Geometric phase in a quantum open system

Fernando C. Lombardo




F.M. Cucchietti J-F. Zhang P.I. Villar R. Laflamme

ICFO IQC BSC IQC PI

Departamento de Física Juan José Giambiagi
Facultad de Ciencias Exactas y Naturales,
Universidad de Buenos Aires

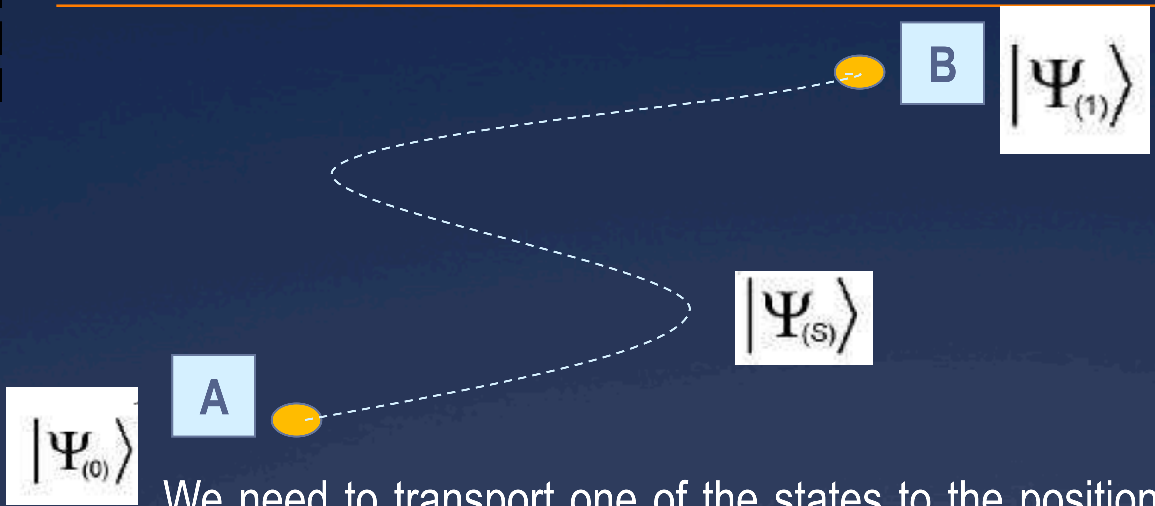
Motivation

- Geometric phases (GPs), arising from cyclic evolutions in a curved space, appear in a wealth of physical settings
- Motivated by the need of an experimentally realistic definition for quantum computing applications, the quantum GP was generalized to open systems
- In the kinematical approach an initial state is evolved cyclically, but coupled to an environment, leading a correction to the GP

Motivation

- We consider a bath that can be tuned near a quantum phase transition
- We demonstrate how the criticality information imprinted in the decoherence factor translates into the GP
- The experiments are done with a NMR quantum simulator
- The GP is measured in a tomographic manner, by measuring the off-diagonal elements of the reduced density matrix of the system

Geometric Quantum Phases: Introduction



Which is the angle between these two vectors? (which is the relative phase between them?)

We need to transport one of the states to the position of the other, and when they are close, measure the angle. But, we do not have to introduce extra-phases when transporting the state.

Direct path: Geodesic

Evolution: parallel transport

Geometric Quantum Phases: Introduction

Let's suppose an infinitesimal transformation $|\psi(s)\rangle \Rightarrow |\psi(s+ds)\rangle$

No extra-phases $\Rightarrow$ $\mathrm{Arg}\{\langle\psi(\mathrm{s})|\psi(\mathrm{s}+\mathrm{ds})\rangle\} = 0$

$$\mathrm{Im}\{\langle\psi(\mathrm{s})|\psi(\mathrm{s}+\mathrm{ds})\rangle\} = \mathrm{Im}\{\langle\psi(\mathrm{s})|\mathrm{d}|\psi(\mathrm{s})\rangle\} = 0$$

(at first order)

$$\langle\psi(s)|d|\psi(s)\rangle = 0$$

Gauge transformation

$$|\tilde{\psi}(s)\rangle = e^{i\alpha(s)}|\psi(s)\rangle \qquad \langle\tilde{\psi}(s)|\mathrm{d}|\tilde{\psi}(s)\rangle = \langle\psi(s)|\mathrm{d}|\psi(s)\rangle + i\frac{\mathrm{d}\alpha}{ds}ds$$

• • • • • • • • •

Geometric Quantum Phases: Introduction

Integrating in a closed loop we get a gauge invariant quantity

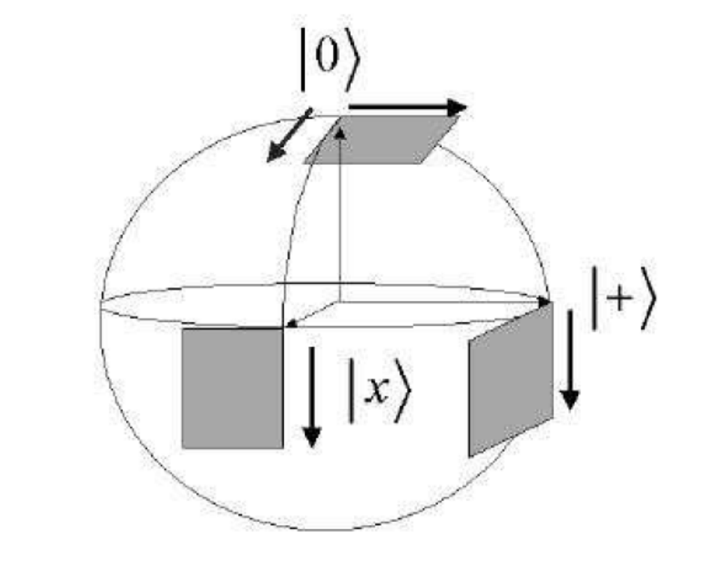
Geometric Phase

$$\gamma = \int_i^f \langle\psi(s)|\frac{d}{ds}|\psi(s)\rangle ds$$

Why there is a total phase in a closed loop if the phase difference in each infinitesimal step is zero?

CURVATURE!! of the parameter space

Example: Bloch Sphere



½ spin - particle (SU(2))

|0> → |+> = |0> + |1>

|x> = |0> + i |1>

|0> ← |x> = |0> + i |1>

There is an angle π/2 between the initial and the final state (it is the surface covered by the state vector during the parallel transport)

• • • • • • • • •

Berry phase: adiabatic, cyclic evolution of pure states (M. Berry, Proc. R. Soc. London, 84)

The GP has been extended to the case of nonadiabatic evolutions (Aharonov & Anandan, PRL 58, 1987 and PRD 38, 1988)

GP for mixed states undergoing a cyclic unitary evolution (Sjoqvist et al, PRL 85, 2000)

GP for mixed states in a non-unitary evolutions (Tong et al, PRL 93, 2004)

Geometric Phases and Decoherence

Closed quantum systems → Geometric Phases (GP)

Open quantum systems → Corrections to the unitary GP

Information from the environment

Decoherence

Effects

Shift of the interference fringes

The decoherence time is crucial because for $t > t_D$ the interference pattern disappears and the phase can not be measured any longer

We have studied the correction to the unitary GP. We estimated the decoherence time for different open quantum systems.

SPIN-BOSON and SPIN-SPIN MODELS

F.L. & P. Villar, PRA 74, 042311 (2006)

F. L. & P. Villar, Int. J. of Quantum Computation, (2008)

Some points

Due to its global properties, the GP is propitious to construct fault tolerant quantum gates

GPs depend on the geometry of the space the states traverse, GPs can provide information on this space. This property can be used to discover the coordinates of quantum phase transitions in the parameter space

GPs can be studied for entangled bipartite systems. It has been shown that entanglement could affects the geometric phase (Vedral ’07)

