Quantum finite size effects for dyonic magnons in $AdS_4 \times \mathbb{CP}^3$

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- All-loop Bethe ansatz, S-matrix and Y-system(TBA)...
- However, We need to check more till before perfect proof.

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- In our work, we answer for these questions.

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  - We consider these finite size effects at strong coupling regime.

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- **ALGEBRAIC CURVE $\rightarrow$ Semi-classical effects in string theory**
- **EXACT S-MATRIX $\rightarrow$ Lüscher F-term correction**

- Consider $\mathrm{SU}(2)_\mathrm{A} \times \mathrm{SU}(2)_\mathrm{B} \subset \mathrm{SU}(4)_\mathrm{R}$
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- **Dilatation operator at two loop is integrable spin chain Hamiltonian.**
  - Two excitation $A_i, B_i \Rightarrow$ Two decoupled Heigenberg $XXX_{\frac{1}{2}}$ Hamiltonian

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  - Using $L(x)$, construct the monodromy $\Omega(x)$ ($\mathrm{Tr}\Omega(\mathrm{x})$ give the transfer matrix $T(x)$.)
  - Diagonalization of the monodromy $\rightarrow$ Chracteristic equations give eigenvalues.

$$\Omega(x) \sim \mathrm{diag}\left(\mathrm{e}^{\mathrm{i}\hat{\mathrm{p}_1}}, \mathrm{e}^{\mathrm{i}\hat{\mathrm{p}_2}}, \mathrm{e}^{\mathrm{i}\hat{\mathrm{p}_3}}, \mathrm{e}^{\mathrm{i}\hat{\mathrm{p}_4}}, \mathrm{e}^{\mathrm{i}\tilde{\mathrm{p}_1}}, \mathrm{e}^{\mathrm{i}\tilde{\mathrm{p}_2}}, \mathrm{e}^{\mathrm{i}\tilde{\mathrm{p}_3}}, \mathrm{e}^{\mathrm{i}\tilde{\mathrm{p}_4}}\right),$$

  - where $\hat{p}$ denotes the eigenvalues corresponding to AdS_4 and $\tilde{p}$ to $\mathbb{CP}^4$.

All-loop Bethe Ansatz equations - Gromov and Vieira

$$
\begin{aligned}
1 &= \prod_{j=1}^{K_2} \frac{u_{1,k} - u_{2,j} + \frac{i}{2}}{u_{1,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{1 - 1/x_{1,k} x_{4,j}^{+}}{1 - 1/x_{1,k} x_{4,j}^{-}} \prod_{j=1}^{K_{\bar{4}}} \frac{1 - 1/x_{1,k} x_{\bar{4},j}^{+}}{1 - 1/x_{1,k} x_{\bar{4},j}^{-}} \,, \\
1 &= \prod_{j\neq k}^{K_2} \frac{u_{2,k} - u_{2,j} - i}{u_{2,k} - u_{2,j} + i} \prod_{j=1}^{K_1} \frac{u_{2,k} - u_{1,j} + \frac{i}{2}}{u_{2,k} - u_{1,j} - \frac{i}{2}} \prod_{j=1}^{K_3} \frac{u_{1,k} - u_{3,j} + \frac{i}{2}}{u_{1,k} - u_{3,j} - \frac{i}{2}} \,, \\
1 &= \prod_{j=1}^{K_2} \frac{u_{3,k} - u_{2,j} + \frac{i}{2}}{u_{3,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{3,k} - x_{4,j}^{+}}{x_{3,k} - x_{4,j}^{-}} \prod_{j=1}^{K_{\bar{4}}} \frac{x_{3,k} - x_{\bar{4},j}^{+}}{x_{3,k} - x_{\bar{4},j}^{-}} \\
\left(\frac{x_{4,k}^{+}}{x_{4,k}^{-}}\right)^{L} &= \prod_{j\neq k}^{K_4} \frac{u_{4,k} - u_{4,j} + i}{u_{4,k} - u_{4,j} - i} \prod_{j=1}^{K_1} \frac{1 - 1/x_{4,k}^{-} x_{1,j}}{1 - 1/x_{4,k}^{+} x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{4,k}^{-} - x_{3,j}}{x_{4,k}^{+} - x_{3,j}} \\
&\times \prod_{j=1}^{K_4} \sigma_{\mathrm{BES}}(u_{4,k}, u_{4,j}) \prod_{j=1}^{K_{\bar{4}}} \sigma_{\mathrm{BES}}(u_{4,k}, u_{\bar{4},j}) \,, \\
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\end{aligned}
$$

