What is a Quantum Field on a noncommutative space-time?

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An (nontrivial) example: A quantum scalar field on a noncommutative geon space-time

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En homenaje a Manolo Asorey en su cumpleaños

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$\Downarrow$

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Joint work with AP Balachandran, G Marmo, M Martone arXiv:1009.5117

1. What are geons?

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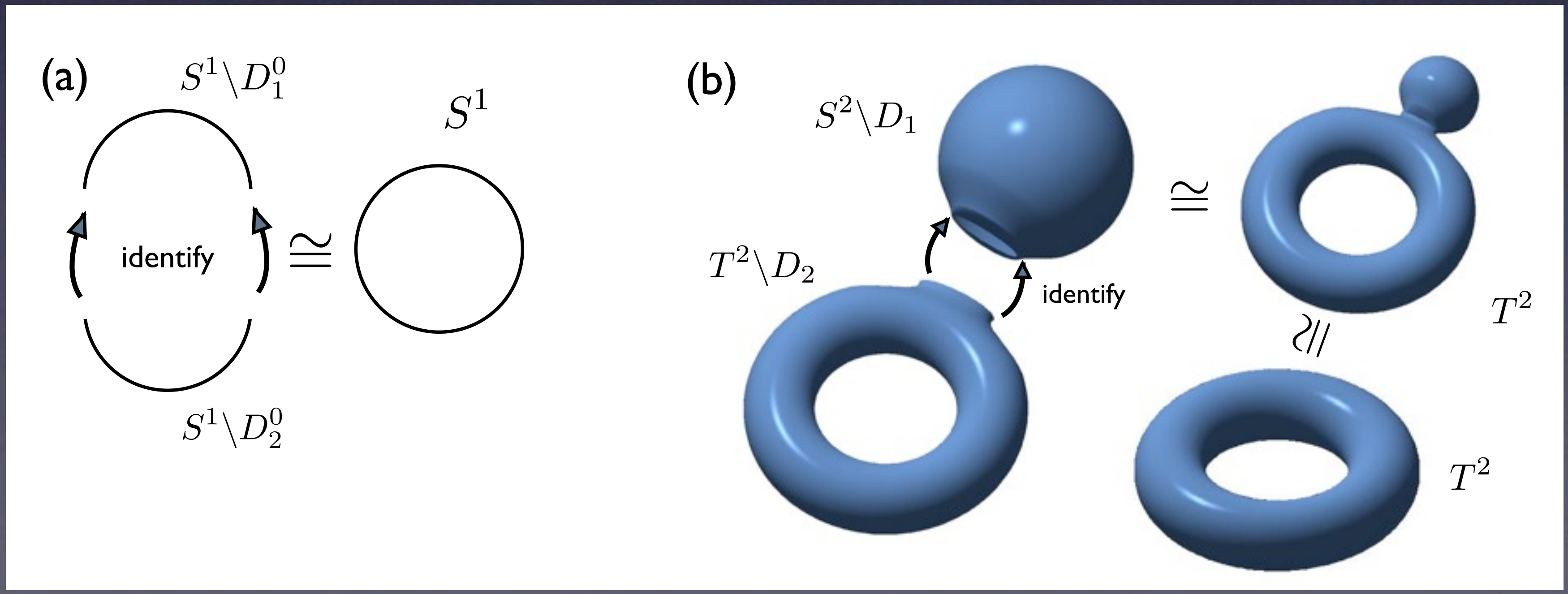

