GRAVITATIONAL WAVES FROM/AFTER REHEATING

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GARCÍA-BELLIDO & DGF, **PRL 2007**
GARCÍA-BELLIDO, DGF & SASTRE, **PRD 2008**
FENU, DGF, DURRER & GARCÍA-BELLIDO, **JCAP 2009**
DUFAUX, DGF & GARCÍA-BELLIDO, **PRD 2010**

BENASQUE 2010, August 17$^{\mathrm{th}}$ 2010, SPAIN

GRAVITATIONAL WAVES (GW): PROBING the EARLY UNIVERSE ($t \lesssim 1$ s)

1. **WEAKNESS of GRAVITY**:

   ADVANTAGE: GW DECOUPLE upon Production
   DISADVANTAGE: DIFFICULT DETECTION

2. **ADVANTAGE**: GW $\rightarrow$ Probe for Early Universe

   $$\rightarrow \begin{cases} \textbf{Decouple} \rightarrow \text{Spectral Form Retained} \\ \textbf{Specific HEP} \;\Leftrightarrow\; \text{Specific GW} \end{cases}$$

3. **Physical Processes**: $\begin{cases} \text{Inflation} \\ \text{Reheating} \\ \text{Phase Transitions} \\ \text{Turbulence} \end{cases}$

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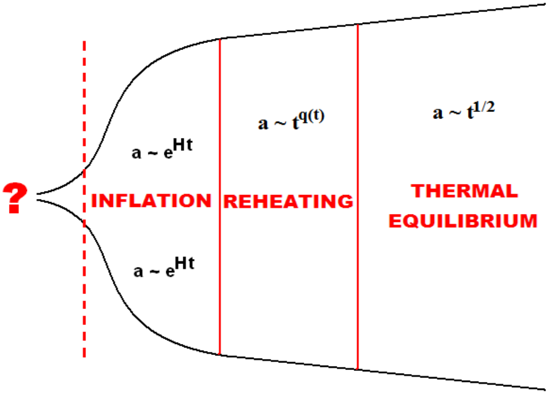
INFLATION $\longrightarrow$ **REHEATING** $\longrightarrow$ **BIG BANG THEORY**



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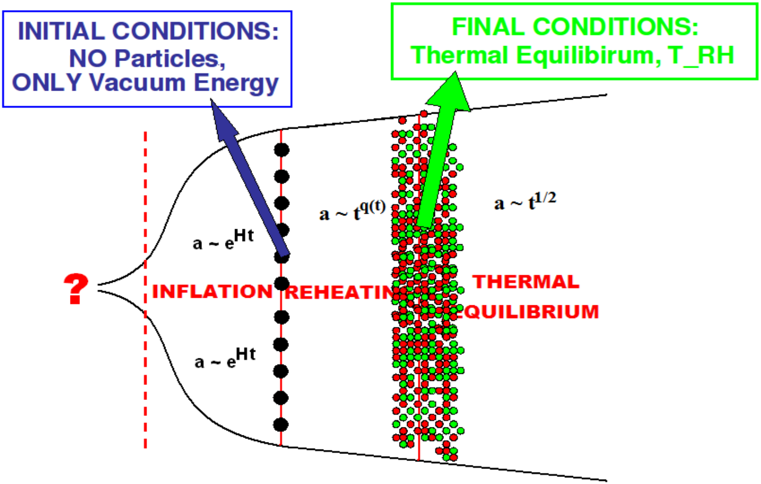

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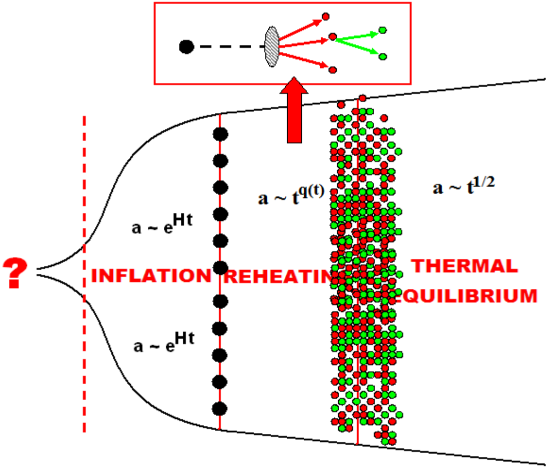

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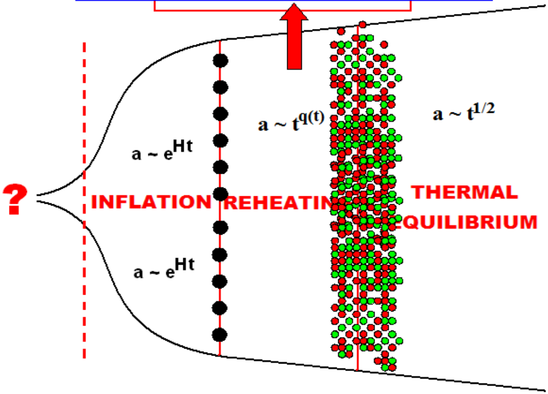
$$\mathcal{L} = \mathcal{L}(\phi, \varphi_i, \psi_j, A_\mu, h_{\mu\nu}, \ldots)$$



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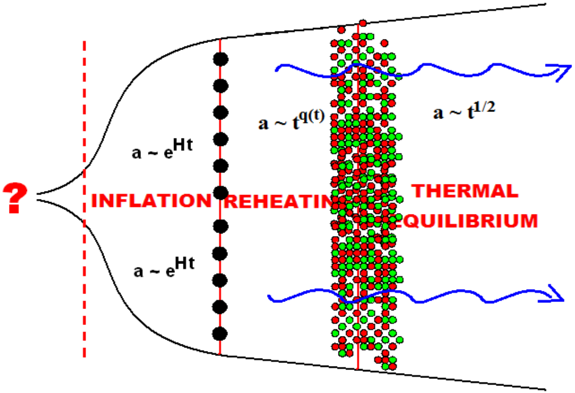


SCALAR REHEATING: SIMPLE EXAMPLES

$$V(\phi,\chi) = \quad \frac{1}{4}\lambda\phi^4 \quad + \quad \frac{1}{2}m_\chi^2\chi^2 \quad + \quad \frac{1}{2}g^2\phi^2\chi^2 \quad \text{(Chaotic)}$$

$$V(\phi,\chi) = \quad \frac{1}{2}\mu^2\phi^2 \quad + \quad \frac{\lambda}{4}(\chi^2 - v^2)^2 \quad + \quad \frac{1}{2}g^2\phi^2\chi^2 \quad \text{(Hybrid)}$$

$$\begin{cases} \ddot{\phi}(t) + 3H\dot{\phi} + V'(\phi) = 0 & (\textbf{Inflaton Zero-Mode} : \text{Damped Oscillator}) \\ \Box\phi_k + F(\int dq\phi_q\chi_{|k-q|})\phi_k + ... = 0 & (\textbf{Inflaton Fluctuations}) \\ \Box\chi_k + F(\int dq\chi_q, \phi_{|k-q|})\chi_k + ... = 0 & (\textbf{Matter Fluctuations}) \end{cases}$$

DYNAMICS:

Non-Linear, Non-Perturbative and Far-From-Equilibrium

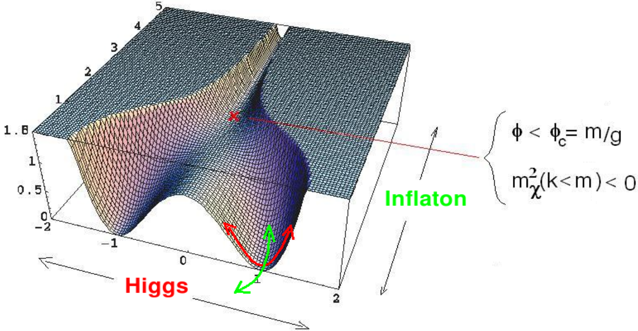
Reheating (Hybrid Scenarios): SPINODAL INSTABILITY

$$\left.\begin{array}{l} \ddot{\phi}(t) + (\mu^2 + g^2|\chi|^2)\phi(t) = 0 \\ \\ \ddot{\chi}_k + \left(k^2 + m^2\left(\frac{\phi^2}{\phi_c^2} - 1\right) + \lambda|\chi|^2\right)\chi_k = 0 \end{array}\right\}$$

$$(k < m = \sqrt{\lambda}v)$$

$$\chi_k, n_k \sim e^{\sqrt{m^2-k^2}t}$$

Hybrid Preheating



Scalar Reheating: Observable Phenomena

PHENOMENA FROM (SCALAR) REHEATING:

SUB-HORIZON GRAVITATIONAL WAVES

SUPER-HORIZON GRAVITATIONAL WAVES

EXPECTATIONs of (p)REHEATING: SubH-GW

Physics of (p)REHEATING: $\ddot{\varphi}_k + \omega^2(k,t)\varphi_k = 0$

$$\begin{cases} \textbf{Hybrid Preheating}: & \omega^2 = k^2 + m^2(1 - V\,t) < 0 \quad (\text{Tachyonic}) \\ \textbf{Chaotic Preheating}: & \omega^2 = k^2 + \Phi^2(t)\sin^2\mu t \quad (\text{Periodic}) \end{cases}$$

At $\mathbf{k}_i$: $\varphi_{k_i}, n_{k_i} \sim e^{\mu(k,t)t} \Rightarrow$ Inhomogeneities: $\begin{cases} L_i \sim 1/k_i \\ \delta\rho/\rho \gtrsim 1 \\ v \approx c \end{cases}$

(p)REHEATING: VERY EFFECTIVE GW GENERATOR

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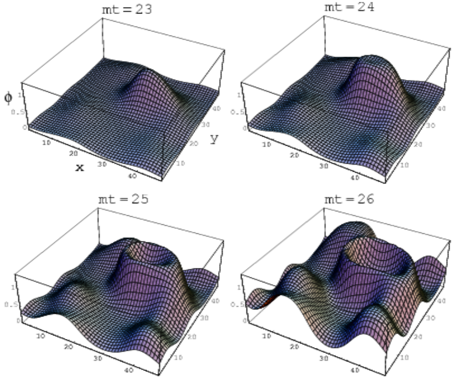
GW from RELATIVISTIC WAVES of MATTER



G[a]-Bellido et al '02 (Hybrid Scenario: $\lambda \approx 0.1$, $VEV = 10^{-3} M_p$)

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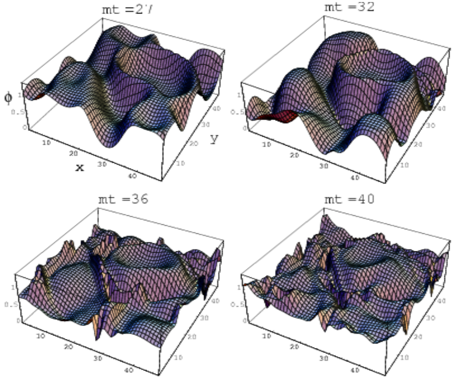

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Lattice Simulations: Dynamics

- Scalars ($n_k \gg 1$): $\quad \Box\phi + V_{,\phi} = 0,\ \Box\chi_a + V_{,\chi_a} = 0$

Semi-classical regime $\pi_k \approx \kappa\phi_k + ...$ **(Squeezed States)**

- FRW: $H^2 = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)\ , \left\{ \begin{array}{l} \rho = \langle \rho_\phi + \rho_\chi + ... \rangle \\ p = \langle p_\phi + p_\chi + ... \rangle \end{array} \right.$

- GW: $h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\rm TT}_{ij}, \qquad \Pi_{ij} = T_{ij} - \langle T_{ij} \rangle_{\rm FRW}$

$ds^2 = a^2(-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \qquad {\rm TT}: \left\{ \begin{array}{l} h_{ii} = 0 \\ h_{ij,j} = 0 \end{array} \right.$

TT dof carry energy out of the source!!!

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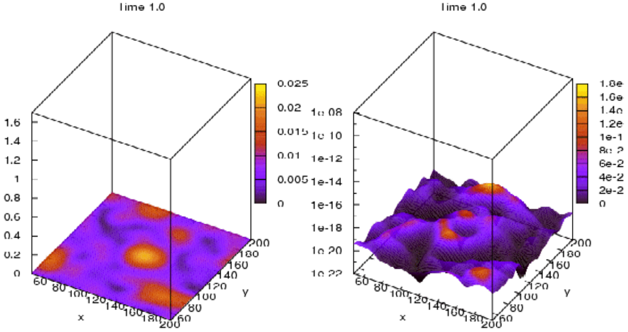
Bubble Nucleation and Collisions (Animation by Alfonso Sastre)



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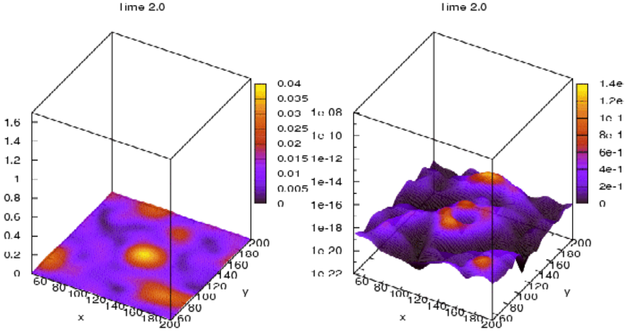

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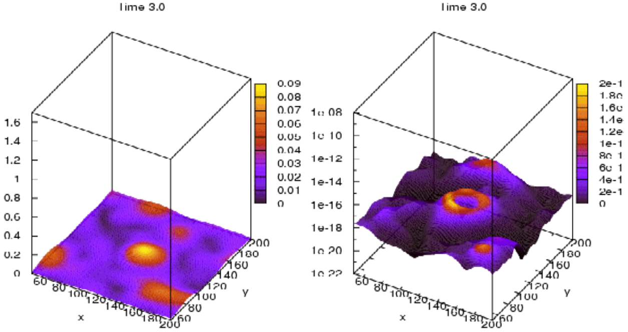

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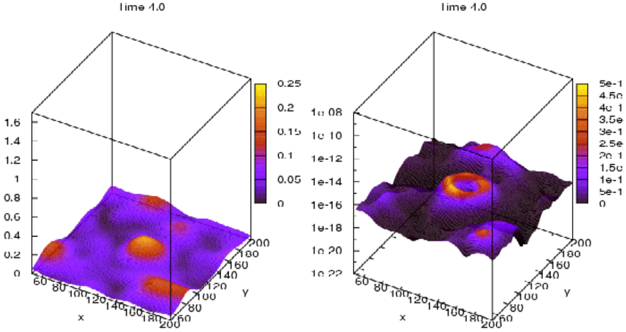

