Quantum contextuality, bounded speed of information and bounded memory

Adán Cabello
University of Sevilla
Spain

Quantum Coherence and Decoherence,
Centro de Ciencias de Benasque "Pedro Pascual", Benasque,
September 10, 2010

Plan

- Contextuality
- Recent experiments on quantum contextuality
- “Macroscopic” quantum contextuality
- Quantum nonlocality via local contextuality
- Memory cost of quantum contextuality

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Physical compatibility

- <u>Compatible observables</u> are those which can be measured simultaneously [without disturbing each other].
- [In a sequential measurements scenario] <u>Compatible measurements</u> do not change previous results. [Operationally] Two consecutive measurements of A give the same result if what is measured in between is compatible with A [for any initial state].
- [In QM, compatibility = commutativity].

Contexts

- Contexts are sets of mutually compatible observables.
- The same observable can belong to different contexts:
  - A, B form context #1.
  - A, a form context #2.
  - B and a can be incompatible.

Contextuality (of results)

- A physical system is contextual when the result of a measurement depends on which *compatible* observables are measured, even though the probabilities do not (probabilities are noncontextual).

Contextuality and quantum contextuality

- Contextuality is a resource for information processing.
- Most types of contextuality can be classically simulated.
- Quantum nonlocality is an example of contextuality which cannot be classically simulated unless one permit arbitrarily fast signaling.
- But there are other types of quantum contextuality.

Kochen-Specker theorem

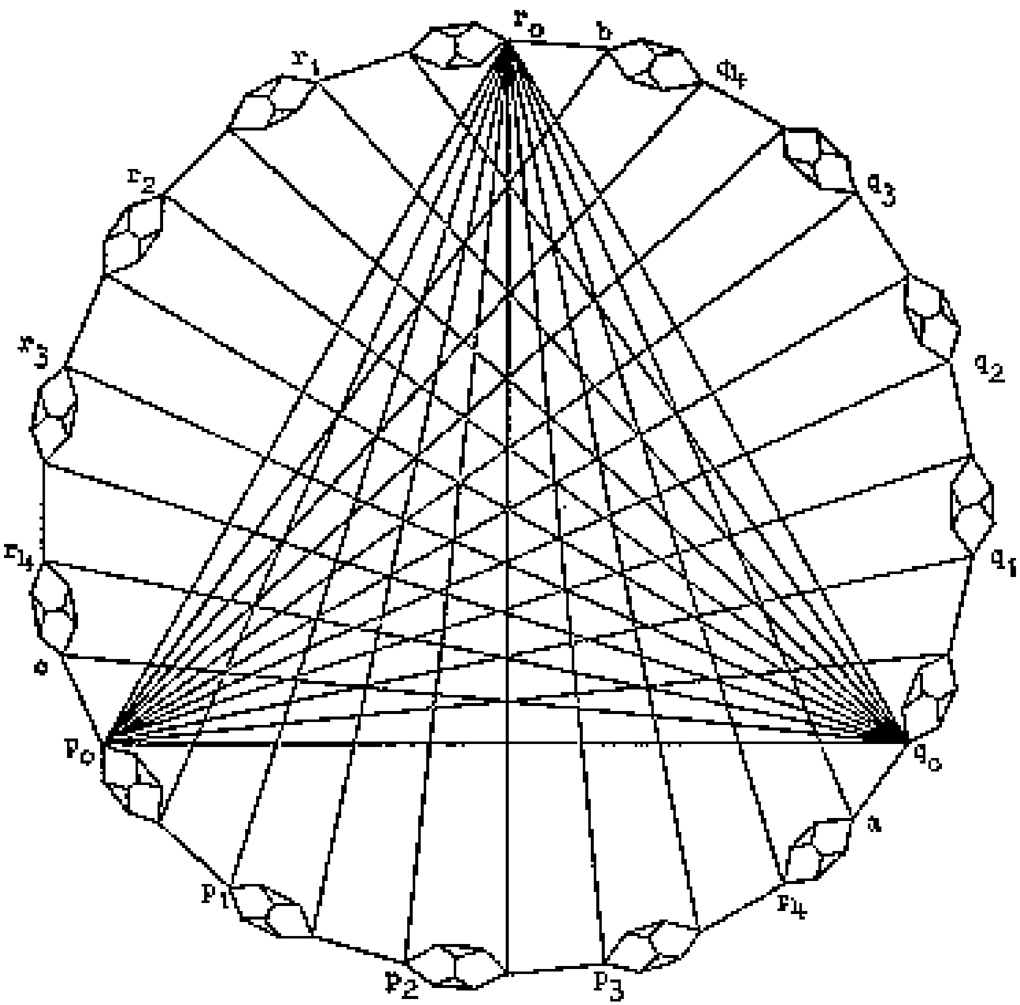
For <u>*any physical system*</u>, <u>*in any state*</u>, there exist a <u>*universal*</u> <u>*finite*</u> set of observables such that it is impossible to pre-assign them noncontextual results (i.e., independent of which other <u>*compatible*</u> observables are jointly measured) respecting the predictions of QM.

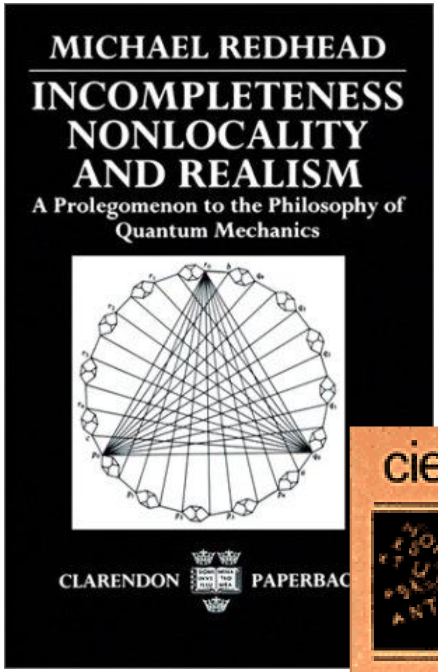
(any physical system in which observables can belong to more than one context, i.e., those represented in QM by a Hilbert space of dimension $d > 2$)

E. P. Specker, A. Specker, and S. Kochen, Zürich, early 1963.

S. Kochen and E.P. Specker, J. Math. Mech. **17**, 59 (1967).

The original 117-vector proof of the KS theorem



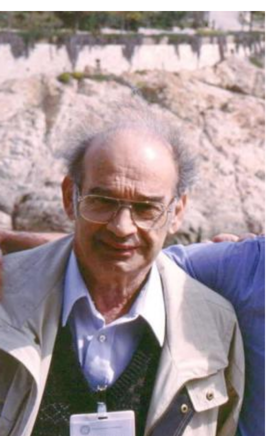


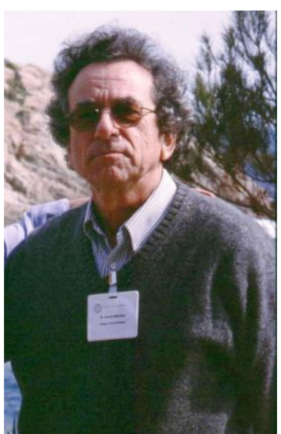
S. Kochen and E.P. Specker, J. Math. Mech. **17**, 59 (1967).

Peres-Mermin proof of the KS theorem

The proof is valid for any state of two qubits.

				$\prod$
	$X_1 X_2$	X_2	X_1	$= \mathbb{1}$
	$Y_1 Y_2$	$Z_1 X_2$	$X_1 Z_2$	$= \mathbb{1}$
	$Z_1 Z_2$	Z_1	Z_2	$= \mathbb{1}$
$\prod$	$= -\mathbb{1}$	$= \mathbb{1}$	$= \mathbb{1}$	





A. Peres, Phys. Lett. A **151**, 107 (1990).

N. D. Mermin, Phys. Rev. Lett. **65**, 3373 (1990).

A Kochen-Specker experiment?

- "The whole notion of an experimental test of KS misses the point" [N. D. Mermin, see Phys. Rev. Lett. **80**, 1797 (1998)].
- "How to test a contradiction?" (R. Clifton, private communication to K. Svozil).
- "The KS theorem, by its mathematical nature, is not empirically testable" [C. Held, in *Stanford Encyclopedia of Philosophy* (2006)].

Inequality for noncontextual theories

$$A, a, \alpha, B, b, \beta, C, c, \gamma \in \{-1, +1\}$$

$a\alpha + BC$	$ac + B\beta$	$\alpha\beta - Cc$
-2	± 2	0
-2	0	± 2
0	± 2	± 2
0	0	0
$+2$	± 2	0
$+2$	0	± 2

$$A(a\alpha + BC) + b(ac + B\beta) + \gamma(\alpha\beta - Cc) \in \{-4, 0, +4\}$$

$$\boxed{\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4}$$

A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008).

State-independent violation

$$\boxed{\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4}$$

$$A = \sigma_z^{(1)}, \quad B = \sigma_z^{(2)}, \quad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)},$$
$$a = \sigma_x^{(2)}, \quad b = \sigma_x^{(1)}, \quad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)},$$
$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)}, \quad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)}, \quad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}.$$

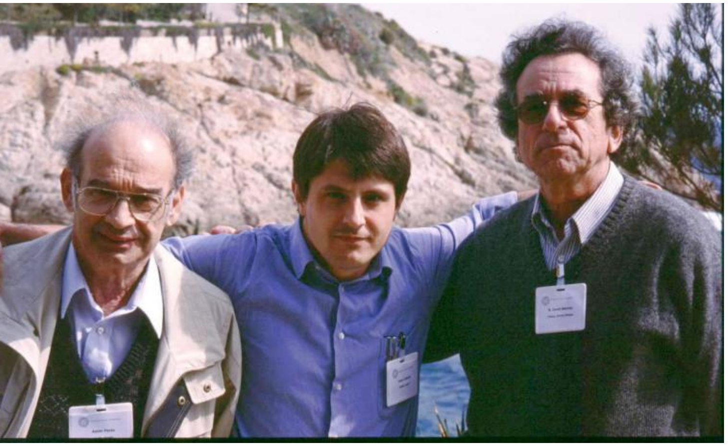


Photo by Enrique Rico, 2002.

$$S_{\mathrm{QM}} = 6$$

for any state!!!

A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008).

Universality

For <u>any</u> physical system there exist a noncontextual inequality violated by <u>any</u> state.

(any physical system which admits a nontrivial noncontextual description, i.e., represented by a Hilbert space of dimension $d > 2$)

P. Badziąg, I. Bengtsson, A. Cabello, and I. Pitowsky, Phys. Rev. Lett. **103**, 050401 (2009).

State-independent quantum contextuality for <u>continuous variables</u>.

R. Plastino and A. Cabello, Phys. Rev. A **82**, 022114 (2010).

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From theorems to experiments

1967: A conflict between two different descriptions of the world: QM and noncontextual hidden-variable theories.

