Perturbations, Irreversibility and chaos: Universal response

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PRL **104**, 254101 (2010) PHYSICAL REVIEW LETTERS week ending 25 JUNE 2010

Universal Response of Quantum Systems with Chaotic Dynamics

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(Received 16 February 2010; published 23 June 2010)

The elusive nature of the Lyapunov regime in the Loschmidt echo

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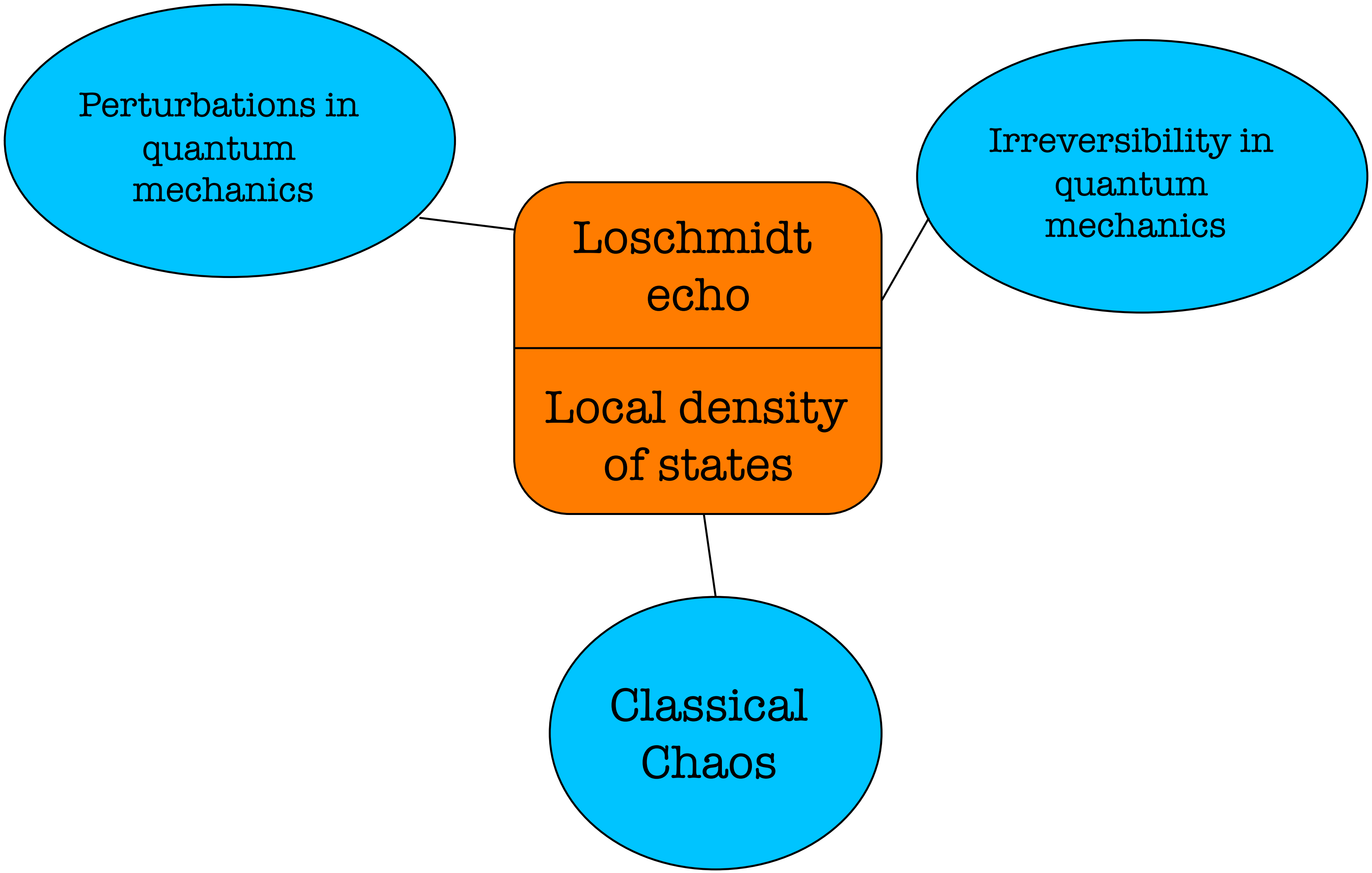
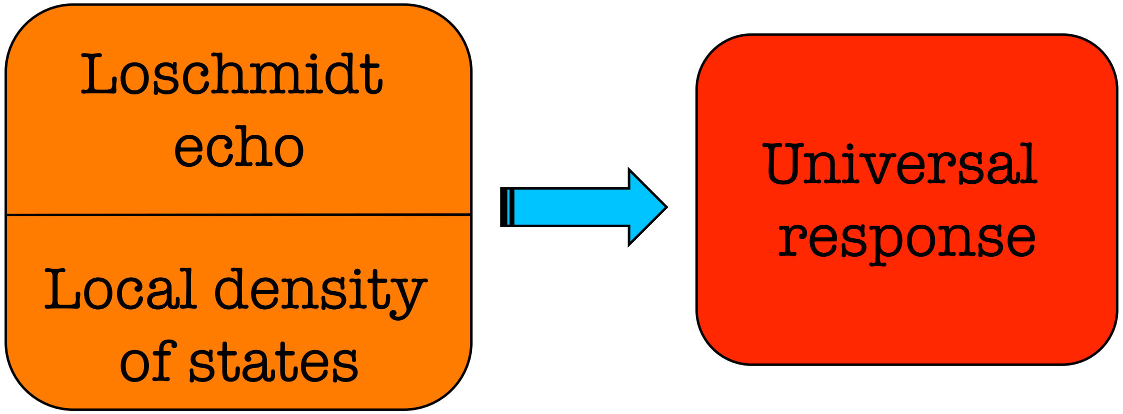
(Dated: August 29, 2010)

Overview


Perturbations in quantum mechanics
Loschmidt echo
Local density of states
Irreversibility in quantum mechanics
Classical Chaos

Overview


Loschmidt echo
Local density of states
Universal response

Outline

- Introduction: What is the Loschmidt Echo?

  Regimes of the LE.

  Local density of states (LDOS).
- Universal behavior of the LDOS:

  Semiclassical approach

  Examples
- Universal behavior of the LE:

  LE and chaos-Problems in Lyapunov regime

  Non diagonal part of the LE
- Conclusions/final remarks

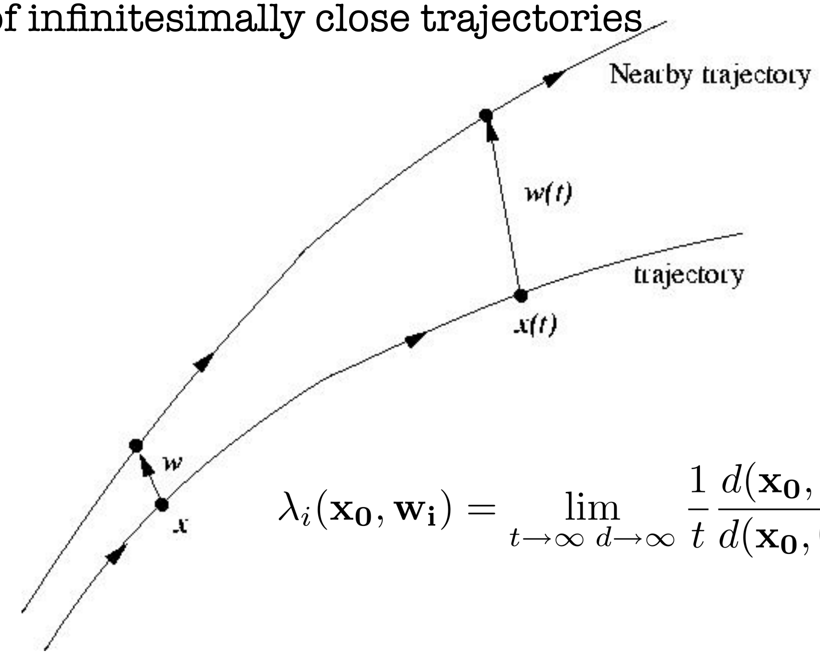
Introduction

Chaos

Classical Mechanics

Lyapunov Exponents

characterizes the rate of separation of infinitesimally close trajectories



$$\lambda_i(\mathbf{x_0}, \mathbf{w_i}) = \lim_{t\to\infty} \lim_{d\to\infty} \frac{1}{t} \frac{d(\mathbf{x_0}, t)}{d(\mathbf{x_0}, 0)}$$

Quantum Mechanics

$$|\psi_0\rangle \quad |\bar{\psi}_0\rangle$$

$$d(\psi_0\rangle, |\bar{\psi}_0\rangle) = |\langle\psi_0|\bar{\psi}_0\rangle|^2$$

$$d(t) = |\langle\psi_0(t)|\bar{\psi}_0(t)\rangle|^2$$

$$= |\langle\psi_0|U^t U|\bar{\psi}_0\rangle|^2 = d(0)$$

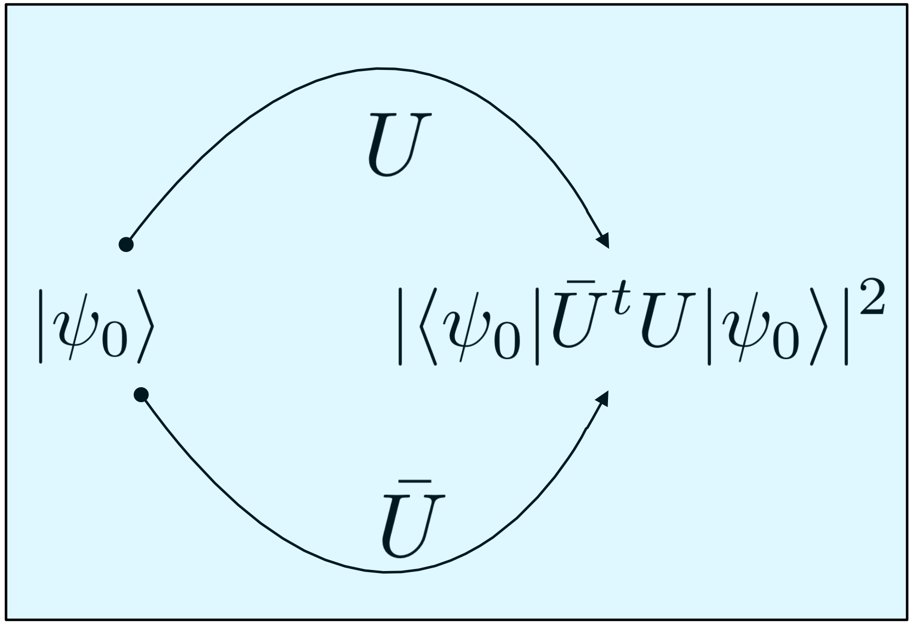
So $d(t) = d(0)$

Introduction

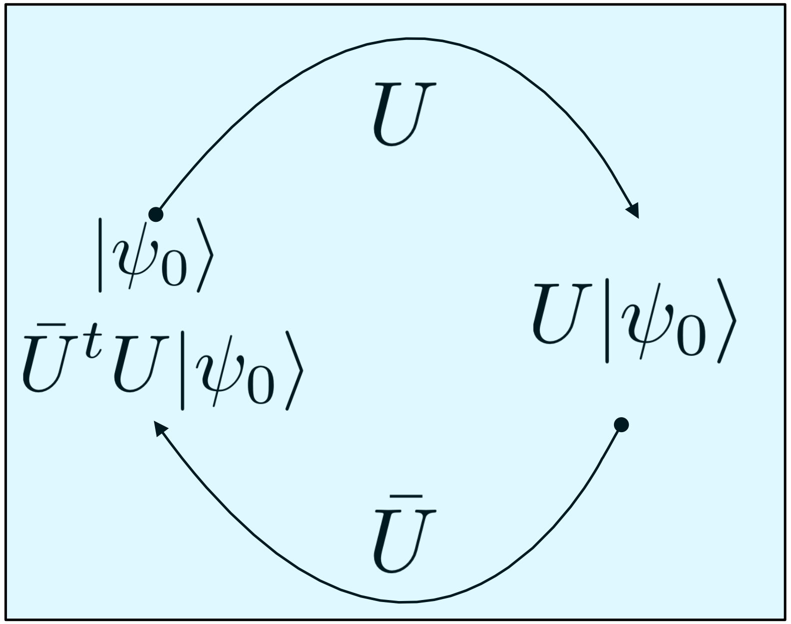
In 1984 A. Peres proposed:

$$\boxed{M(t) = |\langle\psi_0|\bar{U}^t U|\psi_0\rangle|^2}$$

$$U = \exp(-iHt/\hbar) \quad \bar{U} = \exp[-i(H+V)t/\hbar]$$



Sensitivity to perturbations



Irreversibility

Introduction

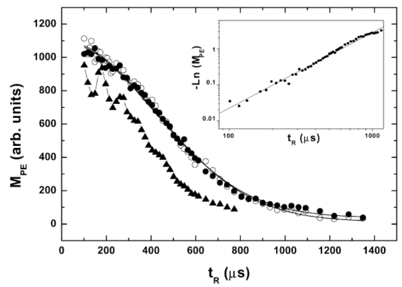
$$M(t) = |\langle\psi_0|\bar{U}^t U|\psi_0\rangle|^2$$