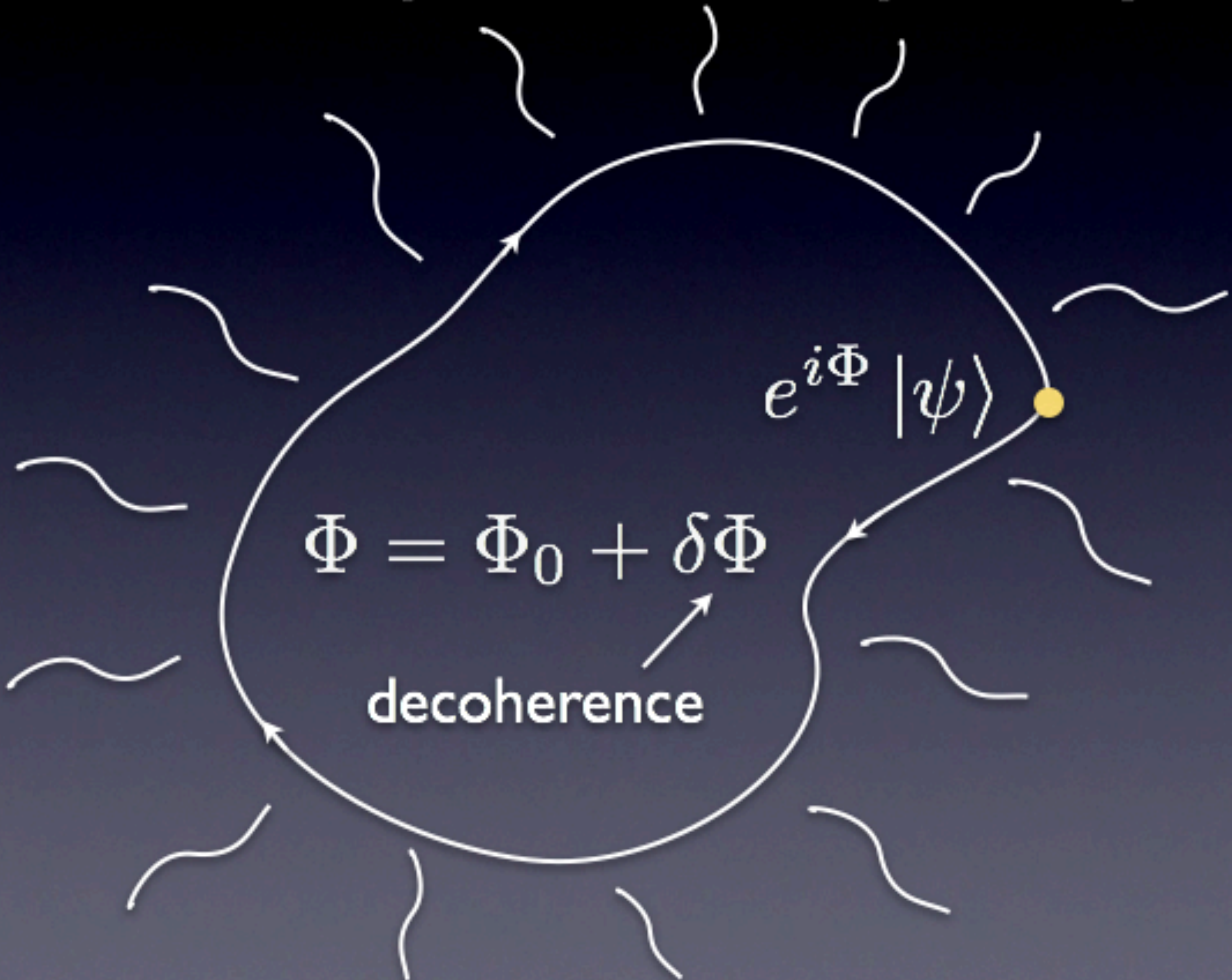
• • • • • • • • •

The first attempt

The idea is to estimate how the unitary GP is corrected by the presence of the environment and study under what conditions this correction can be measured

• • • • • • • • •

Geometric phase in open systems



Geometric Phases for Non Unitary Evolution (Mixed States)

$$\Phi = \arg\{\sum_k \sqrt{\varepsilon_k(0)\varepsilon_k(\tau)}\langle\Psi_k(0)|\Psi_k(\tau)\rangle e^{-\int_0^\tau dt\langle\Psi_k|\frac{\partial}{\partial t}|\Psi_k\rangle}\}$$

τ is the time after which the system completes a cyclic evolution

$$\tau = \frac{2\pi}{\Omega}$$

It is gauge invariant and corresponds to the unitary geometric phase when the state is pure and the system closed

Tong, Sjoqvist, Kwek & Oh. PRL 93, 080405 (2004)

The central result of Tongs' equation is to extract from the global phase acquired during the evolution, by a proper choice of the “parallel transport condition”, the purification independent part which can be termed a geometric phase (GP)

Spin-Boson Model: Geometric Phase and Decoherence

$$H_{SB} = \underbrace{\frac{1}{2}\hbar\Omega\sigma_z}_{H_S} + \frac{1}{2}\sigma_z\sum_k \lambda_k (g_k a_k^{\dagger} + g_k^{*} a_k) + \sum_k \hbar\omega_k a_k^{\dagger} a_k$$

This model captures many of the elements of decoherence theories and sheds some insight into the modification of the GPs due to the presence of an environment. Is has been used to model decoherence in quantum computers (Viola & Lloyd '98) and for a proposal of observing GPs in a superconducting nanocircuit (Vedral '00)

$$[\sigma_z, H_{\rm int}] = 0$$

Purely dep

no dissipa

$$\dot{\rho}_{\rm r} =$$

I(w) ~w^n is the environment spectral density,

$$\mathcal{D}(s) = \int_0^s ds' \int_0^\infty d\omega I(\omega) \coth\left(\frac{\omega}{2k_B T}\right) \cos(\omega(s - s'))$$

We evaluated the correction to the unitary GP for different environments:

$$I(\omega) = \frac{2}{\pi} M \gamma_0 \omega \left(\frac{\omega}{\Lambda}\right)^{\alpha-1} e^{-\frac{\omega^2}{\Lambda^2}}$$

Frequency cutoff

α=1 Ohmic Environment

High Temperature

$$\delta\Phi = \pi^2 \gamma_0 \left(\frac{\pi k_B T}{\hbar\Omega}\right) \sin^2(\theta_0)$$

Zero Temperature

$$\delta\Phi = \pi\gamma_0 \left(\log\left(\frac{2\pi\Lambda}{\Omega}\right) - 1\right) \sin^2(\theta_0)$$

α>1 Supra Ohmic Environment

High Temperature

$$\delta\Phi = \pi\gamma_0 \left(\frac{2 k_B T}{\hbar\Lambda}\right) \sin^2(\theta_0)$$

Zero Temperature

$$\delta\Phi = \pi\gamma_0 \sin^2(\theta_0)$$

F.L. & P. Villar, PRA 74, 042311 (2006)

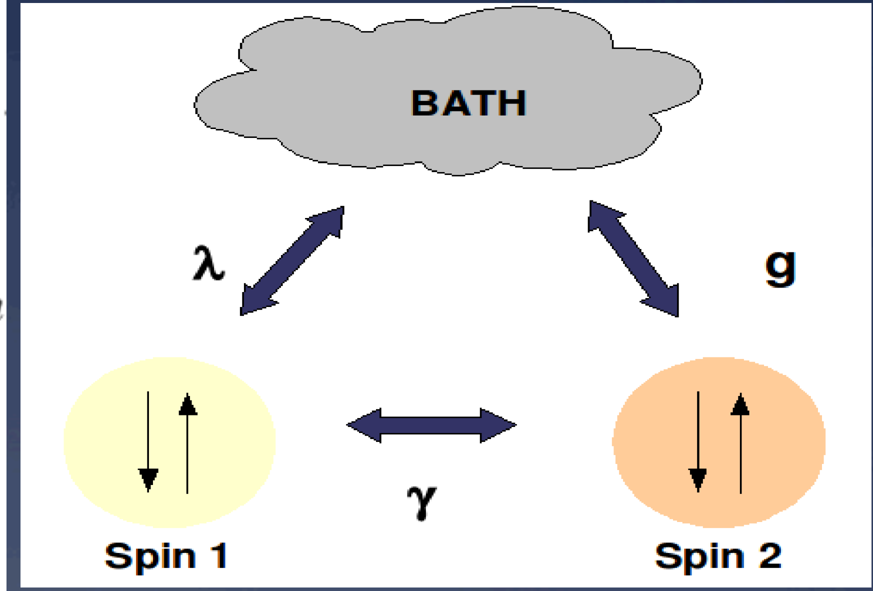
GPs, entanglement and decoherence

We have computed the GP of a bipartite two-level system coupled to an external environment, whether oscillators or spin environment

$$H_S = \frac{\hbar\Omega_1}{2}\sigma_z^1 + \frac{\hbar\Omega_2}{2}\sigma_z^2 + \gamma\sigma_z^1 \otimes \sigma_z^2$$

$$H_I = \sigma_z^1 \otimes \sum_{n=1}^{N} \lambda_n q_n + \sigma_z^2 \otimes \sum_{n=1}^{N} g_n q_n$$

