Lieb–Liniger Bose gas as the non-relativistic limit of the sinh–Gordon model

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Finite Size Technology in
Low Dimensional Quantum Systems

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Outline

- Introduction
  - Lieb–Liniger model and non-linear Schrödinger equation
  - sinh–Gordon model
  - The double limit
- sinh–Gordon ingredients
  - form factors
  - Thermodynamical Bethe Ansatz
  - finite temperature expectation values
- Local correlators for the Bose gas
- Algebraic BA form factors

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More details:

- M. K., G. Mussardo, A. Trombettoni,
  Phys.Rev.Lett. **103**, 210404 (2009), arXiv:0909.1336,
- M. K., G. Mussardo, A. Trombettoni,
  Phys. Rev. **A81**, 043606 (2010) arXiv:0912.3502,
- M. K., G. Mussardo, B. Pozsgay,
  J. Stat. Mech. P05014 (2010), arXiv:1002.3387

Cold atom experiments

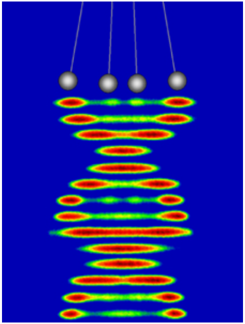

Figure: Newton's cradle of cold atoms

/David Weiss, Penn State University/

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$$H_{\mathrm{LL}} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + 2\lambda\sum_{i<j}\delta(x_i - x_j)$$

- Tonks–Girardeau gas
- experiments with ultracold atoms
  theorist's dream: the interaction can be tuned
- correlation functions: hard!
  bosonisation, quantum MC, BA solution, weak coupling…
- Idea: start from a relativistic theory!
  $\Rightarrow$ form factor approach
- problem: finite density $\Rightarrow$ thermodynamic approach

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1D Bose gas

$$H_{\text{NLS}} = \int \mathrm{d}x \left\{ \frac{\hbar^2}{2m} \frac{\partial \psi^\dagger(x)}{\partial x} \frac{\partial \psi(x)}{\partial x} + \lambda\, \psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x) \right\}$$

Quantum non-linear Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + \lambda\, |\psi(x,t)|^2 \psi(x,t)$$

the density $\rho(x,t) = \psi(x,t)^\dagger \psi(x,t)$ is conserved
$\Rightarrow$ fix particle number:

$$|\psi_N(\alpha_1,\dots,\alpha_N)\rangle = \frac{1}{N!} \int \mathrm{d}x_1 \dots \int \mathrm{d}x_N\, \chi_N(x_1,\dots,x_N|\alpha_1,\dots,\alpha_N)\, \psi^\dagger(x_1)\dots\psi^\dagger(x_N)|0\rangle$$

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1D Bose gas

Bethe wavefunction:

$$\chi_N(x_1, x_2, \ldots, x_N) = N \sum_P a(P)\, e^{i \sum_{j=1}^N P(k_j) x_j}$$

$$a(Q) = \frac{k - l - i\frac{2m}{\hbar^2}\lambda}{k - l + i\frac{2m}{\hbar^2}\lambda}\, a(P)$$

Lieb–Liniger S-matrix /E. H. Lieb, W. Liniger, Phys. Rev. **130**, 1605 (1963)/

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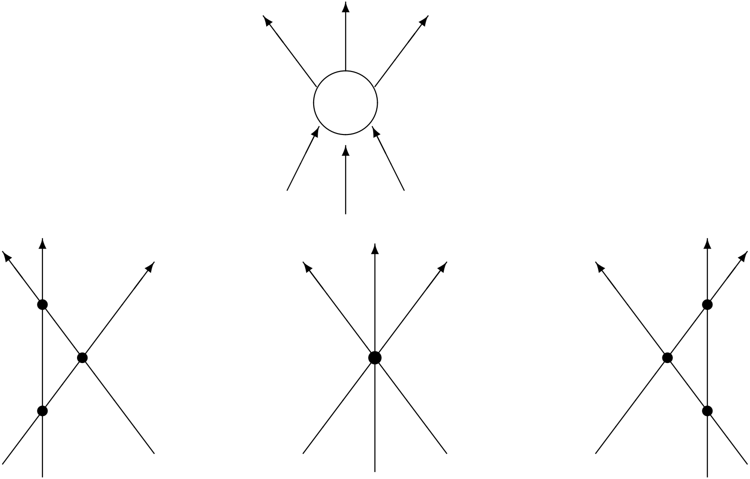
The sinh–Gordon model

