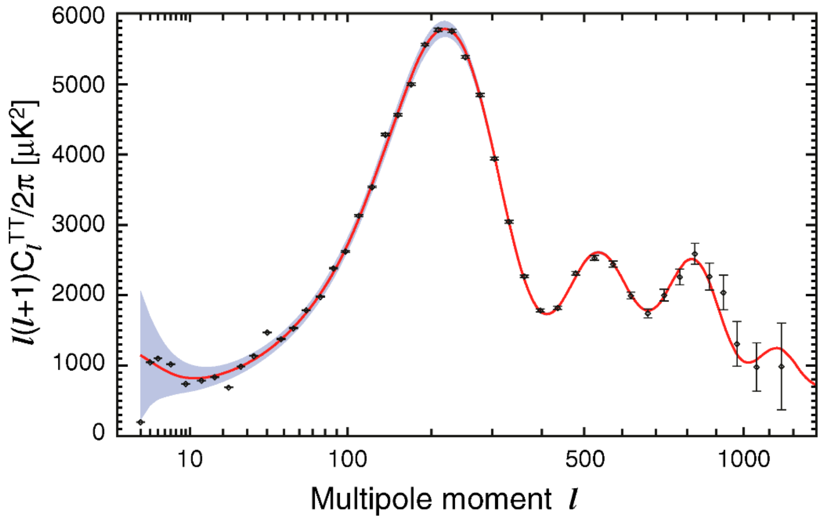
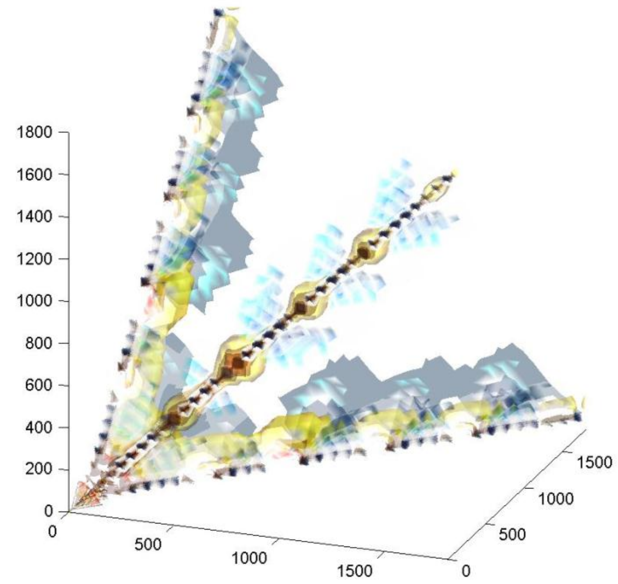
CMB Prospects





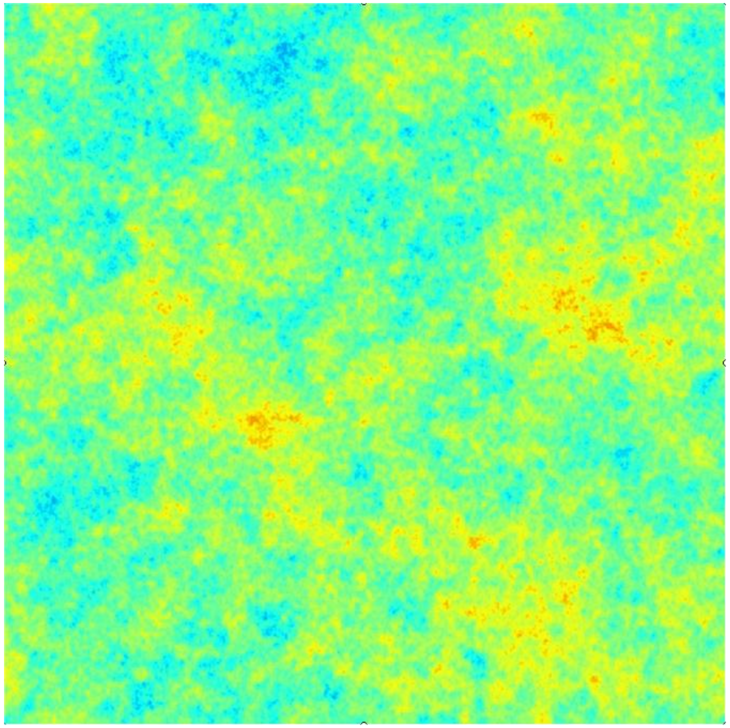
University of Sussex

Antony Lewis

http://cosmologist.info/

CMB temperature

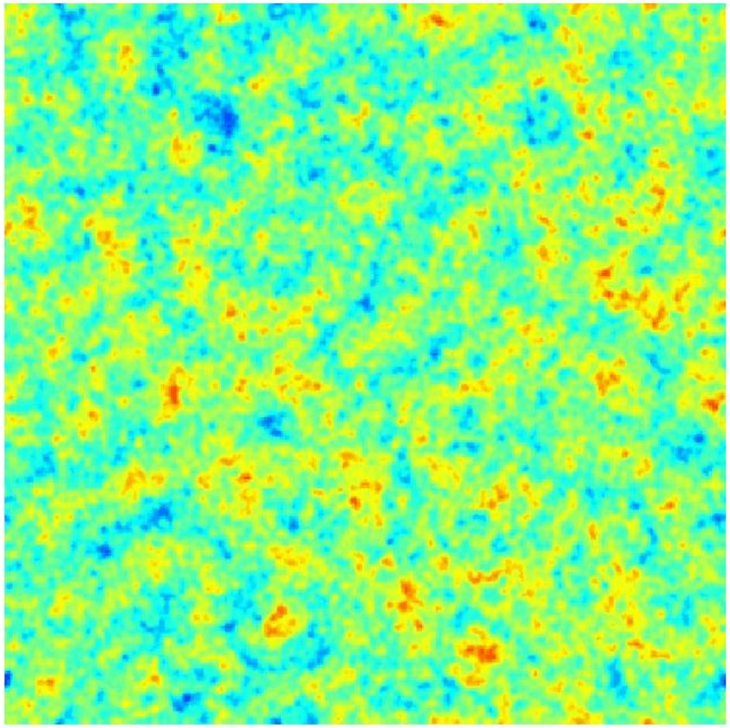
Hot big bang

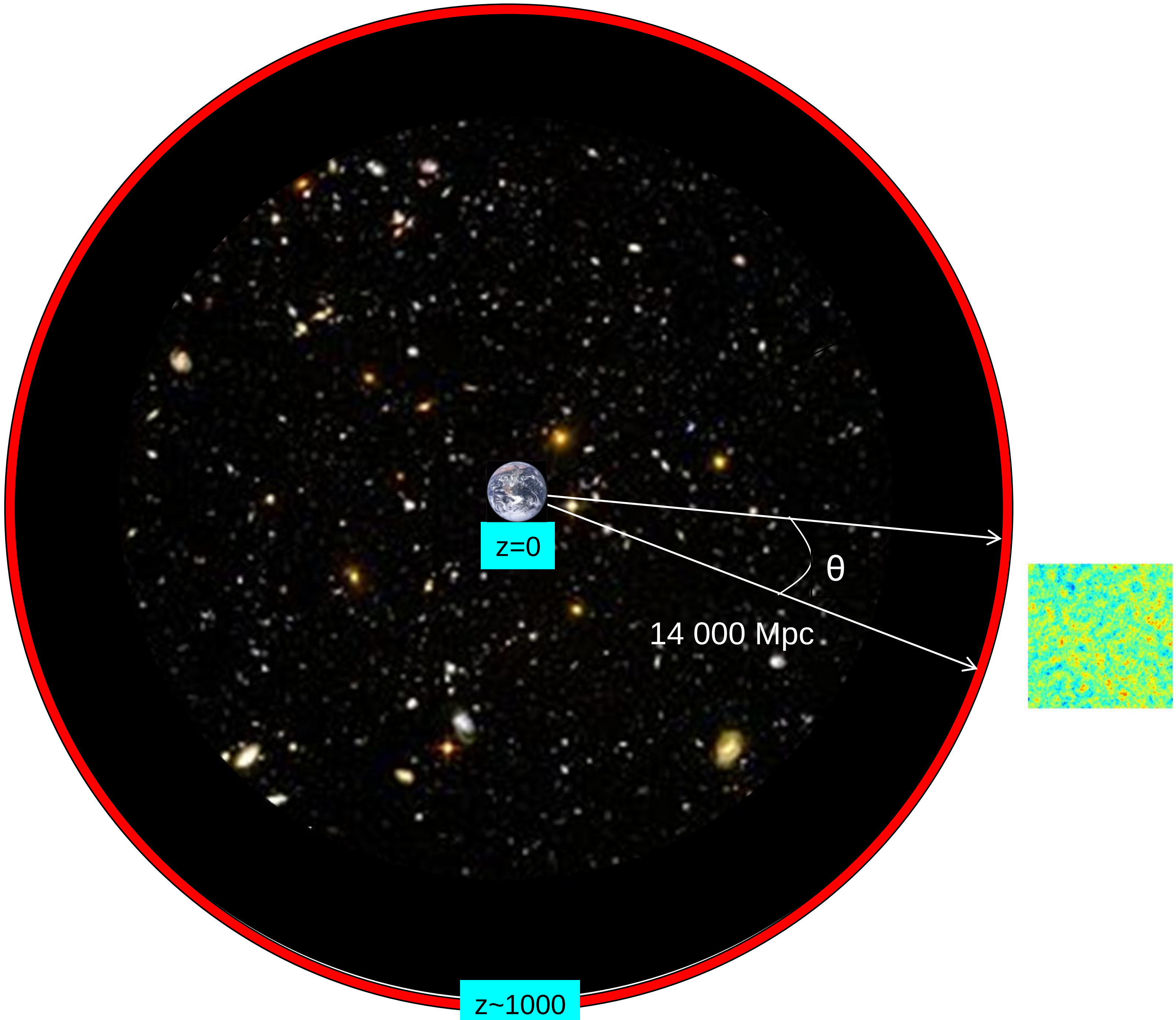


gravity+
pressure+
diffusion

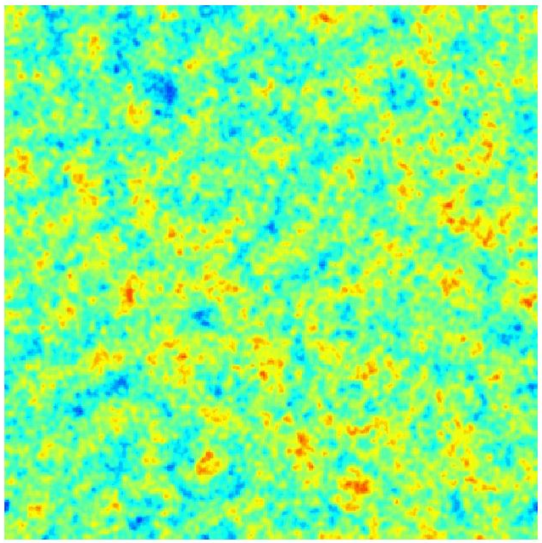
→

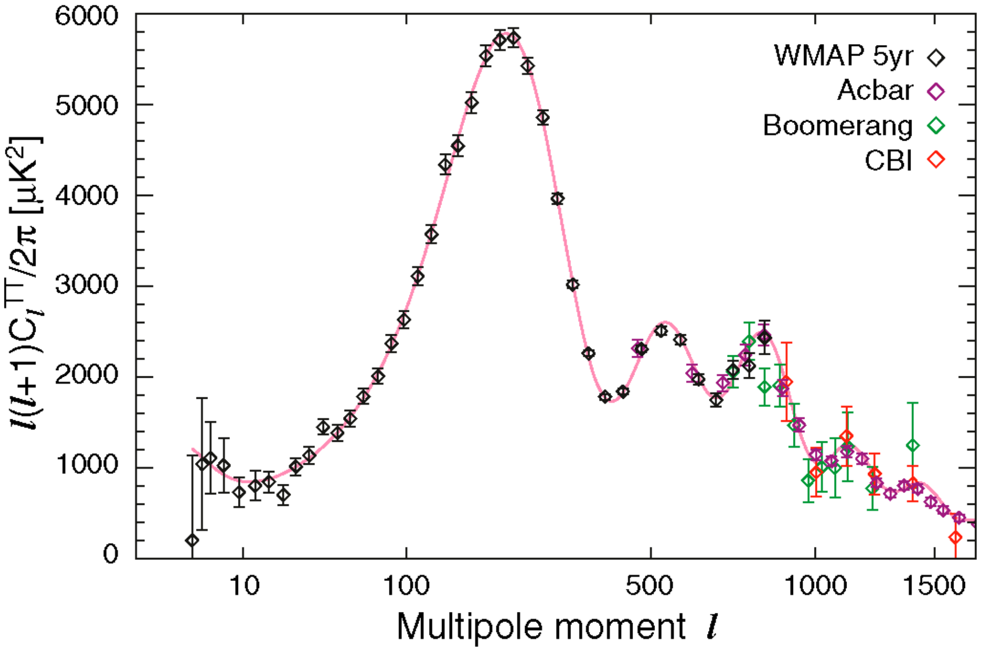
Last scattering surface




z=0
θ
14 000 Mpc
z~1000

Observed CMB temperature power spectrum

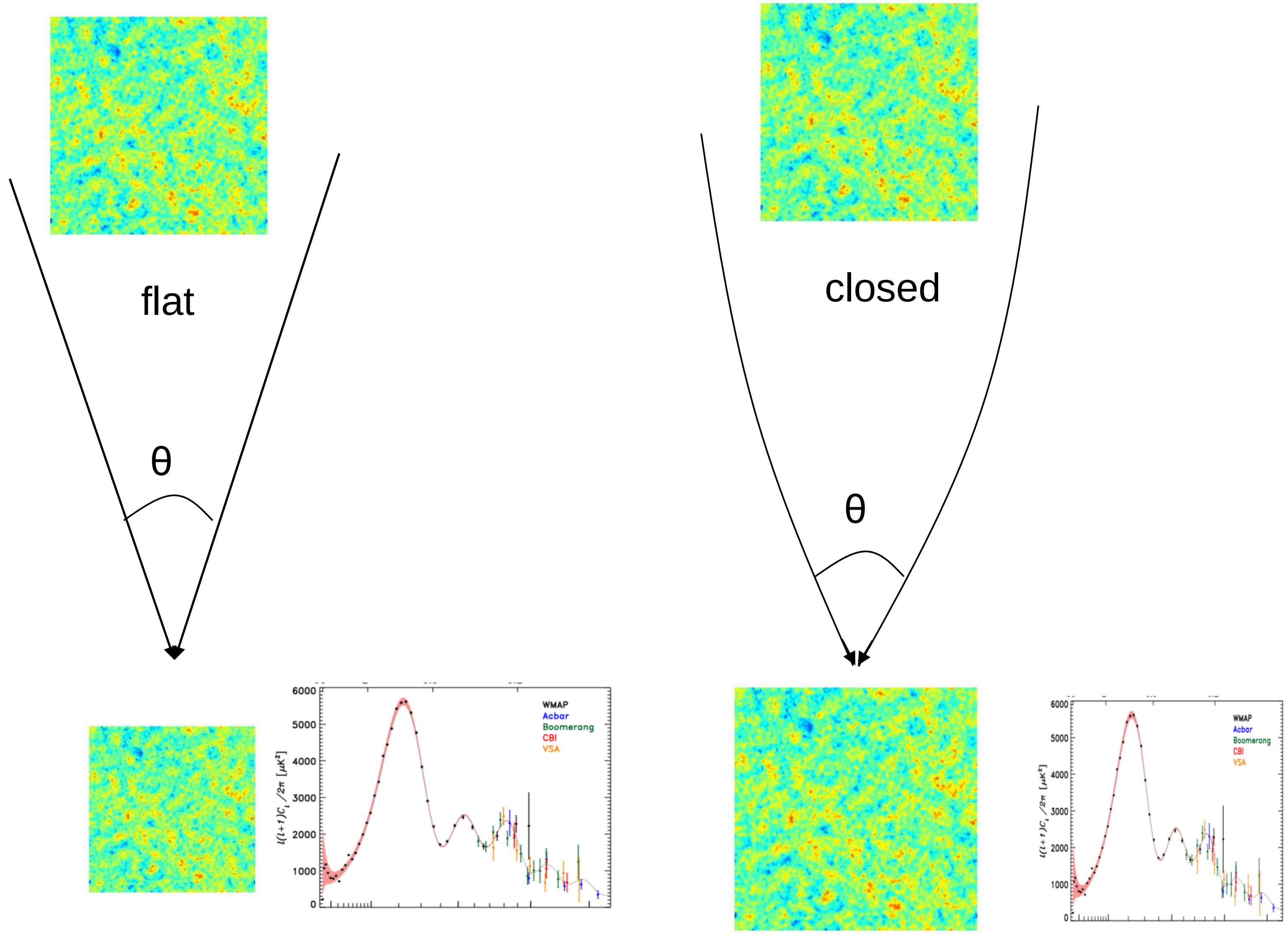




WMAP team

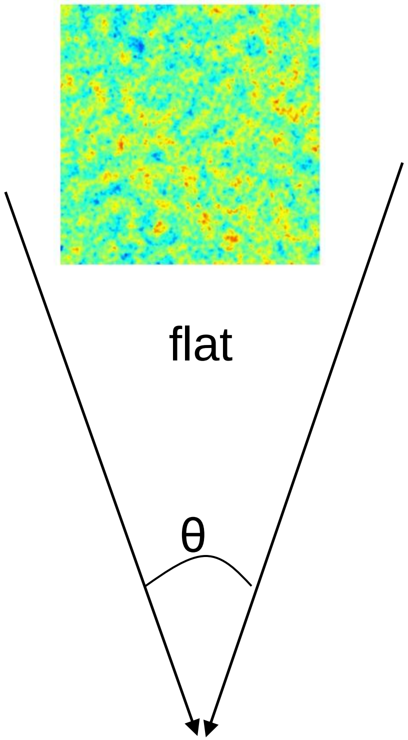
Observations → **Constrain theory of early universe + evolution parameters and geometry**

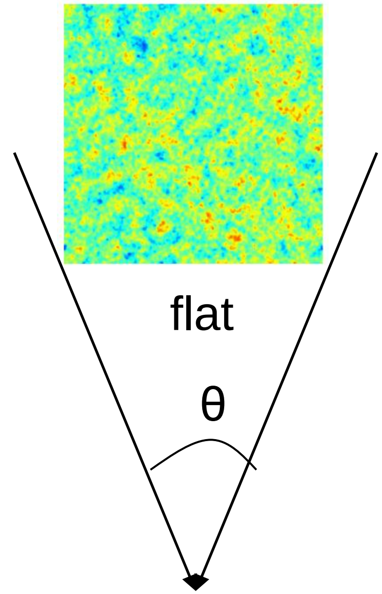
e.g. Geometry: curvature



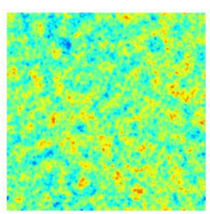
We see:

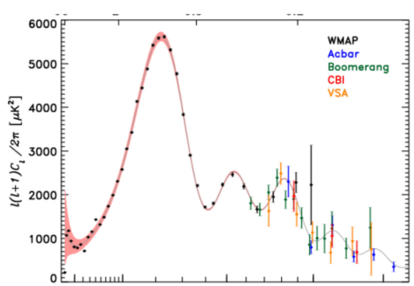
or is it just closer??