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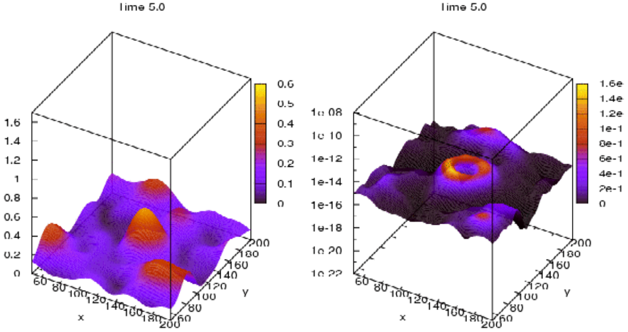

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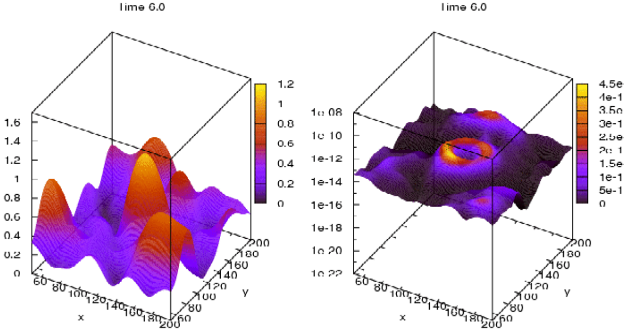

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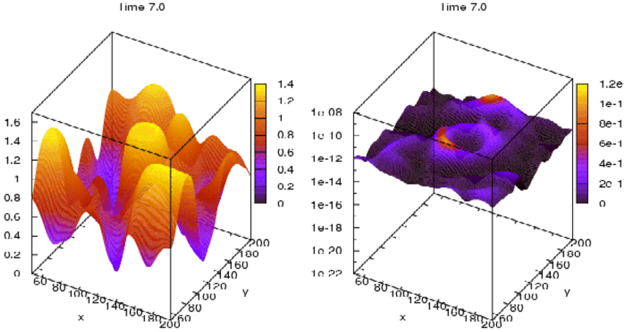

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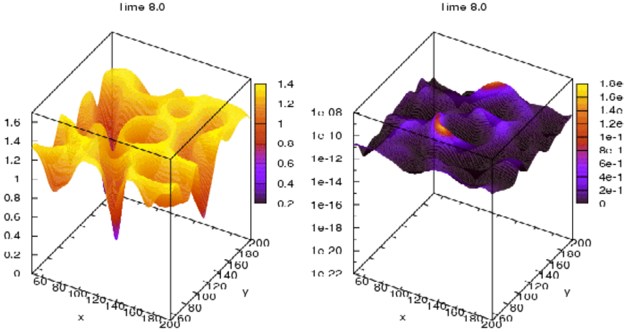

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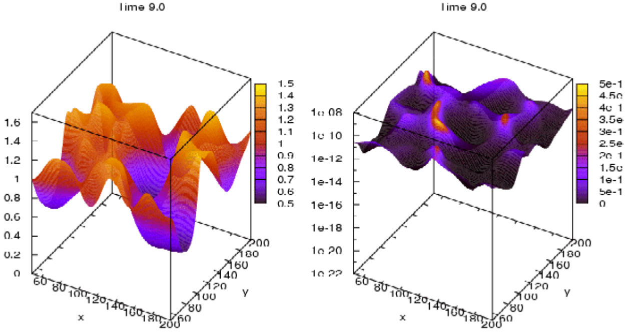

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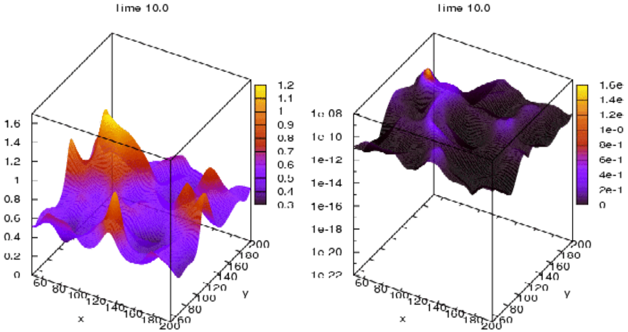

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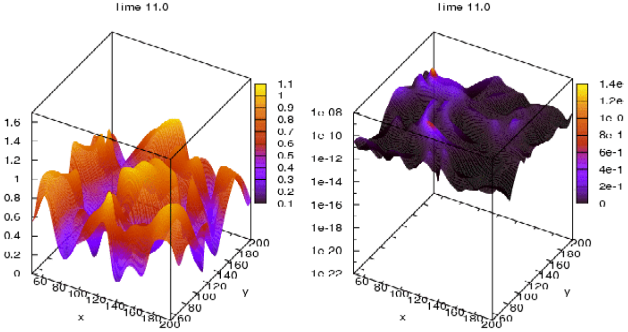

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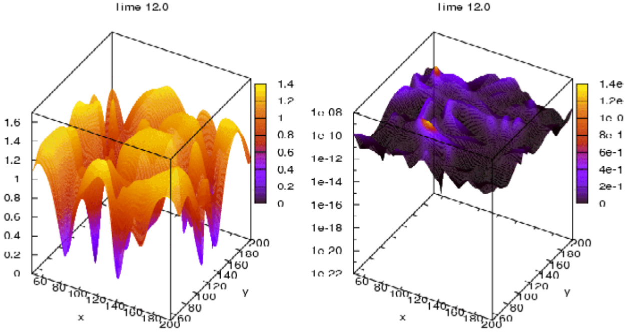

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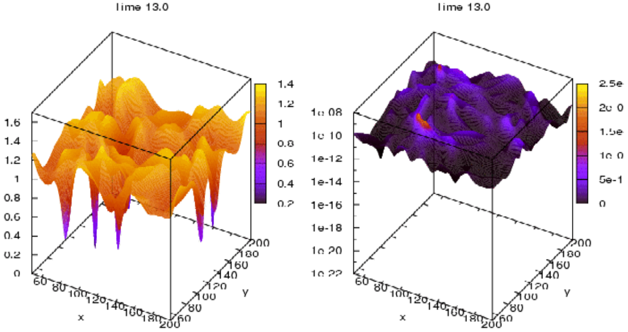

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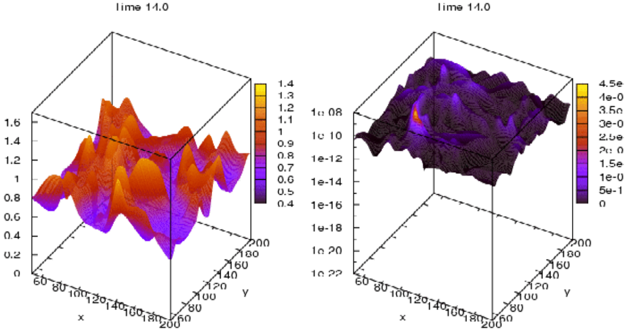

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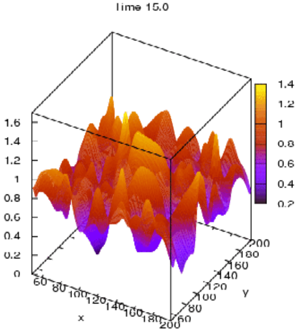