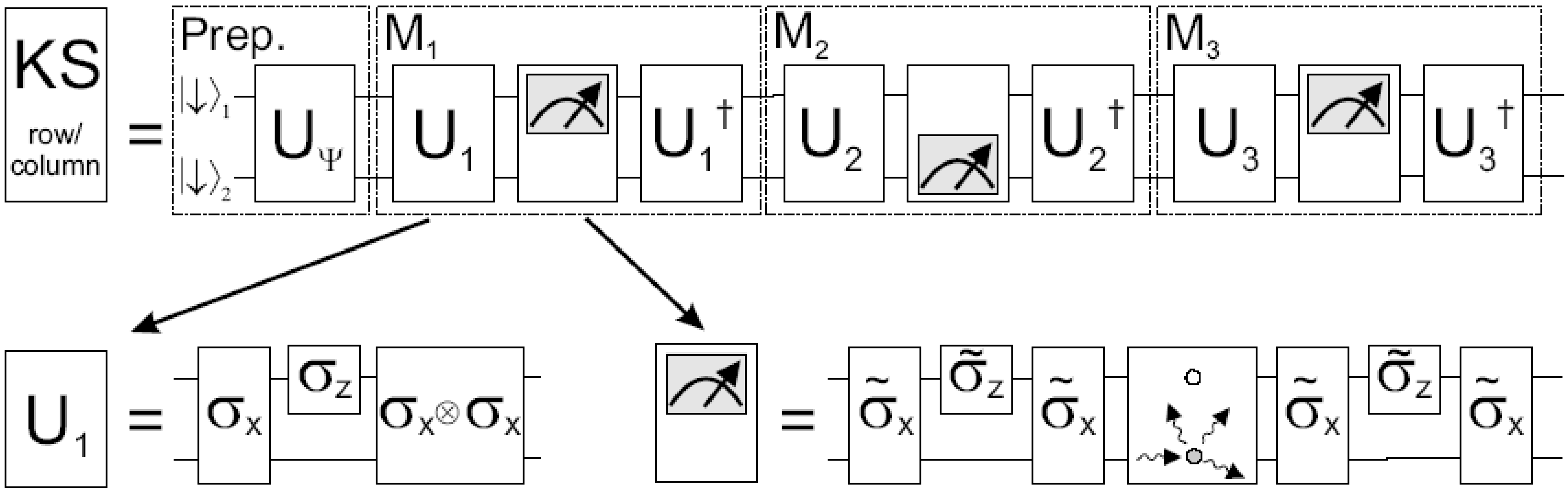
2008: A tool to test whether state-independent contextuality is a property of nature.

Sequential measurements?

- “Repeatable tests [i.e., measurements like *A*, *A*... on the same system] exist mostly in the imagination of theorists”.

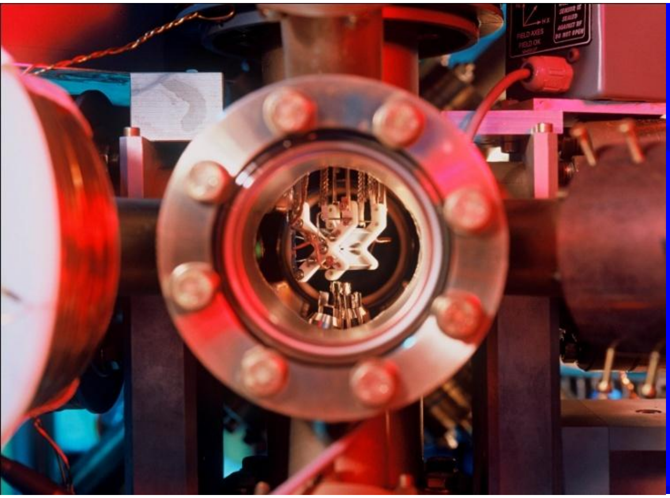
A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, 1993), p. 29.

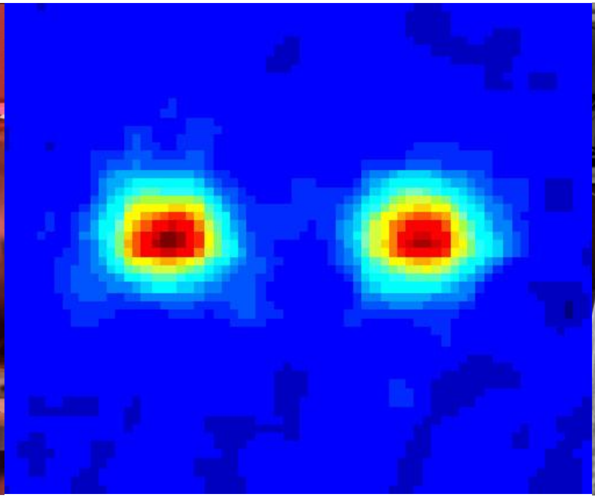
Measuring 3 observables sequentially on two ions