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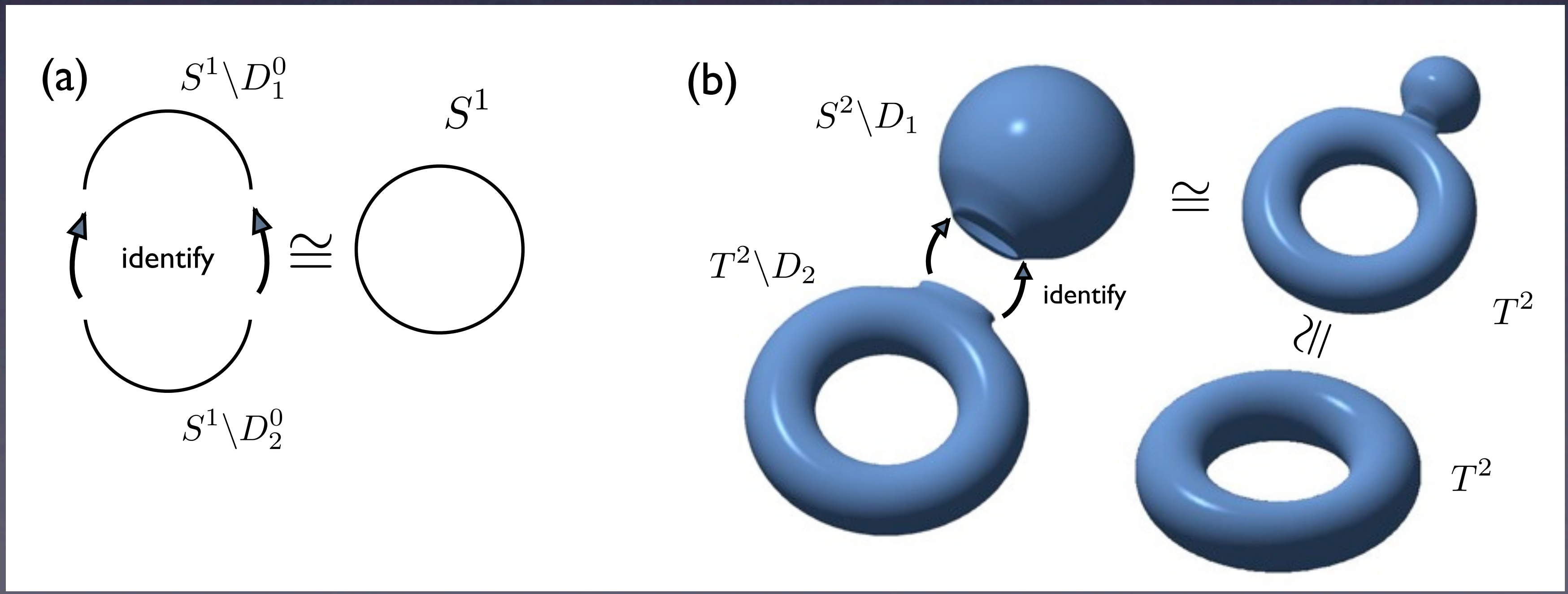

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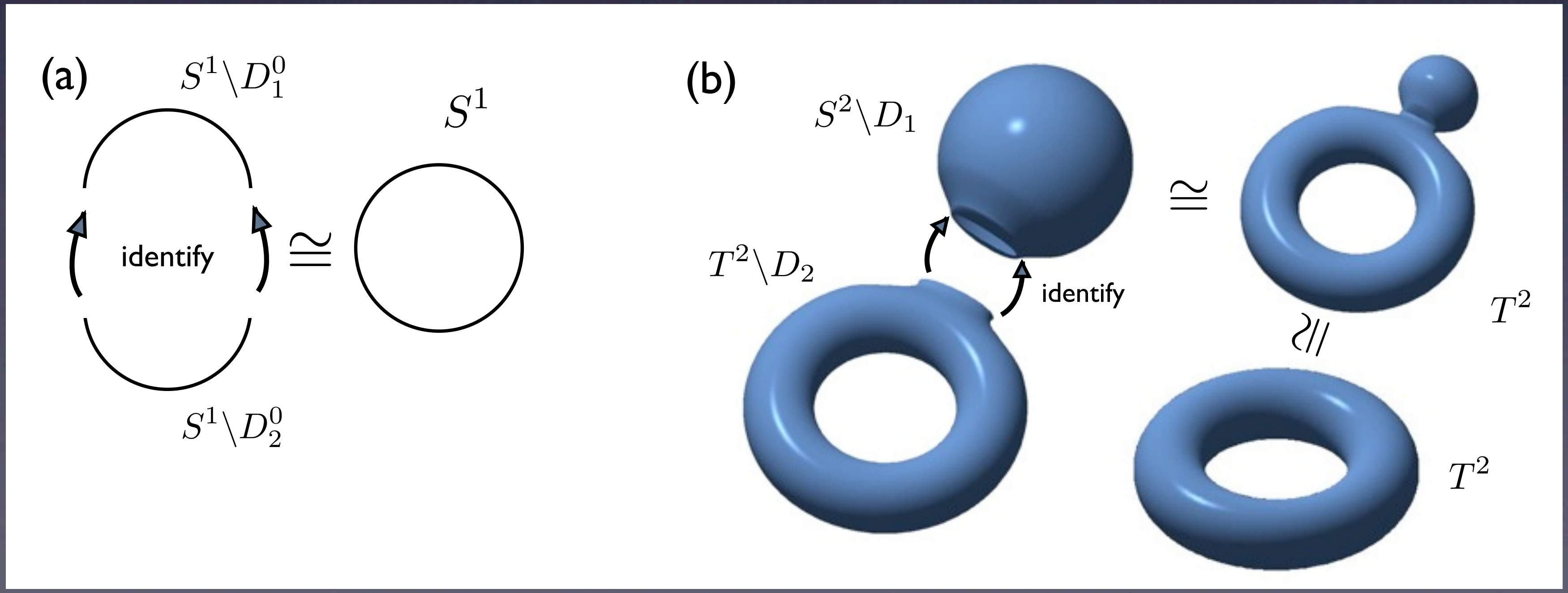
Prime manifolds $\quad \mathcal{M} = \#_\alpha \mathcal{P}_\alpha \bmod S^d$