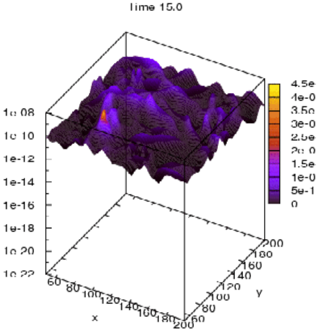

Hybrid (p)Reheating (Part II)

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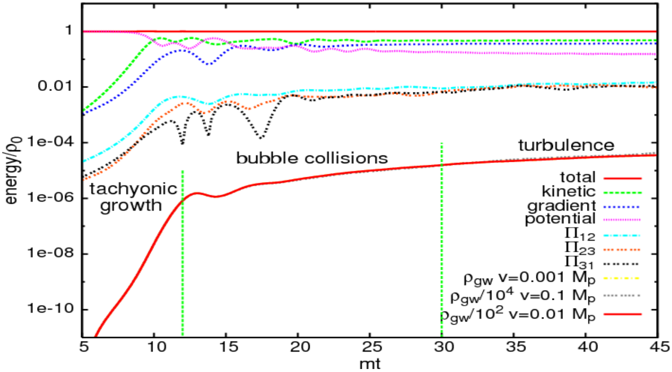
3 stages: **Exp. Instabilities** $\rightarrow$ **Bubble Collisions** $\rightarrow$ **Turbulence**



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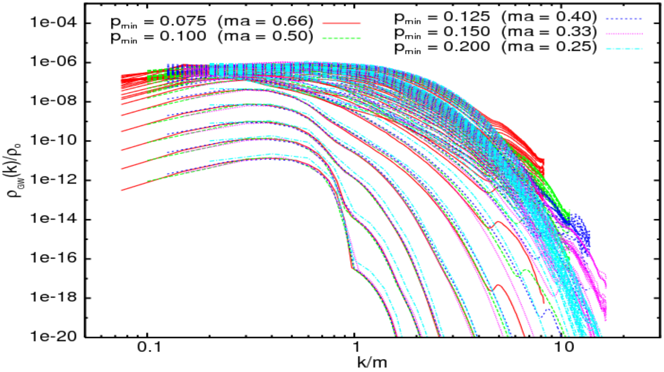
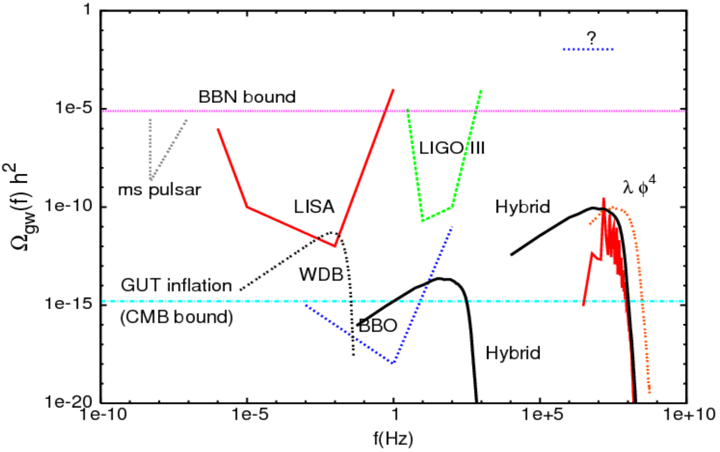


Today's Signal (GW RedShifted)

$$\Omega_{\mathrm{GW}}(f) \propto (v/M_p)^2\,, \qquad f_* \propto \lambda^{1/4} \times k_*$$



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SUPER-HORIZON GRAVITATIONAL WAVES

Hybrid Reheating = Phase Transition

HYBRID REHEATING:

GLOBAL PHASE TRANSITIONS

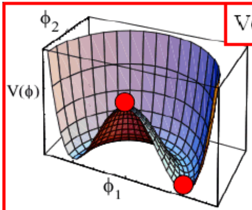
$\chi < \chi_c \longrightarrow m^2_\phi < 0$

$\phi_k \sim e^{\sqrt{m^2 - k^2}\, t}$

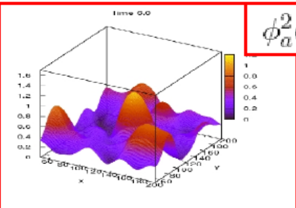
$\Phi^T = (\phi_1, \phi_2, ..., \phi_N)$ **(SSB)**

$\Phi^T \Phi = \sum_a \phi_a^2 = v^2$ **(V.E.V.)**

$V(\phi_1, \phi_2, ..., \phi_N)$



$\phi_a^2(\mathbf{x})$

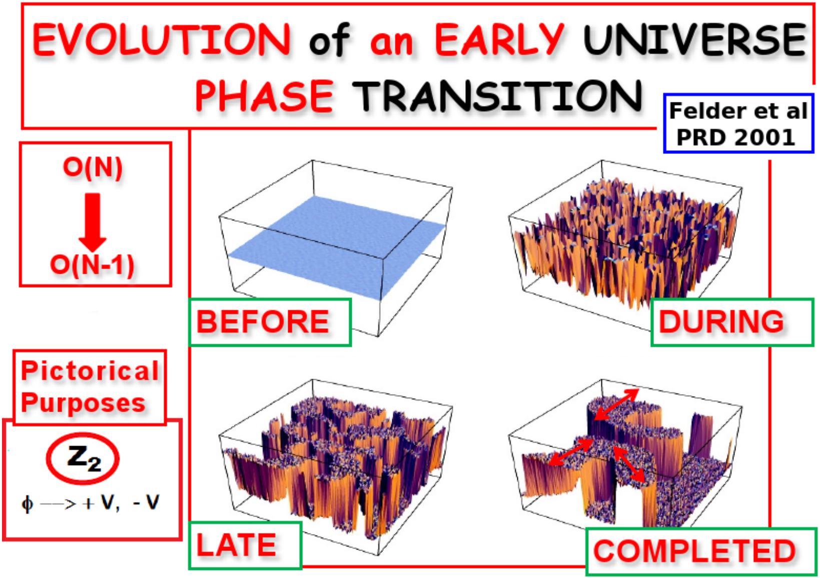


SYMMETRY BREAKING

INHOMOGENEITIES

(RELATIVISTIC WAVES of MATTER)

Hybrid Reheating = Phase Transition



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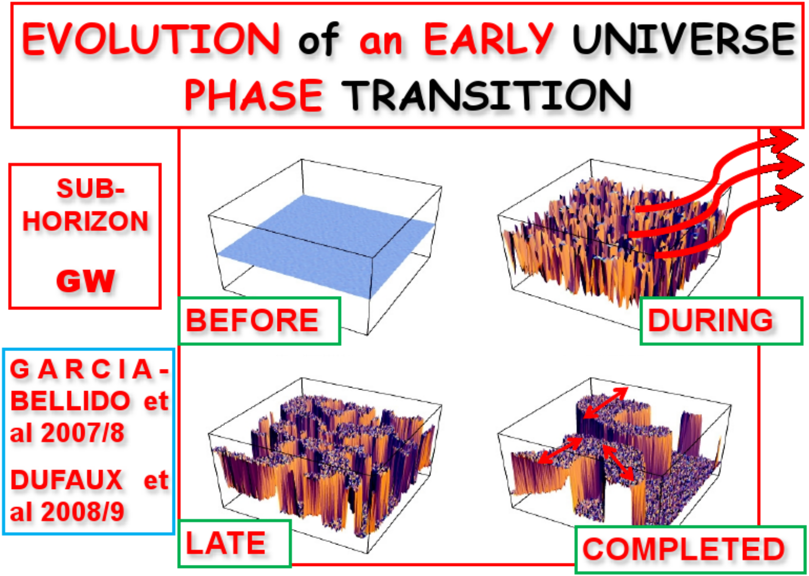