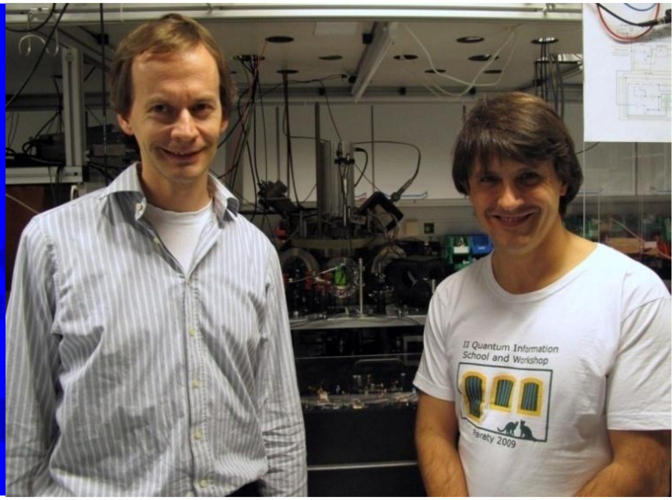


G. Kirchmair *et al.*, Nature (London) **460**, 494 (2009).

Innsbruck KS experiment with two Ca ions



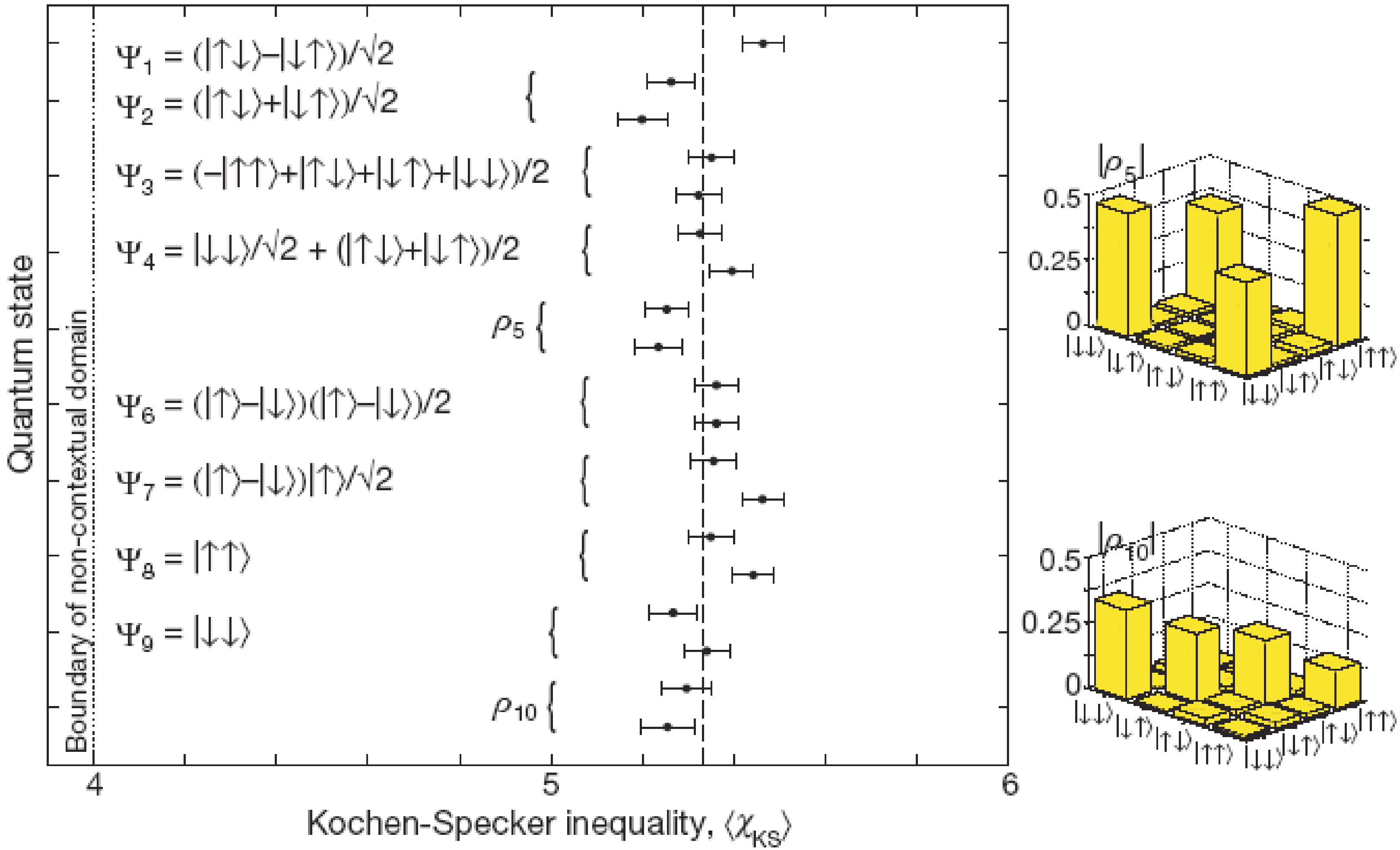




G. Kirchmair F. Zähringer R. Gerritsma M. Kleinmann O. Gühne R. Blatt C. Roos

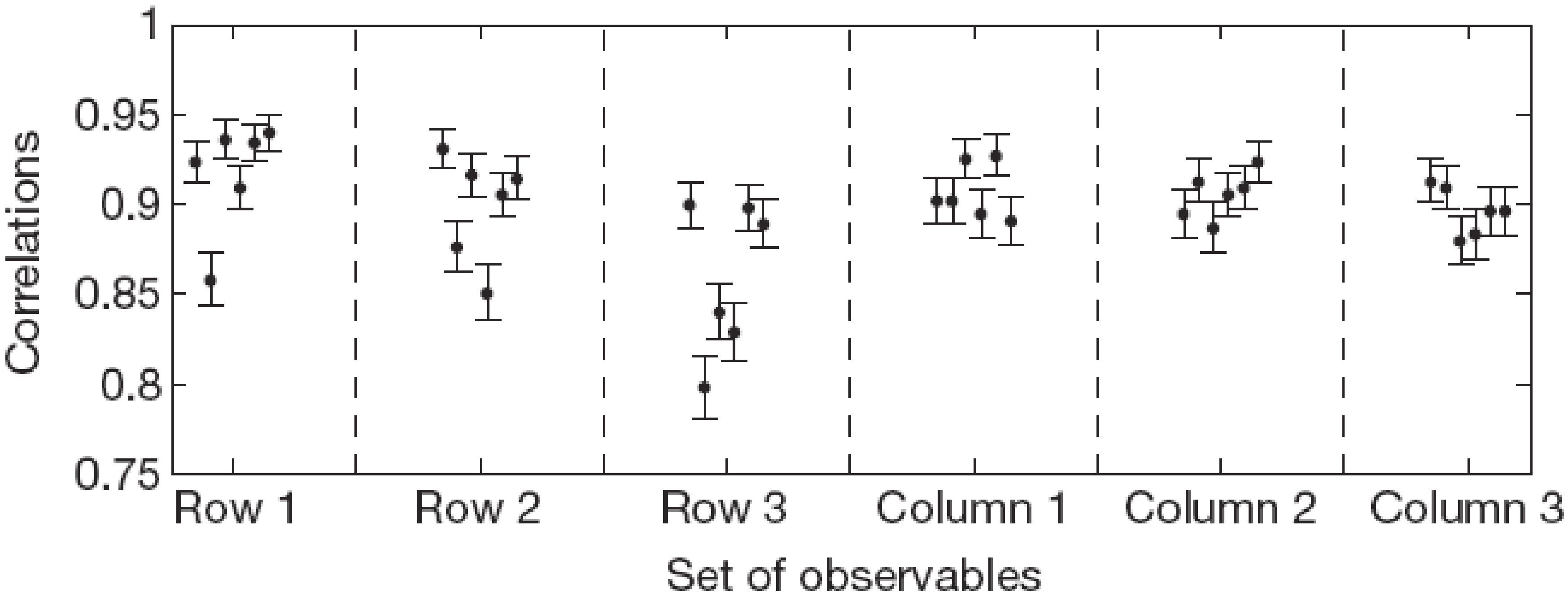
G. Kirchmair *et al.*, Nature (London) **460**, 494 (2009).

Experimental state-independent violation



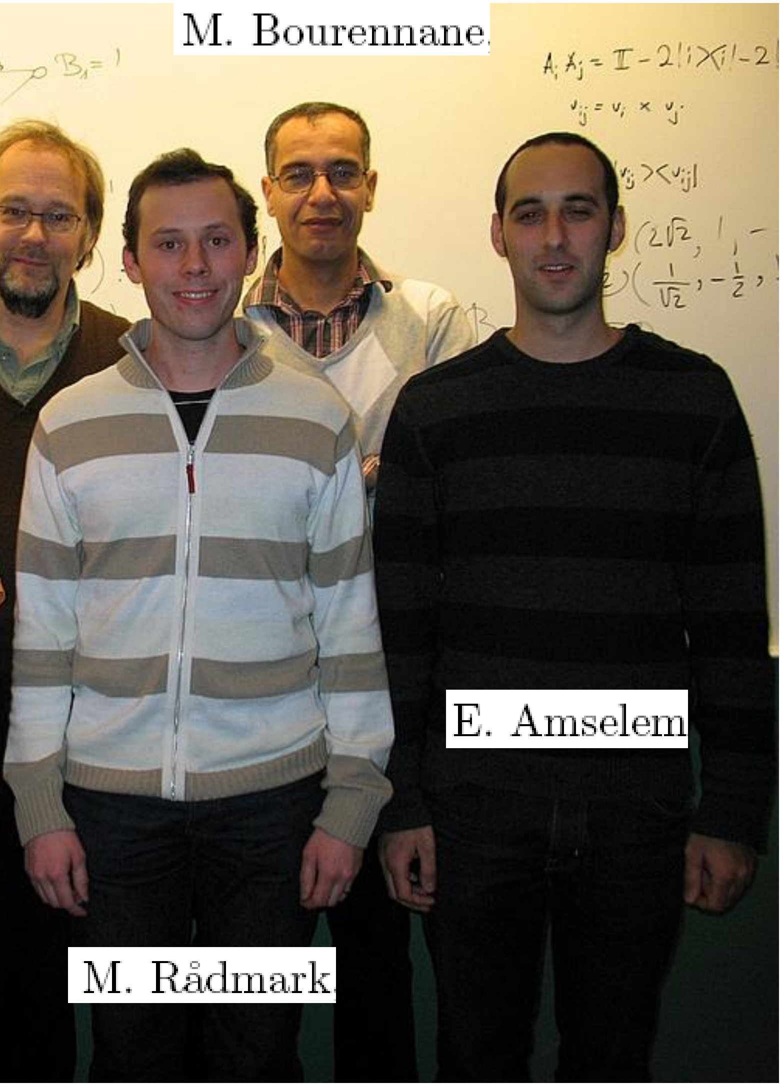
G. Kirchmair *et al.*, Nature (London) **460**, 494 (2009).

Temporal order (*ABC*, *ACB*, *BAC*…) does not matter



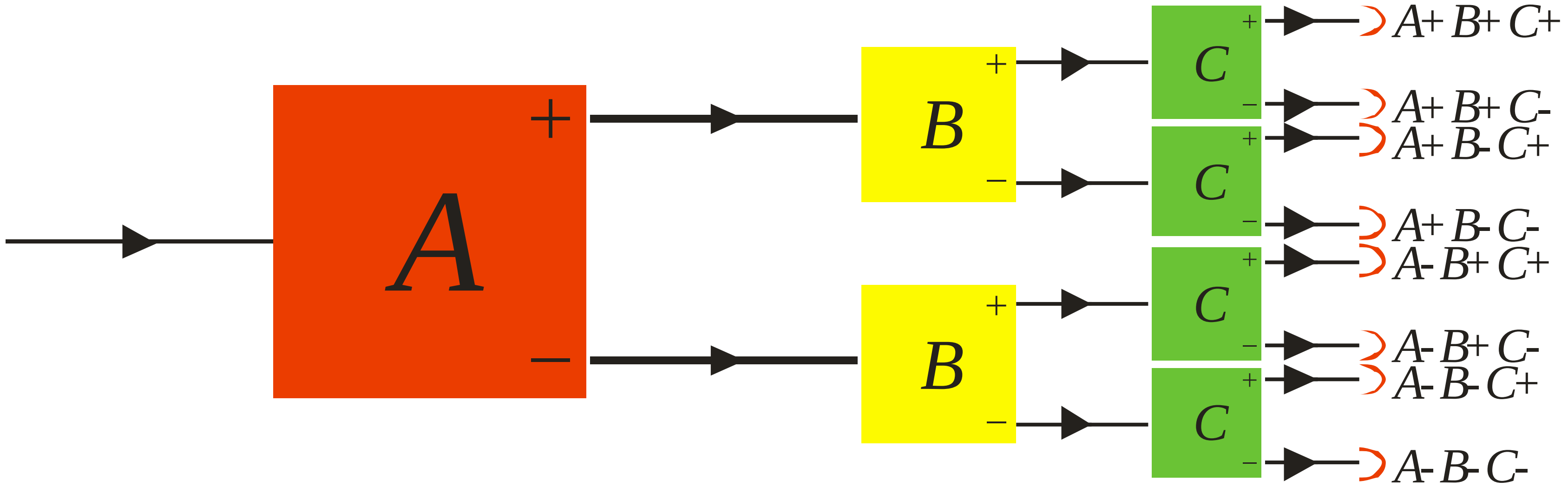
G. Kirchmair *et al.*, Nature (London) **460**, 494 (2009).

Stockholm KS experiment with single photons



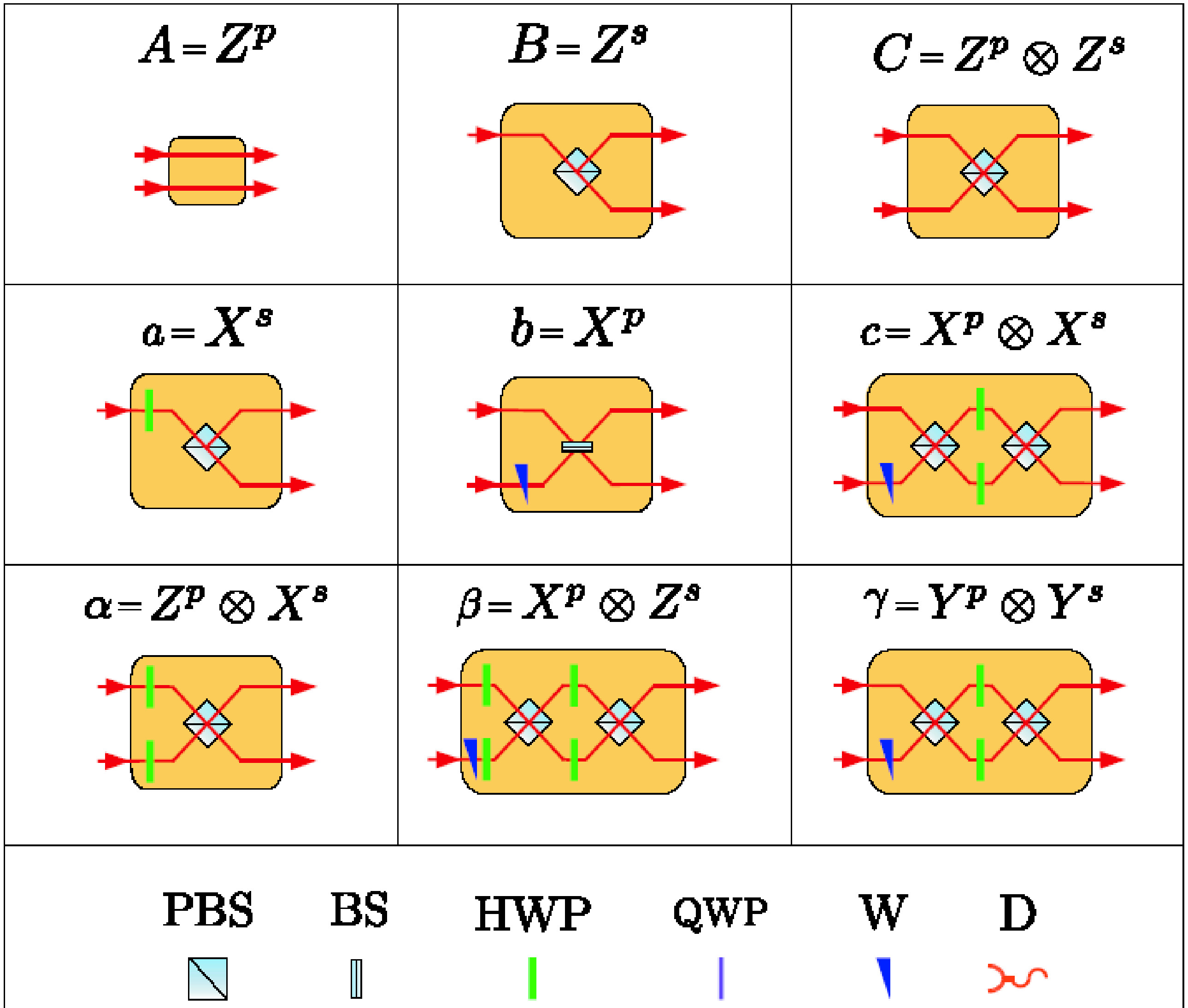
Measuring 3 observables sequentially on one photon

