

Abundances of Rare Objects and primordial non-Gaussianity

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IPMU = Institute for Physics and Mathematics of the Universe



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New building open in Jan 10

21 Full-time faculty members
40+ postdocs



OUTLINE

- Excursion set approach -- moving barrier in non-Gaussian models
- Doroshkevich's formula in local f_{nl} model
- First crossing problem -- path integral or Edgeworth expansion?
- Abundances of another extremes

Signatures of primordial non-Gaussianity on LSS

* Bispectrum

* Halo mass function

* Scale dependent halo bias

* peculiar velocity (redshift-space distortion)

(TYL, Desjacques & Sheth 2010; Schmdit 2010; TYL, Nishimichi & Yoshida 2010)

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Need a good description of the halo mass function

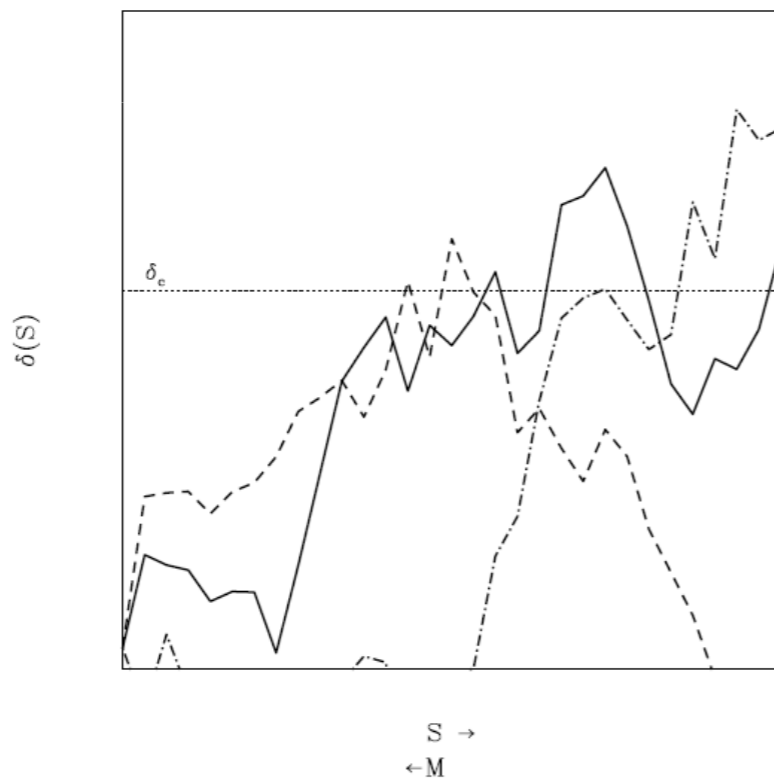
Excursion set approach

Excursion set theory

Bond, Cole, Efsthathiou and Kaiser (1991)

Peacock and Heavens (1990)

- study the evolution of $\delta(R)$ as a function of R
 - at $R=\infty$, $\delta(R)=0$. Lowering R , $\delta(R)$ evolves stochastically
 - use $S=\sigma^2(R)$ as “time”. At $R=\infty$, $S=0$. As R decreases, S increases



First-passage
time problem

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Assumption: halo formation depends only on the smoothed density field

Halo mass function and primordial non-Gaussianity

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OR

Full calculations of first crossing barrier based on path-integral (Maggiore & Riotto 2009+; D'Amico et al. 2010; De Simone et al. 2010) or bivariate Edgeworth expansion (TYL & Sheth 2009)

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ST mass function (moving barrier) describes the mass function from simulations (N-body, Gaussian initial conditions) better than the PS mass function (constant barrier).

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Distribution of ellipticity and prolateness

ST moving barrier: take the most probable values of e and p

$$B(\sigma) = \sqrt{a}\delta_c [1 + \beta(\sigma/\sqrt{a}\delta_c)^{2\gamma}] \quad \text{where } a=0.7, \beta=0.4, \gamma=0.6$$

Question: Is the moving barrier the same for non-Gaussian models?

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OR

$$g(e, p|\delta; f_{nl}) = g(e, p|\delta; f_{nl} = 0)?$$

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Define:

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$\{x, y, z, \Phi_{12}, \Phi_{23}, \Phi_{31}\}$ is an independent set of components of the shear tensor

Important: $\langle y^3 \rangle = \langle z^3 \rangle = \langle \Phi_{ij}^3 \rangle_{i \neq j} = 0$

up to 1st order of f_{nl} and local f_{nl} **ONLY**

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$$\begin{aligned}
 p(\vec{\lambda}) &= p(\vec{\lambda}|\delta_l)p(\delta_l) = p_0(\vec{\lambda}|\delta_l)p_0(\delta_l) \left[1 + \frac{\sigma S_3}{6} H_3(\nu) \right] \\
 &= p_0(\vec{\lambda}) \left[1 + \frac{\sigma S_3}{6} H_3(\nu) \right] \quad \text{TYL, Sheth \& Desjacques 2009}
 \end{aligned}$$