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{c\,\partial t}\right)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{\mu^2}{g^2}(\cosh(g\,\phi) - 1)$$

Factorised scattering



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- The exact S-matrix ($E_i = mc^2 \cosh\theta_i,\ p_i = mc \sinh\theta_i$):

$$S(\theta_1, \theta_2; \alpha) = \frac{\sinh(\theta_1 - \theta_2) - i\,\sin(\alpha\pi)}{\sinh(\theta_1 - \theta_2) + i\,\sin(\alpha\pi)}$$

- Renormalised coupling and mass:

$$\alpha = \frac{\hbar c\, g^2}{8\pi + \hbar c\, g^2}$$

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The limit: S-matrix

$$g \to 0,\ c \to \infty, \quad g\, c \to \frac{4\sqrt{\lambda}}{\hbar} = \text{fixed}$$

$$S(\theta;\alpha) = \frac{\sinh(\theta) - i\,\sin(\alpha\pi)}{\sinh(\theta) + i\,\sin(\alpha\pi)} \ \longrightarrow\ \frac{p - i\frac{2m}{\hbar}\lambda}{p + i\frac{2m}{\hbar}\lambda} = S_{\mathrm{LL}}(p;\lambda)$$

The limit: the Lagrangian

What about the fields?

$$\phi(x,t) = \sqrt{\frac{\hbar^2}{2m}}\left(\psi(x,t)\, e^{-i\frac{mc^2}{\hbar}t} + \psi^\dagger(x,t) e^{+i\frac{mc^2}{\hbar}t}\right)$$

$$\mathcal{L}_{\text{shG}} \longrightarrow \mathcal{L}_{\text{NLS}} = -\frac{\hbar^2}{2m}\frac{\partial\psi^\dagger}{\partial x}\frac{\partial\psi}{\partial x} + i\frac{\hbar}{2}\left(\psi^\dagger\frac{\partial\psi}{\partial t} - \frac{\partial\psi^\dagger}{\partial t}\psi\right) - \lambda\, :(\psi^\dagger\psi)^2:$$

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What do we want to compute? Local correlators in the LL model

$$\langle \psi^{\dagger\, k} \psi^k \rangle_{n,T} = n^k \, g_k(\gamma, \tau)$$

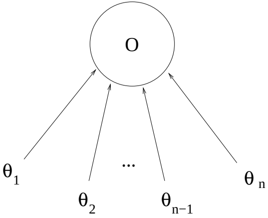
$$\gamma = \frac{2m}{\hbar^2} \frac{\lambda}{n}\,,$$

$$\tau = \frac{T}{T_{\mathrm{D}}}\,, \qquad T_{\mathrm{D}} = \frac{\hbar^2 n^2}{2 m k_{\mathrm{B}}}$$

Form factors I: bootstrap

$$F_n^{\mathcal{O}}(\theta_1, \theta_2, \ldots, \theta_n) = \langle 0|\mathcal{O}(0,0)|\theta_1, \ldots, \theta_n\rangle_{\text{in}}$$

Crossing: $F_n^{\mathcal{O}}(\theta_1 + i\pi, \theta_2, \ldots, \theta_n) = \langle \theta_1|\mathcal{O}(0,0)|\theta_2, \ldots, \theta_n\rangle_{\text{in}}$



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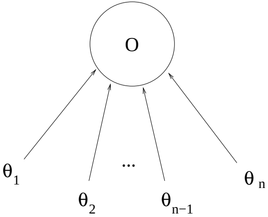
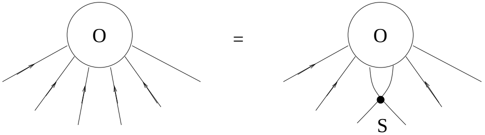


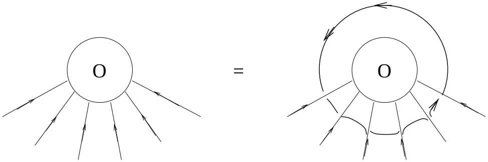
Form factors I: bootstrap

Watson's equations:

$$F_n(\ldots,\theta_i,\theta_{i+1},\ldots) = S(\theta_i-\theta_{i+1})\,F_n(\ldots,\theta_{i+1},\theta_i,\ldots)$$

