Finite-size technology in low-dimensional quantum systems

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Yangian symmetry & bound states in AdS/CFT boundary scattering

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Joint work with

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0. Introduction

The bulk worldsheet S-matrix
is the key to exploiting
integrability in AdS5/CFT4

[Staudacher 04, Beisert 05]

$\Rightarrow$

- all-orders asymptotic Bethe ansatz equations: anomalous dimensions of $\mathcal{N} = 4$ Yang-Mills

[Beisert & Staudacher 05]

- Lüscher formula: finite-size effects

[Bajnok & Janik 08]

The S-matrix in fundamental (4-dim) rep
is completely determined (up to scalar factor) from
su(2|2) symmetry {Beisert 05}

This S-matrix has an even bigger symmetry:
Yangian {Beisert 07}

Just a mathematical curiosity?

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is completely determined (up to scalar factor) from
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Yangian [Beisert 07]

Just a mathematical curiosity?

No!

S-matrices in *higher-dim* reps (i.e., for bound states)
cannot be completely determined from su(2|2) symmetry;
but *can* be determined using also Yangian symmetry!

[Arutyuonov & Frolov 08; de Leeuw '08; Arutyuonov, de Leeuw & Torrielli '09]

For conventional factorizable S-matrices,
there is a well-known "fusion" procedure
for constructing higher-dim S-matrices
from elementary ones

[Kulish, Reshetikhin & Sklyanin '81]

For AdS/CFT S-matrices,
this fusion procedure seems to fail

The symmetry approach (su(2|2) + Yangian) is the only
known procedure of obtaining higher-dimensional
AdS/CFT S-matrices

These S-matrices are useful for deriving TBA equations, etc.

These results are for **bulk** (2-particle) S-matrices

Can they be extended to **boundary** S-matrices?

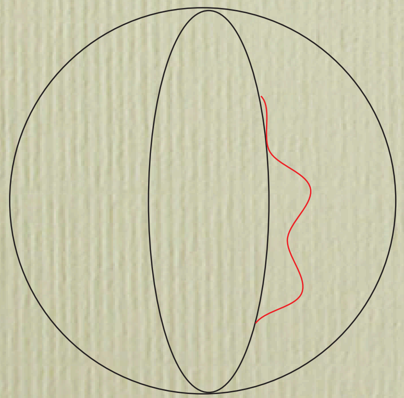
- Robustness of Yangian symmetry
- Applications to boundary problems

Simplest case: open superstring attached to Y=0 maximal giant graviton brane

[Berenstein & Vazquez 05, Hofman & Maldacena 07]

$$W = \Phi_1 + i\Phi_2\,,\ Y = \Phi_3 + i\Phi_4\,,\ Z = \Phi_5 + i\Phi_6$$

$$J = J_{56} \to \infty$$



$$S^5: \qquad |W|^2 + |Y|^2 + |Z|^2 = 1$$

$$\downarrow \qquad Y = 0$$

$$S^3: \qquad |W|^2 + |Z|^2 = 1$$

Determinant-like operators in dual CFT4

[Balasubramanian, Berkooz, Naqvi & Strassler 02, ...]

Without string attached:

$$\epsilon^{j_1 \cdots j_N}_{i_1 \cdots i_N} Y^{i_1}_{j_1} \cdots Y^{i_N}_{j_N}$$

gauge invariant, even though no trace

With string attached:

vacuum: $$\epsilon^{j_1 \cdots j_N}_{i_1 \cdots i_N} Y^{i_1}_{j_1} \cdots Y^{i_{N-1}}_{j_{N-1}} (Z \cdots Z)^{i_N}_{j_N}$$

excitations: $$\epsilon^{j_1 \cdots j_N}_{i_1 \cdots i_N} Y^{i_1}_{j_1} \cdots Y^{i_{N-1}}_{j_{N-1}} (Z \cdots Z \chi Z \cdots Z)^{i_N}_{j_N}$$

Boundary S-matrix describes how excitations reflect from ends

Main results:

- The fundamental boundary S-matrix *does* have Yangian symmetry!
- *Can* use Yangian symmetry to determine the boundary S-matrix in higher-dim (bound state) rep

Outline

1. Review of fundamental bulk S-matrix
   & its Yangian symmetry

2. Review of fundamental *boundary* S-matrix

* 3. Yangian symmetry of fundamental *boundary* S-matrix

4. Review of bulk S-matrix in 2-particle bound state rep

* 5. Determine *boundary* S-matrix in 2-particle bound state rep

6. Discussion

1. Fundamental bulk S-matrix & its Yangian symmetry

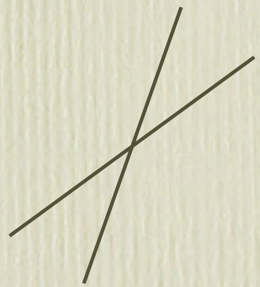
Fundamental representation: 2 bosons + 2 fermions

