Theoretical Cosmology

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- What is the observational basis of homogeneity and isotropy in cosmology?
  Isotropy – CMB
  Homogeneity – RM
- What are very large scale galaxy catalogs really measuring?
  What are very large scale N-body simulations simulating? – RD
- How can we test general relativity in cosmology?
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What are very large scale galaxy catalogs really measuring?

(See: Yoo et al. '09, Yoo '10, Bonvin and RD, in preparation '11)
For each galaxy in a catalog we measure

$$(z, \theta, \phi) = (z, \mathbf{n}) \quad + \text{info about mass, spectral type...}$$

We can count the galaxies inside a redshift bin and small solid angle, $N(z, \mathbf{n})$ and measure its fluctuation,

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}.$$

This quantity is directly measurable. On small scales where fluctuations in the spacetime geometry can be neglected it is simply related to the density contrast $\delta = (\rho(\mathbf{x}, t) - \bar{\rho}(t))/\bar{\rho}(t)$.

On large scales, however, we have to take into account that

- the measured redshift is not simply the background redshift $\bar{z}$,
- not only the number of galaxies but also the volume is distorted
- the angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.

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We first define the redshift density fluctuation $\delta_z(\mathbf{n}, z)$ by

$$\delta_z(\mathbf{n}, z) = \frac{\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\frac{N(\mathbf{n},z)}{V(\mathbf{n},z)} - \frac{\bar{N}(z)}{V(z)}}{\frac{\bar{N}(z)}{V(z)}}$$

This together with the volume fluctuations, results in the directly observed number fluctuations

$$\Delta(\mathbf{n}, z) = \delta_z(\mathbf{n}, z) + \frac{\delta V(\mathbf{n}, z)}{V(z)}$$

Both these terms are in principle measurable and therefore gauge invariant. The calculation, especially of the second term is however quite involved.
The first term is obtained easily by considering

$$\begin{aligned} \delta_z(\mathbf{n}, z) &= \frac{\bar{\rho}(\bar{z}) + \delta\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} = \frac{\bar{\rho}(z - \delta z) + \delta\rho(\mathbf{n}, z) - \bar{\rho}(z)}{\bar{\rho}(z)} \\ &= \frac{\delta\rho}{\bar{\rho}} - \frac{d\rho}{dz}\frac{\delta z}{\bar{\rho}} = \delta(\mathbf{n}, z) - 3\frac{\delta z}{1+z} \end{aligned}$$

With

$$\frac{\delta z}{1+z} = -(\mathbf{n} \cdot \mathbf{V} + \Psi)(\mathbf{n}, z) - \int_{t_S}^{t_O} (\dot{\Phi} + \dot{\Psi}) dt$$

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we arrive at

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For the volume we use

$$\begin{aligned} dV &= \sqrt{-g}\,\epsilon_{abcd}\,u^a \mathrm{d}x^b \mathrm{d}x^c \mathrm{d}x^d \\ &= \sqrt{-g}\,\epsilon_{abcd}u^a \frac{\partial x^b}{\partial z}\frac{\partial x^c}{\partial\theta_S}\frac{\partial x^d}{\partial\varphi_S}\left|\frac{\partial(\theta_S,\varphi_S)}{\partial(\theta_O,\varphi_O)}\right|\mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O = v\,\mathrm{d}z\,\mathrm{d}\theta_O\,\mathrm{d}\varphi_O \end{aligned}$$

To first order in the perturbations one finds

$$v = \frac{a^3r^2\sin\theta_O}{H}\left[1-3\phi+(\cot\theta_O+\frac{\partial}{\partial\theta})\delta\theta+\frac{\partial\delta\phi}{\partial\phi}-\mathbf{v}\cdot\mathbf{n}+2\frac{\delta r}{r}-\frac{d\,\delta r}{dt}+\frac{1}{H}\frac{d\,\delta z}{dt}\right]$$

The lengthy calculation of $\delta\theta$, $\delta\phi$ δr along the perturbed geodesic finally yields

$$\Delta(\mathbf{n},z) = \delta - 2\Phi + \Psi - \mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\left[\dot\Phi + \partial_r\Psi - \frac{d(\mathbf{V}\cdot\mathbf{n})}{dt}\right] +$$
$$\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left(\Psi+\mathbf{V}\cdot\mathbf{n}+\int_{t_S}^{t_O}dt(\dot\Phi+\dot\Psi)\right)+\frac{1}{r}\int_{t_S}^{t_O}dt\left[2-\frac{t-t_S}{(t_O-t)}\Delta_{\mathbb{S}^2}\right](\Phi+\Psi).$$

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We can now expand $\Delta(\mathbf{n}, z)$ in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \quad C_\ell(z) = \langle |a_{\ell m}|^2(z) \rangle.$$

