Quantum gases in the unitary limit and .....

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Outline

- The unitary limit of quantum gases
- S-matrix based approach to thermodynamics
- Application to the unitary limit
- Hubbard model.

(unitary gases work done with Pye-ton How, 2010, JSTAT)

Motivations:

- Intriguing examples of scale invariant theories with z=2 dynamical exponent (Schrodinger symmetry).

- experimental realizations: cold atoms tuned through a Feshbach resonance.

- surface of neutron stars.

- non-relativistic AdS/CFT description? Is there a bound on shear viscosity to entropy density?

$$\eta/s \geq \frac{\hbar}{4\pi k_B}$$???

Unitary limit of quantum gases

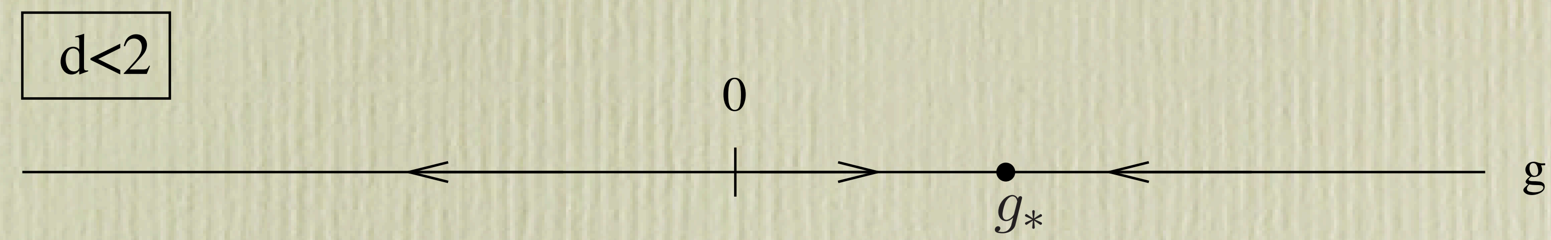
Model actions for bosons and fermions:

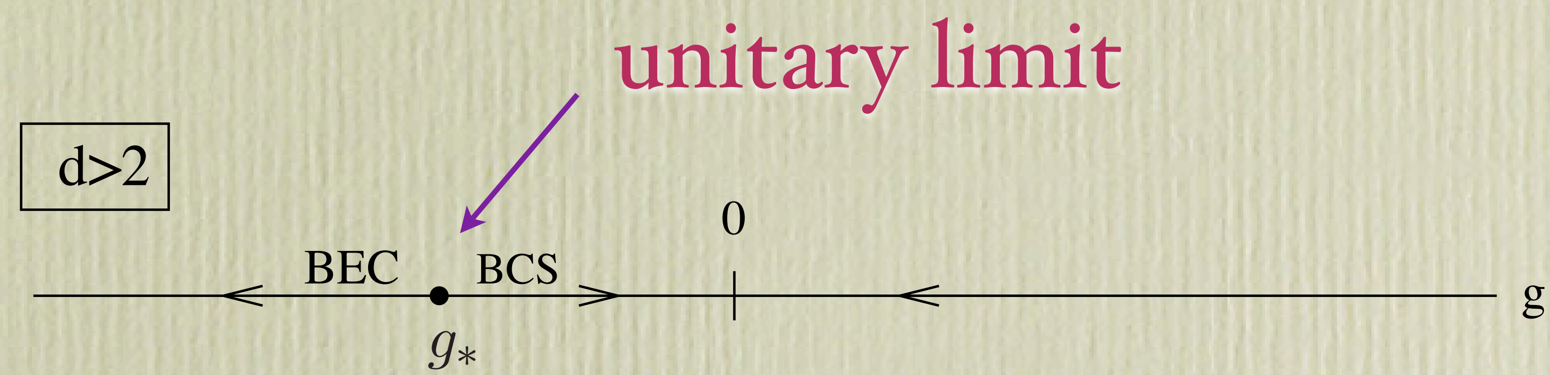
$$S = \int d^d\mathbf{x}dt \left(i\phi^\dagger \partial_t \phi - \frac{|\vec{\nabla}\phi|^2}{2m} - \frac{g}{4}(\phi^\dagger\phi)^2 \right)$$

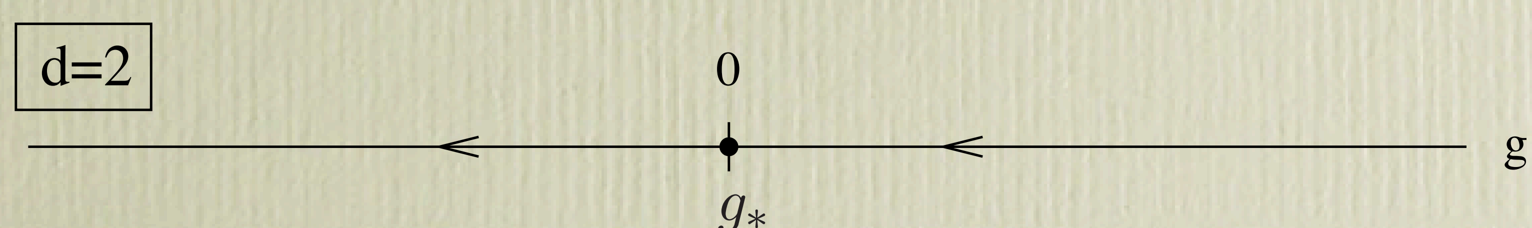
$$S = \int d^d\mathbf{x}dt \left(\sum_{\alpha=\uparrow,\downarrow} i\psi_\alpha^\dagger \partial_t \psi_\alpha - \frac{|\vec{\nabla}\psi_\alpha|^2}{2m} - \frac{g}{2}\psi_\uparrow^\dagger\psi_\uparrow\psi_\downarrow^\dagger\psi_\downarrow \right)$$

Renormalization group:

flows to low energy:







d=3 case:

- at the fixed point, a=scattering length diverges.
- z=2 scale invariant theory. Only energy scales are the chemical potential and temperature.
- On BEC side, the 2-fermion bound state can condense.
- BCS side well described by BCS theory at small coupling. (no bound state on this side.)
- in unitary limit: Very strongly coupled.   No small parameter like na^3
- new methods are needed.

motivate the method with the:

d=1 case

S-matrix:

$$S = \frac{k - k' - ig/4}{k - k' + ig/4}$$

Unitary limit:

$$g \longrightarrow \infty$$

$$S \to -1$$

Turns out to be a free fermion. Difficult to see perturbatively, but clear from the TBA.

Thermodynamic Bethe Ansatz in 1d

free energy:
$$\mathcal{F} = -\frac{1}{\beta} \int dk \log\left(1 + e^{-\beta\varepsilon(k)}\right)$$

$$\varepsilon(k) = \omega_k - \frac{1}{\beta} \int dk' K(k,k') \log\left(1 + e^{-\beta\varepsilon(k')}\right)$$

$\beta = 1/T$

$$K = -i\partial_k \log S$$

$\omega_k = k^2/2m$ = single particle energy

In the unitary limit, just a free fermion.