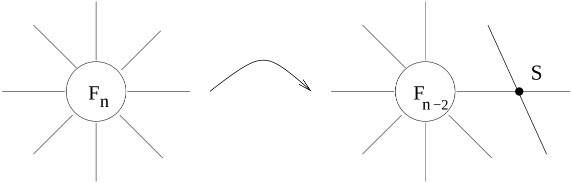
$$F_n(\theta_1+2\pi i,\theta_2,\ldots,\theta_n) = F_n(\theta_2,\ldots,\theta_n,\theta_1)$$





Form factors I: bootstrap

$$-i \operatorname*{Res}_{\tilde{\theta}\to\theta+i\pi} F_n(\tilde{\theta},\theta,\theta_1,\dots,\theta_{n-2}) = \left[1-\prod_{i=1}^{n-2} S(\theta-\theta_i)\right] F_{n-2}(\theta_1,\dots,\theta_{n-2})$$



Form factors II: sinh–Gordon

$$F_n(\theta_1,\ldots,\theta_n) = H_n\, Q_n(e^{\theta_1},\ldots,e^{\theta_n}) \prod_{i<j}^{n} \frac{F_{\min}(\theta_i-\theta_j)}{e^{\theta_i}+e^{\theta_j}}$$

$$F_{\min}(i\pi+\theta)F_{\min}(\theta) = \frac{\sinh\theta}{\sinh\theta+\sinh(i\pi\alpha)}$$

$$\begin{array}{llllll} Q_{2n-1} & \longrightarrow & \phi\,, & :\phi^3:\,, & \ldots, & :\phi^{2n-1}: \\ Q_{2n} & \longrightarrow & :\phi^2:\,, & :\phi^4:\,, & \ldots, & :\phi^{2n}: \end{array}$$

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Form factors III: the exponential operator

/A. Koubek, G. Mussardo, Phys. Lett. B **311** 193 (1993)/

$$F_n(k) = \langle 0|e^{kg\phi}|\theta_1, \theta_2, \ldots, \theta_n\rangle =$$

$$\frac{\sin(k\pi\alpha)}{\sin(\pi\alpha)} \left(\frac{4\sin(\pi\alpha)}{F_{\min}(i\pi)}\right)^{\frac{n}{2}} \det M_n(k) \prod_{i<j}^{n} \frac{F_{\min}(\theta_i - \theta_j)}{e^{\theta_i} + e^{\theta_j}} ,$$

where

$$[M_n(k)]_{i,j} = \sigma^{(n)}_{2i-j} \frac{\sin\left((i-j+k)\pi\alpha\right)}{\sin(\pi\alpha)}$$

$$\sigma^{(n)}_k = \sum_{i_1<\cdots<i_k}^{n} e^{\theta_{i_1}} \ldots e^{\theta_{i_n}}$$

Form factors IV: ϕ^m

- Trick: the $\mathcal{O}(k^m)$ term in the series expansion of $e^{kg\phi}$ ...?
- Not enough:

$$F_n^{:\phi^m:} = \langle 0| :\phi^m: |\theta_1,\dots,\theta_n\rangle \stackrel{!}{=} 0 \qquad \text{for } n < m$$

- "Operator mixing":

$$\tilde{\phi}^2 = \,:\phi^2:\,,$$

$$\tilde{\phi}^4 = \,:\phi^4: - 4\,\frac{\pi^2\alpha^2}{g^2}\,:\phi^2:\,,$$

$$\tilde{\phi}^6 = \,:\phi^6: - 20\,\frac{\pi^2\alpha^2}{g^2}\,:\phi^4: + 16\,\frac{\pi^4\alpha^4}{g^4}\,:\phi^2:$$

- $F_k^{:\phi^k:}(\theta_1,\dots,\theta_k) = 2^k k! \left(\frac{\pi^2\alpha^2}{N g^2 \sin(\pi\alpha)}\right)^{\frac{k}{2}} \prod_{i<j}^k F_{\min}(\theta_1,\dots,\theta_k)$

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Thermodynamical Bethe Ansatz

Bethe–Yang equations:

$$m_i c \sinh(\theta_i) L + \hbar \sum_{j \neq i} \delta(\theta_i - \theta_j) = 2\pi n_i \hbar \,, \qquad n_i \in \mathbb{Z}\,,$$

where $\delta(\theta) = -i \log S(\theta)$

$$\rho(\theta) = \frac{mc}{2\pi\hbar} \cosh(\theta) + \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \varphi(\theta - \theta') \frac{\rho(\theta')}{1 + e^{\varepsilon(\theta')}} \,,$$

$$n = \frac{N}{L} = \int_{-\infty}^{\infty} d\theta \, \frac{\rho(\theta)}{1 + e^{\varepsilon(\theta)}} \,,$$

$$\varepsilon(\theta) = \frac{mc^2}{k_B T} \cosh(\theta) - \frac{\mu}{k_B T} - \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \varphi(\theta - \theta') \log\left(1 + e^{-\varepsilon(\theta')}\right)$$