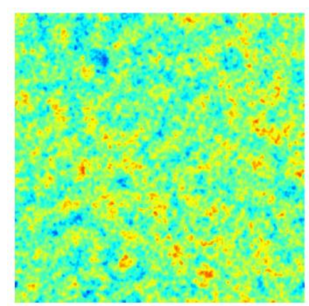


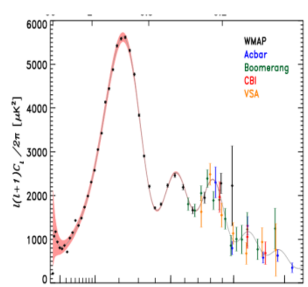


We see:

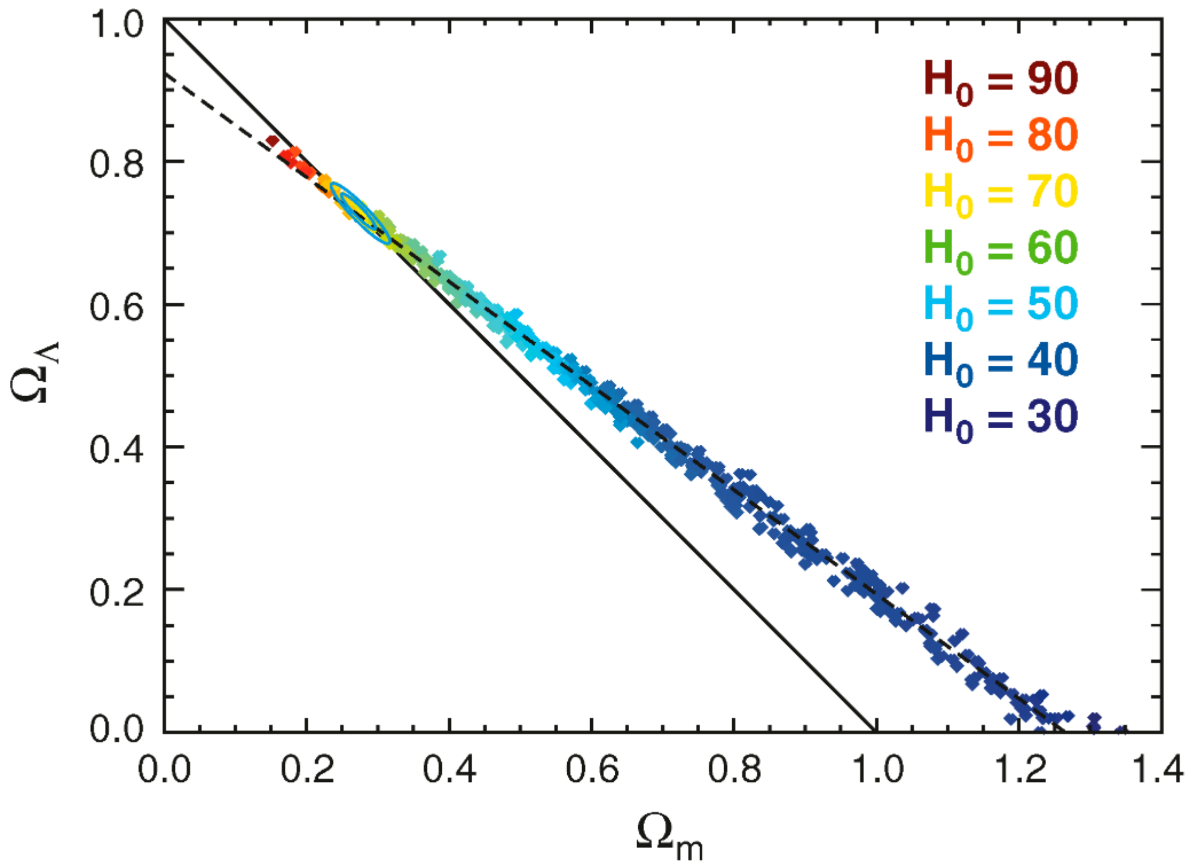








Degeneracies between parameters



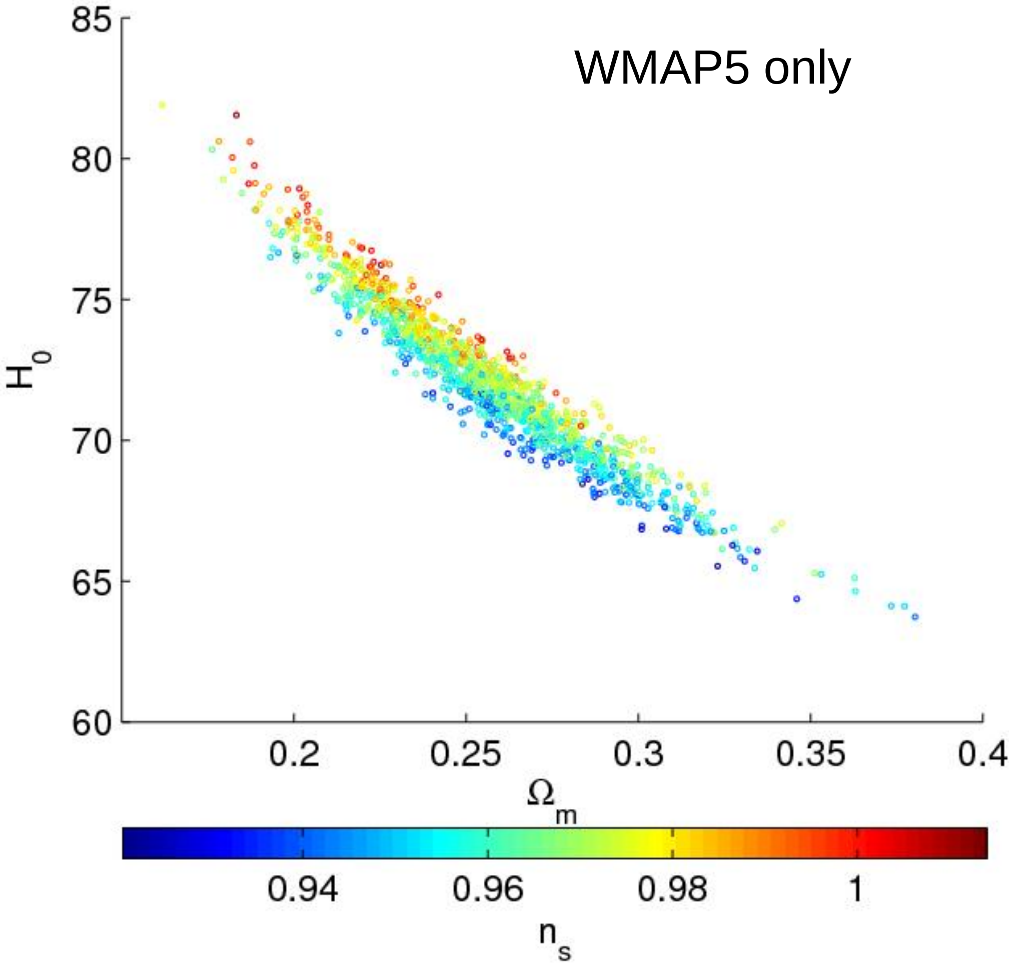
WMAP 7

Need other information to break remaining degeneracies

Constrain *combinations* of parameters accurately

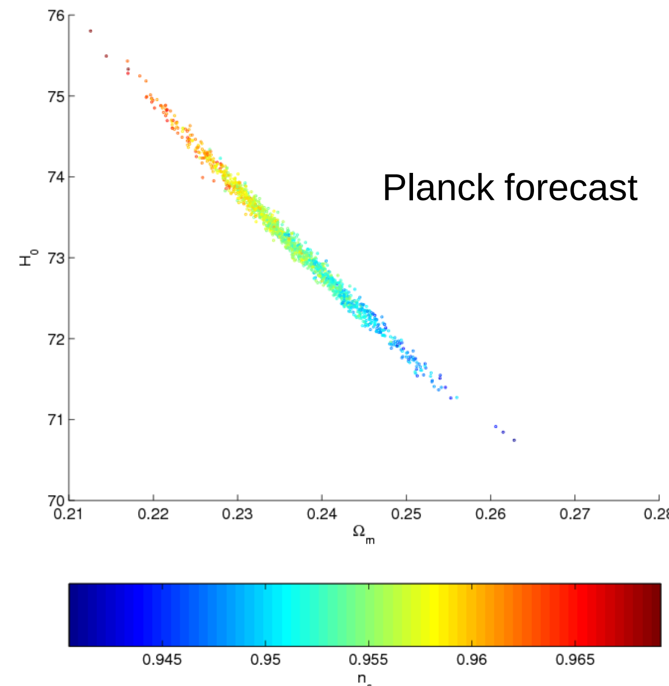
Assume Flat, w=-1

$$\left(\frac{\Omega_m}{0.254}\right)\left(\frac{h}{0.72}\right)^{3.15} = 1.00 \pm 0.03$$



$$\left(\frac{\Omega_m}{0.254}\right)\left(\frac{h}{0.72}\right)^{-3.15} = 1.03 \pm 0.23$$

Use other data to break remaining degeneracies



CMB anisotropies: theory

Linear perturbation theory with $ds^2 = a(\eta)^2 \left[(1+2\Psi)d\eta^2 - (1-2\Phi)d\mathrm{x}^2\right]$

Using the geodesic equation in the Conformal Newtonian Gauge:

$$E(\eta_0) = a(\eta)E(\eta)\left[1 + \Psi(\eta) - \Psi_0 + \int_{\eta}^{\eta_0} d\eta(\Psi' + \Phi')\right]$$

All photons redshift the same way, so $kT \sim E$.

Recombination fairly sharp at background time η_*: ~ *constant temperature* surface

$$\begin{aligned} T(\hat{\mathrm{n}}, \eta_0) &= (a_* + \delta a)T_* \left[1 + \Psi(\eta_*) - \Psi_0 + \hat{\mathrm{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \\ &= T_0 \left[1 + \frac{\delta a}{a_*} + \Psi(\eta_*) - \Psi_0 + \hat{\mathrm{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta(\Psi' + \Phi')\right] \end{aligned}$$

$\rho_\gamma \propto T^4 \propto a^4$

$$\Rightarrow \quad \frac{\Delta T_0}{T}(\hat{\mathbf{n}}) = \frac{\Delta_\gamma(\eta_*)}{4} + \Psi(\eta_*) - \Psi_0 + \hat{\mathbf{n}} \cdot (\mathbf{v}_o - \mathbf{v}) + \int_{\eta_*}^{\eta_0} d\eta (\Psi' + \Phi')$$

Temperature perturbation at recombination

Sachs-Wolfe

Doppler

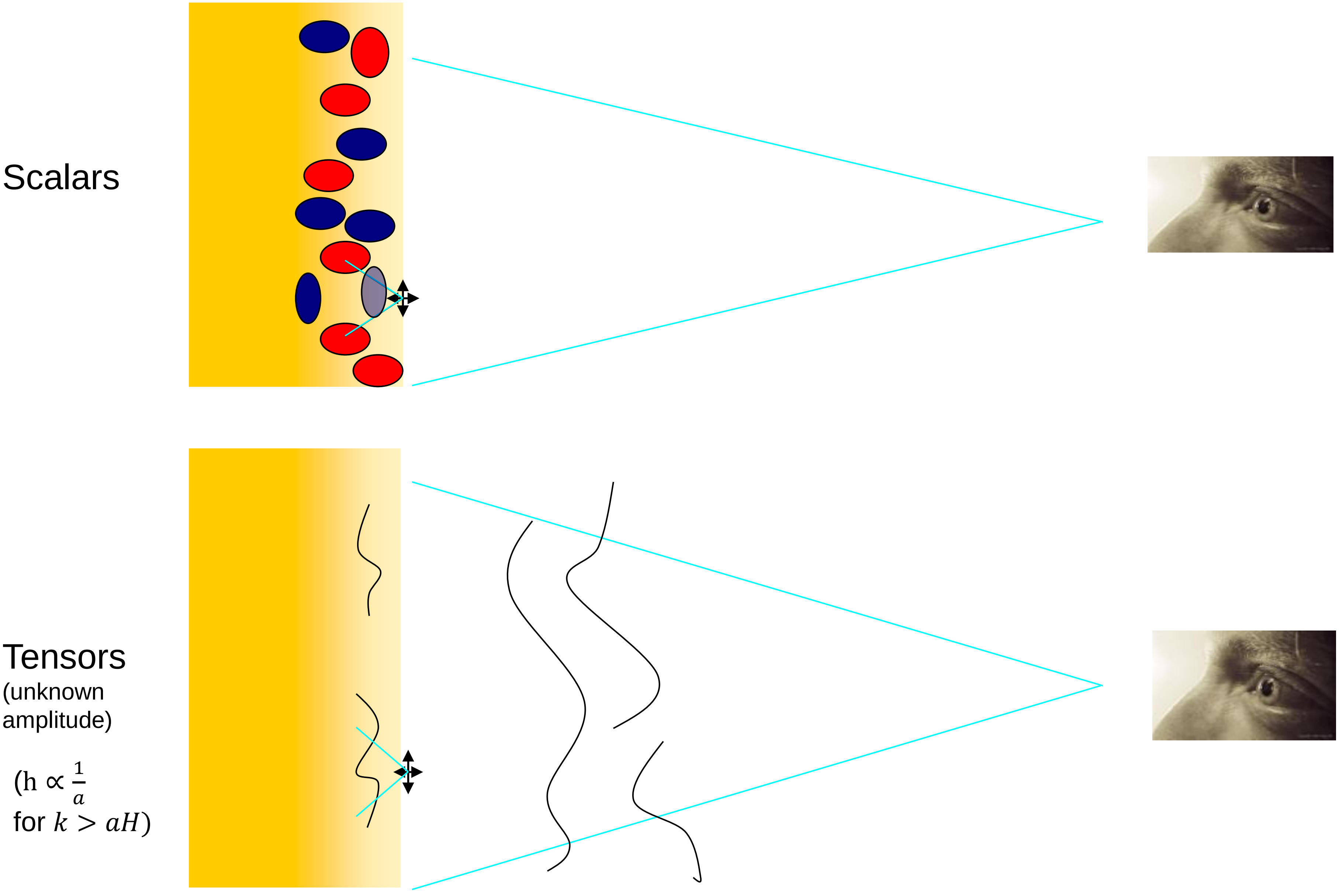
ISW

Poisson Eq: $\Delta_\gamma \sim -\left(\frac{k}{H}\right)^2 \Psi + ..$

Big overdensities – see cold

Small overdensities – see hot

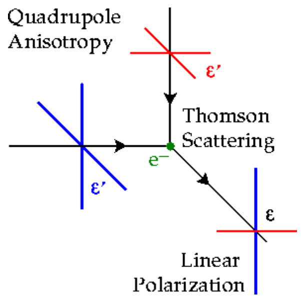
Complication: recombination is not sharp



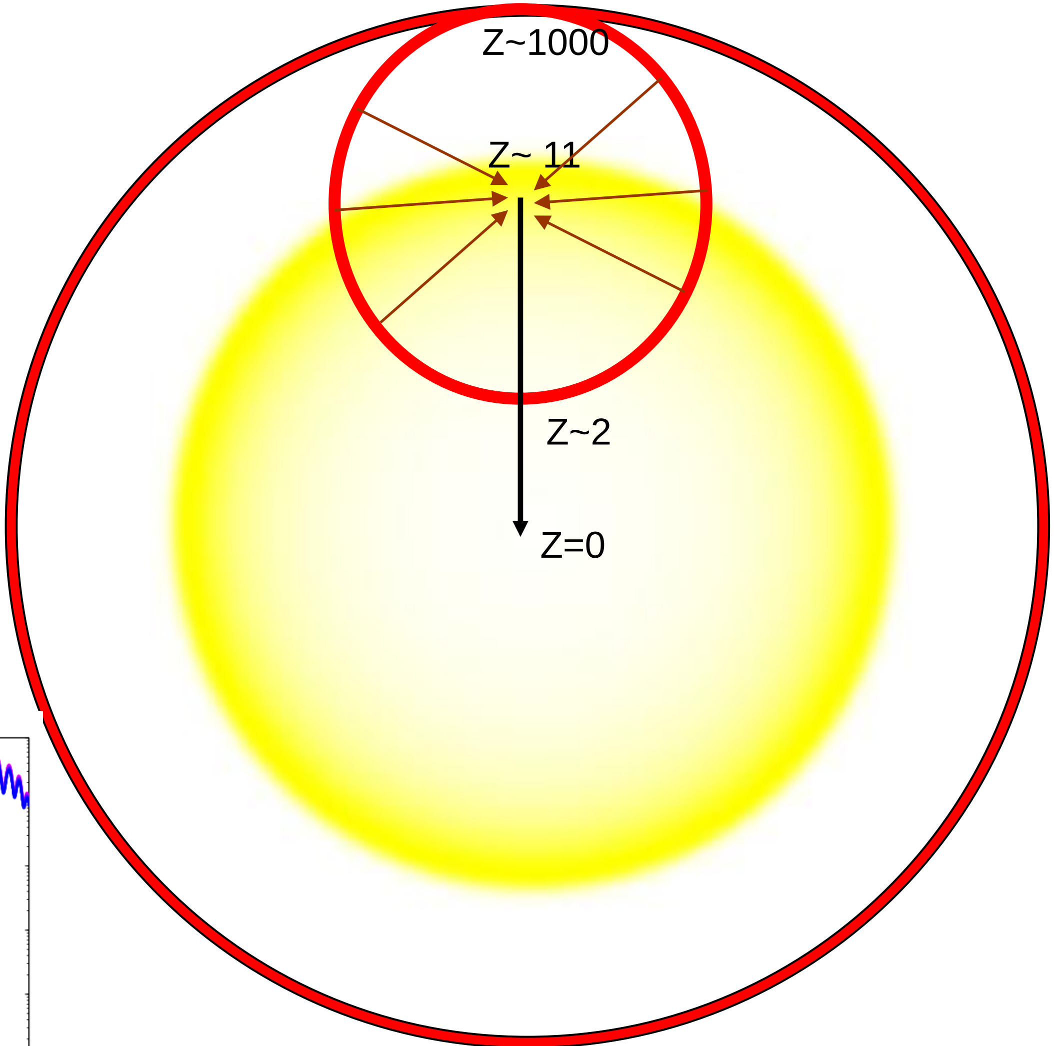
Line-of-sight averaging; Silk damping; polarization

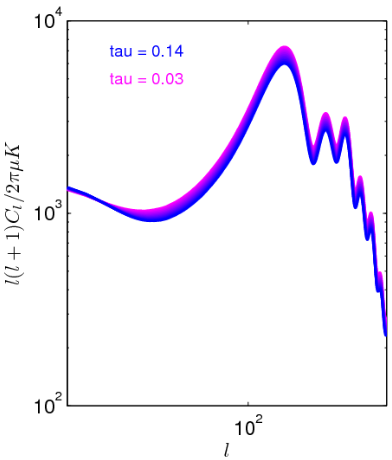
Also reionization

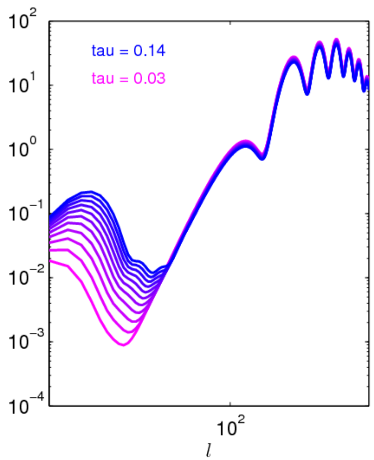
- Damping by $e^{-\tau}$
- Large-scale polarization



Hu astro-ph/9706147



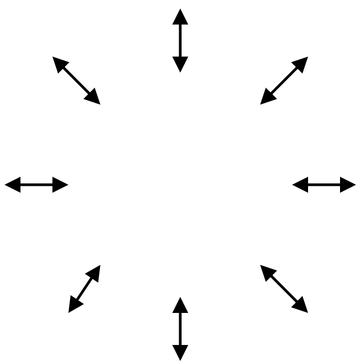




CMB polarization: E and B modes

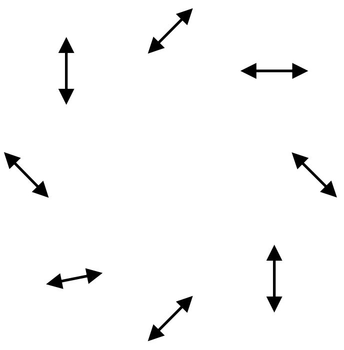
“gradient” modes
E polarization

e.g.

