

Lieb–Liniger Bose gas as the non-relativistic limit of the sinh–Gordon model

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*Finite Size Technology in
Low Dimensional Quantum Systems*

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Outline

- Introduction
 - Lieb–Liniger model and non-linear Schrödinger equation
 - sinh–Gordon model
 - The double limit
- sinh–Gordon ingredients
 - form factors
 - Thermodynamical Bethe Ansatz
 - finite temperature expectation values
- Local correlators for the Bose gas
- Algebraic BA form factors

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More details:

- M. K., G. Mussardo, A. Trombettoni,
Phys.Rev.Lett. **103**, 210404 (2009), `arXiv:0909.1336`,
- M. K., G. Mussardo, A. Trombettoni,
Phys. Rev. **A81**, 043606 (2010) `arXiv:0912.3502`,
- M. K., G. Mussardo, B. Pozsgay,
J. Stat. Mech. P05014 (2010), `arXiv:1002.3387`

Cold atom experiments

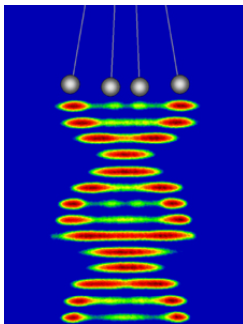


Figure: Newton's cradle of cold atoms

/David Weiss, Penn State University/

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$$H_{\text{LL}} = -\frac{\hbar^2}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + 2\lambda \sum_{i < j} \delta(x_i - x_j)$$

- Tonks–Girardeau gas
- experiments with ultracold atoms
theorist's dream: the interaction can be tuned
- correlation functions: **hard!**
bosonisation, quantum MC, BA solution, weak coupling. . .
- Idea: start from a relativistic theory!
⇒ form factor approach
- problem: finite density ⇒ thermodynamic approach

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1D Bose gas

$$H_{\text{NLS}} = \int dx \left\{ \frac{\hbar^2}{2m} \frac{\partial \psi^\dagger(x)}{\partial x} \frac{\partial \psi(x)}{\partial x} + \lambda \psi^\dagger(x) \psi^\dagger(x) \psi(x) \psi(x) \right\}$$

Quantum non-linear Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + \lambda |\psi(x, t)|^2 \psi(x, t)$$

the density $\rho(x, t) = \psi(x, t)^\dagger \psi(x, t)$ is conserved

\Rightarrow fix particle number:

$$|\psi_N(\alpha_1, \dots, \alpha_N)\rangle = \frac{1}{N!} \int dx_1 \dots \int dx_N \chi_N(x_1, \dots, x_N | \alpha_1, \dots, \alpha_N) \psi^\dagger(x_1) \dots \psi^\dagger(x_N) |0\rangle$$

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1D Bose gas

Bethe wavefunction:

$$\chi_N(x_1, x_2, \dots, x_N) = N \sum_P a(P) e^{i \sum_{j=1}^N P(k_j) x_j}$$

$$a(Q) = \frac{k - l - i \frac{2m}{\hbar^2} \lambda}{k - l + i \frac{2m}{\hbar^2} \lambda} a(P)$$

Lieb–Liniger S-matrix (E. H. Lieb, W. Liniger, Phys. Rev. 130, 1605 (1963))

$$S_{LL}(p_1, p_2; \lambda) = \frac{p_1 - p_2 - i \frac{2m}{\hbar} \lambda}{p_1 - p_2 + i \frac{2m}{\hbar} \lambda}$$

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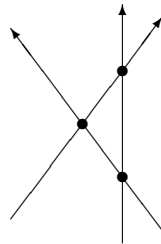
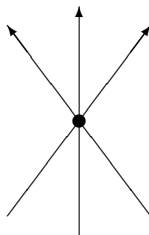
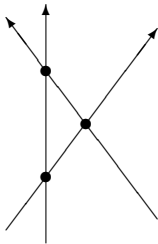
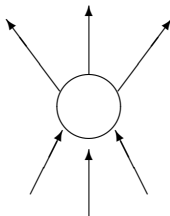
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The sinh–Gordon model

$$\mathcal{L} = \frac{1}{2} \left(\frac{\partial \phi}{c \partial t} \right)^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{\mu^2}{g^2} (\cosh(g \phi) - 1)$$

