Zero Discord , One Goblin and Two Demons

Aharon Brodutch

With the help and guidance of Daniel Terno

Benasque 2010

MACQUARIE UNIVERSITY
SYDNEY ~ AUSTRALIA

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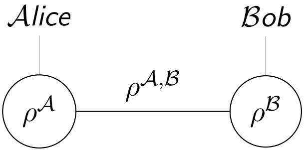
The "right" quantum measurement will make no change to the probability distribution, but the wrong one will make it more random

Bipartite systems



- In a multipartite (quantum) system some measurements require entanglement resources, and some are impossible.
- We will restrict ourselves to local measurements $\Pi^{\mathcal{A}/\mathcal{B}}$ or bi-local measurements $\Pi^{\mathcal{A}} \otimes \Pi^{\mathcal{B}}$
- The "best" measurement is one which commutes with the system. $[\Pi^{\mathcal{A}}, \rho^{\mathcal{AB}}]$ etc.. The same is true for each subsystem.

Discord

Π^A:

$$\rho^{\mathcal{AB}} \xrightarrow{\Pi^{\mathcal{A}}} \rho^{\mathcal{AB}}_{\Pi^{\mathcal{A}}}$$

- We can quantify the effect of a measurement by the change in some known quantity. Mutual information $I(\rho^{\mathcal{AB}})$ and Entropy $S(\rho^{\mathcal{AB}})$ are good candidates.
- Discord is the change in one of these quantities due to a measurement on one side: $\Pi^{\mathcal{A}}$
- To give more meaning it's better to optimize over the best measurement.

Types of discord

- $D_1^{\Pi^A}(\rho^{\mathcal{AB}}) = I(\rho^{\mathcal{AB}}) - I(\rho^{\mathcal{AB}}_{\Pi^{\mathcal{A}}})$ - The change in MI (Zurek 00)
- $D_2^{\Pi^A}(\rho^{\mathcal{AB}}) = S(\rho^{\mathcal{AB}}_{\Pi^{\mathcal{A}}}) - S(\rho^{\mathcal{AB}})$ -The change in entropy (Zurek 03)
- $D_3(\rho^{\mathcal{AB}}) = D_1^{\tilde{\Pi}^A}(\rho^{\mathcal{AB}}) = D_2^{\tilde{\Pi}^A}(\rho^{\mathcal{AB}})$; $[\tilde{\Pi}^{\mathcal{A}}, \rho^{\mathcal{A}}] = 0$
- Other types?
- When minimizing, all types vanish simultaneously! (Brodutch & Terno 10)

Calculating zero discord

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- Calculating discord requires optimization. **It's hard work**
- The marginals are diagonal in the zero discord basis.

$$\rho^{\mathcal{B}} = \begin{pmatrix} a_1 & 0 & \dots & 0 \\ 0 & a_2 & \dots & 0 \\ . & \dots & \dots & 0 \\ \vdots & & a_j & \\ 0 & \dots & \dots & a_d \end{pmatrix} \qquad (1)$$

Calculating zero discord

Given a density matrix $\rho^{\mathcal{AB}}$

1. Find the marginal $\rho^{\mathcal{B}} = tr_{\mathcal{A}}\rho^{\mathcal{AB}}$
2. Find the eigenstates $|j\rangle^{\mathcal{B}}$
3. Use the eigenstates to build a projector basis $\Pi_j^{\mathcal{B}} = |j\rangle^{\mathcal{B}} \langle j|$
4. Calculate the discord in this basis $\delta(\mathcal{A}:\mathcal{B})_{\{\Pi_j^{\mathcal{B}}\}} = H(\mathcal{B}) - H(\mathcal{A},\mathcal{B}) + H(\mathcal{A}|\{\Pi_j^{\mathcal{B}}\})$
5. If and only if $\delta(\mathcal{A}:\mathcal{B})_{\{\Pi_j^{\mathcal{B}}\}} = 0$, the state is a zero discord state.

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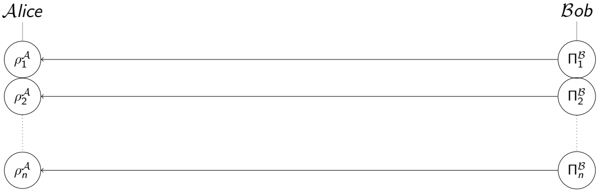
The zero discord basis

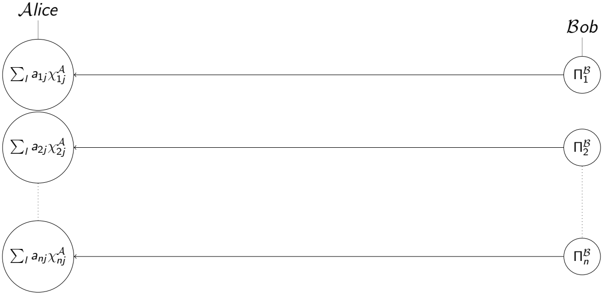
$$\rho_{\mathcal{A},\mathcal{B}} = \sum_j a_j \rho_j^{\mathcal{A}} \otimes \Pi_j^{\mathcal{B}}$$



The zero discord basis

$$\rho^{\mathcal{AB}} = \sum_i \sum_l C_{ij} \chi_{ij}^{\mathcal{A}} \otimes \Pi_i^{\mathcal{B}} \tag{2}$$

.