The formalism: a TBA-like approach in any dimension

density: $$n = -\partial_\mu \mathcal{F} = \int \frac{d^d\mathbf{k}}{(2\pi)^d} f(\mathbf{k})$$

Making a Legendre transformation in the chemical potential and occupation number f, one can show there exists a functional F where the free energy is given by:

variational principle: $$\frac{\delta F}{\delta f} = 0$$

Starting point:

$$Z = Z_0 + \frac{1}{2\pi}\int dE\, \mathrm{e}^{-\beta E}\, \mathrm{Tr}\,\mathrm{Im}\,\partial_{\mathrm{E}} \log \widehat{\mathrm{S}}(\mathrm{E})$$

(Dashen, Ma, Bernstein, 1969)

Can derive:

$$\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_1$$

$$\mathcal{F}_0 = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\left(\overbrace{(\omega_{\mathbf{k}} - \mu) f}^{\text{energy}} - \frac{1}{\beta}\overbrace{[(f-1)\log(1-f) - f\log f]}^{\text{entropy}}\right)$$

F = E - TS
(see Landau-Lifshitz)

$$\mathcal{F}_1 = -\frac{1}{2}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\int \frac{d^2\mathbf{k}'}{(2\pi)^2}\; f(\mathbf{k}')\, G(\mathbf{k},\mathbf{k}')\, f(\mathbf{k})$$

(keep only 2-body terms)

$$2\pi\delta\left(E - \omega_{\mathbf{k}} - \omega_{\mathbf{k}'}\right) V\; G(\mathbf{k},\mathbf{k}') = -i < \mathbf{k},\mathbf{k}'|\log \widehat{S}(E)|\mathbf{k},\mathbf{k}' >$$

Final result. Variational principle gives:

$$f(\mathbf{k}) = \frac{1}{e^{\beta\varepsilon(\mathbf{k})} + 1}$$

$$\varepsilon(\mathbf{k}) = \omega_{\mathbf{k}} - \mu - \int \frac{d^d\mathbf{k}}{(2\pi)^d}\; G(\mathbf{k}, \mathbf{k}')\frac{1}{e^{\beta\varepsilon(\mathbf{k}')} + 1}$$

pseudo-energy integral eqn

$$\mathcal{F} = -T \int \frac{d^2\mathbf{k}}{(2\pi)^2} \left[\log(1 + e^{-\beta\varepsilon}) + \frac{\beta}{2}\frac{1}{e^{\beta\varepsilon} + 1}(\varepsilon - \omega + \mu)\right]$$

(different signs for bosons)

$\mu =$ chemical potential

Structure of the kernel G

$$G = -\frac{i}{2\mathcal{I}} \log\left(\frac{1/g_R - i\mathcal{I}/2}{1/g_R + i\mathcal{I}/2}\right)$$

S-matrix

$$L = i\int \frac{d^d\mathbf{p}}{(2\pi)^d}\frac{1}{E-\omega_{\mathbf{p}}-\omega_{\mathbf{K}-\mathbf{p}}+2i\epsilon} \quad = \mathcal{I} + i\gamma$$

(1-loop integral)

renormalized coupling:

$$g_R = \frac{g}{1-g\gamma/2}$$

E and $\mathbf{K}$ are the total energy and momentum of the 2 particles

!! Non -perturbative, well-defined expansion in 1/g !!

Application to 3d unitary gas

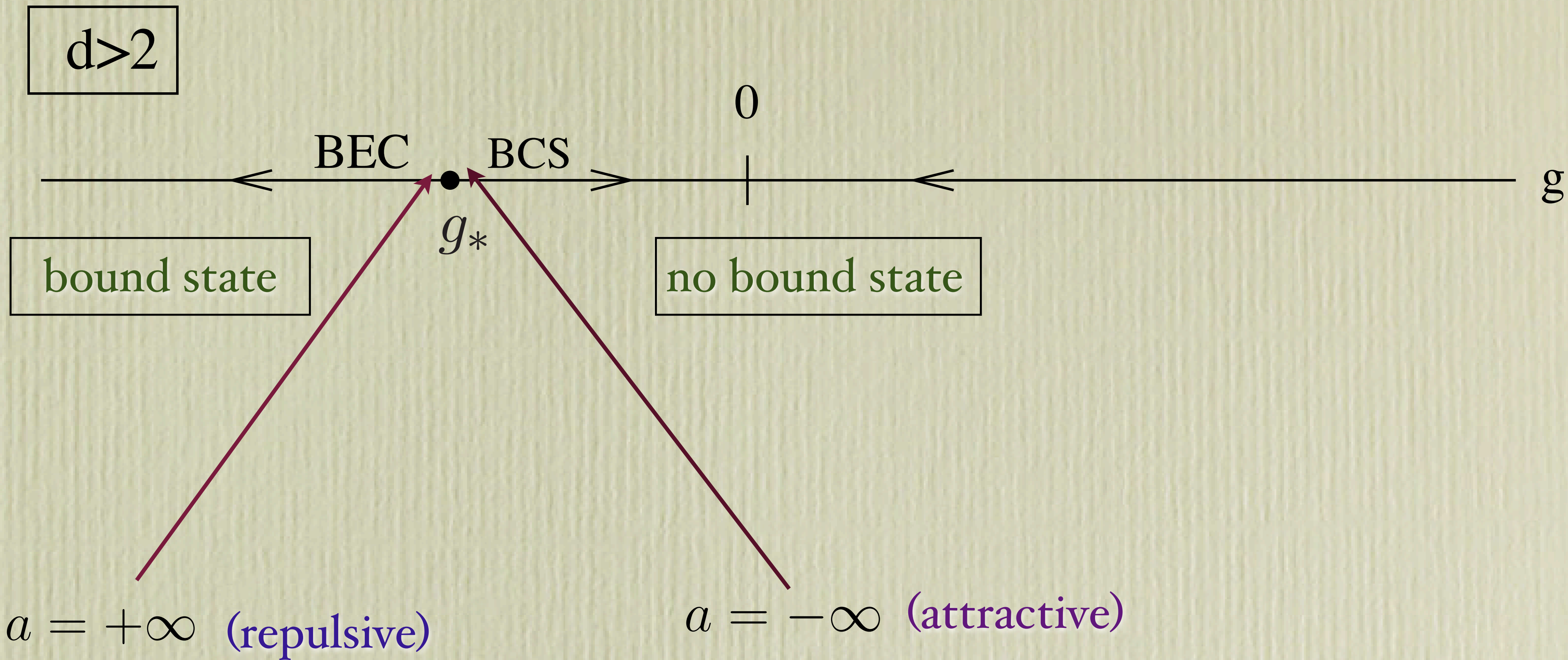
$$G(\mathbf{k}, \mathbf{k}') = -i \frac{8\pi}{m|\mathbf{k} - \mathbf{k}'|} \log \left(\frac{1/g_R - im|\mathbf{k} - \mathbf{k}'|/16\pi}{1/g_R + im|\mathbf{k} - \mathbf{k}'|/16\pi} \right)$$

$$g_R = \frac{g}{1 - g/g_*}$$

In the unitary limit, $g \longrightarrow g_*$

scattering length: $a = m g_R / 2\pi$ $\quad a \longrightarrow \pm\infty$

S-matrix: $S \longrightarrow -1$



$$G \to \mp \frac{8\pi^2}{m|\mathbf{k} - \mathbf{k}'|}$$

-/+ corresponds to repulsive/attractive

(for small g , G = -g)

Unitary limit in 2d

- Formally define it as S=-1, i.e. coupling goes to infinity.
- not an RG fixed point in usual sense, but still scale invariant. Occurs at very low energies (infinitely attractive) or very high energy. (infinitely repulsive).

