Scale dependent non-Gaussianity

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CB, Sami Nurmi, Gianmassimo Tasinato & David Wands; 0911.2780 [astro-ph.CO] + JCAP
CB, Mischa Gerstenlauer, Sami Nurmi, Gianmassimo Tasinato & David Wands; 1007.4277 [astro-ph.CO]
CB, Kari Enqvist, Tomo Takahashi; 1007.5148 [astro-ph.CO]

Benasque Modern Cosmology wokshop 16th August '10, 30 min

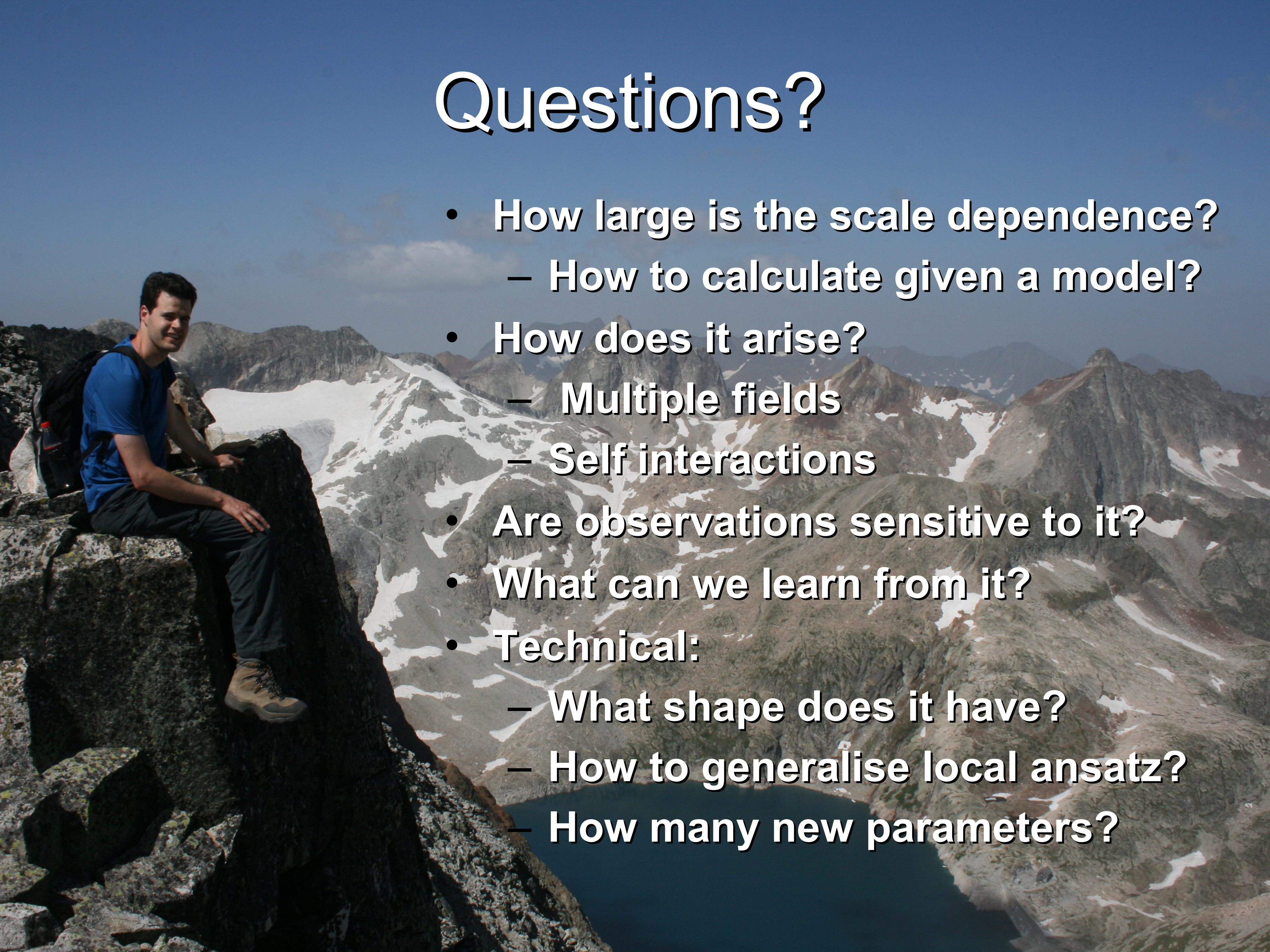
So far data is consistent with Gaussianity