Zamolodchikov-Faddeev (ZF) operators [Arutyunov, Frolov & Zamaklar 06]

$$A_i^\dagger(p) \quad i \in \{1, \ldots, 4\}$$

Bulk S-matrix

$$A_i^\dagger(p_1)\, A_j^\dagger(p_2) = S^{AA\;\; i' j'}_{\quad\;\; i\, j}(p_1, p_2)\, A_{j'}^\dagger(p_2)\, A_{i'}^\dagger(p_1)$$



su(2|2) generators:

rotation	$\mathbb{L}_a{}^b, \quad \mathbb{R}_\alpha{}^\beta$	$a, b \in \{1, 2\}$
supersymmetry	$\mathbb{Q}_\alpha{}^a, \quad \mathbb{Q}^{\dagger\alpha}_a$	$\alpha, \beta \in \{3, 4\}$
central	$\mathbb{C}\,, \mathbb{C}^\dagger\,, \mathbb{H}$	

su(2|2) algebra:

$$
\begin{aligned}
\left[\mathbb{L}_a{}^b\,, \mathbb{J}_c\right] &= \delta_c^b \mathbb{J}_a - \frac{1}{2}\delta_a^b \mathbb{J}_c\,, \quad \left[\mathbb{R}_\alpha{}^\beta\,, \mathbb{J}_\gamma\right] = \delta_\gamma^\beta \mathbb{J}_\alpha - \frac{1}{2}\delta_\alpha^\beta \mathbb{J}_\gamma \\
\left[\mathbb{L}_a{}^b\,, \mathbb{J}^c\right] &= -\delta_a^c \mathbb{J}^b + \frac{1}{2}\delta_a^b \mathbb{J}^c\,, \quad \left[\mathbb{R}_\alpha{}^\beta\,, \mathbb{J}^\gamma\right] = -\delta_\alpha^\gamma \mathbb{J}^\beta + \frac{1}{2}\delta_\alpha^\beta \mathbb{J}^\gamma \\
\left\{\mathbb{Q}_\alpha{}^a\,, \mathbb{Q}_\beta{}^b\right\} &= \epsilon_{\alpha\beta}\epsilon^{ab}\mathbb{C}\,, \quad \left\{\mathbb{Q}^{\dagger\alpha}_a\,, \mathbb{Q}^{\dagger\beta}_b\right\} = \epsilon^{\alpha\beta}\epsilon_{ab}\mathbb{C}^\dagger \\
\left\{\mathbb{Q}_\alpha{}^a\,, \mathbb{Q}^{\dagger\beta}_b\right\} &= \delta_b^a \mathbb{R}_\alpha{}^\beta + \delta_\alpha^\beta \mathbb{L}_b{}^a + \frac{1}{2}\delta_b^a \delta_\alpha^\beta \mathbb{H}
\end{aligned}
$$

Action of su(2|2) generators on ZF operators:

$$\mathbb{L}_a{}^b\, A_c^\dagger(p) = (\delta_c^b\delta_a^d - \frac{1}{2}\delta_a^b\delta_c^d)A_d^\dagger(p) + A_c^\dagger(p)\,\mathbb{L}_a{}^b\,, \qquad \mathbb{L}_a{}^b\, A_\gamma^\dagger(p) = A_\gamma^\dagger(p)\,\mathbb{L}_a{}^b$$

$$\mathbb{R}_\alpha{}^\beta\, A_\gamma^\dagger(p) = (\delta_\gamma^\beta\delta_\alpha^\delta - \frac{1}{2}\delta_\alpha^\beta\delta_\gamma^\delta)A_\delta^\dagger(p) + A_\gamma^\dagger(p)\,\mathbb{R}_\alpha{}^\beta\,, \quad \mathbb{R}_\alpha{}^\beta\, A_c^\dagger(p) = A_c^\dagger(p)\,\mathbb{R}_\alpha{}^\beta$$

$$\begin{aligned}
\mathbb{Q}_\alpha{}^a\, A_b^\dagger(p) &= e^{-ip/2}\left[a\,\delta_b^a\, A_\alpha^\dagger(p) + A_b^\dagger(p)\,\mathbb{Q}_\alpha{}^a\right] \\
\mathbb{Q}_\alpha{}^a\, A_\beta^\dagger(p) &= e^{-ip/2}\left[b\,\epsilon_{\alpha\beta}\epsilon^{ab} A_b^\dagger(p) - A_\beta^\dagger(p)\,\mathbb{Q}_\alpha{}^a\right] \\
\mathbb{Q}_a^{\dagger\alpha}\, A_b^\dagger(p) &= e^{ip/2}\left[c\,\epsilon_{ab}\epsilon^{\alpha\beta} A_\beta^\dagger(p) + A_b^\dagger(p)\,\mathbb{Q}_a^{\dagger\alpha}\right] \\
\mathbb{Q}_a^{\dagger\alpha}\, A_\beta^\dagger(p) &= e^{ip/2}\left[d\,\delta_\beta^\alpha\, A_a^\dagger(p) - A_\beta^\dagger(p)\,\mathbb{Q}_a^{\dagger\alpha}\right]
\end{aligned}$$

$$a = \sqrt{\frac{g}{2l}}\eta\,, \quad b = \sqrt{\frac{g}{2l}}\frac{i}{\eta}\left(\frac{x^+}{x^-} - 1\right)\,, \quad c = -\sqrt{\frac{g}{2l}}\frac{\eta}{x^+}\,, \quad d = \sqrt{\frac{g}{2l}}\frac{x^+}{i\eta}\left(1 - \frac{x^-}{x^+}\right)$$

$$\frac{x^+}{x^-} = e^{ip}\,, \quad x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{2li}{g}\,, \quad \eta = e^{ip/4}\sqrt{i(x^- - x^+)} \qquad l = 1$$

Determine S-matrix by demanding that su(2|2) generators $\mathbb{J}^A$ commute with 2-particle scattering; i.e., consider

$$\mathbb{J}^A\, A_i^\dagger(p_1) A_j^\dagger(p_2)|0\rangle$$

Can first exchange $A^\dagger$'s, then move $\mathbb{J}^A$ to the right $\mathbb{J}^A|0\rangle = 0$

