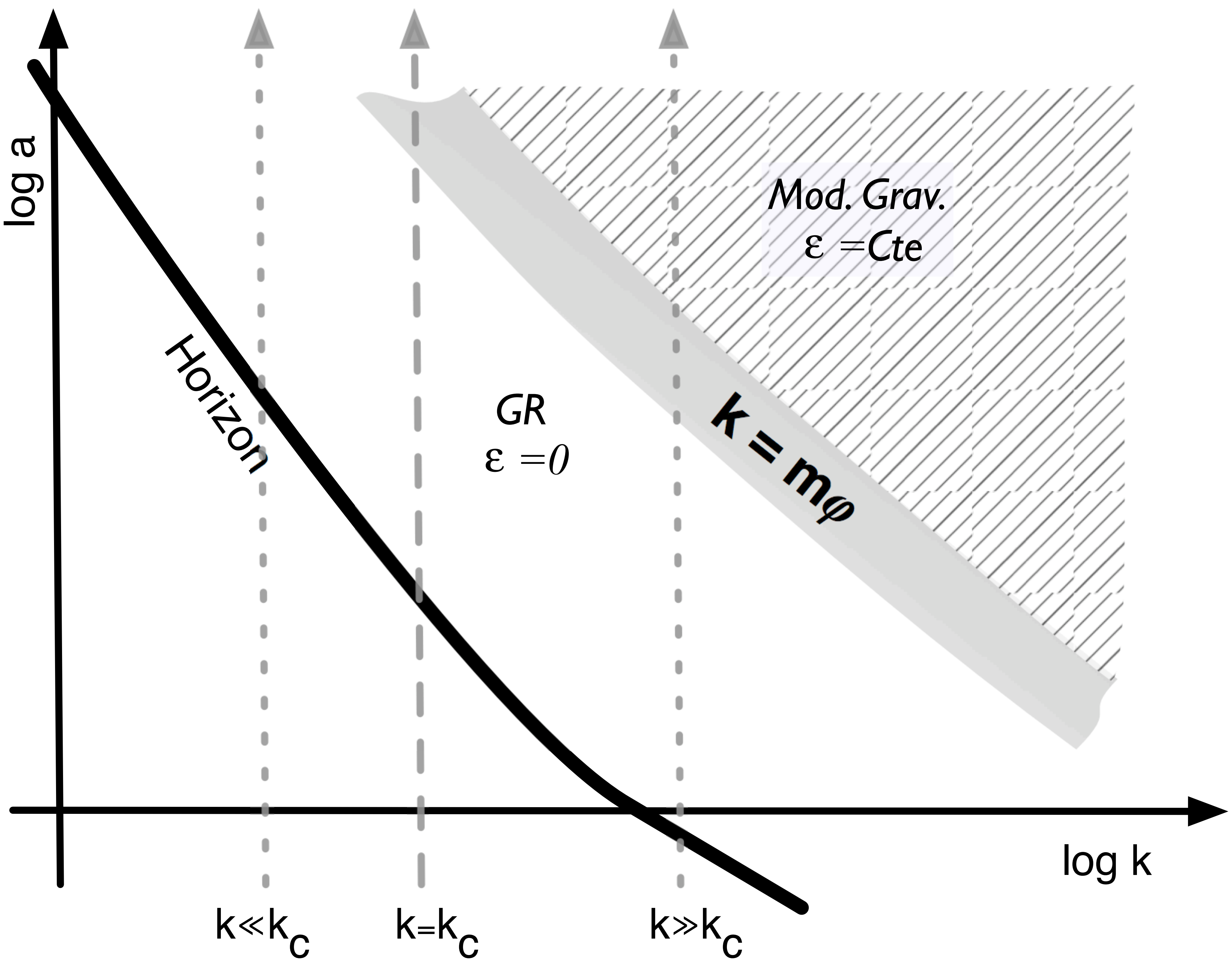
$$k \ll m_\varphi$$

non-modified gravity regime

$$k \gg m_\varphi$$

modified gravity regime (and ε is finite)

Evolution of structure: from GR to modified gravity dynamics



The linear growth rate

$$f_k \equiv \frac{\mathrm{d}\log D_+}{\mathrm{d}\log a}$$

$$\frac{\mathrm{d}f_k}{\mathrm{d}\Omega_m} = \frac{3/2\,\Omega_m(f_k+1+\epsilon(k,\Omega_m)-f_k(2+f_k)}{3\,\Omega_m(\Omega_m-1)}$$