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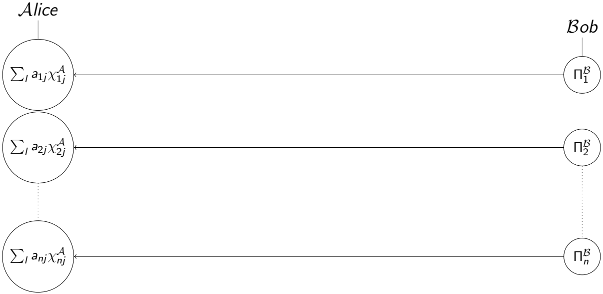

So $\rho^{\mathcal{AB}}$ is diagonal in the basis $\{\chi_{ij}^{\mathcal{A}} \otimes \Pi_i^{\mathcal{B}}\}_{ij}$

Double zero discord

Double zero discord: $\delta(\mathcal{A}:\mathcal{B}) = \delta(\mathcal{B}:\mathcal{A}) = 0$

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Locally diagonalizable $\Leftrightarrow$ Double zero discord

Locally distinguishable states

Non-locality without entanglement[1]

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Examples:

State	Discord	Distinguishability
Equal mixture of locally orthogonal states	DZero	Local
Equal mixture of the 9 states (NLWE)	DZero	Non-local
$\|00\rangle\langle 00\|$ and $\|{++}\rangle\langle{++}\|$	Non-zero	Impossible
A singlet and a $\|11\rangle$ state	Non-zero	Local

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But the eigenbasis of a zero discord density matrix $\{\chi_{il}^{\mathcal{A}} \otimes \Pi_i^{\mathcal{B}}\}_{il}$ defines a locally distinguishable set of states.

[1] Bennett et al (1999)

Maxwell's demon and the Szilard engine

- Maxwell's demon is given a d dimensional system with density matrix ρ.
- It makes a (non-degenerate orthogonal) measurement in the basis $\{\Pi_j\}$
- It then uses the pure state Π_i obtained to extract work $W = k_B T[log(d)]$.
- But since the measuring device now has entropy $H(\rho{:}\{\Pi_j\})$ the net gain is:

$$k_B T[log(d) - H(\rho : \{\Pi_j\})] \tag{3}$$

Set the stage for Alice and Bob

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The players

- Alice the demon - can implement a Szilard engine on her side.
- Bob the demon - can implement a Szilard engine on her side .
- Charlie the all-knowing goblin - knows the initial density matrix and can send information to both Alice and Bob.

[2] W.H.Zurek, (2003) , Horodecki et al (2005)

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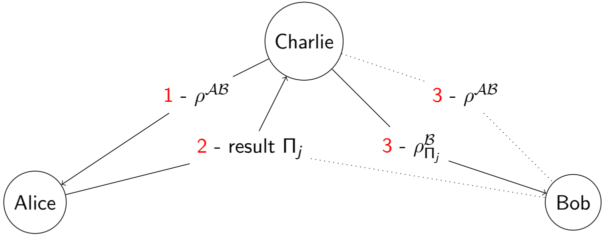
The rules

1. Zurek's rules[2] - Charlie sends the complete density matrix to Alice. Alice can send information to Charlie. Bob can only receive information from charlie.
2. Local rules - Charlie can only send local information to Alice or Bob. Alice can send information to Charlie.

[2] W.H.Zurek, (2003) , Horodecki et al (2005)

Alice and Bob start your engines!

Zurek's rules.

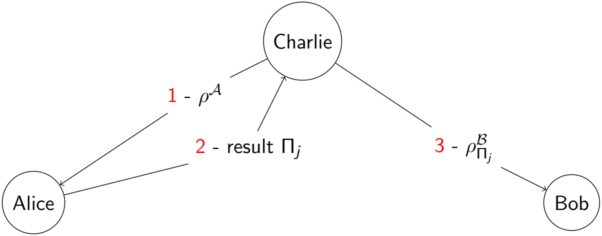


Using the best possible strategy

$$W_z = K_B T[log(d^2) - min_{\{\Pi_i^{\mathcal{A}}\}}[H(\mathcal{A}) + H(\mathcal{B}|\{\Pi_i^{\mathcal{A}}\})] \quad (4)$$

Alice and Bob start your engines!