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$\varepsilon(\theta)$ is the dressed energy of the excitations over the physical vacuum:
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The limit of the TBA

$$\tilde{\rho}(p) = \frac{1}{mc}\,\rho\left(\frac{p}{mc}\right), \qquad \tilde{\varepsilon}(p) = \varepsilon\left(\frac{p}{mc}\right),$$

$$\tilde{\mu} = \mu - mc^2$$

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Finite T expectation values

- At equilibrium: $\langle\mathcal{O}\rangle_{T,n} = \dfrac{\mathrm{Tr}\left(e^{-\frac{H-\mu N}{k_B T}}\mathcal{O}\right)}{\mathrm{Tr}\left(e^{-\frac{H-\mu N}{k_B T}}\right)}$
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$$\langle\mathcal{O}(x,t)\rangle_{T,n} = \sum_{k=0}^{\infty}\frac{1}{k!}\int_{-\infty}^{\infty}\frac{d\theta_1}{2\pi}\dots\frac{d\theta_k}{2\pi}\left(\prod_{i=1}^{k}\frac{1}{1+e^{\varepsilon(\theta_i)}}\right) \langle\theta_k,\dots,\theta_1|\mathcal{O}(0,0)|\theta_1,\dots,\theta_k\rangle_{\text{conn}}$$

- Connected form factor:

$$\langle\theta_k,\dots,\theta_1|\mathcal{O}|\theta_1,\dots,\theta_k\rangle_{\text{conn}} = FP\left(\lim_{\eta_i\to 0}\langle 0|\mathcal{O}|\theta_1,\dots,\theta_k,\theta_k-i\pi+i\eta_k,\dots,\theta_1-i\pi+i\eta_1\rangle\right)$$

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Local correlators in the LL model

$$\langle \psi^{\dagger\, k} \psi^k \rangle_{n,T} = n^k\, g_k(\gamma, \tau)$$

$$\gamma = \frac{2m}{\hbar^2} \frac{\lambda}{n}\,,$$

$$\tau = \frac{T}{T_{\mathrm{D}}}\,, \qquad T_{\mathrm{D}} = \frac{\hbar^2 n^2}{2mk_{\mathrm{B}}}$$

- $g_1 = 1$
- g_2: Hellmann–Feynman thm
- g_3: known at $T = 0$

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The method

▸ Fields

$$\langle :\phi^{2k}: \rangle \longrightarrow \left(\frac{\hbar^2}{2m}\right)^k \binom{2k}{k} \langle \psi^{\dagger\, k} \psi^k \rangle$$

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To do:

1. find the form factors of $:\phi^{2k}:$
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3. take the double limit of them
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$$\langle\psi^\dagger\psi\rangle_{n,T} = \int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}f(p)\frac{1}{\hbar} + \int_{-\infty}^{\infty}\frac{\mathrm{d}p_1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}p_2}{2\pi}f(p_1)f(p_2)\frac{1}{\hbar}\,\tilde{\varphi}(p_{12}) + \int_{-\infty}^{\infty}\frac{\mathrm{d}p_1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}p_2}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}p_3}{2\pi}f(p_1)f(p_2)f(p_3)\frac{1}{\hbar}\,\tilde{\varphi}(p_{12})\tilde{\varphi}(p_{23}) + \dots\,,$$

$$\tilde{\rho}(p) = \frac{1}{2\pi\hbar} + \int_{-\infty}^{\infty}\frac{\mathrm{d}p'}{2\pi}\,\tilde{\varphi}(p-p')\,f(p)\tilde{\rho}(p')$$

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Asymptotic results ($T = 0$)

- For $\gamma \gg 1$, leading order for any k

$$g_k(\gamma) = \frac{k!}{2^k}\left(\frac{\pi}{\gamma}\right)^{k(k-1)} I_k + \dots$$

where $I_k = \int_{-1}^{1} \mathrm{d}x_1 \dots \int_{-1}^{1} \mathrm{d}x_k \prod_{i<j}^{k} x_{ij}^2$
/D. M. Gangardt, G. V. Shlyapnikov, Phys. Rev. Lett. **90**, 010401 (2003)/