$$\boxed{\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4}$$

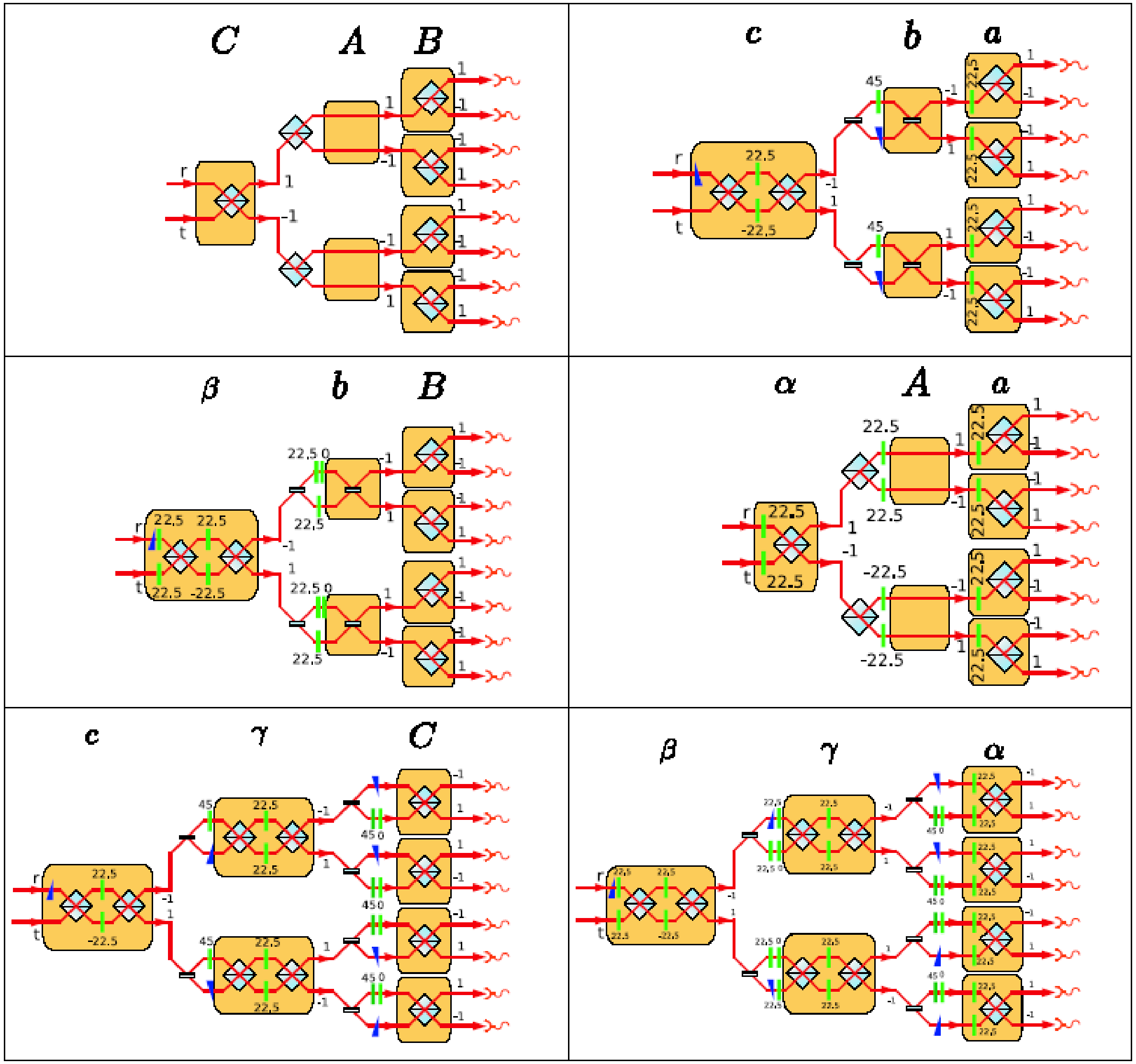


E. Amselem, M. Rådmark, M. Bourennane, and A. Cabello, Phys. Rev. Lett. **103**, 160405 (2009).

The 9 observables



The 6 contexts



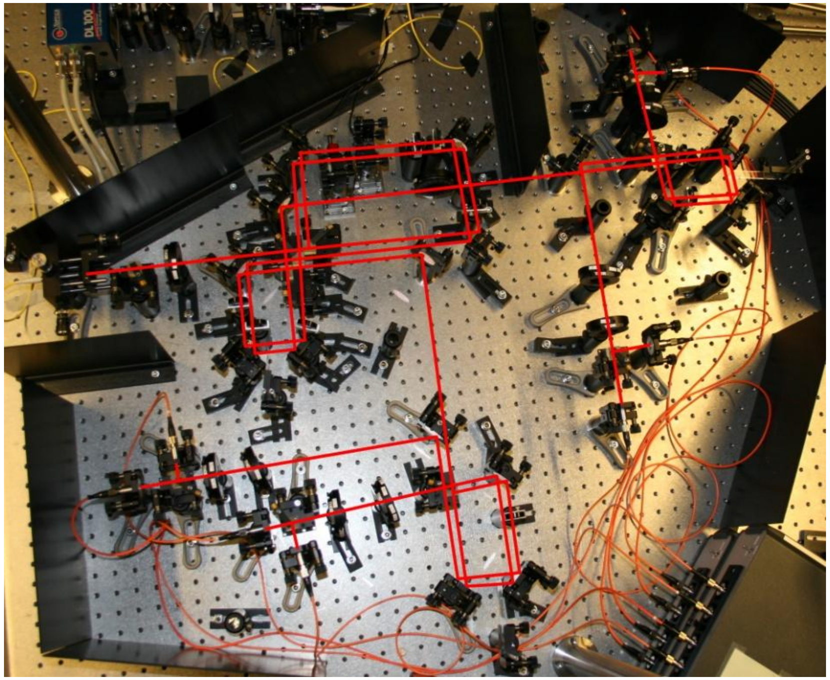
Stockholm KS experiment with single photons

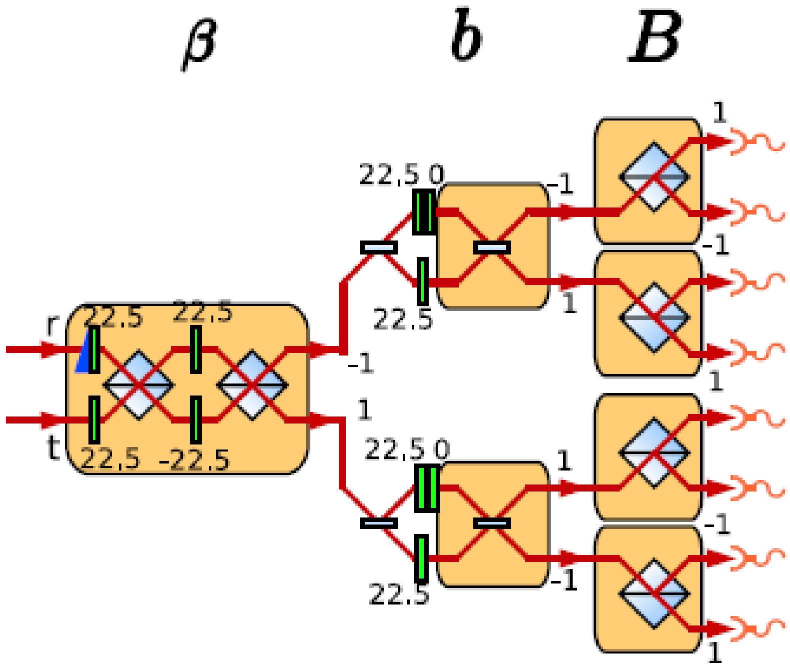
$$\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4$$

$$A = \sigma_z^{(1)}, \quad B = \sigma_z^{(2)}, \quad C = \sigma_z^{(1)} \otimes \sigma_z^{(2)},$$

$$a = \sigma_x^{(2)}, \quad b = \sigma_x^{(1)}, \quad c = \sigma_x^{(1)} \otimes \sigma_x^{(2)},$$

$$\alpha = \sigma_z^{(1)} \otimes \sigma_x^{(2)}, \quad \beta = \sigma_x^{(1)} \otimes \sigma_z^{(2)}, \quad \gamma = \sigma_y^{(1)} \otimes \sigma_y^{(2)}.$$