So why is this field so popular? Komatsu et al; Decadel review, '09

1. Observations are sensitive to non-Gaussianity, so constraints are tight
2. Non-detection already constrains classes of models
3. Constraints are expected to tighten significantly with Planck - topical
4. LSS constraints are competitive and promising
   lots of talks here!
5. In the next two years we could move from no detection to a very high significance detection
6. So how much can we learn, and what should we look for with Planck and beyond?

Some scale dependence is expected

- Analogous to the power spectrum, fNL (local) should have a mild scale dependence
- Observational and theoretical interest
- Breaks degeneracy between early universe models
  - As well as the trispectrum
- Predictions should come first
  - Avoid posterior detections


Questions?
• How large is the scale dependence?
– How to calculate given a model?
• How does it arise?
– Multiple fields
– Self interactions
• Are observations sensitive to it?
• What can we learn from it?
• Technical:
– What shape does it have?
– How to generalise local ansatz?
– How many new parameters?

The primordial curvature perturbation ζ

$$\zeta = \frac{5}{3}\Phi = -\frac{H}{\dot{\phi}}\delta\phi = -\frac{H}{\dot{\rho}}\delta\rho$$ relates to the inflaton fluctuations

For adiabatic perturbations it is conserved on super horizon scales

$$\zeta \rightarrow \frac{\Delta T}{T}$$ relates to the temperature perturbations on the CMB

Minimal local ansatz of non-Gaussianity

- Simplest and reasonably well motivated

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL} \zeta_G^2(\mathbf{x})$$

- f_{NL} is an amplitude, one number
- Useful description, but not exact

General definition of f_{NL}

- Power spectrum $\langle \zeta_{k_1} \zeta_{k_2} \rangle = P_\zeta(k)(2\pi)^3 \delta^3(k_1 + k_2)$
- Bispectrum

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = B_\zeta(k_1, k_2, k_3)(2\pi)^3 \delta^3(k_1 + k_2 + k_3)$$

$$\frac{6}{5} f_{NL}(k_1, k_2, k_3) \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms}}$$

- Function of 3 wavenumbers
  - side lengths of a triangle
- Usually reduced to an amplitude times scale-independent shape function
- Why scale-independent?

Definition of scale dependent f_{NL}

For the equilateral triangle (one k)

$$n_{f_{NL}} \equiv \frac{\partial \log |f_{NL}|}{\partial \log k}$$

- In general f_{NL} trivariate function, so definition needs care
- However $n_{f_{NL}}$ is independent of the shape provided one scales the triangle preserving the shape
  - Hence the above is a useful definition of a new observable Byrnes, Nurmi, Tasinato and Wands, '09
  - Not much change if the shape and size of triangle are changed together

Observable parameters, bispectrum and trispectrum

We define 3 non-linearity parameters

$$B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{6}{5} f_{NL}\left[P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1)\right]$$

$$T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \tau_{NL}\left[P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)P_\zeta(k_3)P_\zeta(k_4)+(11\text{ perms})\right] + \frac{54}{25} g_{NL}\left[P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4)+(3\text{ perms})\right]$$

Seery & Lidsey '06; Byrnes, Sasaki & Wands '06

Note that τ_{NL} and g_{NL} both appear at leading order in the trispectrum
The coefficients have a different k dependence, $P_\zeta \propto k^{-3}$

