http://arxiv.org/pdf/0705.4264

The Standard Model

Antonio Pich

IFIC, CSIC – Univ. Valencia

- Gauge Invariance: QED, QCD
- Electroweak Unification: $SU(2)_L \otimes U(1)_Y$
- Symmetry Breaking: Higgs Mechanism
- Electroweak Phenomenology
- Flavour Dynamics

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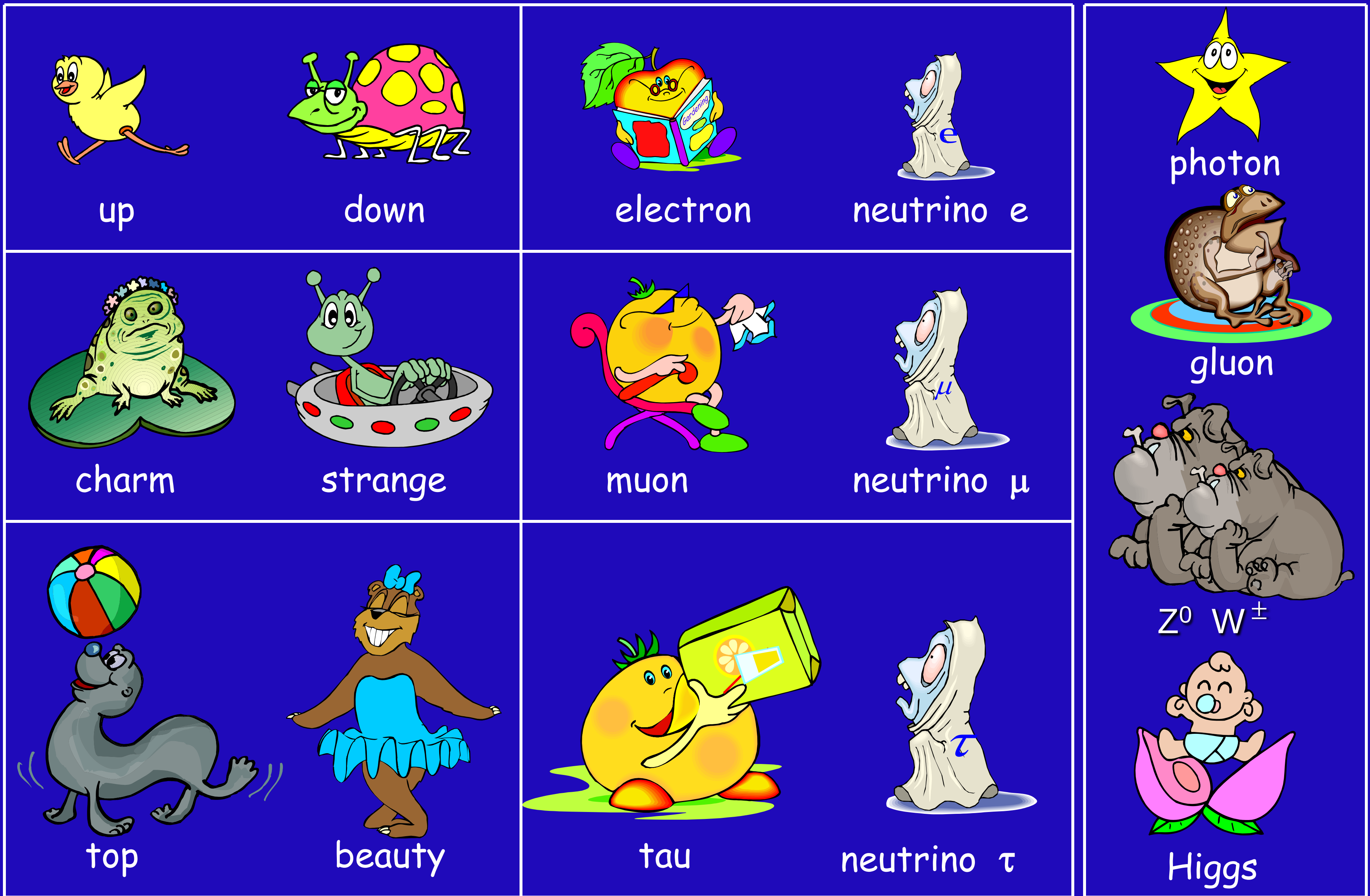
Barcelona, Spain, 31 August – 11 September 2010