1.1. Prime manifolds
d = 2

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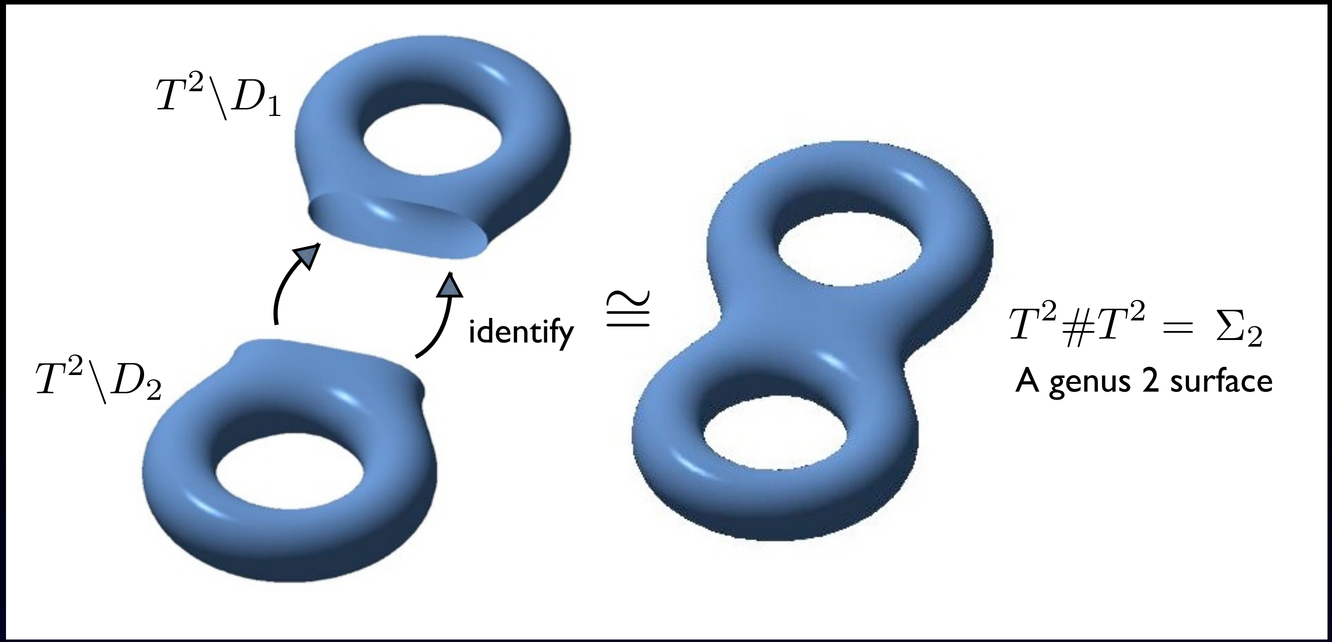