Or first move $\mathbb{J}^A$ to the right, and then exchange $A^\dagger$'s

Consistency $\Rightarrow$ linear equations for S-matrix elements

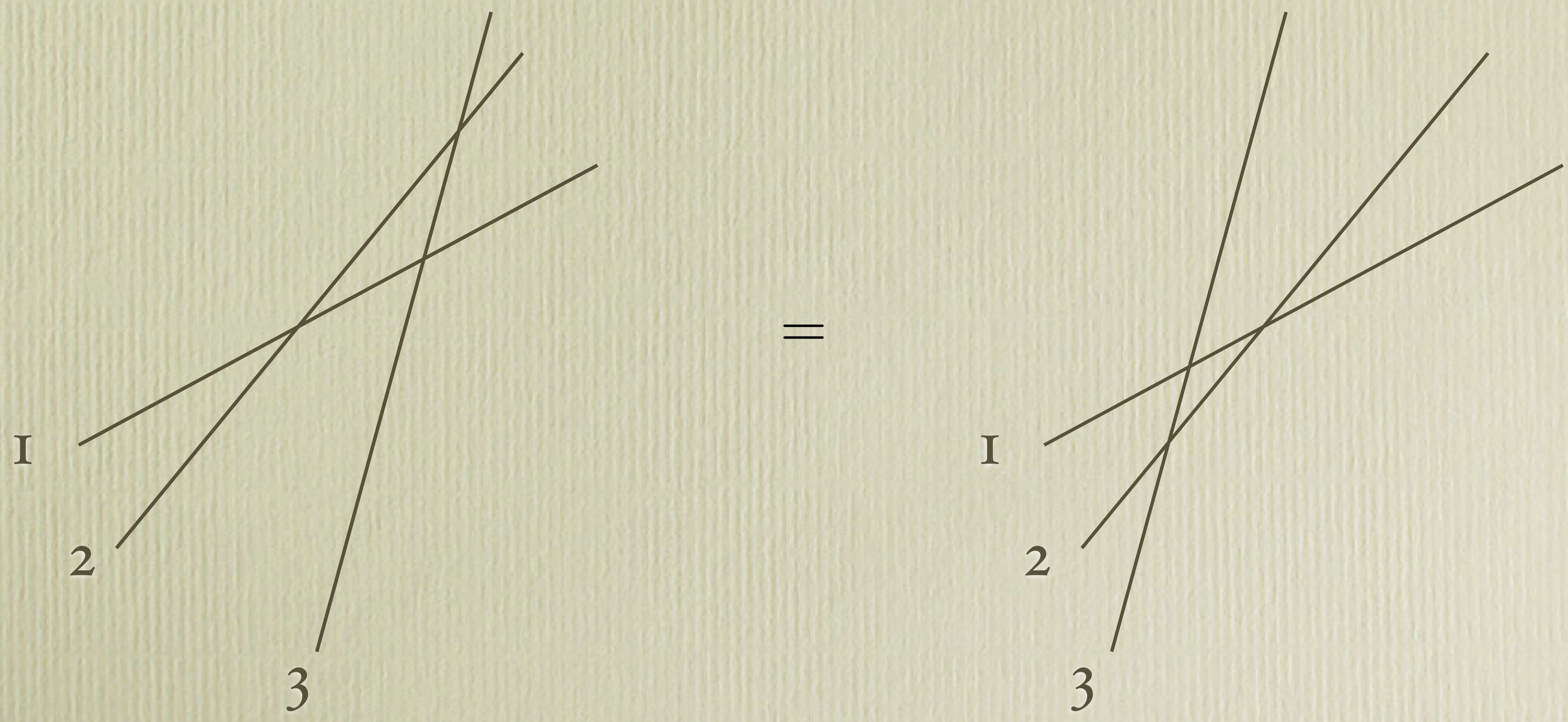
$$
\begin{aligned}
S^{a\,a}_{a\,a} &= \mathcal{A}\,, \quad S^{\alpha\,\alpha}_{\alpha\,\alpha} = \mathcal{D}\,, \\
S^{a\,b}_{a\,b} &= \frac{1}{2}(\mathcal{A}-\mathcal{B})\,, \quad S^{b\,a}_{a\,b} = \frac{1}{2}(\mathcal{A}+\mathcal{B})\,, \\
S^{\alpha\,\beta}_{\alpha\,\beta} &= \frac{1}{2}(\mathcal{D}-\mathcal{E})\,, \quad S^{\beta\,\alpha}_{\alpha\,\beta} = \frac{1}{2}(\mathcal{D}+\mathcal{E})\,, \\
S^{\alpha\,\beta}_{a\,b} &= -\frac{1}{2}\epsilon_{ab}\epsilon^{\alpha\beta}\,\mathcal{C}\,, \quad S^{a\,b}_{\alpha\,\beta} = -\frac{1}{2}\epsilon^{ab}\epsilon_{\alpha\beta}\,\mathcal{F}\,, \\
S^{a\,\alpha}_{a\,\alpha} &= \mathcal{G}\,, \quad S^{\alpha\,a}_{a\,\alpha} = \mathcal{H}\,, \quad S^{a\,\alpha}_{\alpha\,a} = \mathcal{K}\,, \quad S^{\alpha\,a}_{\alpha\,a} = \mathcal{L}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A} &= S_0\,\frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\,\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2}\,, \\
\mathcal{B} &= -S_0\left[\frac{x_2^- - x_1^+}{x_2^+ - x_1^-} + 2\,\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_2^- + x_1^+)}{(x_1^- - x_2^+)(x_1^- x_2^- - x_1^+ x_2^+)}\right]\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2}\,,\ldots
\end{aligned}
$$

$$
x_i^{\pm} = x^{\pm}(p_i)\,, \quad \eta_1 = \eta(p_1)e^{ip_2/2}\,, \quad \eta_2 = \eta(p_2)\,, \quad \tilde{\eta}_1 = \eta(p_1)\,, \quad \tilde{\eta}_2 = \eta(p_2)e^{ip_1/2}
$$

S-matrix satisfies Yang-Baxter equation

$$S_{12}^{AA}(p_1,p_2)\,S_{13}^{AA}(p_1,p_3)\,S_{23}^{AA}(p_2,p_3) = S_{23}^{AA}(p_2,p_3)\,S_{13}^{AA}(p_1,p_3)\,S_{12}^{AA}(p_1,p_2)$$



Yangian

[Drinfeld 85, ...]