Quarks | Leptons | Bosons



FREE Dirac fermion:

$$\mathcal{L} = \bar{\psi}\, (i\gamma^\mu \partial_\mu - m)\, \psi$$

Phase Invariance: $\psi \;\to\; \psi' = \mathrm{e}^{iQ\theta}\, \psi \quad ; \quad \bar{\psi} \;\to\; \bar{\psi}' = \mathrm{e}^{-iQ\theta}\, \bar{\psi}$

Absolute phases are not observable in Quantum Mechanics

GAUGE PRINCIPLE: $\theta = \theta(x)$

Phase Invariance should hold LOCALLY

BUT $\partial_\mu \psi \;\to\; \mathrm{e}^{iQ\theta}\, (\partial_\mu + i\, Q\, \partial_\mu \theta)\, \psi$

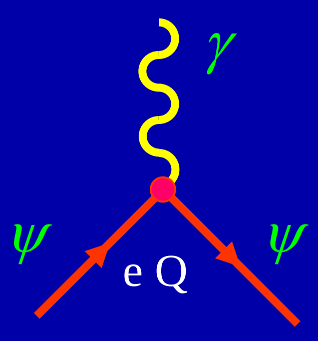
SOLUTION: Covariant Derivative

$$D_\mu \psi \equiv (\partial_\mu + i\, e\, Q\, A_\mu)\, \psi \;\;\to\;\; \mathrm{e}^{iQ\theta}\, D_\mu \psi$$

One needs a spin-1 field A_μ satisfying $A_\mu \;\to\; A_\mu - \frac{1}{e}\, \partial_\mu \theta$

QUANTUM ELECTRODYNAMICS

$$\mathcal{L} = \bar\psi\,(i\gamma^\mu D_\mu - m)\,\psi$$
$$= \bar\psi\,(i\gamma^\mu \partial_\mu - m)\,\psi \;-\; e\,Q\,A_\mu\,(\bar\psi\gamma^\mu\psi)$$



Kinetic term:

$$\mathcal{L}_K = -\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} \quad\Longrightarrow\quad \partial_\mu F^{\mu\nu} = eQ\,(\bar\psi\gamma^\nu\psi)$$

Maxwell

Mass term:

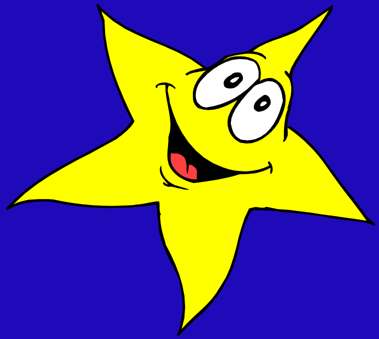
[exp: $m_\gamma < 1\cdot 10^{-18}$ eV]

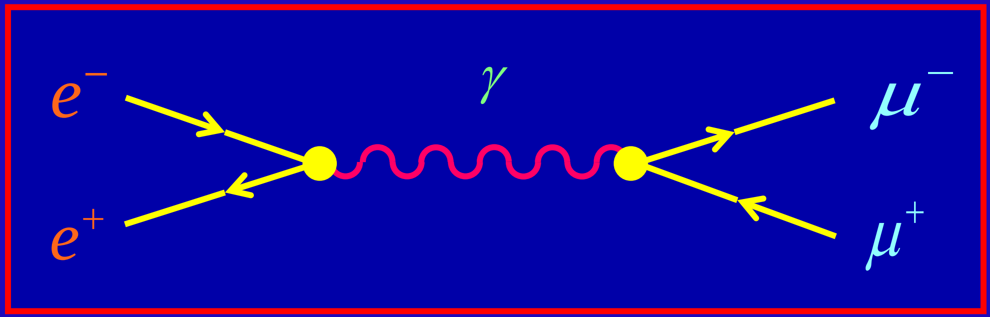
$$\mathcal{L}_M = \frac{1}{2}\,m_\gamma^2\,A^\mu A_\mu$$