This quantity is important for studies in:

- Quantum chaos
- Quantum computer and quantum information
- Mesosocopic physics
- Decoherence
- Quantum phase transition
- Experiments: NMR, Elastic waves, microwave billiards

Introduction

NMR Pastawski 2000

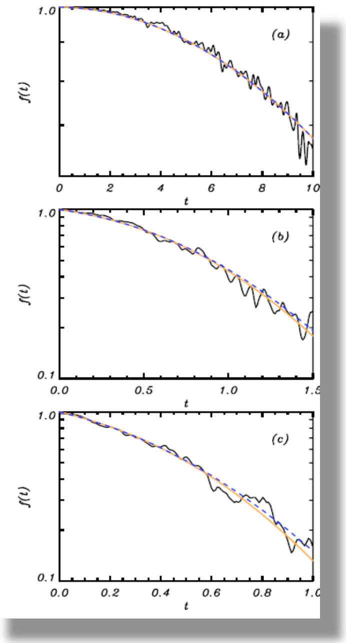


Microwave cavity PRL 05 Stockmann

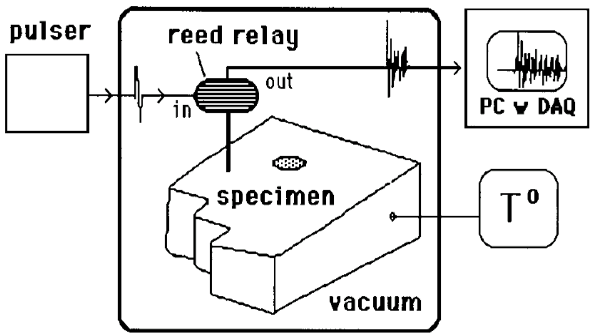
- Electromagnetic cavity: equivalence of Helmholtz and Schrodinger eq.

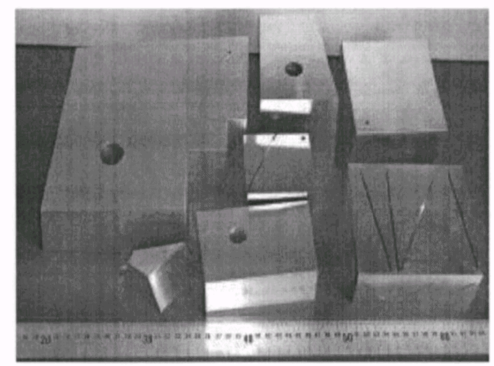
200 mm

438 mm



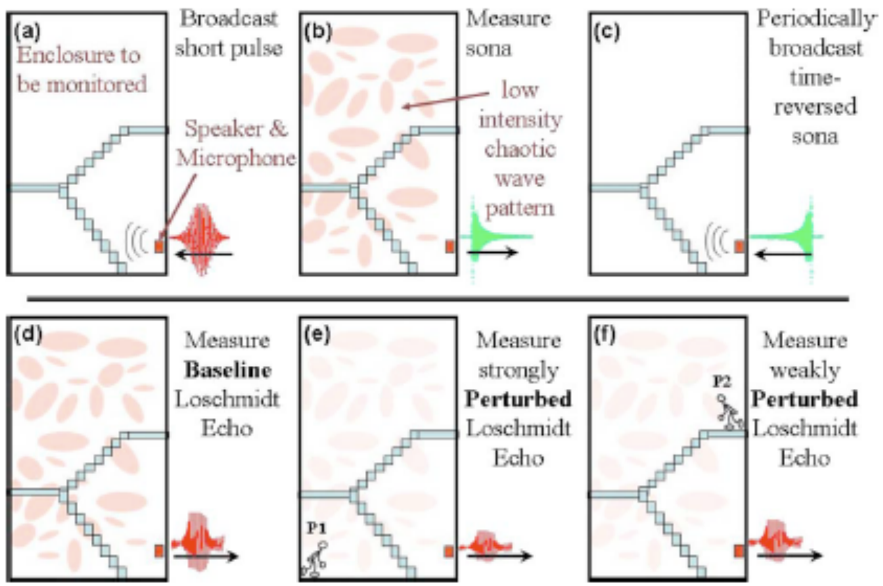
Elastic waves Lobkis PRL 03





Introduction

Elastic waves (2009) B. Taddese et al



BEC- (2009) Ullah and Hoogerland

Experimental observation of Loschmidt time reversal of a Quantum Chaotical System

Arif Ullah and M.D. Hoogerland

Department of Physics, University of Auckland,

Private Bag 92019, Auckland, New Zealand

Introduction

$$M(t) = |\langle \psi_0 | \bar{U}^t U | \psi_0 \rangle|^2$$

Regimes of the Loschmidt echo:

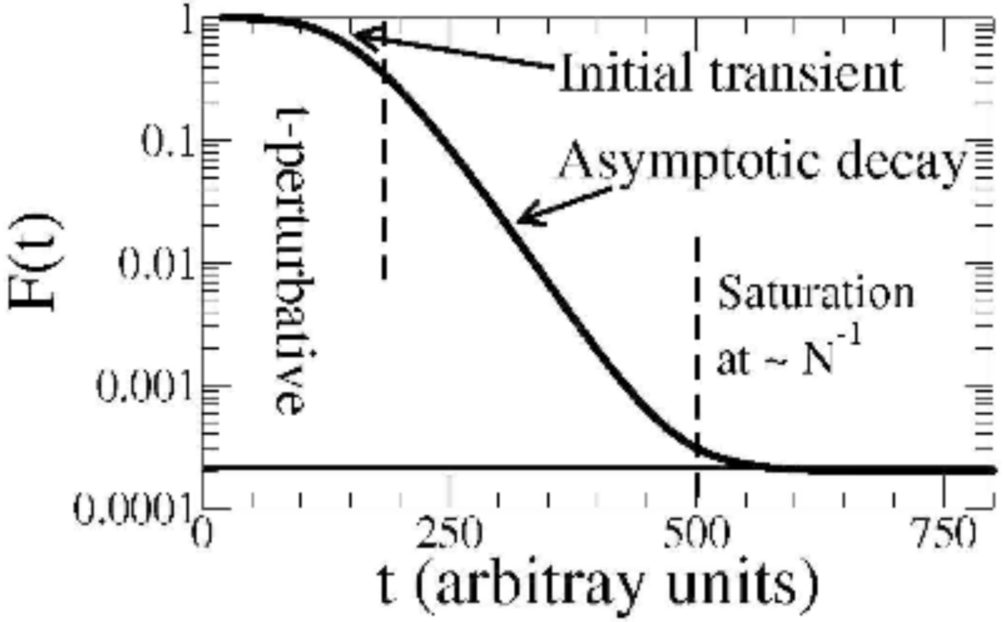
- Time regimes
- Perturbation regimes
- Initial condition

T. Gorin, T. Prosen, T. H. Seligman, and M. Znidaric (2006)
Ph. Jacquod and C. Petitjean (2009)

Introduction

Time regimes:

- Short times: gaussian decay
- Exponential decay
- Saturation (1/N)



Introduction

Perturbation regimes:

- Gaussian regime
- FGR regime
- Lyapunov regime

$$H(\epsilon) = H_0 + \epsilon\, V$$

$$V > \Delta \qquad \Gamma_{\text{LDOS}} > \lambda$$

$$\exp(-at^2) \;\Big|\; \exp(-\Gamma_{\text{LDOS}}\; t) \;\Big|\; \exp(-\lambda t)$$

$$\xrightarrow{\qquad\qquad \epsilon \qquad\qquad}$$

$$\Gamma_{\text{LE}} = min(\Gamma_{\text{LDOS}}, \lambda)$$

Introduction

Local density of states

or ‘the strenght function’

ANNALS OF MATHEMATICS
Vol. 62, No. 3, November, 1955
Printed in U.S.A.

CHARACTERISTIC VECTORS OF BORDERED MATRICES WITH INFINITE DIMENSIONS

BY EUGENE P. WIGNER

(Received April 18, 1955)

Introduction

The statistical properties of the characteristic values of a matrix the elements of which show a normal (Gaussian) distribution are well known (cf. [6] Chapter XI) and have been derived, rather recently, in a particularly elegant fashion.[1] The present problem arose from the consideration of the properties of the wave functions of quantum mechanical systems which are assumed to

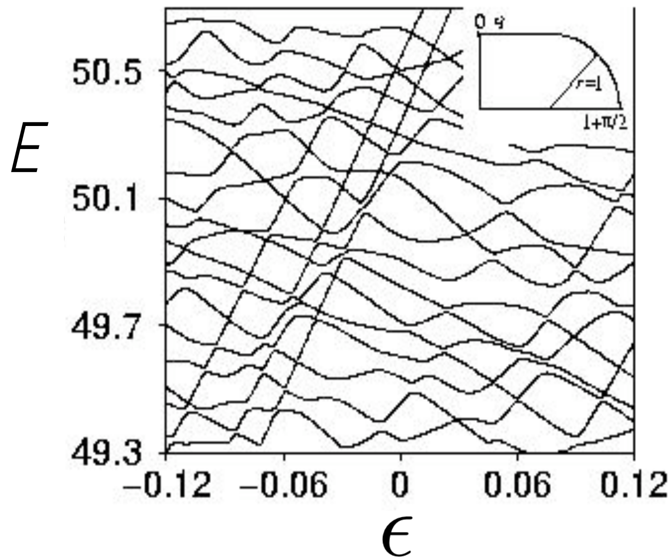
Introduction

Local density of states

$$H(\epsilon) = H_0 + \epsilon\, V$$

$$|\psi_i(\epsilon)\rangle$$

$$E_i(\epsilon)$$



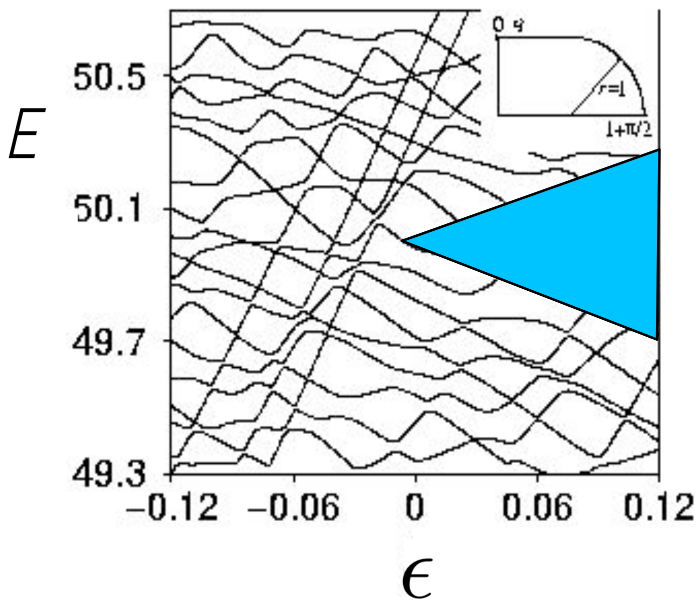
$$\rho_i(E,\epsilon) = \sum_j |\langle\psi_j(\epsilon)|\psi_i(0)\rangle|^2 \delta(E - [E_j(\epsilon) - E_i(0)])$$