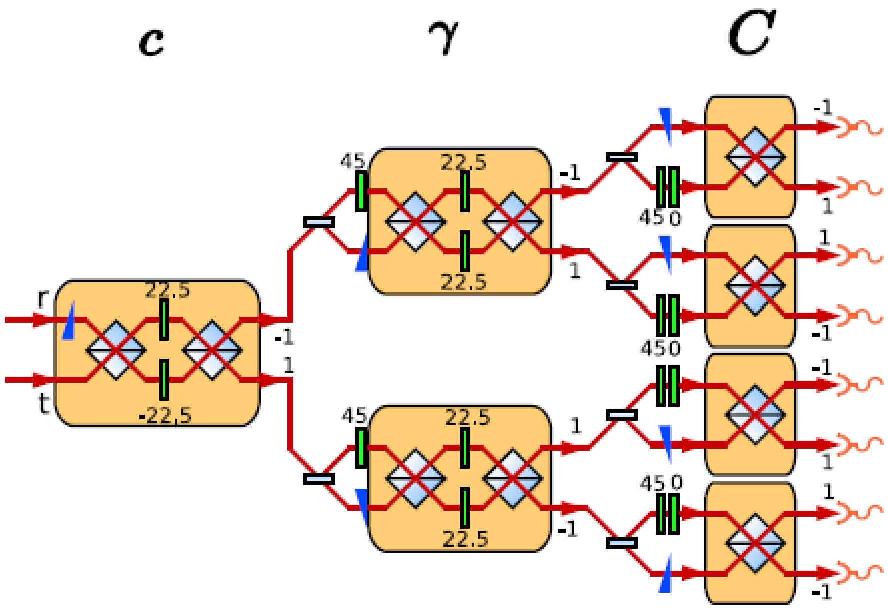




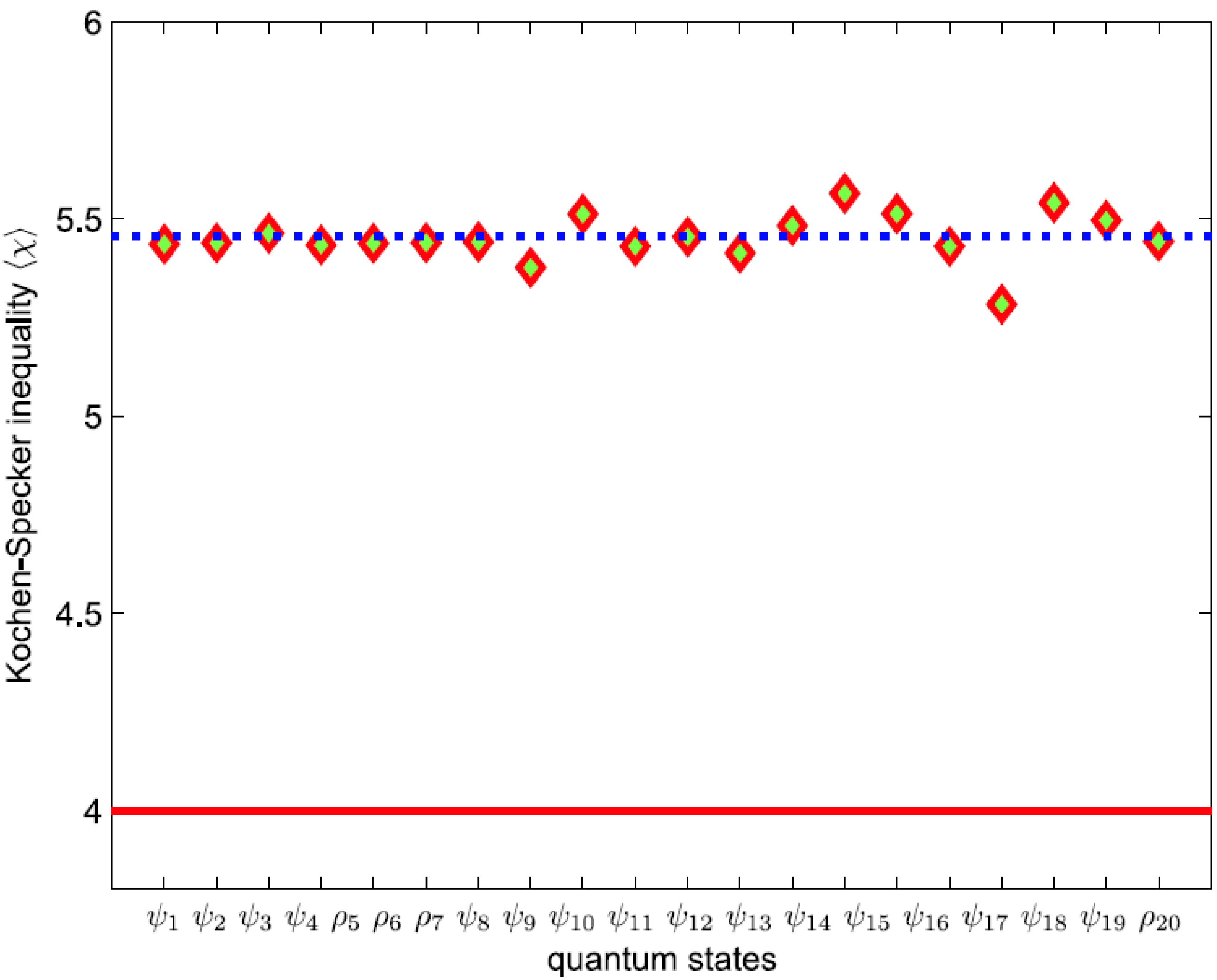
Stockholm KS experiment with single photons

$$\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4$$

$$\begin{aligned} A &= \sigma_z^{(1)}, & B &= \sigma_z^{(2)}, & C &= \sigma_z^{(1)} \otimes \sigma_z^{(2)}, \\ a &= \sigma_x^{(2)}, & b &= \sigma_x^{(1)}, & c &= \sigma_x^{(1)} \otimes \sigma_x^{(2)}, \\ \alpha &= \sigma_z^{(1)} \otimes \sigma_x^{(2)}, & \beta &= \sigma_x^{(1)} \otimes \sigma_z^{(2)}, & \gamma &= \sigma_y^{(1)} \otimes \sigma_y^{(2)}. \end{aligned}$$



State-independent contextuality for single photons

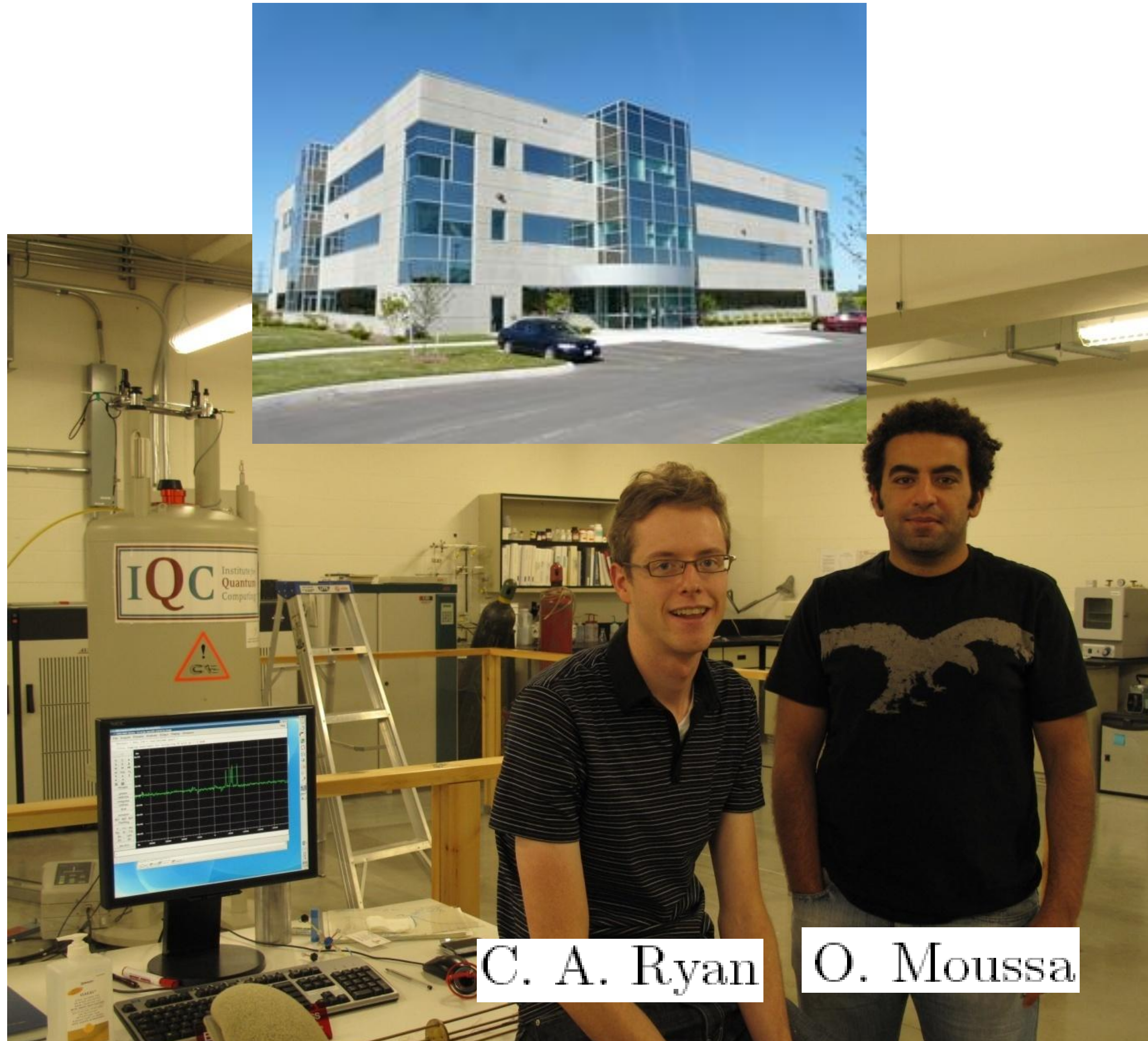


$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|t\rangle|H\rangle + |r\rangle|V\rangle)$$
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|t\rangle|H\rangle - |r\rangle|V\rangle)$$
$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|t\rangle|V\rangle + |r\rangle|H\rangle)$$
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|t\rangle|V\rangle - |r\rangle|H\rangle)$$
$$\rho_5 = \frac{13}{16}|\psi_1\rangle\langle\psi_1| + \frac{1}{16}\sum_{j=2}^{4}|\psi_j\rangle\langle\psi_j$$
$$\rho_6 = \frac{5}{8}|\psi_1\rangle\langle\psi_1| + \frac{1}{8}\sum_{j=2}^{4}|\psi_j\rangle\langle\psi_j|$$
$$\rho_7 = \frac{7}{16}|\psi_1\rangle\langle\psi_1| + \frac{3}{16}\sum_{j=2}^{4}|\psi_j\rangle\langle\psi_j$$
$$|\psi_8\rangle = |t\rangle|H\rangle$$
$$|\psi_9\rangle = |t\rangle|V\rangle$$
$$|\psi_{10}\rangle = |r\rangle|H\rangle$$
$$|\psi_{11}\rangle = |r\rangle|V\rangle$$
$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}}|t\rangle(|H\rangle + |V\rangle)$$
$$|\psi_{13}\rangle = \frac{1}{\sqrt{2}}|t\rangle(|H\rangle + i|V\rangle)$$
$$|\psi_{14}\rangle = \frac{1}{\sqrt{2}}(|t\rangle + |r\rangle)|H\rangle$$
$$|\psi_{15}\rangle = \frac{1}{\sqrt{2}}(|t\rangle + i|r\rangle)|H\rangle$$
$$|\psi_{16}\rangle = \frac{1}{2}(|t\rangle + |r\rangle)(|H\rangle + |V\rangle)$$
$$|\psi_{17}\rangle = \frac{1}{2}(|t\rangle + i|r\rangle)(|H\rangle + |V\rangle)$$
$$|\psi_{18}\rangle = \frac{1}{2}(|t\rangle + |r\rangle)(|H\rangle + i|V\rangle)$$
$$|\psi_{19}\rangle = \frac{1}{2}(|t\rangle + i|r\rangle)(|H\rangle + i|V\rangle)$$
$$\rho_{20} = \frac{1}{4}\sum_{j=1}^{4}|\psi_j\rangle\langle\psi_j|$$