Not Gauge Invariant ⟹ $m_\gamma = 0$

Gauge Symmetry ⟹ QED Dynamics

Successful Theory

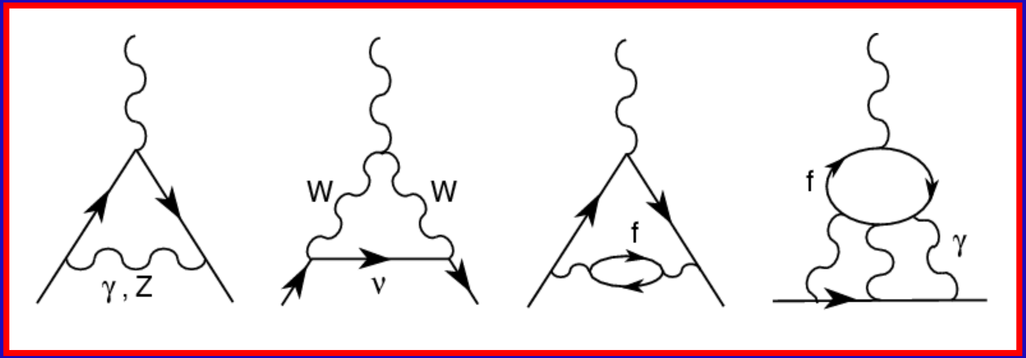




Anomalous Magnetic Moment

$$\mu_l \equiv g_l \, \frac{e}{2\, m_l}$$

$$a_l \equiv \frac{1}{2}\,(g_l - 2)$$



$$a_e = (1\,159\,652\,180.85 \pm 0.76) \times 10^{-12} \quad \Rightarrow \quad \alpha^{-1} = 137.035\,999\,710 \pm 0.000\,000\,096$$

$$\Rightarrow \quad a_\mu^{\mathrm{th}} = (11\,659\,188 \pm 10) \times 10^{-10}$$

$$[\,\mathrm{Exp:} \quad (11\,659\,208.0 \pm 6.3) \times 10^{-10}\,]$$

QUANTUM CHROMODYNAMICS

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

FREE QUARKS:

$$\mathcal{L} = \bar{\mathbf{q}}\,[\,i\gamma^\mu\partial_\mu - m\,]\,\mathbf{q}$$

$N_C = 3$

SU(3) Colour Symmetry: $\mathbf{q} \to \mathbf{U}\,\mathbf{q} \quad ; \quad \bar{\mathbf{q}} \to \bar{\mathbf{q}}\,\mathbf{U}^\dagger$

$$\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\mathbf{U} = \mathbf{1} \quad ; \quad \det\mathbf{U} = 1 \quad ; \quad \mathbf{U} = \exp\left\{ i\,\frac{\lambda^a}{2}\,\theta_a \right\}$$

Gauge Principle: Local Symmetry $\theta_a = \theta_a(x)$

$$\mathbf{D}^\mu\mathbf{q} \equiv (\mathbf{I}_3\,\partial^\mu + i\,g_s\,\mathbf{G}^\mu)\,\mathbf{q} \quad \to \quad \mathbf{U}\;\mathbf{D}^\mu\mathbf{q}$$

$$\mathbf{D}^\mu \to \mathbf{U}\;\mathbf{D}^\mu\,\mathbf{U}^\dagger \quad ; \quad \mathbf{G}^\mu \to \mathbf{U}\,\mathbf{G}^\mu\,\mathbf{U}^\dagger + \frac{i}{g_s}(\partial^\mu\mathbf{U})\;\mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{1}{2}\,(\lambda^a)_{\alpha\beta}\;G^\mu_a(x)$$

8 Gluon Fields

Infinitesimal SU(3) Transformation:

$$\delta \mathrm{q}^{\alpha} = i\ \delta\theta_a \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} \mathrm{q}^{\beta} \qquad ; \qquad \delta G_a^{\mu} = -\frac{1}{g_s}\, \partial^{\mu}(\delta\theta_a) - f^{abc}\, \delta\theta_b\, G_c^{\mu}$$