$$H_B = \sum_{n=1}^{N} \hbar\omega_n a_n^\dagger a_n,$$

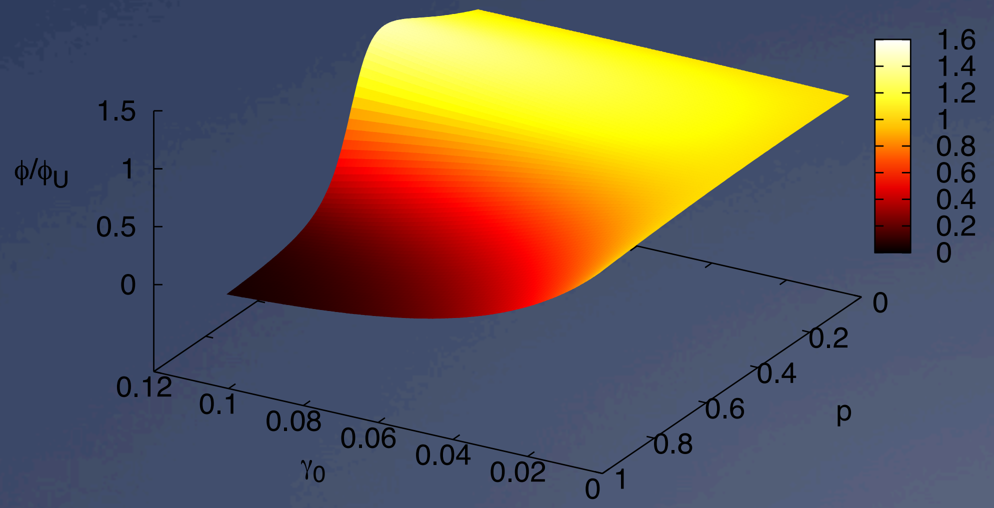


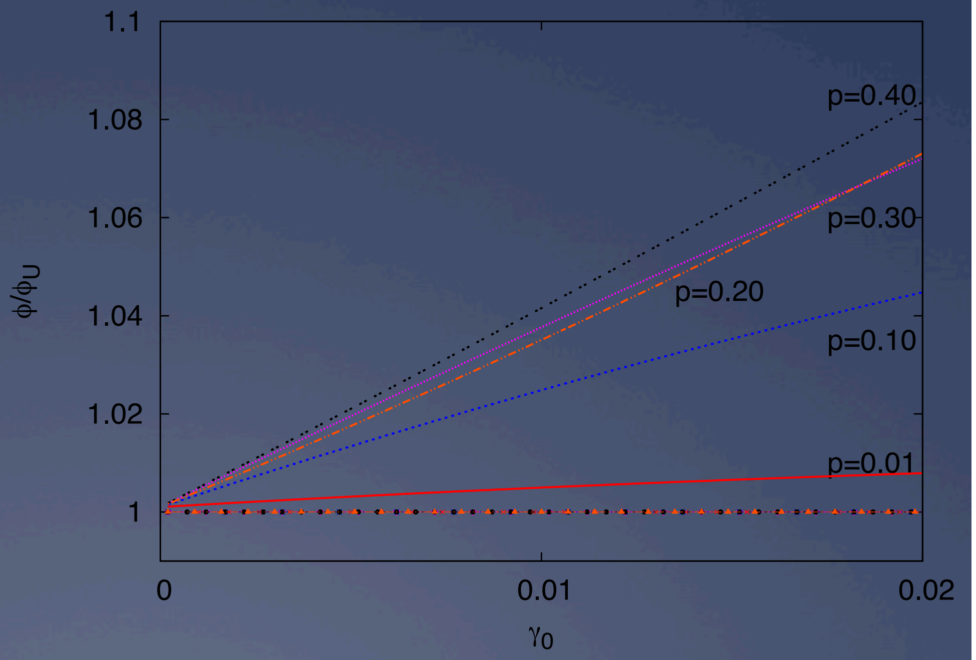
Initial density matrix for two possible cases

$$\rho_r(0) = \frac{1-r}{4} I + r|\phi\rangle\langle\phi|$$

$$|\phi\rangle \begin{cases} |\vartheta\rangle = \sqrt{1-p}|00\rangle + \sqrt{p}|11\rangle \\ |\mu\rangle = \sqrt{1-p}|01\rangle + \sqrt{p}|10\rangle \end{cases}$$

$$\delta\phi_G \approx 32\pi\gamma_0 p(1-3p+2p^2)$$





Main Results

We have checked that the correction to the GP is ZERO for the case of a maximally entangled state. This is due to the topological origin of the phase for this kind of bipartite states

We have found a steady relation between the GP and the concurrence for a Bell state in the isolated situation $\Phi^U = \pi C$

The correction to the unitary GP would be proportional to a winding number, which might imply that the OPEN GP still has some of its geometric character. However, this correction also depends on the spectrum and the total OPEN GP is not simply a geometric quantity

F.L. & P.I. Villar, Phys. Rev. A 81, 022115 (2010)

New stuff: arXiv:1006.1468

What happens to $\delta\Phi$ if the environment is near a critical point?

New stuff

Quantum geometric phase in systems with a critical environment

The main idea is to show the relation between quantum phase transitions, decoherence, and the geometric phase

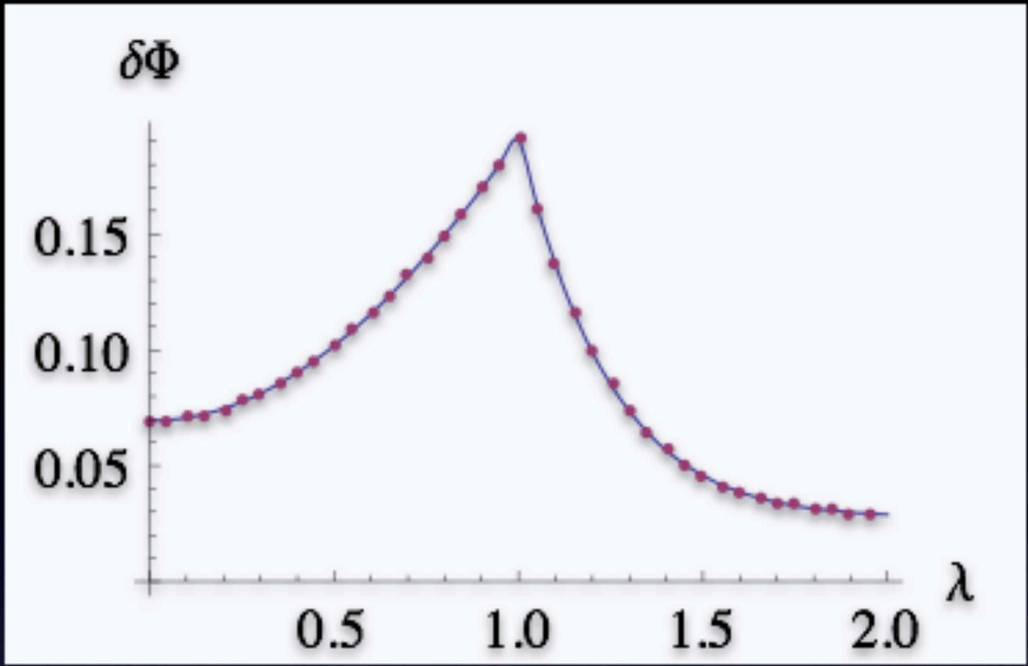
We compute the correction to the GP of a spin $\frac{1}{2}$ coupled to an environment that produces dephasing only: ising spin chain and Landau-Zener examples

• • • • • • • • •

Ising spin chain

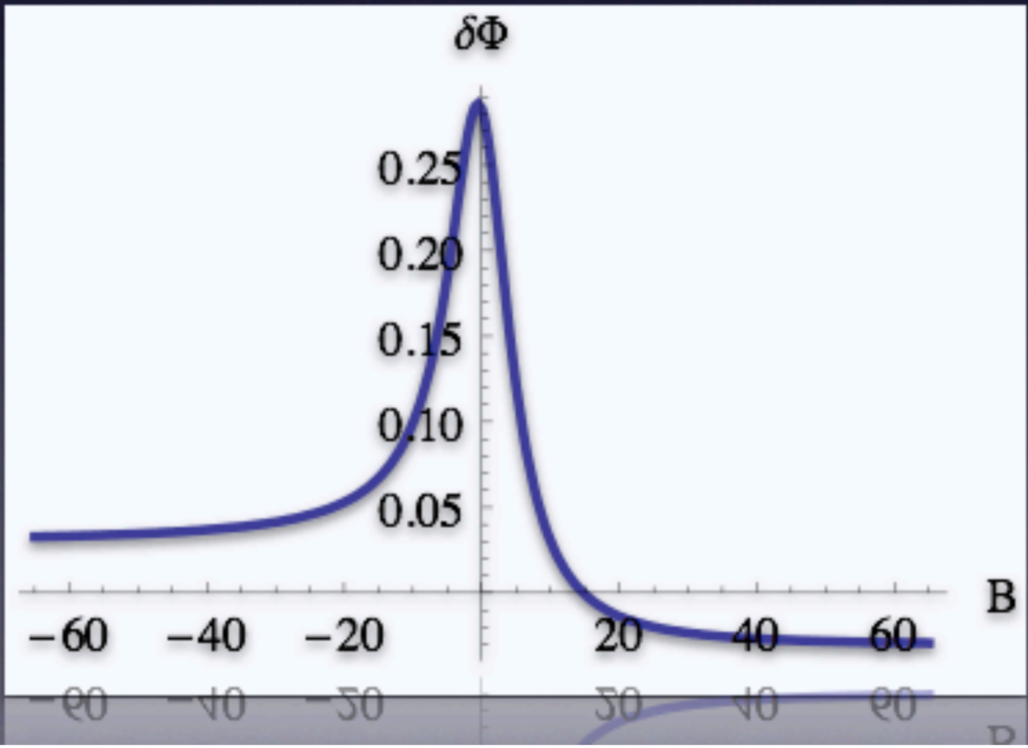
We studied a paradigmatic example of quantum criticality: the Ising sping chain with a homogeneous transverse field

1000 spins. A full quantum simulation of a large enough critical system in on the edge of current technology