Exact S-matrix - Ahn and Nepomechie

$$\begin{aligned} S^{AA}(p_1,p_2) &= S^{BB}(p_1,p_2) = S_0(p_1,p_2)\widehat{S}(p_1,p_2) \\ S^{AB}(p_1,p_2) &= S^{BA}(p_1,p_2) = \tilde{S}_0(p_1,p_2)\widehat{S}(p_1,p_2) \end{aligned}$$

$$\begin{aligned} S_0(p_1,p_2) &= \frac{1-\frac{1}{x_1^+ x_2^-}}{1-\frac{1}{x_1^- x_2^+}}\sigma(p_1,p_2) \\ \tilde{S}_0(p_1,p_2) &= \frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\sigma(p_1,p_2) \end{aligned}$$

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  - Big magnon $\rightarrow$ Dressed solution
- These magnon solutions can be reproduced in Algebraic Curve too.

- The eigenvalues $e^{ip(x)}$ are the zeroes of the characteristic polynomial of $\Omega(x)$, and thereby define the algebraic curve in the complex parameter. The degree of the polynomial specifies the number of sheets, which in the case of $\mathrm{AdS}_4 \times \mathbb{CP}^3$ string is eight.

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- We study classical string solutions and their one-loop effects using this algebraic curve.
- Different string solutions are mapped to different sets of eigenvalues of the monodromy.

Algebraic curve and Semi-classical effects

- String solutions living in mostly $\mathbb{CP}^3$ are mapped to the following quasi-momenta.

$$
\begin{aligned}
q_1 &= \frac{\frac{\Delta}{2g}x}{x^2-1} \\
q_2 &= \frac{\frac{\Delta}{2g}x}{x^2-1} \\
q_3 &= \frac{\frac{\Delta}{2g}x}{x^2-1} + G_u(0) - G_u\left(\frac{1}{x}\right) + G_v(0) - G_v\left(\frac{1}{x}\right) + G_r(x) - G_r(0) + G_r\left(\frac{1}{x}\right) \\
q_4 &= \frac{\frac{\Delta}{2g}x}{x^2-1} + G_u(x) + G_v(x) - G_r(x) + G_r(0) - G_r\left(\frac{1}{x}\right) \\
q_5 &= G_u(x) - G_u(0) + G_u\left(\frac{1}{x}\right) - G_v(x) + G_v(0) - G_v\left(\frac{1}{x}\right).
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\end{aligned}
$$

- And $(q_6, q_7, q_8, q_8, q_{10}) = -(q_5, q_4, q_3, q_2, q_1)$

Algebraic curve and Semi-classical effects

- These quasi-momenta satisfy following analytic properties:

$$\lim_{x\to\infty}\begin{pmatrix} q_1(x) \\ q_2(x) \\ q_3(x) \\ q_4(x) \\ q_5(x) \end{pmatrix} \simeq \frac{1}{2gx}\begin{pmatrix} \Delta \\ \Delta \\ J_1 \\ J_2 \\ J_3 \end{pmatrix}.$$

$$\begin{pmatrix} q_1\,(1/x) \\ q_2\,(1/x) \\ q_3\,(1/x) \\ q_4\,(1/x) \\ q_5\,(1/x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \pi m \\ \pi m \\ 0 \end{pmatrix} + \begin{pmatrix} -q_2(x) \\ -q_1(x) \\ -q_4(x) \\ -q_3(x) \\ +q_5(x) \end{pmatrix}.$$

$$\lim_{x\to\pm1}\begin{pmatrix} q_1(x) \\ q_2(x) \\ q_3(x) \\ q_4(x) \\ q_5(x) \end{pmatrix} \simeq \frac{1}{2(x\mp1)}\begin{pmatrix} \alpha_\pm \\ \alpha_\pm \\ \alpha_\pm \\ \alpha_\pm \\ 0 \end{pmatrix}.$$