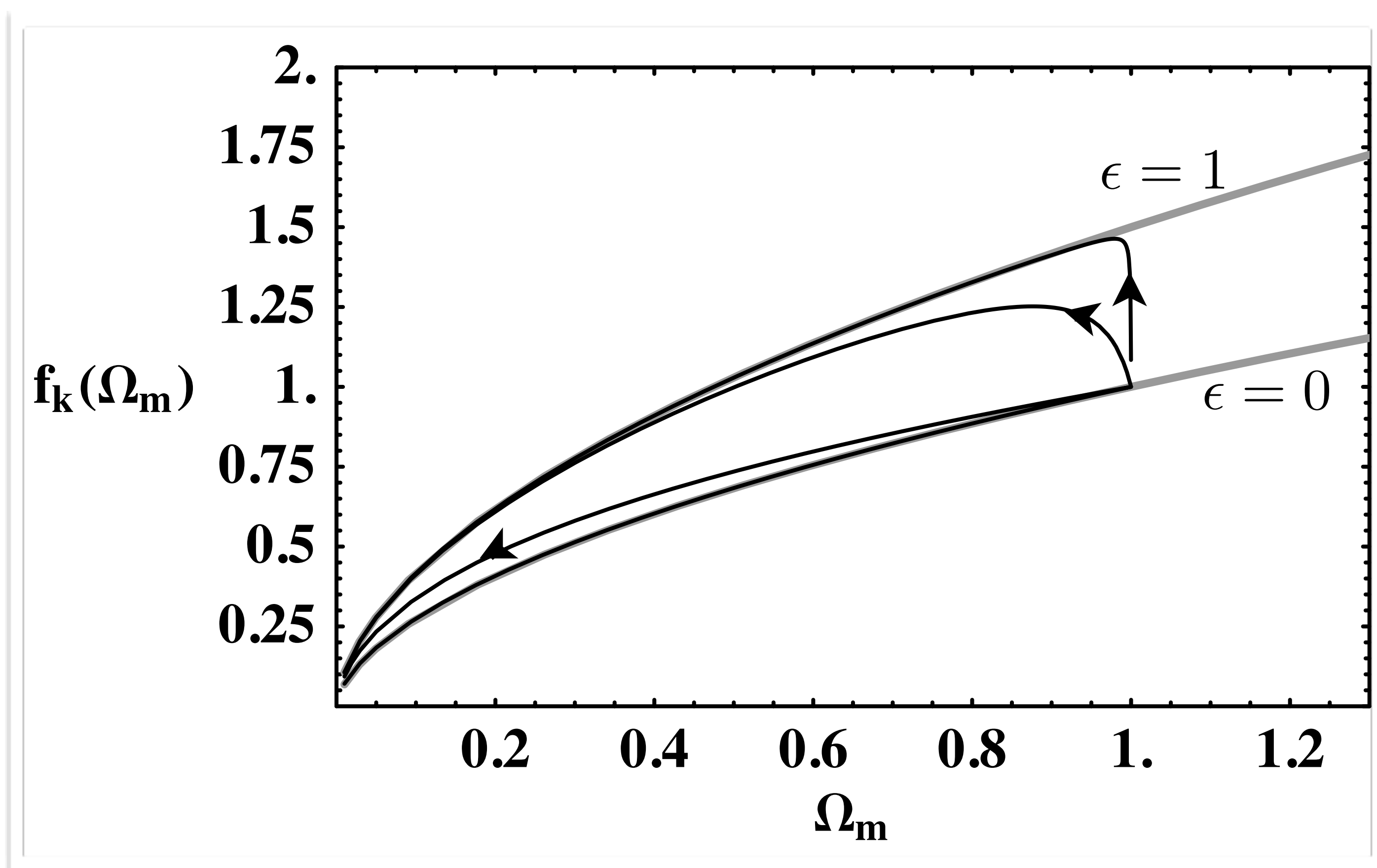
$$\begin{aligned} f_k(\Omega_m) \;=\; & \frac{1}{4\,{}_2F_1\left(\frac{f_+}{3},\frac{1}{3}(f_++2);\frac{1}{6}(4f_++7);1-\frac{1}{\Omega_m}\right)(4f_++7)\,\Omega_m} \\ & \times\left\{12\,{}_2F_1\left[\frac{1}{3}(f_++3),\frac{1}{3}(f_++5);\frac{1}{6}(4f_++13);1-\frac{1}{\Omega_m}\right](\epsilon+f_++1)(\Omega_m-1)\right. \\ & \left.+\,4\,{}_2F_1\left(\frac{f_+}{3},\frac{1}{3}(f_++2);\frac{1}{6}(4f_++7);1-\frac{1}{\Omega_m}\right)(6\epsilon+5f_++6)\,\Omega_m\right\} \end{aligned}$$

$$\epsilon = Cte$$

$$f_+ = \frac{(25+24\epsilon)^{1/2}-1}{4}$$

$$f(\Omega_m) \approx f_+\,\Omega_m^{\frac{2(2+f_+)}{4f_++7}}\,.$$

dependence at variance with other types of models, see also di Porto, Amendola '07.



The mode coupling evolution

$$\begin{aligned} \delta'(k) + \tilde{\theta}(k) &= -\alpha(\mathbf{k}_1, \mathbf{k}_2)\, \delta(k_1)\, \tilde{\theta}(k_2) \frac{f_{k_2}}{f_k} \\ \tilde{\theta}'(k) - \left(1 - \frac{3}{2}\frac{\Omega_m}{f_k^2}(1+\epsilon(k))\right)\tilde{\theta}(k) + \frac{3}{2}\frac{\Omega_m}{f_k^2}(1+\epsilon(k))\delta(k) &= -\beta(\mathbf{k}_1, \mathbf{k}_2)\tilde{\theta}(k_1)\tilde{\theta}(k_2)\frac{f_{k_1} f_{k_2}}{f_k^2} \end{aligned}$$