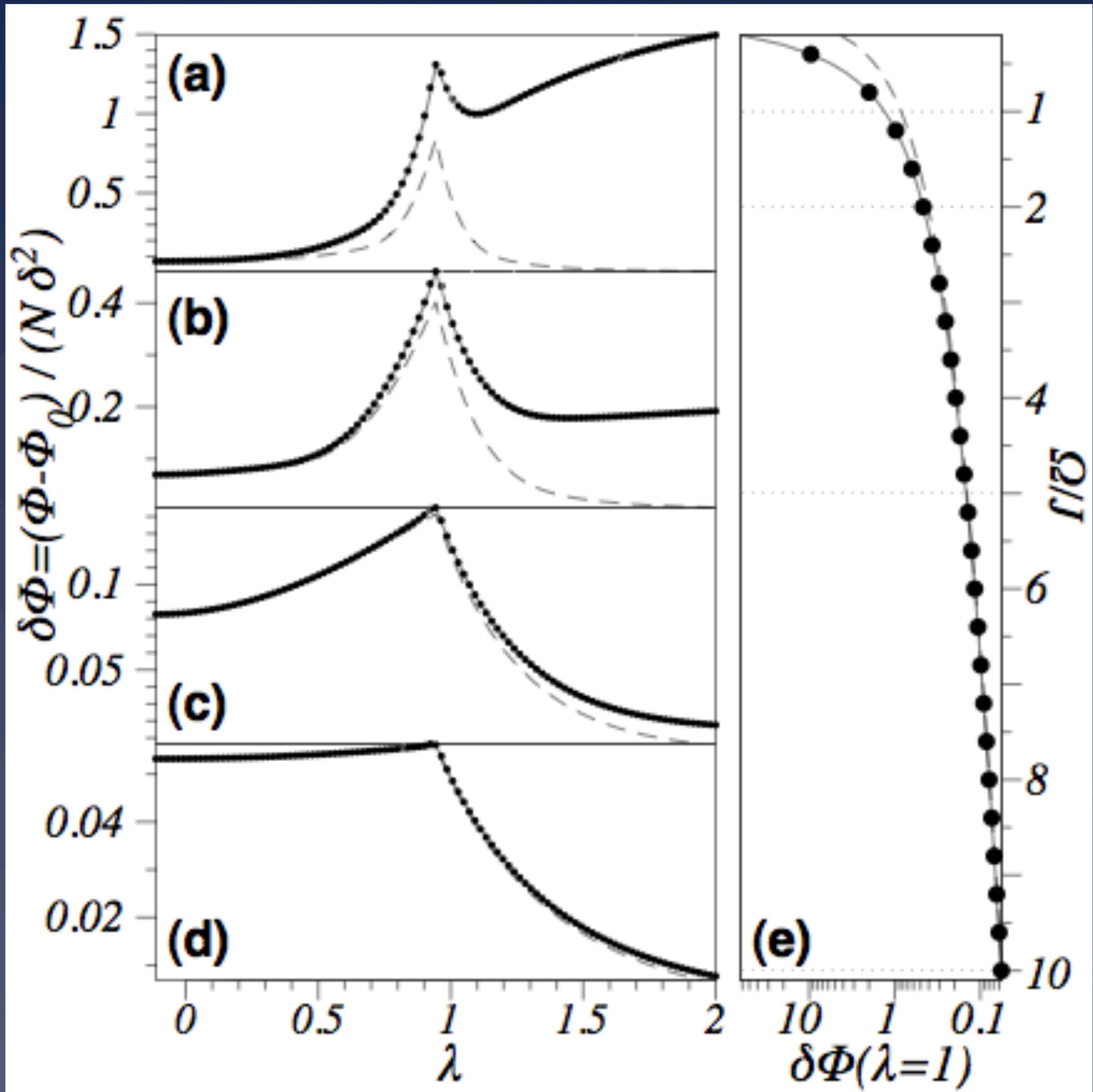
Ising chain in a transverse field, 1000 spins

At the point where the environment is critical, the GP becomes singular in the thermodynamical limit



Landau Zener qubit

GP of the central spin in presence of an Ising spin chain as a function of the transverse field of strength λ, for different self-energies of the system. We can see that the third order approximation to the exact formula works properly



Third order expansion

$$
\begin{aligned}
|r(t)|^2 &= 1 - R_{(2)}(t)\delta^2 - R_{(3)}(t)\delta^3 + \mathcal{O}(\delta^4) \\
\varphi(t) &= \varphi_{(1)}(t)\delta + \varphi_{(2)}(t)\delta^2 + \varphi_{(3)}(t)\delta^3 + \mathcal{O}(\delta^4)
\end{aligned}
$$

$$
\begin{aligned}
\Phi &\simeq \pi(1+\cos\theta) + \cos\theta\sin^2\theta\left[\delta^2\frac{1}{4}\Omega\int_0^\tau R_{(2)}(t)\ dt\right. \\
&+ \left.\frac{1}{24}\delta^3\left(3R_{(2)}(\tau)\varphi_{(1)}(\tau) + \varphi^3_{(1)}(\tau) + 6\Omega\int_0^\tau R_{(3)}(t)\ dt - 6\int_0^\tau R_{(2)}(t)\frac{\partial\varphi_{(1)}}{\partial t}(t)\ dt\right)\right]
\end{aligned}
$$

$$
r(t) = \prod_{k>0} R_k(t)e^{i(\varphi_k(t)-\varepsilon_k t)} = Re^{i\varphi}
$$

$$
\varepsilon_k = 2\sqrt{1+\lambda^2-2\lambda\cos(k)}
$$

Critical environment: a toy model

Model $$H = \Omega Z_{\mathcal{S}} \otimes I_{\mathcal{E}} + Z_{\mathcal{S}} \otimes H_{\mathcal{S}\mathcal{E}} + I_{\mathcal{S}} \otimes H_{\mathcal{E}}$$

$Z_{\mathcal{S}} \otimes H_{\mathcal{S}\mathcal{E}}$ Dephasing spin-bath interaction

Product initial state for the system-bath $$\rho(0) = |\psi_0\rangle \langle\psi_0| \otimes |\varepsilon(0)\rangle \langle\varepsilon(0)|$$

where $$|\psi_0\rangle = \sin(\theta/2)\,|0\rangle + \cos(\theta/2)\,|1\rangle$$

For the isolated case, the spin follows an evolution around the Bloch sphere, reaching again the initial state for $\tau = 2\pi/\Omega$

Dynamical phase $$\Phi_d = -\pi\cos(\theta)$$

Geometric Phase (gauge invariant) $$\Phi_g = \pi(1 - \cos(\theta))$$

When coupled to an environment:

$$\rho_{\mathrm{r}}(t) = \sin^2(\theta/2)\,|0\rangle\langle 0| + \cos^2(\theta/2)\,|1\rangle\langle 1| + \frac{\sin\theta}{2}e^{-i2\Omega t}r(t)\,|0\rangle\langle 1| + \frac{\sin\theta}{2}e^{i2\Omega t}r^*(t)\,|1\rangle\langle 0|$$

Decoherence factor
(complex)

$$r(t) \quad = \quad \langle \varepsilon_0(t)|\varepsilon_1(t)\rangle$$

$$r(t) = |r(t)|e^{-i\varphi}$$

Geometric phase

$$\Phi = \int_0^\tau dt \left(\Omega - \frac{\partial\varphi}{\partial t}\right) \sin^2\!\left(\frac{\theta_t^+}{2}\right) +$$

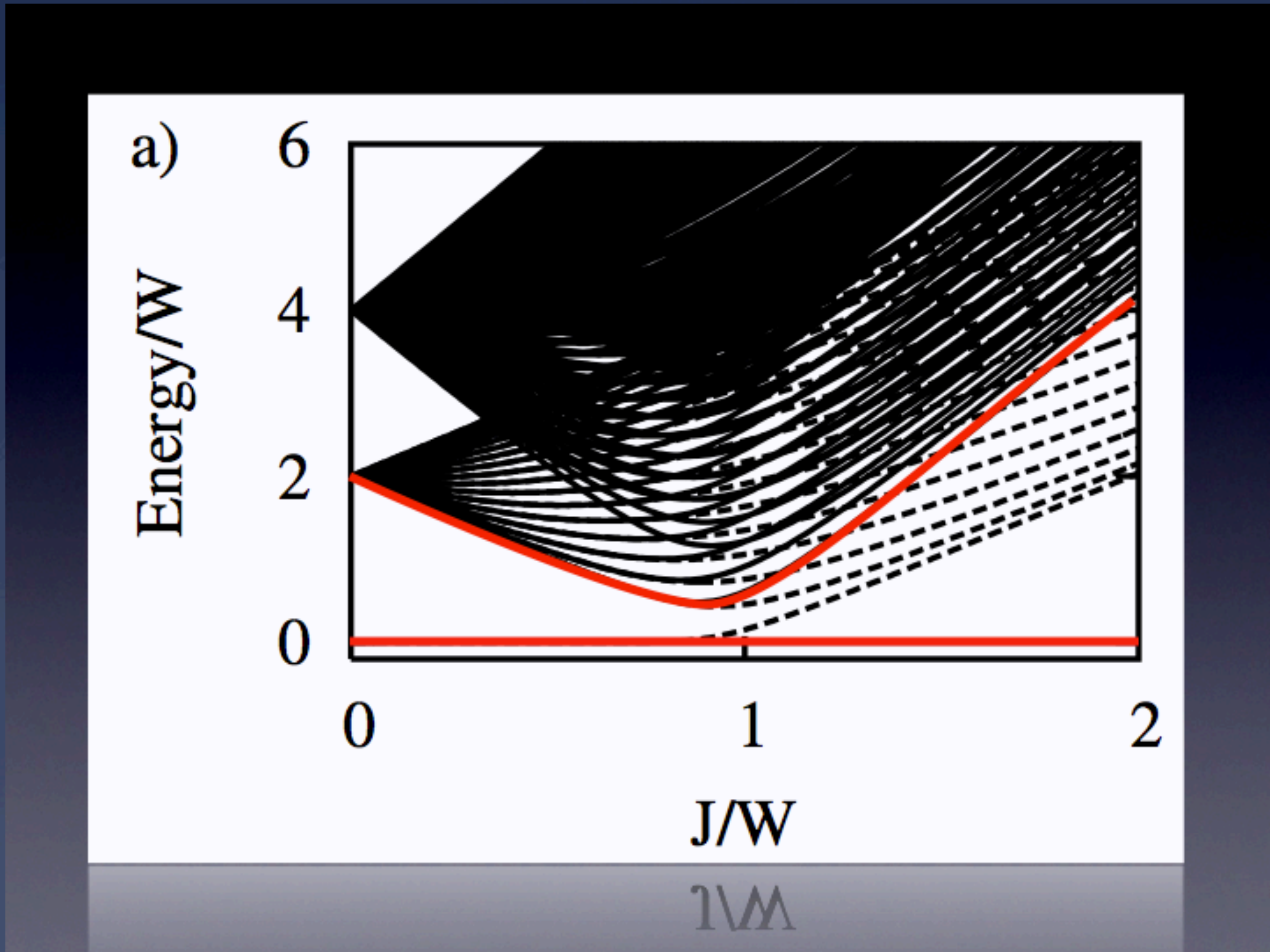
$$\tan^{-1} \frac{\sin\varphi(\tau)\sin(\frac{\theta_\tau^+}{2})\sin(\frac{\theta}{2})}{\cos\varphi(\tau)\sin(\frac{\theta_\tau^+}{2})\sin(\frac{\theta}{2}) + \cos(\frac{\theta_\tau^+}{2})\cos(\frac{\theta}{2})},$$

$$\cos(\theta_t^+/2) = \frac{2(\epsilon_+ - \sin^2(\theta/2))}{\sqrt{|r(t)|^2\sin^2(\theta) + 4(\epsilon_+ - \sin^2(\theta/2))^2}}$$

$$\sin(\theta_t^+/2) = \frac{|r(t)|\sin(\theta)}{\sqrt{|r(t)|^2\sin^2(\theta) + 4(\epsilon_+ - \sin^2(\theta/2))^2}}.$$

What is the most relevant feature of the quantum phase transition?