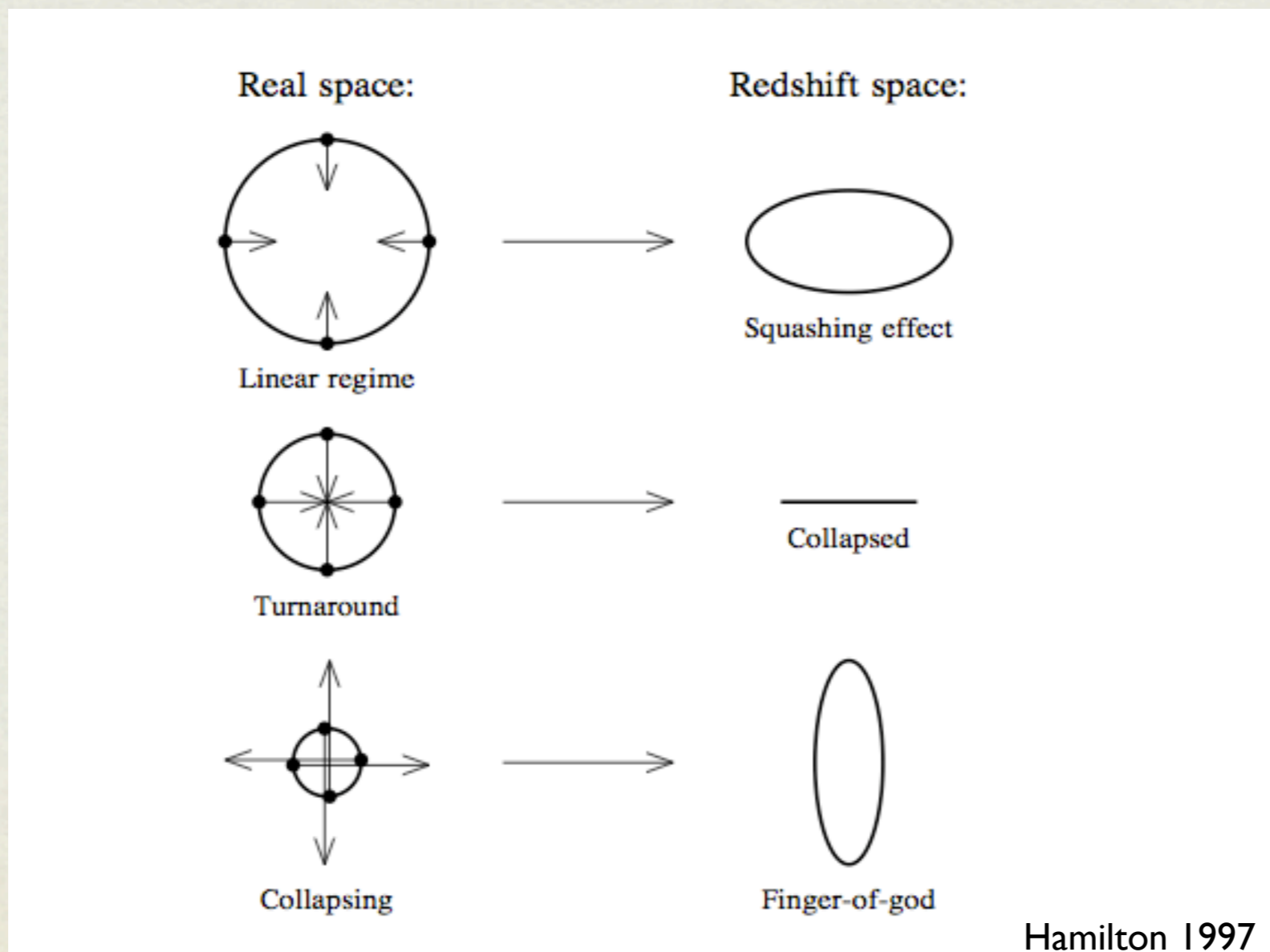
Generation of Doroshkevich's formula to the local fnl model

Applications

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- ▶ Redshift space distortion in f_{nl} model (TYL, Desjacques & Sheth 2010)

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- ✓ ellipsoidal collapse model gets the correct answer (Ohta et al. 2004; TYL & Sheth 2008)
- ✓ The Generalized Doroshkevich's formula in local f_{nl} type can apply to compute the effect on redshift space distortion (TYL, Desjacques & Sheth 2010)

□ The zeroth order gives the original Kaiser factor:

$$\langle \delta_s^2 \rangle \approx \langle (\delta_r^{(1)})^2 \rangle = \left(1 + \frac{2}{3} f_1 + \frac{1}{5} f_1^2 \right) \sigma^2$$

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□ The first order has the effect of f_{nl}

$$\langle \delta_s^2 \rangle^{(2)} = 2 \frac{\sigma S_3}{6} \sigma^3 \left[3\nu_2 + \left(\nu_2 + \frac{2}{3} \right) f_1 - \frac{44}{45} f_1^2 + \frac{4}{9} f_1^3 + \frac{\nu_2}{3} f_1 f_2 + \nu_2 f_2 \right]$$

$\propto f_{nl}$

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- first crossing probability across moving barrier (height depends on smoothing scale/mass)

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←
analogy to n-point path integral neglecting correlation walk

Iterative solution (TYL & Sheth 2009):

$$s f_0(s, b) = \left[b - s \frac{\partial b}{\partial s} \right] \frac{e^{-b^2/2s}}{\sqrt{2\pi s}} - \sum_{i=2}^{\infty} \frac{s^i}{i!} \frac{\partial^i b}{\partial s^i} \int_0^s dS f_0(S, B) \frac{e^{-(b-B)^2/2(s-S)}}{\sqrt{2\pi(s-S)}} (S/s - 1)^{i-1}$$

Path Integral (neglecting correlation) (De Simone et al. 2010):

$$\mathcal{F}^{(0)}(S) = \frac{B(S)}{\sqrt{2\pi S^{3/2}}} e^{-B^2(S)/(2S)}, \quad (68)$$

while the higher orders give

$$\mathcal{F}^{(a)}(S) = -\frac{B'(S)}{\sqrt{2\pi S}} e^{-B(S)^2/(2S)}, \quad (69)$$

$$\mathcal{F}^{(b)}(S) = \frac{B''(S)}{4\pi} \times \left\{ \sqrt{2\pi S} e^{-B(S)^2/(2S)} - \pi B(S) \text{Erfc} \left[\frac{B(S)}{\sqrt{2S}} \right] \right\}. \quad (70)$$

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New computation for new barrier.