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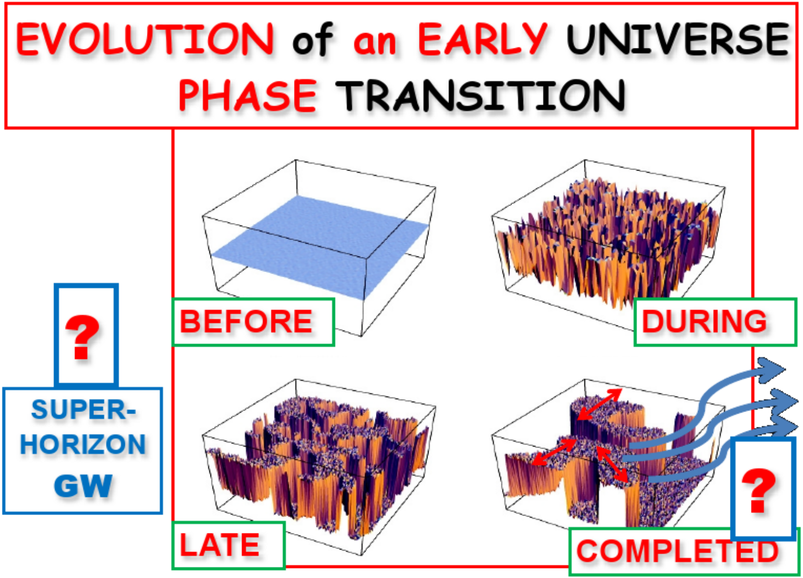
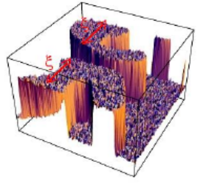


Aftermath of Hybrid Reheating: NLSM $\longrightarrow$ GW

After the PHASE TRANSITION (NON-Linear SIGMA MODEL)



UNIVERSE EXPANDING (CAUSAL HORIZON)

FIELD SELF-ORDERS

$$(\xi \uparrow\uparrow \,,\ \xi < 1/H)$$

$$\mathbf{O(N) \rightarrow O(N\text{-}1):} \begin{cases} \sum_a \phi_a^2 = v^2 & \text{(CONSTRAINT)} \\ \Box \phi_a + V'(\phi_a) = 0 & \text{(EOM)} \end{cases} \Rightarrow \boxed{\Box \phi_a + (\partial_\mu \phi_b \cdot \partial^\mu \phi_b)\, \phi_a = 0}$$

LARGE-N LIMIT: $(N \geq 4)$

$$\boxed{\phi_a(\mathbf{k},\eta) = (k\eta)^{\frac{1}{2}-\gamma}\, C_1(\mathbf{k})\, J_{\gamma+1}(k\eta)} \qquad (a = \eta^\gamma)$$

$(k\eta_* < 1$, Super-Horizon Scales$)$

Aftermath of Hybrid Reheating: NLSM $\rightarrow$ GW

GRAVITATIONAL WAVE BACKGROUND

$\phi_a(\mathbf{k},\eta) \rightarrow T_{\mu\nu}(\phi_a) \rightarrow \Pi^{TT}_{\mu\nu}(\phi_a) \rightarrow \Box h_{\mu\nu} = 16\pi G\, \Pi^{TT}_{\mu\nu}$

FIELD FLUCTUATIONS | STRESS tensor | ANISOTROPIC (TT) STRESS tensor | GW EQUATIONS (TT metric perturb.)

$$\rho_{GW} = \frac{<\dot{h}_{\mu\nu}\dot{h}^{\mu\nu}>}{16\pi G} = \int \frac{d\rho_{GW}(k,\eta)}{d\log k} d\log k \Rightarrow \Omega_{GW}(k,\eta) \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}(k,\eta)}{d\log k}$$

TECHNICALLY

$$\left\langle \phi_a(\mathbf{k},\eta)\, \phi_a(\mathbf{k},\eta) \right\rangle \longrightarrow \left\langle \Pi^{TT}_{\mu\nu}(\phi_a)\Pi^{TT}_{\mu\nu}(\phi_a) \right\rangle \longrightarrow \left\langle \dot{h}_{\mu\nu}\, \dot{h}_{\mu\nu} \right\rangle$$

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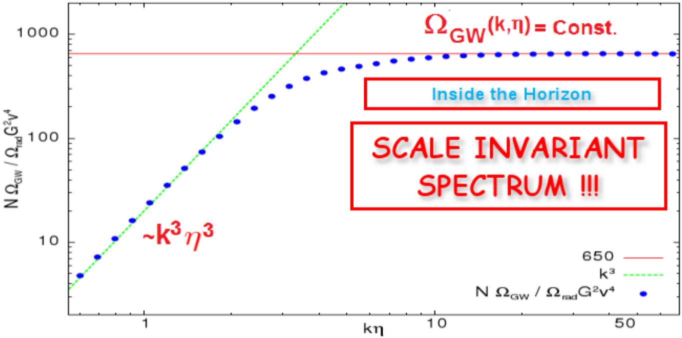

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SCALE INVARIANT SPECTRUM !!!

(FREQ. INDEPENDENT)

$$\Omega_{\rm GW}(k,\eta_0) \simeq \frac{651}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm Pl}}\right)^4$$

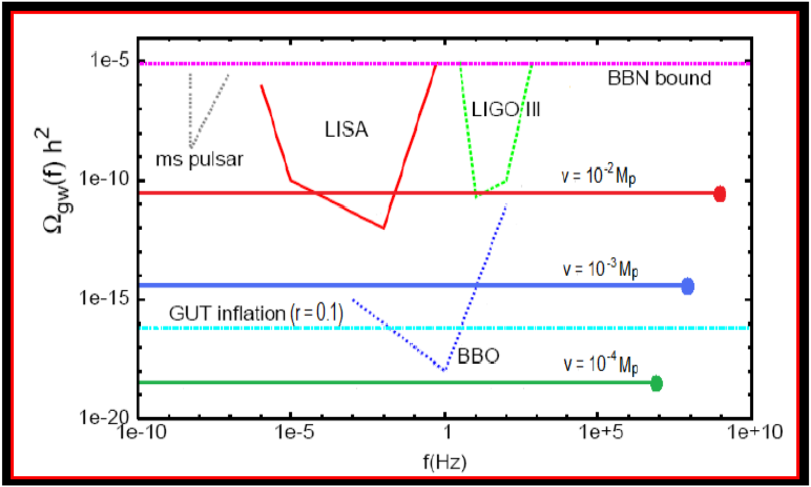
$$\mathcal{R} \equiv \frac{\Omega_{\rm GW}(k,\eta_0)}{\Omega^{\rm (inf)}_{\rm GW}(k,\eta_0)} \simeq \frac{356}{N}$$

Jones-Smith et al, 2008

Fenu, DGF, Durrer, Garcia-Bellido 2009

Aftermath of Hybrid Reheating: Scale Inv SubH GW

GRAVITATIONAL WAVE BACKGROUND



Summary: GW from Scalar (p)Reheating

1. GW are Early Universe Ideal Probe: decoupled upon production $\Rightarrow$ spectral signature retained till today $\Rightarrow$ GWB: "photo" of very Early Universe

2. GW from Reheating: Form, freq. Peak and Amplitude
$\longrightarrow$ Specific Model of Inflation (Disadvantage/Advantage)

3. Scalar Reheating Models: GW (high amplitude, too high frequency)

4. Hybrid Reheating Models: GW (high amplitude, within BBO)

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Summary: Scale-Inv GW from Global PhT.