The kernel becomes a constant and the integral equation is transcendentally algebraic!

$$G(|\mathbf{k}|) = \mp \frac{4\pi}{m}$$

4 cases: attractive/repulsive bosons/fermions

Results

Scale invariance implies the scaling form:

$$\mathcal{F} = -\zeta(5/2)\left(\frac{mT}{2\pi}\right)^{3/2} T\, c(\mu/T)$$

c=1 for
free boson
at zero chem.pot.

Critical points must occur at fixed values of μ/T.
These points can be expressed as:

$$n\lambda_{T_c}^{d} = \text{constant}$$

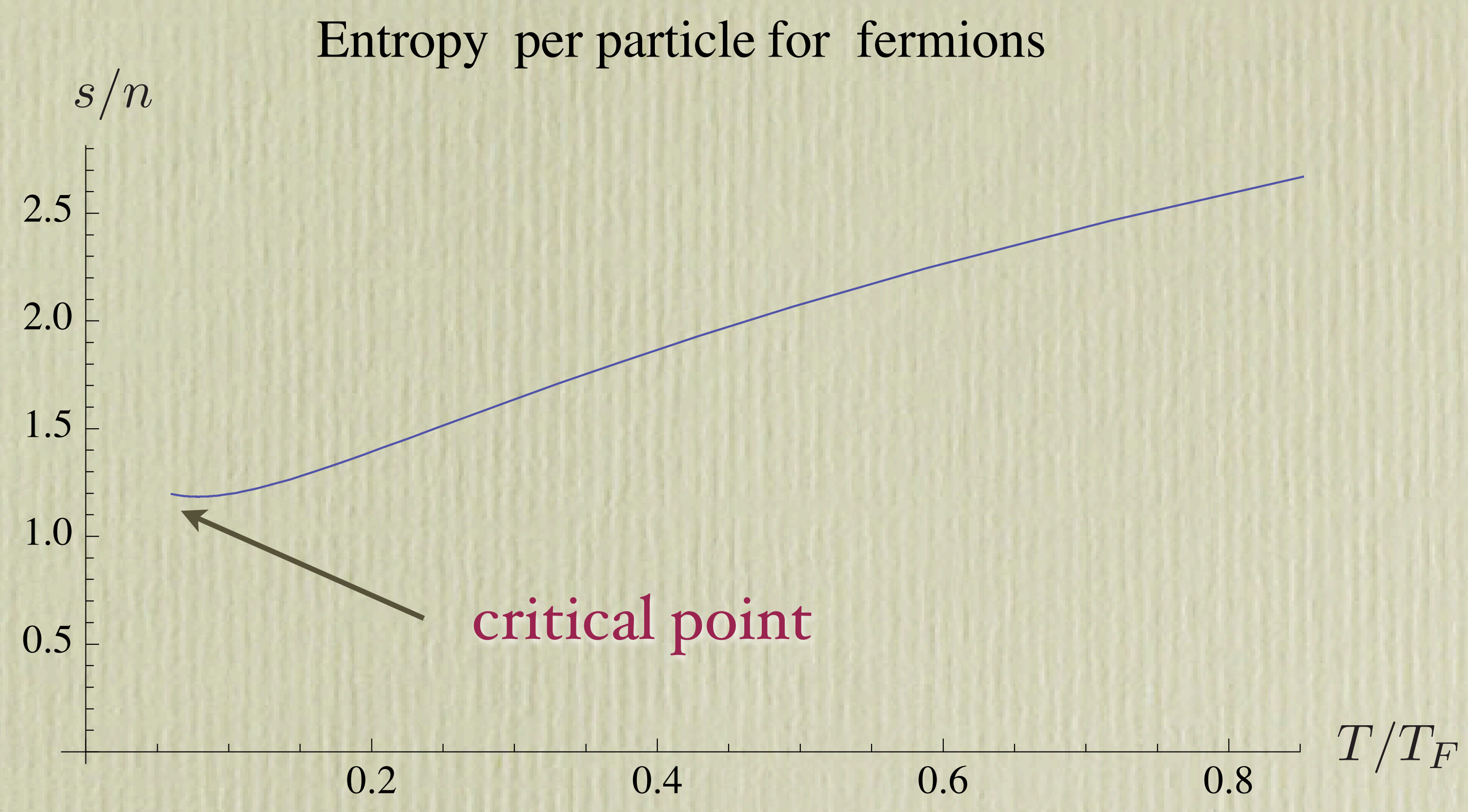
(bosons)

$\lambda_T = \sqrt{2\pi/mT}$.

$$T_c/T_F = \text{constant}$$

(fermions)