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Monte-Carlo:

Easy for Gaussian initial conditions with no
correlation between steps.

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- d. Useful for halo merger, f_{nl} models
- e. Advantages: fast, no need to rerun for different barriers

Monte-Carlo simulation on correlated walk

Characteristic function:

$$\mathcal{M}(\vec{t}) = \exp \langle \exp(i\vec{t} \cdot \vec{x}) \rangle_c$$

\vec{x} is a N-dim vector, x_i is the height of the (uncorrelated) random walk at step i , whose variance is s_i

Its Fourier transform is the multivariate normal distribution

$$\mathcal{F}_g(\vec{x}) = \frac{1}{(2\pi)^{n/2} |M|} \exp\left(-\frac{1}{2|M|} x^T M^{-1} x\right)$$

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Write $\mathcal{M}(t) = \mathcal{G}(t, x) \mathcal{M}_g(t)$

correction due to correlated walk

$$\mathcal{G}(t, x) = \exp \left[\sum_{m < l} (\langle x_m x_l \rangle - \sigma_m / \sigma_l) (it_m)(it_l) + \sum_{m=0}^n \frac{\langle x_m^3 \rangle}{3!} (it_m^3) \right. \\ \left. + \sum_{m < l} \frac{\langle x_m^2 x_l \rangle}{2} (it_m)^2 (it_l) + \sum_{m < l < k} \langle x_m x_l x_k \rangle (it_m)(it_l)(it_k) + \dots \right]$$

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Fourier transform is derivative operator on $F_g(x)$

Expression of nth order derivative of F_g :

$$\frac{\partial^n}{\partial x_{i_1} \dots \partial x_{i_k}} \mathcal{F}_g(x) = (-1)^k \mathcal{F}_g(x) \times \text{correction}$$

Path-integral: compute the cumulative probability and
calculate first cross probability

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- i. Correlated steps with moving barrier
- ii. Moving barrier with other types of primordial non-Gaussianity
- iii. Another type of moving barrier

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- Void-in-cloud: two barriers problem δ_c and δ_v

Two barriers problem (TYL, Sheth & Desjacques)

Ordinary first crossing prob.

conditional first crossing prob.

$$\mathcal{F}(s, \delta_v, \delta_c) = f(s, \delta_v) - \int_0^s dS_1 \mathcal{F}(S_1, \delta_c, \delta_v) f(s, \delta_v | S_1, \delta_c)$$

first crossing prob. of δ_c at S , without crossing δ_v for all $S' < S$

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Swapping δ_v and δ_c  recurrence relation between F and f

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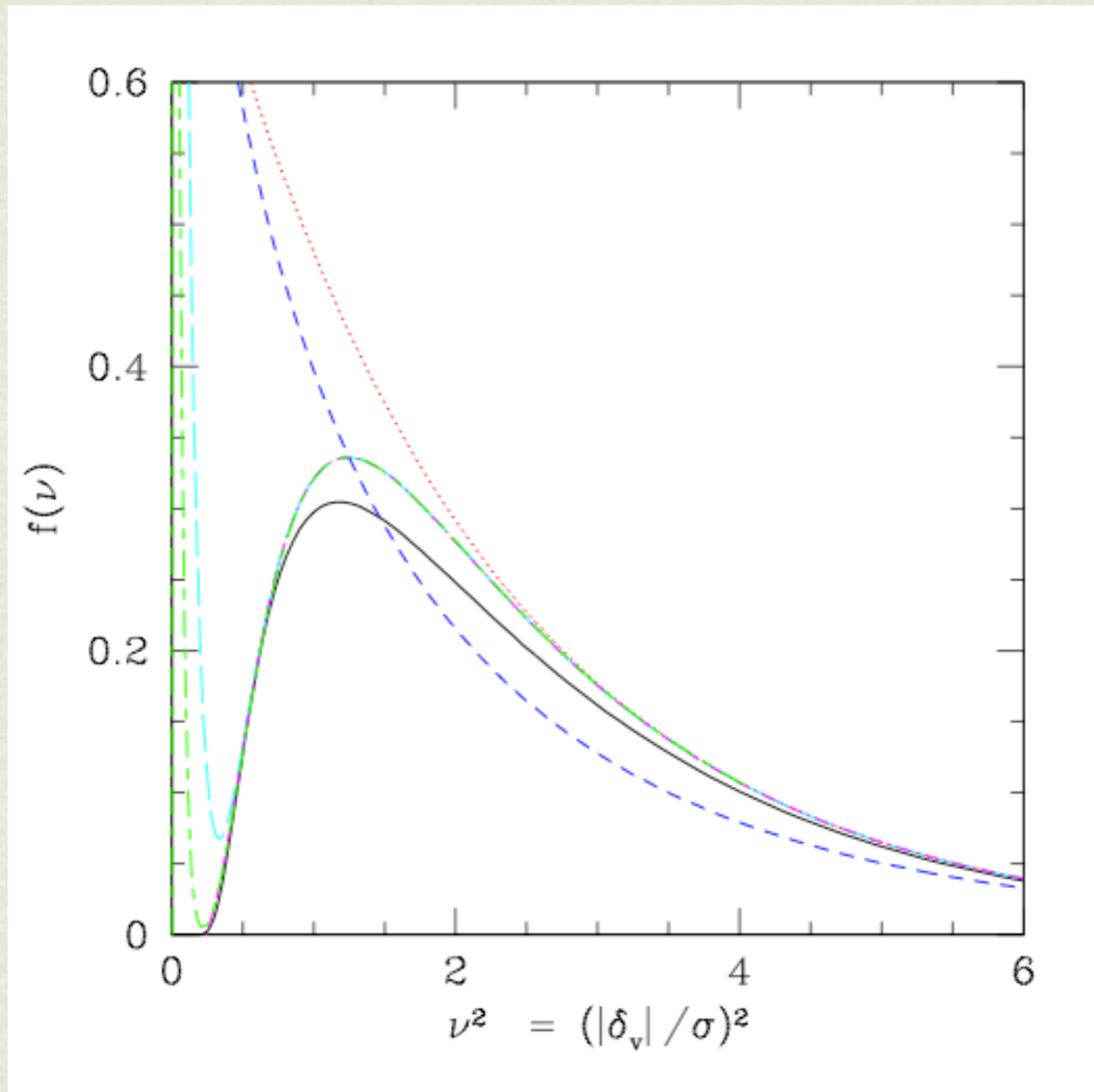
swapping δ_v and δ_c  recurrence relation between \mathcal{F} and f