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Algebraic curve and Semi-classical effects

- Three kinds of GM solutions in $\mathbb{CP}^3$
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  - $G_u = G_v = G_r = G_{\mathrm{magnon}} \to$ Big GM (dressed solution)
- In our case, resolvents are :

$$G_{\mathrm{magnon}} = -i \log \left(\frac{x - X^+}{x - X^-} \right).$$

Algebraic curve and Semi-classical effects

- Small magnon

$$
\begin{aligned}
q_1 &= -q_{10} = \frac{\alpha x}{x^2-1} \\
q_2 &= -q_9 = \frac{\alpha x}{x^2-1} \\
q_3 &= -q_8 = \frac{\alpha x}{x^2-1} - i\log\left(\frac{X^+}{X^-}\right) + i\log\left(\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\right) + \tau \\
q_4 &= -q_7 = \frac{\alpha x}{x^2-1} - i\log\left(\frac{x-X^+}{x-X^-}\right) + \tau \\
q_5 &= -q_6 = -i\log\left(\frac{x-X^+}{x-X^-}\right) + i\log\left(\frac{X^+}{X^-}\right) - i\log\left(\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\right).
\end{aligned}
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- Pair of small magnon

$$
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q_1 &= -q_{10} = \frac{\alpha x}{x^2-1} \\
q_2 &= -q_9 = \frac{\alpha x}{x^2-1} \\
q_3 &= -q_8 = \frac{\alpha x}{x^2-1} - 2i\log\left(\frac{X^+}{X^-}\right) + 2i\log\left(\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\right) - p \\
q_4 &= -q_7 = \frac{\alpha x}{x^2-1} - 2i\log\left(\frac{x-X^+}{x-X^-}\right) - p \\
q_5 &= -q_6 = 0.
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$$\Omega_n^{ij} = -\kappa_{ij}\delta_{i,1} + 2g \lim_{x\to\infty} x\delta_n^{ij} q_1(x)$$

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  - $\kappa_{ij} = 2$ for $(i,j) = (1,10), (2,9)$ and $\kappa_{ij} = 1$ for other pairs.

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  - Off-shell frequencies

$$\Omega^{ij}(y) = \Omega_n^{ij}\big|_{n\to\frac{q_i(y)-q_j(y)}{2\pi}}.$$

- Fluctuation frequencies for magnon solutions
  - On-shell frequencies

$$\Omega_n^{ij} = -\kappa_{ij}\delta_{i,1} + 2g \lim_{x\to\infty} x\delta_n^{ij} q_1(x)$$

  - $\kappa_{ij} = 2$ for $(i,j) = (1,10), (2,9)$ and $\kappa_{ij} = 1$ for other pairs.
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$$\Omega^{ij}(y) = \Omega_n^{ij}\big|_{n\to\frac{q_i(y)-q_j(y)}{2\pi}}.$$

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- There are 16 polarization modes for type IIA superstring in $AdS_4 \times \mathbb{CP}^3$.
  - 8 light modes which are $(i,5)$ or $(i,6)$ pairs and 8 heavy modes for other pairs

- All Ω_{ij}'s can be written in terms of only Ω_{15} and Ω_{45}.

Algebraic curve and Semi-classical effects

- All Ω_{ij}'s can be written in terms of only Ω_{15} and Ω_{45}.
- Light modes

$$\begin{aligned}
\Omega_{i5}(x) &= \Omega_{i6}(x) \\
\Omega_{25}(x) &= \Omega_{15}(0) - \Omega_{15}\left(\frac{1}{x}\right) \\
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- These fluctuation frequencies are all the same : small, pair of small and big magnon.