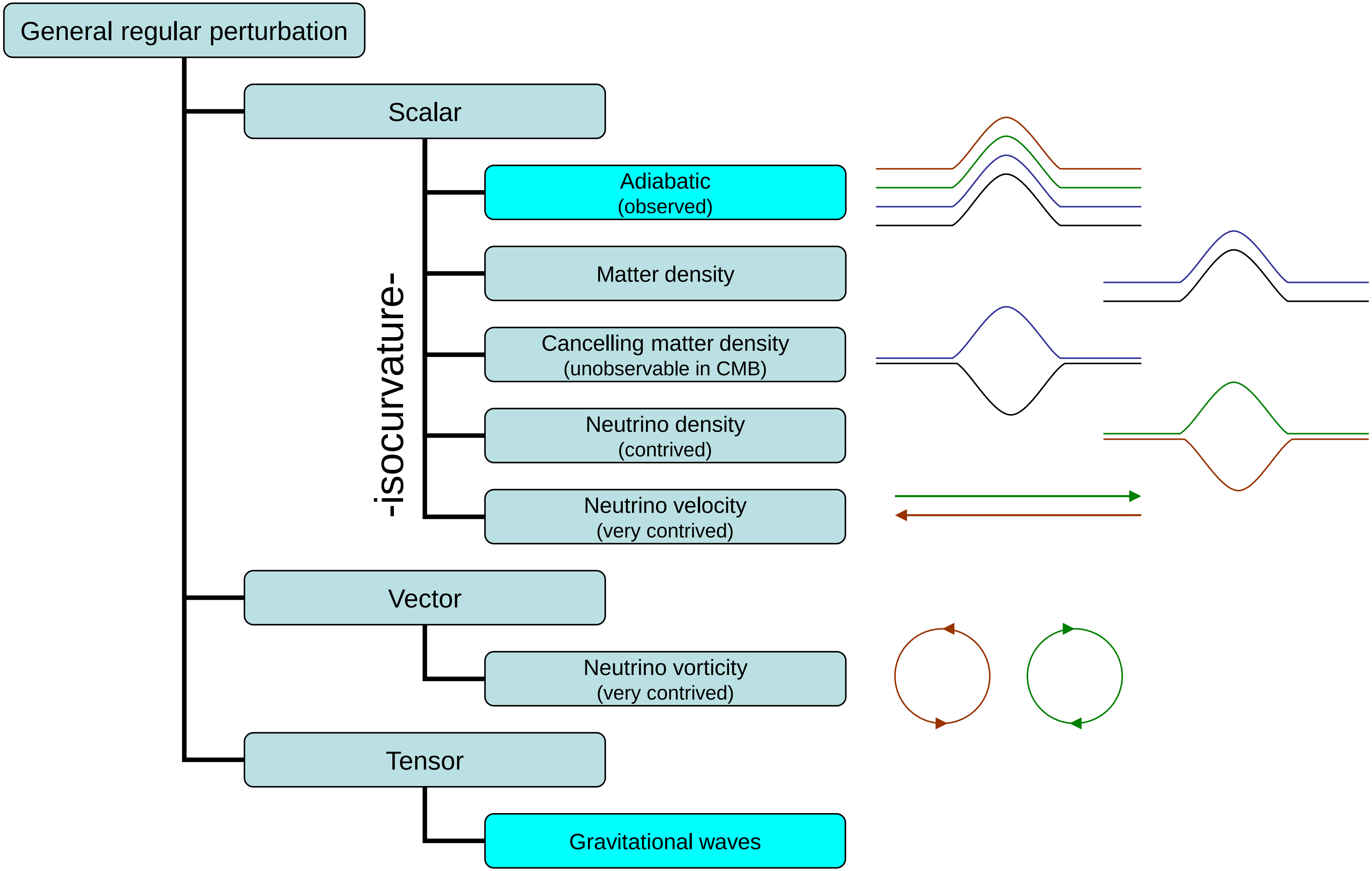


e.g. cold spot

“curl” modes
B polarization



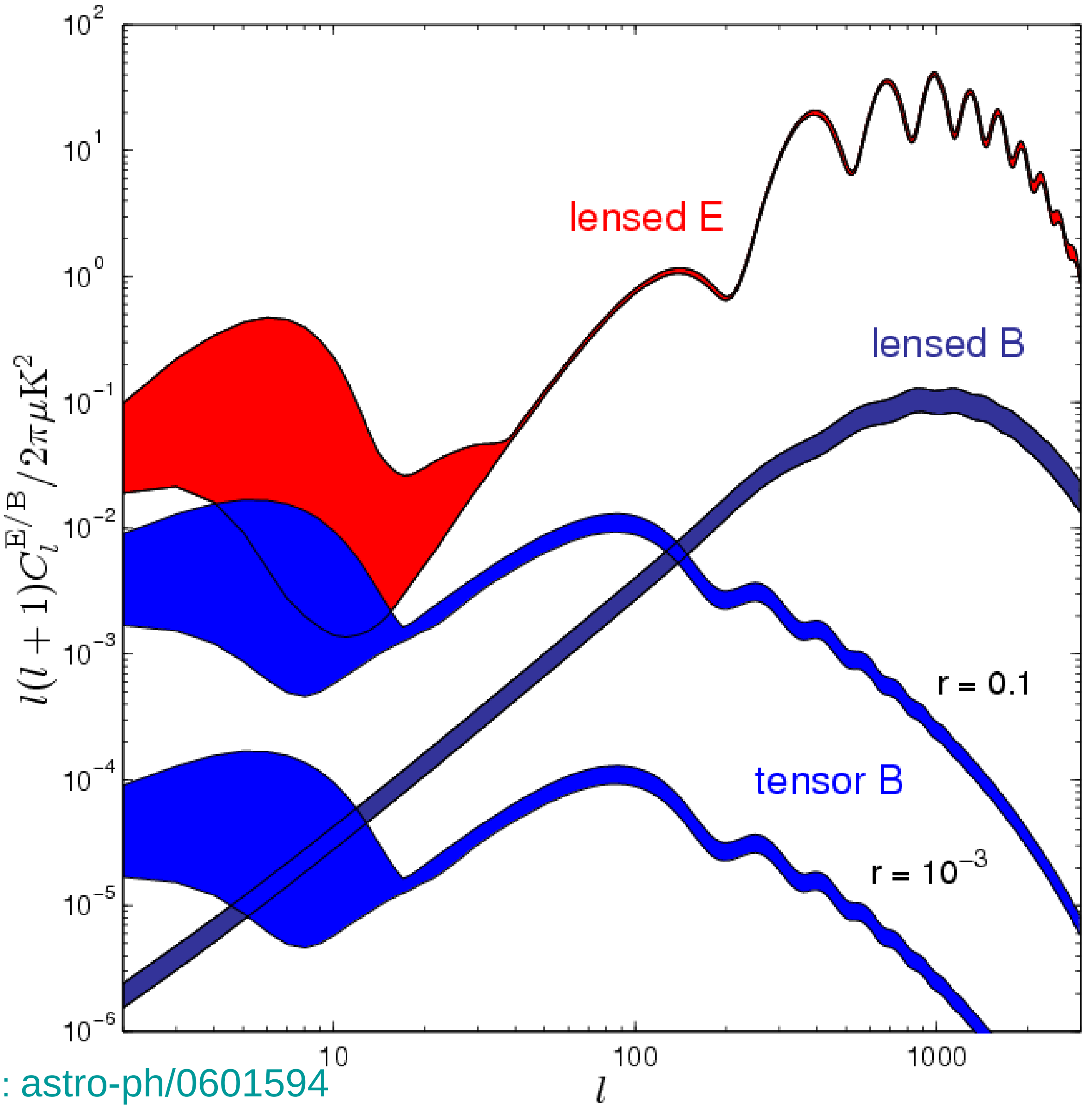
Possible regular initial perturbations



+ possible sources, e.g. strings

Polarization power spectra

Current 95% indirect limits for LCDM given WMAP+2dF+HST



Lewis, Challinor : astro-ph/0601594

Can we calculate the power spectra accurately enough?

- Linear theory (+ lensing) very well understood

- But depends on background evolution of x_e

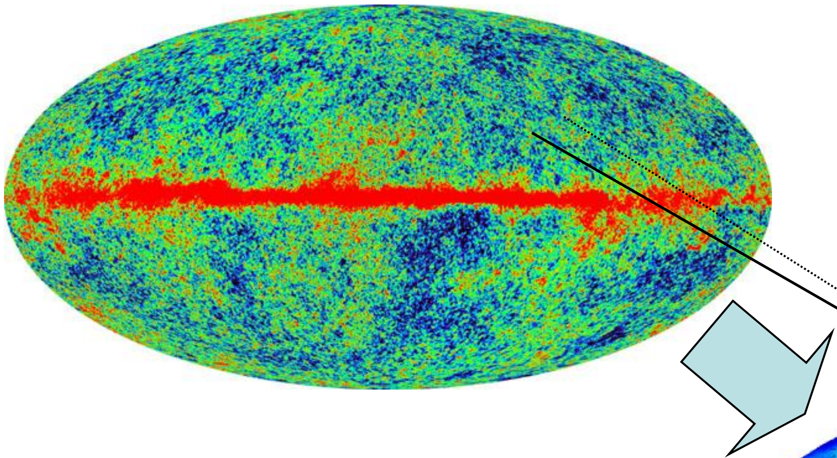
Recombination:

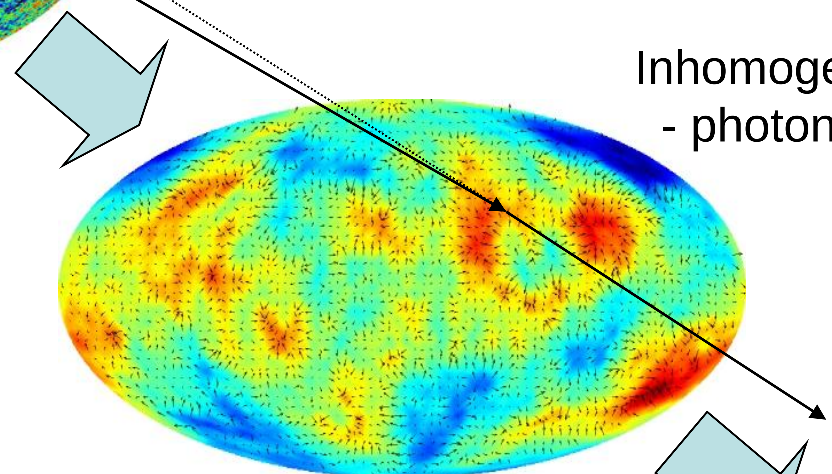
- Leading uncertainty, potentially percent-level errors
- Now independent codes, HyRec and CosmoRec (Chluba et al 2010, Ali-Hamoud et al 2011)
- Is there plausibly anything important that is still forgotten?

Reionization:

- Detailed shape unknown and not predictable in detail
- But E polarization not really sensitive, not a big issue
- Can constrain models from data

CMB Lensing

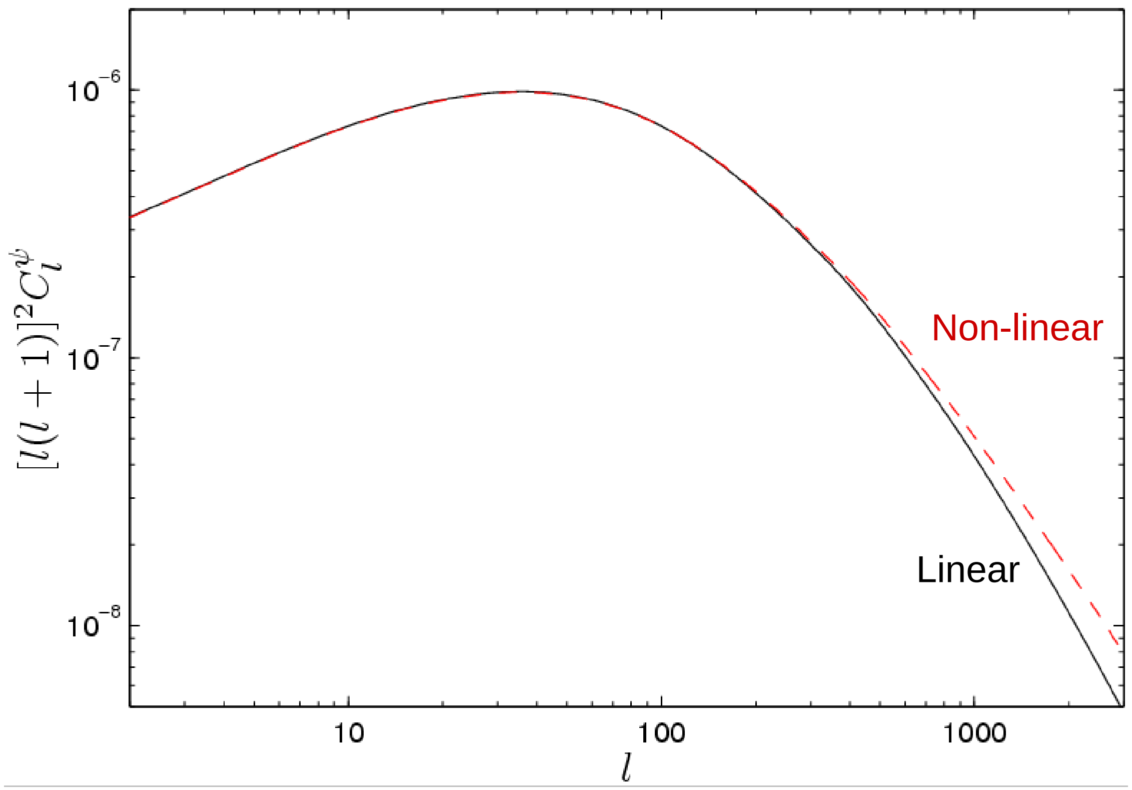




$$\psi(\hat{\mathbf{n}}) \equiv -2\int_0^{\chi_*} \mathrm{d}\chi\, \frac{f_K(\chi_* - \chi)}{f_K(\chi_*)f_K(\chi)} \Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi)$$

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha}) \qquad \boldsymbol{\alpha} = \boldsymbol{\nabla}\boldsymbol{\psi}$$

Deflection angle power spectrum



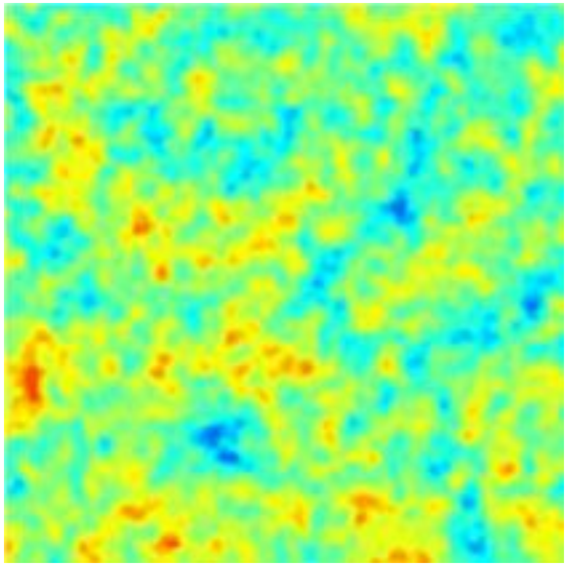
Deflections O(10^{-3}), but coherent on degree scales → important!

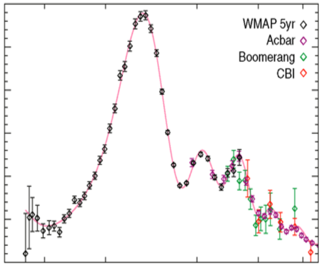
Why lensing is important

- Known effect, significant amplitude ($\sim 10^{-3}$)
- Modifies the power spectra on small-scales ($\sim 10^{-2}$)
- Lensing of E gives B-mode polarization (confusion for tensors/strings)
- Produces significant squeezed-shape bispectrum
- Large squeezed-shape trispectrum
- Non-Gaussianities measure lensing potentials
  - break degeneracies, constraint dark energy, m_ν, Ω_K ...

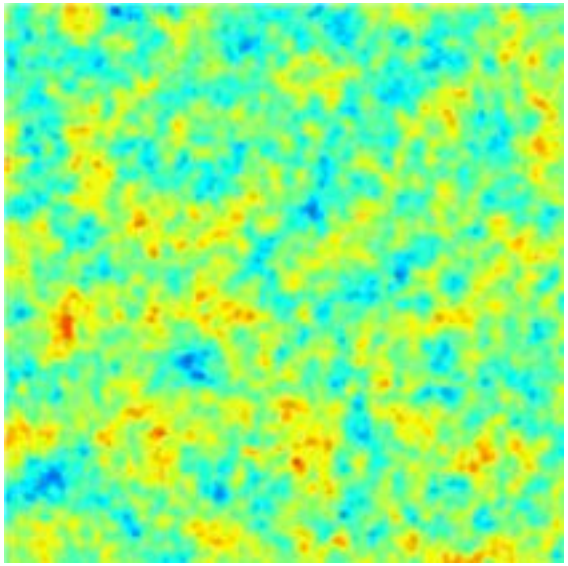
Beyond the power spectrum

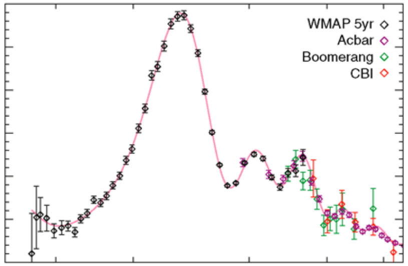
Magnified



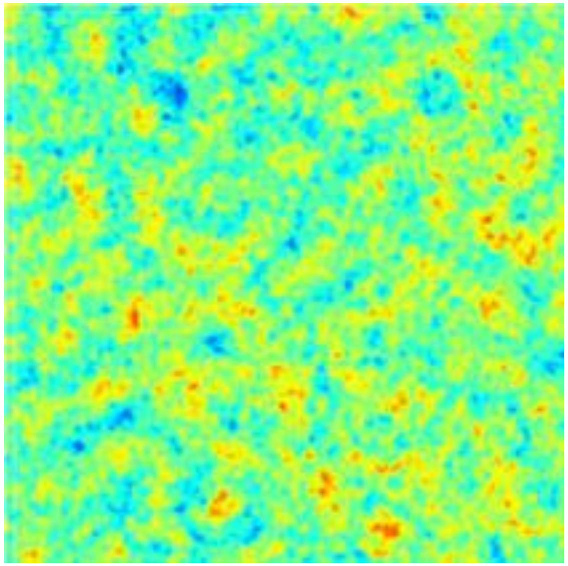


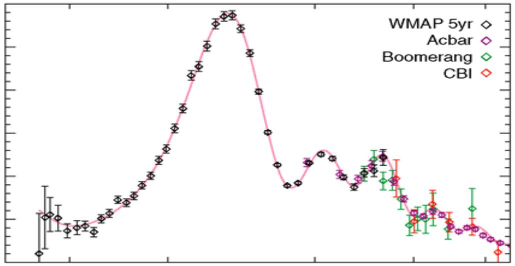
Unlensed



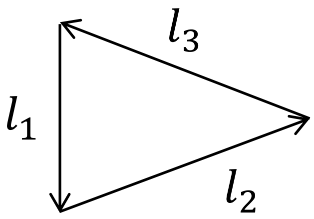


Demagnified





Bispectrum



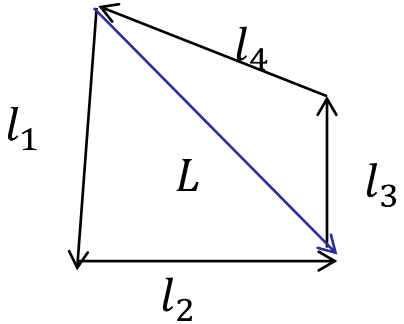
Flat sky approximation: $\langle \Theta(l_1)\Theta(l_2)\Theta(l_3) \rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$

If you know $T(l_1), T(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $T(l_3)$ is more likely

Trispectrum

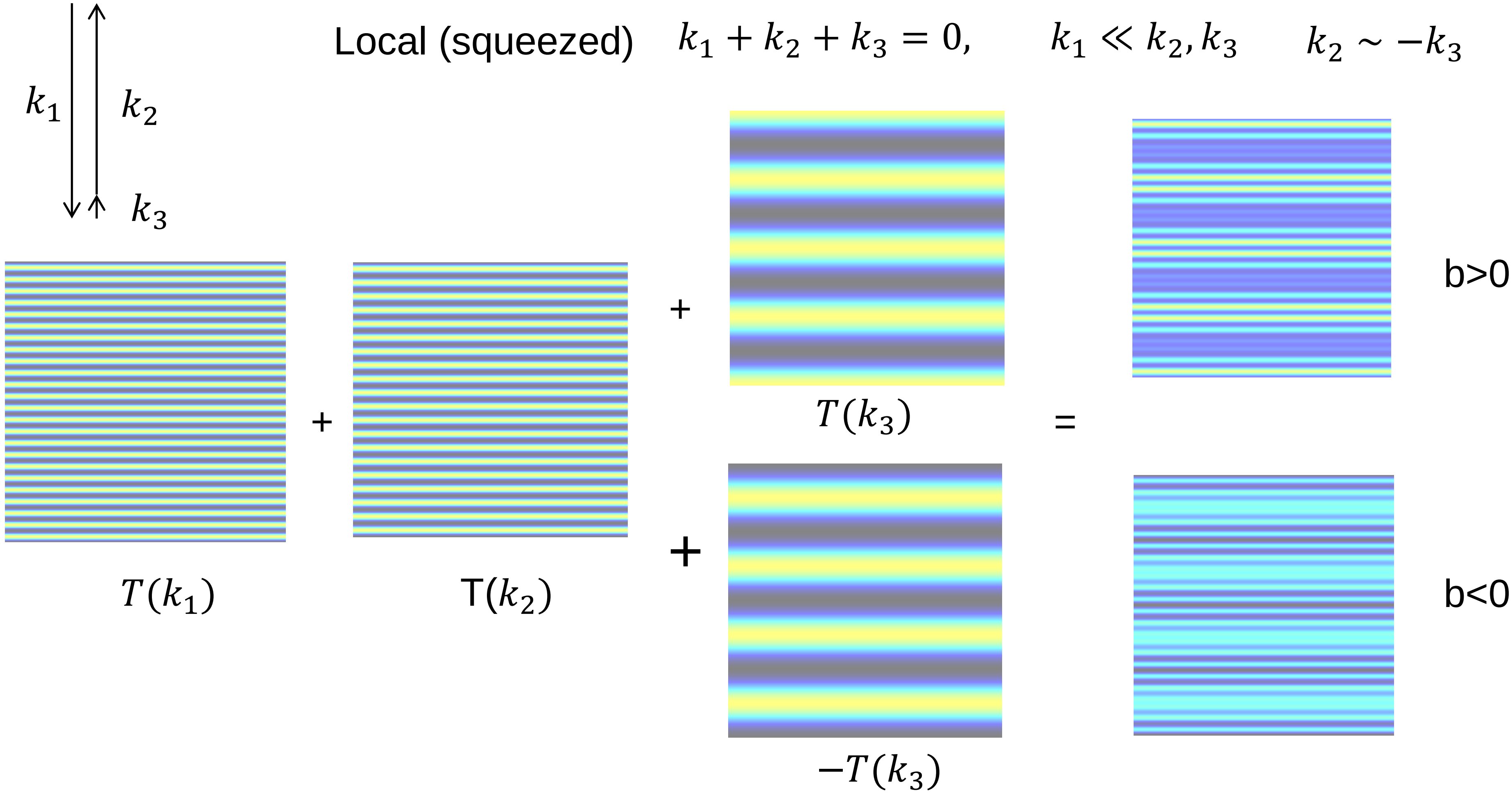
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4) \rangle_C = (2\pi)^{-2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{l}_3 + \mathbf{l}_4) T(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4)$$

$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4) \rangle_C = \frac{1}{2}\int \frac{d^2\mathbf{L}}{(2\pi)^2} \delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{L})\delta(\mathbf{l}_3 + \mathbf{l}_4 - \mathbf{L}) \mathbb{T}^{(\ell_1 \ell_2)}_{(\ell_3 \ell_4)}(L) + \text{perms.}$$