Constraints $|f_{NL}| \lesssim 100, \quad |\tau_{NL}| \lesssim 10^5, \quad |g_{NL}| \lesssim 10^6$

WMAP7; Desjacques & Seljak '09; Smidt et al '10 a

Planck forecasts $|f_{NL}| \lesssim 10, \quad |\tau_{NL}| \lesssim 10^3, \quad |g_{NL}| \lesssim 10^5$

Smidt et al '10 b

Simple extension of local fNL

- The multivariate local model

$$\zeta(x) = \zeta_{G,\phi}(x) + \zeta_{G,\chi}(x) + f_\chi \zeta^2_{G,\chi}(x) + g_\chi \zeta^3_{G,\chi}(x)$$

phi is the Gaussian inflaton field,
chi generates non-Gaussianity (uncorrelated to phi)

applies to mixed inflaton and curvaton/modulated reheating scenarios, provided f_χ is a constant

– Two-component hybrid inflation CB, Choi, Hall '08

Bispectrum has the usual local shape – not changed

$$P_\zeta(k) = P_{\zeta_\phi}(k) + P_{\zeta_\chi}(k), \quad P_\zeta \propto k^{n-4}, \; P_{\zeta_\chi} \propto k^{n_\chi - 4}$$

$$f_{NL}(k) = \frac{5}{3}\frac{B_{\zeta_\chi}}{3P_\zeta^2} = \frac{5}{3}\frac{P_{\zeta_\chi}(k)^2}{P_\zeta(k)^2} f_\chi \propto \left(\frac{k}{k_p}\right)^{2(n_\chi - n)}$$

- So a scale dependence of f_{NL} is simple and natural
- Trispectrum $$n_{\tau_{NL}} = n_{g_{NL}} = \frac{3}{2} n_{f_{NL}} = 3(n_\chi - n)$$

Mixed inflaton-curvaton scenario

- The inflaton phi has Gaussian perturbations, the curvaton field chi (quadratic V) is non-Gaussian assume a small field model of inflation $\epsilon \ll \eta_{\phi\phi}$

$$n - 1 = 2(1 - w_\chi)\eta_{\phi\phi}$$

$$f_{NL}(k) = w_\chi^2(k) f_\chi$$

$$n_{f_{NL}} = 2(n_\chi - n) = -4(1 - w_\chi)\eta_{\phi\phi}$$

where $w_\chi(k) = P_{\zeta_\chi}(k)/P_\zeta(k)$

- New consistency relation $n_{f_{NL}} = -2(n-1) \simeq 0.1$

- Trispectrum $\tau_{NL} = \left(\frac{6}{5} f_{NL}\right)^2 \frac{1}{w_\chi}$ $n_{\tau_{NL}} = -3(n-1)$

Observational prospects

- Planck could reach a tight constraint
- Predicted to reach $\Delta n_{f_{NL}} = 0.1$ for $f_{NL} = 50$
- CMBPol has double this sensitivity
- LSS maybe best?

    Ask Giannantonio, Huterer, Porciani, Shandera ...

- Error bar is inversely proportional to the fiducial value of f_{NL}

- So its possible that Planck will provide the first detection of non-Gaussianity, and simultaneously detect its scale dependence!

Sefusatti, Ligouri, Yadav, Jackson, Pajer; '09

Single-field scale dependence I

- Models where any single field generates the perturbations
  - Not assumed to be the inflaton
- fNL can be scale dependent
- Arises from the non-linearity of the field evolution just after horizon exit
- Only exception is a free test field (quadratic potential)
  - has a linear equation of motion
- The assumption that f_{NL} is scale independent is only valid in the simplest toy models!
- Example is the simplest curvaton scenario
- Including the inflaton field fluctuations or self interactions will generate a scale dependence