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- Inversion symmetry, large x asymptotics and synchronization of poles at $x = \pm 1$ give set of equation.
- From these equations, we obtain $\delta\Delta = \Omega_{15}(y)$.
- The above argument is same in case of $(4,5)$ polarization.

- The leading part of 1-loop energy shift is given by the sum of fluctuation frequencies.

$$\begin{aligned}\delta\Delta_{one-loop} &= \frac{1}{2}\sum_{ij}\sum_{n}(-1)^{F_{ij}}\,\Omega_{ij}^{n}\\ &= \int\frac{dx}{2\pi i}\partial_x\Omega(x)\sum_{ij}\gamma_{ij}(-1)^{F_{ij}}\,e^{-i\left(q_i-q_j\right)}.\end{aligned}$$

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- When we evaluate the above integral by using saddle-point approximation, heavy modes can be suppressed because of the factor 2 in exponent.
- So we need to compute $\sum_{ij}(-1)^{F_{ij}}\,e^{-i(q_i-q_j)}$ where the sum over (i,j) pairs include only the light modes.

Algebraic curve and Semi-classical effects

- $\mathbb{CP}^2$ magnon

$$\sum_{ij}(-1)^{F_{ij}}\, e^{-i\left(q_i-q_j\right)} = e^{\frac{-i\alpha x}{x^2-1}}\left[2\,\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\,\frac{x-X^+}{x-X^-}\,\frac{X^+}{X^-}+2\,\frac{\frac{1}{x}-X^-}{\frac{1}{x}-X^+}\,\frac{x-X^-}{x-X^+}\,\frac{X^-}{X^+}-\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\sqrt{\frac{X^-}{X^+}}\right.$$
$$\left.-\frac{x-X^-}{x-X^+}\sqrt{\frac{X^+}{X^-}}-\left(\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\right)^2\frac{x-X^+}{x-X^-}\left(\frac{X^-}{X^+}\right)^{\frac{3}{2}}-\frac{\frac{1}{x}-X^-}{\frac{1}{x}-X^+}\left(\frac{x-X^+}{x-X^-}\right)^2\left(\frac{X^+}{X^-}\right)^{\frac{3}{2}}\right].$$

Algebraic curve and Semi-classical effects

- $\mathbb{CP}^2$ magnon

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$$-\frac{x-X^-}{x-X^+}\sqrt{\frac{X^+}{X^-}}-\left(\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\right)^2\frac{x-X^+}{x-X^-}\left(\frac{X^-}{X^+}\right)^{\frac{3}{2}}$$
$$\left.-\frac{\frac{1}{x}-X^-}{\frac{1}{x}-X^+}\left(\frac{x-X^+}{x-X^-}\right)^2\left(\frac{X^+}{X^-}\right)^{\frac{3}{2}}\right].$$

- $\mathbb{RP}^3$ magnon

$$\sum_{ij}(-1)^{F_{ij}}\, e^{-i\left(q_i-q_j\right)} = 2e^{\frac{-i\alpha x}{x^2-1}}\left[2-\left(\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\right)^2\frac{X^-}{X^+}-\left(\frac{x-X^-}{x-X^+}\right)^2\frac{X^+}{X^-}\right].$$

Algebraic curve and Semi-classical effects

- $\mathbb{CP}^2$ magnon

$$\sum_{ij}(-1)^{F_{ij}}\, e^{-i(q_i-q_j)} = e^{\frac{-i\alpha x}{x^2-1}}\left[2\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\frac{x-X^+}{x-X^-}\frac{X^+}{X^-}+2\frac{\frac{1}{x}-X^-}{\frac{1}{x}-X^+}\frac{x-X^-}{x-X^+}\frac{X^-}{X^+}-\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\sqrt{\frac{X^-}{X^+}}\right.$$
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- Big magnon

$$\sum_{ij}(-1)^{F_{ij}}\, e^{-i(q_i-q_j)} = 4e^{\frac{-i\alpha x}{x^2-1}}\left[1-\frac{\frac{1}{x}-X^+}{\frac{1}{x}-X^-}\frac{x-X^-}{x-X^+}\right].$$

S-matrix and Lüscher's F-term

- Crucial point of the S-matrix in $\mathrm{AdS}_4/\mathrm{CFT}_3$ is that there are two types of fundamental excitations, types A and B, leading 4 kinds of S-matrices S^{AA} ,S^{BB} ,S^{AB} and S^{BA}.