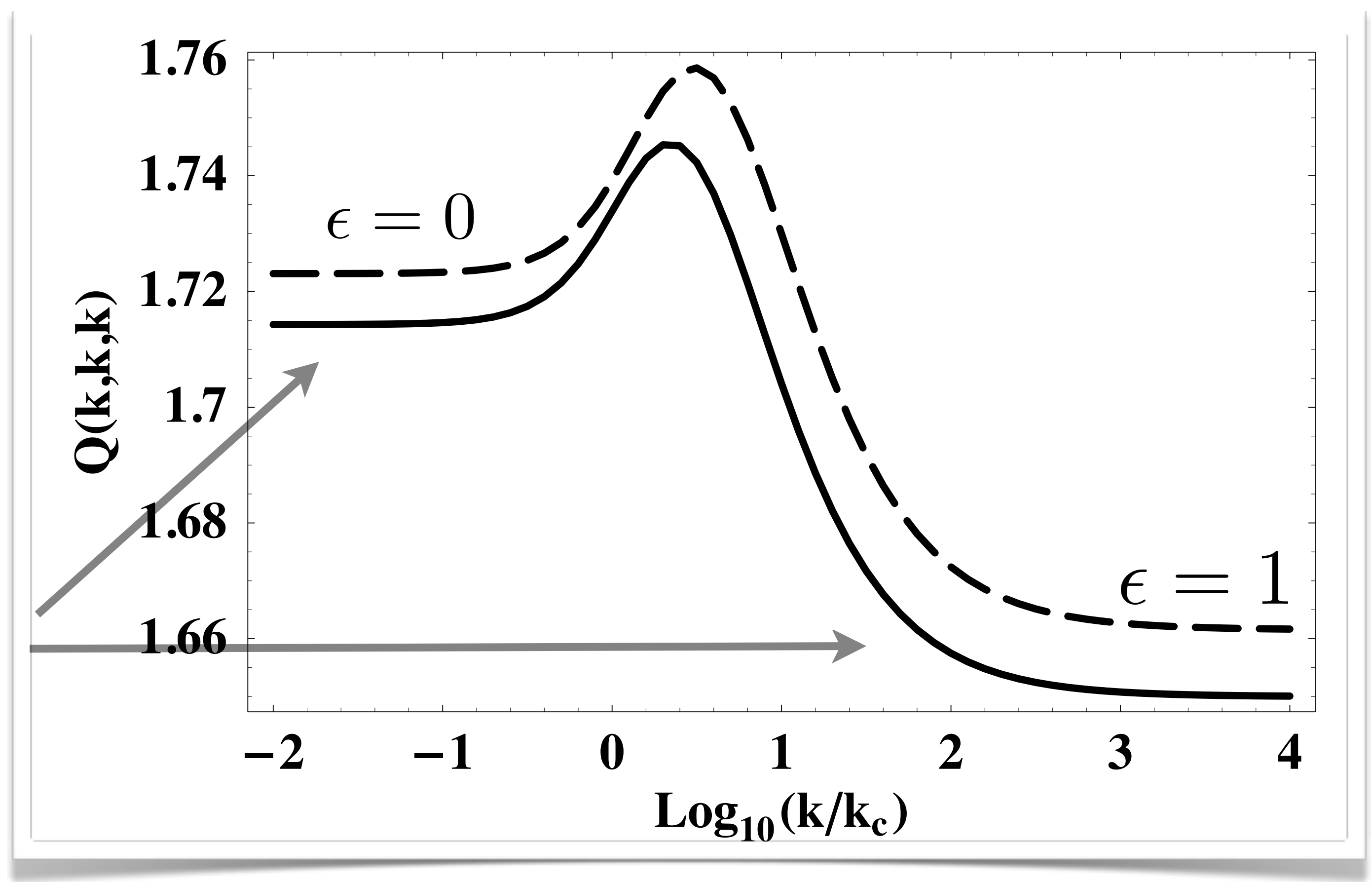
effectively : $\dfrac{\Omega_m}{f^2} \rightarrow \dfrac{\Omega_m}{f_k^2}(1+\epsilon(k))$

$$F_2^{(s)} = \left(\frac{3\nu_2}{4} - \frac{1}{2}\right) + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1.\mathbf{k}_2}{k_2^2} + \left(\frac{3}{2} - \frac{3\nu_2}{4}\right)\frac{(\mathbf{k}_1.\mathbf{k}_2)^2}{k_1^2\, k_2^2}$$

for fixed ξ

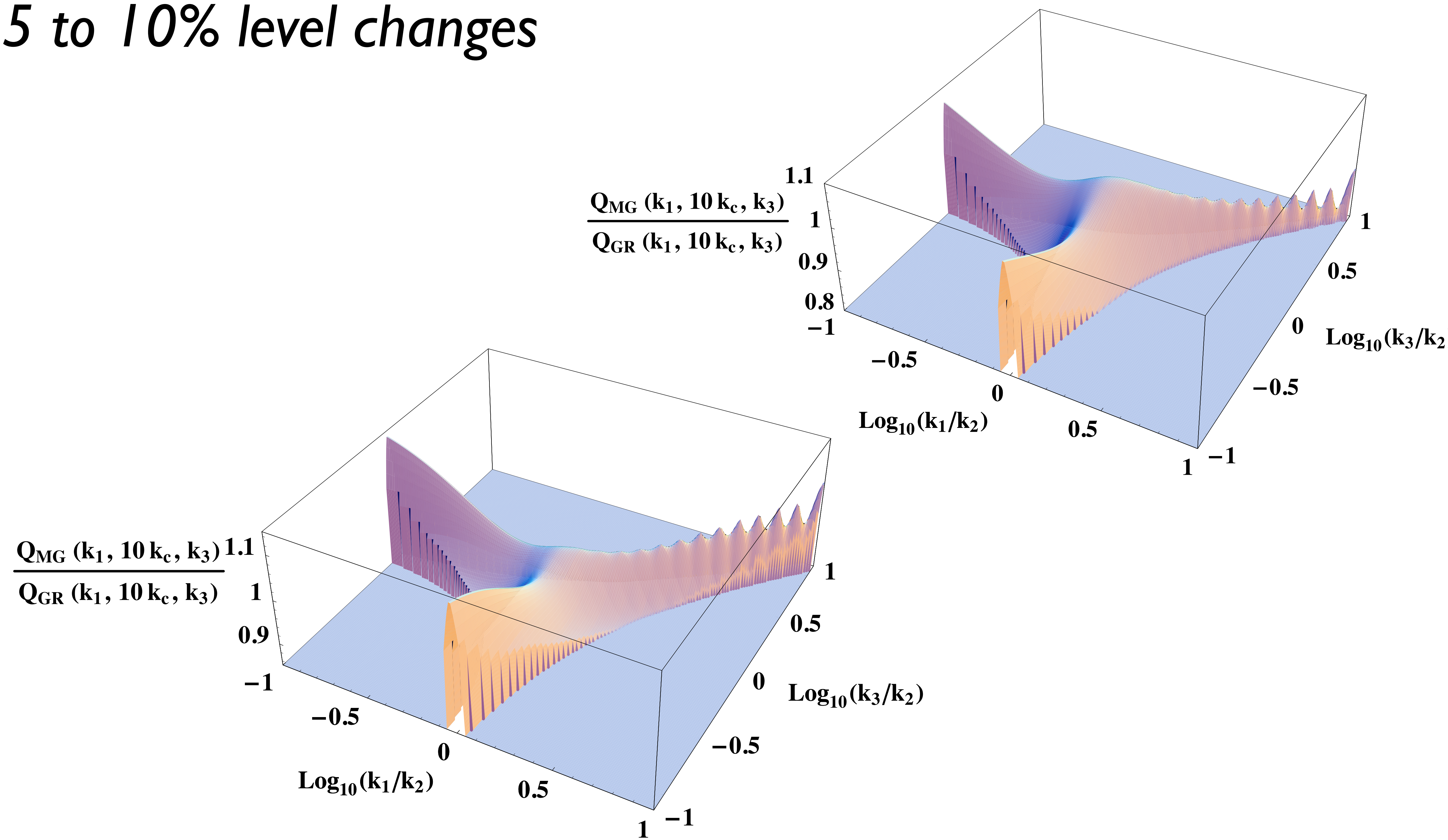
$$f_+ = \frac{(25+24\epsilon)^{1/2} - 1}{4}$$