1. NLSM + large-N limit: Self-Ordering Scalar Fields after SSB (Global PhT).
2. $k\eta_* \ll 1 \rightarrow k\eta \gg 1 : \Omega_{GW}(k,\eta) = const.$
   Observable at LIGO, LISA, BBO,...
3. For $VEV = M_I$, then $\Omega_{GW}/\Omega_{GW}^{\rm inf} \sim \mathcal{O}(10) - \mathcal{O}(100)$
   Scale-Inv GW is not any more a smoking gun of inflation.

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BACK SLIDES

Aftermath Hybrid Reheating: Matter Perturbations

NON-GAUSSIANITY AFTER GLOBAL PhT

A. Jaffe '93: $\delta(\mathbf{x},\eta) = \frac{2\pi G}{5}\eta^2 \int d\eta'\, \partial_i T_{0i}(\mathbf{x},\eta')\,,$ (MATTER DOMINATION)

Aftermath Hybrid Reheating: Matter Perturbations

NON-GAUSSIANITY AFTER GLOBAL PhT

$$\delta(\mathbf{x},\eta) = \frac{2\pi G}{5}\eta^2 \int d\eta' \, \partial_i T_{0i}(\mathbf{x},\eta') \, ,$$

(MATTER DOMINATION)

Matter Density Perturbations

C

Energy-Momentum Tensor

$$\boxed{T_{0i} = (\partial_0\phi^a)(\partial_i\phi^a)}$$

(BI-LINEAR)

$$\left(\phi^a(\mathbf{k},\eta) = C_1(\mathbf{k})\,(k\eta)^{\frac{1}{2}-\gamma} J_\nu(k\eta)\right)$$

Aftermath Hybrid Reheating: Matter Perturbations

NON-GAUSSIANITY AFTER GLOBAL PhT

$$\delta(\mathbf{x},\eta) = \frac{2\pi G}{5}\eta^2 \int d\eta'\, \partial_i (\partial_0 \phi^a)(\partial_i \phi^a)$$

$$\boxed{\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle = (2\pi)^3\, \delta_D(\mathbf{k}+\mathbf{k}')\, P^\sigma(k),}$$

2-Point Correlator

$$\boxed{P^\sigma(k,\eta) \equiv \frac{C^2\eta^4}{A^2}\frac{k}{N} g_2,}$$

Power Spectrum

$$\boxed{g_2 \equiv \int \frac{d^3 v}{(2\pi)^3}\, [\mathcal{I}(v, |\hat{\mathbf{k}} - \mathbf{v}|)]^2\, (\hat{\mathbf{k}}\cdot \mathbf{v})\, [2(\hat{\mathbf{k}}\cdot \mathbf{v}) - 1]}$$

Aftermath Hybrid Reheating: Matter Perturbations

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$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = (2\pi)^3\, \delta_D(\mathbf{k}+\mathbf{k}')\, P^\sigma(k),$$

2-Point Correlator

$$P^\sigma(k,\eta) \equiv \frac{C^2\eta^4}{A^2}\frac{k}{N}g_2,$$

Power Spectrum

Physical Constraints

$$\frac{v}{N^{1/4}} = \left(\frac{p_\sigma A^2 \Delta^2_{\mathcal{R}}}{8G^2_{\mathrm{sw}} g_2}\right)^{1/4} M_{\mathrm{Pl}} \lesssim \frac{M_{\mathrm{Pl}}}{2000}$$

$$\begin{pmatrix} G_{\mathrm{sw}} = 10 \\ p_\sigma = 0.1 \end{pmatrix}$$

NON-GAUSSIANITY AFTER GLOBAL PhT

$$\boxed{\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)}$$

(3-Point Correlator) (Bispectrum)

Aftermath Hybrid Reheating: Non-Gaussianity

NON-GAUSSIANITY AFTER GLOBAL PhT

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

(3-Point Correlator)

$$B(k_1,k_2,k_3) = \frac{C^3\eta^6}{A^3N^2}g_3(k_1,k_2,k_3)$$

$$g_3(k_1,k_2,k_3) \equiv \int \frac{d^3v}{(2\pi)^3} H(\mathbf{k}_2+\mathbf{v},\mathbf{v}) \times H(\mathbf{v},\mathbf{k}_1-\mathbf{v})H(\mathbf{k}_1-\mathbf{v},\mathbf{k}_2+\mathbf{v})$$

Aftermath Hybrid Reheating: Non-Gaussianity

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$$\boxed{\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3)}$$

(Bispectrum)

Comparing with

Local Model

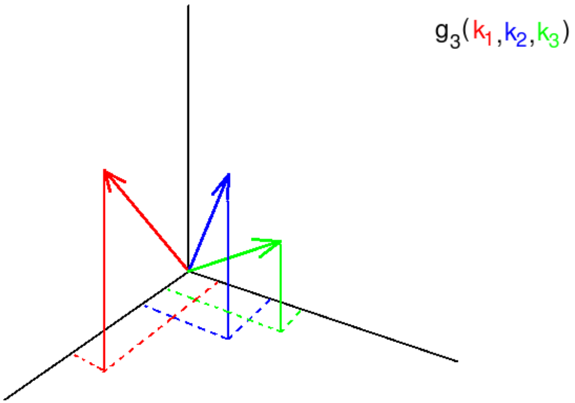
$$\boxed{f_{\rm nl}^{\sigma} \simeq 3\, G_{\rm sw}^{-3} \left(\frac{v}{5 \times 10^{15}\,{\rm GeV}} \right)^6 \left(\frac{N}{5} \right)^{-2}}$$

(for equilateral configurations)

$$\left. \begin{array}{l} v \sim \text{GUT} \\ N = 5 \end{array} \right\} \longrightarrow \boxed{f_{\rm nl} \sim O(1)} \qquad \begin{pmatrix} G_{\rm sw} = 10 \\ p_\sigma = 0.1 \end{pmatrix}$$

Aftermath Hybrid Reheating: Non-Gaussianity

NON-GAUSSIANITY AFTER GLOBAL PhT



Aftermath Hybrid Reheating: Non-Gaussianity

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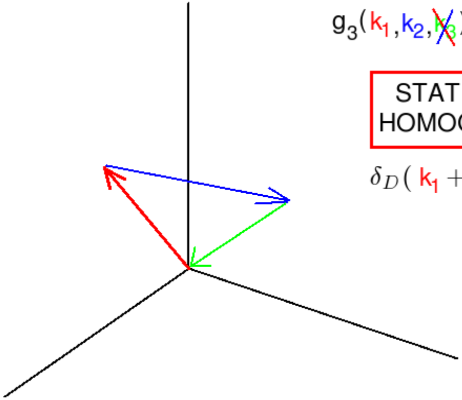