Generators $\mathbb{J}^A\,, \hat{\mathbb{J}}^A\,, \ldots$

$$[\mathbb{J}^A\,, \mathbb{J}^B] = f_C^{AB}\,\mathbb{J}^C\,, \quad [\mathbb{J}^A\,, \hat{\mathbb{J}}^B] = f_C^{AB}\,\hat{\mathbb{J}}^C$$

+ Jacobi & Serre relations

Nontrivial coproduct

$$\Delta(\hat{\mathbb{J}}^A) = \hat{\mathbb{J}}^A \otimes \mathbb{I} + \mathbb{I} \otimes \hat{\mathbb{J}}^A + \frac{\alpha}{2} f_{BC}^A\,\mathbb{J}^B\,\mathbb{J}^C$$

Motivation from QISM: g-invariant R-matrix

monodromy matrix $$T_a(u) = R_{a1}(u) \cdots R_{aN}(u)$$

algebra $$R_{ab}(u-v)\, T_a(u)\, T_b(v) = T_b(v)\, T_a(u)\, R_{ab}(u-v)$$

coproduct

1 site → 2 sites: $$R_{a1}(u) \to R_{a1}(u) R_{a2}(u)$$

i.e.,

$$\Delta(T_a(u)) = T_a(u) \otimes T_a(u)$$

transfer matrix $$t(u) = \mathrm{tr}_a\, T_a(u)$$

$$[t(u)\,, t(v)] = 0$$

Familiar fact:

small u expansion of *transfer* matrix

$$\ln t(u) = \sum_{n=0}^{\infty} \frac{u^n}{n!} H_n$$

$H_0 =$ momentum, $\quad H_1 =$ Hamiltonian, etc.

$$[H_n\,, H_m] = 0$$

Familiar fact:

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$$[H_n\,, H_m] = 0$$

Less familiar fact:

large u expansion of *monodromy* matrix

$$\ln T_a(u) = -\frac{1}{u} t_A \mathbb{J}^A + \frac{1}{u^2} t_A \hat{\mathbb{J}}^A + \ldots$$

coproduct for $T_a(u) \Rightarrow$ coproduct for $\hat{\mathbb{J}}^A$

For su(2|2): Evaluation representation

[Beisert 07]

$$\hat{\mathbb{J}}^A = -\frac{1}{2} igu\, \mathbb{J}^A \qquad u = \frac{1}{2}\left(x^+ + \frac{1}{x^+} + x^- + \frac{1}{x^-}\right)$$

Nontrivial coproduct, e.g.

$$\Delta(\hat{\mathbb{L}}_2{}^1) = \hat{\mathbb{L}}_2{}^1 \otimes \mathbb{I} + \mathbb{I} \otimes \hat{\mathbb{L}}_2{}^1$$
$$+\frac{1}{2}\mathbb{L}_2{}^c \otimes \mathbb{L}_c{}^1 - \frac{1}{2}\mathbb{L}_c{}^1 \otimes \mathbb{L}_2{}^c - \frac{1}{2}\mathbb{Q}_2^{\dagger\gamma} \otimes \mathbb{Q}_\gamma{}^1 - \frac{1}{2}\mathbb{Q}_\gamma{}^1 \otimes \mathbb{Q}_2^{\dagger\gamma}$$

Can re-express in terms of ZF operators:

$$\begin{aligned}
\hat{\mathbb{L}}_2{}^1\, A_1^\dagger(p) \quad &= \quad -\frac{1}{2} igu\, A_2^\dagger(p) + A_1^\dagger(p)\, \hat{\mathbb{L}}_2{}^1 - \frac{1}{2} A_1^\dagger(p)\, \mathbb{L}_2{}^1 + \frac{1}{2} A_2^\dagger(p)\, (\mathbb{L}_1{}^1 - \mathbb{L}_2{}^2) \\
&\quad + \frac{1}{2} c A_4^\dagger(p)\, \mathbb{Q}_3{}^1 - \frac{1}{2} c A_3^\dagger(p)\, \mathbb{Q}_4{}^1 - \frac{1}{2} a A_3^\dagger(p)\, \mathbb{Q}_2^{\dagger 3} - \frac{1}{2} a A_4^\dagger(p)\, \mathbb{Q}_2^{\dagger 4}\,, \\
\hat{\mathbb{L}}_2{}^1\, A_2^\dagger(p) \quad &= \quad A_2^\dagger(p)\, \hat{\mathbb{L}}_2{}^1 + \frac{1}{2} A_2^\dagger(p)\, \mathbb{L}_2{}^1\,, \cdots
\end{aligned}$$

For su(2|2): Evaluation representation [Beisert 07]

$$\hat{\mathbb{J}}^A = -\frac{1}{2} igu\, \mathbb{J}^A \qquad u = \frac{1}{2}\left(x^+ + \frac{1}{x^+} + x^- + \frac{1}{x^-}\right)$$

Nontrivial coproduct, e.g.