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- We can write these S-matrices as

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\begin{aligned}
S^{AA}(p_1,p_2) &= S^{BB}(p_1,p_2) = S_0(p_1,p_2)\hat{S}(p_1,p_2) \\
S^{AB}(p_1,p_2) &= S^{BA}(p_1,p_2) = \tilde{S}_0(p_1,p_2)\,\hat{S}(p_1,p_2)
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- Here, $\hat{S}$ is the $su(2|2)$-invariant SYM S-matrix and

$$\begin{aligned} S_0(p_1,p_2) &= \frac{1-\frac{1}{x_1^+x_2^-}}{1-\frac{1}{x_1^-x_2^+}}\sigma(p_1,p_2) \\ \tilde{S}_0(p_1,p_2) &= \frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\sigma(p_1,p_2)\,. \end{aligned}$$

S-matrix and Lüscher's F-term

- The relevant S-matrix elements for elementary particles are

$$
\begin{aligned}
a_1(p_1, p_2) &= \frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2} \\
a_2(p_1, p_2) &= \frac{\left(x_1^- - x_1^+\right)\left(x_2^- - x_2^+\right)\left(x_2^- - x_1^+\right)}{\left(x_1^- - x_2^+\right)\left(x_2^- x_1^- - x_2^+ x_1^+\right)}\frac{\eta_1\eta_2}{\tilde{\eta}_1\tilde{\eta}_2} \\
a_6(p_1, p_2) &= \frac{x_1^+ - x_2^+}{x_1^- - x_2^+}\frac{\eta_2}{\tilde{\eta}_2}
\end{aligned}
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S-matrix and Lüscher's F-term

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- The Lüscher's F-term is written as follows.

$$\delta E_F = -\sum_b (-1)^{F_b} \int \frac{dq}{2\pi} \left[1 - \frac{\varepsilon'_Q(p)}{\varepsilon'_1(q^*)} \right] e^{-iq^* L} \left(S_{ba}^{ba}(q^*, p) - 1 \right).$$

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- The leading one-loop contribution from the point of view of S-matrix of dyonic magnon comes from the Lüscher's F-term which is correction effect that result from scattering between virtual and physical particles.

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- In our problem, physical particles are Q-boundstates of magnon or dyonic giant magnons. Virtual particles are composed of fundamental excitations and their boundstates.

S-matrix and Lüscher's F-term

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- The leading one-loop contribution from the point of view of S-matrix of dyonic magnon comes from the Lüscher's F-term which is correction effect that result from scattering between virtual and physical particles.
- In our problem, physical particles are Q-boundstates of magnon or dyonic giant magnons. Virtual particles are composed of fundamental excitations and their boundstates.
- But, leading contribution of this scattering is only from scattering between fundamental virtual magnons and physical dyonic magnons.

- We only show the Small magnon case, but procedures for other magnons are the same.

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- The dispersion relation for the small magnon is as follows.

$$\Delta - J/2 = \varepsilon_Q(p) = \sqrt{\frac{Q^2}{4} + 16g^2 \sin^2 \frac{p}{2}}$$

$$\begin{aligned} y^{\pm} &= x \pm \frac{ix^2}{4g(x^2-1)} \\ q^* &= \frac{x}{g(x^2-1)} \\ q &= \frac{i}{2}\frac{x^2+1}{x^2-1} \\ \varepsilon_Q'(p) &= g\left(\frac{X^+ + X^-}{X^+X^- + 1}\right) \\ \varepsilon_1'(q^*) &= g\left(\frac{2x}{x^2+1}\right) \end{aligned}$$

S-matrix and Lüscher's F-term

- After above computations, we obtained

$$\delta E^{F}_{one-loop} = \int \frac{dx}{2\pi i} \partial_x \Omega\left(x\right) e^{-\frac{ixJ}{g\left(x^2-1\right)}} \sum \left(-1\right)^{F_b} \left(S^{b1_Q}_{b1_Q}\left(q^*, p\right)\right)$$

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- Here, $\Omega\left(x\right)$ is exactly the same with off-shell energy in algebraic curve.