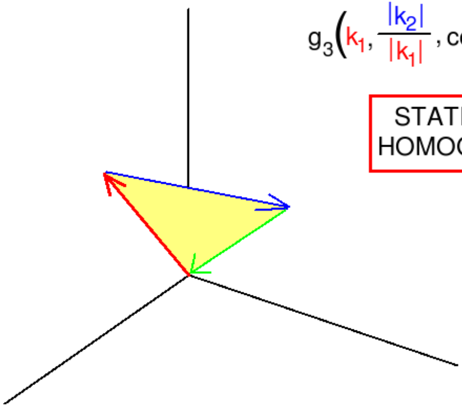
$$g_3(k_1, k_2, \cancel{k_3}) = g_3(k_1, k_2)$$

STATISTICAL HOMOGENEITY

$$\delta_D(\, k_1 + k_2 + k_3 \,)$$

Aftermath Hybrid Reheating: Non-Gaussianity

NON-GAUSSIANITY AFTER GLOBAL PhT



$$g_3\left(k_1, \frac{|k_2|}{|k_1|}, \cos(k_1,k_2), \triangle\right)$$

STATISTICAL HOMOGENEITY

Aftermath Hybrid Reheating: Non-Gaussianity

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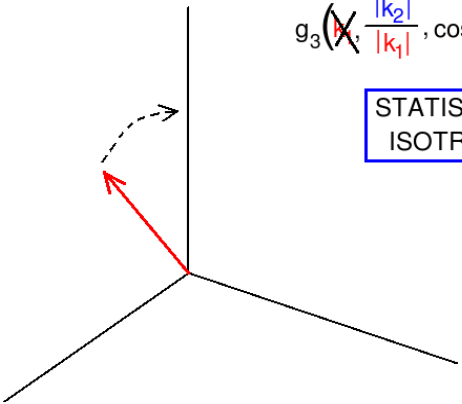
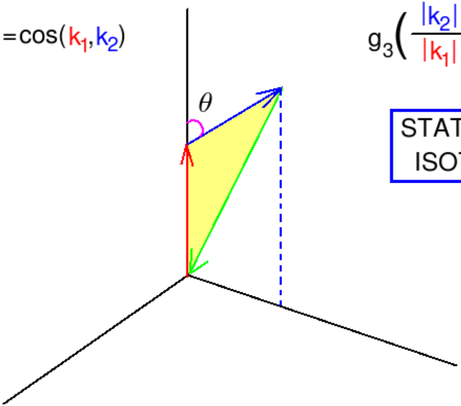
$\boxed{\text{STATISTICAL ISOTROPY}}$



Aftermath Hybrid Reheating: Non-Gaussianity

NON-GAUSSIANITY AFTER GLOBAL PhT

$\cos(\theta) = \cos(k_1, k_2)$



$g_3\left(\frac{|k_2|}{|k_1|}, \theta, \ldots\right)$

STATISTICAL ISOTROPY

Aftermath Hybrid Reheating: Non-Gaussianity

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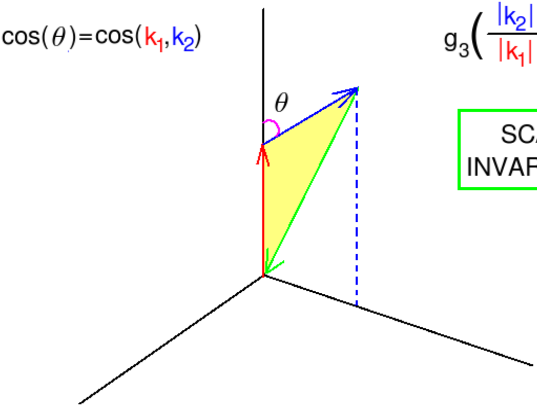

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SCALE INVARIANCE

Aftermath Hybrid Reheating: Non-Gaussianity

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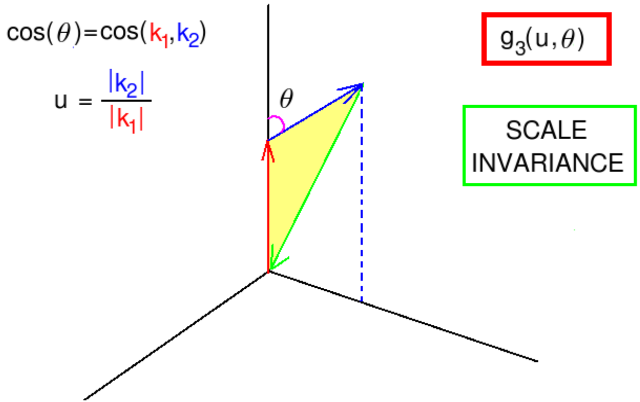

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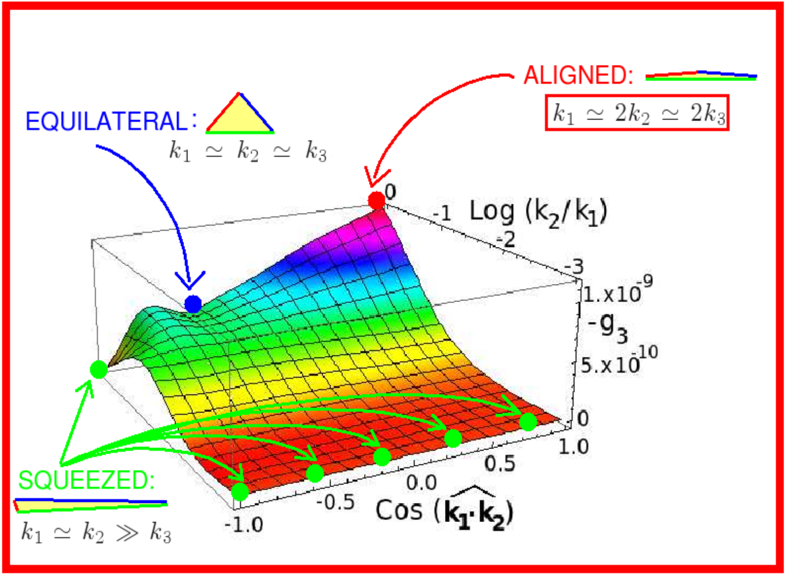
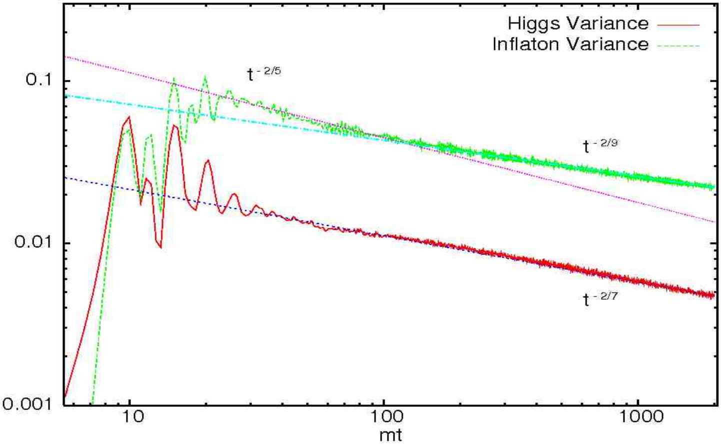