Near its critical point, the spectrum of a quantum critical system is characterized by the closing of the energy gap between the ground and the excitated state



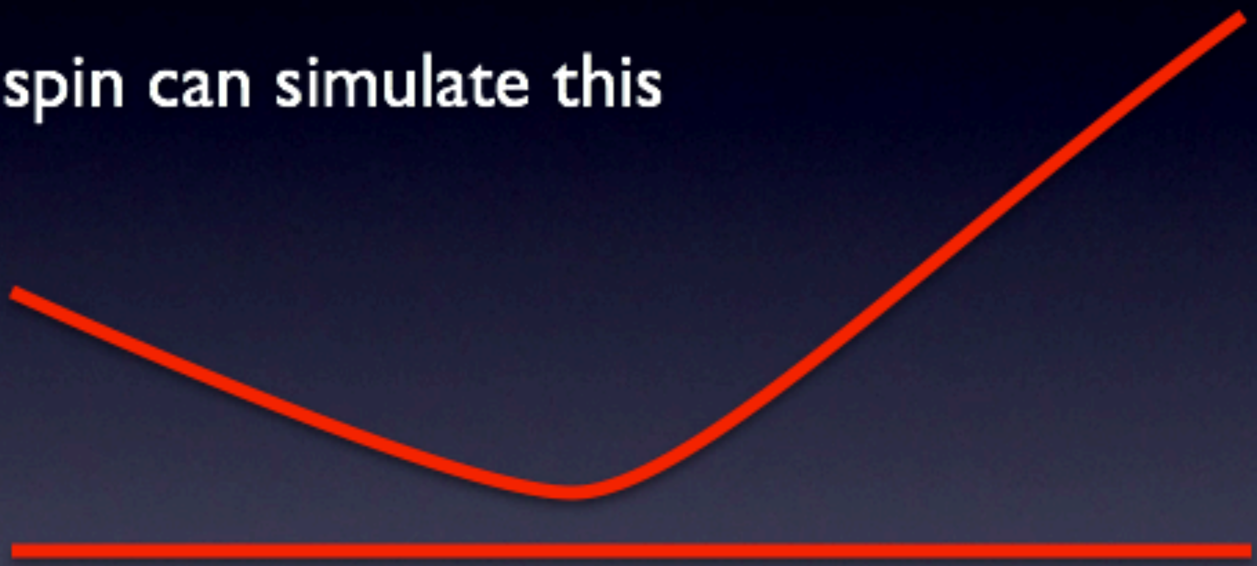
In the thermodynamical limit, the gap closes with a power law

$$\sim |\lambda - \lambda_c|^{z\nu}$$

For all finite size systems there is a minimun gap Δ near λ_c

It is possible to describe qualitatively the effects of a critical environment: as long as the excitations are small, and one is interested in qualitative behaviour, a small energy expansion of the decoherence factor can justify considering just two levels with appropriate dynamics

A single spin can simulate this



$$H_{\mathcal{E}} = \lambda|\lambda|^{z\nu-1}\Delta\ Z_{\mathcal{E}} + \Delta\ X_{\mathcal{E}}$$

$\lambda_c = 0$ Critical point

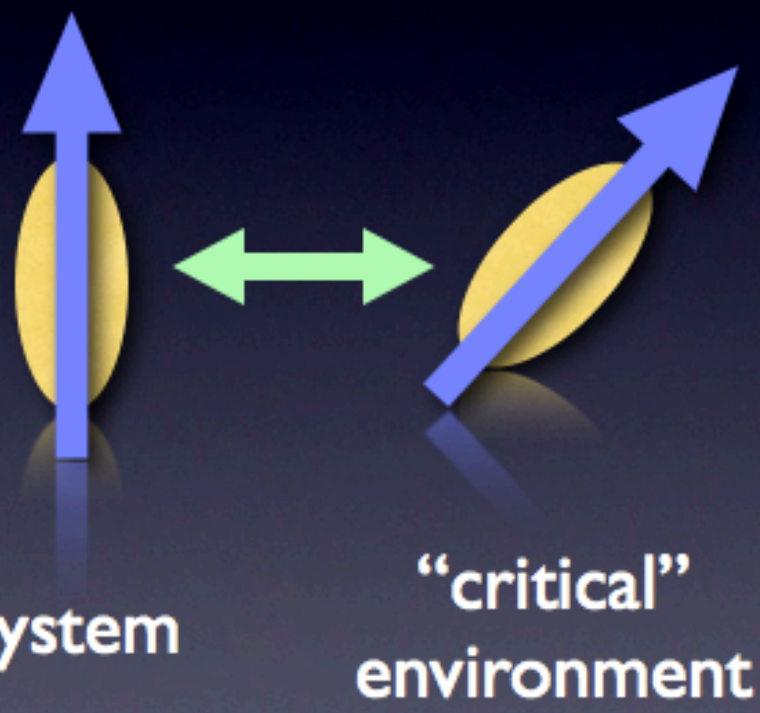
$B = \lambda\Delta$ Dimensionless transverse field

For $z\nu = 1$ we have a correct qualitative description of the creation of topological excitations during a finite speed quench (B. Damski, PRL (2005))

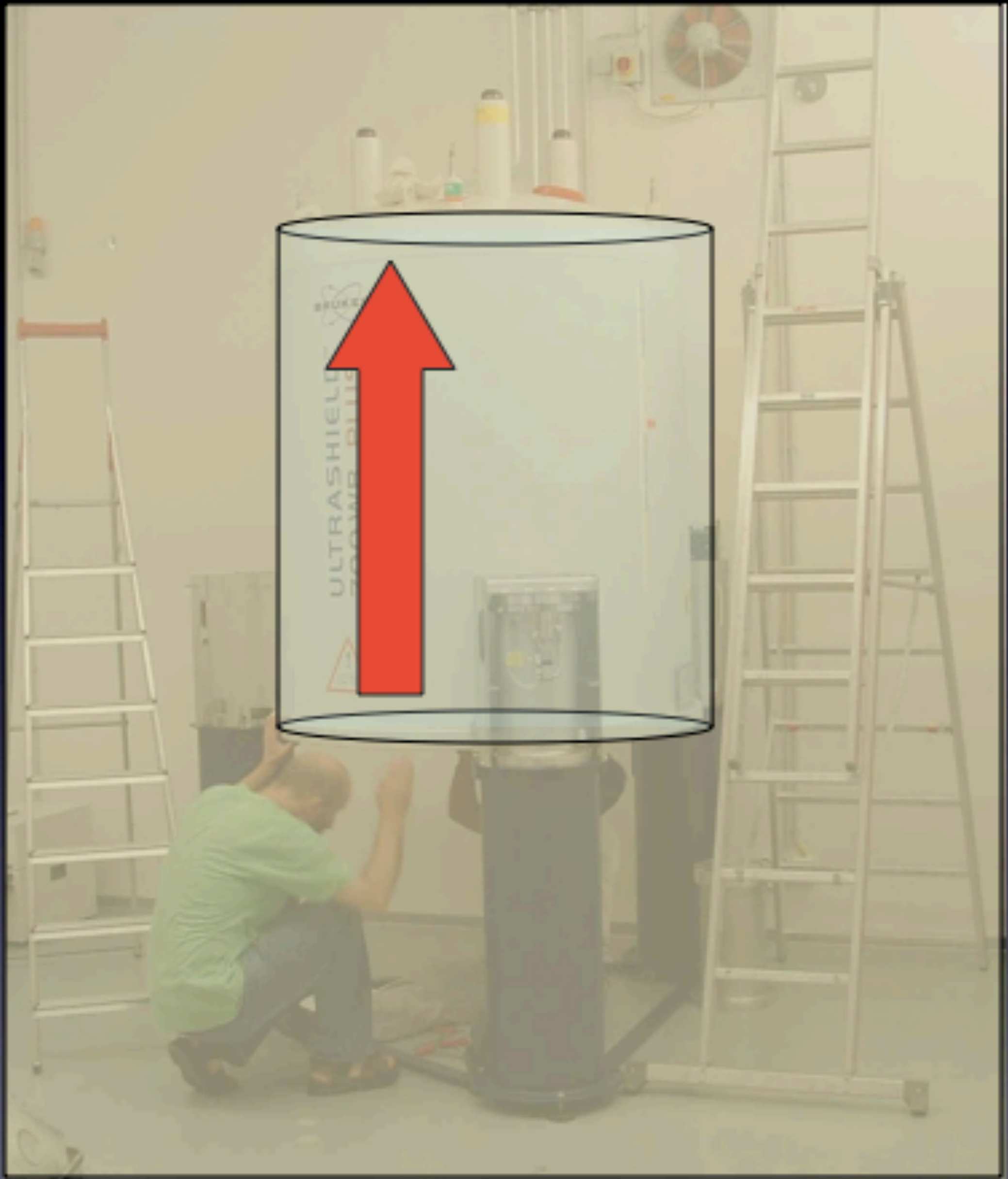
How to (quantum) simulate a system + critical environment with a few qubits?