$$\mathcal{F}(s, \delta_v, \delta_c) = f(s, \delta_v)$$

$$+ \sum_{n=1}^{\infty} (-1)^n \int_0^{S_0} dS_1 \dots \int_0^{S_{n-1}} dS_n \prod_{m=0}^{n-1} f(S_m, \delta_m | S_{m+1}, \delta_{m+1}) f(S_n, \delta_n)$$

where $S_0 \equiv s$, $\delta_n = \delta_v$ (n even) or δ_c (n odd)

Void abundances



(TYL, Sheth & Desjacques)

Effect of f_{nl}

$$f(\delta_n, s) = f_0(\delta_n, s) \left[1 + \frac{\sigma S_3}{6} H_3 \left(\frac{\delta_n}{\sigma} \right) \right]$$

$$f(s, \delta_v | S, \delta_c) = f_0(s, \delta_v | S, \delta_c) \left[1 + \frac{\sigma S_3}{6} \zeta(s, \delta_v, S, \delta_c) \right]$$

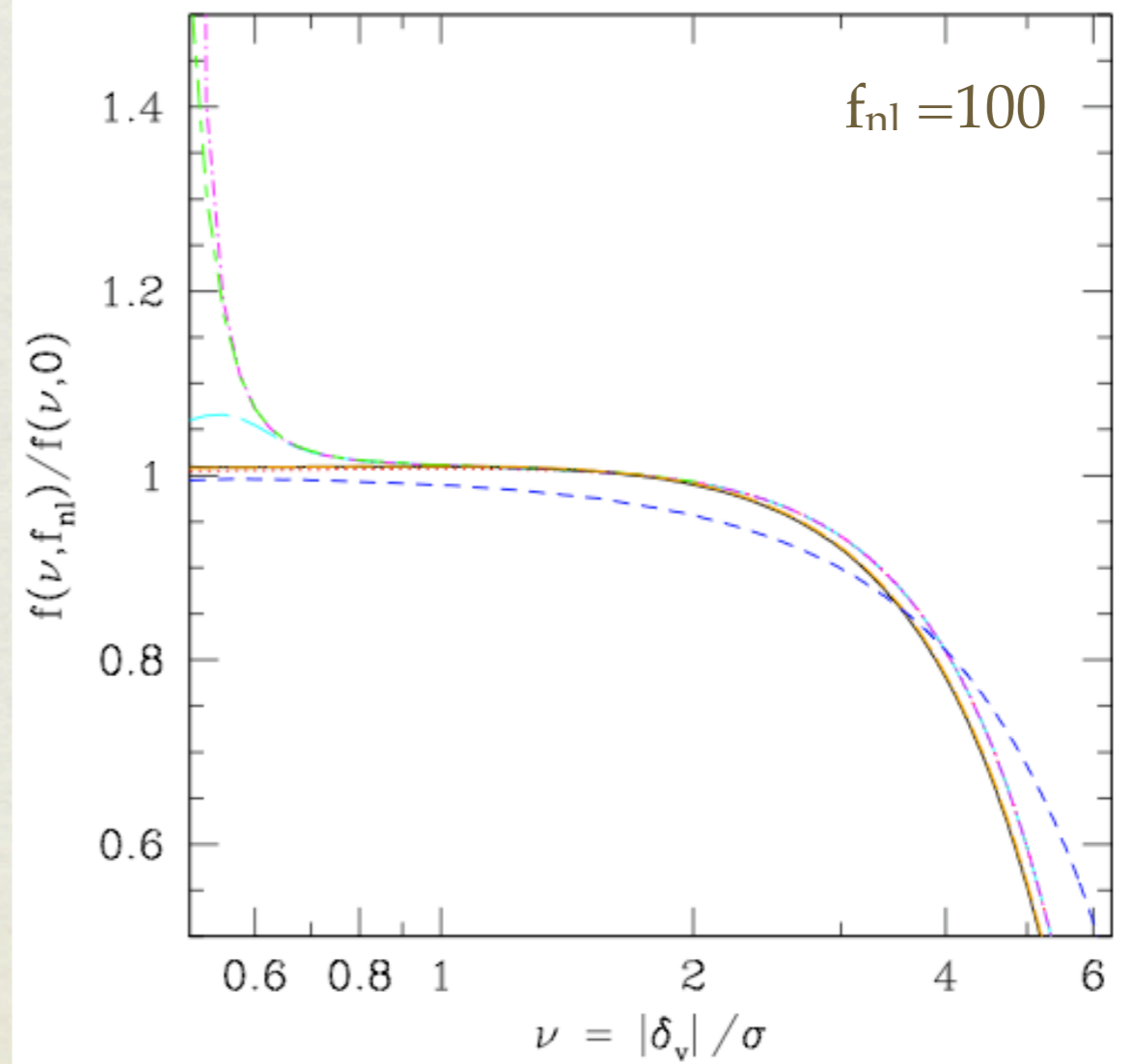