Non Abelian Group:

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i\ f^{abc}\,\frac{\lambda^c}{2}$$

- δG_a^{μ} depends on G_a^{μ}
- Universal g_s
- No Colour Charges

Kinetic Term:

$$\mathbf{G}^{\mu\nu} \equiv -\frac{i}{g_s}[\mathbf{D}^\mu, \mathbf{D}^\nu] = \partial^\mu \mathbf{G}^\nu - \partial^\nu \mathbf{G}^\mu + i\, g_s [\mathbf{G}^\mu, \mathbf{G}^\nu] \quad \rightarrow \quad \mathbf{U}\; \mathbf{G}^{\mu\nu}\; \mathbf{U}^\dagger$$

$$\mathbf{G}^{\mu\nu} \equiv \frac{\lambda^a}{2} G_a^{\mu\nu} \quad ; \quad G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s\, f^{abc}\, G_b^\mu\, G_c^\nu$$

$$\mathcal{L}_K = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\mathbf{G}_{\mu\nu}) = -\frac{1}{4} G_a^{\mu\nu}\, G_{\mu\nu}^a$$

Mass Term:

$$\mathcal{L}_M = \frac{1}{2}\, m_G^2\; G_a^\mu\, G_\mu^a$$

Not Gauge Invariant ➡ $m_G = 0$

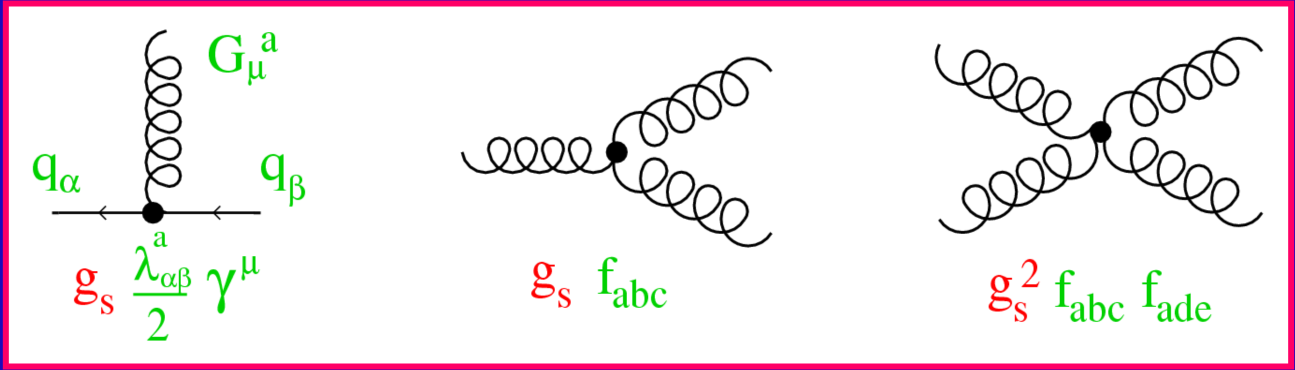
Massless Gluons

$$\mathcal{L}_{\rm QCD} \;=\; -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{G}^{\mu\nu}\,\mathbf{G}_{\mu\nu}) \;+\; \bar{\mathbf{q}}\,[\,i\,\gamma^\mu\,\mathbf{D}_\mu - m_q\,]\,\mathbf{q}$$

$$= -\frac{1}{4}\left(\partial^\mu G^\nu_a - \partial^\nu G^\mu_a\right)\left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\right) + \sum_q \bar{q}_\alpha\,[\,i\,\gamma^\mu\partial_\mu - m_q\,]\,q_\alpha$$

$$-\;\frac{1}{2}\sum_q g_s\,[\bar{q}_\alpha\,(\lambda^a)_{\alpha\beta}\,\gamma^\mu q_\beta\,]\,G^a_\mu$$

$$+\;\frac{1}{2}\,g_s\,f_{abc}\,(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)\,G^\mu_b\,G^\nu_c \;-\; \frac{1}{4}\,g_s^2\,f_{abc}\,f_{ade}\,G^\mu_b\,G^\nu_c\,G^d_\mu\,G^e_\nu$$