Factorised scattering



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- The exact S-matrix ($E_i = mc^2 \cosh \theta_i$, $p_i = mc \sinh \theta_i$):

$$S(\theta_1, \theta_2; \alpha) = \frac{\sinh(\theta_1 - \theta_2) - i \sin(\alpha\pi)}{\sinh(\theta_1 - \theta_2) + i \sin(\alpha\pi)}$$

- Renormalised coupling and mass:

$$\alpha = \frac{\hbar c g^2}{8\pi + \hbar c g^2}$$

$$\mu^2 = \frac{m^2 c^2}{\hbar^2} \frac{\pi \alpha}{\sin(\pi \alpha)}$$

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The limit: S-matrix

$$g \rightarrow 0, \quad c \rightarrow \infty, \quad gc \rightarrow \frac{4\sqrt{\lambda}}{\hbar} = \text{fixed}$$

$$S(\theta; \alpha) = \frac{\sinh(\theta) - i \sin(\alpha\pi)}{\sinh(\theta) + i \sin(\alpha\pi)} \longrightarrow \frac{p - i\frac{2m}{\hbar} \lambda}{p + i\frac{2m}{\hbar} \lambda} = S_{\text{LL}}(p; \lambda)$$

The limit: the Lagrangian

What about the fields?

$$\phi(x, t) = \sqrt{\frac{\hbar^2}{2m}} \left(\psi(x, t) e^{-i\frac{mc^2}{\hbar} t} + \psi^\dagger(x, t) e^{+i\frac{mc^2}{\hbar} t} \right)$$

$$\mathcal{L}_{\text{shG}} \longrightarrow \mathcal{L}_{\text{NLS}} = -\frac{\hbar^2}{2m} \frac{\partial \psi^\dagger}{\partial x} \frac{\partial \psi}{\partial x} + i \frac{\hbar}{2} \left(\psi^\dagger \frac{\partial \psi}{\partial t} - \frac{\partial \psi^\dagger}{\partial t} \psi \right) - \lambda : (\psi^\dagger \psi)^2 :$$

$$H = \int dx \left\{ \frac{\hbar^2}{2m} \frac{\partial \psi^\dagger}{\partial x} \frac{\partial \psi}{\partial x} + \lambda : (\psi^\dagger \psi)^2 : \right\}$$

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What do we want to compute?

Local correlators in the LL model

$$\langle \psi^{\dagger k} \psi^k \rangle_{n,T} = n^k g_k(\gamma, \tau)$$

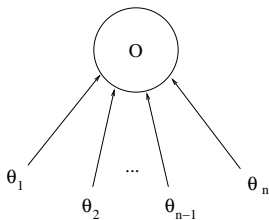
$$\gamma = \frac{2m\lambda}{\hbar^2 n},$$

$$\tau = \frac{T}{T_D}, \quad T_D = \frac{\hbar^2 n^2}{2mk_B}$$

Form factors I: bootstrap

$$F_n^{\mathcal{O}}(\theta_1, \theta_2, \dots, \theta_n) = \langle 0 | \mathcal{O}(0, 0) | \theta_1, \dots, \theta_n \rangle_{\text{in}}$$

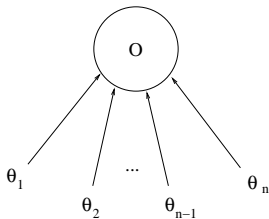
Crossing: $F_n^{\mathcal{O}}(\theta_1 + i\pi, \theta_2, \dots, \theta_n) = \langle \theta_1 | \mathcal{O}(0, 0) | \theta_2, \dots, \theta_n \rangle_{\text{in}}$



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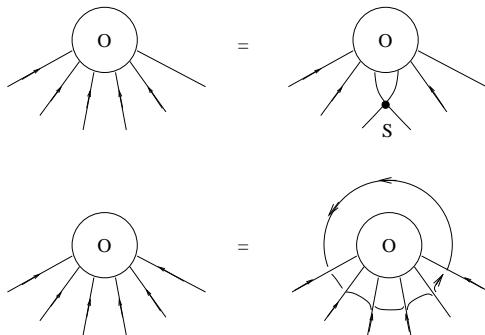