$$\Delta(\hat{\mathbb{L}}_2{}^1) = \hat{\mathbb{L}}_2{}^1 \otimes \mathbb{I} + \mathbb{I} \otimes \hat{\mathbb{L}}_2{}^1$$
$$+\frac{1}{2}\mathbb{L}_2{}^c \otimes \mathbb{L}_c{}^1 - \frac{1}{2}\mathbb{L}_c{}^1 \otimes \mathbb{L}_2{}^c - \frac{1}{2}\mathbb{Q}_2^{\dagger\gamma} \otimes \mathbb{Q}_\gamma{}^1 - \frac{1}{2}\mathbb{Q}_\gamma{}^1 \otimes \mathbb{Q}_2^{\dagger\gamma}$$

Can re-express in terms of ZF operators:

$$\hat{\mathbb{L}}_2{}^1\, A_1^\dagger(p) = -\frac{1}{2} igu\, A_2^\dagger(p) + A_1^\dagger(p)\,\hat{\mathbb{L}}_2{}^1 - \frac{1}{2}A_1^\dagger(p)\,\mathbb{L}_2{}^1 + \frac{1}{2}A_2^\dagger(p)\,(\mathbb{L}_1{}^1 - \mathbb{L}_2{}^2)$$
$$+\frac{1}{2}cA_4^\dagger(p)\,\mathbb{Q}_3{}^1 - \frac{1}{2}cA_3^\dagger(p)\,\mathbb{Q}_4{}^1 - \frac{1}{2}aA_3^\dagger(p)\,\mathbb{Q}_2^{\dagger 3} - \frac{1}{2}aA_4^\dagger(p)\,\mathbb{Q}_2^{\dagger 4},$$

$$\hat{\mathbb{L}}_2{}^1\, A_2^\dagger(p) = A_2^\dagger(p)\,\hat{\mathbb{L}}_2{}^1 + \frac{1}{2}A_2^\dagger(p)\,\mathbb{L}_2{}^1, \cdots$$

Yangian generators $\hat{\mathbb{J}}^A$ commute with 2-particle scattering!

2. Fundamental boundary S-matrix

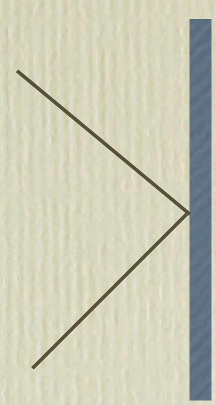
Boundary operator $\mathbf{B}$

[Ghoshal & Zamolodchikov 93;
Ahn & N. 08]

$$|0\rangle_B = \mathbf{B}|0\rangle$$

Boundary S-matrix

$$A_i^\dagger(p)\,\mathbf{B} = R^{A\,i'}_{i}(p)\,A^\dagger_{i'}(-p)\,\mathbf{B}$$



Y=0 brane preserves only an su(1|2) subalgebra

[Hofman &
Maldacena 07]

$$\mathbb{L}_1{}^1\,, \quad \mathbb{L}_2{}^2\,, \quad \mathbb{H}\,, \quad \mathbb{R}_\alpha{}^\beta\,, \quad \mathbb{Q}_\alpha{}^1\,, \quad \mathbb{Q}_1^{\dagger\alpha} \qquad \alpha\,,\beta \in \{3\,,4\}$$

Determine boundary S-matrix by demanding that unbroken su(1|2) generators $\mathbb{J}^A$ commute with 1-particle reflection:

$$\mathbb{J}^A \, A_i^\dagger(p)|0\rangle_B$$

Can first reflect $A^\dagger$, then move $\mathbb{J}^A$ to the right $\mathbb{J}^A|0\rangle_B = 0$

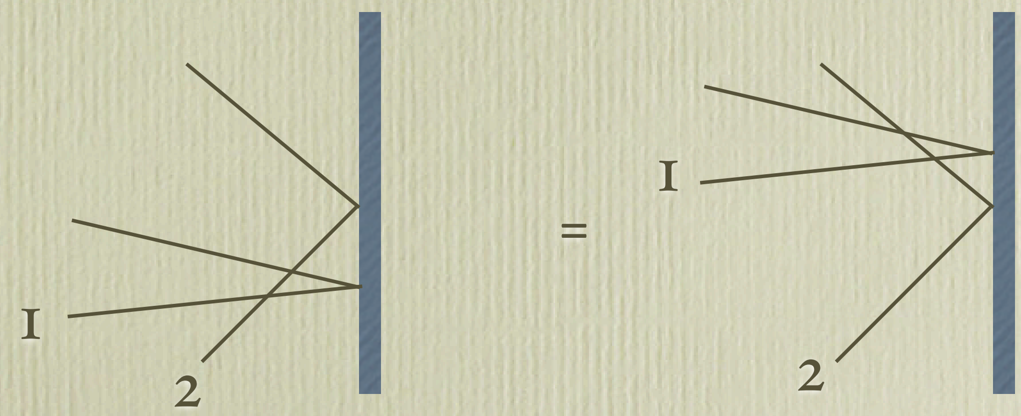
Or first move $\mathbb{J}^A$ to the right, and then reflect $A^\dagger$

Consistency $\Rightarrow$ linear equations for S-matrix elements

$$R^A(p) = r(p)\,\mathrm{diag}(e^{-ip/2}\,, -e^{ip/2}\,, 1\,, 1)$$

S-matrix satisfies boundary Yang-Baxter equation [Cherednik 84, ...]

$$S_{12}^{AA}(p_1,p_2)\,R_1^A(p_1)\,S_{21}^{AA}(p_2,-p_1)\,R_2^A(p_2)$$
$$= R_2^A(p_2)\,S_{12}^{AA}(p_1,-p_2)\,R_1^A(p_1)\,S_{21}^{AA}(-p_2,-p_1)$$