Fermions



$$T_c/T_F \approx 0.1.$$

consistent with lattice Monte Carlo

Bosons

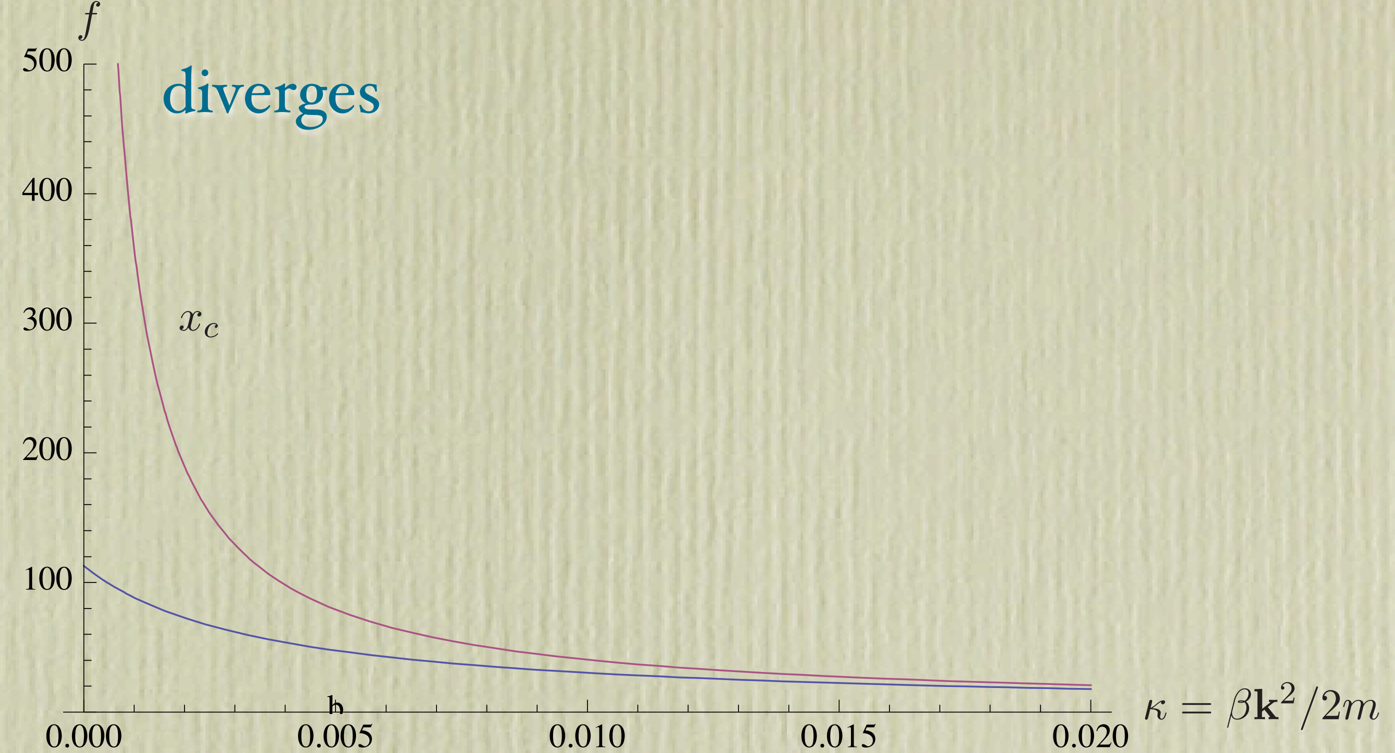
Evidence for an interacting version of BEC (new)

$$n_c \lambda_T^3 = 1.325, \qquad\qquad (\mu/T = x_c = -1.2741)$$

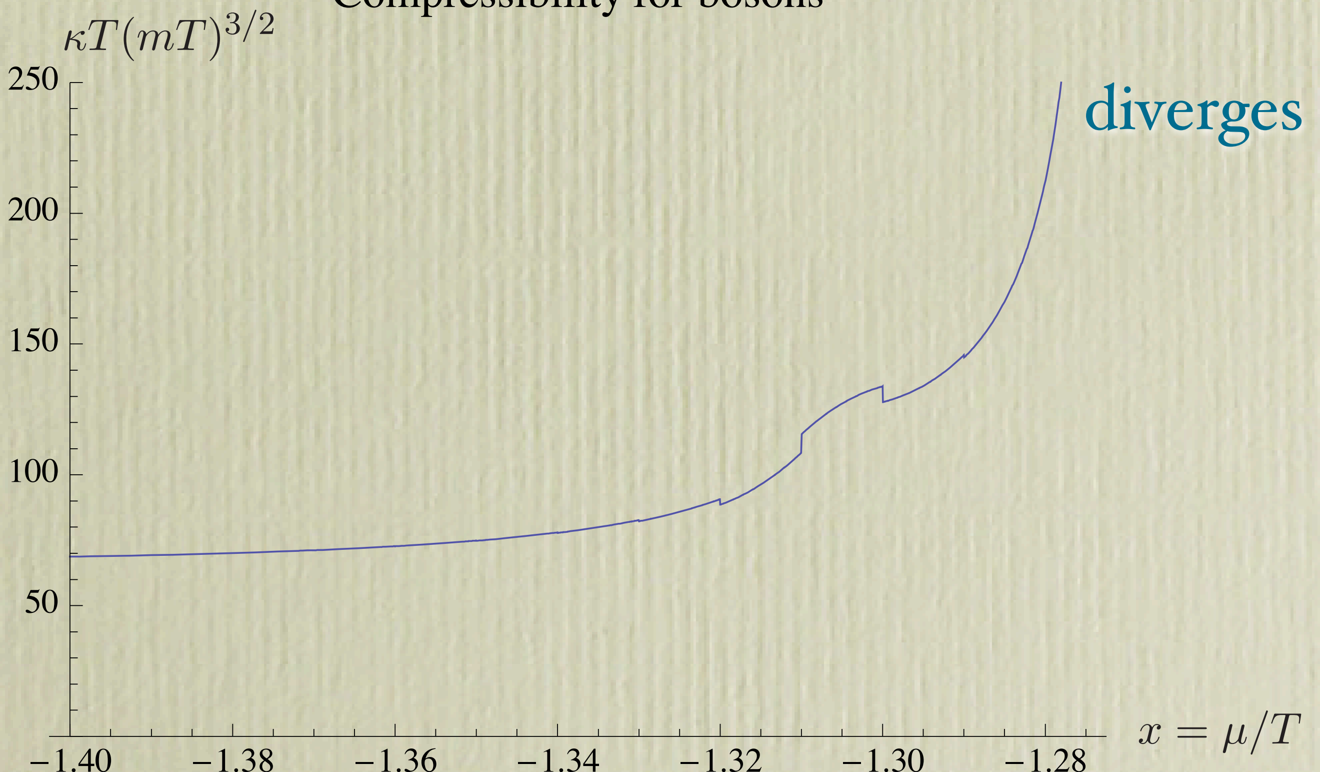
compare with non-interacting BEC:

$$x_c = 0 \text{ and } n_c \lambda_T^3 = \zeta(3/2) = 2.61,$$

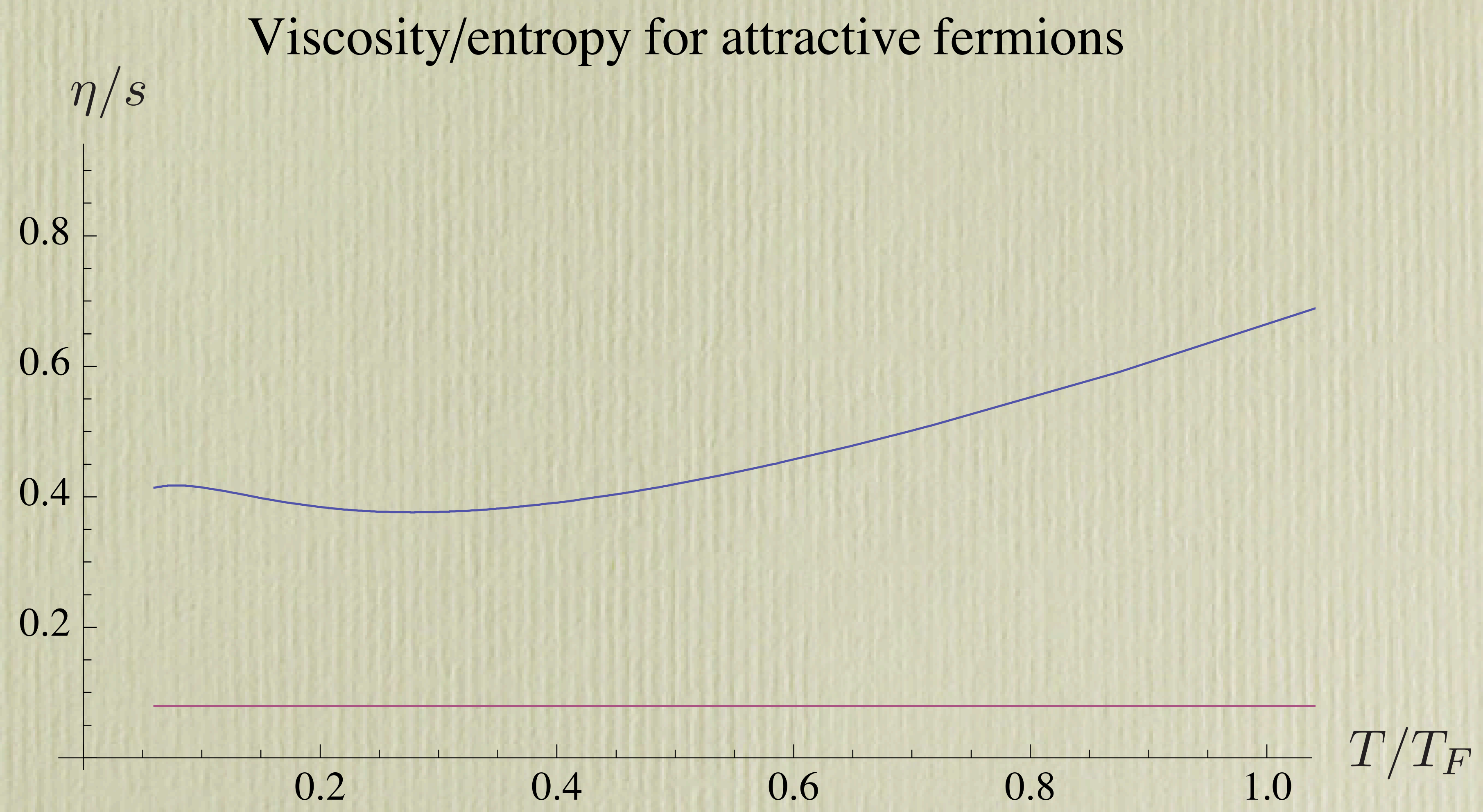
Occupation number for bosons



Compressibility for bosons

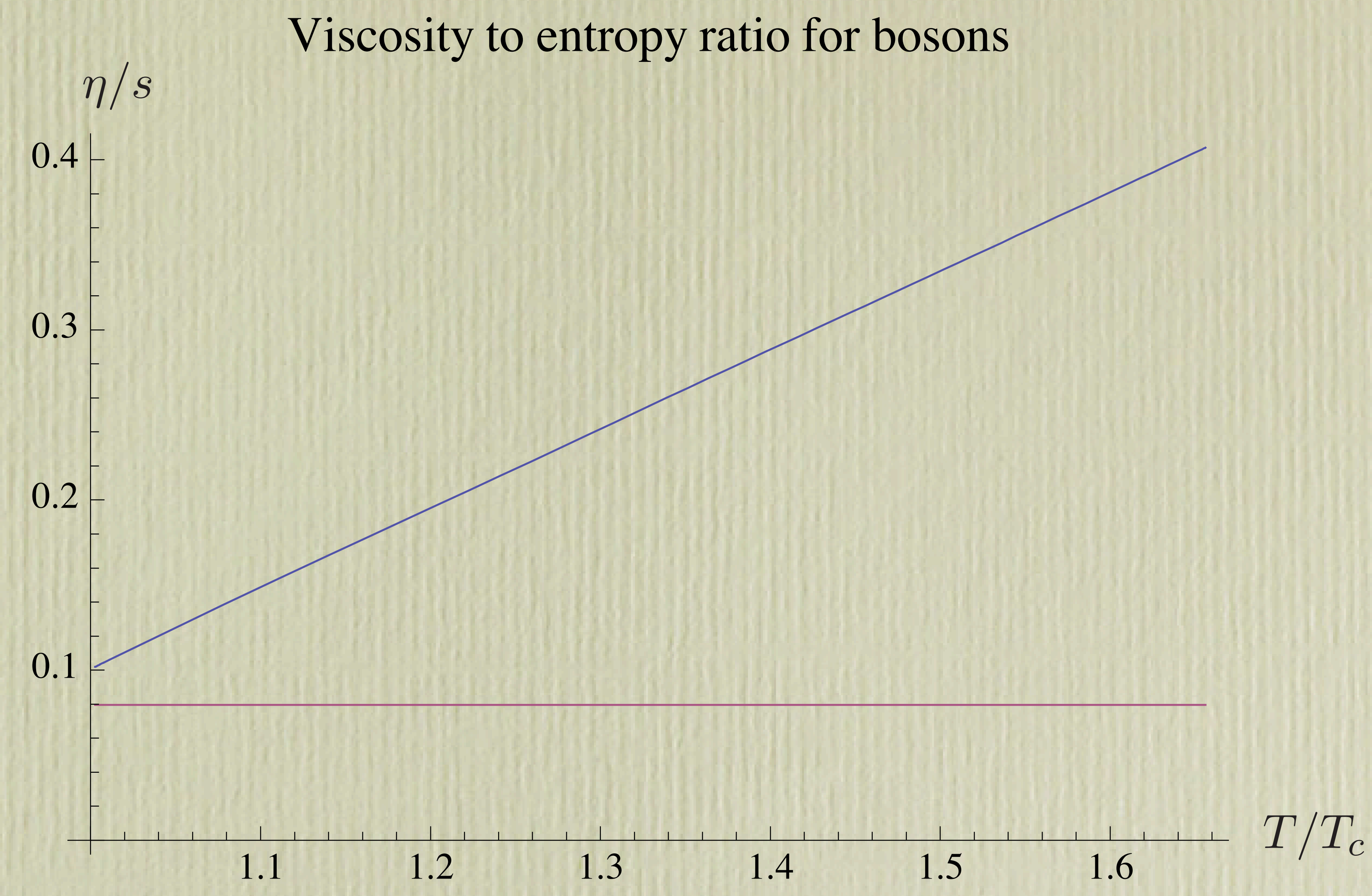


Viscosity to entropy density ratio



$$\boxed{\frac{\eta}{s} > 4.72 \frac{\hbar}{4\pi k_B}}$$

In good agreement with experiments

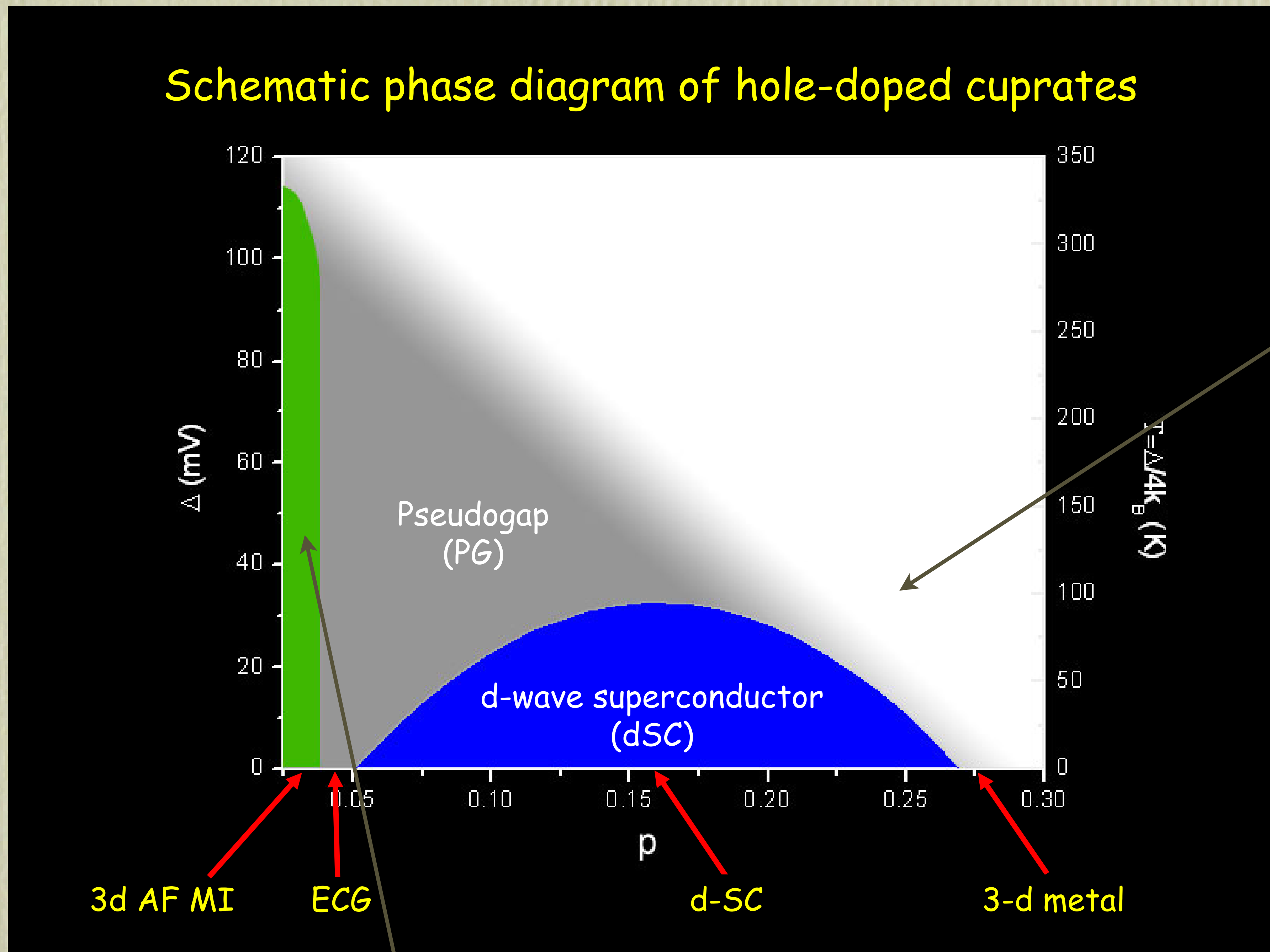


$$\frac{\eta}{s} > 1.26 \frac{\hbar}{4\pi k_B}$$

a more perfect fluid than fermions

High Temperature Superconductivity