Introduction

Local density of states

$$\rho_i(E,\epsilon) = \sum_j |\langle \psi_j(\epsilon)|\psi_i(0)\rangle|^2 \delta(E - [E_j(\epsilon) - E_i(0)])$$

$$\Gamma_{\mathrm{LDOS}} \sim \epsilon^2$$

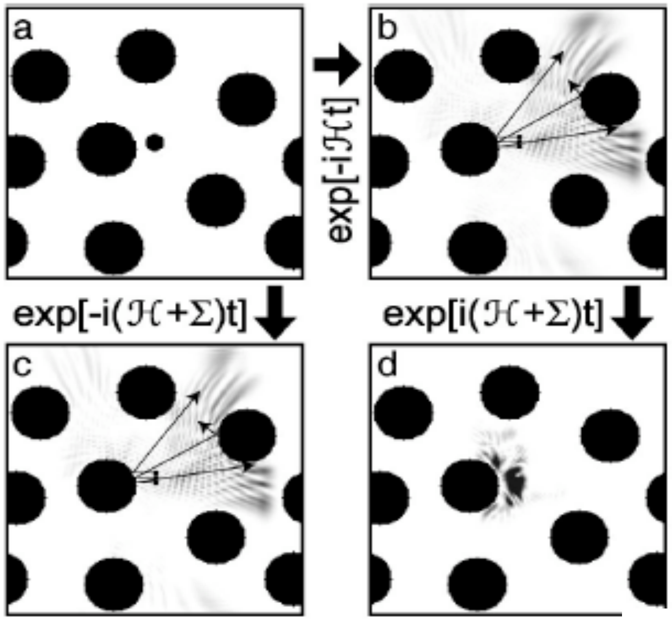


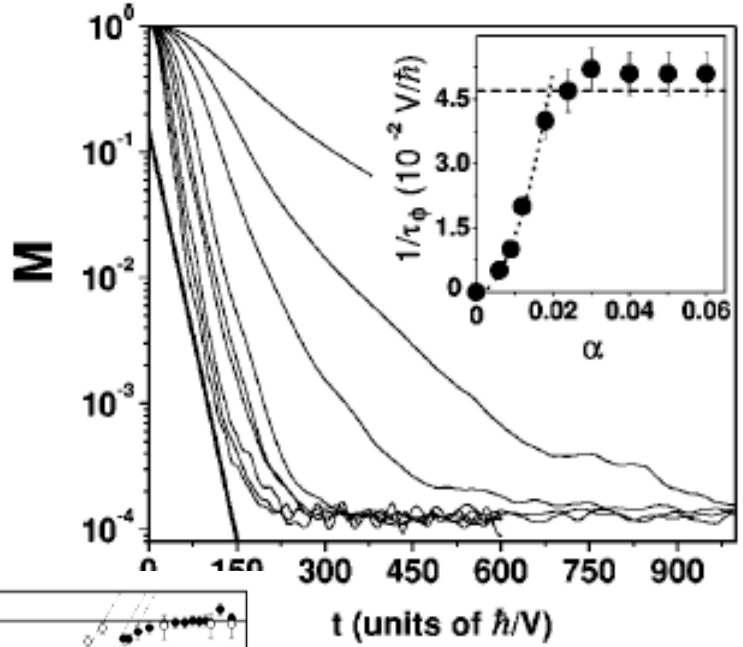
$$\Gamma_{\mathrm{LDOS}}$$

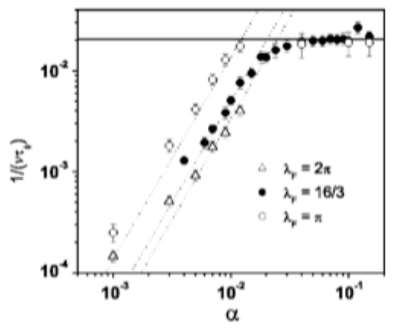
Introduction

Example: LE in the Lorentz gas

F. Cucchietti, H. Pastawski, DAW (2000)







Introduction

But ...

RAPID COMMUNI

PHYSICAL REVIEW E **69**, 025201(R) (2004)

Stability of quantum motion: Beyond Fermi-golden-rule and Lyapunov decay

Wen-ge Wang,[1,2] G. Casati,[3,4,1] and Baowen Li[1]
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[3]*Center for Nonlinear and Complex Systems, Università degli Studi dell'Insubria and Istituto Nazionale per la Fisica della Materia, Unità di Como, Via Valleggio 11, 22100 Como, Italy*
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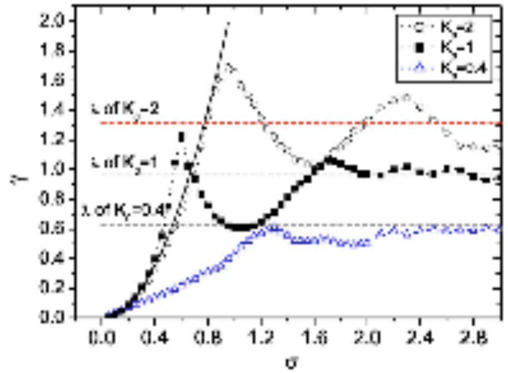
(Received 22 September 2003; published 27 February 2004)



PHYSICAL REVIEW E **80**, 046216 (2009)

Loschmidt echo and the local density of states

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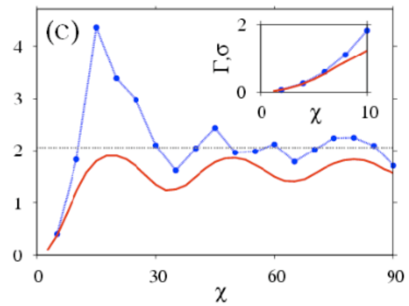
Departamento de Física "J. J. Giambiagi," FCEN, UBA, Pabellón 1, Ciudad Universitaria, C1428EGA Buenos Aires, Argentina
(Received 7 August 2009; published 26 October 2009)



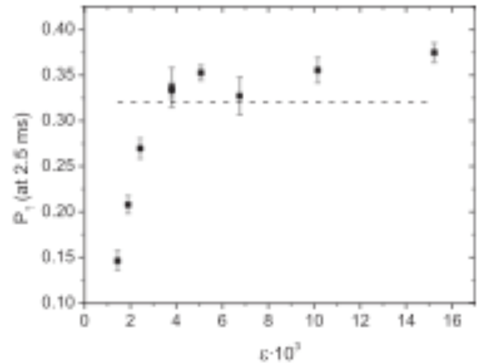
Introduction

But ...

PRL **97**, 104102 (2006) PHYSICAL REVIEW LETTERS week ending 8 SEPTEMBER 2006

Decay of Quantum Correlations in Atom Optics Billiards with Chaotic and Mixed Dynamics

M. F. Andersen,* A. Kaplan,† T. Grünzweig, and N. Davidson
Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 20 April 2004; published 8 September 2006)



PRL **98**, 057006 (2007) PHYSICAL REVIEW LETTERS week ending 2 FEBRUARY 2

Fidelity and Quantum Chaos in the Mesoscopic Device for the Josephson Flux Qubit

Ezequiel N. Pozzo and Daniel Domínguez
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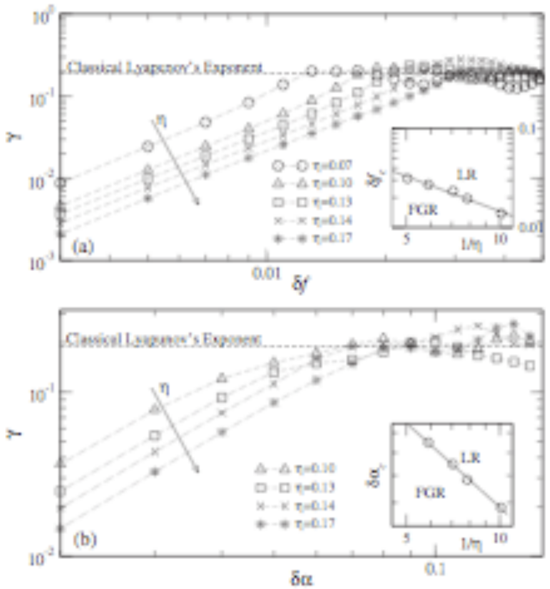
(Received 7 February 2006; published 2 February 2007)



Universal behavior of the LDOS

$$\hat{H}(x)|\phi_j(x)\rangle = \hbar\,\omega_j(x)|\phi_j(x)\rangle$$

$$\boxed{\rho_i(\omega, \delta x) = \sum_j |\langle\phi_j(x)|\phi_i(x_0)\rangle|^2 \delta(\omega - \omega_{ij})}$$

$$\delta x = x - x_0 \qquad \omega_{ij} = \omega_j(x) - \omega_i(x_0)$$

$$\boxed{\bar{\rho}(\omega, \delta x) = \frac{1}{n}\sum_{i=1}^{n} \rho_i(\omega, \delta x)}$$

Universal behavior of the LDOS

$$\mathcal{F}[\bar{\rho}](t,\delta x) = \frac{1}{n}\sum_{i=1}^{n} e^{-i\omega_i(x_0)t}\underbrace{\langle\phi_i(x_0)|e^{i\hat{H}(x)t/\hbar}|\phi_i(x_0)\rangle}_{\text{Survival probability}}$$

Survival probability

Let us evaluate last sum semiclassically:

$$\mathcal{F}[\bar{\rho}](t,\delta x) \approx \int dq dp W(q,p)\exp[-\mathrm{i}\Delta S_t(q,p,\delta x)/\hbar]$$

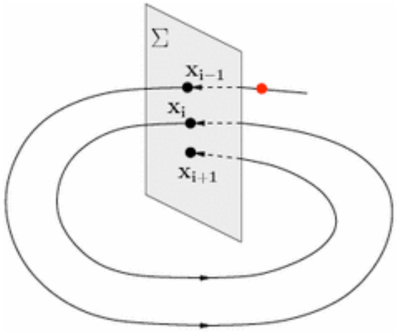
Dephasing representation (Vanicek 2006)

the action difference evaluated along the unperturbed orbit starting at (q, p)

$$W(q,p) = (1/n)\sum W_i(q,p)$$

Wigner distribution of state i

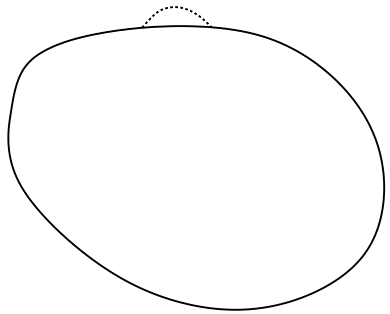
Universal behavior of the LDOS



The last integral can be solved:

Goussev et al 2008

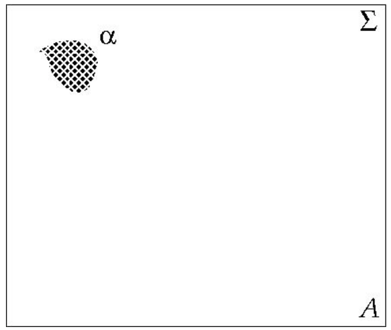
$$\boxed{\mathcal{F}[\bar{\rho}](t,\delta x) \approx e^{-\gamma|t|}}$$



$$\gamma = \eta\left(1 - \Re\left\langle e^{-\mathrm{i}\Delta S(q,p,\delta x)/\hbar}\right\rangle\right)$$

$$\eta = \frac{\alpha}{\tau\mathcal{A}}$$

Probability to hit the perturbed region

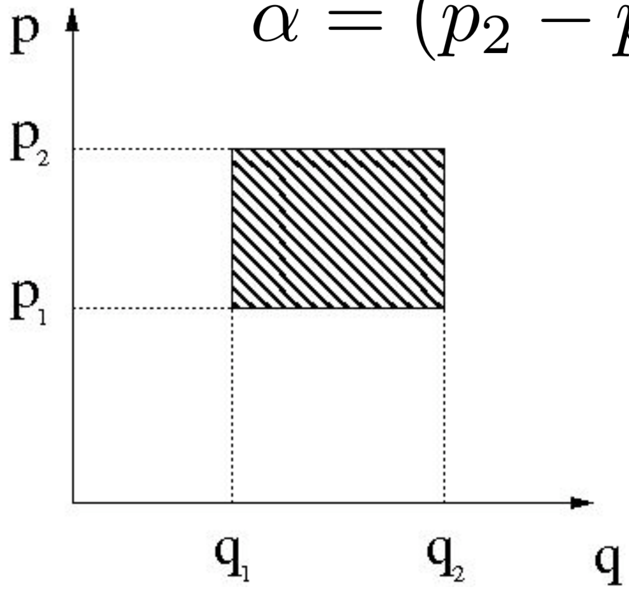


Universal behavior of the LDOS

$$\left\langle e^{-\mathrm{i}\Delta S(q,p,\delta x)/\hbar} \right\rangle = \frac{1}{\alpha} \int_{p_1}^{p_2} \int_{q_1}^{q_2} e^{-\mathrm{i}\Delta S(q,p,\delta x)/\hbar} dq dp$$

$$\alpha = (p_2 - p_1)(q_2 - q_1)$$

$$\eta = \frac{\alpha}{\tau \mathcal{A}}$$

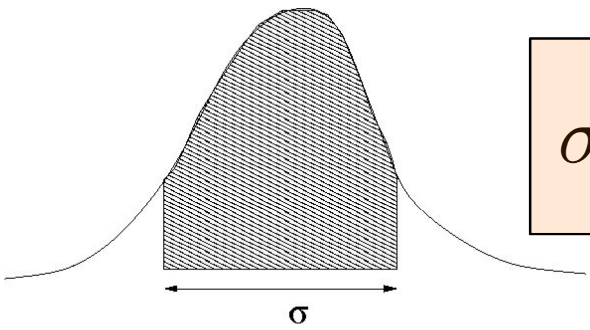


Universal behavior of the LDOS

$$\boxed{\mathcal{F}[\bar{\rho}](t, \delta x) \approx e^{-\gamma|t|}}$$

$$\boxed{\bar{\rho}(\omega, \delta x) \approx L(\gamma, \omega) = \frac{\gamma}{\pi(\omega^2 + \gamma^2)}}$$

The LDOS is a lorentzian distribution!!!!



$$\boxed{\sigma_{sc} = \tan\left(0.7\frac{\pi}{2}\right)\gamma \approx 1.963\gamma}$$

Universal behavior of the LDOS

- How does this semiclassical aproximation works to describe the LDOS width?
- In real systems: is the LDOS a BW distribution?

Universal behavior of the LDOS

Example 1: Cat maps

Why we study the LE in cat maps?