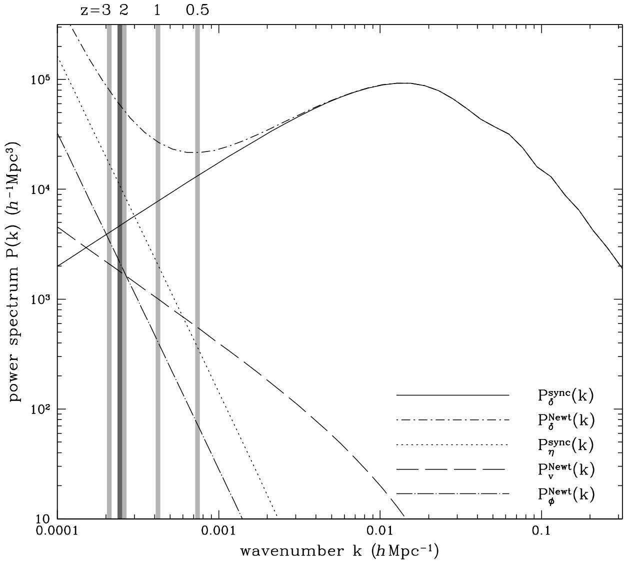
$$\langle a_{\ell m}(z) a^*_{\ell' m'}(z') \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell(z, z') \ .$$

The transversal power spectrum at redshift z is now given by the$C_\ell(z, z)$ and the longitudinal power spectrum can be obtained from $C_\ell(z, z')$ which probably can be approximated by $C_\ell(z) f(\Delta r)$ where $\Delta r = r(z) - r(z')$.

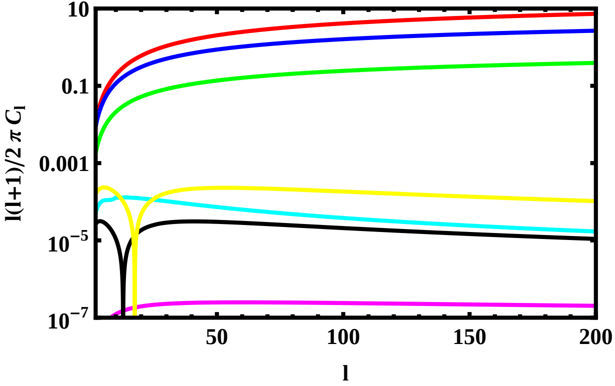
$$P_{\rm long}(k) = \int e^{ik\Delta r} f(\Delta r) d\Delta r \ .$$

What are very large scale galaxy catalogs really measuring?

Theoretical power spectra in synchronous and Newtonian gauge (from Yoo et al. ’09)

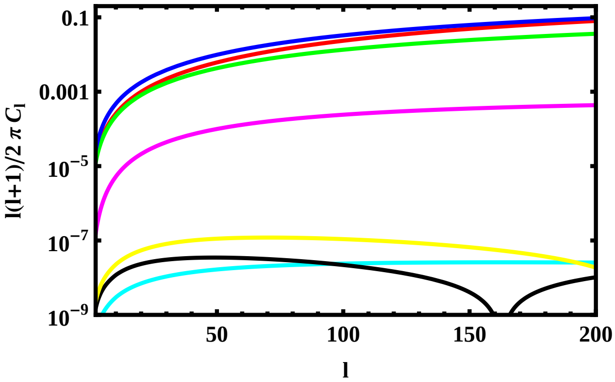


Contributions to the transverse power spectrum at redshift $z = 0.1$ (from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), C_ℓ^{Dz} (blue), C_ℓ^{LL} (magenta), C_ℓ^{VV} (cyan), C_ℓ^{zV} (black), C_ℓ^{DV} (yellow).

Contributions to the transverse power spectrum at redshift $z = 2$
(from Bonvin & RD '11)



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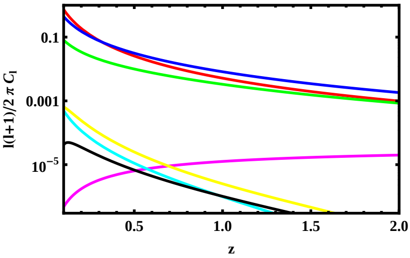
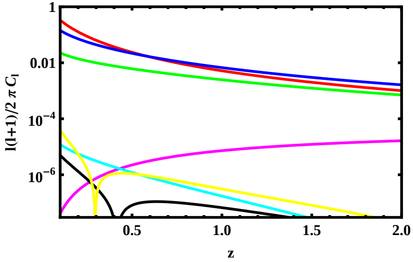
Contributions to the transverse power spectrum at redshift $\ell = 10$ and $\ell = 50$ (from Bonvin & RD '11)