Start here



Not here (doping a Mott insulator)

Hubbard Model Gas

$$H = -t \sum_{<i,j>,\alpha=\uparrow,\downarrow} \left(c^{\dagger}_{\mathbf{r}_i,\alpha} c_{\mathbf{r}_j,\alpha}\right) - t' \sum_{<i,j>',\alpha=\uparrow,\downarrow} \left(c^{\dagger}_{\mathbf{r}_i,\alpha} c_{\mathbf{r}_j,\alpha}\right) + U \sum_{\mathbf{r}} n_{\mathbf{r}\uparrow} n_{\mathbf{r}\downarrow}$$

diagonalize free part

treat as local

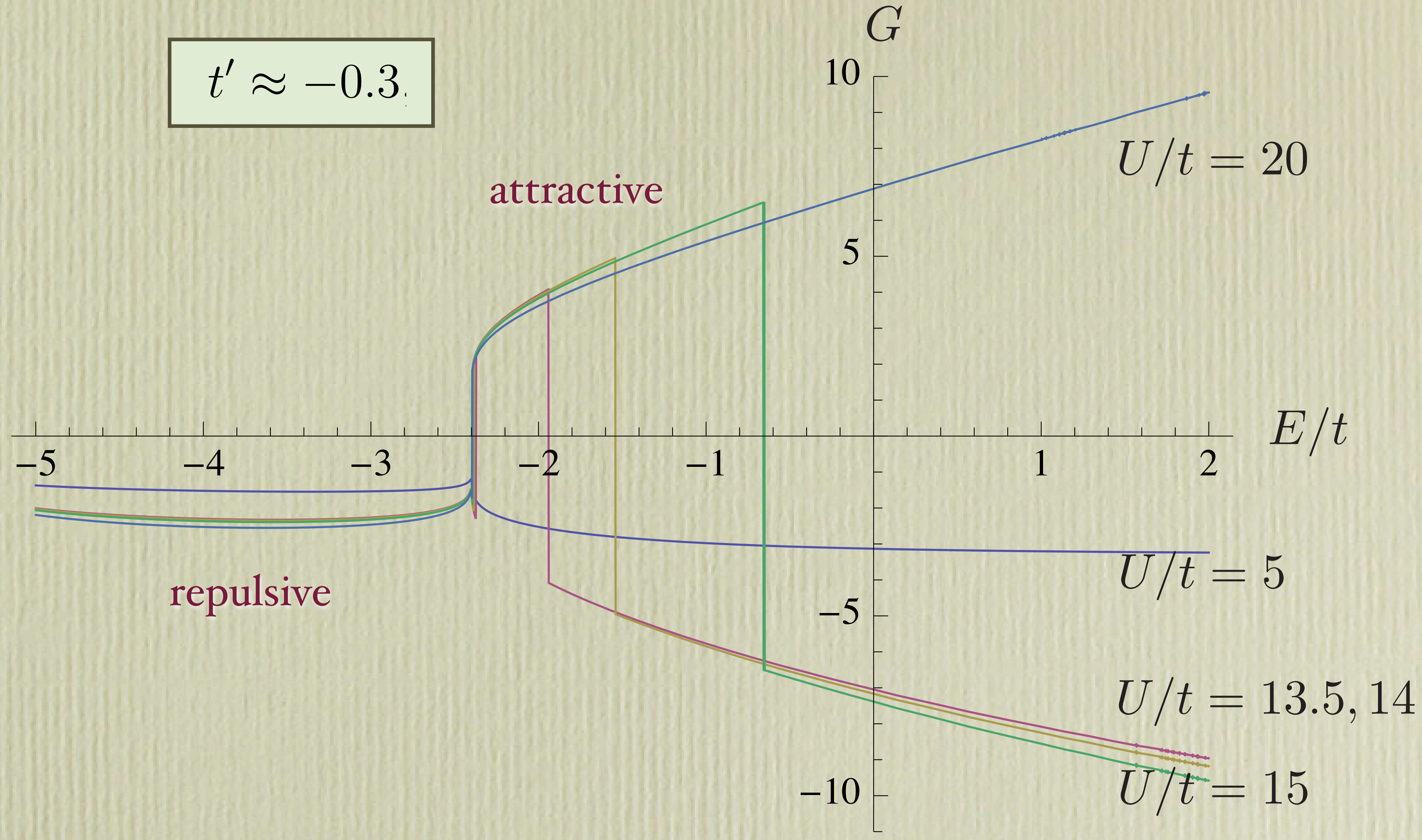
*free, single particle energies:

$$\omega_{\mathbf{k}} = -2t\left(\cos(k_x a) + \cos(k_y a)\right) - 4t' \cos(k_x a)\cos(k_y a)$$

* can treat as a gas with coupling

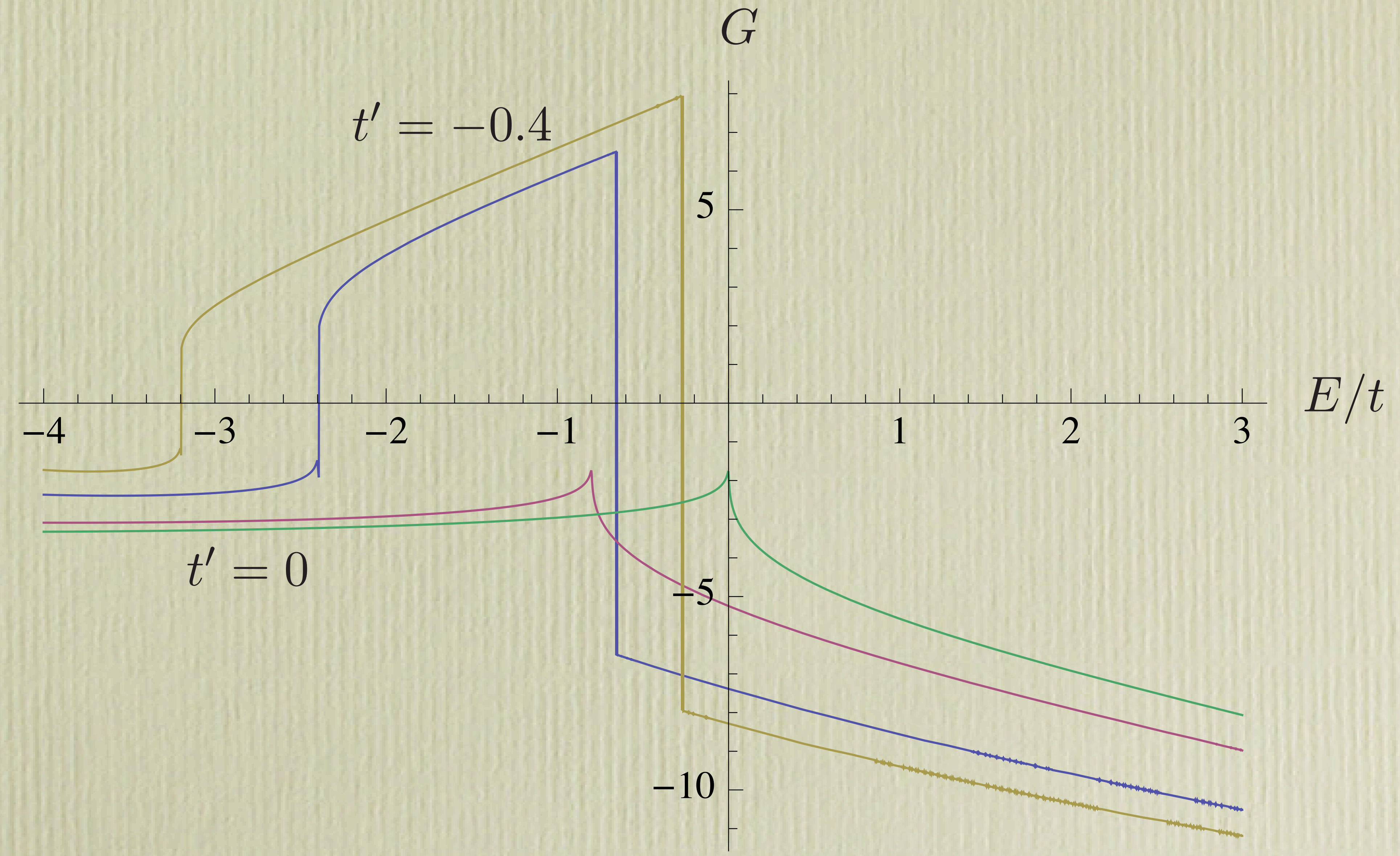
$$\boxed{g = U/t}$$

Cuprates: $t' \approx -0.3, \qquad g \approx 13$



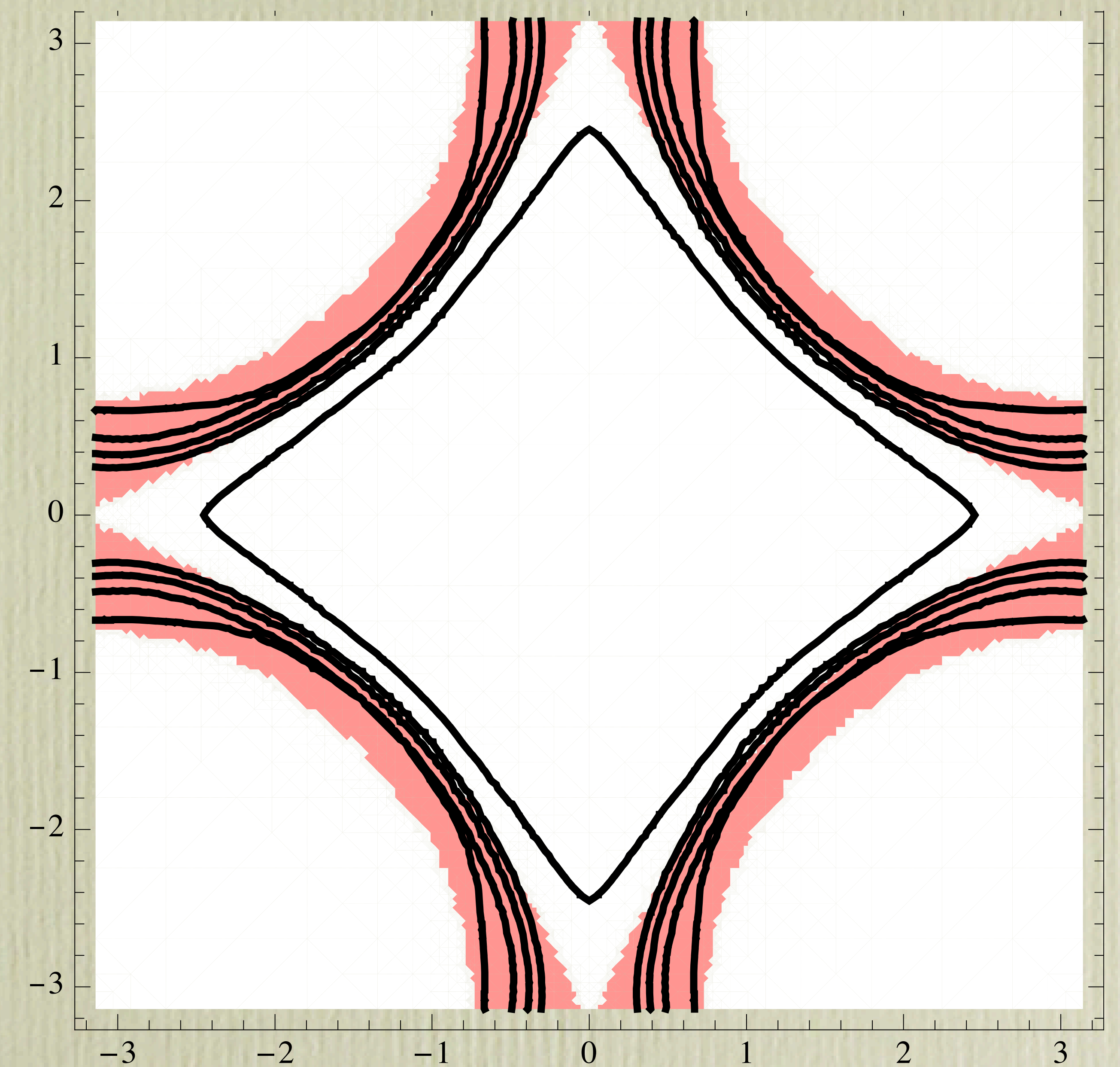
Conclusion: for t'=-0.3, g must be greater than 12.8 for an attractive band to exist.