- Completely chaotic system
- We can use different maps with differenet Lyapunov exponent
- Simple to perturb
- Simple quantization

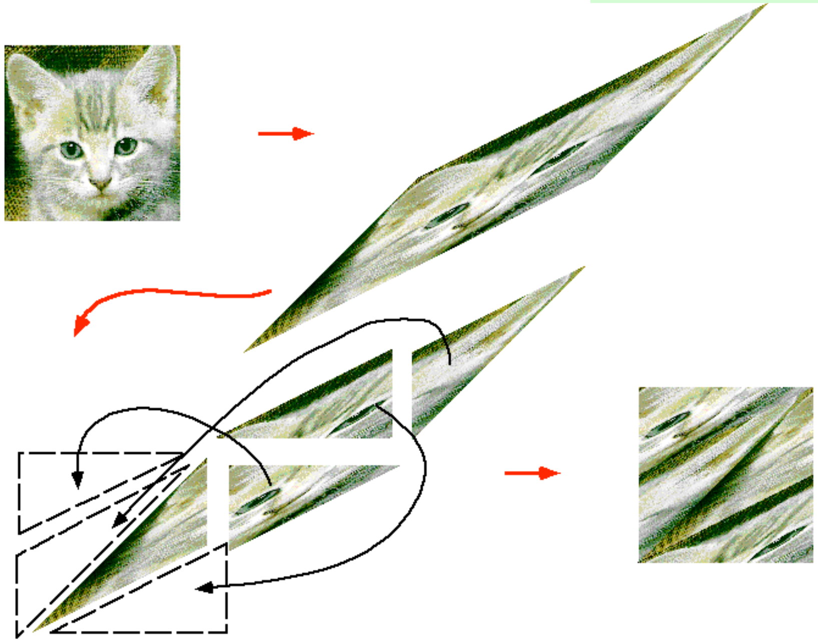
Universal behavior of the LDOS

Example 1: Cat maps

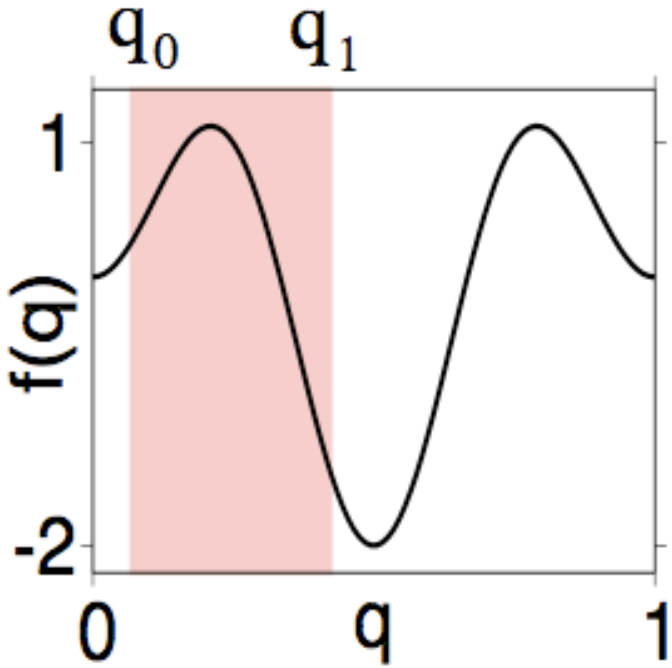
$$
\begin{aligned}
q' &= a_{11}\, q + a_{12}\, p \\
p' &= a_{21}\, q + a_{22}\, p + \epsilon(q,k)
\end{aligned}
$$

$\epsilon(q,k) = (k/2\pi)[\cos(2\pi q) - \cos(4\pi q)$

Perturbation: shear in momentum



Universal behavior of the LDOS



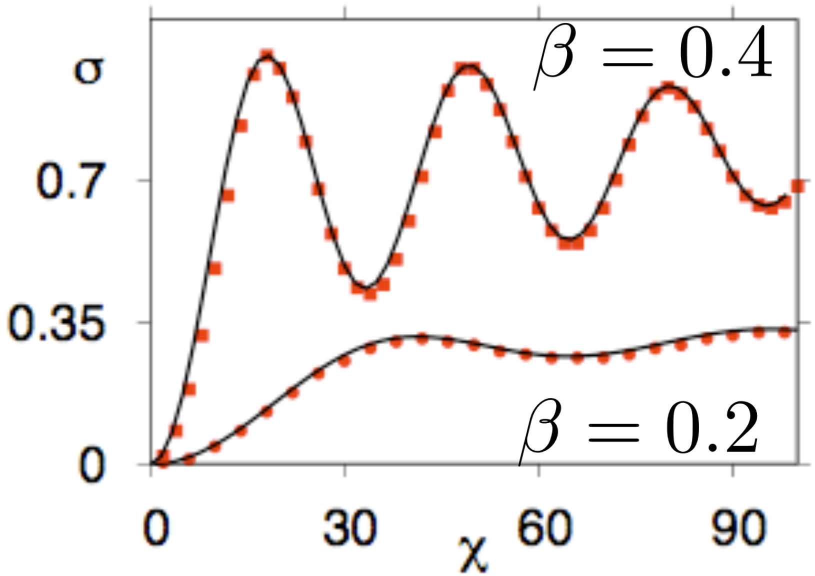
$$\beta = q_1 - q_0$$

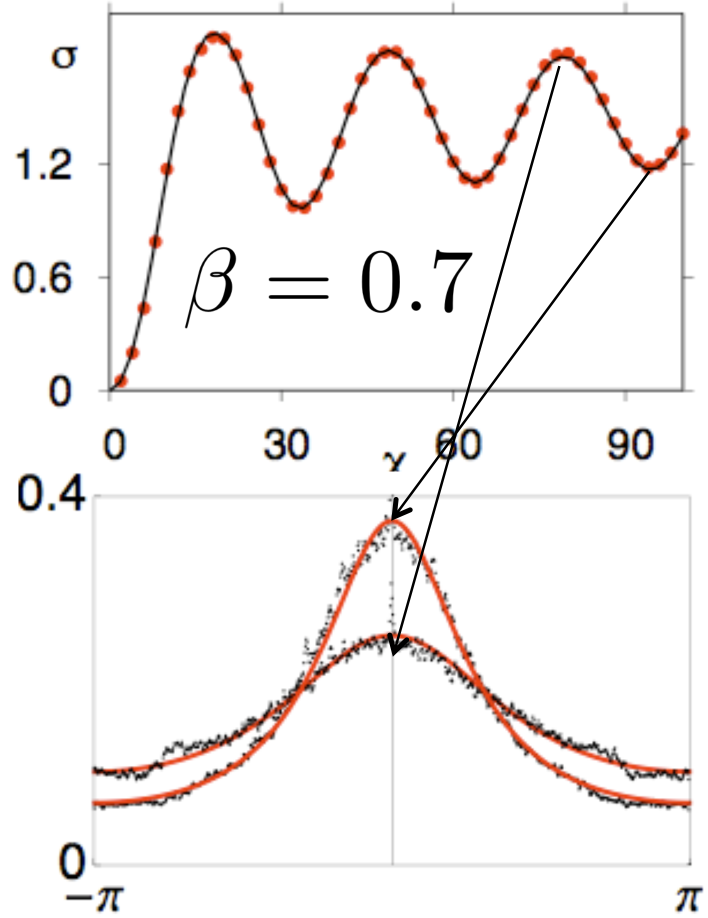
$$f(q) \equiv 2\pi\epsilon(q,k)/k$$

$$\Delta S(q,\delta k) = (\delta k/4\pi^2)[\sin(2\pi q) - \sin(4\pi q)/2]$$

Universal behavior of the LDOS

Local Perturbations

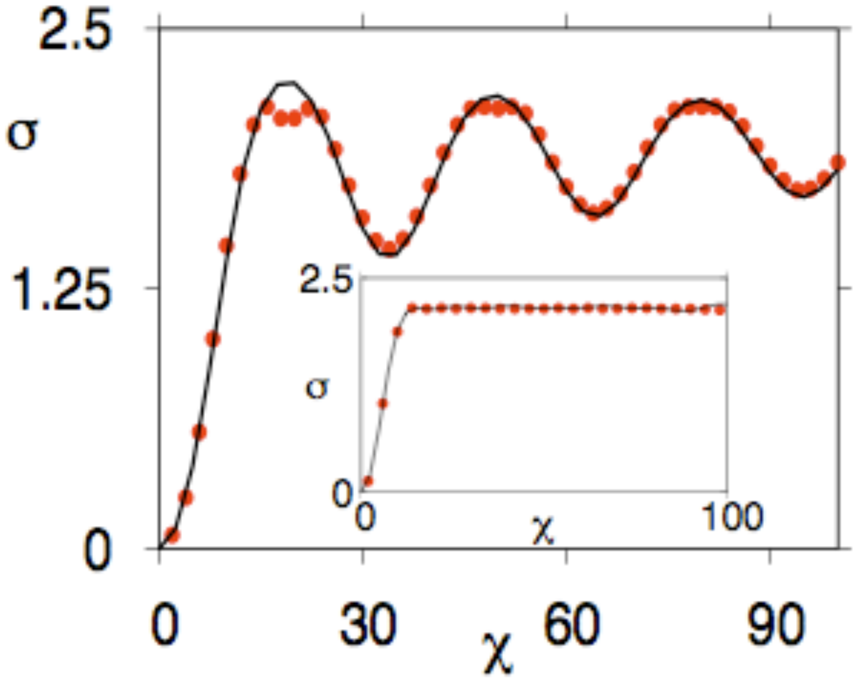




Universal behavior of the LDOS

Global Perturbations

$$\beta = 1$$



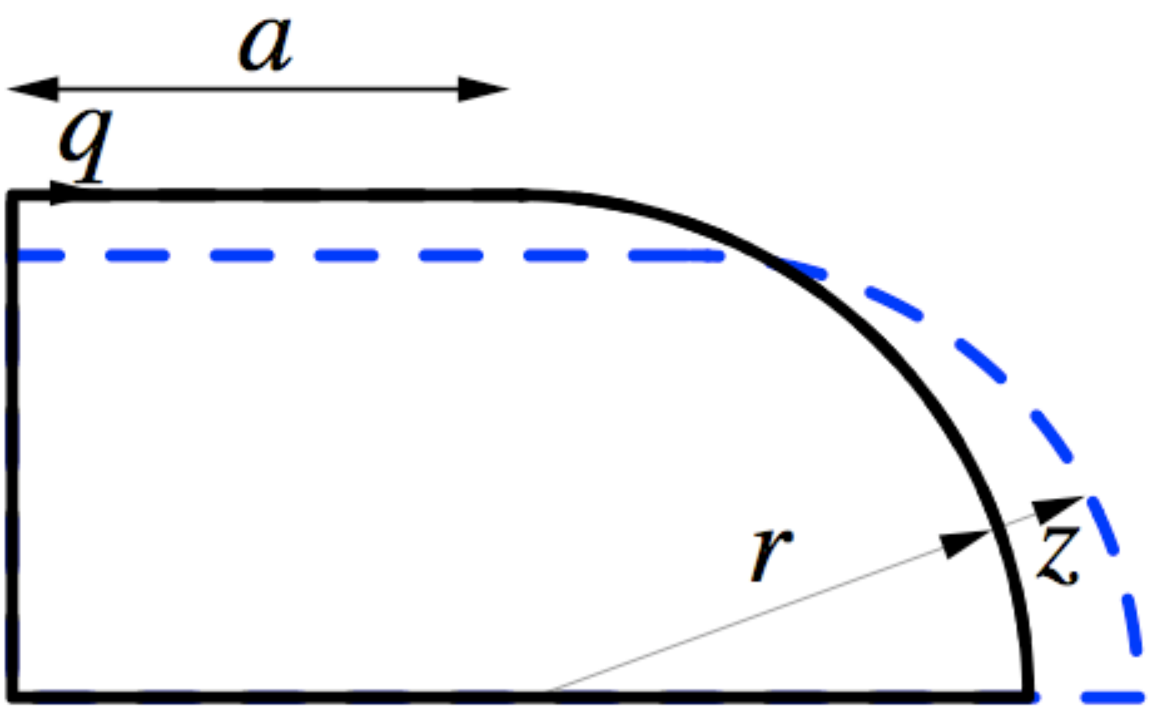
Perturbation: shear in Momentum and position

$$\begin{bmatrix} q' \\ p' \end{bmatrix} = G \begin{bmatrix} q + \bar{\epsilon}(p) \\ p + \epsilon(q) \end{bmatrix}$$