N-spectra…

Local (squeezed) $k_1 + k_2 + k_3 = 0, \quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$



Modulation of small-scale power by large-scale modes

Local primordial spatial modulation

$$\chi(\mathbf{x}) = \chi_0(\mathbf{x})[1 + \phi(\mathbf{x})]$$

(small) modulating field

Gaussian and statistically homogeneous

Gives squeezed non-Gaussianity

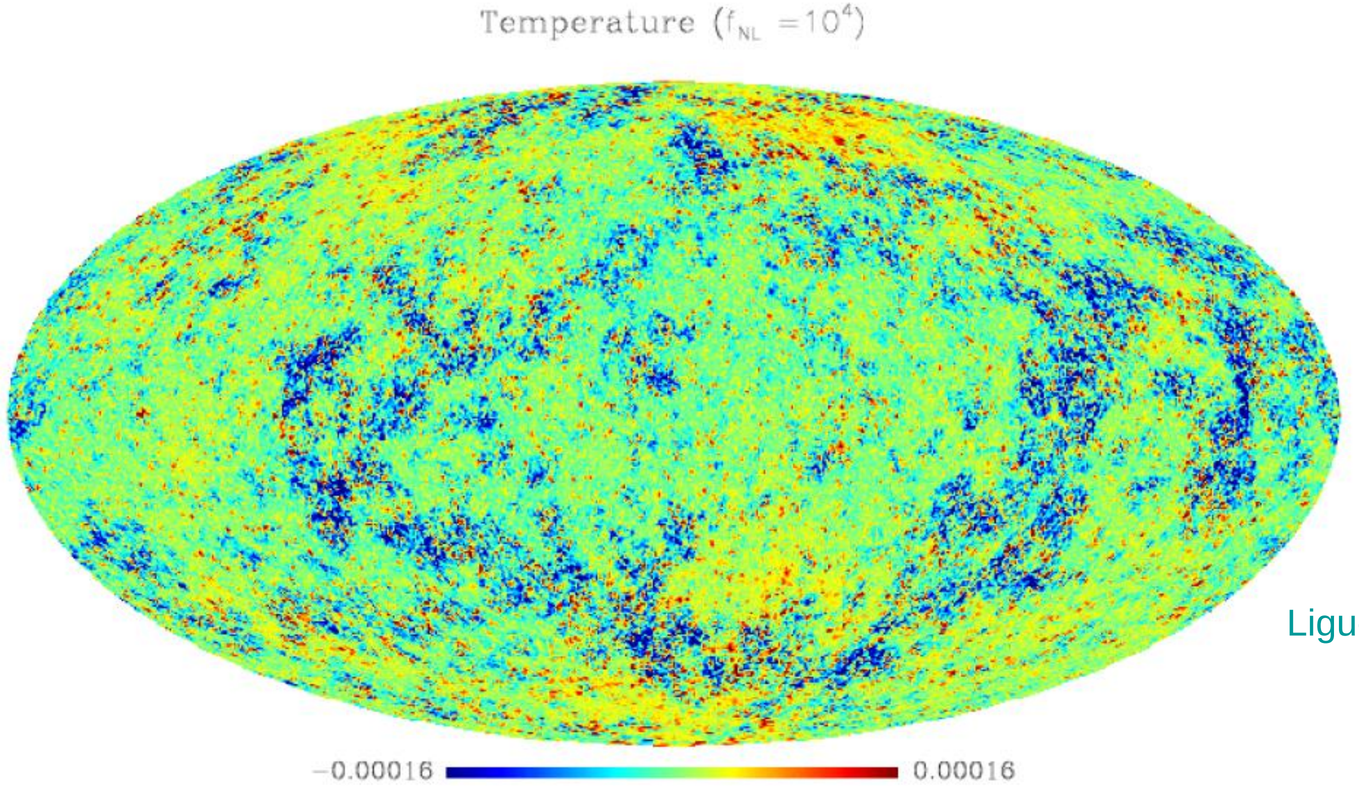
$$\langle \chi\chi\chi \rangle \sim \langle \chi_0\chi_0\chi_0\phi \rangle + \cdots \qquad \sim P_{\chi_0\chi_0} P_{\chi_0\phi}$$

$$\langle \chi\chi\chi\chi \rangle \sim \langle \chi_0\chi_0\chi_0\chi_0 \rangle + \langle \chi_0\chi_0\chi_0\chi_0\phi\phi \rangle + \cdots$$

$$\sim (Gaussian) + \; P_{\chi_0\chi_0} P_{\chi_0\chi_0} P_{\phi\phi}$$

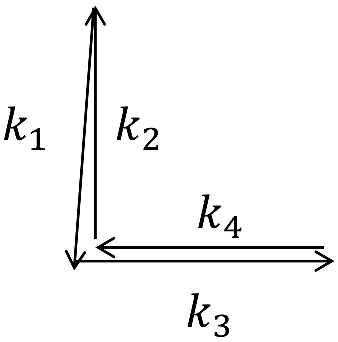
Since $P_{\chi_0\phi}^2 \leq P_{\chi_0\chi_0} P_{\phi\phi}$ $\qquad P_{\chi_0\chi_0} \langle \chi\chi\chi\chi \rangle_{squeezed} \geq \langle \chi\chi\chi \rangle_{squeezed}^2$

In conventional definitions $\tau_{NL} \geq \left(\frac{6 f_{NL}}{5}\right)^2$ (also L by L if quasi local)

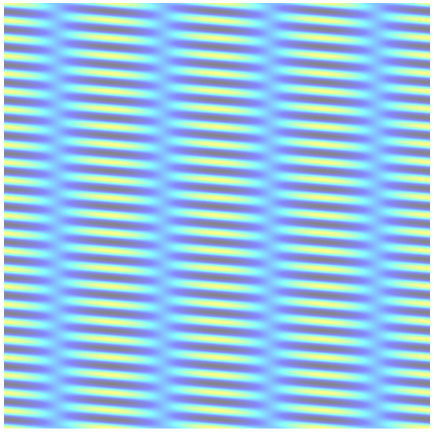


Liguori et al 2007

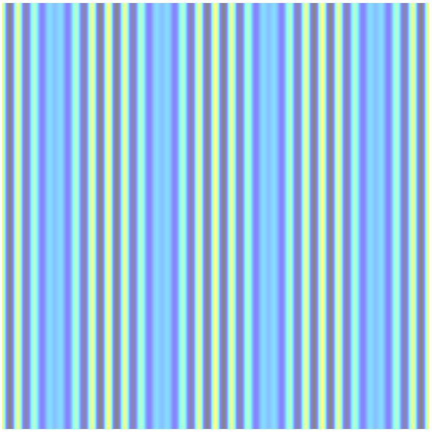
Squeezed trispectrum ~ power spectrum of modulation field



$T(k_1)$+T(k_2)

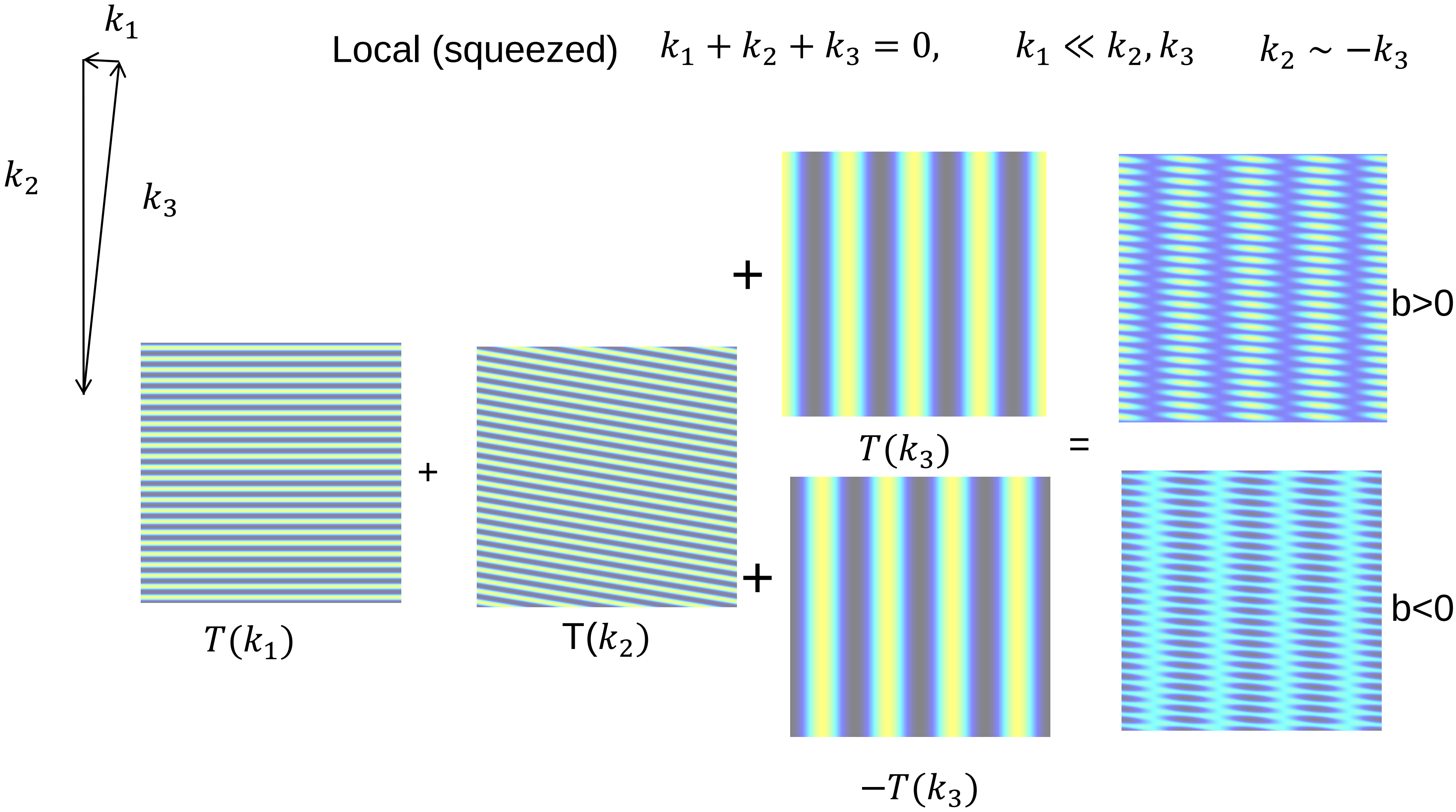


$T(k_3)$+T(k_4)



Small scale power is modulated by mode with $K = k_1 + k_2 = -(k_3 + k_4)$
- may or may not be correlated to large scale T modes

Local (squeezed) $k_1 + k_2 + k_3 = 0, \quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$

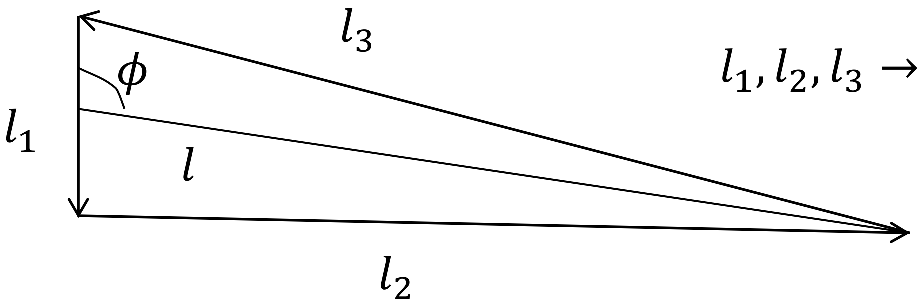


Possible direction-dependent modulation.

Local f_{NL} is isotropic, but e.g. CMB lensing is not:

$$
\begin{aligned}
b_{l_1 l_2 l_3} &\approx -C_{l_1}^{T\psi}\left[(\mathbf{l}_1 \cdot \mathbf{l}_2)\tilde{C}_{l_2}^{TT} + (\mathbf{l}_1 \cdot \mathbf{l}_3)\tilde{C}_{l_3}^{TT}\right] \\
&\approx l_1^2 C_{l_1}^{T\psi}\left[\frac{(\mathbf{l}_1 \cdot \mathbf{l}_2)^2}{l_1^2 l_2^2}\frac{\mathrm{d}\tilde{C}_{l_2}^{TT}}{\mathrm{d}\ln l_2} + \tilde{C}_{l_2}^{TT}\right].
\end{aligned}
$$

Shape decomposition of squeezed triangles



$l_1, l_2, l_3 \to l_1, l, \phi$

$$b_{l_1 l_2 l_3} = \sum_m b^m_{l_1 l} e^{im\phi}$$

Local isotropic modulations: $m = 0$

CMB lensing: $m = 0$ + $m = 2$

Looks like $f_{NL} \sim 9$
~ 2σ signal

Orthogonal to f_{NL}
~ 4σ signal

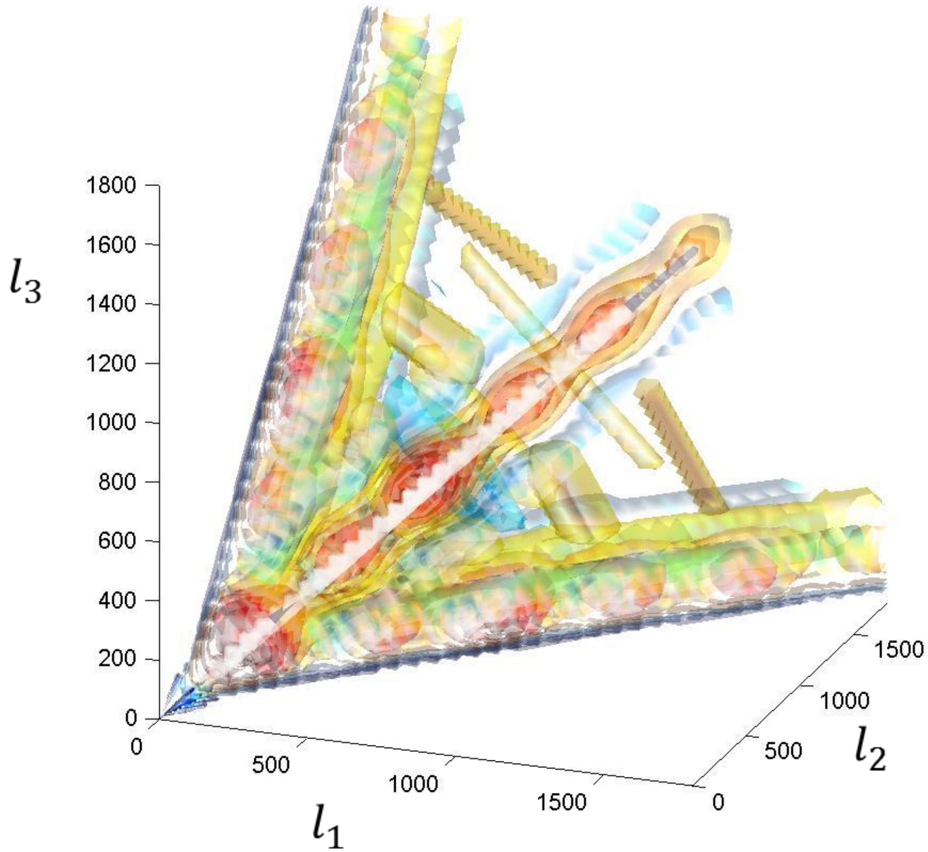
Angular dependence can be used to isolate and subtract different signals

- also l_1 dependence and phase of l dependence can be distinctive

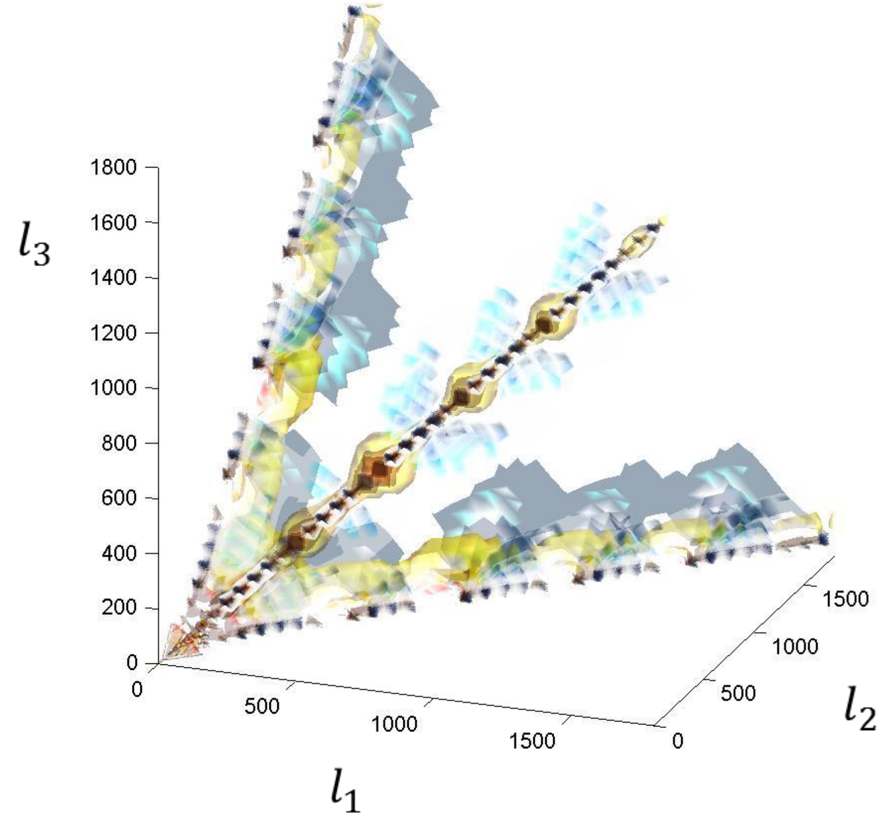
Lensing also modifies primordial signals. For $m = 0$ just like the power spectrum, otherwise:

$$\tilde{b}^m_{l_1 l} \approx \int r dr J_m(lr) \int dl' l' b^m_{l_1 l'} e^{-l'^2 \sigma^2(r)/2} \sum_n I_n[l'^2 C_{\mathrm{gl},2}(r)/2] J_{2n+m}(l'r)$$

$b_{l_1 l_2 l_3}$



Local f_{NL}



CMB lensing

Lewis, Challinor & Hanson 2010

Anisotropy estimators – just reconstruct the 'modulating' field

Following Hu et al 2000-2003

Munshi & Heavens 2009

Hanson & Lewis, 0908.0963

First treat modulation field h as fixed. If other fields Gaussian:

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$

Maximum likelihood:

$$\frac{\delta\mathcal{L}}{\delta\mathrm{h}^\dagger} = -\frac{1}{2}\hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\mathrm{Tr}\left[(C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}\right] = 0$$

First iteration solution: Quadratic Maximum Likelihood (QML)

$$\hat{\mathrm{h}} = \mathcal{F}^{-1}[\tilde{\mathrm{h}} - \langle\tilde{\mathrm{h}}\rangle].$$

$$\tilde{\mathrm{h}} = \mathcal{H}_0 = \frac{1}{2}\bar{\Theta}^\dagger\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}\bar{\Theta} = \frac{1}{2}\sum_{lm,l'm'}\left[\frac{\delta C^{\hat{\Theta}\hat{\Theta}}_{lm,\,l'm'}}{\delta\mathrm{h}^\dagger}\right]\bar{\Theta}^*_{lm}\bar{\Theta}_{l'm'},$$

$$\bar{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1}|_0\hat{\Theta}$$

CMB lensing

Reconstruct lensing potential, ψ_{lm}

Bispectrum measured by $C_l^{T\psi} = \langle T_{lm}^* \ \psi_{lm} \rangle$ (Probes ISW, some info on dark energy)

Trispectrum measured by $C_l^{\psi\psi} = \langle \psi_{lm}{}^* \psi_{lm} \rangle$ (All scales, most of the information)

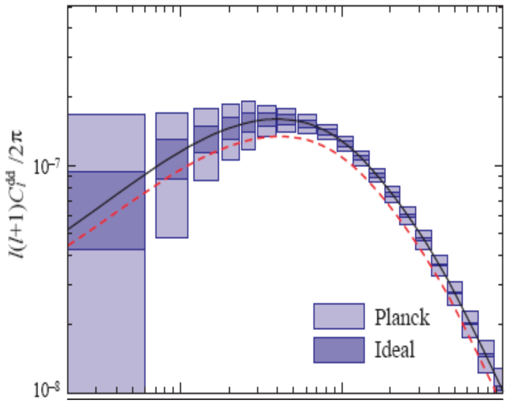
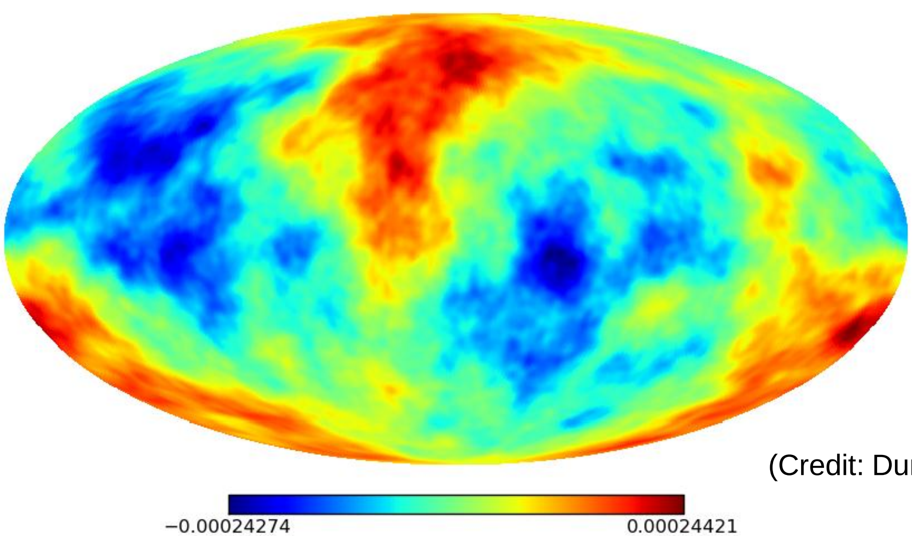


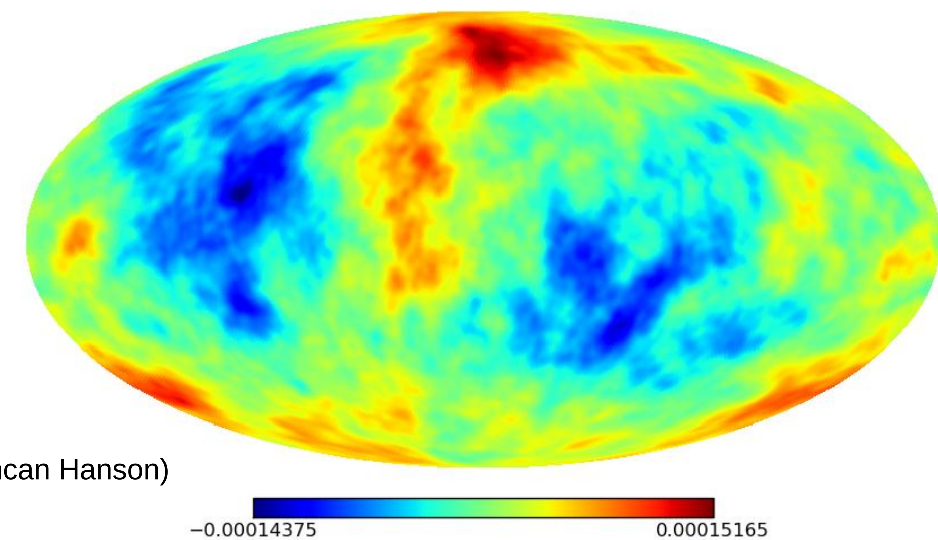
FIG. 3. CMB lensing power spectra for the fiducial $w = -1$ model (solid) and the degenerate $w = -2/3$ model (dashed) of Fig. 1. Boxes represent 1σ errors on band powers assuming the Planck and ideal experiments of Tab. I. Top: deflection power spectra. Bottom: cross correlation of deflection and temperature fields.