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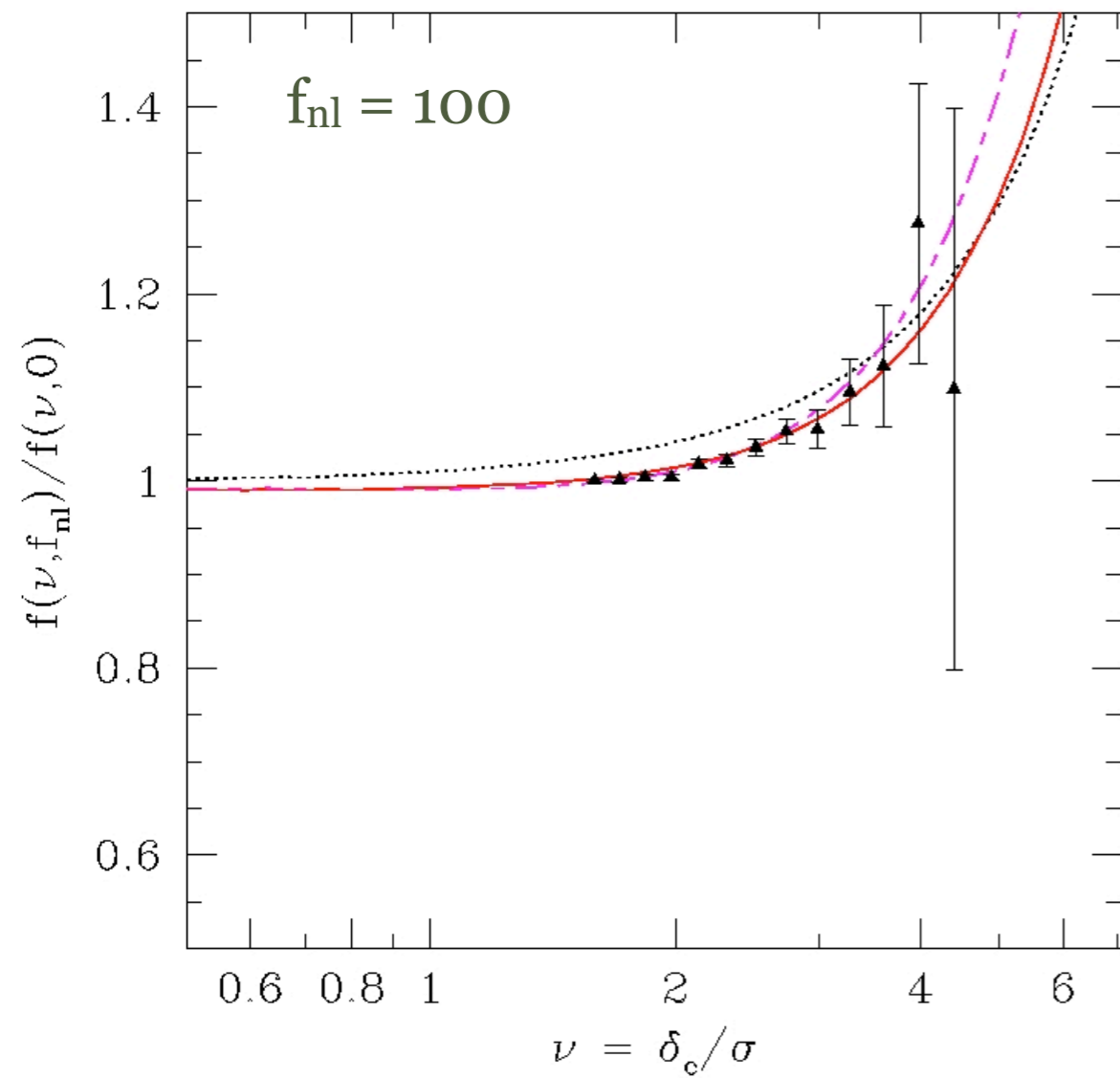
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$$\begin{aligned} \mathcal{F}(s, \delta_v, \delta_c) = & \mathcal{F}_0(s, \delta_v, \delta_c) + \frac{\sigma S_3}{6} \left\{ f_0(s, \delta_v) H_3 \left(\frac{\delta_v}{\sqrt{s}} \right) \right. \\ & \left. + \sum_{n=1}^{\infty} (-1)^n \int_0^{S_0} dS_1 \dots \int_0^{S_{n-1}} dS_n \left[\prod_{m=0}^{n-1} f_0(S_m, \delta_m | S_{m+1}, \delta_{m+1}) \right] f_0(S_n, \delta_n) \left[\sum_{m=0}^{n-1} \zeta(S_m, \delta_m, S_{m+1}, \delta_{m+1}) + H_3 \left(\frac{\delta_n}{\sqrt{S_n}} \right) \right] \right\}. \end{aligned} \quad (54)$$