Form factors I: bootstrap

Watson's equations:

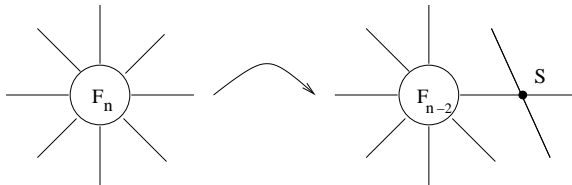
$$F_n(\dots, \theta_i, \theta_{i+1}, \dots) = S(\theta_i - \theta_{i+1}) F_n(\dots, \theta_{i+1}, \theta_i, \dots)$$

$$F_n(\theta_1 + 2\pi i, \theta_2, \dots, \theta_n) = F_n(\theta_2, \dots, \theta_n, \theta_1)$$



Form factors I: bootstrap

$$-i \operatorname{Res}_{\tilde{\theta} \rightarrow \theta + i\pi} F_n(\tilde{\theta}, \theta, \theta_1, \dots, \theta_{n-2}) = \left[1 - \prod_{i=1}^{n-2} S(\theta - \theta_i) \right] F_{n-2}(\theta_1, \dots, \theta_{n-2})$$



Form factors II: sinh–Gordon

$$F_n(\theta_1, \dots, \theta_n) = H_n Q_n(e^{\theta_1}, \dots, e^{\theta_n}) \prod_{i < j}^n \frac{F_{\min}(\theta_i - \theta_j)}{e^{\theta_i} + e^{\theta_j}}$$

$$F_{\min}(i\pi + \theta) F_{\min}(\theta) = \frac{\sinh \theta}{\sinh \theta + \sinh(i\pi \alpha)}$$

$$\begin{aligned} Q_{2n-1} &\longrightarrow \phi, \quad : \phi^3 :, \quad \dots, \quad : \phi^{2n-1} : \\ Q_{2n} &\longrightarrow : \phi^2 :, \quad : \phi^4 :, \quad \dots, \quad : \phi^{2n} : \end{aligned}$$

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Form factors III: the exponential operator

/A. Koubek, G. Mussardo, Phys. Lett. B **311** 193 (1993)/

$$F_n(k) = \langle 0 | e^{kg\phi} | \theta_1, \theta_2, \dots, \theta_n \rangle = \frac{\sin(k\pi\alpha)}{\sin(\pi\alpha)} \left(\frac{4 \sin(\pi\alpha)}{F_{\min}(i\pi)} \right)^{\frac{n}{2}} \det M_n(k) \prod_{i < j}^n \frac{F_{\min}(\theta_i - \theta_j)}{e^{\theta_i} + e^{\theta_j}},$$

where

$$[M_n(k)]_{i,j} = \sigma_{2i-j}^{(n)} \frac{\sin((i-j+k)\pi\alpha)}{\sin(\pi\alpha)}$$

$$\sigma_k^{(n)} = \sum_{i_1 < \dots < i_k}^n e^{\theta_{i_1}} \dots e^{\theta_{i_k}}$$

Form factors IV: ϕ^m

- Trick: the $\mathcal{O}(k^m)$ term in the series expansion of $e^{kg\phi} \dots$?
- Not enough:

$$F_n^{:\phi^m}: = \langle 0 | : \phi^m : | \theta_1, \dots, \theta_n \rangle \stackrel{!}{=} 0 \quad \text{for } n < m$$

- “Operator mixing”:

$$\tilde{\phi}^2 = : \phi^2 : ,$$

$$\tilde{\phi}^4 = : \phi^4 : - 4 \frac{\pi^2 \alpha^2}{g^2} : \phi^2 : ,$$

$$\tilde{\phi}^6 = : \phi^6 : - 20 \frac{\pi^2 \alpha^2}{g^2} : \phi^4 : + 16 \frac{\pi^4 \alpha^4}{g^4} : \phi^2 :$$

- $F_k^{:\phi^k}: (\theta_1, \dots, \theta_k) = 2^k k! \left(\frac{\pi^2 \alpha^2}{Ng^2 \sin(\pi\alpha)} \right)^{\frac{k}{2}} \prod_{i < j}^k F_{\min}(\theta_1, \dots, \theta_k)$