Hu: astro-ph/0108090

True (simulated)



Reconstructed (Planck noise, Wiener filtered)



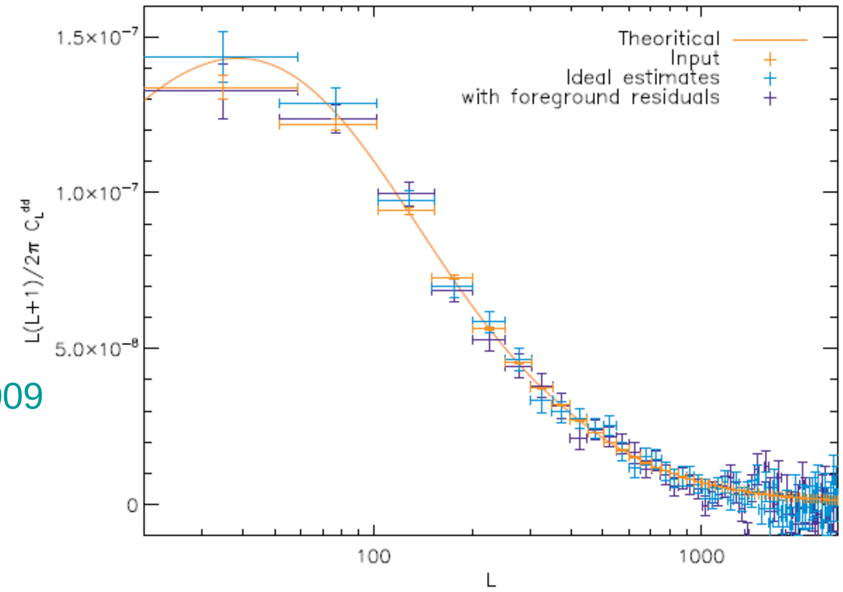
(Credit: Duncan Hanson)

What does a reconstruction of lensing ψ_{lm} and hence estimate of $C_l^{\psi\psi}$ do for us?

Probe 0.5<~ z <~ 6: depends on geometry and matter power spectrum

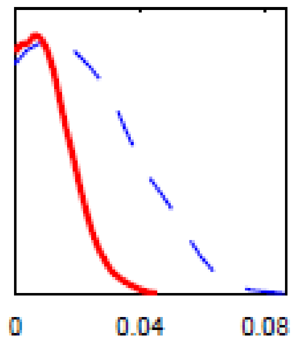
Can break degeneracies in the linear CMB power spectrum

- Better constraints on neutrino mass, dark energy, Ω_K, …



Perotto et al. 2009 (simulation)

Neutrino mass fraction with and without lensing (Planck only)



Perotto et al. 2006

Local modulation

Reconstruct $\phi_{lm}(\chi)$ – modulation field at distance χ

General bispectra

Similar construction, but a bit more complicated

- Numerically challenging unless separable; or use modes (Ferguson et al)

Anisotropic primordial power spectrum

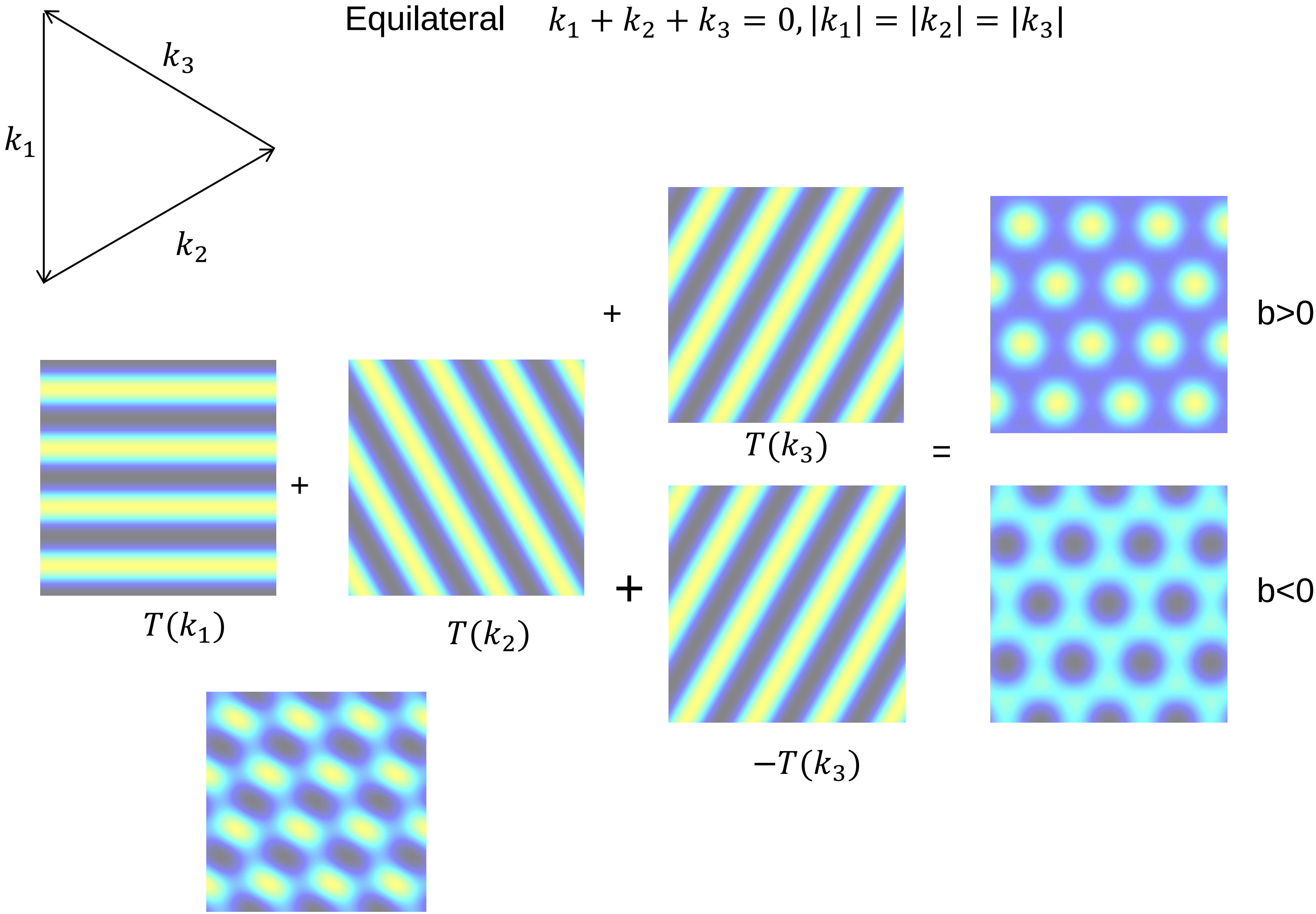
$$\mathcal{P}_\chi(\mathbf{k}) = \mathcal{P}_\chi(k)[1 + a(k)g(\hat{\mathbf{k}})]$$

e.g.
Ackerman et.al. astro-ph/0701357
Gumrukcuoglu et al 0707.4179

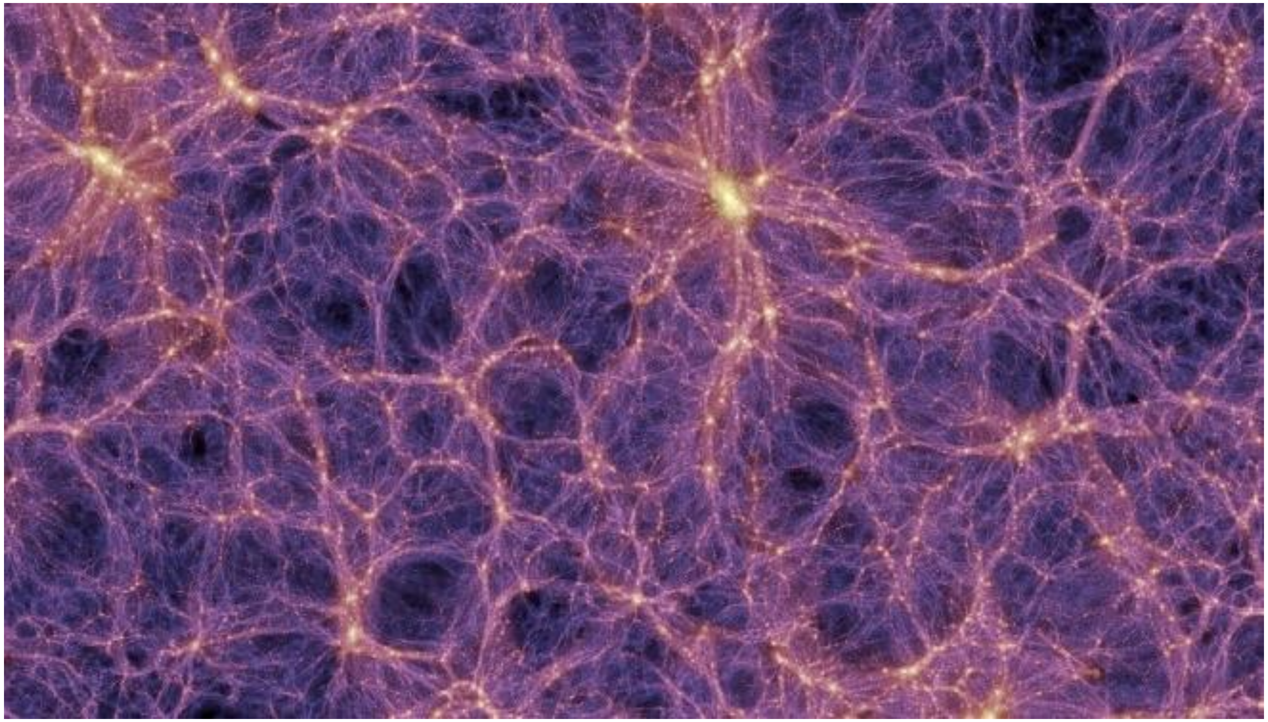
- Would show up in trispectrum, or just reconstruct g

(there is *not* any evidence for primordial $g \neq 0$ in WMAP)

Many other possibilities..

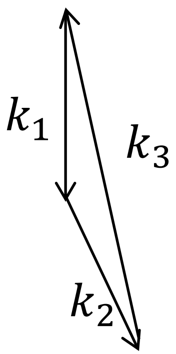


Equilateral $k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3|$
k_1
k_2
k_3
+
+
+
=
$T(k_1)$
$T(k_2)$
$T(k_3)$
$-T(k_3)$
b>0
b<0

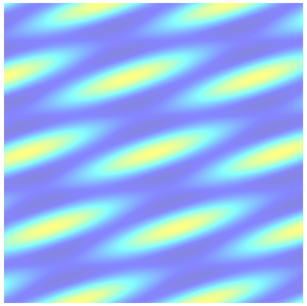


Millennium simulation

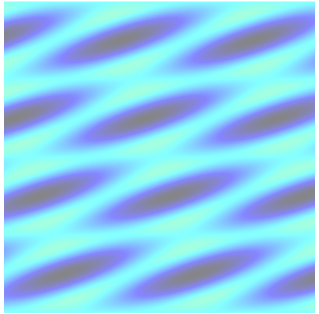
Near-equilateral to flattened:



b>0



b<0



- In general can measure full dependence on shape, but need specific models to have S/N > 1

e.g. ‘orthogonal’ shape changes sign of b between equilateral and flattened

Non-Gaussianities from non-linear effects till recombination

Pitrou, Uzan, Bernardeau (2010) claim local and equilateral components with $f_{NL} \sim 5$

Important for Planck and beyond. Do we believe this?

Total signal is about 2σ, so not easy to check directly against the data, but still important bias if neglected.

Things we might expect:

Isotropic local power modulation via large-scale modulation of Silk scale
(but shouldn't this give negative f_{NL}?)

Large-scale modulation of sound horizon
- squeezed contribution out of phase with primary signal $\propto \frac{dC_l}{d\ln l}$

Equilateral contribution from non-linear growth of density perturbations

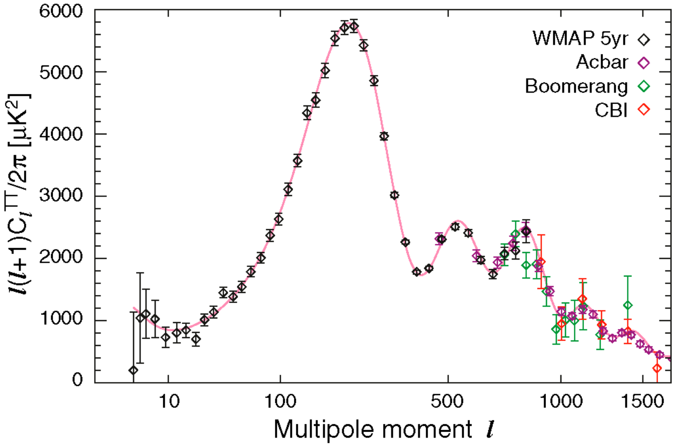
Can anyone give a physical explanation of the squeezed signal?

‘Anomalies’ in WMAP

Some will never be measured better.. but Planck will give check on WMAP

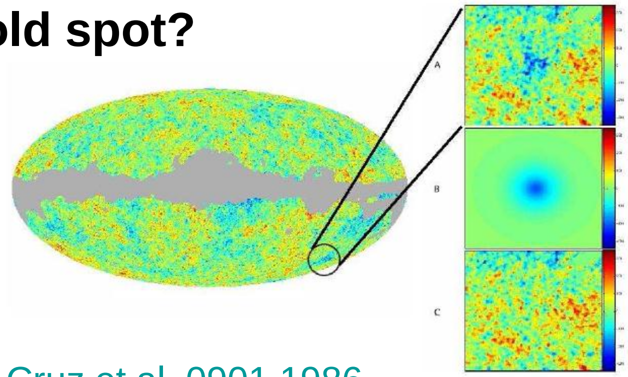
.. and polarization measurements can give good consistency check on models

(e.g. Dvorkin et at 0711.2321)



Low quadrupole?

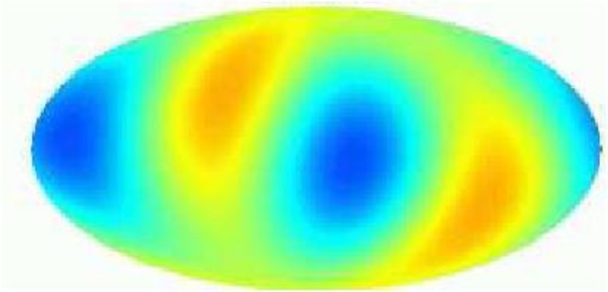
Cold spot?



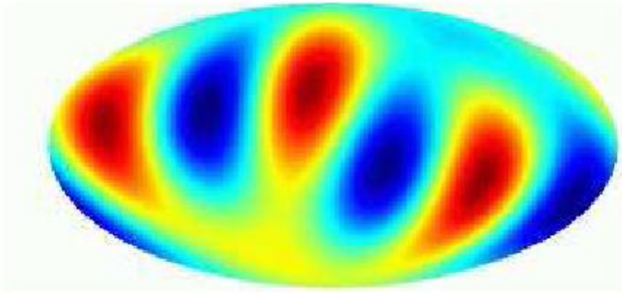
Cruz et al, 0901.1986

Alignments?

Tegmark et al.



Quadrupole



Octopole

etc.

Conclusions

- CMB is still by far the cleanest probe of early-universe physics and cosmological parameters

- Measure some parameters very accurately, but degeneracies and cosmic variance limitations

- Power spectrum and relation to parameters well understood (recombination? reionization?)

- Polarization can cleanly identify non-scalar signals and give powerful consistency checks on results from the temperature alone

- Statistical anisotropy/non-Gaussianities

Many possibilities

- primordial signals cosmic variance limited to around $f_{NL} \sim 2$

- some signals definitely present and easily detectable

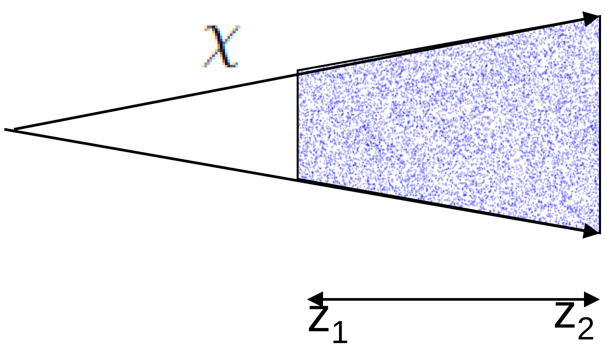
    - Lensing; valuable new information, break degeneracies
    - Non-linear effects at recombination (??)
    - Local effects (SZ, point sources, foregrounds…???)