$$\nu_2(\epsilon) = \frac{2(8+9\xi)}{3(4+3\xi)} \quad \text{with} \quad \xi = \frac{1+\epsilon}{f_+^2},$$



The resulting shape of the bispectrum (ns=-1, ns=-2)

5 to 10% level changes



The analysis of a fully working model

The mass is determined dynamically
The couplings are also determined dynamically

$$m_\varphi^2(\bar{\phi}) \;\to\; m_\varphi^2(\bar{\phi}+\delta\phi) \quad \text{which implies cubic couplings, etc.}$$
$$\beta_\varphi(\bar{\phi}) \;\to\; \beta_\varphi(\bar{\phi}+\delta\phi)$$

These functions can be computed explicitly (given values of A_2 and λ)

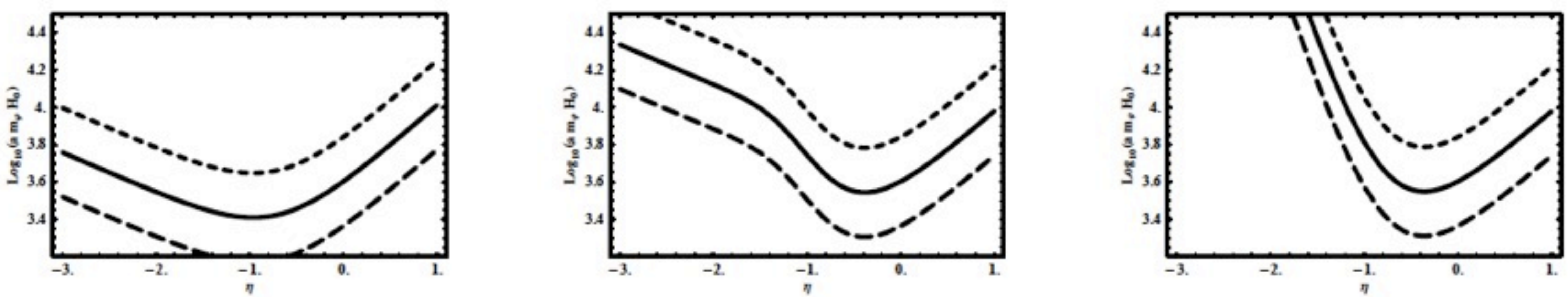


FIG. 4: The effective mass of the field φ as a function of time η in units of $1/H_0$. The numerical application corresponds to the model (53) with $\lambda = 1$ (left panel), $\lambda = 10$ (middle panel) and $\lambda = 1000$ (right panel),$A_2 = 5.6\,10^5$ (solid line) and the dashed lines to cases where A_2 is 1/3 or 3 times larger. In this case the value of k_c is $k_c = 4\,10^3 H_0/c \approx 1.3hMpc^{-1}$. Gravity is modified for modes that are above the solid line at time η.

A new Euler equation (up to second order)

$$\frac{1}{H}\dot{\theta}^{(2)}+(2+\frac{\dot{H}}{H^2})\theta^{(2)}+\frac{3}{2}\Omega_m(1+\epsilon(k))\delta_{\rm m}^{(2)} = -\beta(\mathbf{k}_1,\mathbf{k}_2)\theta^2-\left[\mathcal{S}_{\rm Eul.}(\mathbf{k}_1,\mathbf{k}_2)+\mathcal{S}_{\rm Intr.}(\mathbf{k}_1,\mathbf{k}_2)\right](\delta_{\rm m}^{(1)})^2$$

$$\mathcal{S}_{\rm Eul.}(\mathbf{k}_1,\mathbf{k}_2) = \frac{(\mathbf{k}_2.\mathbf{k})}{k_1^2}\frac{a^2m^2(\bar{\phi})}{k_2^2}S(k_1)\eta(k_2)$$