Single field II

- In models with large non-Gaussianity the single field is isocurvature during inflation

$$n_{f_{NL}} \sim \frac{\sqrt{r_T}}{f_{NL}} \frac{V'''}{3H^2} \qquad r_T = \frac{P_T}{P_\zeta}$$

$$\tau_{NL} = \left(\frac{6}{5} f_{NL}\right)^2 \Rightarrow n_{\tau_{NL}} = 2 n_{f_{NL}}$$

$$n_{g_{NL}} \sim \frac{r_T}{g_{NL}} \frac{V''''}{3H^2} \sim \frac{\mathcal{P}_\zeta^{-1}}{g_{NL}} V''''$$

- Model dependent size, could be large
- Neither spectral index nor its running probe higher derivatives of the isocurvature's field potential
- Only way to probe self-interactions?

Interacting curvaton scenario I

$$V(\chi) = \frac{1}{2}m^2\chi^2 + \lambda m^4 \left(\frac{\chi}{m}\right)^p$$

Strength of self interaction (at horizon exit, *)

$$s = 2\lambda \left(\frac{\chi_*}{m}\right)^{p-2}$$

In the limit of s=0 recover scale invariance

Energy density of curvaton at time of decay

$$r_{dec} \equiv \frac{\rho_\chi}{4\rho_{rad} + 3\rho_\chi} \qquad f_{NL} \sim \frac{1}{r_{dec}}$$