Local Rules.



Alice's best strategy is to measure in the eigenbasis.

$$W_l = K_B T[log(d^2) - [H(\mathcal{A}) + H(\mathcal{B}|\{\Pi_i^{\mathcal{A}}\})] \quad (5)$$

with $\{\Pi_i^{\mathcal{A}}\}$ being the eigenbasis of $\rho^{\mathcal{A}}$.

Charlie the goblin is a non-local demon

If Charlie the goblin is a non-local demon he can use a non-local measurement and do more work.

$$W_{nl} = K_B T[log(d^2) - H(\mathcal{AB})]$$

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The difference between what work the non-local goblin and the two local demons can do is given by

$$\Delta_z = min_{\{\Pi_i^{\mathcal{A}}\}}[H(\mathcal{A}) + H(\mathcal{B}|\{\Pi_i^{\mathcal{A}}\})] - H(\mathcal{A},\mathcal{B})$$

or

$$\Delta_I = [H(\mathcal{A}) + H(\mathcal{B} : \{\Pi_i^{\mathcal{A}}\})] - H(\mathcal{AB}))]; \ \{\Pi_i^{\mathcal{A}}\} = \{eigenbasis(\rho^{\mathcal{A}})\}$$

The deficit

- $min_{\{\Pi^{\mathcal{A}}\}}[D_2^{\Pi^A}(\rho^{\mathcal{AB}})] = min_{\{\Pi^{\mathcal{A}}\}}[S(\rho^{\mathcal{AB}}_{\Pi^{\mathcal{A}}}) - S(\rho^{\mathcal{AB}})] = \Delta_z$
- $D_3(\rho^{\mathcal{AB}}) = D_2^{\tilde{\Pi}^A}(\rho^{\mathcal{AB}}) = \Delta_l$
- $[\tilde{\Pi}^{\mathcal{A}}, \rho^{\mathcal{A}}] = 0$
- Again zero discord implies a very local situation. Alice does not need any information about Bob's side to archive the optimal measurement.

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- $D_1 \leq D_2 \leq D_3$

Distributed quantum gates

Distributed quantum gates

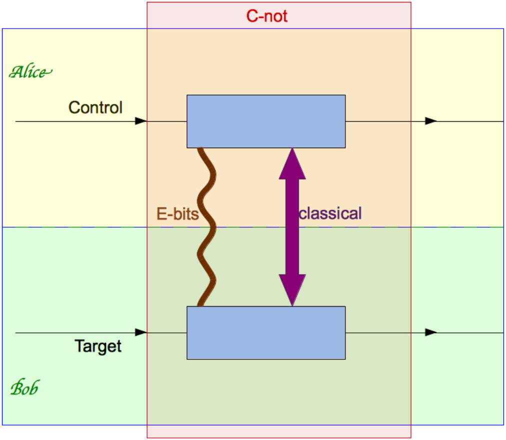
A CONTROLLED NOT GATE

Distributed quantum gates

A CONTROLLED NOT GATE
with discord at the input/output

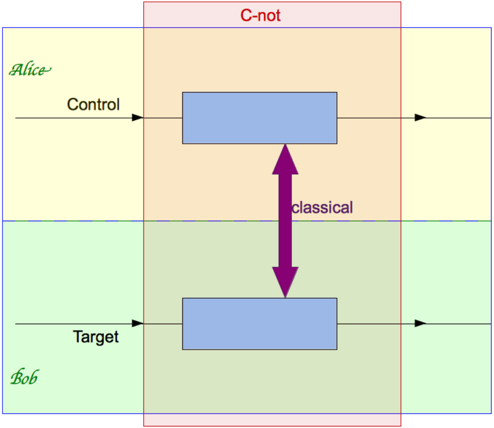
The classical c-not gate (control,target)	The quantum c-not gate (control,target)
Input $\rightarrow$ *output*	*Input* $\rightarrow$ *output*
$00 \rightarrow 00$	$\|0\psi\rangle \rightarrow \|0\psi\rangle$
$01 \rightarrow 01$	$\|X_+0\rangle \rightarrow [\|11\rangle + \|00\rangle]/\sqrt{2}$
$10 \rightarrow 11$	$\|\psi X_+\rangle \rightarrow \|\psi X_+\rangle$
$11 \rightarrow 10$	$\|1X_-\rangle \rightarrow \|0X_-\rangle$

The distributed c-not



2 classical bits and 1 e-bit

The distributed c-not



What can we do without entanglement?