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- Here, $\Omega(x)$ is exactly the same with off-shell energy in algebraic curve.
- The remaining piece is to compute S-matrix elements for dyonic magnons.

$$S^{AAb1_Q}_{b1_Q} = \prod_{k=1}^{Q} \left(\frac{1 - \frac{1}{y^+ x_k^-}}{1 - \frac{1}{y^- x_k^+}} \sigma_{BES}(y, x_k)\, \tilde{a}_b(y, X_k) \right)$$

$$S^{ABb1_Q}_{b1_Q} = \prod_{k=1}^{Q} \left(\frac{y^- - x_k^+}{y^+ - x_k^-} \sigma_{BES}(y, x_k)\, \tilde{a}_b(y, X_k) \right).$$

S-matrix and Lüscher's F-term

- The S-matrix of small magnon is

$$\sum (-1)^{F_b} \left(S_{b1_Q}^{b1_Q}(q^*, p) \right) = \sum_b (-1)^{F_b} \left(S_{b1_Q}^{AAb1_Q} + S_{b1_Q}^{ABb1_Q} \right).$$

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- From those, we obtain the following integration form.

$$\begin{aligned} \delta\Delta_{F-term} \quad = \quad & \int \frac{dx}{2\pi i} \partial_x \Omega(x)\, e^{-i\Delta \frac{x}{2g\left(x^2-1\right)}} \left[\left(\frac{x - \frac{1}{X^+}}{x - \frac{1}{X^-}} \right) e^{\frac{ip}{2}} + \left(\frac{x - X^-}{x - X^+} \right) e^{\frac{ip}{2}} \right. \\ & + \left(\frac{x - \frac{1}{X^-}}{x - \frac{1}{X^+}} \right) \left(\frac{x - X^-}{x - X^+} \right)^2 e^{\frac{ip}{2}} + \left(\frac{x - \frac{1}{X^+}}{x - \frac{1}{X^-}} \right)^2 \left(\frac{x - X^+}{x - X^-} \right) e^{\frac{ip}{2}} \\ & \left. -2 \left(\frac{x - \frac{1}{X^+}}{x - \frac{1}{X^-}} \right) \left(\frac{x - X^+}{x - X^-} \right) - 2 \left(\frac{x - \frac{1}{X^-}}{x - \frac{1}{X^+}} \right) \left(\frac{x - X^-}{x - X^+} \right) \right]. \end{aligned}$$

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& + \left(\frac{x - \frac{1}{X^-}}{x - \frac{1}{X^+}} \right) \left(\frac{x - X^-}{x - X^+} \right)^2 e^{\frac{ip}{2}} + \left(\frac{x - \frac{1}{X^+}}{x - \frac{1}{X^-}} \right)^2 \left(\frac{x - X^+}{x - X^-} \right) e^{\frac{ip}{2}} \\
& \left. -2 \left(\frac{x - \frac{1}{X^+}}{x - \frac{1}{X^-}} \right) \left(\frac{x - X^+}{x - X^-} \right) - 2 \left(\frac{x - \frac{1}{X^-}}{x - \frac{1}{X^+}} \right) \left(\frac{x - X^-}{x - X^+} \right) \right].
\end{aligned}$$

- These are exactly matched with Al.Curve result.