Turbulence

Results for Hybrid Reheating: Turbulence

Turbulece: $var(\phi) \sim t^{-2p}, \qquad n(k,t) = t^{-\gamma p} n_o(t^{-p}k)$ (M,T '04)



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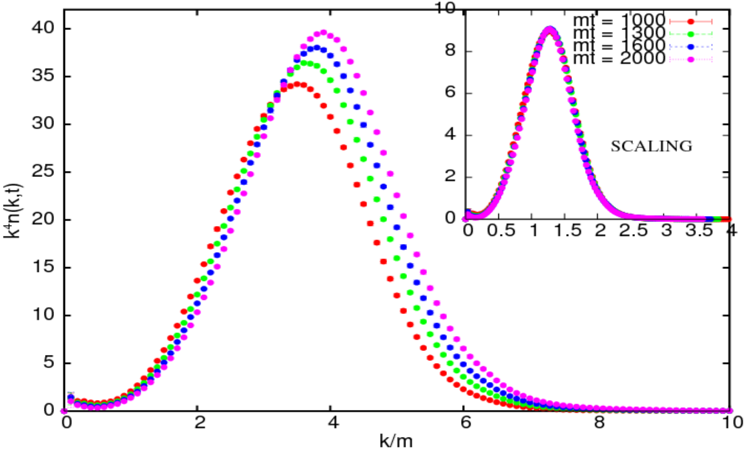

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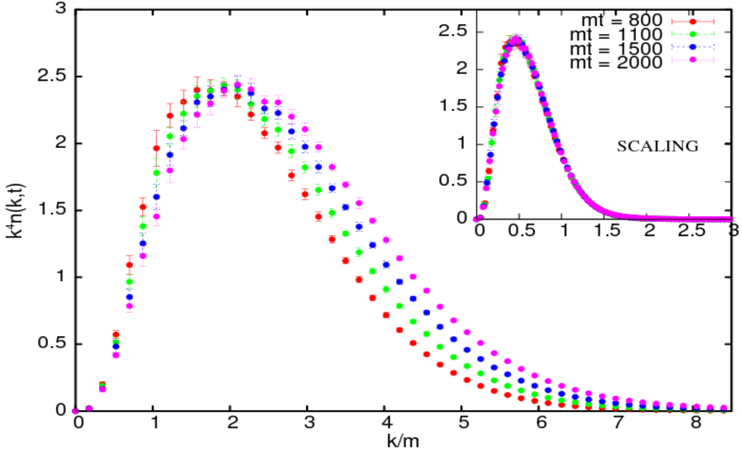


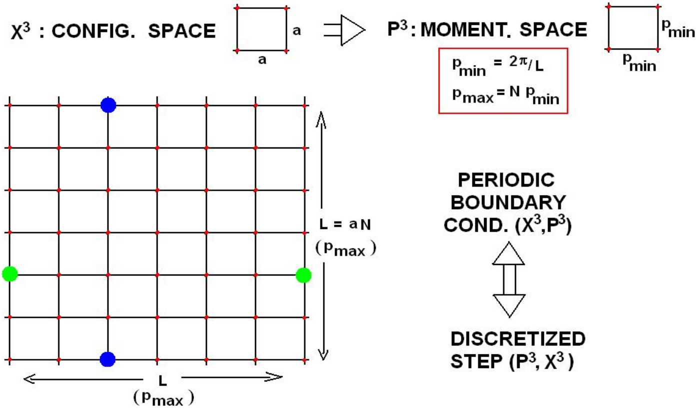
Back Slides

Lattice Techniques

Lattice Simulations: Numerics

$$\partial_\mu O(x) \to (O(x+\mu) - O(x-\mu))/2a_\mu$$

$$\partial_\mu \partial_\mu O(x) \to (O(x+2\mu) + O(x-2\mu) - 2O(x))/4a_\mu^2$$



GW extraction

Scalar Source (Configuration Space):

$$\ddot{h}_{ij}(\mathbf{x},t) + 3H\dot{h}_{ij}(\mathbf{x},t) - \tfrac{1}{a^2}\nabla^2 h_{ij}(\mathbf{x},t) = \tfrac{16\pi}{a^2}\ \mathrm{TT}\left\{\nabla_l \phi^a \nabla_m \phi^a\right\}(\mathbf{x},t)$$

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Scalar Source (Fourier):

$$\ddot{h}_{ij}(\mathbf{k},t) + 3H\dot{h}_{ij}(\mathbf{k},t) + \frac{k^2}{a^2}h_{ij}(\mathbf{k},t) = 16\pi\ \Lambda_{ij,lm}(\hat{\mathbf{k}})\left\{\nabla_l\phi^a\nabla_m\phi^a\right\}(\mathbf{k},t)$$

$$\Lambda_{ij,lm} = P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm}, \qquad P_{ij} = \delta_{ij} - k_ik_j/k^2$$

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$$\Lambda_{ij,lm} = P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm}, \qquad P_{ij} = \delta_{ij} - k_ik_j/k^2$$

Solution: $(h_{ij}(t_0) = \dot{h}_{ij}(t_0) = 0)$

$$h_{ij}(\mathbf{k},t) = \Lambda_{ij,lm}(\hat{\mathbf{k}})\int_{t_0}^{t} dt' G(t-t')\Pi^{\rm eff}_{lm}(\mathbf{k},t'), \qquad \Pi^{\rm eff}_{lm} = \nabla_l\phi\,\nabla_m\phi$$

GW extraction (II)

Building the Solution:

1) Non-Physical eq.:
$\ddot{u}_{ij}(\mathbf{x},t) + 3H\dot{u}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}u_{ij}(\mathbf{x},t) = 16\pi \left\{\phi^a{}_{,i}\, \phi^a{}_{,j}\right\}(\mathbf{x},t)$

2) Fourier transform: $u_{ij}(\mathbf{x},t) \rightarrow u_{ij}(\mathbf{k},t)$

3) Proyection: $h_{ij}(\mathbf{k},t) = \Lambda_{ij,lm}(\hat{\mathbf{k}})u_{lm}(\mathbf{k},t)$

GW extraction (II)

Outputs: $\rho_{GW} = \frac{1}{32\pi G}\frac{1}{L^3}\int d^3\mathbf{x}\,\dot{h}_{ij}\dot{h}_{ij} = \frac{1}{32\pi G}\frac{1}{L^3}\int d^3\mathbf{k}|\dot{h}_{ij}(t,\mathbf{k})|^2$

1) Total GW density:
$\rho_{GW} = \frac{1}{32\pi G L^3} \times \int k^2 dk \int d\Omega\, \Lambda_{ij,lm}(\hat{\mathbf{k}})\dot{u}_{ij}(t,\mathbf{k})\dot{u}^*_{lm}(t,\mathbf{k})$

2) Spectrum: $\frac{d\rho}{d\log k} = \frac{1}{8GL^3}k^3\left\langle \Lambda_{ij,lm}(\hat{\mathbf{k}})\dot{u}_{ij}(t,\mathbf{k})\dot{u}^*_{lm}(t,\mathbf{k})\right\rangle_{4\pi}$

3) Snapshots: $h_{ij}(t,\mathbf{x}) = (2\pi)^{-3/2}\int d^3k e^{-i\mathbf{kx}}\Lambda_{ij,lm}(\hat{\mathbf{k}})u_{lm}(t,\mathbf{k})$