$$\bar{\epsilon}(p) = \frac{k}{2\pi}(sin(6\pi p) - cos(4\pi p))$$

Universal behavior of the LDOS

Example 2: Stadium billiard



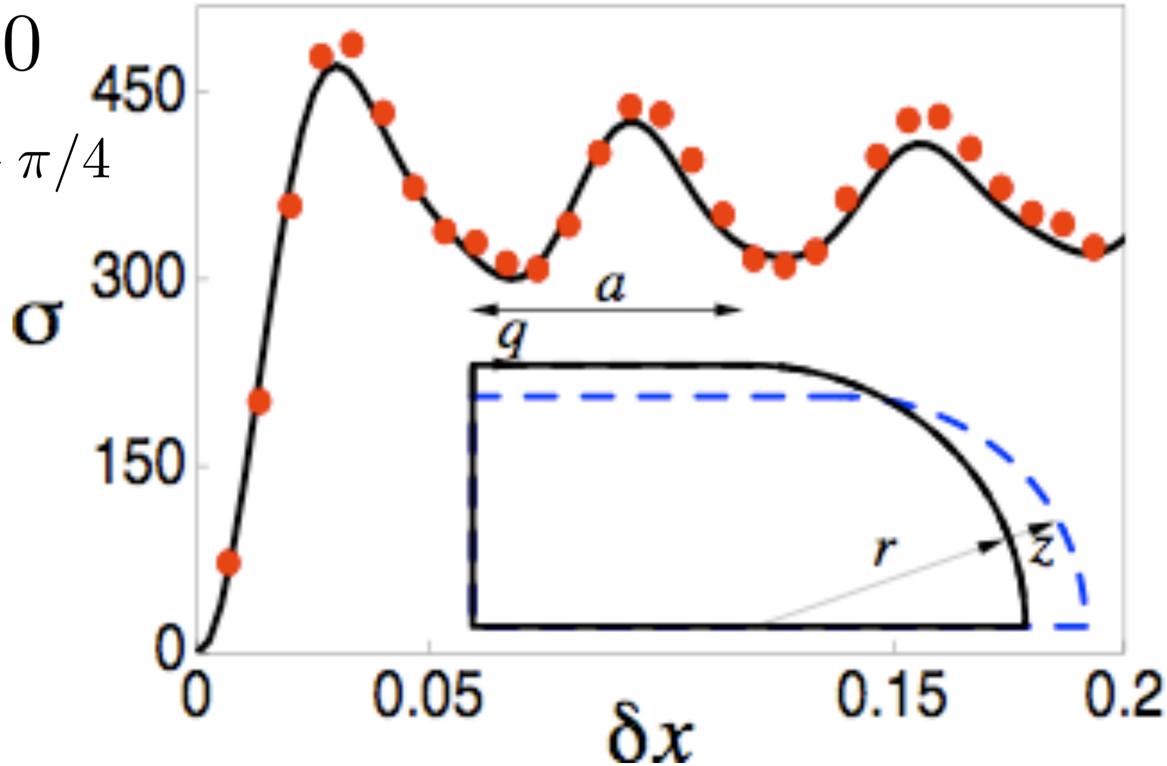
Universal behavior of the LDOS

Example 2: Stadium billiard

wavenumber

$$k = 200$$

$$Area = 1 + \pi/4$$



Universal behavior of the LDOS

We show that the LDOS is a Breit-Wigner distribution under very general perturbations of arbitrary high intensity. This work demonstrates the universal response of quantum systems with classically chaotic dynamics.

Universal behavior of Loschmidt Echo

PHYSICAL REVIEW E **80**, 046216 (2009)

Loschmidt echo and the local density of states

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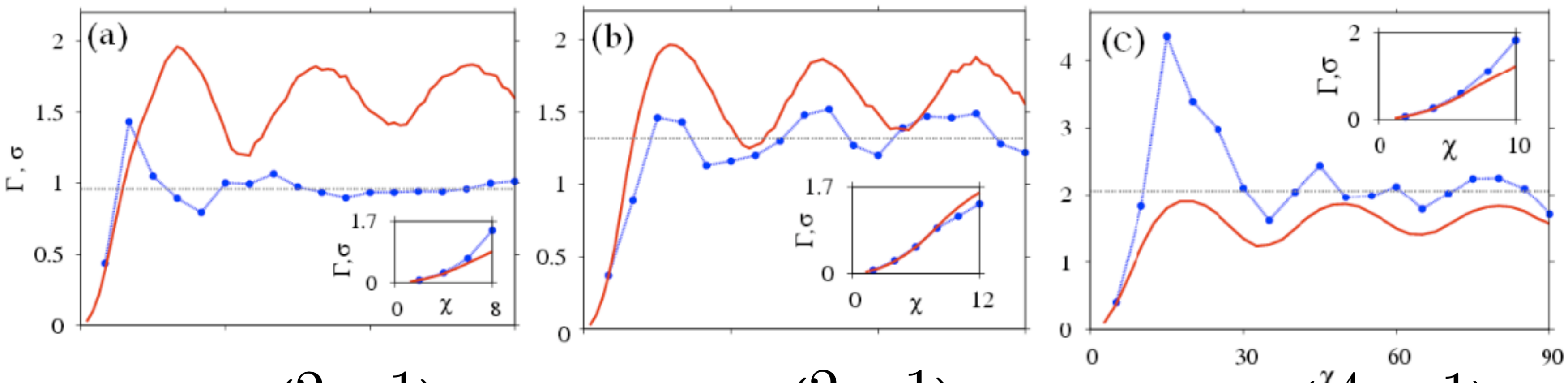
(Received 7 August 2009; published 26 October 2009)



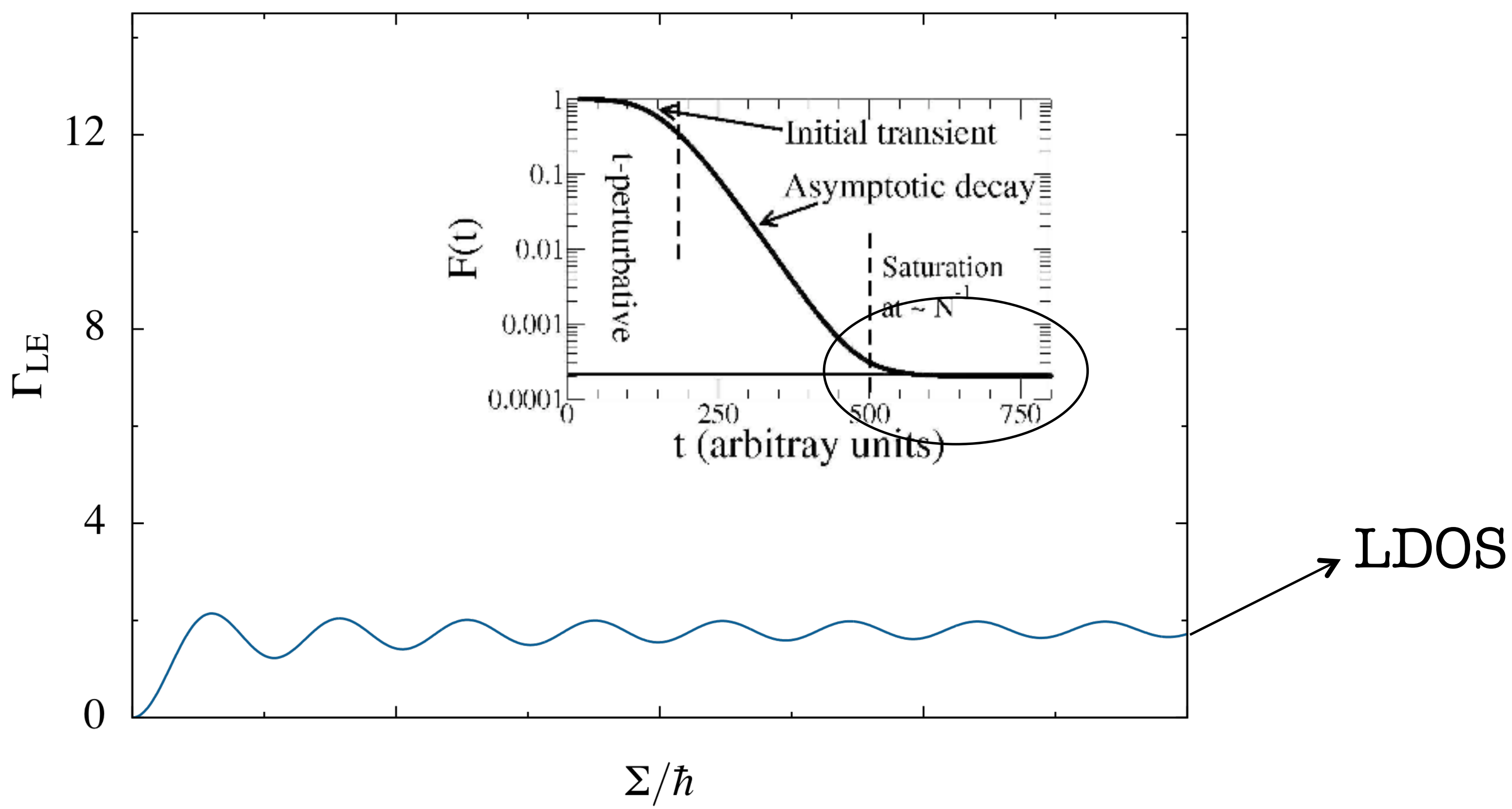
$$G_1 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \qquad G_2 = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \qquad G_3 = \begin{pmatrix} 4 & 1 \\ 15 & 4 \end{pmatrix}$$

$$\lambda_1 = \log(\frac{1}{2}(3+\sqrt{5})) \approx 0.96 \qquad \lambda_2 = \log(2+\sqrt{3})) \approx 1.32 \qquad \lambda_3 = \log(4+\sqrt{15})) \approx 2.06$$

$$\Gamma_{\mathrm{LE}} = min(\Gamma_{\mathrm{LDOS}}, \lambda)$$ Is not true!!!!!!