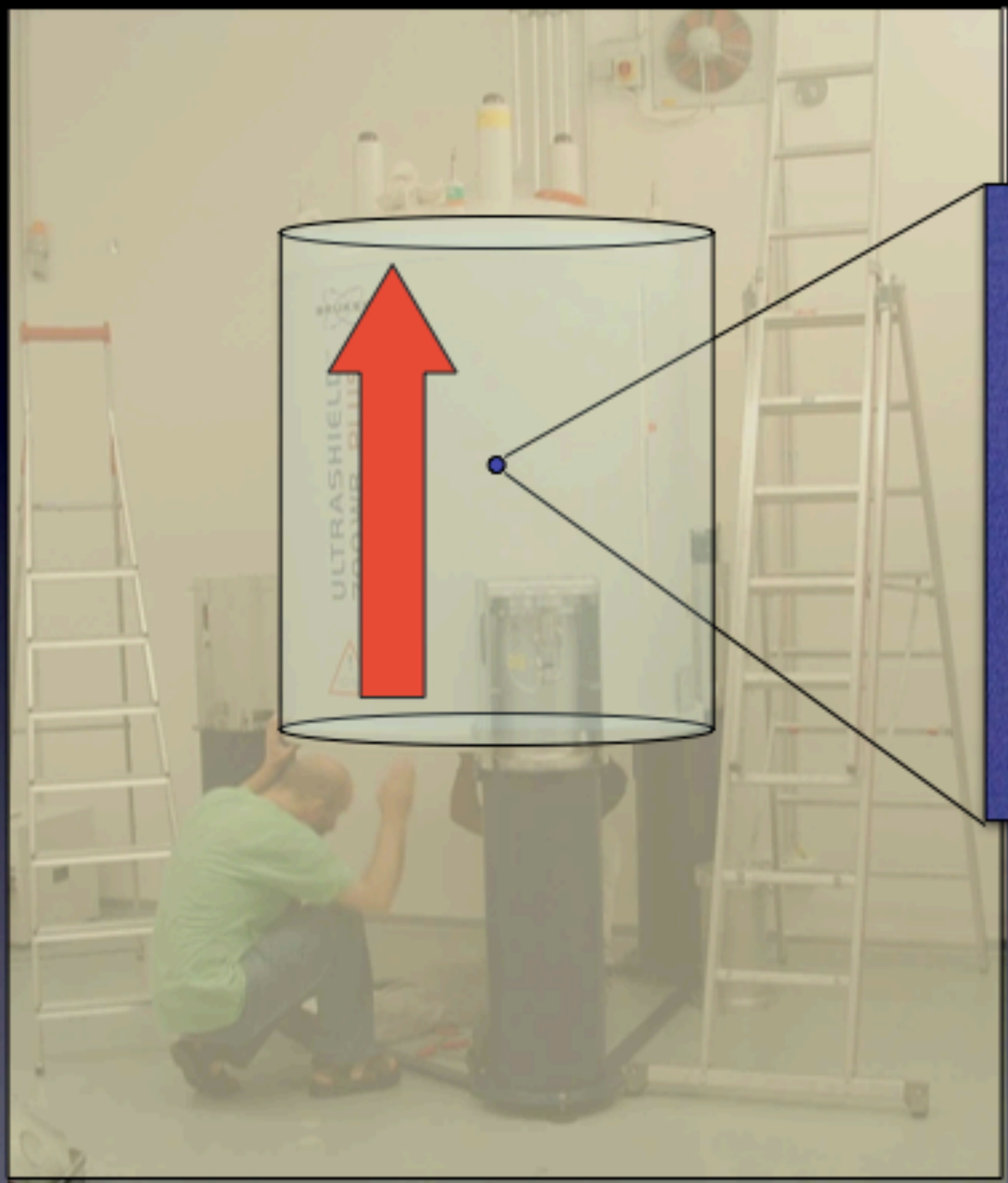
System = 1 spin

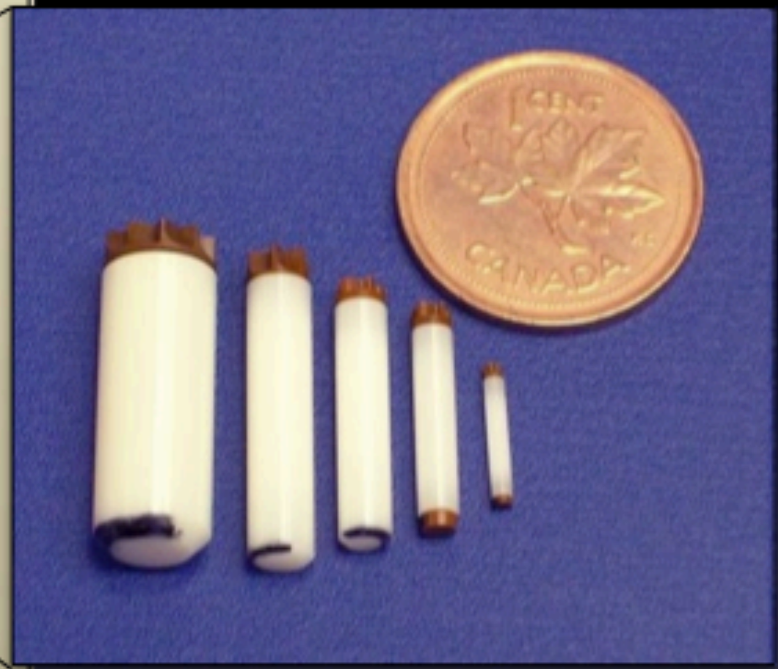
The experiment



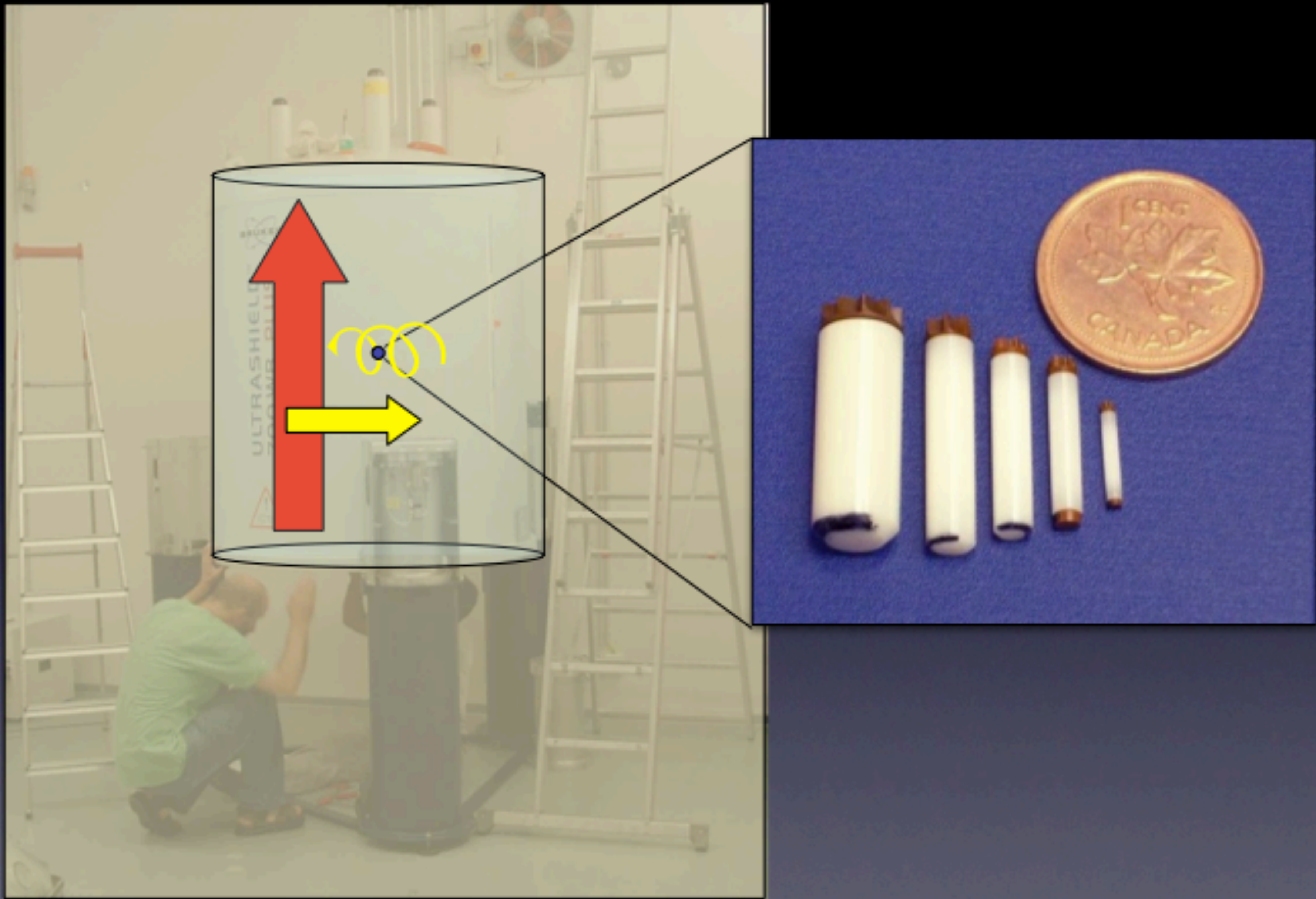
Nuclear
Magnetic
Resonance

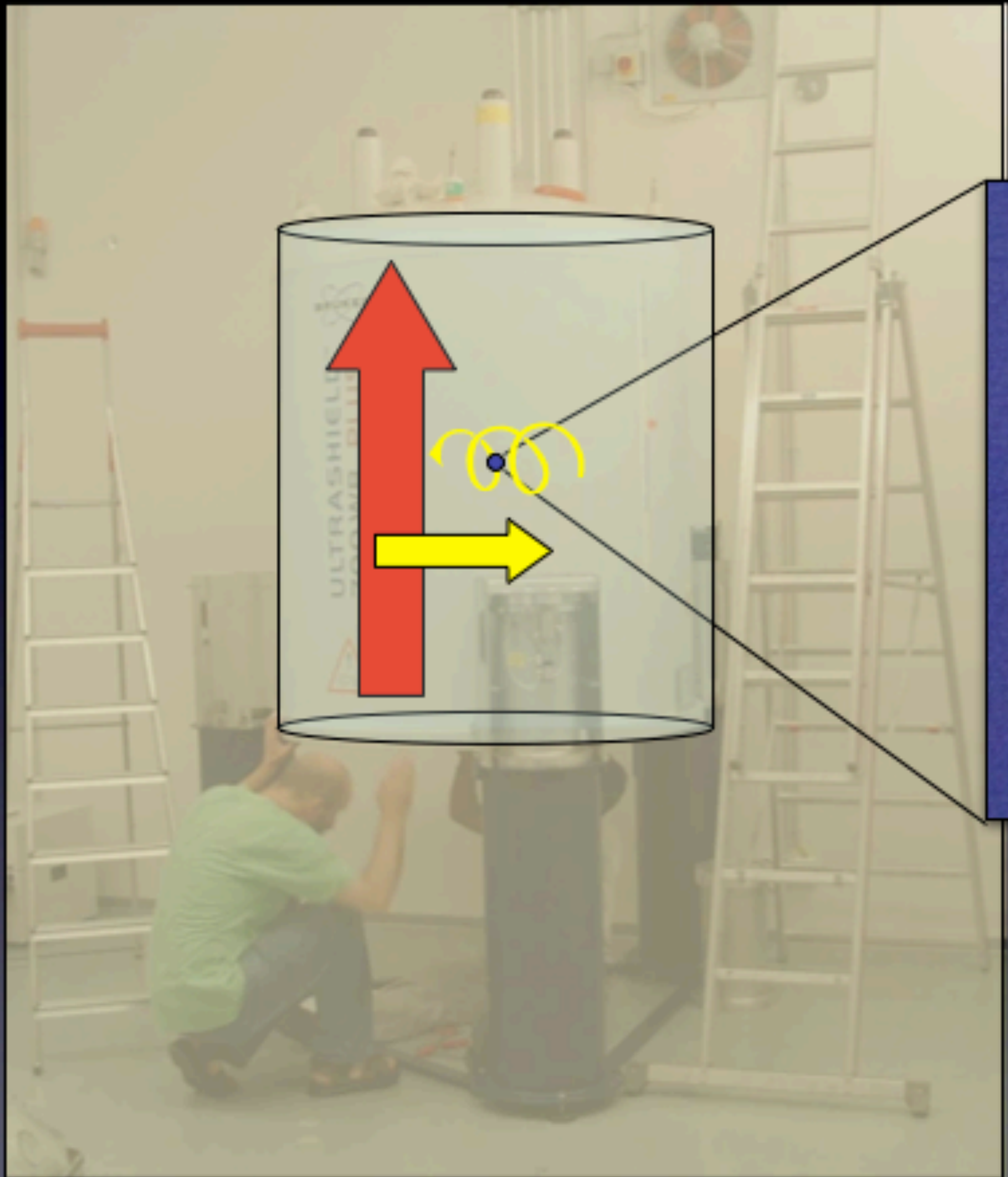

ULTRASHIELD

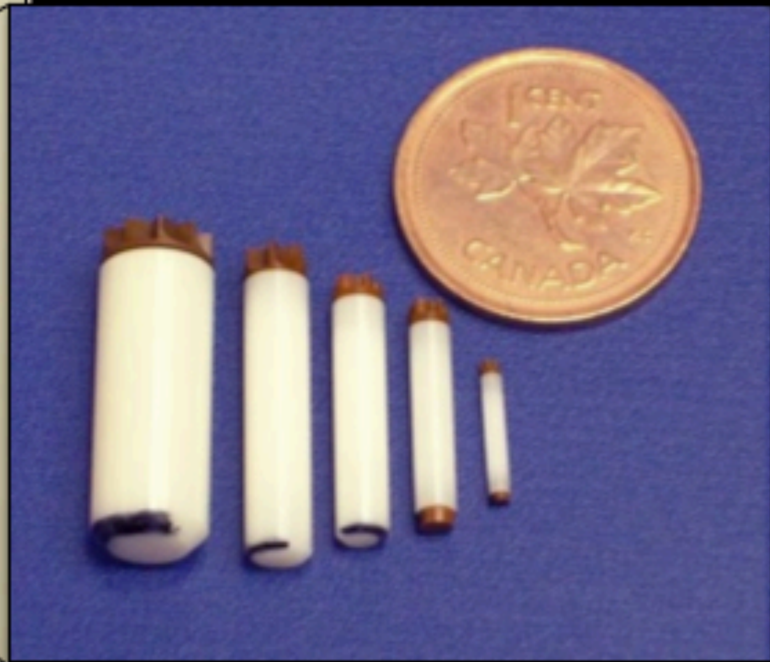




We choose the C^{13} and H^{1} spins in the molecule of