Waterloo KS experiment with NMR

R. Laflamme

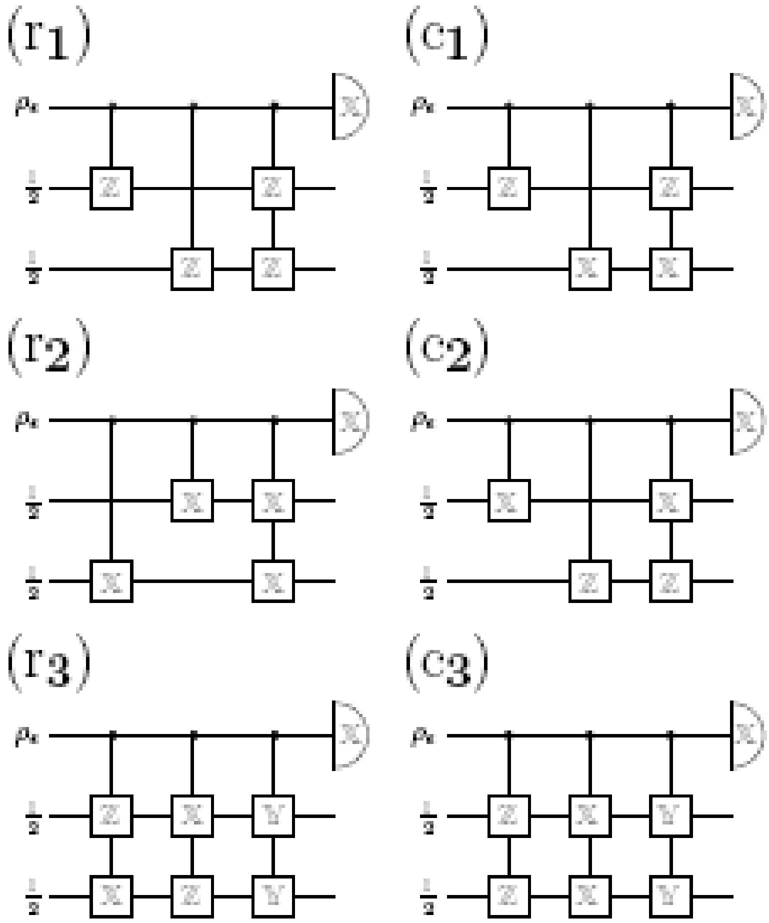
D. G. Cory



O. Moussa, C.A. Ryan, D.G. Cory, and R. Laflamme, Phys. Rev. Lett. **104**, 160501 (2010).

Waterloo KS experiment with NMR

Each row and column can be evaulated through the circuit below



O. Moussa, C.A. Ryan, D.G. Cory, and R. Laflamme, Phys. Rev. Lett. **104**, 160501 (2010).

Recent quantum contextuality experiments

[1] G. Kirchmair, F. Zähringer, R. Gerritsma, M. Kleinmann, O. Gühne, A. Cabello, R. Blatt, and C.F. Roos, Nature (London) **460**, 494 (2009).

[2] H. Bartosik, J. Klepp, C. Schmitzer, S. Sponar, A. Cabello, H. Rauch, and Y. Hasegawa, Phys. Rev. Lett. **103**, 040403 (2009).

[3] E. Amselem, M. Rådmark, M. Bourennane, and A. Cabello, Phys. Rev. Lett. **103**, 160405 (2009).

[4] B. H. Liu, Y. F. Huang, Y. X. Gong, F. W. Sun, Y. S. Zhang, C. F. Li, and G. C. Guo, Phys. Rev. A **80**, 044101 (2009).

[5] O. Moussa, C. A. Ryan, D. G. Cory, and R. Laflamme, Phys. Rev. Lett. **104**, 160501 (2010).

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Compatibility loophole

The NCHV bound of the inequality is obtained under the assumption that all observables in $\langle \mathcal{C}_i \rangle$ are perfectly compatible.

$$\beta := \sum_{i=1} \langle \mathcal{C}_i \rangle \leq b,$$

Imperfections not only reduce the ideal quantum result

$$\beta_{\mathrm{QM}} \rightarrow \beta_{\mathrm{QM}} - \sum_{i=1}^{n} \epsilon_i ,$$

but increase the NCHV bound

$$b \rightarrow b + \sum_{i=1}^{n} \phi_i ,$$

Robustness measure

Assuming that all measurements introduce similar errors,

$$\sum_{i=1}^{n} \epsilon_i = n\epsilon$$

$$\sum_{i=1}^{n} \phi_i = n\phi,$$

and defining an average error in each context as

$$\chi = \epsilon + \phi,$$

The relevant measure of robustness of a violation against imperfections is the maximum tolerated error,

$$\chi_{\max} := \frac{\beta_{\mathrm{QM}} - b}{n}.$$

Inequality based on the PM table

$$\boxed{\langle ABC\rangle + \langle abc\rangle + \langle \alpha\beta\gamma\rangle + \langle Aa\alpha\rangle + \langle Bb\beta\rangle - \langle Cc\gamma\rangle \leq 4}$$

$$\chi_{\max} := 1/3.$$

A more robust inequality

$$\boxed{\begin{aligned}\nu \equiv &\langle XI\ IX\ XX\rangle + \langle XI\ IY\ XY\rangle + \cdots + \langle ZI\ IZ\ ZZ\rangle \\ &+ \langle XX\ YZ\ ZY\rangle + \langle XY\ YX\ ZZ\rangle + \langle XZ\ YY\ ZX\rangle \\ &- \langle XX\ YY\ ZZ\rangle - \langle XY\ YZ\ ZX\rangle - \langle XZ\ YX\ ZY\rangle \leq 9\end{aligned}}$$

$$\nu_{\mathrm{QM}} = 15,$$

$$\chi_{\max} = 0.4.$$

Scaling

- A single system with 2^*n* levels.
- Sequences of three compatible measurements (longer sequences are experimentally difficult).
- Measurements are products of Pauli matrices.

$$\boxed{\sum_{i=1}^{S(n)} \langle \mathcal{C}_i \rangle - \sum_{i=S(n)+1}^{N(n)} \langle \mathcal{C}'_i \rangle \le 2S(n) - N(n).}$$

$$N(n) = \frac{1}{3}(4^n - 1)(4^{n-1} - 1).$$

$$N(n) - S(n) = \frac{1}{6} \sum_{c=0}^{n-2} \sum_{a,b} \binom{n}{c} \binom{n-c}{a} \binom{n-c-a}{b} \times 3^{2n-a-b-2c},$$

$$a, b \ge 0,\ a+b \text{ even},\ \lfloor \tfrac{a}{2} \rfloor + \lfloor \tfrac{b}{2} \rfloor \text{ odd, and } a+b+c \le n.$$

Quantum contextuality grows with “size”

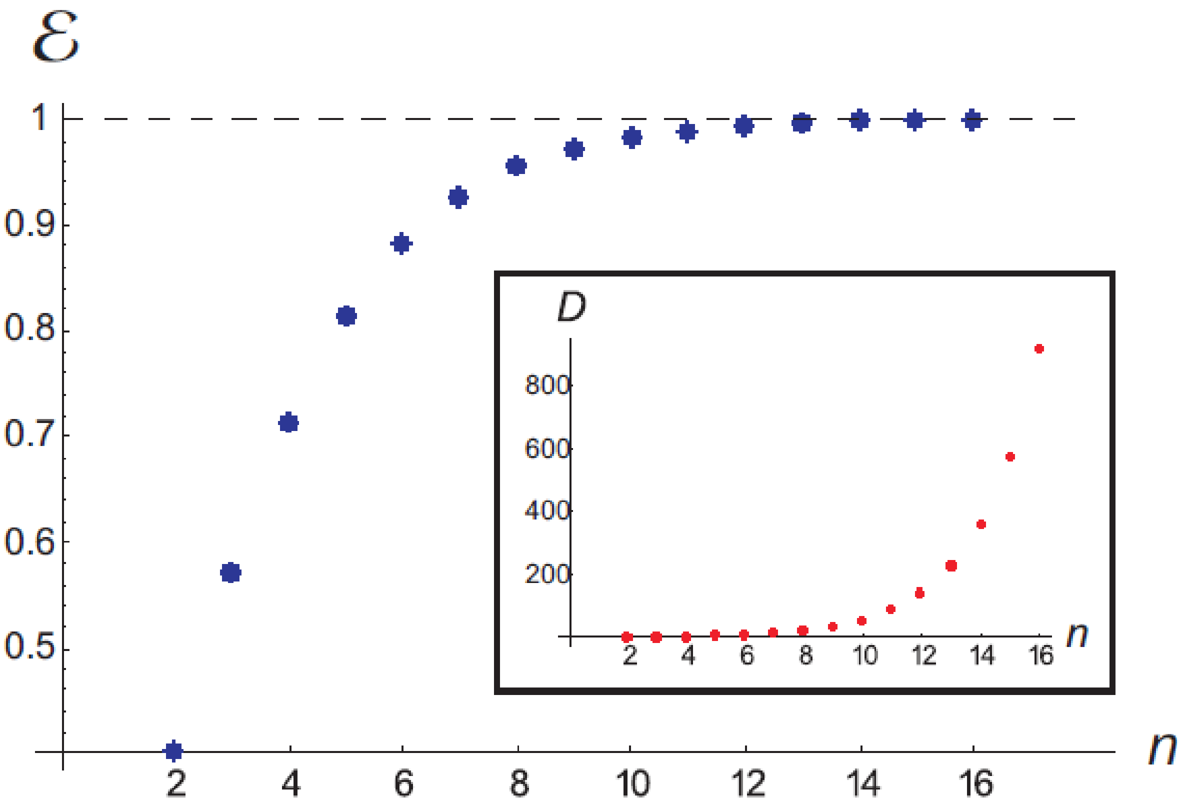

FIG. 1. Tolerated error per correlation (still violating the inequality), ε, and degree of violation, D, of the inequality as a function of the number of qubits, n.

A. Cabello, Phys. Rev. A (September 2010); arXiv:1002.3135.

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Bell's objection to noncontextuality (of results)

"It was tacitly assumed that measurement of an observable must yield the same value independently of what other measurements may be made simultaneously. Thus as well as A say, one might measure *either B or C*, where B and C are orthogonal to [i.e., compatible with] A but not to one another. These different possibilities requires different experimental arrangements; there is no *a priori* reason to believe that the results for A should be the same. The result of an observation may reasonably depend not only on the state of the system (including hidden variables) but also on the complete disposition of the apparatus" [10].

J. S. Bell, Rev. Mod. Phys. **38**, 447 (1966).

J. S. Bell, Found. Phys. **12**, 989 (1982).

- Quantum contextuality (of non spacelike separated systems) can be classically simulated without violating any physical principle.

Photo by Renate Bertlmann, 1989.

Compatibility loophole

- A basic assumption behind the inequalities used for testing noncontextual hidden variable models is that the observables measured on the same individual system (i.e., *A*, *B*, and *C*) are perfectly compatible.

- However, compatibility is not perfect in actual experiments using sequential measurements.

- Therefore, the performed experiments only rule out certain class of noncontextual hidden variable models which obey a kind of extended noncontextuality.

O. Gühne, M. Kleinmann, A. Cabello, J.-Å. Larsson, G. Kirchmair, F. Zähringer, R. Gerritsma, and C.F. Roos, Phys. Rev. A **81**, 022121 (2010).

Finite precision loophole

- “Finite precision measurement nullifies the KS theorem” [D. A. Meyer, Phys. Rev. Lett. **83**, 3751 (1999)].
- “Hidden variables are compatible with physical measurements” [A. Kent, Phys. Rev. Lett. **83**, 3755 (1999)].
- “All the predictions of nonrelativistic QM that are verifiable within any finite precision *can* be simulated classically by NCHV [non-contextual hidden-variable] theories” [R. Clifton and A. Kent, Proc. R. Soc. London, Ser. A **456**, 2101 (2000)].