$$\mathcal{S}_{\rm Intr.}(\mathbf{k}_1,\mathbf{k}_2) = \frac{a^2m^2(\bar{\phi})}{k_2^2}S(k)\tilde{\eta}(k_2)+\frac{a^2m^2(\bar{\phi})}{k_1^2}\frac{a^2m^2(\bar{\phi})}{k_2^2}S(k_1)S(k_2)\mu(k)$$

$$\eta(k) = S(k)\frac{H^2}{m^2(\bar{\phi})}\frac{d(\beta_{\rm eff}(\phi))}{k(\bar{\phi})d\phi}, \qquad \tilde{\eta}(k) = S(k)\frac{H^2}{m^2(\bar{\phi})}\frac{d(A(\phi)\beta_{\rm eff}(\phi))}{k(\bar{\phi})d\phi}$$

(negligible in $\lambda \to \infty$ limit)

$$\mu(k) = \frac{S(k)}{3\Omega_m}\frac{H^2}{m^4(\bar{\phi})}\quad\frac{d^3V_{\rm eff}}{2M_{\rm Pl}^2d\varphi^3}$$

(negligible in $\lambda \to 0$ limit)

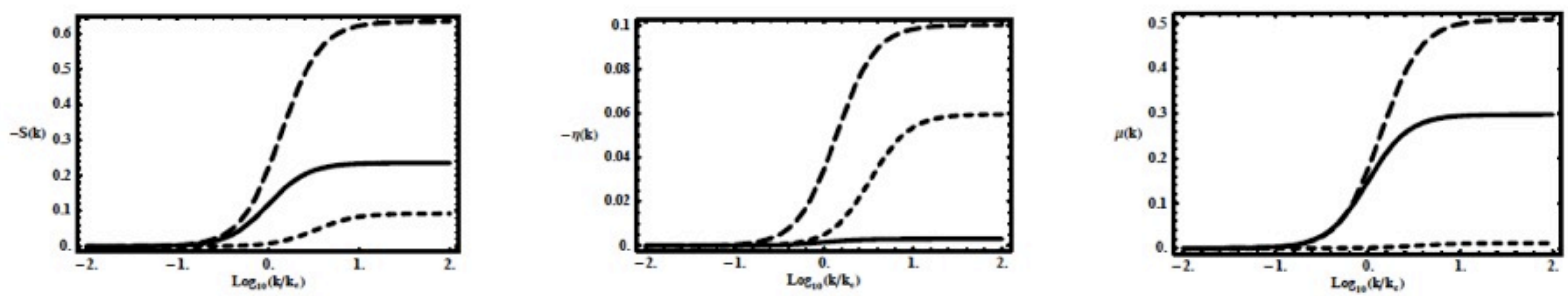


FIG. 5: Dependence on k of the parameters $S(k)$, $\eta(k)$ and $\mu(k)$ for $\eta = 0$ (solid lines), $\eta = -1$ (long dashed) and $\eta = -2$ (short dashed). Note that for the adopted parameters $\eta(k)$ and $\tilde{\eta}(k)$ are undistinguishable.

Equilateral configurations

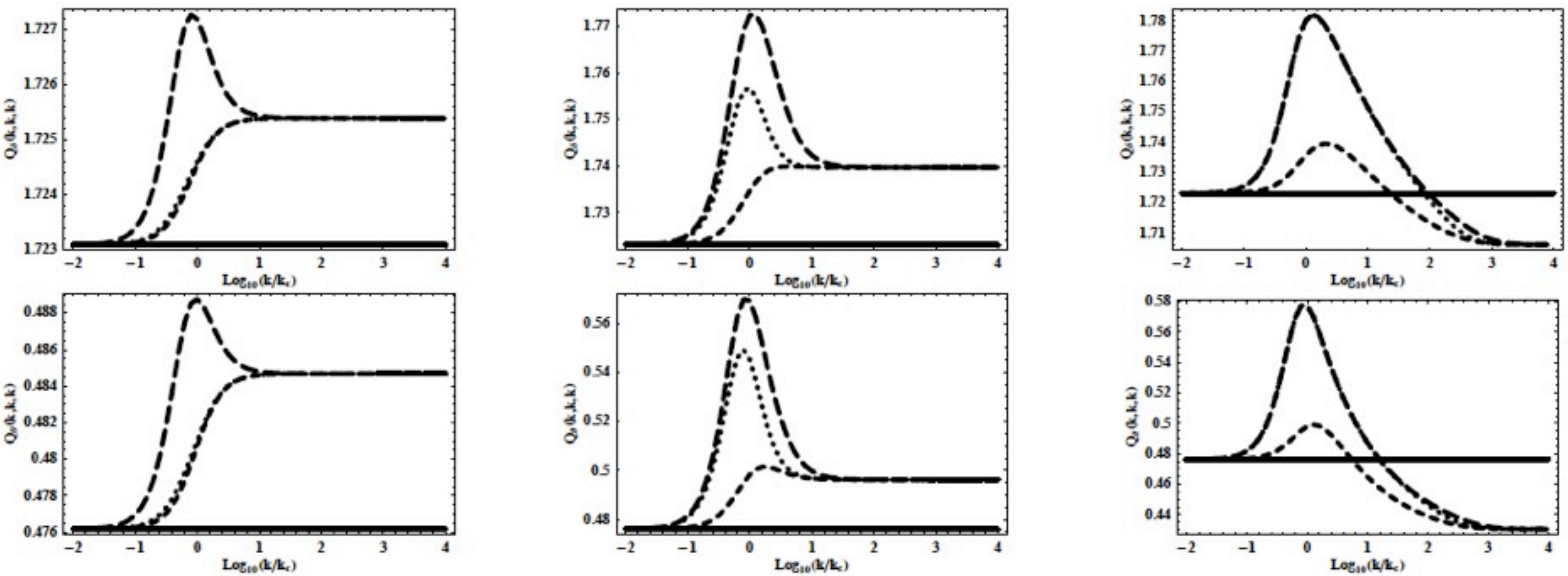
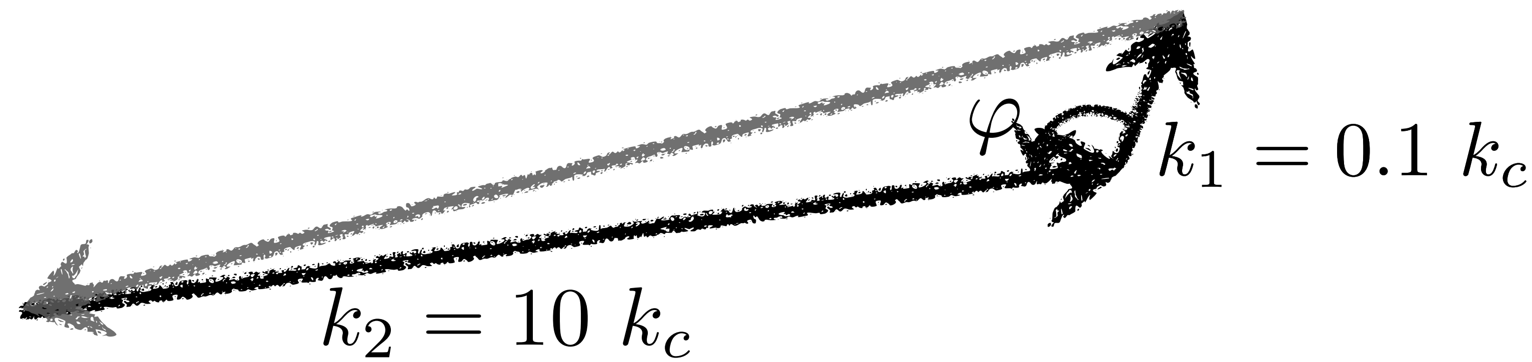