Angular source count density in linear theory

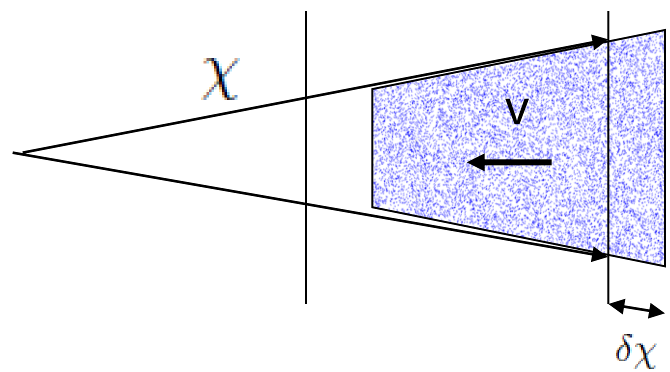
$$\Delta_n(\hat{\mathrm{n}}, z) = \delta_n - \frac{1}{\mathcal{H}}\hat{\mathrm{n}} \cdot \frac{\partial \mathbf{v}}{\partial \chi} - 2\kappa + \left[-\frac{2}{\mathcal{H}\chi} + \frac{\mathrm{d}\ln(a^3 \bar{n}_s)}{\mathcal{H}\mathrm{d}\eta} - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right]\left[-\psi - \int^{\eta_A} (\dot{\phi} + \dot{\psi})\mathrm{d}\eta + \hat{\mathrm{n}} \cdot \mathbf{v}\right] + \frac{2}{\chi}\int^{\eta_A} (\phi + \psi)\mathrm{d}\eta + \frac{1}{\mathcal{H}}\dot{\phi} + \psi - 2\phi$$

Challinor & Lewis 2020

What we think we are seeing



Actually seeing



Think about large-scale modes as modulating small-scale modes:

- Expect largest modulation on scales of acoustic peak, T large hot/cold spots

- Hot spot implies recombination delayed

  - Silk scale is larger, more damping

    $\Delta T > 0$: more time, less small scale power
    $\Delta T < 0$: less time, more small scale power

    $\Rightarrow$ local $f_{NL} > 0$

  - Sound horizon larger, shift in angular scale of smaller perturbations

    - Effect roughly orthogonal to local f_{NL} (total small-power preserved)
    - Looks very similar to lensing, with `magnification' correlated to T

Write general quadratic anisotropy estimator:

$$6X_{lm} \equiv \sum_{l_1 m_1, l_2 m_2} B_{l l_1 l_2} (-1)^{m_1} \begin{pmatrix} l & l_1 & l_2 \\ m & -m_1 & m_2 \end{pmatrix} \bar{\Theta}_{l_1 m_1} \bar{\Theta}^*_{l_2 m_2}$$
$$= \int d\Omega Y^*_{lm} \times \sum_{l_1 l_2} b_{l l_1 l_2} \left[\sum_{m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1} \right] \left[\sum_{m_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2} \right]$$

Bispectrum estimators are basically the cross-correlation of an anisotropy estimator with the temperature

$$\mathcal{E} = \frac{1}{F_{\mathcal{E}}} \bar{\Theta}^\dagger (\mathbf{X} - 3\langle \mathbf{X} \rangle),$$

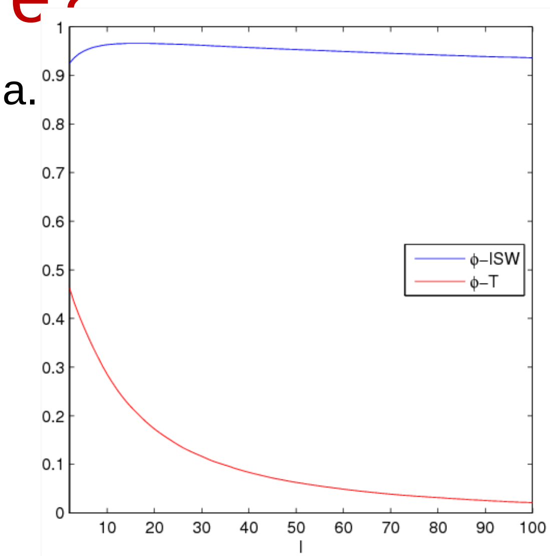
In harmonic space

$$\mathcal{E} = \frac{1}{6F_{\mathcal{E}}} \sum_{l_i m_i} B_{l_1 l_2 l_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \times \left[\bar{\Theta}_{l_1 m_1} \bar{\Theta}_{l_2 m_2} \bar{\Theta}_{l_3 m_3} - 3 C^{-1}_{l_1 m_1 l_2 m_2} \bar{\Theta}_{l_3 m_3} \right].$$

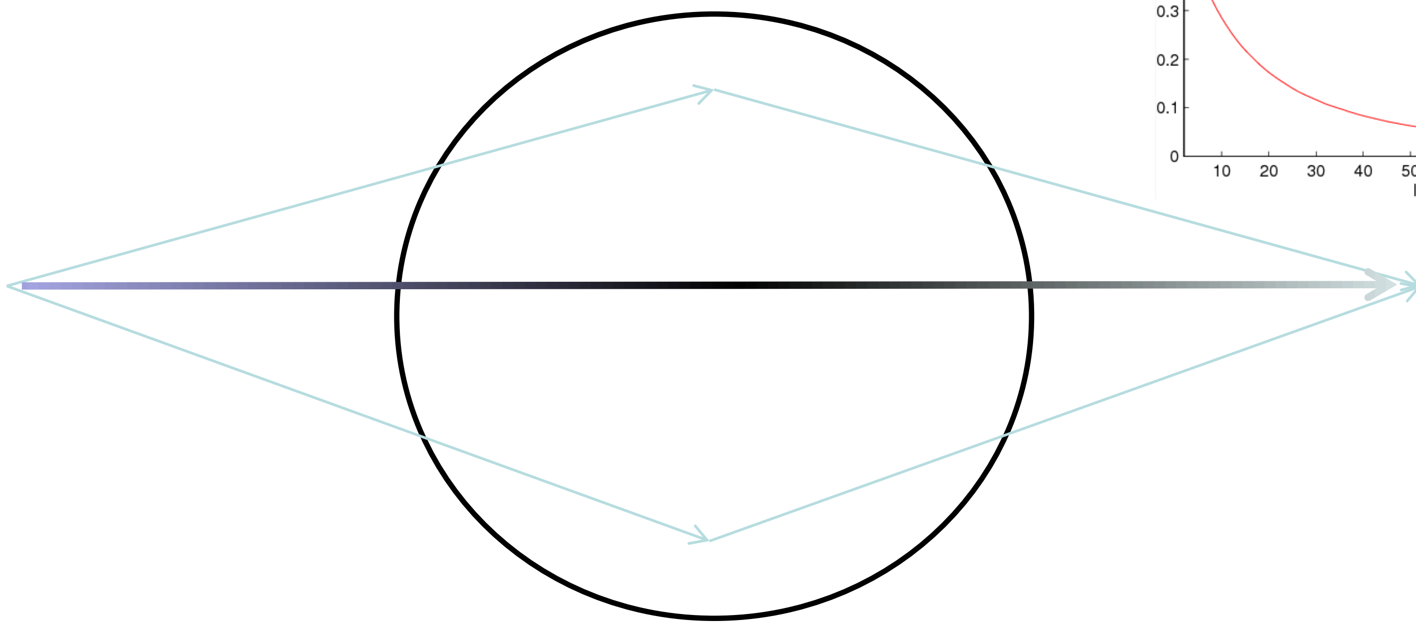
Creminelli et al 2005, Babich 2005, Smith & Zaldarriaga 2006

Why is there a correlation between large-scale lenses and the temperature?

(small-scales: also SZ , Rees-Sciama.



$$\Delta T_{\rm ISW}(\hat{\mathbf{n}}) = 2\int_0^{\chi_*} \mathrm{d}\chi\, \dot{\Psi}(\chi\hat{\mathbf{n}};\eta_0-\chi)$$



Overdensity: magnification correlated with positive Integrated Sachs-Wolfe (net bluesh

Underdensity: demagnification correlated with negative Integrated Sachs-Wolfe (net red

Accurate bispectrum calculation

Assume Gaussian fields. Non-perturbative result:

$$\langle T(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle = C_{l_1}^{T\psi}\left\langle \frac{\delta}{\delta\psi(\mathbf{l}_1)^*}\left(\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\right)\right\rangle$$

Use $\tilde{T}(\boldsymbol{x}) = T(\boldsymbol{x} + \boldsymbol{\nabla}\psi)$ ⇒

$$\frac{\delta}{\delta\psi(\mathbf{l}_1)^*}\tilde{T}(\mathbf{l}) = -\frac{i}{2\pi}\mathbf{l}_1\cdot\widetilde{\boldsymbol{\nabla}T}(\mathbf{l}+\mathbf{l}_1),$$

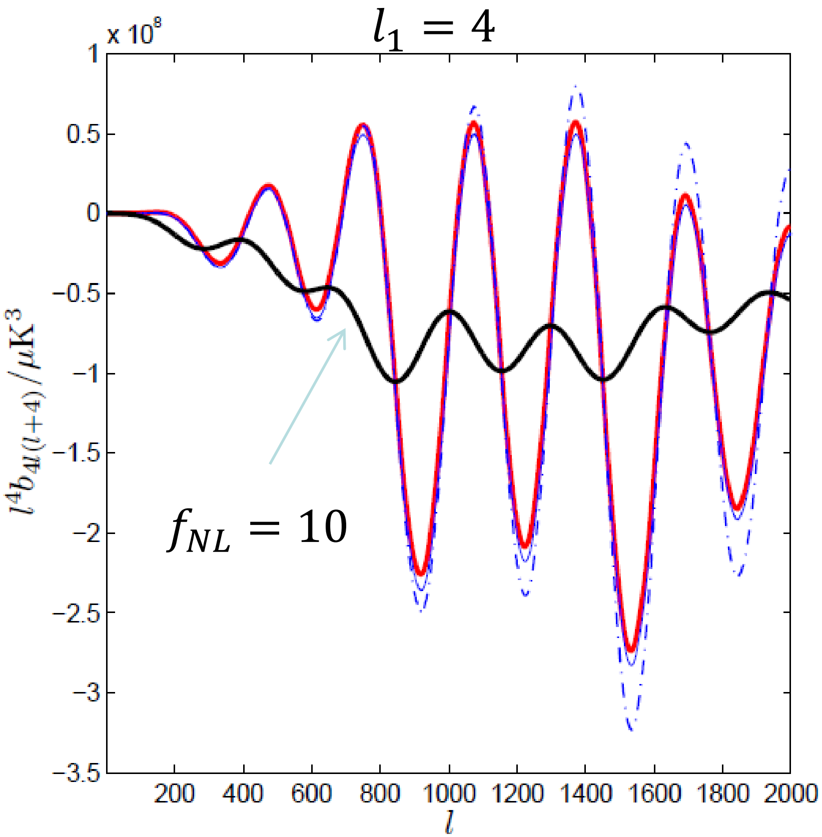
$$\begin{aligned}\langle T(\mathbf{l}_1)\tilde{T}(\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\rangle &= -\frac{i}{2\pi}C_{l_1}^{T\psi}\,\mathbf{l}_1\cdot\left\langle\widetilde{\boldsymbol{\nabla}T}(\mathbf{l}_1+\mathbf{l}_2)\tilde{T}(\mathbf{l}_3)\right\rangle + (\mathbf{l}_2\leftrightarrow\mathbf{l}_3)\\ &= -\frac{1}{2\pi}\delta(\mathbf{l}_1+\mathbf{l}_2+\mathbf{l}_3)C_{l_1}^{T\psi}\left[(\mathbf{l}_1\cdot\mathbf{l}_2)\tilde{C}_{l_2}^{T\nabla T} + (\mathbf{l}_1\cdot\mathbf{l}_3)\tilde{C}_{l_3}^{T\nabla T}\right]\end{aligned}$$

~ Lensed temperature power spectrum

Lensing bispectrum depends on *changes* in the small-scale *lensed* power

$$b_{l_1 l_2 l_3} \approx -C^{T\psi}_{l_1}\left[(\mathbf{l}_1\cdot\mathbf{l}_2)\tilde{C}^{TT}_{l_2} + (\mathbf{l}_1\cdot\mathbf{l}_3)\tilde{C}^{TT}_{l_3}\right]$$
$$\approx l_1^2 C^{T\psi}_{l_1}\left[\frac{(\mathbf{l}_1\cdot\mathbf{l}_2)^2}{l_1^2 l_2^2}\left.\frac{\mathrm{d}\tilde{C}^{TT}_l}{\mathrm{d}\ln l}\right|_{l_2} + \tilde{C}^{TT}_{l_2}\right].$$

$$(\mathbf{l_1} + \mathbf{l_2} + \mathbf{l_3} = 0)$$



- Quite large signal. Expect ~ 5σ with Planck. Cosmic variance ~ 7σ.

- Using lensed power spectra important at 5-20% level: leading-order result (using unlensed spectra) not accurate enough

If lensing is neglected get bias $\Delta f_{NL} \sim 9$ on primordial local models with Planck
(see e.g. Hanson et al 0905.4732, Mangilli 0906.2317)

BUT:

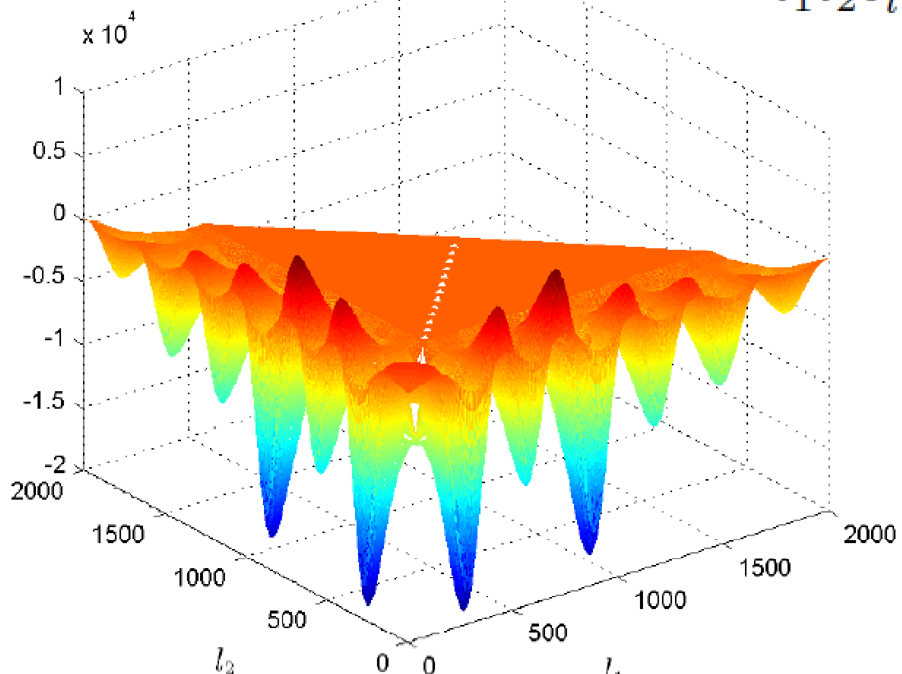
- Lensing bispectrum depends on power difference: has phase shift compared to any adiabatic primordial bispectrum (and different scale dependence)
- Lensing bispectrum is strongly scale dependent (small ISW for larger l_1)

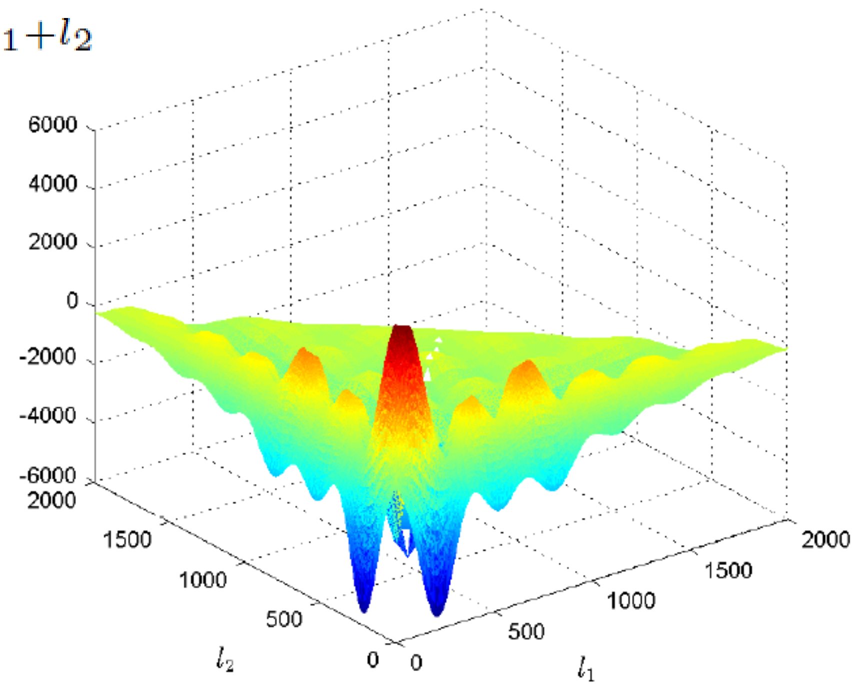
- Lensing bispectrum depends on shape of squeezed triangle ($l_1 \cdot l_2$ factor)

Lensing

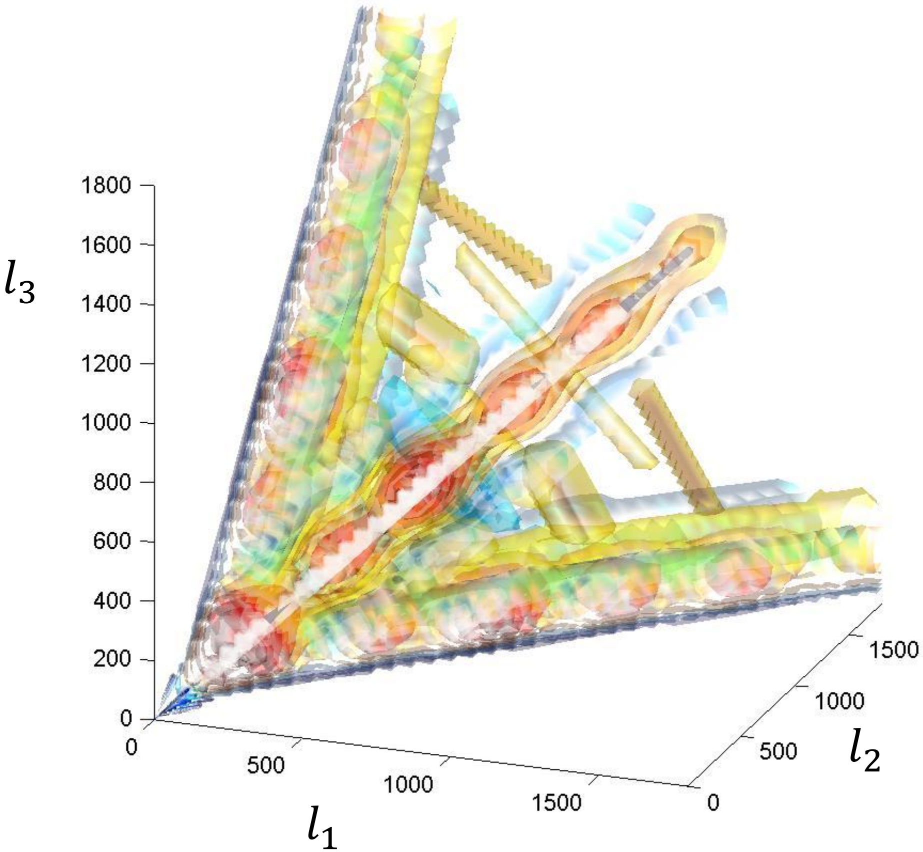
$$l_1^2 l_2^2 b^{TTT}_{l_1 l_2 l_1+l_2}$$

Local f_{NL}

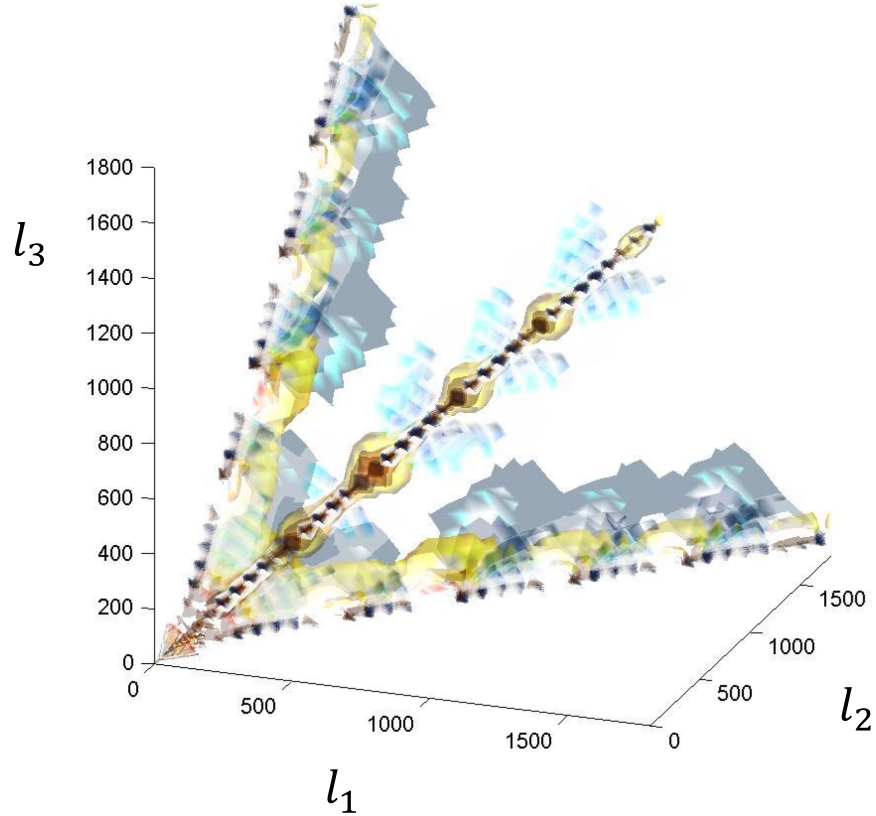




$b_{l_1 l_2 l_3}$



Local f_{NL}



CMB temperature lensing

Lensing bispectrum also squeezed triangles but quite distinctive

Temperature bispectrum correlation with local $f_{NL} \sim 30\%$: in null hypothesis can measu
amplitude using optimized estimator and accurately subtract from f_{NL} estimator