- $$g_2 = \frac{4}{3}\frac{\pi^2}{\gamma^2}\left(1 - \frac{6}{\gamma} - (24 - \frac{8}{5}\pi^2)\frac{1}{\gamma^2}\right) + \mathcal{O}(\gamma^{-5})$$

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g_1 and g_2 at $T = 0$

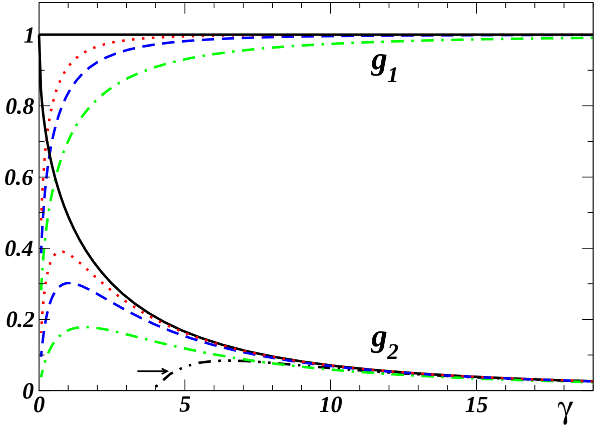


Figure: g_1 and g_2 at $T = 0$ using form factors up to $n = 4$, 6 and 8 particles, respectively with green dot-dashed, blue dashed and red dotted lines.

g_3 at $T = 0$

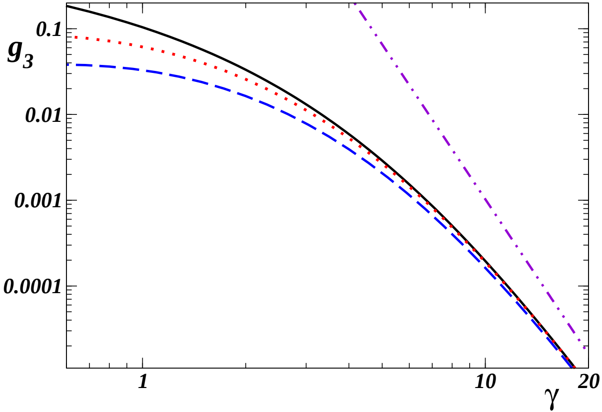


Figure: g_3 at $T = 0$ with form factors up to $n = 6$ and 8 particles. The exact value is given by the solid line whereas the purple dot-dashed line above corresponds to the leading order expression.

Asymptotic results ($T > 0$)

For $\gamma^2 \gg \tau \gg 1$:

$$g_k(\gamma,\tau) = \left(\frac{\tau}{\gamma^2}\right)^{\frac{k(k-1)}{2}} J_k\,,$$

where

$$J_k = \frac{k!}{\pi^{k/2}} \int \mathrm{d}x_1 \ldots \mathrm{d}x_k\, e^{-\sum_{i=1}^k x_i^2} \prod_{i<j}^{k} (x_i - x_j)^2 = \frac{B_k}{2^{k(k-1)/2}}$$

with $B_{k+1} = (k+1)\Gamma(k+2)B_k$, $B_1 = 1$

/D. M. Gangardt, G. V. Shlyapnikov, New J. Phys. **5**, 79 (2003)/

g_2 at $T > 0$

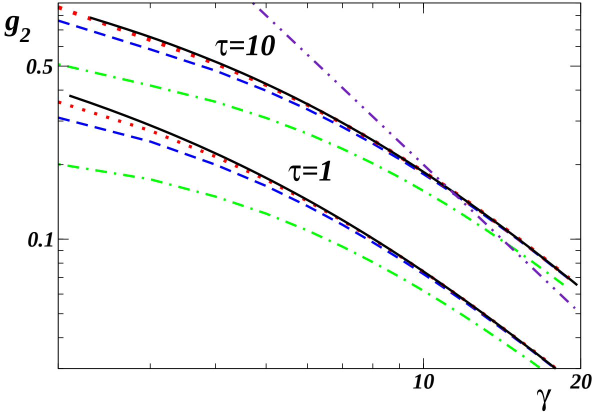
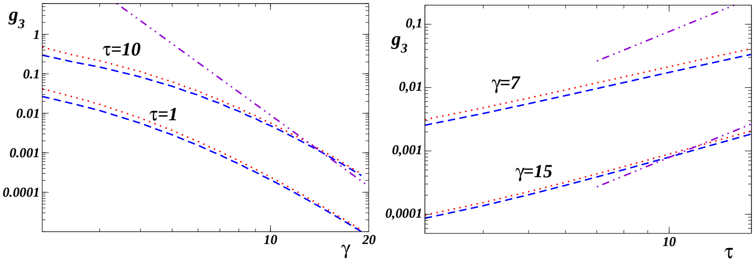


Figure: g_2 at $\tau = 1$ and $\tau = 10$ using form factors up to $n = 4$, 6 and 8 particles. The solid lines show the exact result, while the purple dot-dot-dashed line is the leading order expression.