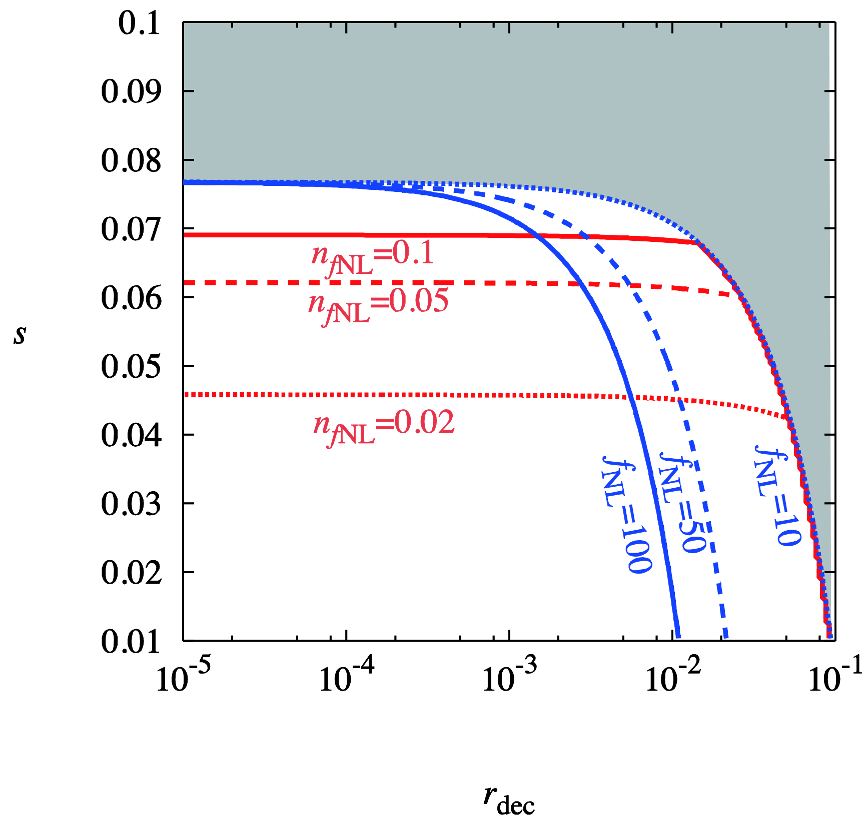
CB, Enqvist, Takahashi; 1007.5148

Interacting curvaton scenario II

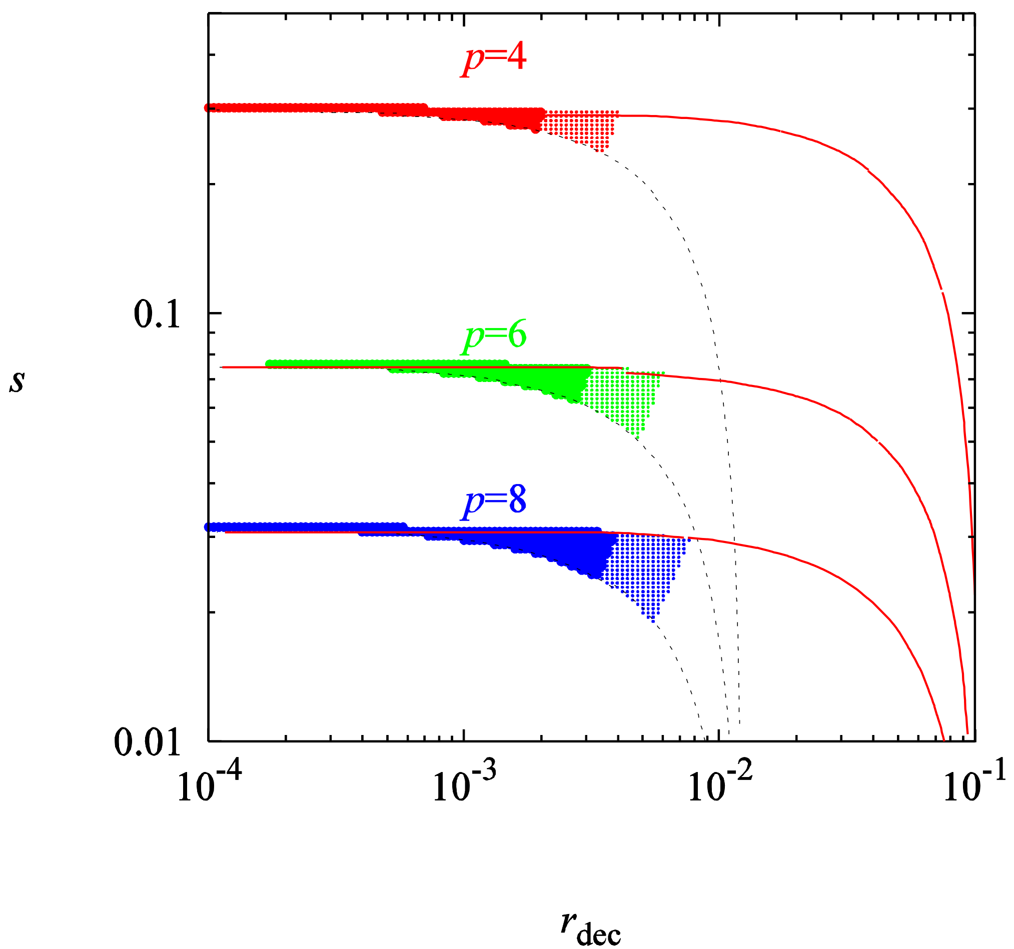
$p = 6 \qquad \eta_{\chi\chi} = 0.005$

$f_{NL} < 10$ is shaded

$n_{f_{NL}} > 0.1$ is possible even for small s



Interacting curvaton scenario III testable region



shaded regions are testable with CMBPol at 1 and $2-\sigma$

top redline $f_{NL=10}$
lower dashed line $f_{NL} = 100$

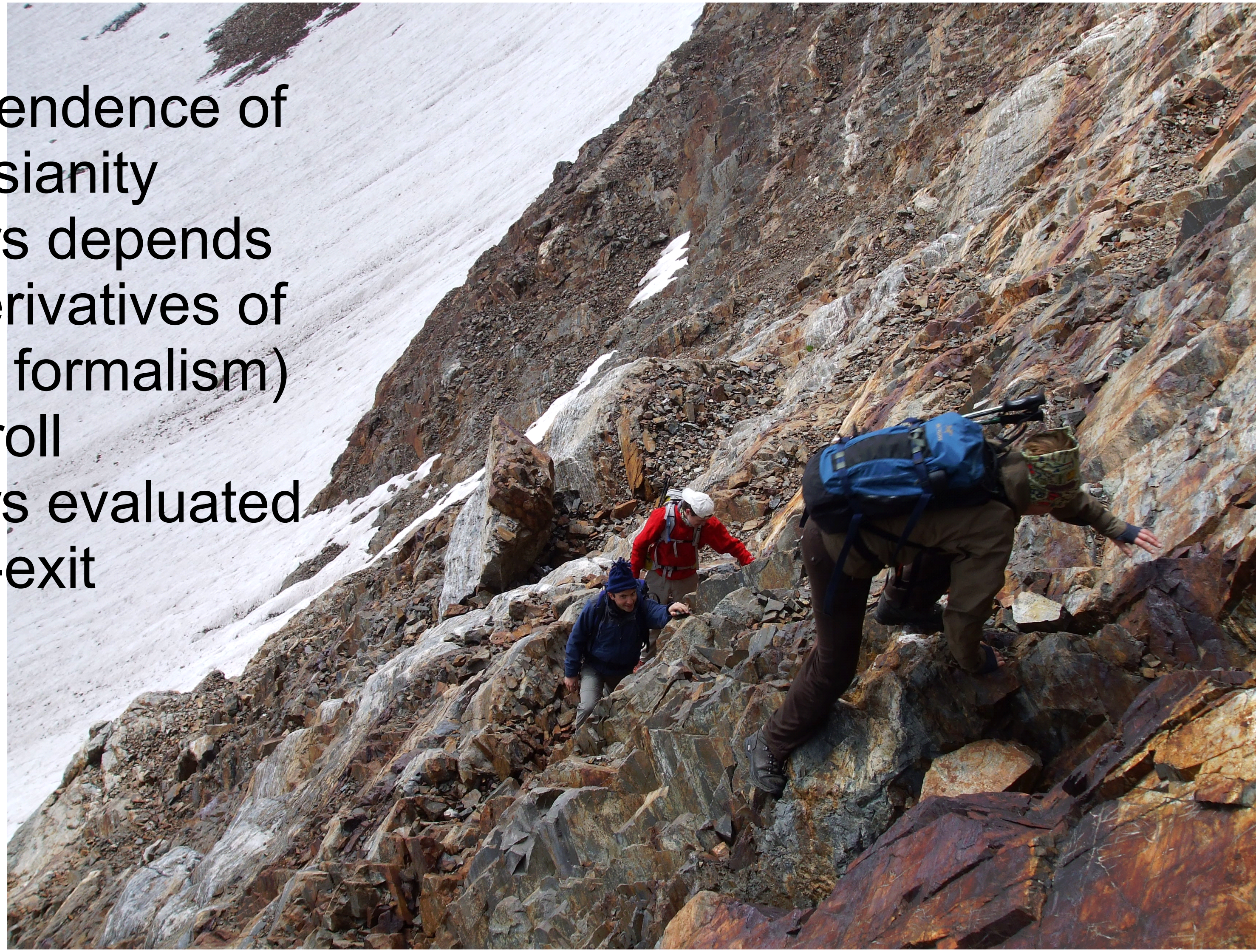
Interacting curvaton scenario IV Summary

- Knowledge of $f_{NL}, \quad n_{f_{NL}}, \quad g_{NL}$
  would give us information on the curvaton parameters
  $m, \quad p, \quad s$
- Even a small self interaction significantly changes the model predictions
  - Makes all of the non-linearity parameters scale dependent
- The curvaton is required to have a quadratic minimum
  - Models which could have a pure self interaction potential (eg modulated reheating) may have larger scale dependence

CB, Gerstenlauer, Nurmi, Tasinato & Wands '10; see also Bernardeau '10

Easy to calculate

Scale dependence of non-Gaussianity parameters depends only on derivatives of N (delta N formalism) and slow-roll parameters evaluated at Hubble-exit



Conclusions

- Non-Gaussianity is an important and topical way of constraining the many models of inflation
- It is not given by just one amplitude
- Should include a scale-dependence
  – New observable
  – Easy to calculate in many models
- Can arise due to:
  a) Multiple field effects
  b) Self interactions of the fields
- The scale dependence could be large

- CB, Nurmi, Tasinato & Wands; 0911.2780 [astro-ph.CO]
- CB, Gerstenlauer, Nurmi, Tasinato & Wands;1007.4277 [astro-ph.CO]
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Loop corrections?