- **Gluon Self – interactions** $\quad G^3\,,\;G^4$
- **Universal Coupling** $\quad g_s$ $\qquad$ **(No Colour Charges)**

EXPERIMENTAL FACTS

Three Families $\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} \; , \; \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} \; , \; \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$

Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \left\{ \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \; (\nu_l)_R \; , \; l^-_R \right\} \; ; \; \left\{ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L , \; (q_u)_R \; , \; (q_d)_R \right\}$$

Charged Currents $W^\pm$
- Left–handed Fermions only
- Flavour Changing: $\nu_l \Leftrightarrow l \; , \; q_u \Leftrightarrow q_d$

Neutral currents $\gamma \, , \, Z$ Flavour Conserving

Universality (Family – Independent Couplings)

$$(\nu_l)_R \quad ?$$


standard
model
E.M
Strong
Weak
Quarks
Leptons

$SU(2)_L \otimes U(1)_Y$

GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 = \sum_j i\, \bar{\psi}_j\, \gamma^\mu \partial_\mu \psi_j$$

$SU(2)_L \otimes U(1)_Y$ **Flavour Symmetry:**

$$\mathrm{U}_L \equiv \exp\left\{ i \frac{\vec{\sigma}}{2} \vec{\alpha} \right\}$$

$$\psi_1 \to e^{i y_1 \beta}\, \mathrm{U}_L\, \psi_1 \quad ; \quad \psi_2 \to e^{i y_2 \beta}\, \psi_2 \quad ; \quad \psi_3 \to e^{i y_3 \beta}\, \psi_3$$

$$\bar{\psi}_1 \to \bar{\psi}_1\, \mathrm{U}_L^\dagger\, e^{-i y_1 \beta} \quad ; \quad \bar{\psi}_2 \to \bar{\psi}_2\, e^{-i y_2 \beta} \quad ; \quad \bar{\psi}_3 \to \bar{\psi}_3\, e^{-i y_3 \beta}$$

4 Massless Gauge Bosons $\quad W_\mu^\pm \, , \; W_\mu^3 \, , \; B_\mu^0$

Gauge Principle: $\vec{\alpha} = \vec{\alpha}(x) \quad , \quad \beta = \beta(x)$

$$\mathbf{D}_\mu \psi_1 \equiv \left[\partial_\mu + i\, g\, \mathbf{W}_\mu(x) + i\, g'\, y_1\, B_\mu(x)\right] \psi_1 \quad \rightarrow \quad \mathrm{e}^{i\, y_1\, \beta(x)}\, \mathbf{U}_{\mathrm{L}}(x)\; \mathbf{D}_\mu \psi_1$$

$$\mathbf{D}_\mu \psi_k \equiv \left[\partial_\mu + i\, g'\, y_k\, B_\mu(x)\right] \psi_k \quad \rightarrow \quad \mathrm{e}^{i\, y_k\, \beta(x)}\, \mathbf{D}_\mu \psi_k \qquad (k = 2, 3)$$

$$B_\mu(x) \quad \rightarrow \quad B_\mu(x) - \frac{1}{g'}\, \partial_\mu \beta(x)$$

$$\mathbf{W}_\mu(x) \quad \rightarrow \quad \mathbf{U}_{\mathrm{L}}(x)\; \mathbf{W}_\mu(x)\; \mathbf{U}_{\mathrm{L}}^\dagger(x) + \frac{i}{g}\, \partial_\mu \mathbf{U}_{\mathrm{L}}(x)\; \mathbf{U}_{\mathrm{L}}^\dagger(x)$$

$$\mathrm{U}(x) \equiv \exp\left\{ i\, \frac{\vec{\sigma}}{2}\, \vec{\alpha}(x) \right\} \quad ; \quad \mathbf{W}_\mu(x) \equiv \frac{\vec{\sigma}}{2}\, \vec{W}_\mu(x) \quad ; \quad \delta W_\mu^i = -\frac{1}{g}\, \partial_\mu(\delta\alpha^i) - \varepsilon^{ijk}\, \delta\alpha^j\, W_\mu^k$$