ACTpol 1006.5049
From 2013

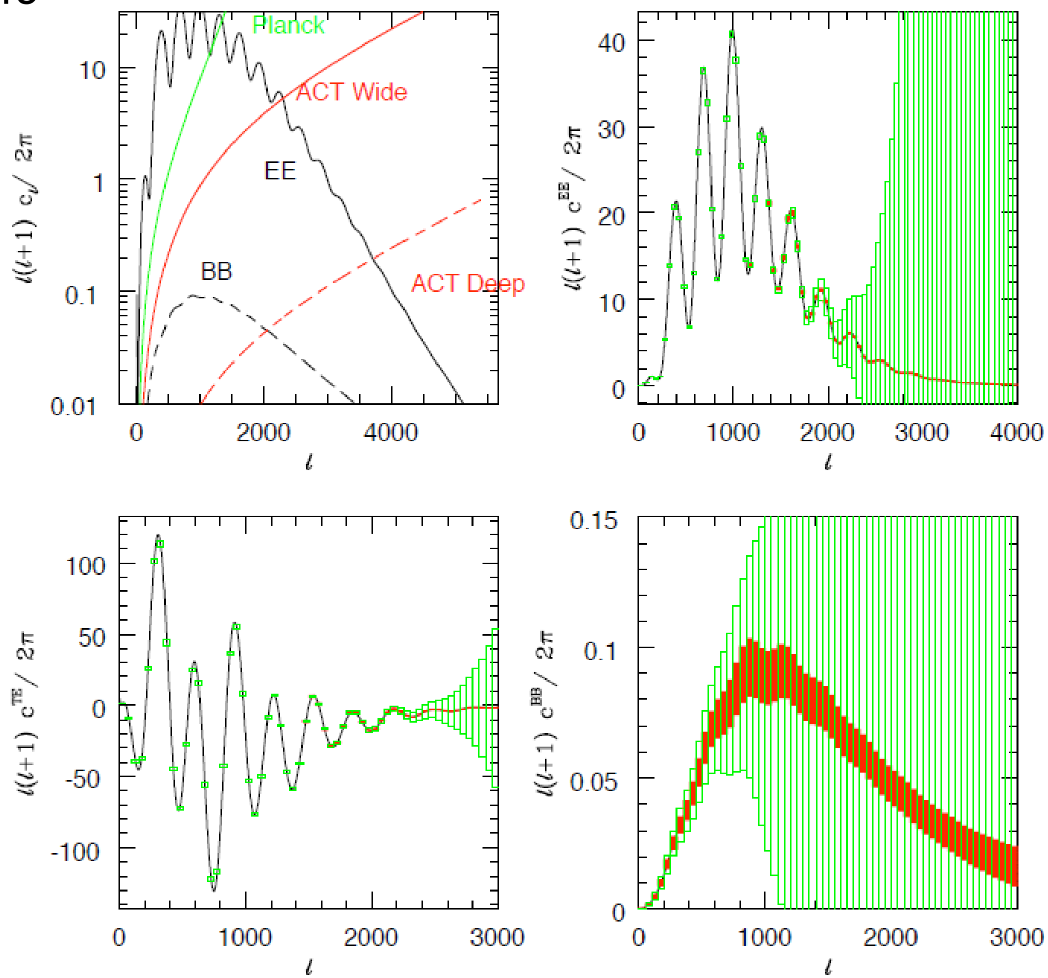


Figure 2. The upper left panel shows the *statistical* noise per multipole for Planck, ACTPol Wide, and ACTPol Deep (§3) and the predicted EE and BB power spectra. The BB spectrum is due to lensing. The other three panels show the EE, BB, and TE spectra with target ACTPol errors (filled red boxes, which are small on a linear scale except in the BB spectrum; Figure 1 shows the EE spectrum on a logarithmic scale) and Planck errors (open green boxes). At low ℓ, when the Planck errors are smaller, only they (open green boxes) are shown. In the BB spectrum the red boxes at $\ell < 850$ are for the ACTPol wide survey (§3), while the red boxes at $\ell > 850$ are for the ACTPol deep survey. The latter three panels are binned with $\Delta\ell = 50$. Atmospheric contamination may preclude ACTPol measurements below $\ell \sim 500$; however, the atmosphere is not polarized at these frequencies, which could enable polarization measurements at lower ℓ than temperature measurements. The y-axes are in μK^2.

Unexpected signals?..

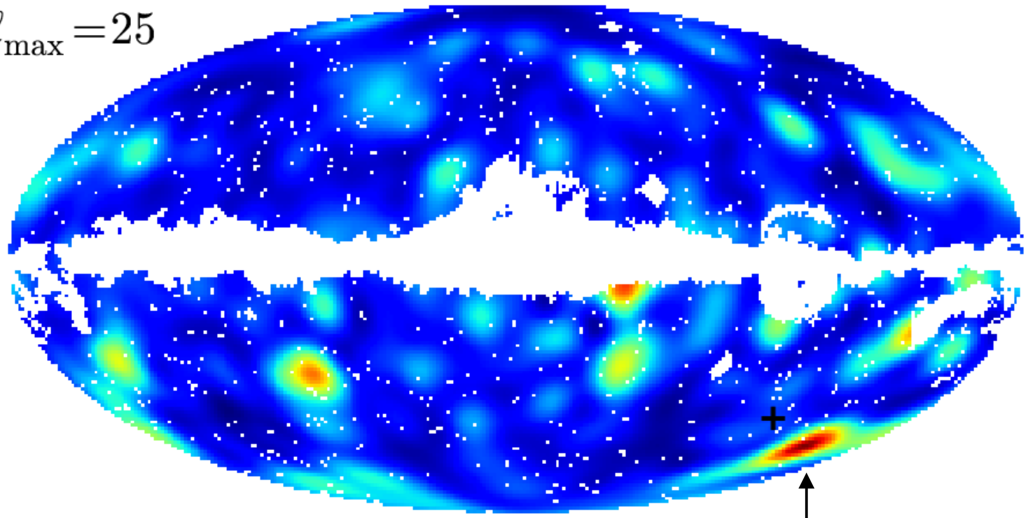
Sky modulation?

Popular modulation model: $\Theta_f(\hat{\mathbf{n}}) = [1 + f(\hat{\mathbf{n}})]\Theta^i_f(\hat{\mathbf{n}})$

QML estimator for f:

$$\tilde{h}^f_{lm} = \int d\Omega Y^*_{lm} \left[\sum_{l_1 m_1}^{l_{\max}} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \left[\sum_{l_2 m_2}^{l_{\max}} C_{l_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$

$\ell_{\max} = 25$

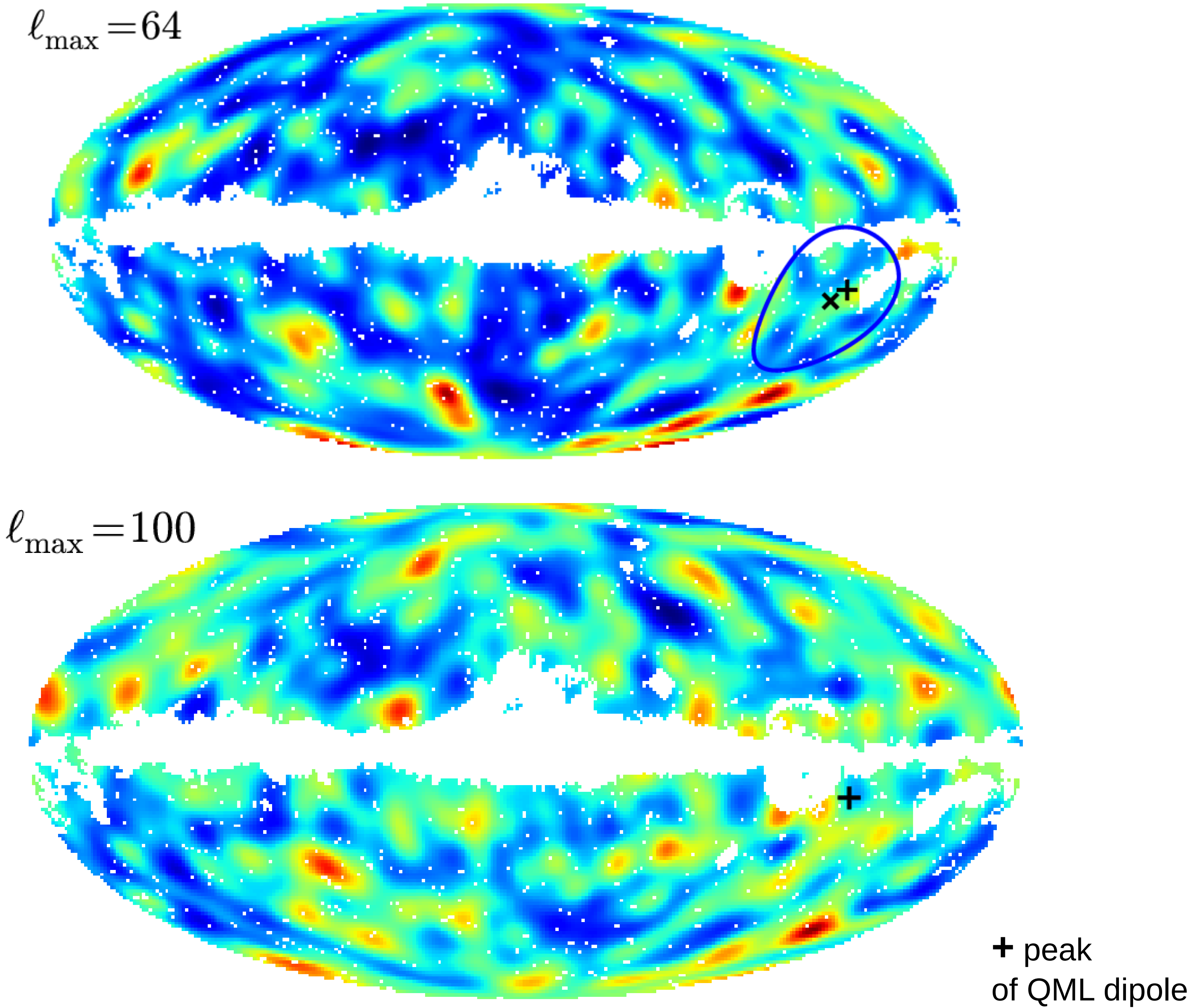


Cold spot?

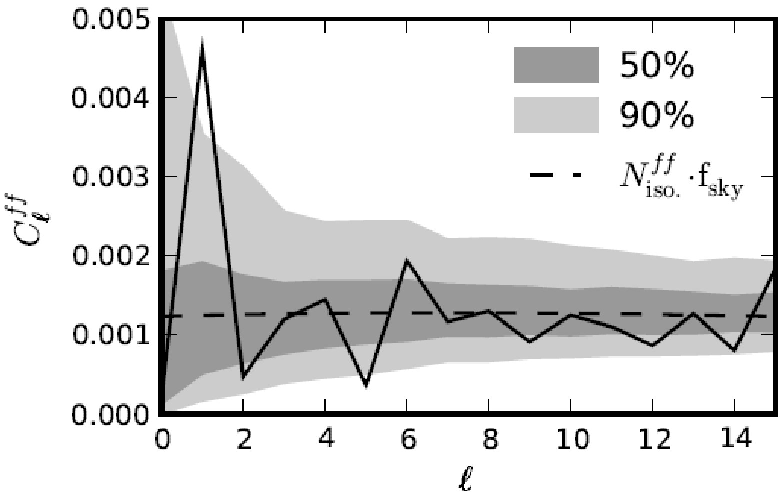
WMAP power reconstruction

(V band, KQ85 mask, foreground cleaned; reconstruction smoothed to 10 degrees)

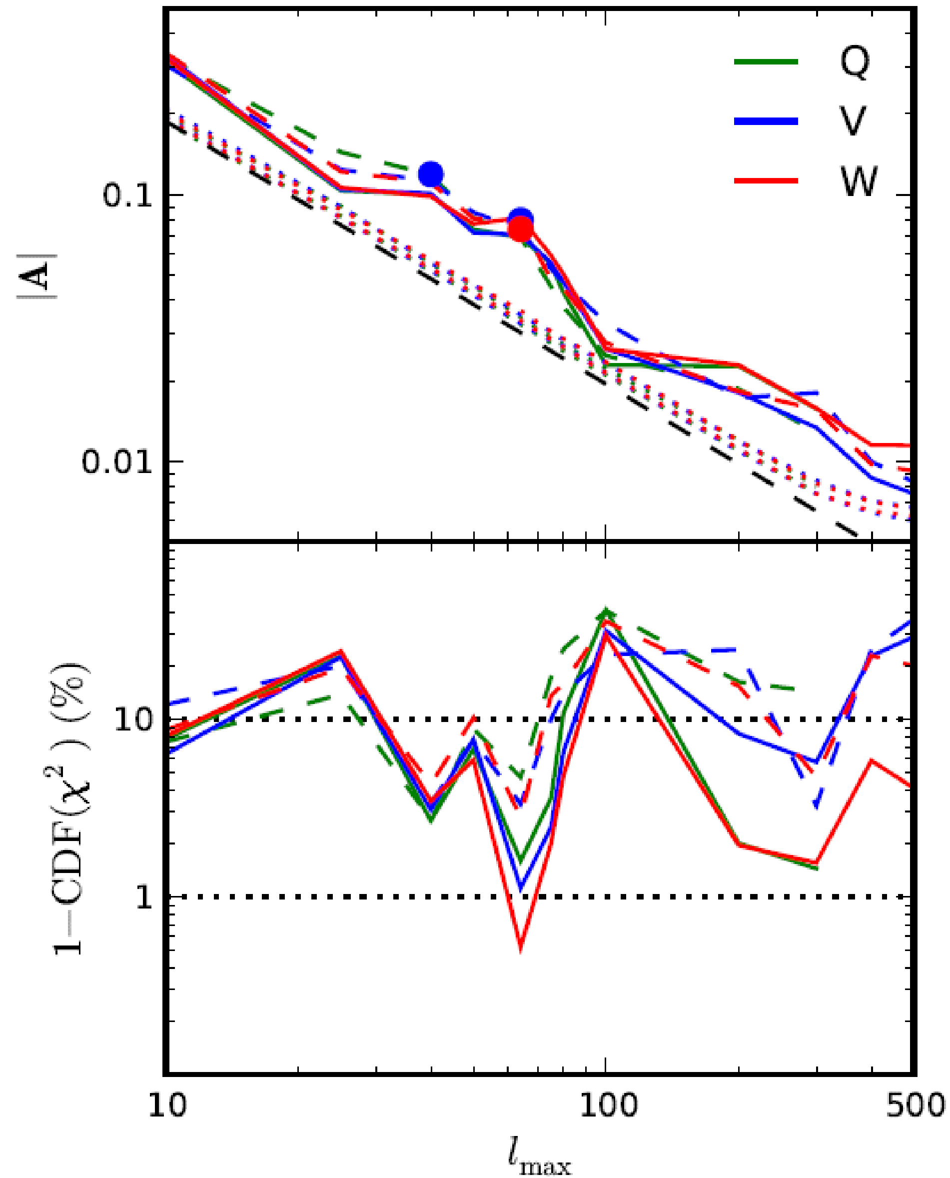
Following Eriksen et al, WMAP, etc..


$\ell_{\rm max} = 64$
$\ell_{\rm max} = 100$
+ peak
of QML dipole

Modulation power spectrum l_{max}=64



Dipole amplitude as function of l_{max}



Only ~1% modulation
allowed on small scales

Consistent with Hirata 2009
- Very small observed anisotropy in
quasar distribution

Unexpected signals?..

Primordial power spectrum anisotropy

Look for direction-dependence in primordial power spectrum:

$$\langle \chi_0(\mathbf{k})\chi_0^*(\mathbf{k}')\rangle \;=\; (2\pi)^3\delta(\mathbf{k}-\mathbf{k}')P_\chi(\mathbf{k})$$

Simple case:

$$\mathcal{P}_\chi(\mathbf{k}) = \mathcal{P}_\chi(k)[1 + a(k)g(\hat{\mathbf{k}})]$$

e.g.
Ackerman et.al. astro-ph/0701357
Gumrukcuoglu et al 0707.4179

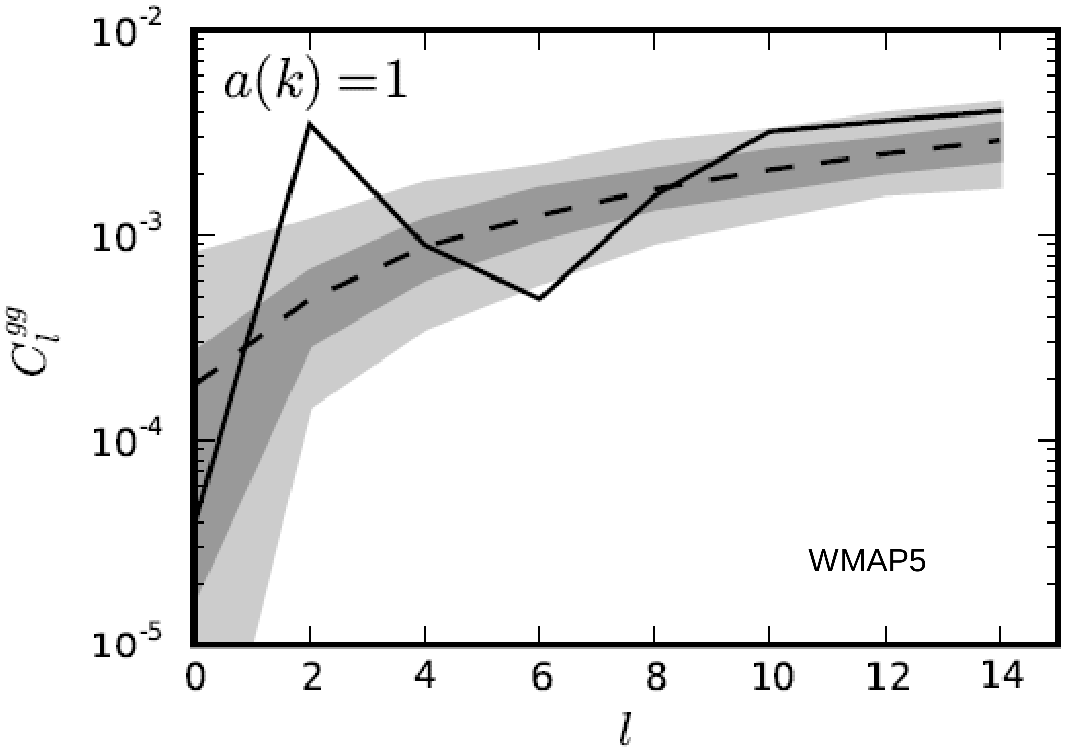
Anisotropic covariance:

$$C_{l_1 m_1 l_2 m_2} = i^{l_1 - l_2}\frac{\pi}{2}\int \mathrm{d}^3\mathbf{k} P_\chi(\mathbf{k})\Delta_{l_1}(k)\Delta_{l_2}(k)Y^*_{l_1 m_1}(\hat{\mathbf{k}})Y_{l_2 m_2}(\hat{\mathbf{k}})$$

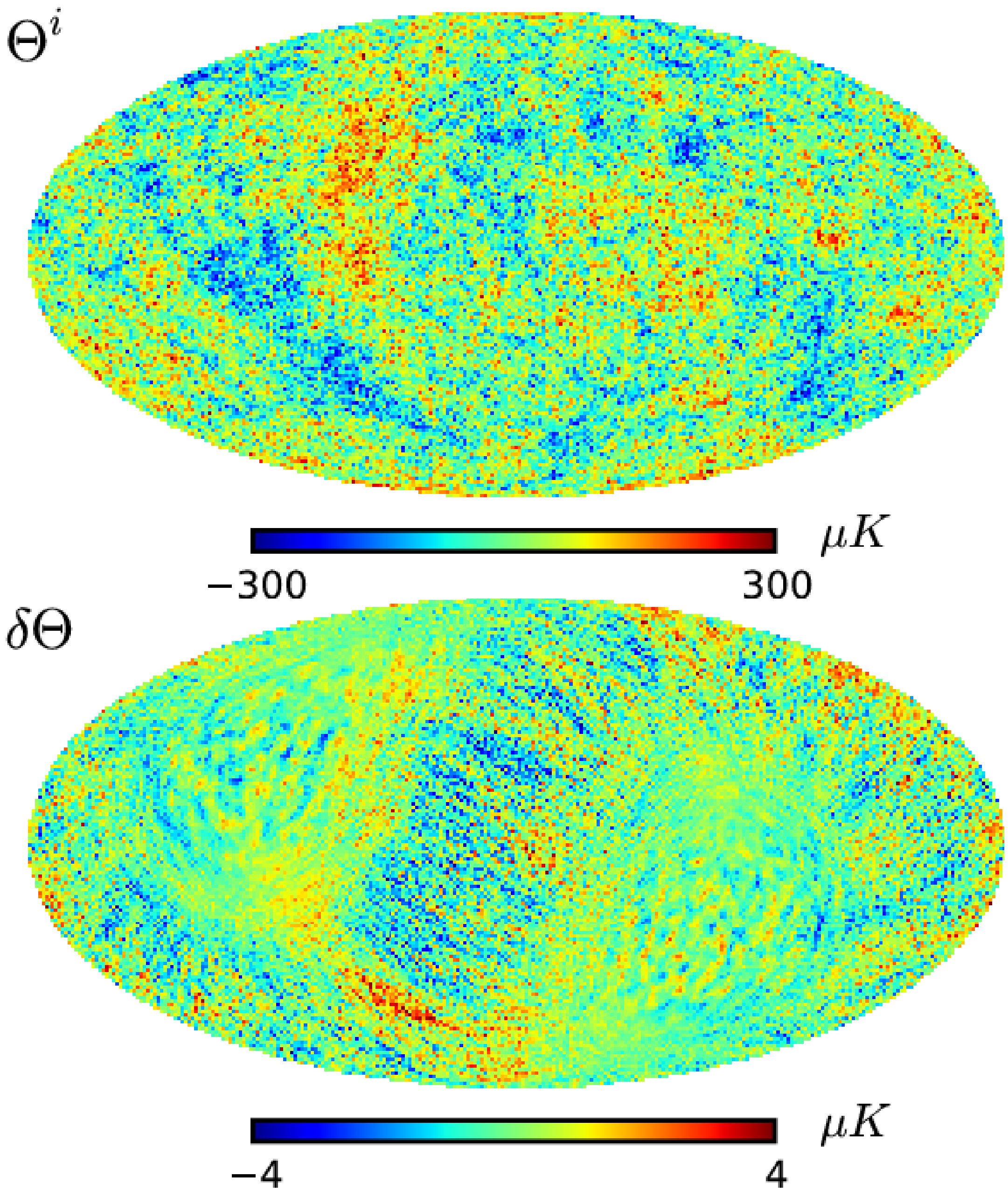
Reconstruct g(k)

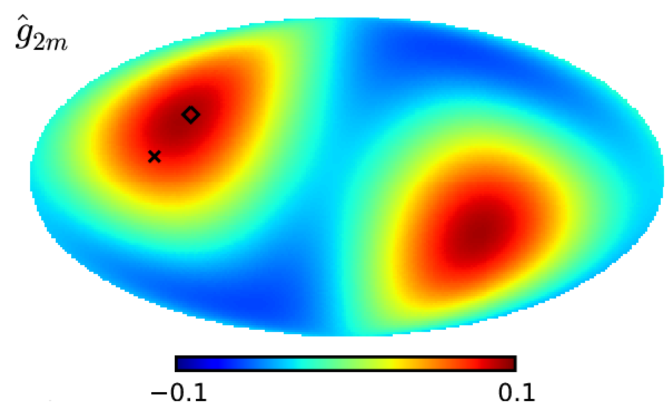
QML estimator:

$$\tilde{h}^g_{lm} = \frac{1}{2}\int \mathrm{d}\Omega Y^*_{lm} \sum_{l_1 l_2} i^{l_1 - l_2} C_{l_1 l_2} \times \left[\sum_{m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right]\left[\sum_{m_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$



Many-sigma quadrupole primordial power anisotropy??