- With extreme parameter values, the bispectrum can be large through a "loop" correction

$$\zeta = \zeta_\phi + \zeta_\chi^2$$

Boubekeur & Lyth; '05
Suyama & Takahashi; '08

- Applying a sharp IR cut-off L

$$f_{NL} = f_{NL}^{\rm loop} \sim \frac{P^3_{\zeta_\chi}(k)}{P^2_\zeta(k)} \ln(kL)$$

- If we take L~1/H

$$n_{f_{NL}} \sim 1/\ln(kL) \sim 0.2$$

Kumar, Leblond & Rajaraman; '09

- Some controversy if this is physical or artifact of the cut off

Non-Gaussianity during slow-roll

- Possible to generate a large non-Gaussianity during slow-roll inflation (with canonical kinetic terms)
- Requires trajectory sensitive to Hubble exit value of the (subdominant) isocurvature field
- Requires curved trajectory during inflation, this breaks the conservation of zeta

Gordon et al '00

- f_{NL} generated on super-horizon scales (in common with nearly all local models)
- Difference from all other local models is that here the non-Gaussianity is generated during inflation

General conditions: CB, Choi & Hall '08 a)
Without slow roll (exact solution): CB & Tasinato '09

Two-component hybrid inflation

$$W = W_0\left(1+\frac{1}{2}\eta_{\varphi\varphi}\frac{\varphi^2}{M_P^2}+\frac{1}{2}\eta_{\chi\chi}\frac{\chi^2}{M_P^2}\right)$$

If we choose initial conditions to maximise f_{NL} then

$$f_{NL}=\frac{5}{24}\eta_{\chi\chi}e^{2N(\eta_{\varphi\varphi}-\eta_{\chi\chi})}, \quad n_\zeta - 1 = \eta_{\varphi\varphi}+\eta_{\chi\chi}$$

N is the number of e-foldings from horizon crossing till the end of inflation; Scales which exit earlier are more non-Gaussian

$$n_{f_{NL}} \equiv \frac{\partial \log|f_{NL}|}{\partial \log k} = -2(\eta_{\varphi\varphi}-\eta_{\chi\chi})$$

$\eta_{\varphi\varphi}$	$\eta_{\chi\chi}$	φ_*	χ_*	f_{NL}	$n_{f_{NL}}$	$n_\zeta - 1$	r
0.04	-0.04	1	6.8×10^{-5}	-123	-0.16	0	0.006
0.08	0.01	1	0.0018	9.27	-0.14	0.09	0.026
-0.01	-0.09	1	3×10^{-6}	-132	-0.276	-0.04	0.0007

First to calculate scale dependence: Byrnes, Choi & Hall '08 b)

Inflaton field

- Can find analytic results using the slow-roll approximation
- Neglecting the non-Gaussianity of the fields at horizon exit, i.e. taking only the local part

$$\frac{6}{5} f_{NL} = \frac{N''}{N'^2} = 2\epsilon - \eta$$

$$n_{f_{NL}} = \frac{6\epsilon\eta - 8\epsilon^2 - \xi^2}{\eta - 2\epsilon}$$

Can see how this arises from the second-order field evolution

$$\delta_2\phi(t_i) = \delta_2(\phi_*) + \frac{H(t_i - t_*)}{\sqrt{2\epsilon}}(8\epsilon^2 - 6\epsilon\eta + \xi^2)\delta_1^2\phi$$