4 Massless Gauge Bosons $\quad W_\mu^\pm \; , \; W_\mu^3 \; , \; B_\mu^0$

CHARGED CURRENTS

$$\sum_j i\,\bar\psi_j\,\gamma^\mu D_\mu \psi_j \quad \longrightarrow \quad -g\;\bar\psi_1\,\gamma^\mu \mathbf{W}_\mu\,\psi_1 \;-\; g'\;B_\mu \sum_j y_j\;\bar\psi_j\,\gamma^\mu\,\psi_j$$

$$\mathbf{W}_\mu \equiv \frac{\vec\sigma}{2}\cdot\vec{W}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^\dagger \\ \sqrt{2}\,W_\mu & -W_\mu^3 \end{pmatrix} \quad ; \quad W_\mu \equiv \left(W_\mu^1 + i\,W_\mu^2\right)/\sqrt{2}$$

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\;W_\mu^\dagger\left[\,\bar q_u\,\gamma^\mu(1-\gamma_5)\,q_d \;+\; \bar\nu_l\,\gamma^\mu(1-\gamma_5)\,l\,\right] \;+\; \text{h.c.}$$

Quark / Lepton Universality

Left – Handed Interaction

NEUTRAL CURRENTS

$$\mathcal{L}_{\rm NC} = -g\, W_\mu^3\; \bar\psi_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - g'\, B_\mu \sum_j y_j\, \bar\psi_j \gamma^\mu \psi_j$$

Massless Fields → Arbitrary Combination

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

A_μ has the QED Interaction IF $\quad g \sin\theta_W = g' \cos\theta_W = e$

$$y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} \quad ; \quad y_2 = Q_u \quad ; \quad y_3 = Q_d$$

Electroweak Unification

$$\mathcal{L}_{\rm NC} = -e\, A_\mu \sum_j \bar\psi_j \gamma^\mu Q_j \psi_j + \mathcal{L}_{\rm NC}^Z$$

$$Q_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

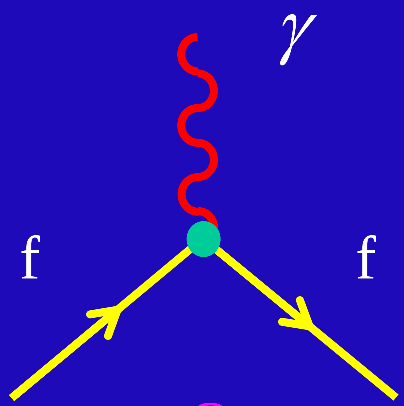
$$\mathcal{L}_{NC}^{Z} = -\frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \left\{ \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - \sin^2\theta_W \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j \right\}$$

$$= -\frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu [v_f - a_f \gamma_5] f$$

	q_u	q_d	ν_l	l^-
$2\,v_f$	$1 - \frac{8}{3}\sin^2\theta_W$	$-1 + \frac{4}{3}\sin^2\theta_W$	1	$-1 + 4\sin^2\theta_W$
$2\,a_f$	1	-1	1	-1