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Thermodynamical Bethe Ansatz

Bethe–Yang equations:

$$m_i c \sinh(\theta_i) L + \hbar \sum_{j \neq i} \delta(\theta_i - \theta_j) = 2\pi n_i \hbar, \quad n_i \in \mathbb{Z},$$

where $\delta(\theta) = -i \log S(\theta)$

$$\rho(\theta) = \frac{mc}{2\pi\hbar} \cosh(\theta) + \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \varphi(\theta - \theta') \frac{\rho(\theta')}{1 + e^{\varepsilon(\theta')}},$$

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$\varepsilon(\theta)$ is the dressed energy of the excitations over the physical vacuum:

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The limit of the TBA

$$\tilde{\rho}(p) = \frac{1}{mc} \rho\left(\frac{p}{mc}\right), \quad \tilde{\varepsilon}(p) = \varepsilon\left(\frac{p}{mc}\right),$$

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Finite T expectation values

- At equilibrium: $\langle \mathcal{O} \rangle_{T,n} = \frac{\text{Tr} \left(e^{-\frac{H-\mu N}{k_B T}} \mathcal{O} \right)}{\text{Tr} \left(e^{-\frac{H-\mu N}{k_B T}} \right)}$

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$$\langle \mathcal{O}(x, t) \rangle_{T,n} = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{-\infty}^{\infty} \frac{d\theta_1}{2\pi} \cdots \frac{d\theta_k}{2\pi} \left(\prod_{i=1}^k \frac{1}{1 + e^{\varepsilon(\theta_i)}} \right) \langle \theta_k, \dots, \theta_1 | \mathcal{O}(0, 0) | \theta_1, \dots, \theta_k \rangle_{\text{conn}}$$

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Local correlators in the LL model

$$\langle \psi^{\dagger k} \psi^k \rangle_{n,T} = n^k g_k(\gamma, \tau)$$

$$\gamma = \frac{2m\lambda}{\hbar^2 n},$$

$$\tau = \frac{T}{T_D}, \quad T_D = \frac{\hbar^2 n^2}{2mk_B}$$

- $g_1 = 1$
- g_2 : Hellmann–Feynman thm
- g_3 : known at $T = 0$

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The method

► Fields

$$\langle : \phi^{2k} : \rangle \longrightarrow \left(\frac{\hbar^2}{2m} \right)^k \binom{2k}{k} \langle \psi^\dagger{}^k \psi^k \rangle$$

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$$g_1 = \langle \psi^\dagger \psi \rangle / n = 1$$

$$\langle \psi^\dagger \psi \rangle_{n,T} = \int_{-\infty}^{\infty} \frac{dp}{2\pi} f(p) \frac{1}{\hbar} + \int_{-\infty}^{\infty} \frac{dp_1}{2\pi} \int_{-\infty}^{\infty} \frac{dp_2}{2\pi} f(p_1) f(p_2) \frac{1}{\hbar} \tilde{\varphi}(p_{12}) + \\ \int_{-\infty}^{\infty} \frac{dp_1}{2\pi} \int_{-\infty}^{\infty} \frac{dp_2}{2\pi} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} f(p_1) f(p_2) f(p_3) \frac{1}{\hbar} \tilde{\varphi}(p_{12}) \tilde{\varphi}(p_{23}) + \dots,$$

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Asymptotic results ($T = 0$)

- For $\gamma \gg 1$, leading order for any k

$$g_k(\gamma) = \frac{k!}{2^k} \left(\frac{\pi}{\gamma}\right)^{k(k-1)} I_k + \dots$$

where $I_k = \int_{-1}^1 dx_1 \dots \int_{-1}^1 dx_k \prod_{i < j}^k x_{ij}^2$

/D. M. Gangardt, G. V. Shlyapnikov, Phys. Rev. Lett. **90**, 010401 (2003)/

- $$g_2 = \frac{4\pi^2}{3\gamma^2} \left(1 - \frac{6}{\gamma} - (24 - \frac{8}{5}\pi^2) \frac{1}{\gamma^2}\right) + \mathcal{O}(\gamma^{-5})$$