carbon-13 labelled chloroform ($CHCL_3$) disolved in d6-acetone


ULTRASHIELD
CANADA





C^{13} : System

H^{1} : Environment

Me

Canada

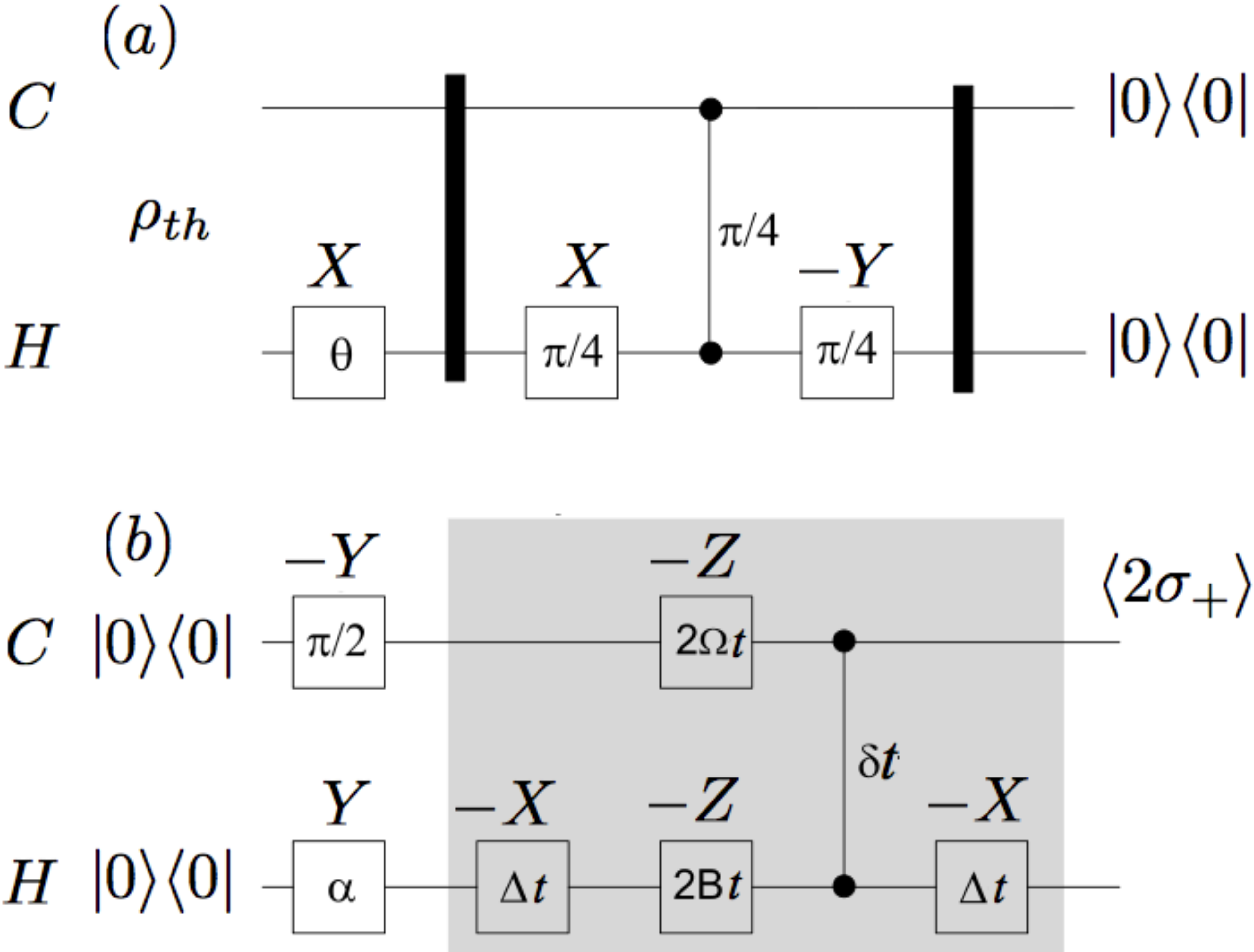
The Hamiltonian to simulate experimentally is

$$H = \Omega Z_{\mathcal{S}} + \delta Z_{\mathcal{S}} Z_{\mathcal{E}} + B Z_{\mathcal{E}} + \Delta X_{\mathcal{E}}$$