IF ν_R do exist: $y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions

Sterile Neutrinos



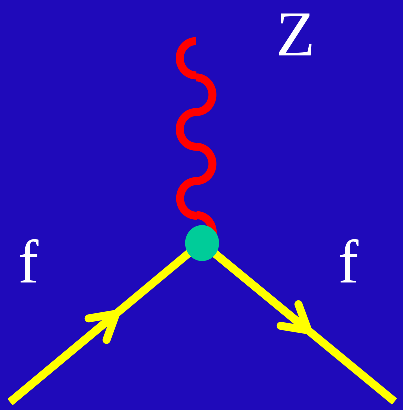
$e\,Q_f$

NEUTRAL

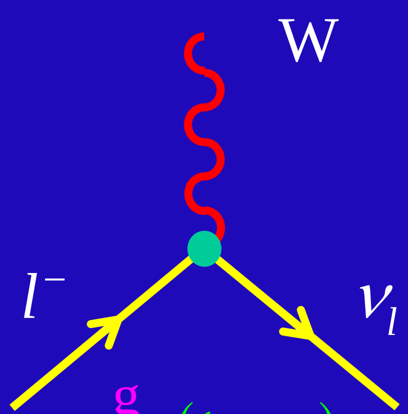
CURRENTS

$$a_f = T_3^f = \pm\frac{1}{2}$$

$$v_f = T_3^f\left(1 - 4\,|Q_f|\,\sin^2\theta_W\right)$$



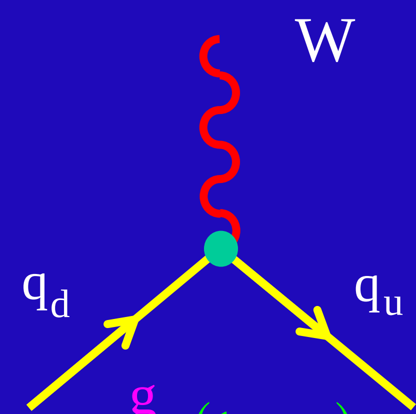
$$\frac{e}{2\,s_\theta c_\theta}\left(v_f - a_f\,\gamma_5\right)$$



$$\frac{g}{2^{3/2}}\left(1-\gamma_5\right)$$

CHARGED

CURRENTS



$$\frac{g}{2^{3/2}}\left(1-\gamma_5\right)$$

$y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions **Sterile Neutrinos**

$$\mathbf{W}_{\mu\nu} \equiv -\frac{i}{g}\left[\mathbf{D}_\mu , \mathbf{D}_\nu\right] \equiv \frac{\vec{\sigma}}{2}\cdot\vec{W}_{\mu\nu} \;\rightarrow\; \mathbf{U}_L\, \mathbf{W}_{\mu\nu}\, \mathbf{U}_L^\dagger \qquad ; \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \;\rightarrow\; B_{\mu\nu}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\varepsilon^{ijk}\, W^j_\mu W^k_\nu$$

$$\mathcal{L}_K = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2}\mathrm{Tr}(\mathbf{W}_{\mu\nu}\, \mathbf{W}^{\mu\nu}) = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_3 + \mathcal{L}_4$$

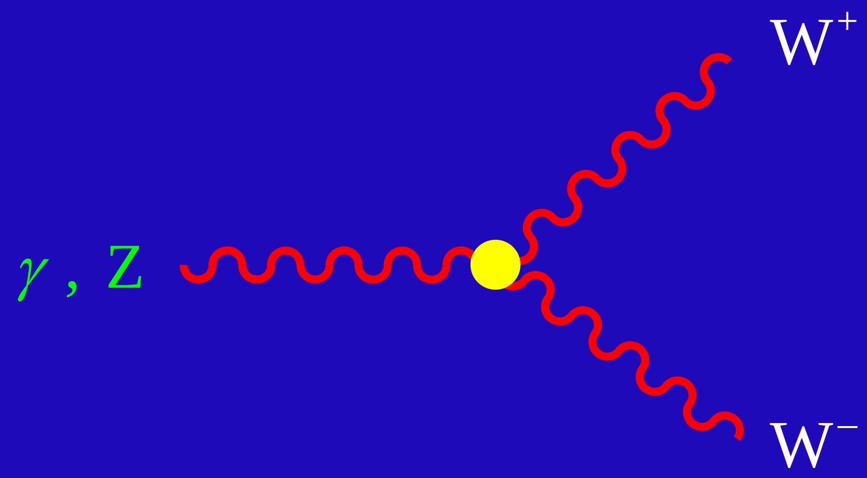
$$\mathcal{L}_3 = i\,e\,\cot\theta_W \left\{(\partial^\mu W^\nu - \partial^\nu W^\mu)\, W^\dagger_\mu Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger})\, W_\mu Z_\nu + W_\mu W^\dagger_\nu (\partial^\mu Z^\nu - \partial^\nu Z^\mu)\right\}$$