- $$g_3 = \frac{16\pi^6}{15\gamma^6} \left(1 - \frac{16}{\gamma}\right) + \mathcal{O}(\gamma^{-8})$$

Asymptotic results ($T = 0$)

- For $\gamma \gg 1$, leading order for any k

$$g_k(\gamma) = \frac{k!}{2^k} \left(\frac{\pi}{\gamma}\right)^{k(k-1)} I_k + \dots$$

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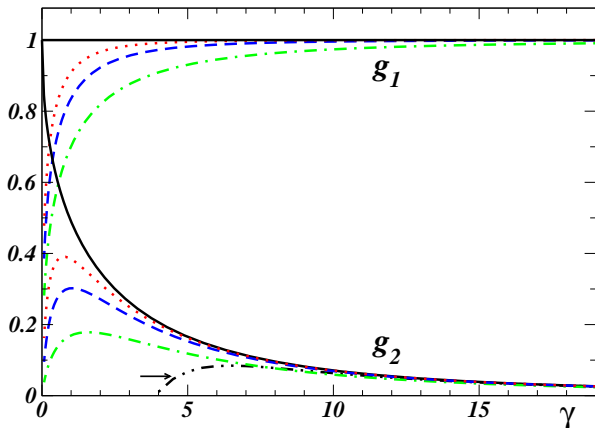
g_1 and g_2 at $T = 0$ 

Figure: g_1 and g_2 at $T = 0$ using form factors up to $n = 4, 6$ and 8 particles, respectively with green dot-dashed, blue dashed and red dotted lines.

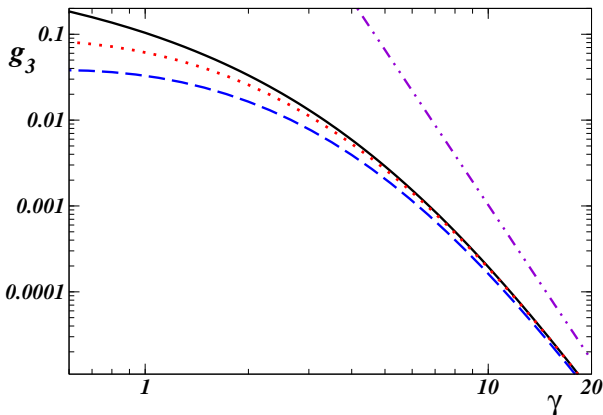
g_3 at $T = 0$ 

Figure: g_3 at $T = 0$ with form factors up to $n = 6$ and 8 particles. The exact value is given by the solid line whereas the purple dot-dashed line above corresponds to the leading order expression.

Asymptotic results ($T > 0$)

For $\gamma^2 \gg \tau \gg 1$:

$$g_k(\gamma, \tau) = \left(\frac{\tau}{\gamma^2} \right)^{\frac{k(k-1)}{2}} J_k,$$

where

$$J_k = \frac{k!}{\pi^{k/2}} \int dx_1 \dots dx_k e^{-\sum_{i=1}^k x_i^2} \prod_{i < j}^k (x_i - x_j)^2 = \frac{B_k}{2^{k(k-1)/2}}$$