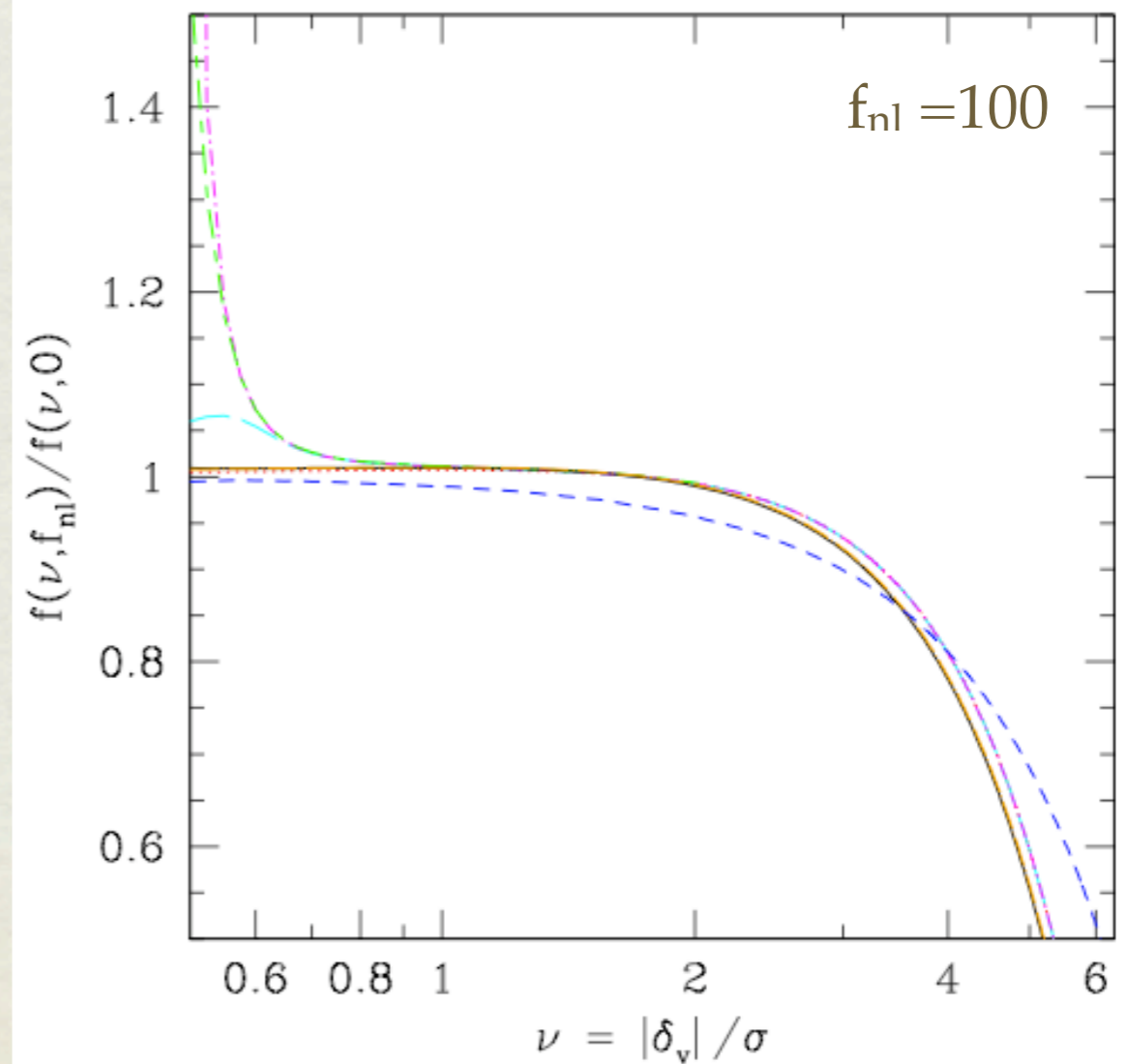
Void Abundance



Halo Abundance



Void Abundance



SUMMARY

- Excursion set theory two ingredients: barrier and first crossing probability
- Distribution of e and p does not change given δ for local type
- Moving barrier (ST02) is unchanged in local type; not necessarily true for other types
- First crossing probability for correlated walk and moving barrier is rough to solve analytically: but easy to implement Monte-Carlo simulations
- Void abundances is also sensitive to f_{n1} and the excursion set approach provides a method to solve the two-barrier problem

PECULIAR VELOCITY

- * peculiar velocity field & redshift space distortions also affected by primordial non-Gaussianity
- * The idea was proposed almost 20 years ago ([Scherrer 1992](#); [Catelan & Scherrer 1995](#); [Schmidt 2010](#)) -- all used linear theory
- * [TYL, Nishimichi & Naoki](#): Pairwise Vel PDF & evolution included
- * Non-vanishing three-point functions:
- * Induces correlation between velocities in parallel and perp directions

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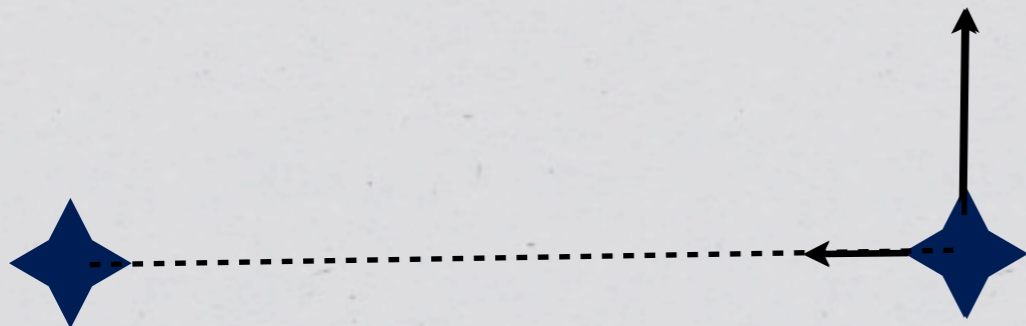
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- * Non-vanishing three-point functions: $\langle v_{\parallel}^3 \rangle$ and $\langle v_{\parallel} v_{\perp}^2 \rangle$
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Evolution of PDF