FIG. 6: The density (top row) and reduced velocity divergence (bottom row) bispectra for equilateral configurations as a function of scale for $\lambda = 1$ left panels, $\lambda = 10$, middle panels and $\lambda = 1000$, right panels. The solid line is the General Relativity prediction. For this configuration the results are independent of scale. The modified gravity model used here corresponds to the long dashed lines. The short dashed line is obtained when the extra couplings that appear in the Euler equation are dropped. They reproduce the large k behavior. The dotted line correspond to the case when only the intrinsic coupling of the ϕ field is preserved (second term of ?). It gives the dominant contribution when λ is large (right panels) but a negligible one when λ is small (left panels).

Squeezed configurations



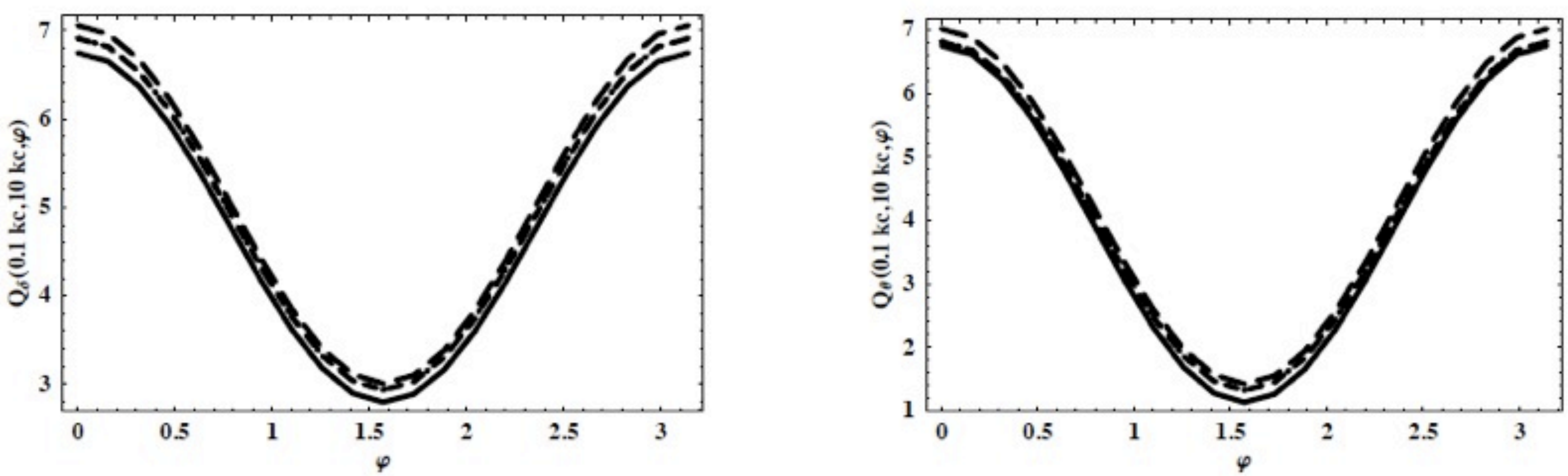
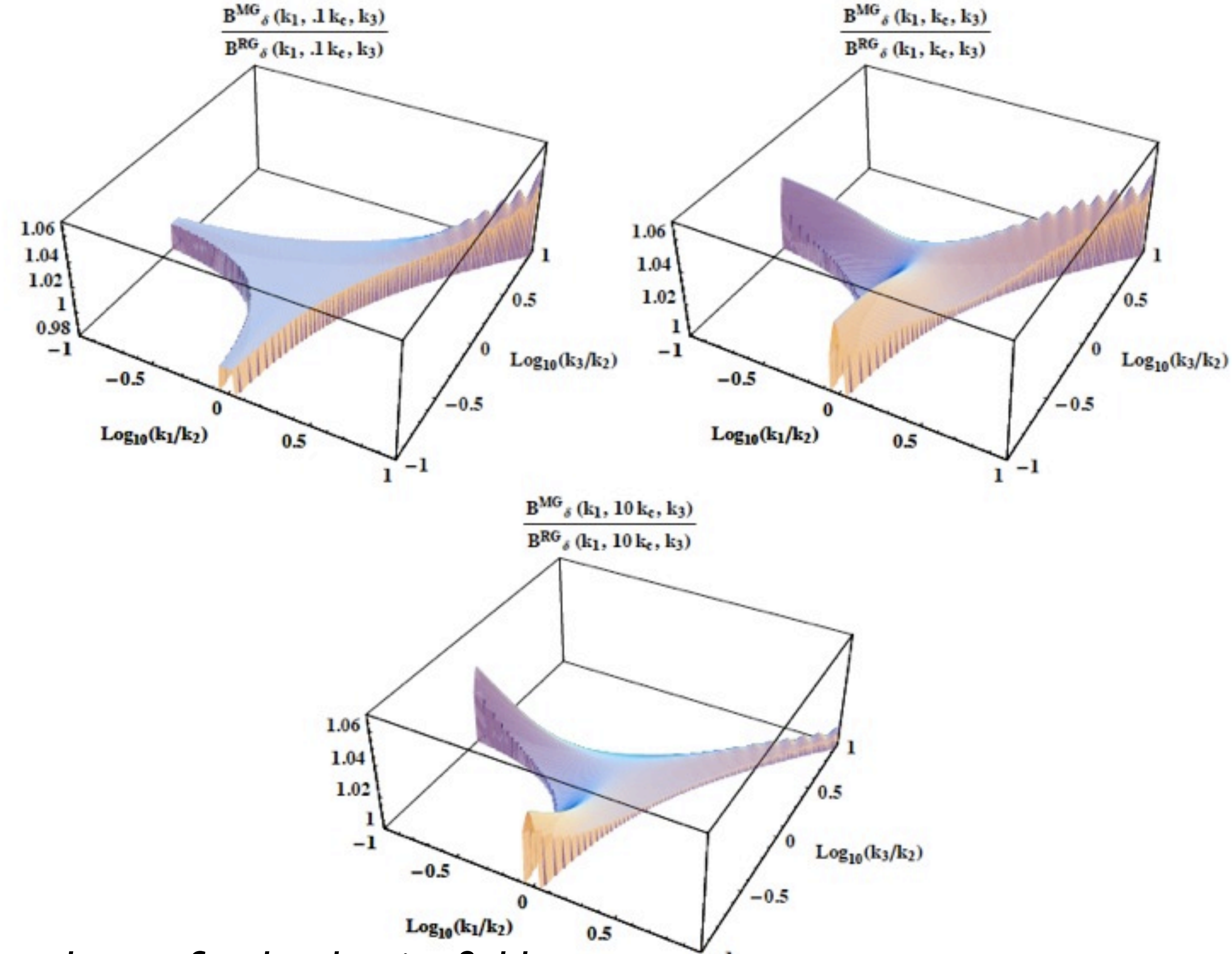
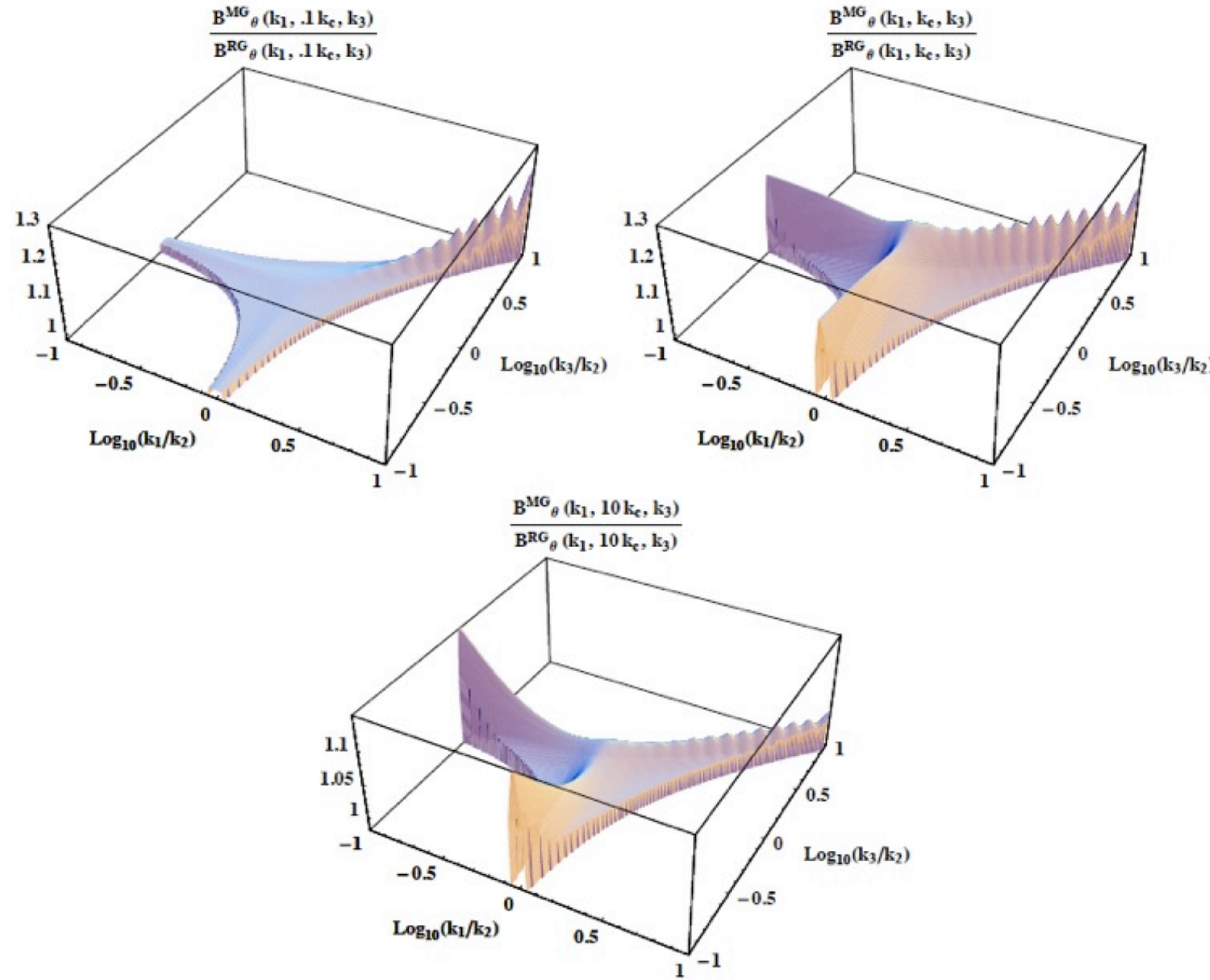