with $B_{k+1} = (k+1)\Gamma(k+2)B_k$, $B_1 = 1$

/D. M. Gangardt, G. V. Shlyapnikov, New J. Phys. **5**, 79 (2003)/

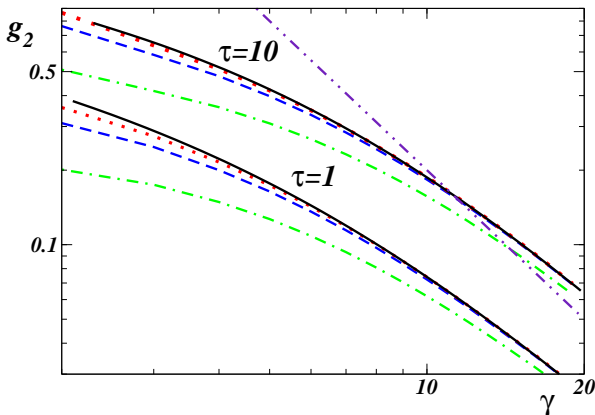
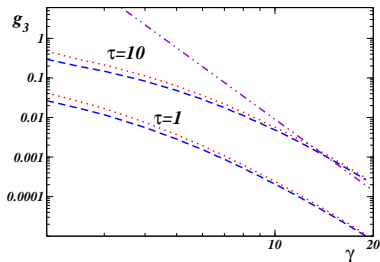
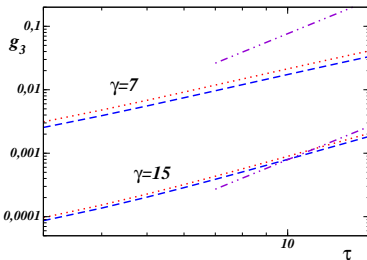
g_2 at $T > 0$ 

Figure: g_2 at $\tau = 1$ and $\tau = 10$ using form factors up to $n = 4, 6$ and 8 particles. The solid lines show the exact result, while the purple dot-dot-dashed line is the leading order expression.

New result: g_3 at $T > 0$



(a) g_3 vs γ at $\tau = 1$ and $\tau = 10$



(b) g_3 vs τ at $\gamma = 7$ and $\gamma = 15$

Figure: The blue dashed and the red dotted lines refer to $n = 6$ and 8 particles, respectively. The purple lines show the asymptotic results.

Algebraic BA form factors from sh-G

We have seen that the

- S-matrices,
- Bethe–Yang and TBA equations,
- fields

can be put in correspondence.

Moreover,

- the Algebraic Bethe Ansatz parameters λ_i are the momenta
- the pseudo-vacuum is the Fock vacuum

⇒ what about the **form factors**?

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Finite volume form factors

$$\langle \theta'_1, \dots, \theta'_l | \mathcal{O}(0, 0) | \theta_1, \dots, \theta_n \rangle_L = \frac{F^{\mathcal{O}}(\theta'_1 + i\pi, \dots, \theta'_l + i\pi, \theta_1, \dots, \theta_n)}{\sqrt{\rho_l(\theta'_1, \dots, \theta'_l)} \sqrt{\rho_n(\theta_1, \dots, \theta_n)}} + \mathcal{O}(e^{-\mu L})$$

where

$$Q_j = mcL \sinh \theta_j + \sum_{k \neq j}^n \frac{1}{i} \log S(\theta_j - \theta_k) = 2\pi I_j, \quad j = 1, \dots, n$$

$$\rho_n(\theta_1, \dots, \theta_n) = \det \frac{\partial Q_j}{\partial \theta_k}$$

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$$F_{\min}(\theta_{jl}) \rightarrow \frac{\lambda_j - \lambda_l}{\lambda_j - \lambda_l + ic} = f(\lambda_j, \lambda_l), \quad F_{\min}(\theta_{jl} + i\pi) \rightarrow 1$$

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$$\left[\text{a lot of prefactors} \right] \times$$

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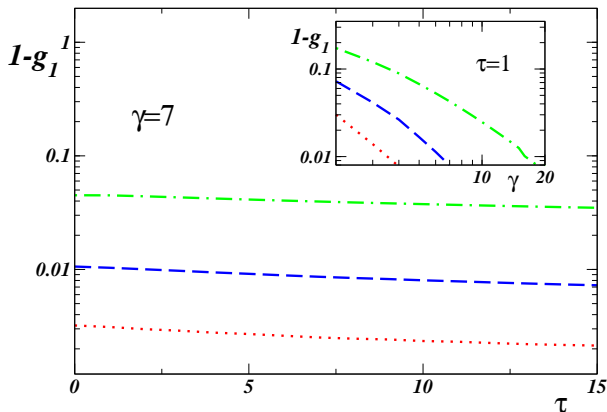


Figure: Deviations $1 - g_1$ from the exact result ($g_1 = 1$) as a function of the scaled temperature τ for a fixed value of $\gamma = 7$.

Inset: $1 - g_1$ vs γ at $\tau = 1$. In both figures form factors are used up to $n = 4$ (green dot-dashed), 6 (blue dashed) and 8 (red dotted) particles.