New result: g_3 at $T > 0$



(a) g_3 vs γ at $\tau = 1$ and $\tau = 10$ (b) g_3 vs τ at $\gamma = 7$ and $\gamma = 15$

Figure: The blue dashed and the red dotted lines refer to $n = 6$ and 8 particles, respectively. The purple lines show the asymptotic results.

Algebraic BA form factors from sh-G

We have seen that the

- S-matrices,
- Bethe–Yang and TBA equations,
- fields

can be put in correspondence.
Moreover,

- the Algebraic Bethe Ansatz parameters λ_i are the momenta
- the pseudo-vacuum is the Fock vacuum

$\Longrightarrow$ what about the form factors?

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B. Pozsgay and G. Takács, Nucl.Phys.**B788**, 167 (2008), Nucl.Phys.**B788**, 209 (2008):

Finite volume form factors

$$\langle\theta_1',\dots,\theta_l'|\mathcal{O}(0,0)|\theta_1,\dots,\theta_n\rangle_L = \frac{F^{\mathcal{O}}(\theta_1'+i\pi,\dots,\theta_l'+i\pi,\theta_1,\dots,\theta_n)}{\sqrt{\rho_l(\theta_1',\dots,\theta_l')}\sqrt{\rho_n(\theta_1,\dots,\theta_n)}} + \mathcal{O}(e^{-\mu L})$$

where

$$Q_j = mcL\sinh\theta_j + \sum_{k\neq j}^{n}\frac{1}{i}\log S(\theta_j-\theta_k) = 2\pi I_j\,, \qquad j=1,\dots,n$$

$$\rho_n(\theta_1,\dots,\theta_n) = \det\frac{\partial Q_j}{\partial\theta_k}$$

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$$F_{\min}(\theta_{jl}) \to \frac{\lambda_j - \lambda_l}{\lambda_j - \lambda_l + ic} = f(\lambda_j, \lambda_l)\,, \qquad F_{\min}(\theta_{jl} + i\pi) \to 1$$

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$$i^{p-q} \binom{p+q}{p}^{-1} \left(\frac{2m}{\hbar^2}\right)^{\frac{p+q}{2}} \lambda^{\frac{2N+p-q}{2}} \prod_{j<k}^{N} f(\lambda_j, \lambda_k) \prod_{j<k}^{N+p-q} f(\lambda'_j, \lambda'_k) \times$$

$$\times \widetilde{\lim} \left\{ (mc)^{-\frac{2N+p-q}{2}} \langle 0 | :\phi^{p+q}: | \theta'_1 + i\pi, \ldots, \theta'_{N+p-q} + i\pi, \theta_1, \ldots, \theta_N \rangle \right\}$$

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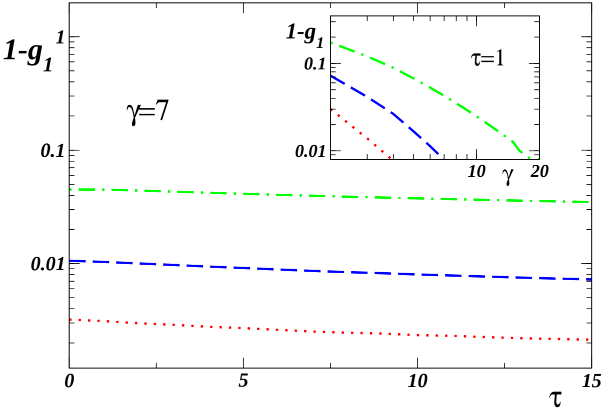

Figure: Deviations $1 - g_1$ from the exact result ($g_1 = 1$) as a function of the scaled temperature τ for a fixed value of $\gamma = 7$.
Inset: $1 - g_1$ vs γ at $\tau = 1$. In both figures form factors are used up to $n = 4$ (green dot-dashed), 6 (blue dashed) and 8 (red dotted) particles.