3. Yangian symmetry of fundamental boundary S-matrix

Assume conserved ("boundary Yangian"):

$$\tilde{\mathbb{Q}} \equiv \hat{\mathbb{L}}_2{}^1 + \frac{1}{2}\left(\mathbb{L}_2{}^1\mathbb{L}_1{}^1 - \mathbb{L}_2{}^1\mathbb{L}_2{}^2 - \mathbb{Q}_2^{\dagger 3}\mathbb{Q}_3{}^1 - \mathbb{Q}_2^{\dagger 4}\mathbb{Q}_4{}^1\right)$$

Action on ZF operators:

$$\begin{aligned}
\tilde{\mathbb{Q}}\, A_1^\dagger(p) &= \Big(-\frac{1}{2}igu + \frac{1}{2} - ad\Big) A_2^\dagger(p) + A_2^\dagger(p)\,(\mathbb{L}_1{}^1 - \mathbb{L}_2{}^2) \\
&\quad + cA_4^\dagger(p)\,\mathbb{Q}_3{}^1 - cA_3^\dagger(p)\,\mathbb{Q}_4{}^1 + A_1^\dagger(p)\,\tilde{\mathbb{Q}} \\
\tilde{\mathbb{Q}}\, A_2^\dagger(p) &= A_2^\dagger(p)\,\tilde{\mathbb{Q}} \\
\tilde{\mathbb{Q}}\, A_3^\dagger(p) &= dA_2^\dagger(p)\,\mathbb{Q}_3{}^1 + A_3^\dagger(p)\,\tilde{\mathbb{Q}} \\
\tilde{\mathbb{Q}}\, A_4^\dagger(p) &= dA_2^\dagger(p)\,\mathbb{Q}_4{}^1 + A_4^\dagger(p)\,\tilde{\mathbb{Q}}
\end{aligned}$$

All the symmetry generators on RHS are conserved !

"coideal property" [Delius, MacKay & Short 01]

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Action on ZF operators:

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\tilde{\mathbb{Q}}\, A_1^\dagger(p) &= \left(-\frac{1}{2}igu + \frac{1}{2} - ad\right) A_2^\dagger(p) + A_2^\dagger(p)\,(\mathbb{L}_1{}^1 - \mathbb{L}_2{}^2) \\
&\quad + cA_4^\dagger(p)\,\mathbb{Q}_3{}^1 - cA_3^\dagger(p)\,\mathbb{Q}_4{}^1 + A_1^\dagger(p)\,\tilde{\mathbb{Q}} \\
\tilde{\mathbb{Q}}\, A_2^\dagger(p) &= A_2^\dagger(p)\,\tilde{\mathbb{Q}} \\
\tilde{\mathbb{Q}}\, A_3^\dagger(p) &= dA_2^\dagger(p)\,\mathbb{Q}_3{}^1 + A_3^\dagger(p)\,\tilde{\mathbb{Q}} \\
\tilde{\mathbb{Q}}\, A_4^\dagger(p) &= dA_2^\dagger(p)\,\mathbb{Q}_4{}^1 + A_4^\dagger(p)\,\tilde{\mathbb{Q}}
\end{aligned}$$

All the symmetry generators on RHS are conserved !

"coideal property" [Delius, MacKay & Short 01]

Boundary Yangian generator $\tilde{\mathbb{Q}}$ commutes with 1-particle reflection:

$$\tilde{\mathbb{Q}}\, A_i^\dagger(p)|0\rangle_B$$

Can first reflect $A^\dagger$, then move $\tilde{\mathbb{Q}}$ to the right $\tilde{\mathbb{Q}}|0\rangle_B = 0$

Or first move $\tilde{\mathbb{Q}}$ to the right, and then reflect $A^\dagger$

Consistency $\Rightarrow$ linear equation which is compatible with boundary S-matrix!

4. Bulk S-matrix in 2-particle bound state rep

2-particle bound states form an 8-dimensional
(atypical totally symmetric) representation of su(2|2):
4 bosons + 4 fermions

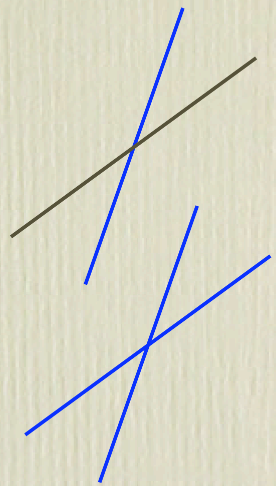
ZF operators

$$B^\dagger_J(p) \qquad J \in \{1, \ldots, 8\}$$

Bulk S-matrices

$$A^\dagger_i(p_1)\, B^\dagger_J(p_2) = S^{AB}{}_{i\,J}^{i'\,J'}(p_1, p_2)\, B^\dagger_{J'}(p_2)\, A^\dagger_{i'}(p_1)$$

$$B^\dagger_I(p_1)\, B^\dagger_J(p_2) = S^{BB}{}_{I\,J}^{I'\,J'}(p_1, p_2)\, B^\dagger_{J'}(p_2)\, B^\dagger_{I'}(p_1)$$



Action of su(2|2) generators on ZF operators:

$$\mathbb{J}^A \, B_J^\dagger(p) = (\mathbb{J}^A)_J{}^K \, B_K^\dagger(p) + B_J^\dagger(p) \, \mathbb{J}^A \qquad \mathbb{J}^A = \mathbb{L}_a{}^b \, , \; \mathbb{R}_\alpha{}^\beta$$

$$\begin{aligned} \mathbb{Q}_\alpha{}^a \, B_J^\dagger(p) &= e^{-ip/2} \left[(\mathbb{Q}_\alpha{}^a)_J{}^K \, B_K^\dagger(p) + (-1)^{\epsilon_J} B_J^\dagger(p) \, \mathbb{Q}_\alpha{}^a \right] \\ \mathbb{Q}_a^{\dagger\alpha} \, B_J^\dagger(p) &= e^{ip/2} \left[(\mathbb{Q}_a^{\dagger\alpha})_J{}^K \, B_K^\dagger(p) + (-1)^{\epsilon_J} B_J^\dagger(p) \, \mathbb{Q}_a^{\dagger\alpha} \right] \end{aligned}$$

$$\epsilon_J = \begin{cases} 0 & \text{for } J = 1, \ldots, 4 \quad \text{bosonic} \\ 1 & \text{for } J = 5, \ldots, 8 \quad \text{fermionic} \end{cases}$$