FIG. 7: The density and reduced velocity divergence bispectra in the squeezed limit for $\lambda = 10$. Conventions are the same as for Fig. 6.



5 % effect change for the density field

FIG. 8: Amplitude of the reduced bispectrum $Q_\delta(k_1, k_2, k_3)$ for the density field for the modified gravity model ($\lambda = 10$) divided by the expected result for standard gravity as a function of k_1/k_2 and k_3/k_2 for respectively $k_2 = .1k_c$, $k_2 = k_c$ and $k_2 = 10k_c$. We assume here that $P(k) \sim k^{-1.5}$.



10 % effect change for the reduced velocity divergence

Conclusions

- Details on astrophysical observations in paper by Brax et al.

- Small-scale non-linear evolution (with RPT ?) has to be taken into account for accurate prediction on the spectra/ bispectra.

- Effects here are more important than for DGP type models *(R. Scoccimarro PRD '09)*

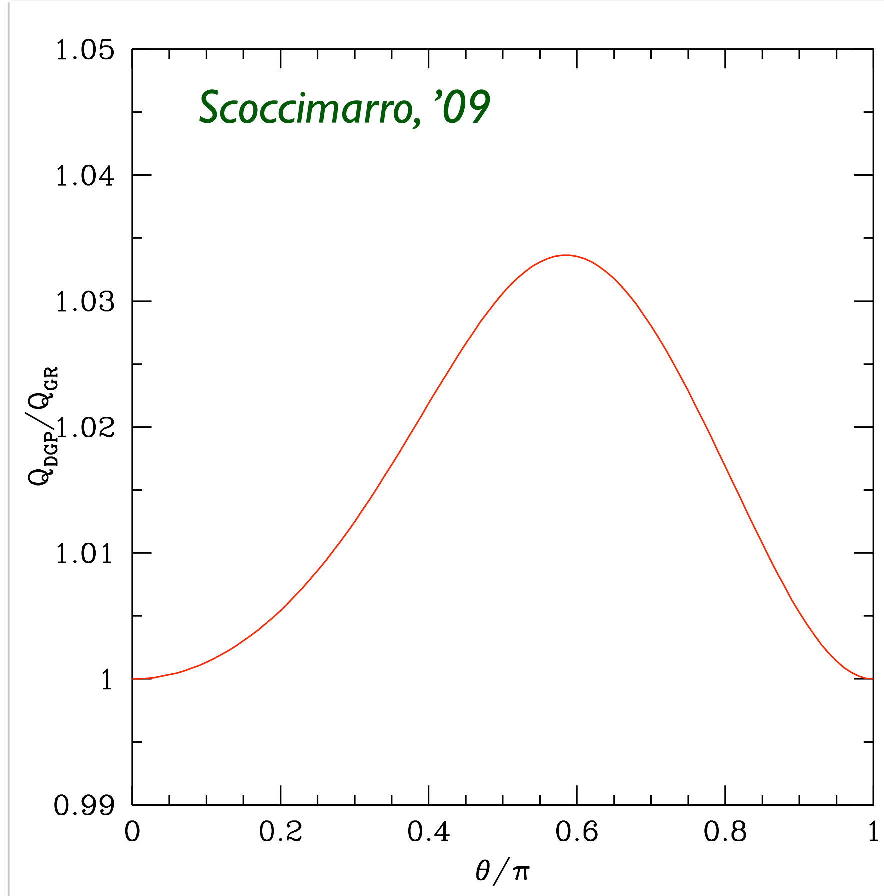


FIG. 10: The ratio of reduced bispectra Q in brane-induced gravity to GR for triangles with $k_1 = 0.1\,h\,\mathrm{Mpc}^{-1}$ and $k_2 = 2k_1$ as a function of angle θ between $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$.

Message 2 : changing strength/form of gravity laws is our best chance to induce significant (although mild) changes in the shape/amplitude of the observable bispectra.