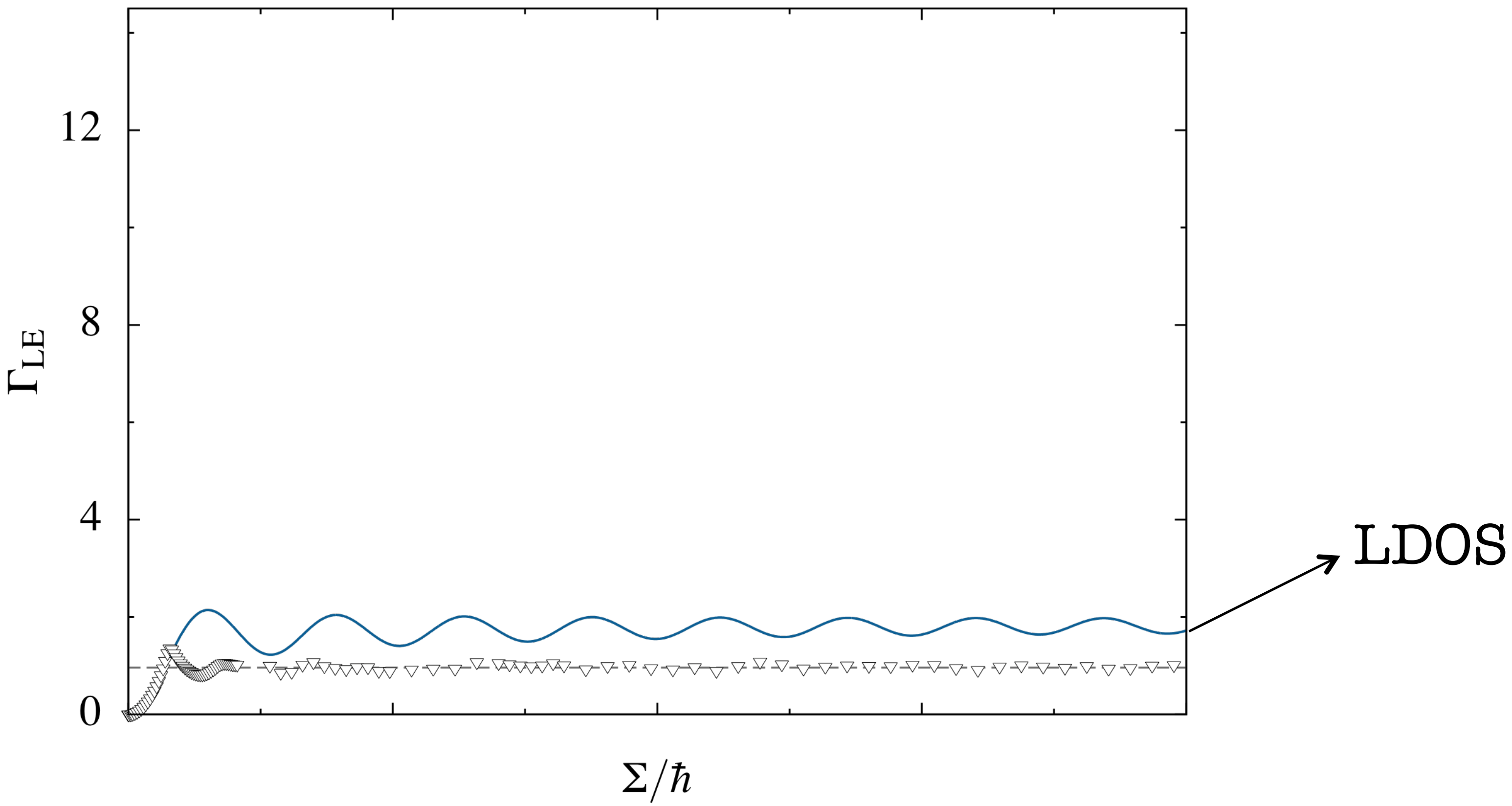
Universal behavior of Loschmidt Echo

N=2^{20}



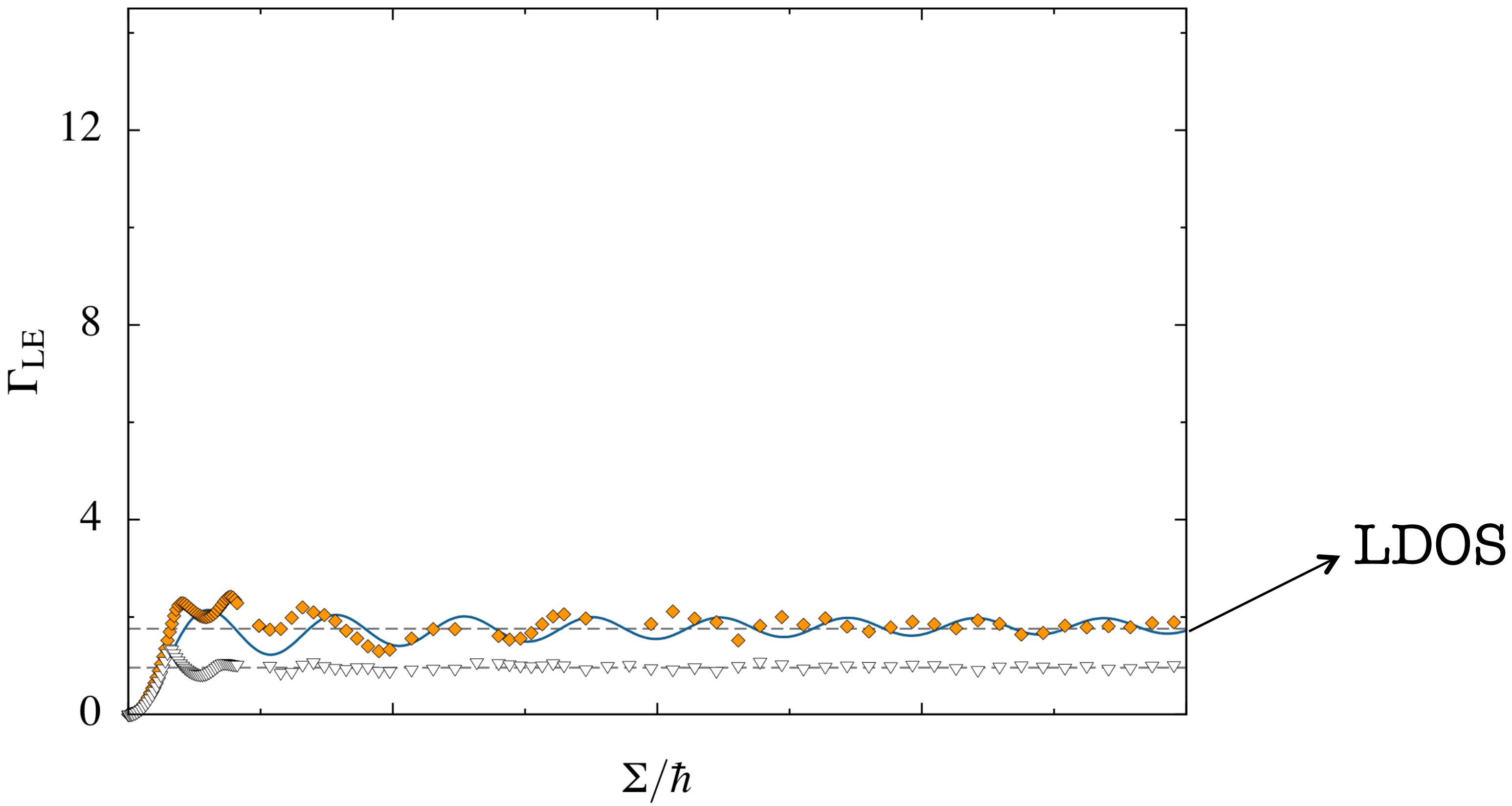
Universal behavior of Loschmidt Echo

$N=2^{20}$



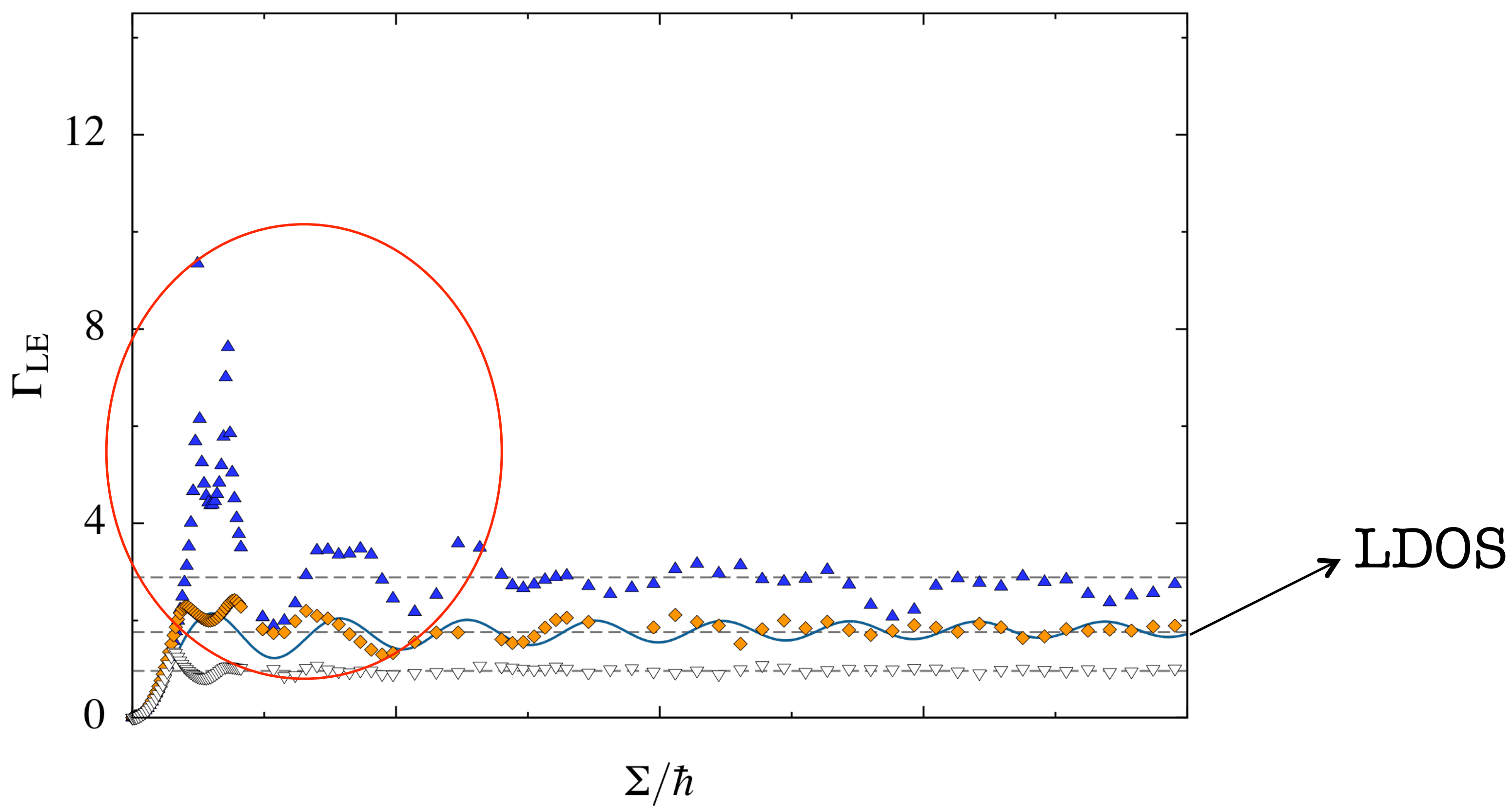
Universal behavior of Loschmidt Echo

$N=2^{20}$



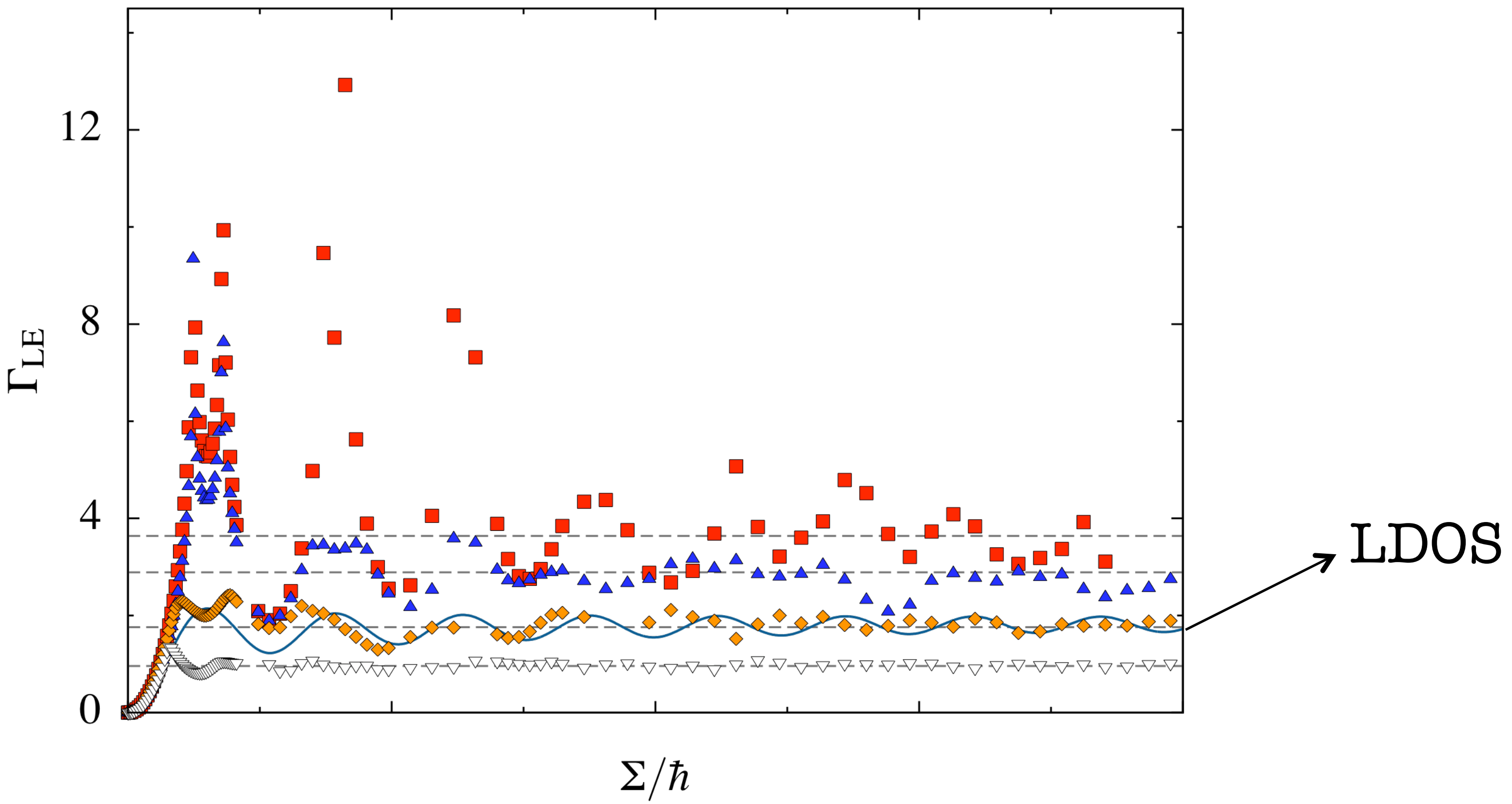
Universal behavior of Loschmidt Echo

$N = 2^{20}$



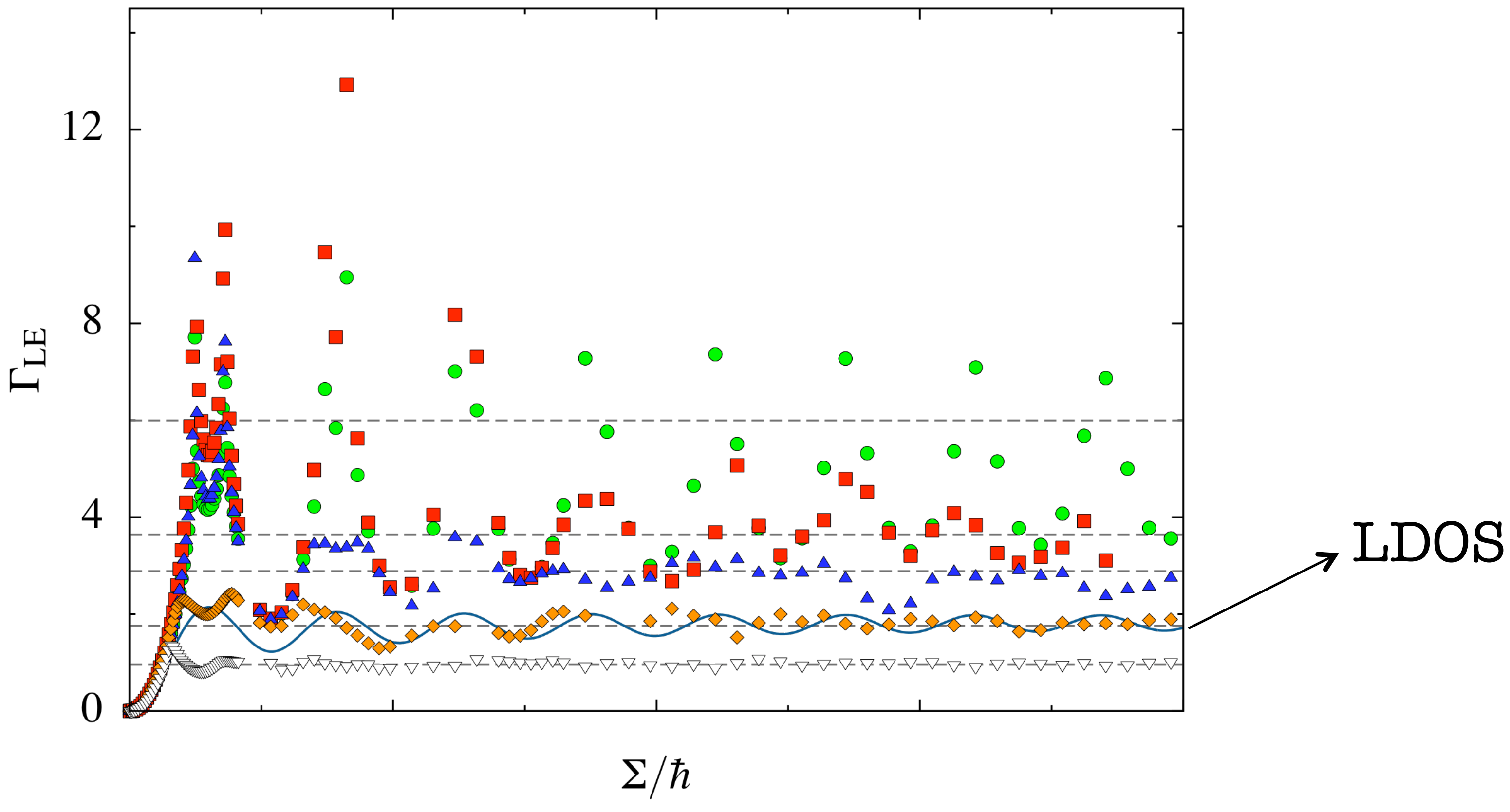
Universal behavior of Loschmidt Echo

N=2^{20}



Universal behavior of Loschmidt Echo

$N=2^{20}$



How can we explain this complex behavior?

Universal behavior of Loschmidt Echo

$$\boxed{M(t) = |O(t)|^2}$$

Dephasing representation (Vanicek 2006)