Direction close to ecliptic!
Also varies with frequency and detector.

Could it be systematics? - beam asymmetries? uncorrected in WMAP maps

Check with analytic model of scan strategy

$$\tilde{\Theta}(\Omega_p) = \sum_s w(\Omega_p, -s) \left[\sum_{lm} B_{ls} \Theta_{lm\, s}Y_{lm}(\Omega_p) \right]$$

Scan strategy

Beam shape multipoles

$$w(\Omega_p, -s) = \sum_{i \in p} e^{-is\alpha_i} / H_p$$
$$= v(\Omega_p, s) / v(\Omega_p, 0)$$

$$v(\Omega_p, s) = \sum_{i \in p} e^{is\alpha_i}$$

WMAP model

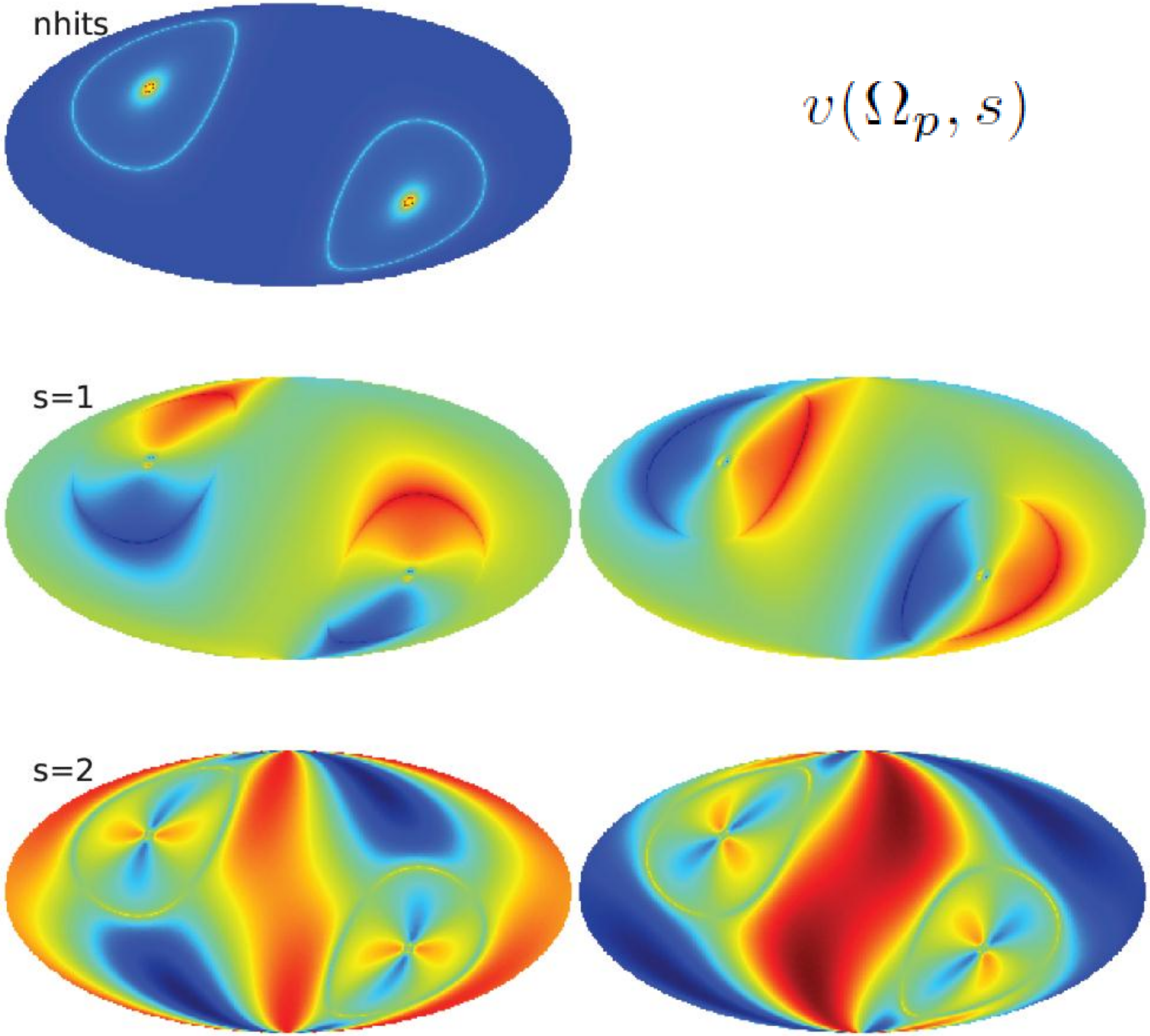
Hirata et al astro-ph/0406004.

(1) a beam at an angle θ_b to the satellite spin axis, which rotates with period τ_s;

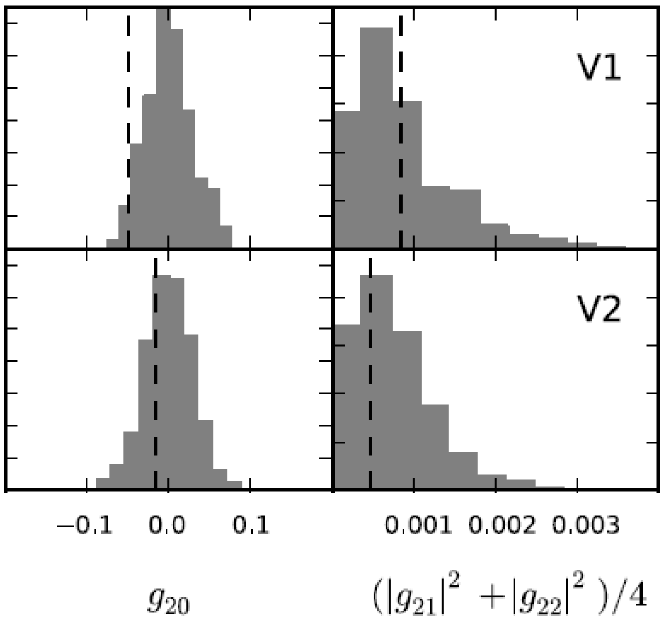
(2) a precession at an angle θ_p to the anti-solar direction, with period τ_p; and

(3) a continuous repointing of the anti-solar direction as the observer orbits the sun.

$$\longrightarrow \quad [v(\Omega_p, s)]_{lm} = \delta_{m0} K P_l(0) P_l(\cos\theta_p)\, {}_sY_{l0}(\theta_b, 0)$$


nhits
$v(\Omega_p, s)$
s=1
s=2

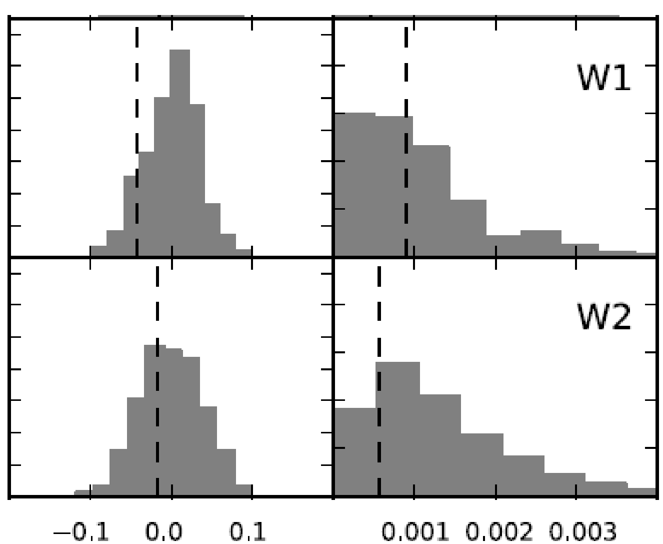
Monte Carlo with subtraction of mean field analytic model of beam asymmetries

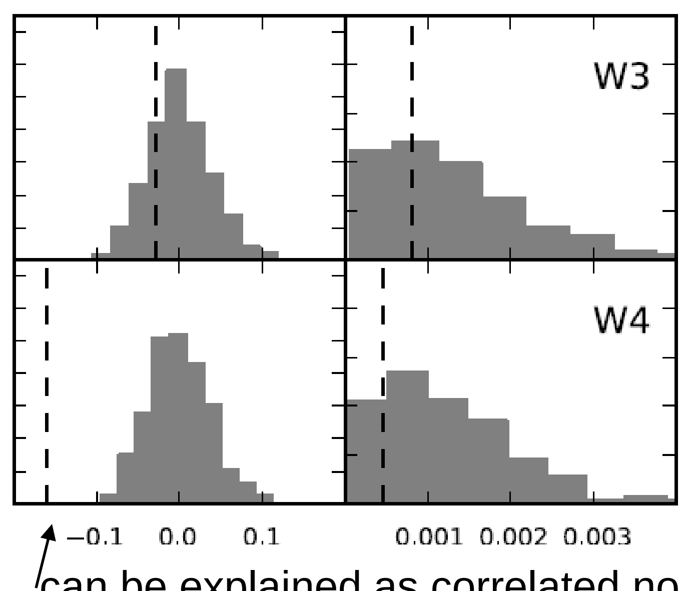


No detection..

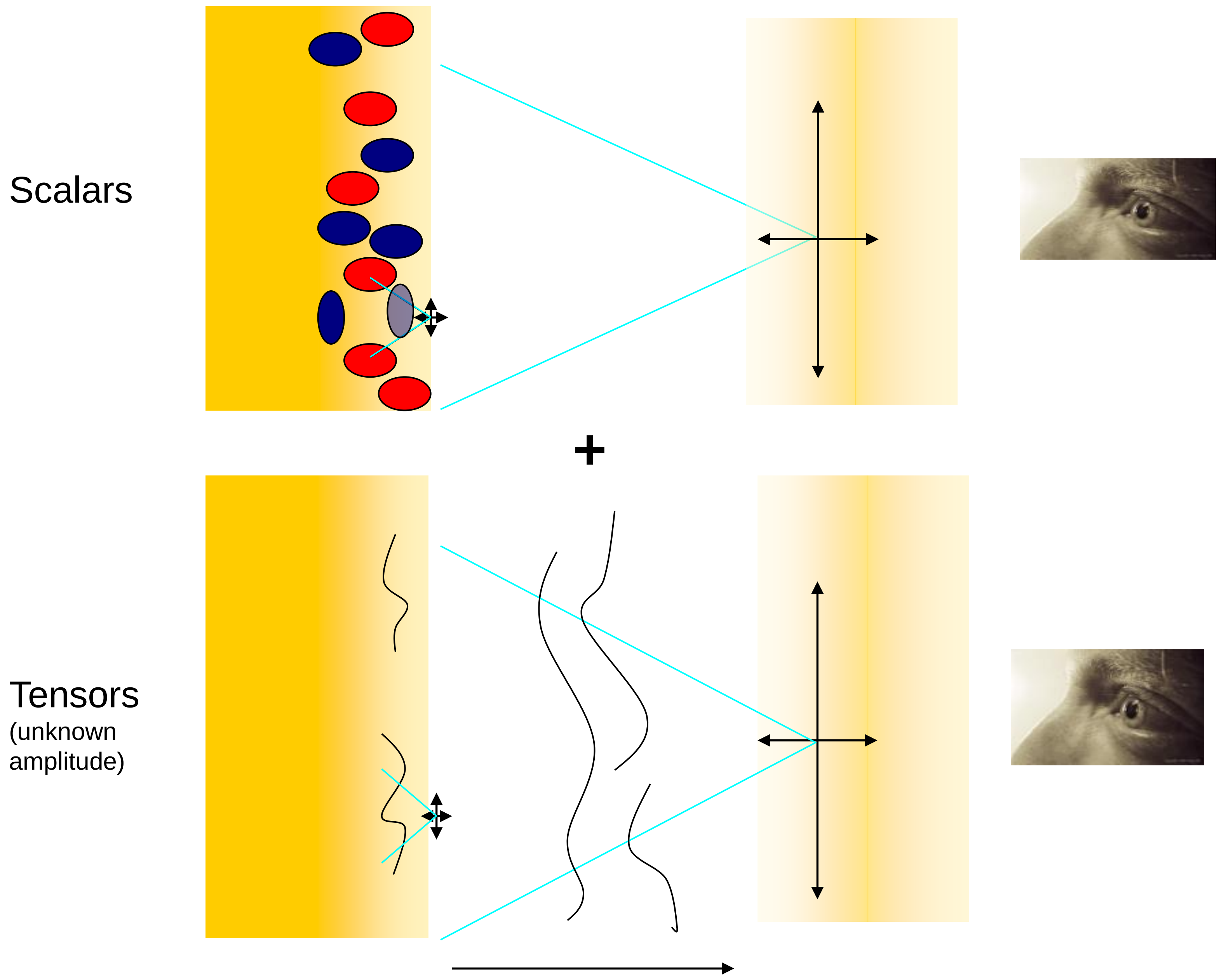
$|g_{2M}| < 0.07$ at 95% confidence.

Consistent with Pullen et al 2010
constraint from large-scale structure 1003.0673





can be explained as correlated noise



Quadrupole generated by anisotropic redshifting of LSS monopole
by gravitational waves along the line of sight

Signal to Noise

Signal quite large, so cosmic variance important as well as noise

$$[F^{(l_1)-1}]_{ij} = [\bar{F}_{l_1}^{\mathrm{lens,lens}\,-1}]_{ij} + \frac{[C_{l_1}^{a^i\psi} C_{l_1}^{a^j\psi} + C_{l_1}^{a^i a^j} C_{l_1}^{\psi\psi}]}{(2l_1+1) C_{l_1}^{a^i\psi} C_{l_1}^{a^j\psi}}$$

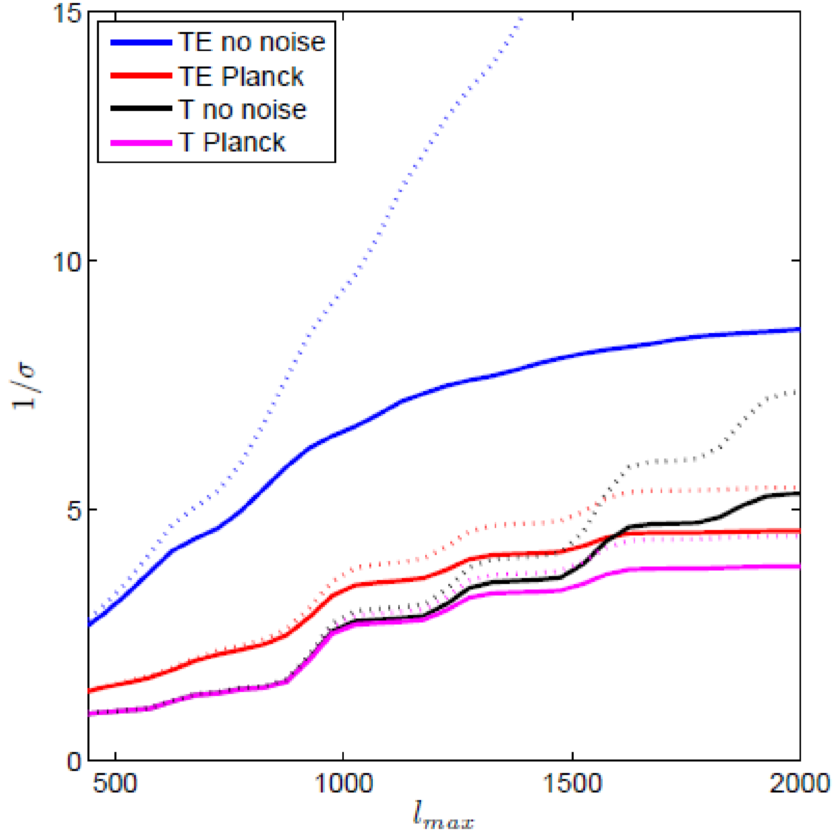
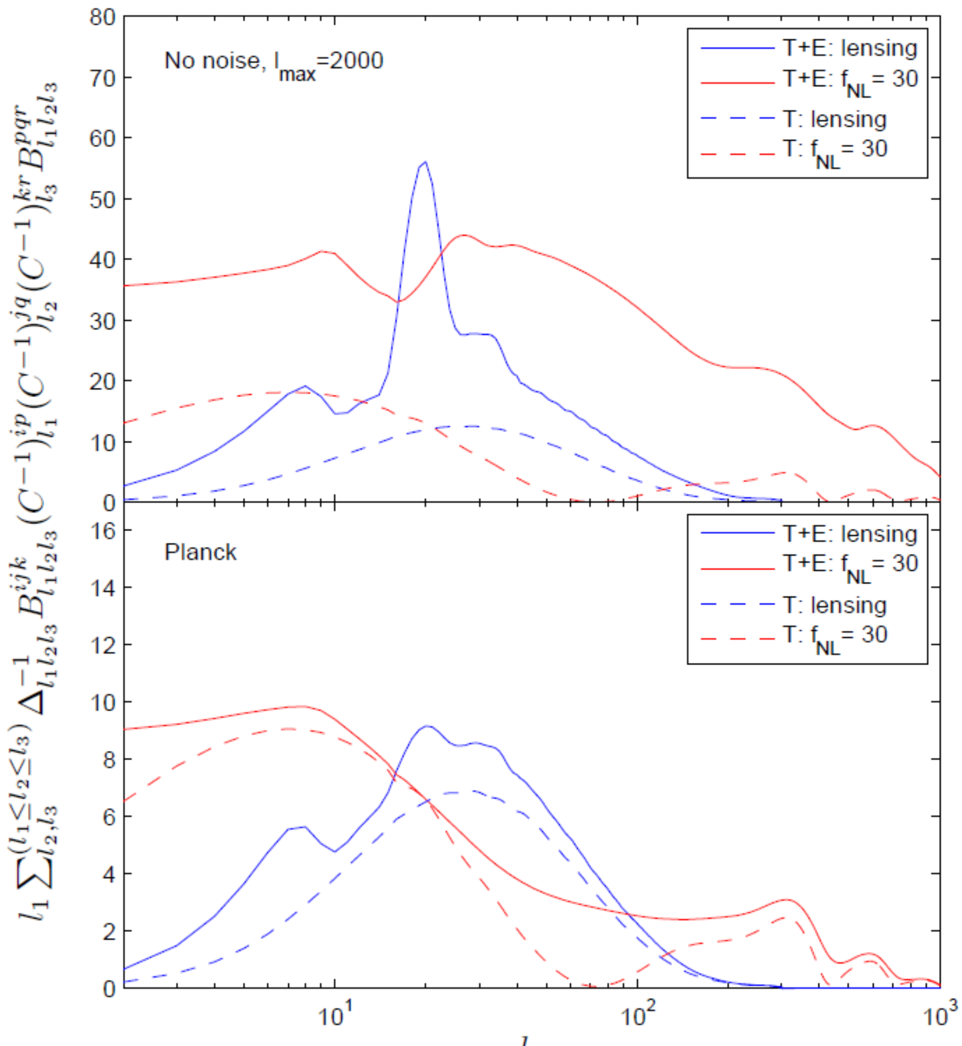


FIG. 10: The Fisher detection significance of the CMB lensing bispectrum as a function of $l_{\max}$ for no noise and Planck, using just the temperature bispectrum or using all the T and E-polarization bispectra. The dotted lines show the (incorrect) results obtained if the signal contribution to the variance is neglected: all results are bounded by the cosmic variance detection limit on a measurement of the low-l cross-correlation spectra $C_l^{T\psi}$ and $C_l^{E\psi}$.

Signal to noise

Contributions to Fisher inverse variance



Lensing signal peaks around $l_1 \sim 30$
- trade-off between size of signal and number of modes

- Cosmic variance limits simply determined by cosmic variance detection limits on $C_l^{T\psi}$ and $C_l^{E\psi}$

Planck $\sim 5\sigma$; Cosmic Variance $\sim 9\sigma$