Action of su(2|2) generators on ZF operators:

$$\mathbb{J}^A\, B_J^\dagger(p) = (\mathbb{J}^A)_J{}^K B_K^\dagger(p) + B_J^\dagger(p)\, \mathbb{J}^A \qquad \mathbb{J}^A = \mathbb{L}_a{}^b\,,\ \mathbb{R}_\alpha{}^\beta$$

$$\mathbb{Q}_\alpha{}^a\, B_J^\dagger(p) = e^{-ip/2}\left[(\mathbb{Q}_\alpha{}^a)_J{}^K B_K^\dagger(p) + (-1)^{\epsilon_J} B_J^\dagger(p)\, \mathbb{Q}_\alpha{}^a\right]$$

$$\mathbb{Q}_a^{\dagger\alpha}\, B_J^\dagger(p) = e^{ip/2}\left[(\mathbb{Q}_a^{\dagger\alpha})_J{}^K B_K^\dagger(p) + (-1)^{\epsilon_J} B_J^\dagger(p)\, \mathbb{Q}_a^{\dagger\alpha}\right]$$

$$\epsilon_J = \begin{cases} 0 & \text{for } J = 1, \ldots, 4 \quad \text{bosonic} \\ 1 & \text{for } J = 5, \ldots, 8 \quad \text{fermionic} \end{cases}$$

Explicit 8 x 8 matrices $(\mathbb{J}^A)_J{}^K,\ (\mathbb{Q}_\alpha{}^a)_J{}^K,\ (\mathbb{Q}_a^{\dagger\alpha})_J{}^K$
can be obtained using superspace formalism

Superspace formalism

[Arutyuonov & Frolov 08]

Represent su(2|2) generators by differential operators on a vector space of analytic functions of two bosonic variables w_a and two fermionic variables θ_α :

$$\mathbb{L}_a{}^b = w_a \frac{\partial}{\partial w_b} - \frac{1}{2}\delta_a^b w_c \frac{\partial}{\partial w_c}\,, \qquad \mathbb{R}_\alpha{}^\beta = \theta_\alpha \frac{\partial}{\partial \theta_\beta} - \frac{1}{2}\delta_\alpha^\beta \theta_\gamma \frac{\partial}{\partial \theta_\gamma}$$

$$\mathbb{Q}_\alpha{}^a = a\,\theta_\alpha \frac{\partial}{\partial w_a} + b\,\epsilon^{ab}\epsilon_{\alpha\beta} w_b \frac{\partial}{\partial \theta_\beta}\,, \qquad \mathbb{Q}_a^{\dagger\alpha} = d\,w_a \frac{\partial}{\partial \theta_\alpha} + c\,\epsilon_{ab}\epsilon^{\alpha\beta}\theta_\beta \frac{\partial}{\partial w_b}$$

Basis vectors $|e_i\rangle$ for fundamental $(l=1)$ rep:

$$|e_a\rangle = w_a\,,\ |e_\alpha\rangle = \theta_\alpha$$

Basis vectors $|e_J\rangle$ for 2-particle bound state $(l=2)$ rep:

$$|e_1\rangle = \frac{w_1 w_1}{\sqrt{2}}\,, \qquad |e_2\rangle = w_1 w_2\,, \qquad |e_3\rangle = \frac{w_2 w_2}{\sqrt{2}}\,, \qquad |e_4\rangle = \theta_3\theta_4$$

$$|e_5\rangle = w_1\theta_3\,, \qquad |e_6\rangle = w_1\theta_4\,, \qquad |e_7\rangle = w_2\theta_3\,, \qquad |e_8\rangle = w_2\theta_4$$

As for S^{AA}, can determine S^{AB} by demanding that su(2|2) generators commute with 2-particle scattering.

Not so for S^{BB}: need 1 additional relation

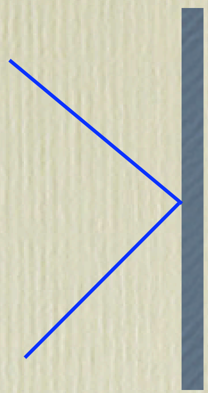
Can obtain by demanding that Yangian generators $\hat{\mathbb{J}}^{A}$ commute with 2-particle scattering

[de Leeuw '08; Arutyuonov, de Leeuw & Torrielli '09]

5. Boundary S-matrix in 2-particle bound state rep

Boundary S-matrix

$$B^{\dagger}_{J}(p)\,\mathbf{B} = R^{B}{}_{J}{}^{J'}(p)\,B^{\dagger}_{J'}(-p)\,\mathbf{B}$$



Again assume that only su(1|2) generators are conserved:

$$\mathbb{L}_1{}^1\,,\quad \mathbb{L}_2{}^2\,,\quad \mathbb{H}\,,\quad \mathbb{R}_\alpha{}^\beta\,,\quad \mathbb{Q}_\alpha{}^1\,,\quad \mathbb{Q}^{\dagger\alpha}_1 \qquad \alpha\,,\beta \in \{3\,,4\}$$

Demanding that $\mathbb{L}_1{}^1, \mathbb{R}_\alpha{}^\beta$ commute with 1-particle reflection

$$\Rightarrow \qquad R^B(p) = \begin{pmatrix} r_1 & & & & & & & \\ & r_2 & & r_5 & & & & \\ & & r_3 & & & & & \\ & r_6 & & r_4 & & & & \\ & & & & r_7 & & & \\ & & & & & r_7 & & \\ & & & & & & r_8 & \\ & & & & & & & r_8 \end{pmatrix}$$

$\mathbb{Q}_\alpha{}^1, \ \mathbb{Q}_1^{\dagger\alpha}$ $\Rightarrow$ 6 independent relations for 7 unknowns

As in the case of S^{BB}, missing 1 relation.