- * Analytical model to describe the evolution of the pairwise velocity PDF:

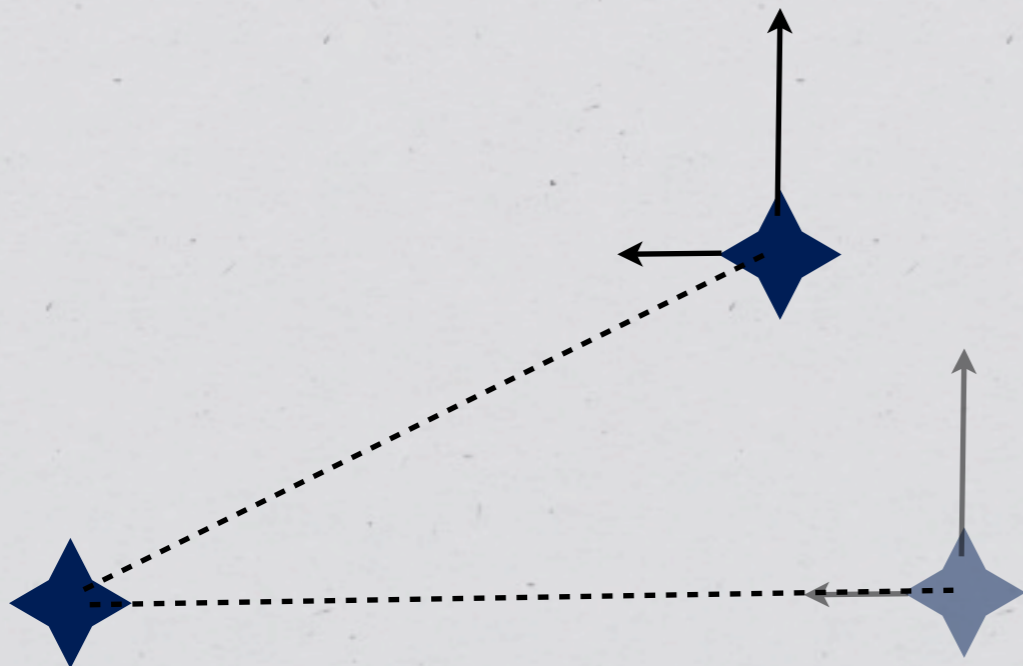
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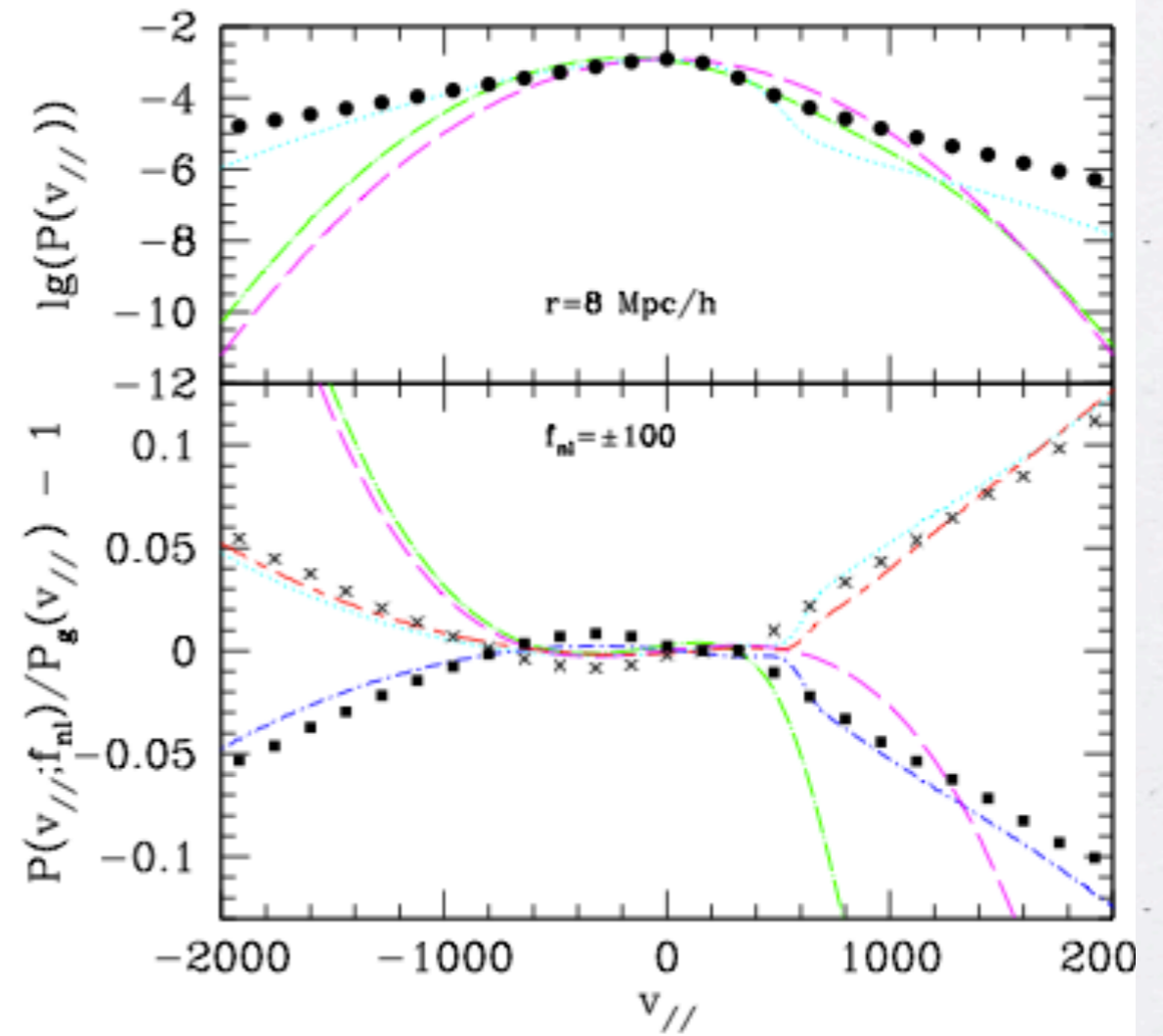
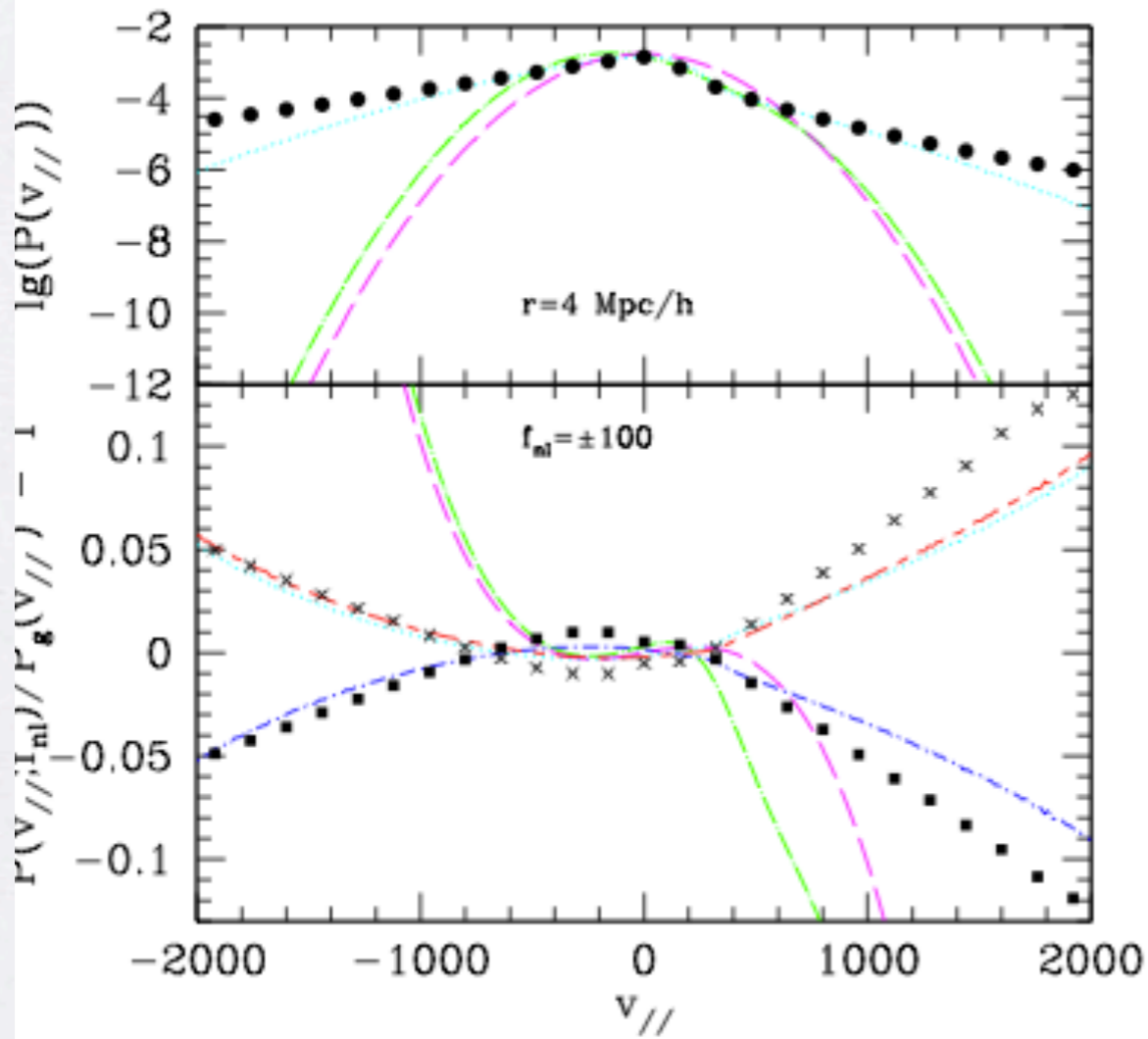
$$r^2 = \left(r_i + \frac{D_0}{\dot{D}_i} v_{\parallel}^i \right)^2 + \left(\frac{D_0}{\dot{D}_i} \right)^2 (v_{\perp a}^{i2} + v_{\perp b}^{i2})$$

$$v_{\parallel} = \frac{\dot{D}_0}{r} \left(\frac{r_i v_{\parallel}^i}{\dot{D}_i} + \frac{D_0}{\dot{D}_i^2} v^{i2} \right)$$

$$|v_{\perp}|^2 = v_{\perp a}^2 + v_{\perp b}^2 = \left(\frac{\dot{D}_0}{\dot{D}_i} v^i \right)^2 - v_{\parallel}^2$$

RESULTS

parallel to the line of separation direction



RESULTS

perpendicular to the line of separation direction