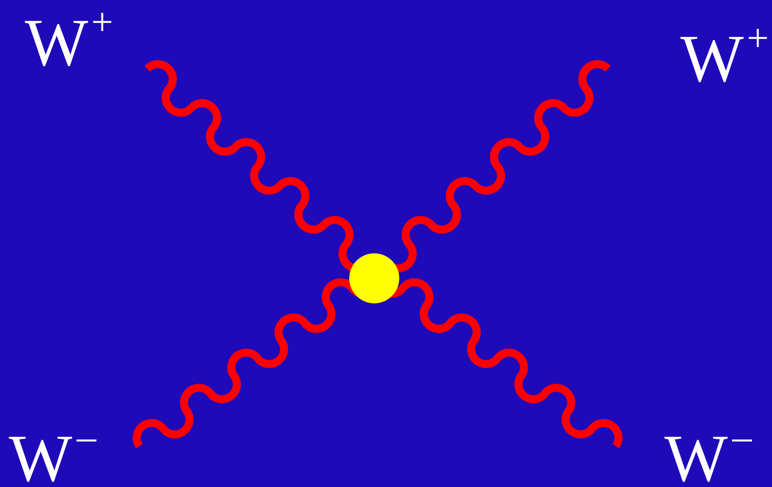
$$+\, i\,e \left\{\left(\partial^\mu W^\nu - \partial^\nu W^\mu\right) W^\dagger_\mu A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu A_\nu + W_\mu W^\dagger_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right\}$$

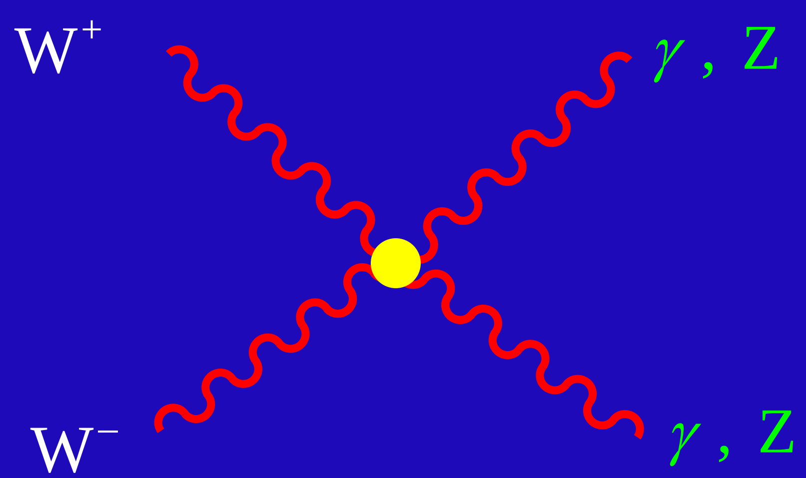
$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W}\left\{\left(W^\dagger_\mu W^\mu\right)^2 - W^\dagger_\mu W^{\mu\dagger} W_\nu W^\nu\right\} - e^2\cot^2\theta_W \left\{W^\dagger_\mu W^\mu Z_\nu Z^\nu - W^\dagger_\mu Z^\mu W_\nu Z^\nu\right\}$$

$$-e^2\cot\theta_W \left\{2 W^\dagger_\mu W^\mu Z_\nu A^\nu - W^\dagger_\mu Z^\mu W_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu Z^\nu\right\} - e^2 \left\{W^\dagger_\mu W^\mu A_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu A^\nu\right\}$$

GAUGE SELF-INTERACTIONS







PROBLEM WITH MASS SCALES

Gauge Symmetry → $m_\gamma = 0$ **Good**

$M_W = M_Z = 0$ **Bad!**

$M_W = 80.40 \text{ GeV}$

$M_Z = 91.19 \text{ GeV}$

Moreover $\mathcal{L}_{m_f} \equiv -m_f \, \bar{f} f = -m_f \, (\bar{f}_L f_R + \bar{f}_R f_L)$

Also Forbidden by Gauge Symmetry → $m_f = 0 \qquad \forall f$

All Particles Massless