Finite precision loophole

- A state-independent proof of KS cannot be made if only unit vectors with rational components would exist in nature.
- Moreover, the rational unit sphere is KS colourable (i.e., admits a NCHV model).
- The rational unit sphere is dense in the sphere of unit vectors.
- No finite precision measurement can distinguish a unit vector from a rational unit vector.
- Finite precision measurement nullifies the KS theorem.

D.A. Meyer, Phys. Rev. Lett. **83**, 3751 (1999).

Finite precision loophole

- Even worse, there exist a set of unit vectors which do not have any orthogonal vectors and is dense in the sphere of unit vectors.

R. Clifton and A. Kent, Proc. R. Soc. London, Ser. A **456**, 2101 (2000).

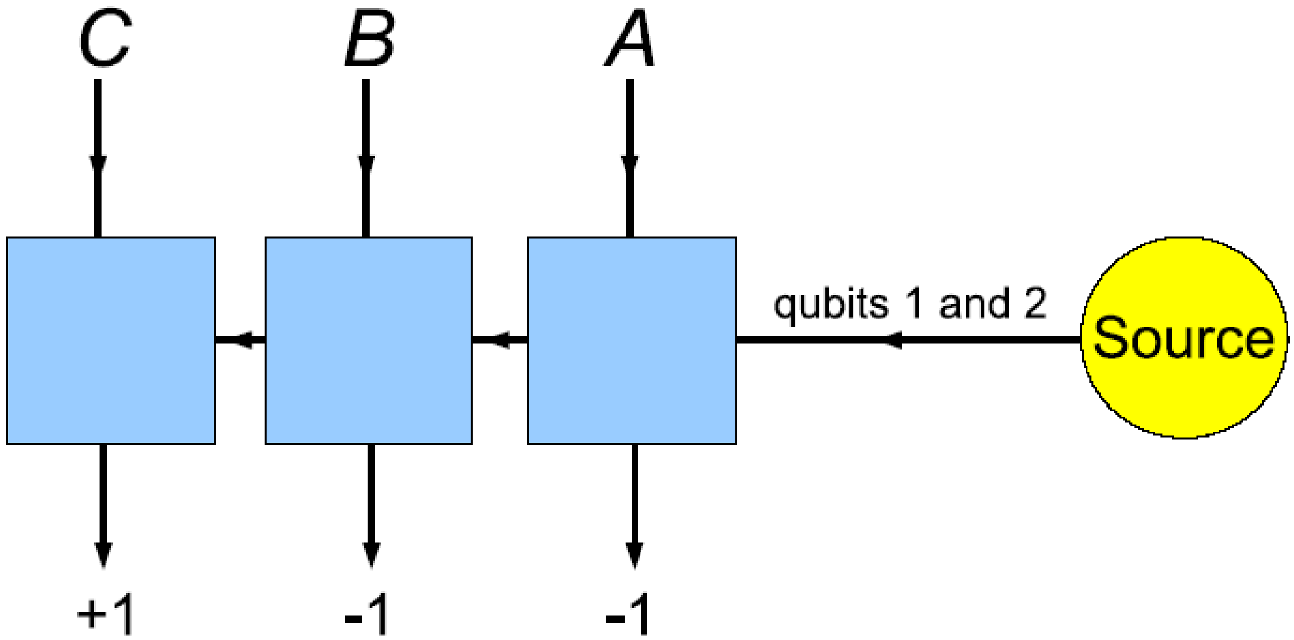
Loophole-free contextuality test

- Perfect compatibility and perfect orthogonality cannot be achieved on measurements on the same system: Use two separated systems.
- Derive a noncontextual inequality in which perfect compatibility is guaranteed by the fact the measurements are performed on separated systems.

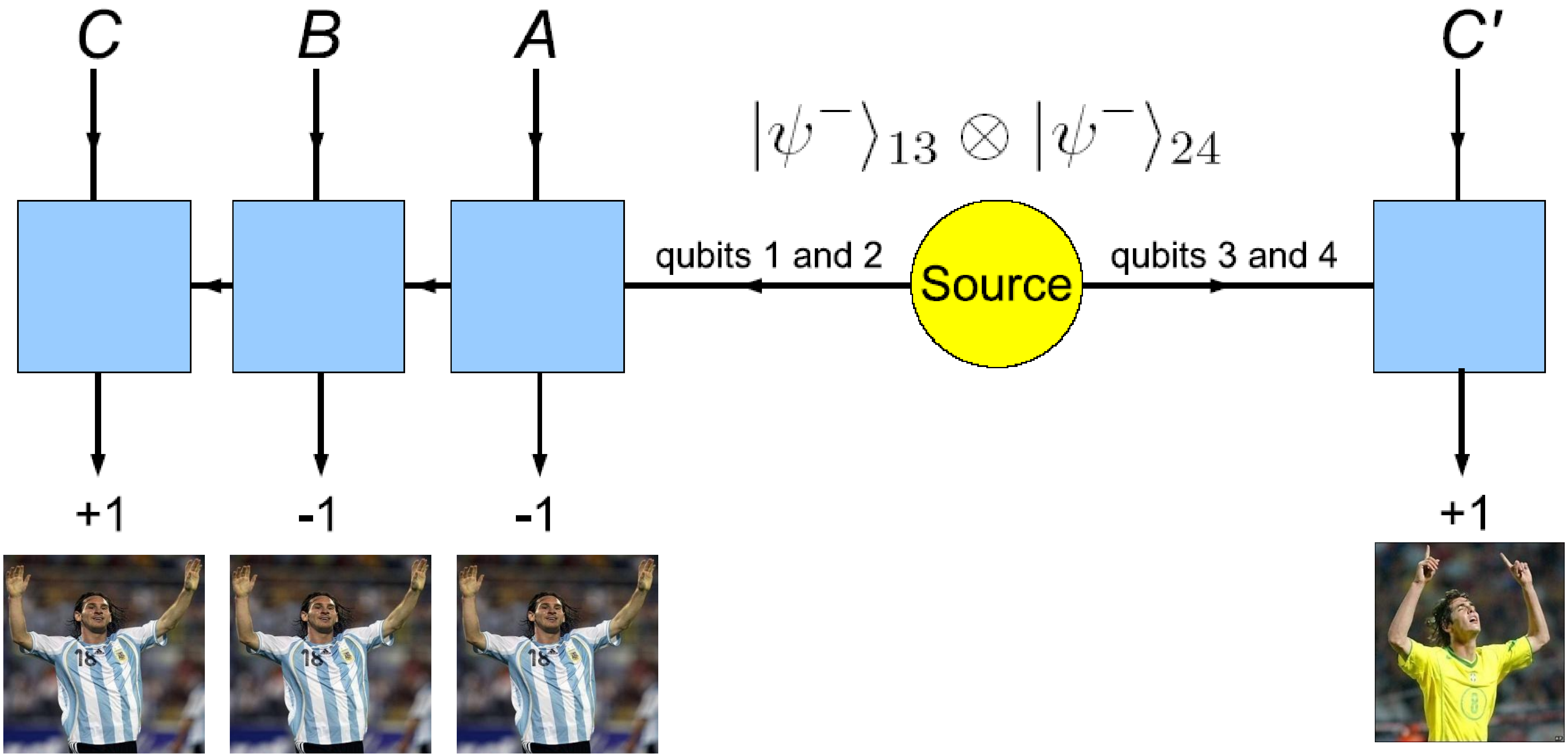
If you do not buy noncontextuality

- Space-like separate the systems.
- Invoke locality instead of noncontextuality.

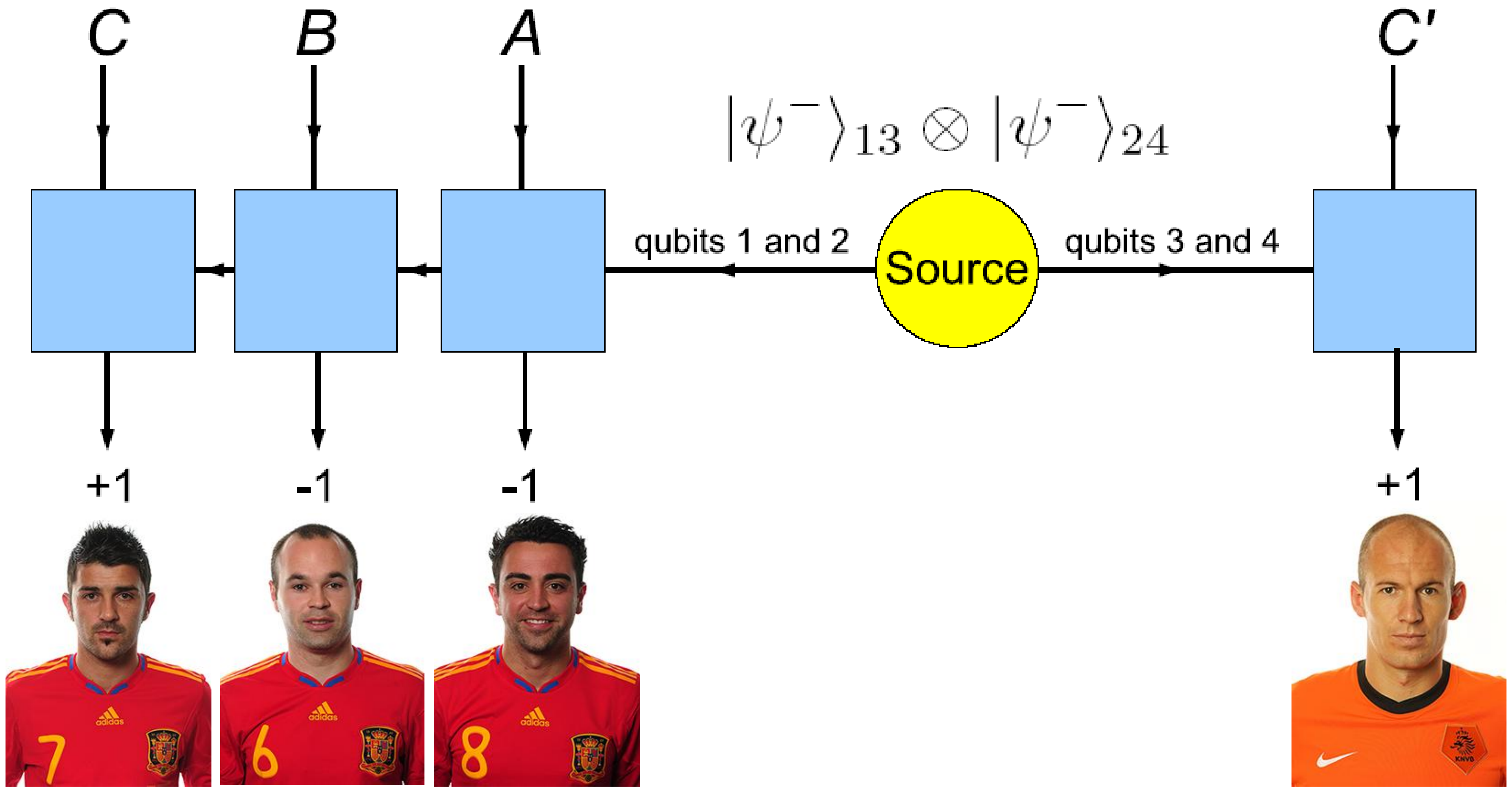
Experiments on contextuality



Nonlocality via local contextuality



Nonlocality via local contextuality



Bell inequality

- Bell inequality:

$$\boxed{\langle S\rangle + \langle \chi\rangle \leq 16.}$$

- Correlations between Alice and Bob:

$$\begin{aligned}\langle S\rangle \equiv & |\langle BB'\rangle_{ABC}| + |\langle BB'\rangle_{bB\beta}| + |\langle CC'\rangle_{ABC}| + |\langle CC'\rangle_{\gamma cC}| \\ & + |\langle aa'\rangle_{bac}| + |\langle aa'\rangle_{Aa\alpha}| + |\langle cc'\rangle_{bac}| + |\langle cc'\rangle_{\gamma cC}| \\ & + |\langle \alpha\alpha'\rangle_{\gamma\beta\alpha}| + |\langle \alpha\alpha'\rangle_{Aa\alpha}| + |\langle \beta\beta'\rangle_{\gamma\beta\alpha}| + |\langle \beta\beta'\rangle_{bB\beta}|.\end{aligned}$$