Decoherence factor

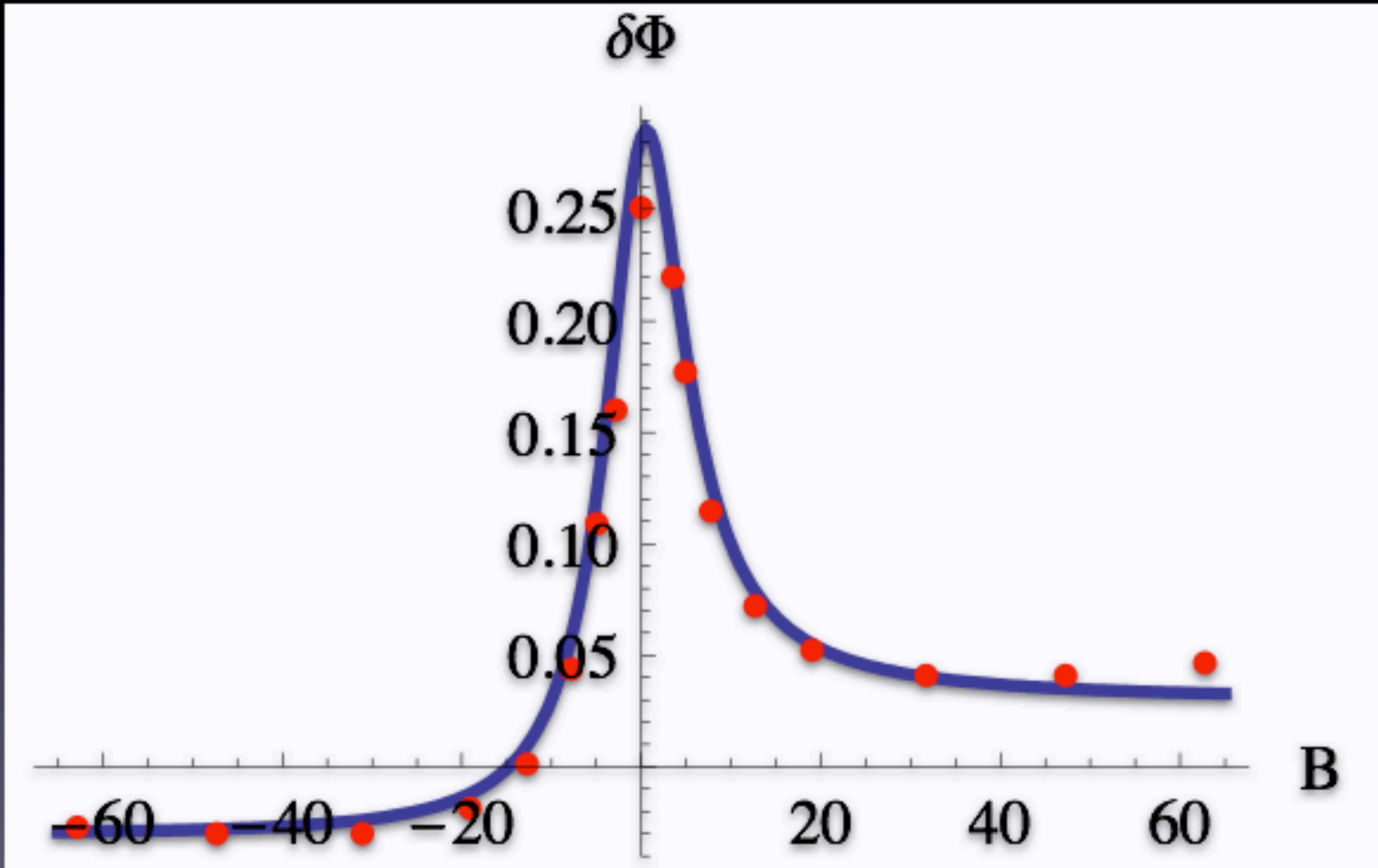
$$\begin{aligned} r(t) &= e^{i\epsilon_-(\lambda)t}\Big[\cos\epsilon_-(\lambda+\delta)t - \\ &\quad i \frac{\epsilon_-^2(\lambda+\delta) - \Delta^2\delta^2}{\epsilon_-(\lambda)\epsilon_-(\lambda+\delta)} \sin\epsilon_-(\lambda+\delta)t\Big] \end{aligned}$$

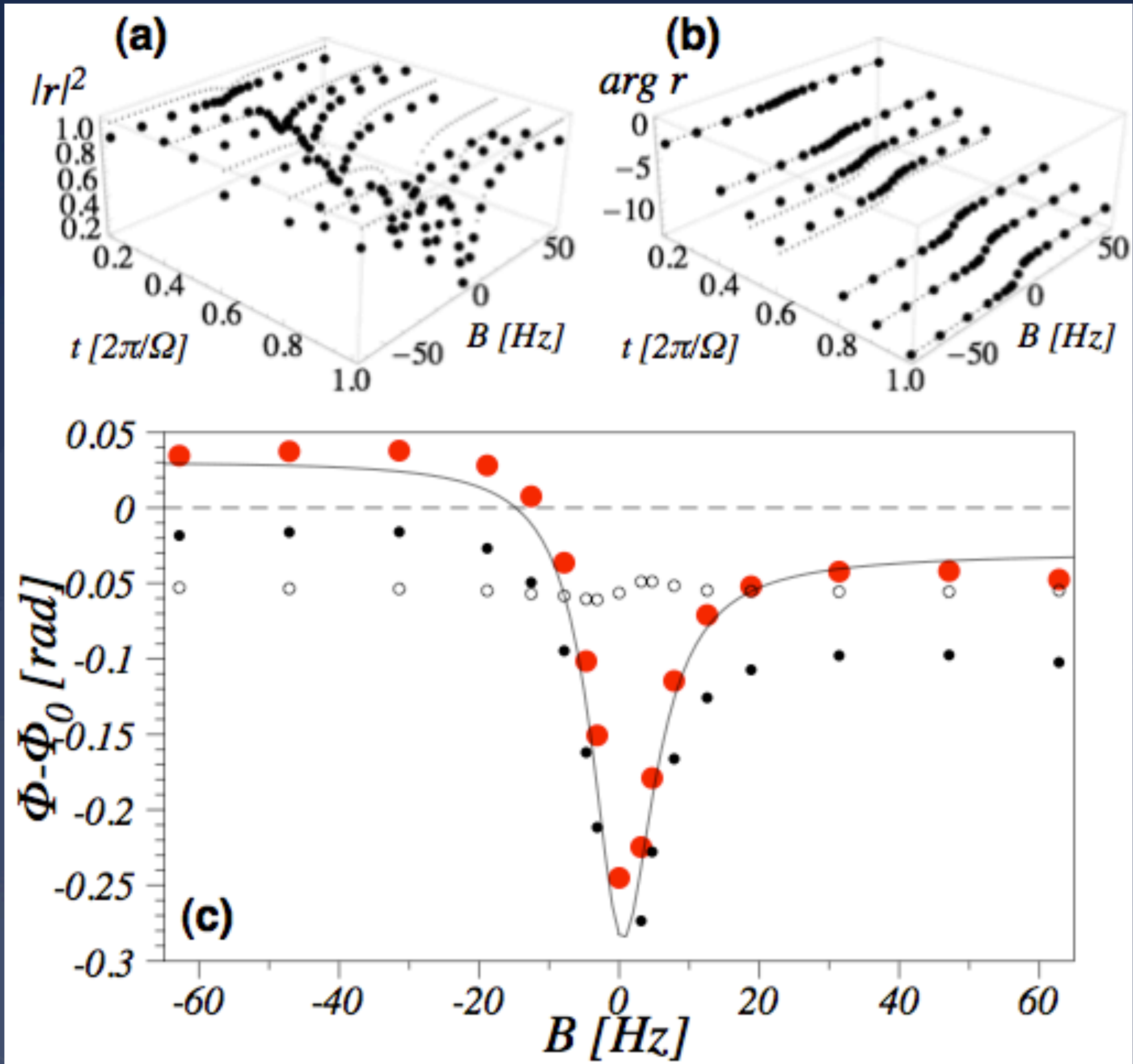
$$\epsilon_\pm = \pm\Delta\sqrt{1+\lambda^{2z\nu}}$$



(a)
C
H
ρ_{th}
X
θ
X
π/4
π/4
−Y
π/4
$|0\rangle\langle 0|$
$|0\rangle\langle 0|$
(b)
C $|0\rangle\langle 0|$
H $|0\rangle\langle 0|$
−Y
π/2
Y
α
−X
Δt
−Z
2Ωt
−Z
2Bt
δt
−X
Δt
$\langle 2\sigma_+\rangle$

Experimental results





2008

Final remarks

We have studied the correction to the unitary GP for a single qubit. We have estimated the decoherence time for different open quantum systems

We computed the GP for a bipartite two-level system coupled to an external environment. We analyzed the dependence of GP with the degree of entanglement between spins in the system

Using a NMR quantum simulator, we have obtained the quantum GP for an open system undergoing nonunitary evolution. The GP in computed in a tomographic manner: we measure the off-diagonal elements of the reduced density matrix, from which we extract the decoherence factor

Our experiment support the observation that when the environment is near a second order quantum phase transition, the correction to the GP becomes SINGULAR

By adding stochastic fields and further spins, we can quantum-simulate more realistic environments and couplings to the system