$$O(t) = \langle\psi|e^{iH_\Sigma t}e^{-iH_0^t}|\psi\rangle = \int dq_0 dp_0 W(q_0, p_0) \exp[-i\Delta S(q_0, p_0, t)/\hbar]$$

$$M(t) = \int dq_0 dp_0 dq_1 dp_1 W(q_0, p_0) W^*(q_1, p_1) \exp[-i(\Delta S(q_0, p_0, t) - \Delta S(q_1, p_1, t))/\hbar]$$

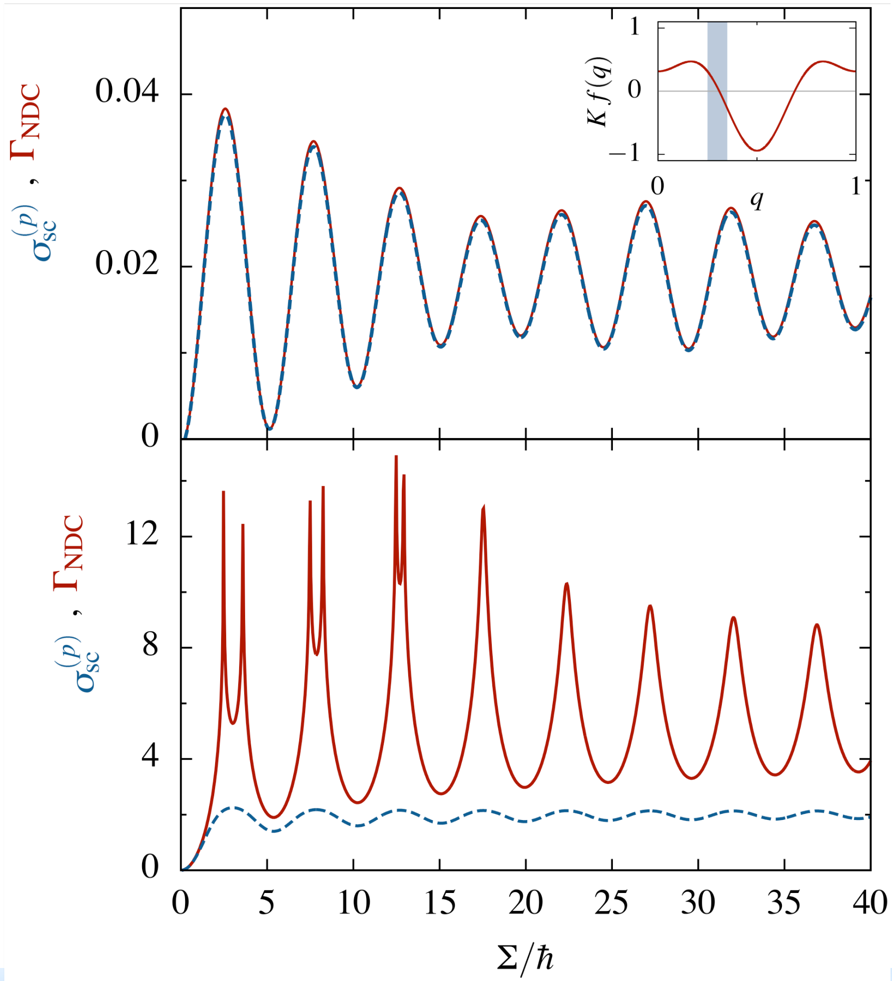
$$\boxed{M(t) = M^{\mathrm{DC}}(t) + M^{\mathrm{NDC}}(t)}$$

$$\boxed{M^{\mathrm{DC}}(t) \sim \exp(-\lambda t)}$$

$$\boxed{M^{\mathrm{NDC}}(t) \sim e^{-\Gamma_{\mathrm{NDC}} t}}$$

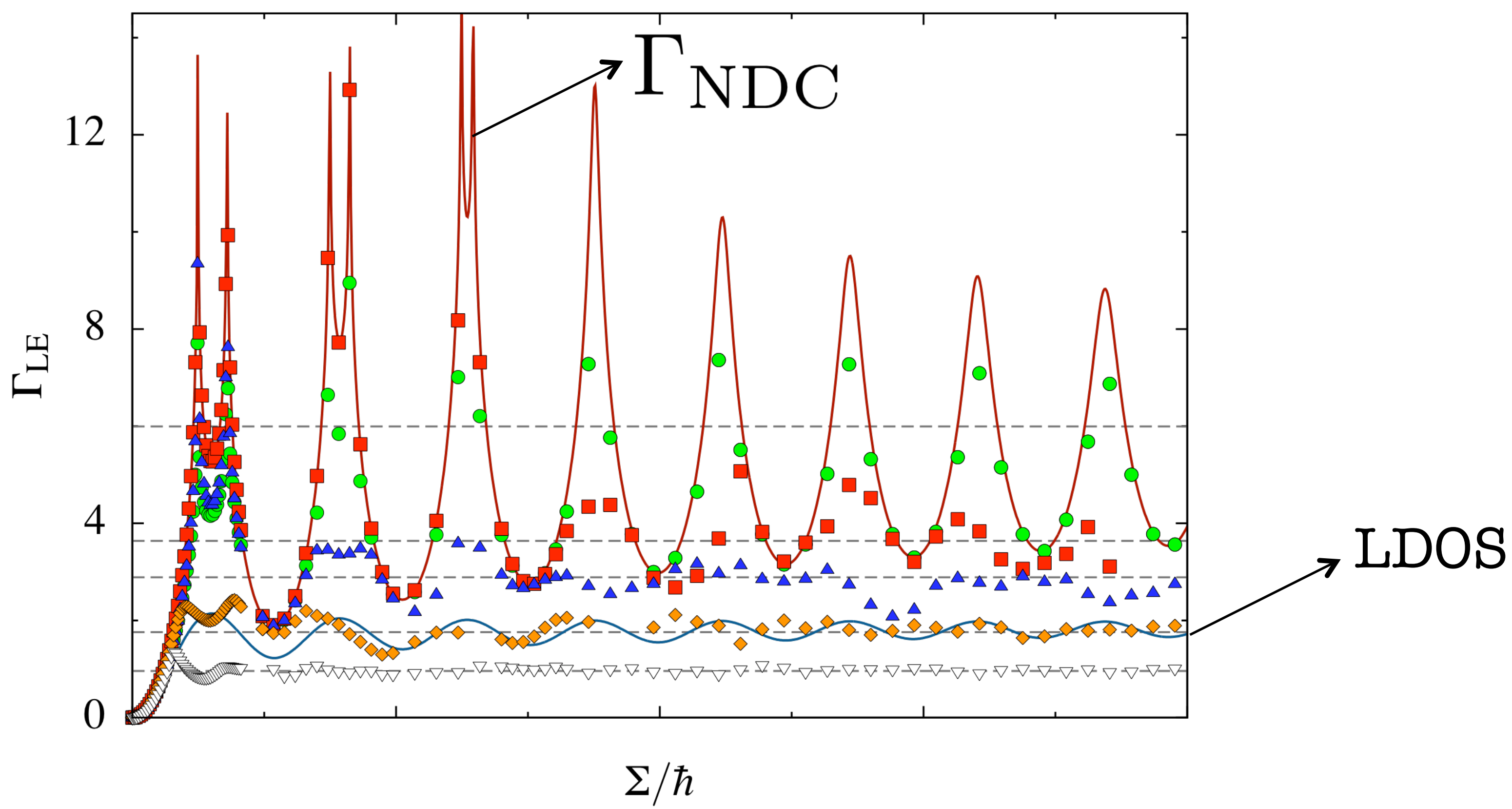
Universal behavior of Loschmidt Echo

$$\Gamma_{\mathrm{NDC}} = -\ln[1 - \alpha(1 - \langle e^{-i\Delta S(q_0,p_0)/\hbar}\rangle)]$$



Universal behavior of Loschmidt Echo

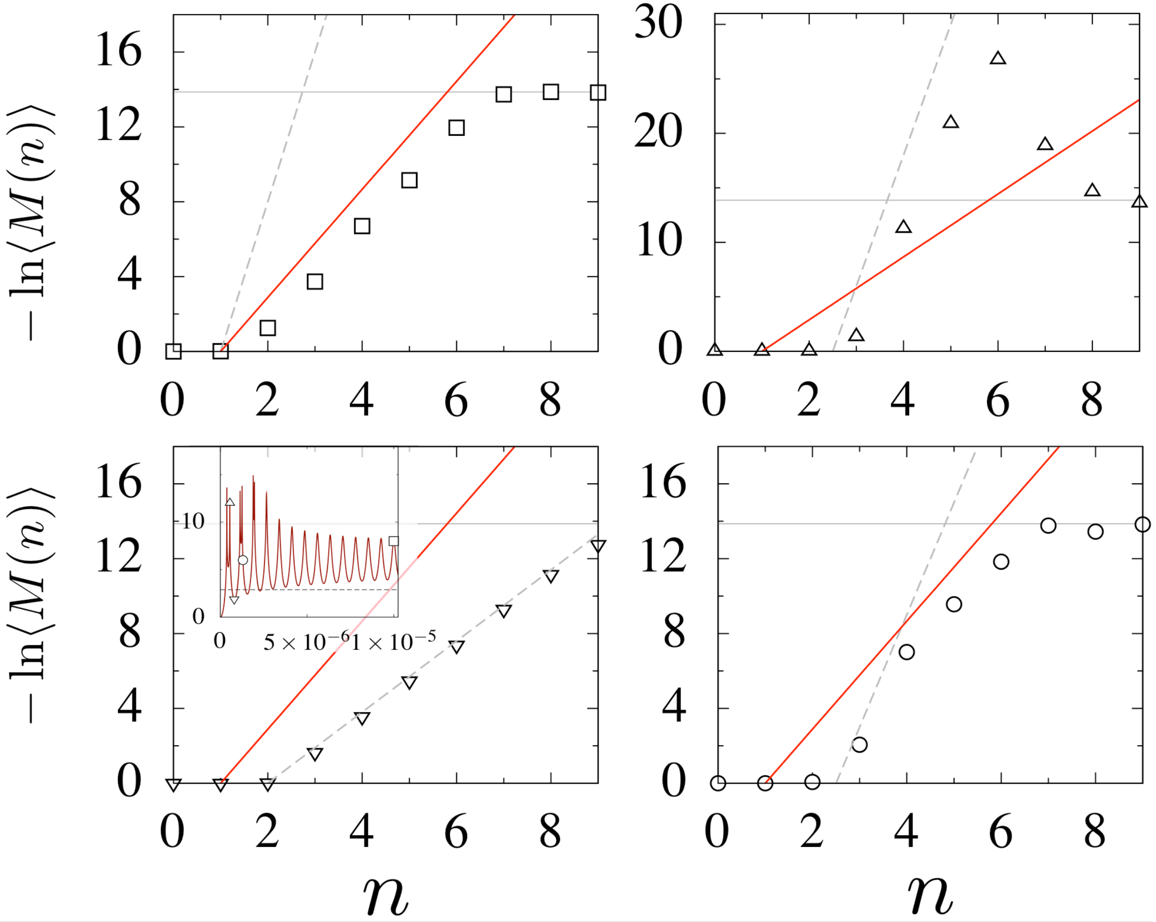
$N=2^{20}$



How can we explain this complex behavior?