S-matrix and Lüscher's F-term

- The S-matrix of pair of small magnon is

$$\begin{aligned}\sum(-1)^{F_b}\left(S_{b1_Q}^{b1_Q}(q^*,p)\right) &= \sum_b(-1)^{F_b}\left(S_{b1_Q}^{AAb1_Q}S_{b1_Q}^{ABb1_Q}+S_{b1_Q}^{BAb1_Q}S_{b1_Q}^{BBb1_Q}\right)\\ &= 2\sum_b(-1)^{F_b}\,S_{b1_Q}^{AAb1_Q}S_{b1_Q}^{ABb1_Q}.\end{aligned}$$

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- From the above S-matrix element, we have

$$\delta\Delta_{F-term} = \int\frac{dx}{2\pi i}\partial_x\Omega(x)\,e^{-i\Delta\frac{x}{2g\left(x^2-1\right)}} \times \left[4-2\left(\frac{x-\frac{1}{X_p^+}}{x-\frac{1}{X_p^-}}\right)^2 e^{ip}-2\left(\frac{x-X_p^-}{x-X_p^+}\right)^2 e^{ip}\right].$$

S-matrix and Lüscher's F-term

- The S-matrix of pair of small magnon is

$$\begin{aligned}\sum (-1)^{F_b} \left(S_{b1_Q}^{b1_Q}(q^*, p)\right) &= \sum_b (-1)^{F_b} \left(S_{b1_Q}^{AAb1_Q} S_{b1_Q}^{ABb1_Q} + S_{b1_Q}^{BAb1_Q} S_{b1_Q}^{BBb1_Q}\right) \\ &= 2\sum_b (-1)^{F_b}\, S_{b1_Q}^{AAb1_Q} S_{b1_Q}^{ABb1_Q}.\end{aligned}$$

- From the above S-matrix element, we have

$$\begin{aligned}\delta\Delta_{F-term} &= \int \frac{dx}{2\pi i} \partial_x \Omega(x)\, e^{-i\Delta \frac{x}{2g\left(x^2-1\right)}} \\ &\times \left[4 - 2\left(\frac{x - \frac{1}{X_p^+}}{x - \frac{1}{X_p^-}}\right)^2 e^{ip} - 2\left(\frac{x - X_p^-}{x - X_p^+}\right)^2 e^{ip}\right].\end{aligned}$$

- We also have the same energy shift with al.curve.

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- Then, the S-matrix of big magnon is

$$\sum (-1)^{F_b} \left(S_{ba}^{ba}\left(q^*, p\right) \right) = \sum_b (-1)^{F_b} \left(S_{b1_Q}^{AAb1_Q} S_{b1_{-Q}}^{AA' b1_{-Q}} + S_{b1_Q}^{BAb1_Q} S_{b1_{-Q}}^{BA' b1_{-Q}} \right).$$

$$\delta\Delta = \int \frac{dx}{2\pi i} \partial_x \Omega\left(x\right) e^{-i\Delta \frac{x}{2g\left(x^2-1\right)}} \times 4 \left[1 - \left(\frac{x - \frac{1}{X^+}}{x - \frac{1}{X^-}} \right) \left(\frac{x - X^-}{x - X^+} \right) e^{ip} \right].$$

S-matrix and Lüscher's F-term

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- Mathematical coincidence OR Physical meaning (??)

Multi dyonic magnons

- Consider N-dyonic A-particles and M-dyonic B-particles.

Multi dyonic magnons

- Consider N-dyonic A-particles and M-dyonic B-particles.
- Quasi-momenta ansatz about these multi-magnons is :

$$
\begin{aligned}
q_1 &= \frac{\alpha x}{x^2-1} \\
q_2 &= \frac{\alpha x}{x^2-1} \\
q_3 &= \frac{\alpha x}{x^2-1} + \sum_{k=1}^{N}\left(G_u^k(0) - G_u^k\left(\frac{1}{x}\right)\right) + \sum_{k=N+1}^{N+M}\left(G_v^k(0) - G_v^k\left(\frac{1}{x}\right)\right) + \sum_{i=1}^{N+M}\tau_i \\
q_4 &= \frac{\alpha x}{x^2-1} + \sum_{k=1}^{N} G_u^k(x) + \sum_{k=N+1}^{N+M} G_v^k(x) + \sum_{i=1}^{N+M}\tau_i \\
q_5 &= \sum_{k=1}^{N}\left(G_u^k(x) - G_u^k(0) + G_u^k\left(\frac{1}{x}\right)\right) + \sum_{k=N+1}^{N+M}\left(-G_v^k(x) + G_v^k(0) - G_v^k\left(\frac{1}{x}\right)\right)
\end{aligned}
$$