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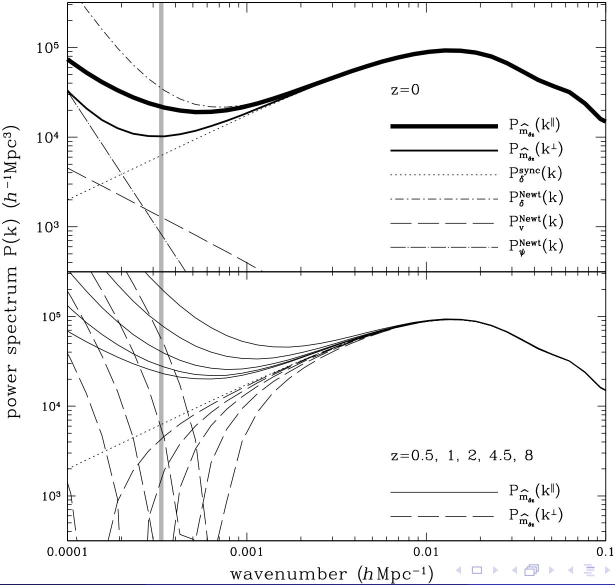
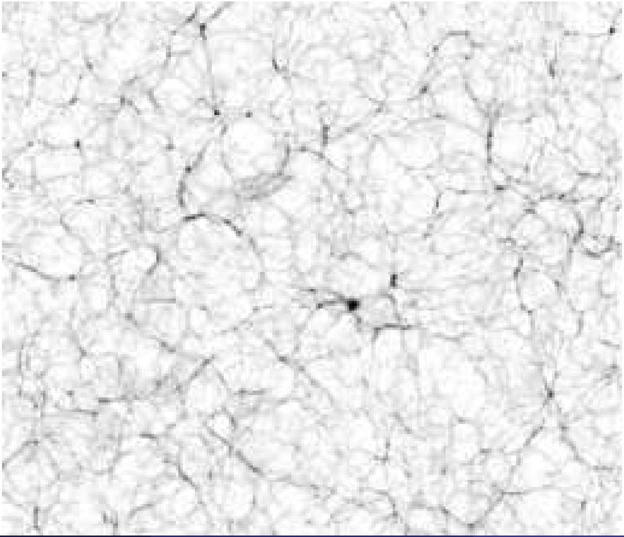
The observable matter power spectrum δ_z (from Yoo '10)



What are Newtonian N-body simulations simulating?

At present we have Hubble size N-body simulations which go out to redshifz $z \simeq 2$.
What are such simulations really calculating?
(Example: slice through 'MareNostrum', by Gottlöber et al. '06)



What are Newtonian N-body simulations simulating?

In principle, Newtonian N-body simulations are solving the Poisson equation in a clever way (initial conditions from linear perturbation theory, Zel'dovich approximation),

$$\Delta\phi = 4\pi G\delta\rho$$

Interestingly enough in linear perturbation theory (which is sufficient on large scales, where relativistic effects are most relevant), this is exactly the 00-constraint equation if we interpret δ as the matter density fluctuations in comoving gauge and ϕ as the Bardeen potential $\Phi = \Psi$ (in absence of anisotropic stresses). Hence the power spectrum obtained from Newtonian N-body simulation, agrees with the one of the density fluctuations in comoving gauge (see also Chisari & Zaldarriaga '11). This is related to the density fluctuation in longitudinal (or Newtonian) gauge via

$$\delta_{cm} = \delta_{Newt} + 3\mathcal{H}k^{-1}V$$

The velocity is obtained from the non-relativistic continuity equation, and from the eqn. of motion

$$\dot\delta = -\nabla\cdot\mathbf{v}, \quad \text{and} \quad \nabla\cdot\dot{\mathbf{v}} + \mathcal{H}\nabla\cdot\mathbf{v} = \Delta\phi$$

Interestingly, for pressureless matter this equation is exactly equal to the energy conservation equation if $\delta = \delta_{cm}$ and $\mathbf{v}$ is the velocity in Newtonian gauge, $\mathbf{v} = i\hat{\mathbf{k}}V$ in Fourier space.

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What do large galaxy surveys really measure:

- Is it possible to isolate some of terms in the formula for $\Delta(z, \mathbf{n})$, e.g. with complementary measurements?
- How can we measure pure volume distortions?
- Info in transversal vs. longitudinal power?
- Is $C_\ell(z, z')$ useful or should we stay with $P(k_\perp)$ and $P(k_\parallel)$?

What do large N-body simulations really calculate:

- Is it surprising that 1st order scalar relativistic perturbations agree with Newtonian gravity?
- Is this sufficient or do we need more?
- What happens at second order?