Universal behavior of Loschmidt Echo

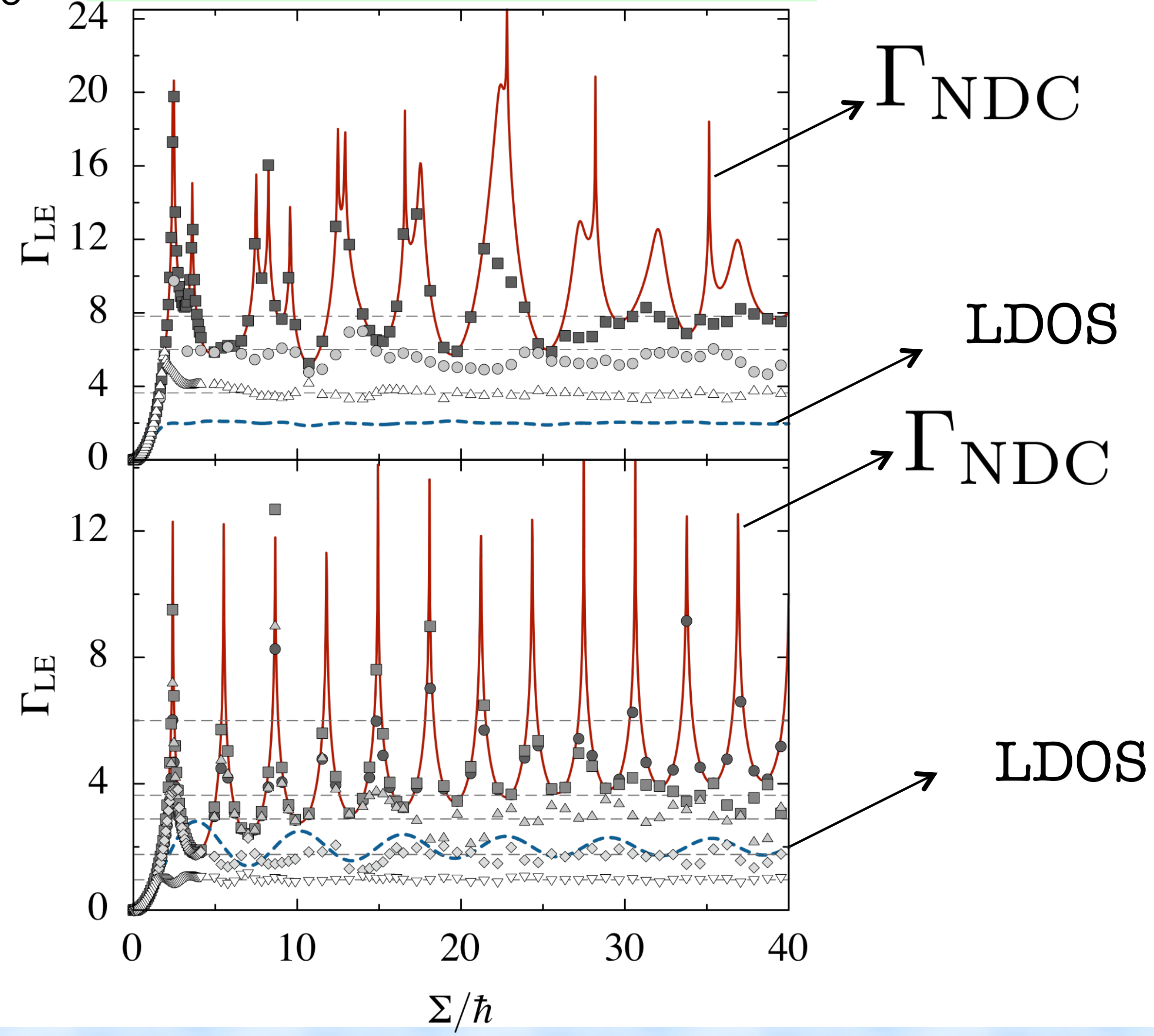
$$M(t) = a\exp(-\lambda t) + b\exp(-\Gamma_{\mathrm{NDC}}(t))$$



Universal behavior of Loschmidt Echo

Shear in momentum and position

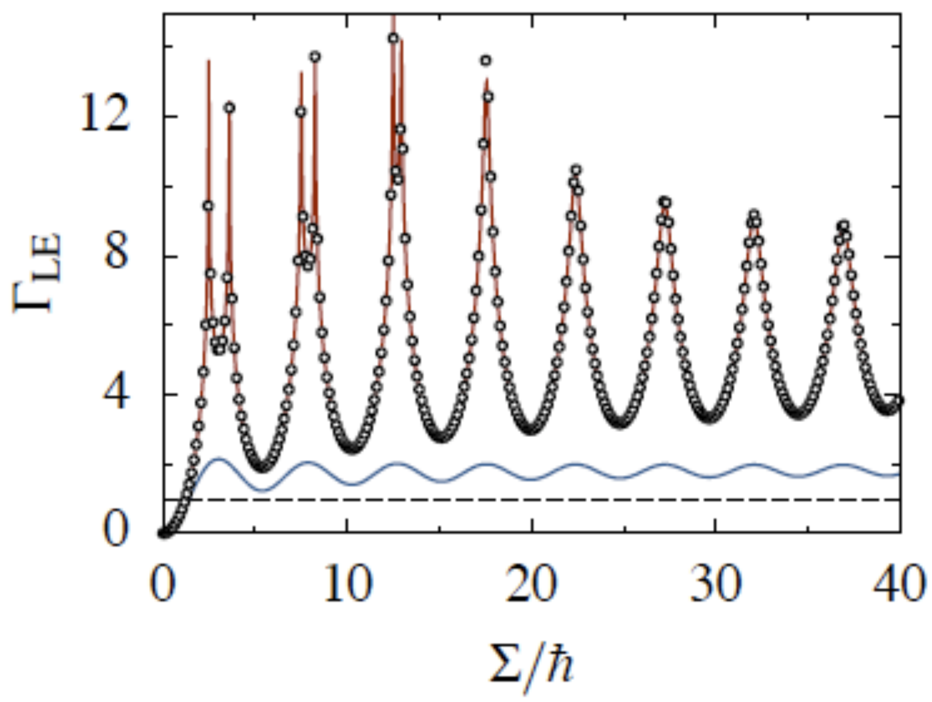
N=2^{20}



Universal behaviour of Loschmidt Echo

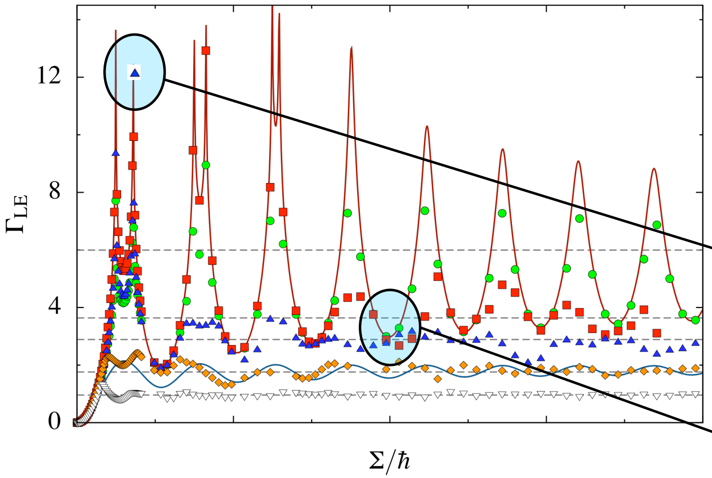
Dependence on the initial state

Position state

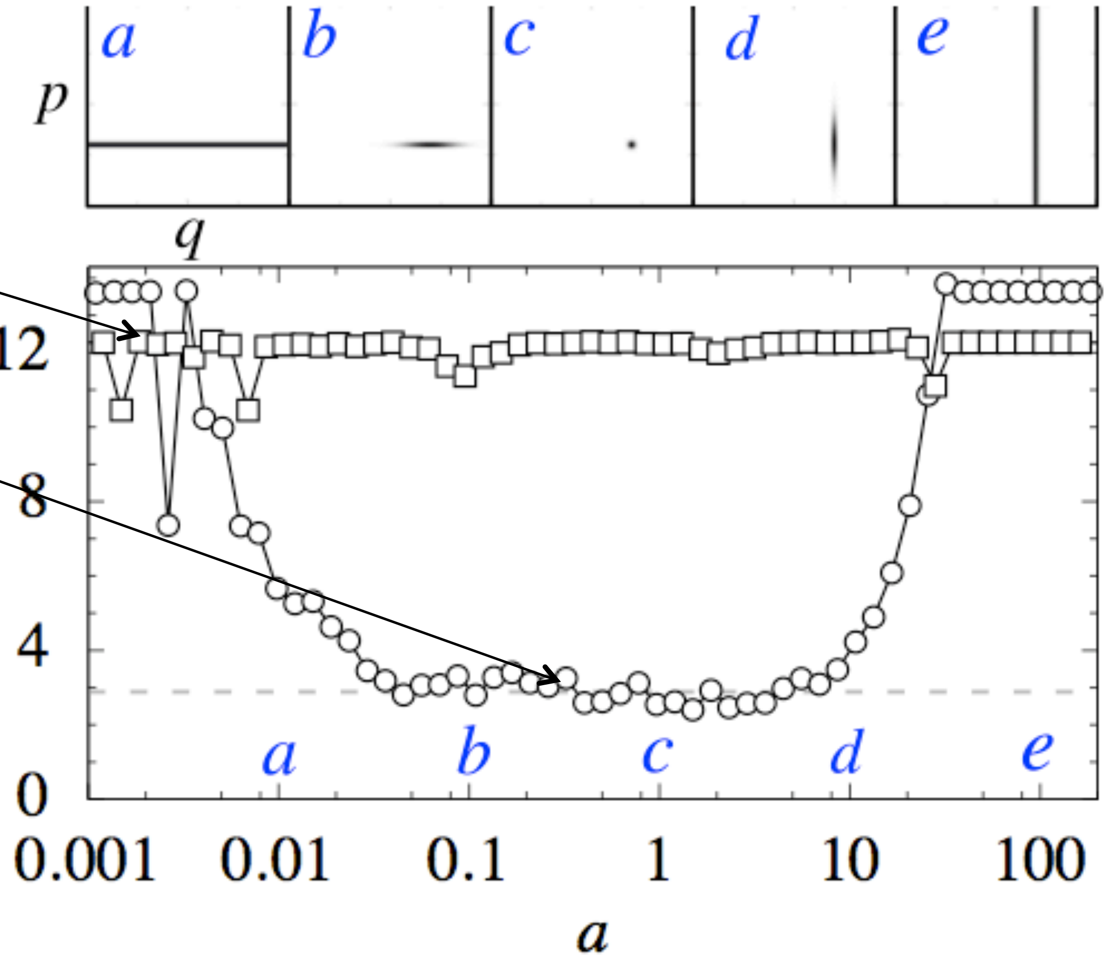


Universal behaviour of Loschmidt Echo

Dependence on the initial state



$$\psi_0(q) = \left(\frac{1}{\pi\hbar a^2}\right)^{1/4} \exp\left[\frac{i}{\hbar}p_0(q-q_0) - \frac{(q-q_0)^2}{2\hbar a^2}\right]$$



Final Remarks

◆ The LDOS is a Breit-Wigner distribution, even for strong perturbations.

◆ We derive a semiclassical expression for the width of the LDOS which is shown to be very accurate for paradigmatic systems of quantum chaos.

◆ Our results demonstrate that quantum systems with classically chaotic dynamics react in a universal way as a consequence of perturbations of classical nature.

Final Remarks

◆ We have demonstrated the importance of the semiclassical non diagonal contribution to the decay of the LE.

◆We have calculated explicitly the decay rate of this term.

◆Contrary to previous assumptions, we have shown that the decay rate of the NDC can – and usually is – very different from the width of the LDOS.

◆Using numerical simulations we have shown that there is an interplay between the DC and the NDC. As the Lyapunov exponent (chaos) increases the importance of the NDC becomes more visible for larger ranges of values of the perturbation strength.

◆When the initial state is not a circular Gaussian state, the NDC clearly dominates the exponential decay of the LE for intermediate times. We have shown this using position states where $\Gamma_{LE} = \Gamma_{NDC}$.