Multi dyonic magnons

- Fluctuation frequencies :

$$\begin{aligned}\Omega_{ij}^{\text{light}}(x) &= \frac{1}{x^2-1}\left(1-\sum_l \alpha_l\left(x\frac{X_l^+ + X_l^-}{X_l^+X_l^- + 1}\right)\right)\\ \Omega_{ij}^{\text{heavy}}(x) &= \frac{2}{x^2-1}\left(1-\sum_l \alpha_l\left(x\frac{X_l^+ + X_l^-}{X_l^+X_l^- + 1}\right)\right).\end{aligned}$$

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- The S-matrix factor :

$$\begin{aligned}
&\sum(-1)^{F_b} S_{\text{multi}} = \sigma_{BES}\left(y, X_1^{Q_1}\right)\cdots\sigma_{BES}\left(y, X_{N+M}^{Q_{N+M}}\right) \\
\times\quad & \frac{\eta\left(X_1^{Q_1}\right)}{\tilde{\eta}\left(X_1^{Q_1}\right)}\cdots\frac{\eta\left(X_{N+M}^{Q_{N+M}}\right)}{\tilde{\eta}\left(X_{N+M}^{Q_{N+M}}\right)} \\
\times\quad & \left(\prod_{i=1}^{N} S_{BDS}\left(y, X_i^{Q_i}\right) + \prod_{i=N+1}^{N+M} S_{BDS}\left(y, X_i^{Q_i}\right)\right) \\
\times\quad & \left(\frac{\eta(y)}{\tilde{\eta}(y)}\right)^{\sum Q_i}\sum_b(-1)^{F_b}\prod_{i=1}^{N+M} s_b(p_i)
\end{aligned}$$

Multi dyonic magnons

- The result is :

$$
\begin{aligned}
\delta\Delta_F^{\mathrm{multi}} &= \int \frac{dx}{2\pi i}\partial_x\Omega_{multi}(x)\, e^{-i\Delta_{total}\frac{x}{2g\left(x^2-1\right)}} \times \prod_{i=1}^{N+M}\left(\frac{x-\frac{1}{X_i^+}}{x-\frac{1}{X_i^-}}\sqrt{\frac{X_i^+}{X_i^-}}\right) \\
&\times \left(\prod_{i=1}^{N}\left(\frac{x-X_i^-}{x-X_i^+}\right)^2\left(\frac{x-\frac{1}{X_i^-}}{x-\frac{1}{X_i^+}}\right)^2 + \prod_{j=N+1}^{N+M}\left(\frac{x-X_j^-}{x-X_j^+}\right)^2\left(\frac{x-\frac{1}{X_j^-}}{x-\frac{1}{X_j^+}}\right)^2\right) \\
&\times \left(1+\prod_{k=1}^{N+M}\left(\frac{x-X_k^+}{x-X_k^-}\right)\left(\frac{x-\frac{1}{X_k^+}}{x-\frac{1}{X_k^-}}\right) - 2\prod_{l=1}^{N+M}\left(\frac{x-X_l^+}{x-X_l^-}\right)\sqrt{\frac{X_l^-}{X_l^+}}\right)
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$$\times \left(\prod_{i=1}^{N}\left(\frac{x-X_i^-}{x-X_i^+}\right)^2\left(\frac{x-\frac{1}{X_i^-}}{x-\frac{1}{X_i^+}}\right)^2 + \prod_{j=N+1}^{N+M}\left(\frac{x-X_j^-}{x-X_j^+}\right)^2\left(\frac{x-\frac{1}{X_j^-}}{x-\frac{1}{X_j^+}}\right)^2\right)$$
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- In the above expression, There are undetermined function α_l.
- But, at strong coupling, these parts in integral form are suppressed.

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- In these computations, we proposed particle interpretation of the big magnon.
- Also, we generalize the leading one-loop energy shifts to multi-dyonic magnons.

Conclusion and Discussion

- Thank you!!