- Correlations among Alice's sequential measurements:

$$\langle \chi\rangle \equiv \langle ABC\rangle + \langle bac\rangle + \langle \gamma\beta\alpha\rangle + \langle Aa\alpha\rangle + \langle bB\beta\rangle - \langle \gamma cC\rangle.$$

Quantum violation

- Quantum violation:

$$\boxed{\langle S \rangle_{\rm QM} + \langle \chi \rangle_{\rm QM} = 18.}$$

- For this entangled state: $\langle BB' \rangle = -1, \quad \langle CC' \rangle = 1, \quad \langle aa' \rangle = -1,$

$$\langle cc' \rangle = 1, \quad \langle \alpha\alpha' \rangle = 1, \quad \langle \beta\beta' \rangle = 1.$$

$$\langle S \rangle_{\rm QM} = 12.$$

- For any state:

$$\langle \chi \rangle_{\rm QM} = 6.$$

Bell inequality

Proof: $\hat{B}'$ is the results LHV assign to B' when no other observable is measured before B'. Any LHV theory must satisfy:

$$\langle A\hat{B}'\hat{C}'\rangle + \langle b\hat{a}'\hat{c}'\rangle + \langle \gamma\hat{\beta}'\hat{\alpha}'\rangle + \langle A\hat{a}'\hat{\alpha}'\rangle + \langle b\hat{B}'\hat{\beta}'\rangle - \langle \gamma\hat{c}'\hat{C}'\rangle \leq 4,$$

which is not directly testable because $\hat{B}'$ and $\hat{C}'$ cannot be measured both first. However,

$$\begin{aligned}
&|\langle A\hat{B}'\hat{C}'\rangle - \langle ABC\rangle| \\
&\quad \leq |\langle A\hat{B}'\hat{C}'\rangle - \langle AB\hat{C}'\rangle| + |\langle AB\hat{C}'\rangle - \langle ABC\rangle| \\
&\quad \leq \langle |A\hat{B}'\hat{C}' - AB\hat{C}'\hat{B}'^2|\rangle + \langle |AB\hat{C}' - ABC\hat{C}'^2|\rangle \\
&\quad = \langle |A\hat{B}'\hat{C}'(1 - B\hat{B}')|\rangle + \langle |AB\hat{C}'(1 - C\hat{C}')|\rangle \\
&\quad \leq 1 - |\langle BB'\rangle| + 1 - |\langle CC'\rangle|
\end{aligned}$$

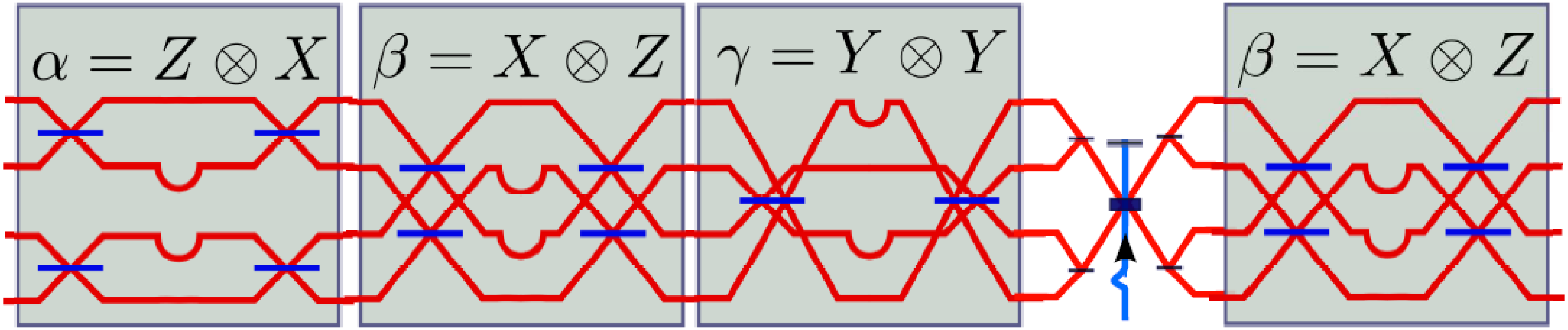
leads to

$$\langle A\hat{B}'\hat{C}'\rangle \geq \langle ABC\rangle + |\langle BB'\rangle_{ABC}| + |\langle CC'\rangle_{ABC}| - 2,$$

where the right-hand side is experimentally testable. Similarly, for $\langle b\hat{a}'\hat{c}'\rangle$, $\langle \gamma\hat{\beta}'\hat{\alpha}'\rangle$, $\langle A\hat{a}'\hat{\alpha}'\rangle$, $\langle b\hat{B}'\hat{\beta}'\rangle$, and $-\langle \gamma\hat{c}'\hat{C}'\rangle$.

A. Cabello, Phys. Rev. Lett. **104**, 220401 (2010).

Experimental proposal



- Two true 4-level entangled systems.
- Time encoding for sequential measurements.

E. Amselem *et al.* (2010).

Morals

- Bell inequalities can also contain sequences of local measurements
- QM violates Bell inequalities even when the correlations *between Alice and Bob* admit a local model.
- The role of entanglement is marginal: The violation is due to local contextuality.
- Contextuality (local or distributed) is the reason why QM violates Bell inequalities (i.e., quantum nonlocality is a subproduct of quantum contextuality).

Plan

- Contextuality
- Recent experiments on quantum contextuality
- “Macroscopic” quantum contextuality
- Quantum nonlocality via local contextuality
- Memory cost of quantum contextuality

Bell's objection to noncontextuality (of results)

"It was tacitly assumed that measurement of an observable must yield the same value independently of what other measurements may be made simultaneously. Thus as well as A say, one might measure *either B or C*, where B and C are orthogonal to [i.e., compatible with] A but not to one another. These different possibilities requires different experimental arrangements; there is no *a priori* reason to believe that the results for A should be the same. The result of an observation may reasonably depend not only on the state of the system (including hidden variables) but also on the complete disposition of the apparatus" [10].

J. S. Bell, Rev. Mod. Phys. **38**, 447 (1966).

J. S. Bell, Found. Phys. **12**, 989 (1982).

- Quantum contextuality (of non spacelike separated systems) can be classically simulated without violating any physical principle.

Photo by Renate Bertlmann, 1989.

Simulating contextuality requires memory

- Every physical system can be seen as an *n*-state machine that generates an output (the result of the measurement) based on its current state and input (the observable being measured) `[A Mealy automaton]`.
- The memory needed is lower bounded by log *n* bits.

G.H. Mealy, Bell Systems Technical J. **34**, 1045 (1955).

C.H. Roth Jr., *Fundamentals of Logic Design* (Thomson, Stanford, CT, 2009).

Example: Mealy automaton for PR boxes

$$
\begin{array}{lcccc}
 & A_0 & A_1 & B_0 & B_1 \\
s_1 \equiv & +1 & +3 & +1 & +2 \\
s_2 \equiv & +1 & -2 & -4 & +2 \\
s_3 \equiv & -4 & +3 & +1 & -3 \\
s_4 \equiv & -4 & -2 & -4 & -3
\end{array}
$$

Memory cost of quantum nonlocality

Quantum violation	Memory (bits/qubit)
CHSH	≤ 0.33
Infinite-setting chained Bell	≤ 0.79

Memory cost of quantum contextuality

Table 1: Memory cost in bits per qubit.

Sequences	PM (9 obs.)	15 obs.
Mutually compatible: $ABBCBC\ldots$	0.79	?
Compatible with the first: $ABaCaBA$	1	>1
Arbitrary: $ABcC\ldots$	>1 (≤ 1.66)	>1

- State-independent quantum contextuality can be classically simulated, but the memory needed is larger than the information carrying capacity of the physical system (Holevo's bound).

M. Kleinmann, O. Gühne, J.R. Portillo, J.-Å. Larsson, and A. Cabello, arXiv:1007.3650.

Memory cost of quantum contextuality

- The density of memory (bits/per qubit) needed to simulate quantum contextuality scales exponentially with the number of qubits [we only consider all products of the 3 Pauli observables].
- Therefore, if we assume that the density of memory is bounded in nature, then we can have a Bell-like theorem of impossibility of classical theories beyound QM based on Realism+Bounded Memory+Freedom rather than on Realism+Locality+Freedom.

Quantum resources for quantum information

	Resource	Simplest example
▪	Superposition	Single qubit + alternative basis
▪	Nonlocality	Pairs of entangled qubits + Bell ineq.

Quantum resources for quantum information

	Resource	*Simplest example*
▪	Superposition	Single qubit + alternative basis
▪	Contextuality	Single qutrit + alternative contexts
▪	Nonlocality	Pairs of entangled qubits + Bell ineq.

Applications of contextuality for QI (I)

- QKD based on proofs of the KS theorem [H. Bechmann-Pasquinucci and A. Peres, PRL **85**, 3313 (2000); K. Svozil, arXiv:0903.0231].
- Random number generation [K. Svozil, PRA **79**, 054306 (2009)].
- Quantum contextuality powered quantum games [N. Aharon and L. Vaidman, PRA **77**, 052310 (2008)].
- Quantum contextuality powered parity-oblivious transfer and multiplexing tasks [E. F. Galvao, quant-ph/0212124; R. W. Spekkens *et al.*, PRL **102**, 010401 (2009)].

Applications of contextuality for QI (II)

- Link between quantum contextuality and quantum computation [R. Raussendorf, arXiv:0907.5449].
- Device-independent secure communication [K. Horodecki, M. Horodecki, P. Horodecki, R. Horodecki, M. Pawlowski, and M. Bourennane, arXiv:1006.0468].
- Increase the number of classical messages which can be sent without error through a classical channel [T. Cubitt, D. Leung, W. Matthews, and A. Winter, PRL **104**, 230503 (2010)].