We try to restrict the input to separable states.

$$|0\rangle |\psi\rangle \rightarrow |0\rangle |\psi\rangle$$
$$|1\rangle |\psi\rangle \rightarrow |1\rangle \sigma_x |\psi\rangle$$

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$$|\psi\rangle\,|X_+\rangle \rightarrow |\psi\rangle\,|X_+\rangle$$
$$|\psi\rangle\,|X_-\rangle \rightarrow \sigma_z\,|\psi\rangle\,|X_-\rangle$$

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$$|\psi\rangle |X_+\rangle \rightarrow |\psi\rangle |X_+\rangle$$
$$|\psi\rangle |X_-\rangle \rightarrow \sigma_z |\psi\rangle |X_-\rangle$$

#	State	#	State
a	$\|1\rangle\|Y_+\rangle \rightarrow \|1\rangle\|Y_-\rangle$	c	$\|Y_+\rangle\|X_-\rangle \rightarrow \|Y_-\rangle\|X_-\rangle$
b	$\|0\rangle\|Y_+\rangle \rightarrow \|0\rangle\|Y_+\rangle$	d	$\|Y_+\rangle\|X_+\rangle \rightarrow \|Y_+\rangle\|X_+\rangle$

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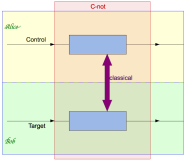

- In an LOCC protocol Alice and Bob can always know what operation they performed.
- The gate should either "flip" or "not flip" a Y_+state on either Alice's or Bob's side.
- But these operations are incompatible, so Alice and Bob can know what the input state was.

example

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a	$\|1\rangle\|Y_+\rangle \rightarrow \|1\rangle\|Y_-\rangle$
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Alice	Bob	
	F	N
F	$\left\{\begin{matrix} a & c \\ & \end{matrix}\right\}$	$\left\{\begin{matrix} & c \\ b & \end{matrix}\right\}$
N	$\left\{\begin{matrix} a & \\ & d \end{matrix}\right\}$	$\left\{\begin{matrix} & \\ b & d \end{matrix}\right\}$

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Alice	Bob	
	F	N
F	$\{a \quad c\}$	$\{c,\ b\}$
N	$\{a,\ d\}$	$\{b \quad d\}$

Alice and Bob now do the reverse operation to what (they know) they did

example

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F	$\{a \ \ c\}$	$\{c, b\}$
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Bob now performs a σ_y

example

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Alice	Bob: F	Bob: N
F	$\begin{Bmatrix} a & c \\ & \end{Bmatrix}$	$\begin{Bmatrix} & c \\ b & \end{Bmatrix}$
N	$\begin{Bmatrix} a & \\ & d \end{Bmatrix}$	$\begin{Bmatrix} & \\ b & d \end{Bmatrix}$

We are now assured that the next operation would be a "flip-flip"

example

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	F	N
F	$\left\{ a \quad c \right\}$	$\left\{ b \quad c \right\}$
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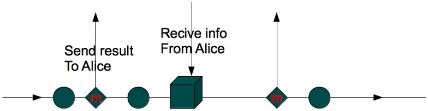
- A protocol which would allow Alice and Bob to implement this gate without entanglement will allow discrimination between the 4 non orthogonal states.
- We can see that a restricted version of the c-not gate with separable input-output states cannot be implemented using LOCC

- A mixture of the states used in the example would give non zero discord for either the input or output.
- A more general scheme can be used to show that a unitary operation which changes the discord (on both sides) cannot be implemented without entanglement. (?)
- Any quantum computation which involves changing the discord of states must have some (possibly hidden) entanglement as a resource. (?)

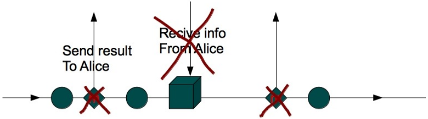
Gates and discord



What can Bob do?

- Perform some operation
- Make a measurement and send information to Alice
- Perform some operation which depends on information received from Alice

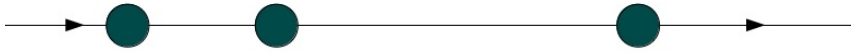
Gates and discord



What can Bob do?

- Perform some operation
- ~~Make a measurement and send information to Alice~~
- ~~Perform some operation which depends on information received from Alice~~

Gates and discord



What can Bob do?

- Perform some operation
- ~~Make a measurement and send information to Alice~~
- ~~Perform some operation which depends on information received from Alice~~

- Discord is a measure of how much a system is changed by a local measurement
- Zero discord is a unique property which can be easily verified.
- Different versions of discord relate to different types of non local advantage.
- Zero discord is common to all types.
- There is no simple relation between local distinguishability and discord.
- A restricted version of the c-not gate with discord inputs cannot be implemented using LOCC
- Unless both input and output ensembles have zero discord.

The end

Questions?