g=15,   t'=0, -0.1, -0.3, -0.4



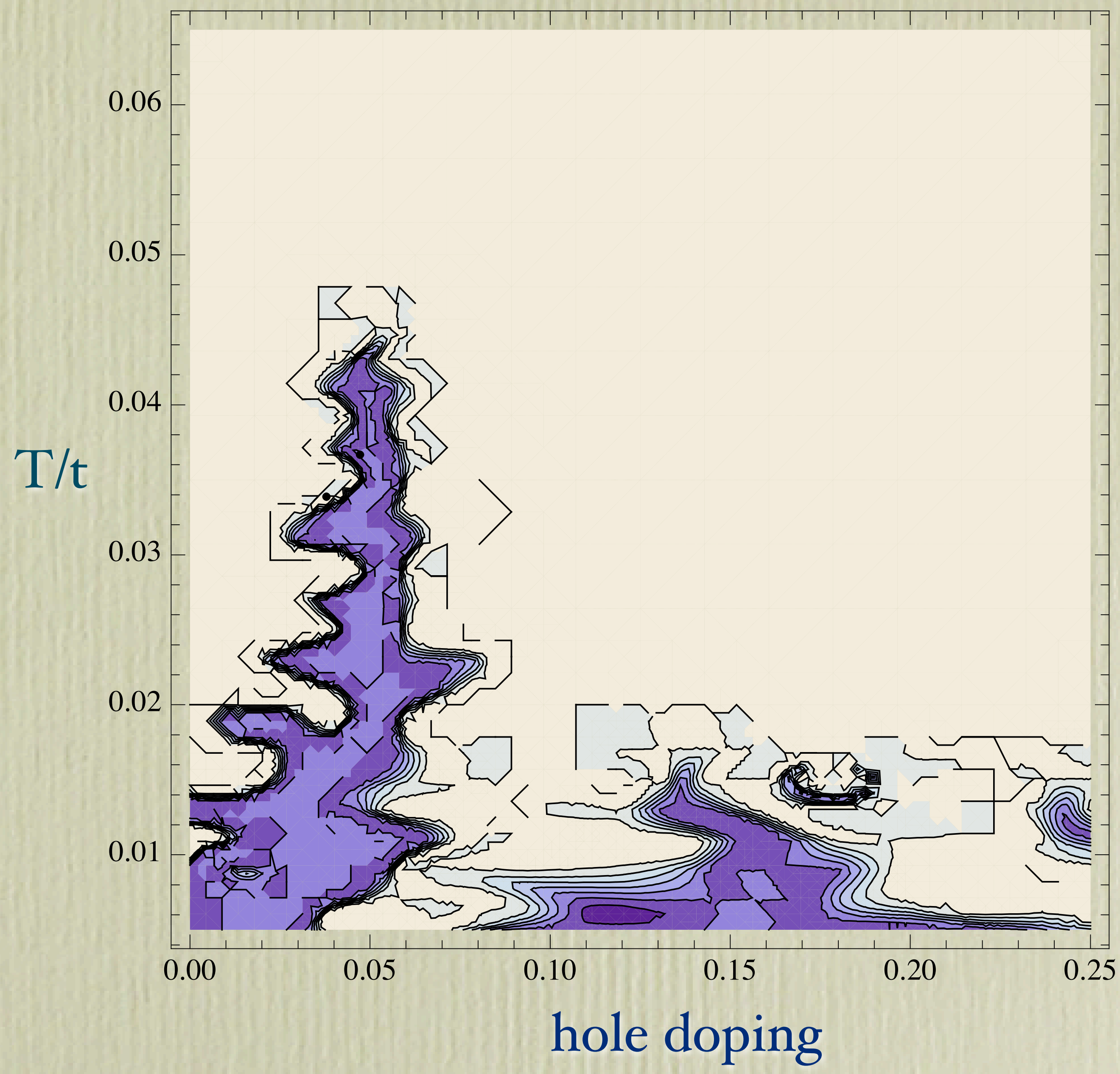
Conclusion: no superconductivity if t' =0

Fermi surfaces for hole doping h=0, .1, .2, .3, .4



Attractive band in pink

?? Can we see the phase transitions ??



Dark regions: no solution to pseudogap equation

$$T_c/t \approx 0.02$$

the End