Can obtain from boundary Yangian $\tilde{\mathbb{Q}}$!

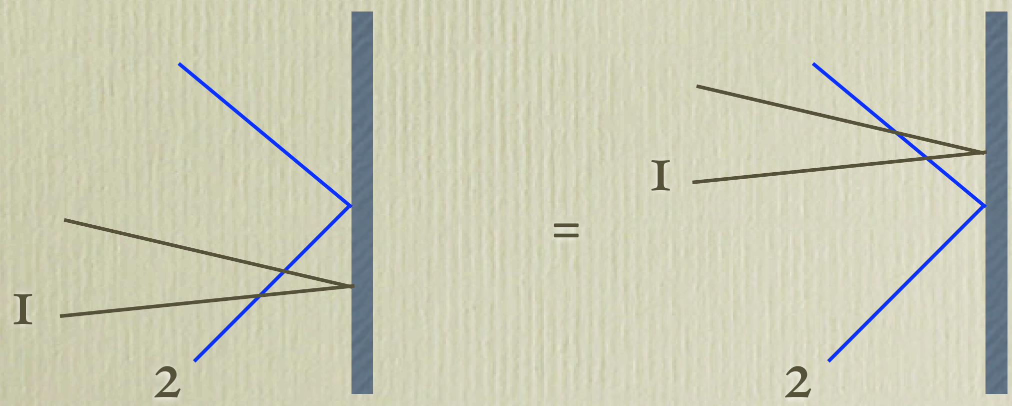
Obtain:

$$r_1 = 1\,, \qquad r_2 = -\frac{\frac{1}{x^-}+x^-}{\frac{1}{x^+}+x^-}\,, \qquad r_3 = e^{ip}\,, \qquad r_4 = \frac{\frac{1}{x^+}+x^+}{\frac{1}{x^+}+x^-}$$

$$r_5 = -r_6 = e^{ip/2}\,\frac{x^- - x^+}{1+x^-x^+}\,, \qquad r_7 = -r_8 = e^{ip/2}$$

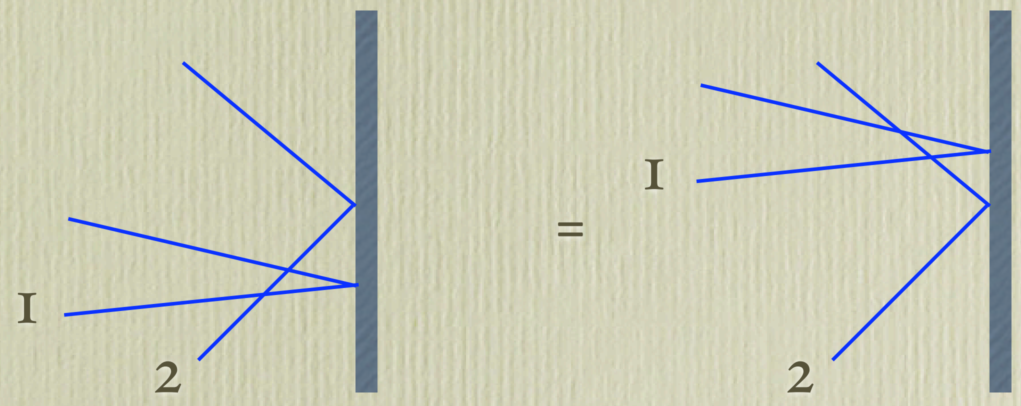
Both boundary Yang-Baxter equations are satisfied:

$$S_{12}^{AB}(p_1,p_2)\,R_1^A(p_1)\,S_{21}^{BA}(p_2,-p_1)\,R_2^B(p_2)$$
$$= R_2^B(p_2)\,S_{12}^{AB}(p_1,-p_2)\,R_1^A(p_1)\,S_{21}^{BA}(-p_2,-p_1)$$



and

$$S_{12}^{BB}(p_1,p_2)\,R_1^B(p_1)\,S_{21}^{BB}(p_2,-p_1)\,R_2^B(p_2)$$
$$= R_2^B(p_2)\,S_{12}^{BB}(p_1,-p_2)\,R_1^B(p_1)\,S_{21}^{BB}(-p_2,-p_1)$$



Remarks:

- S^{BB} is a very complicated 64 x 64 matrix!
- R^B is not diagonal!

6. Discussion

For Y=0 brane:

- Fundamental boundary S-matrix has Yangian symmetry
- Can use this symmetry to determine boundary S-matrix for 2-particle bound states
- Should be possible to extend to general l-particle bound states

Expect similar results for other branes:

- Z=0 [Hofman & Maldacena 07]
- D7 & D5 [Correa & Young 08]

Yangian symmetry in both bulk and boundary scattering

Yangian symmetry may be a generic feature of AdS/CFT worldsheet scattering

Any connection with Yangian symmetry in spacetime scattering?

[Drummond, Henn & Plefka 09;
Beisert, Henn, McLoughlin & Plefka 10]

¡Arriba España!