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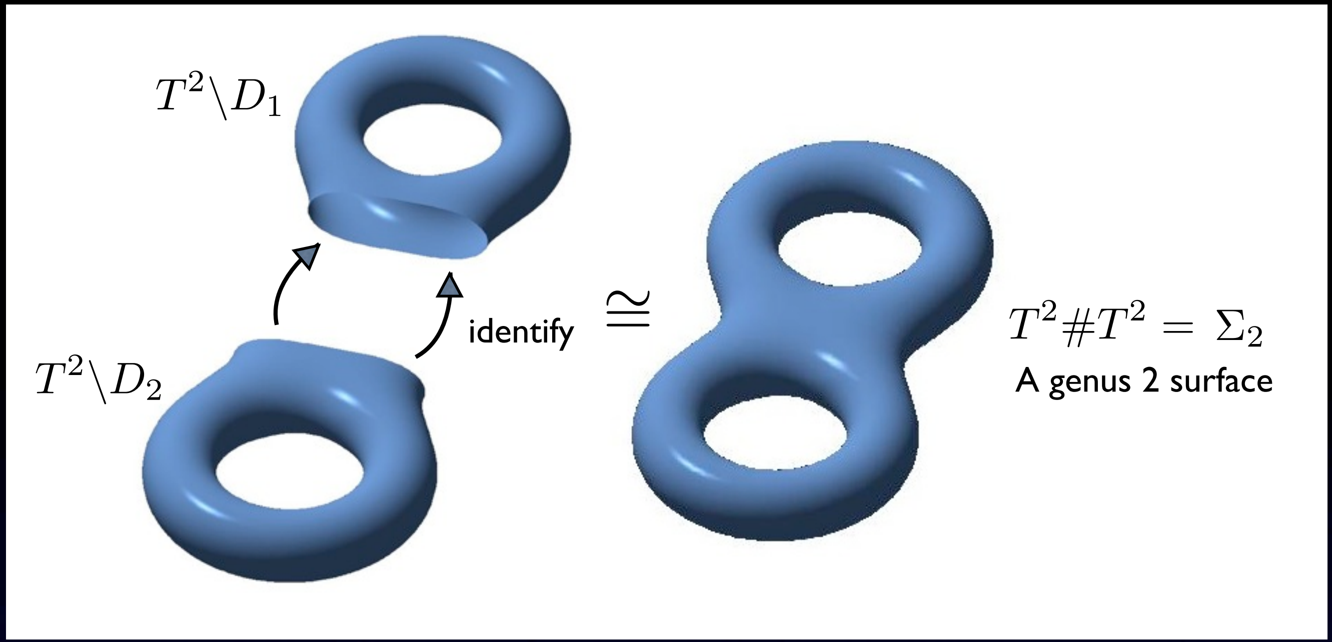

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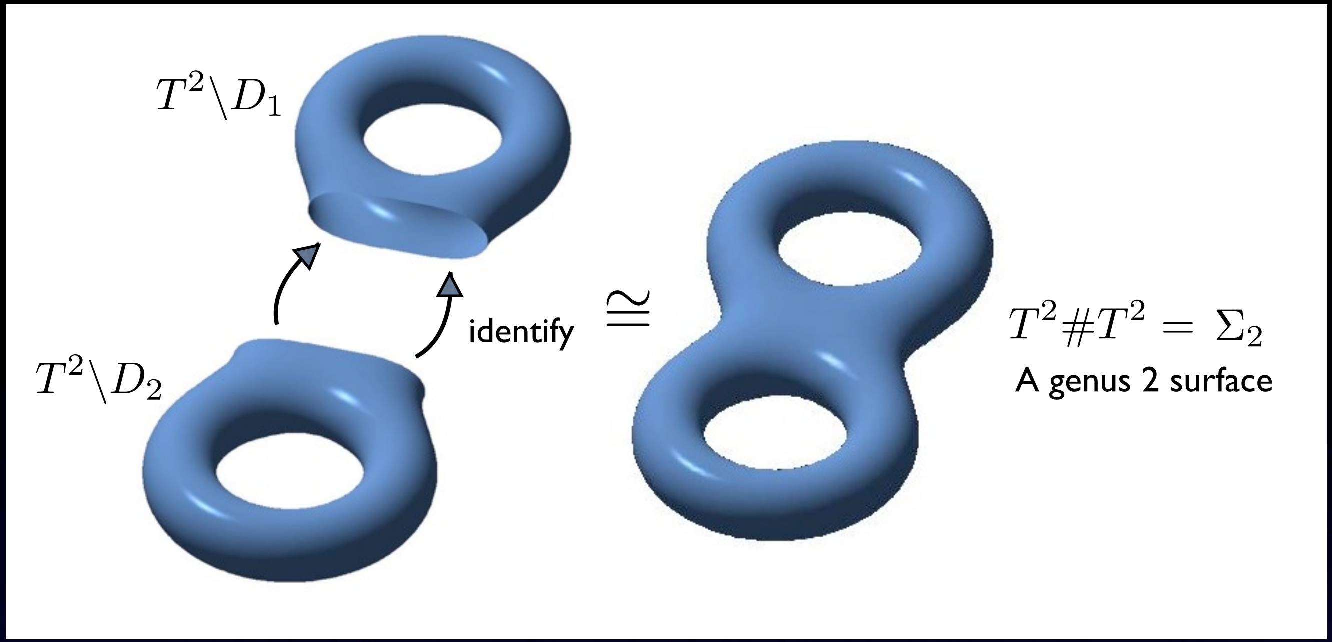

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Spherical Space Forms

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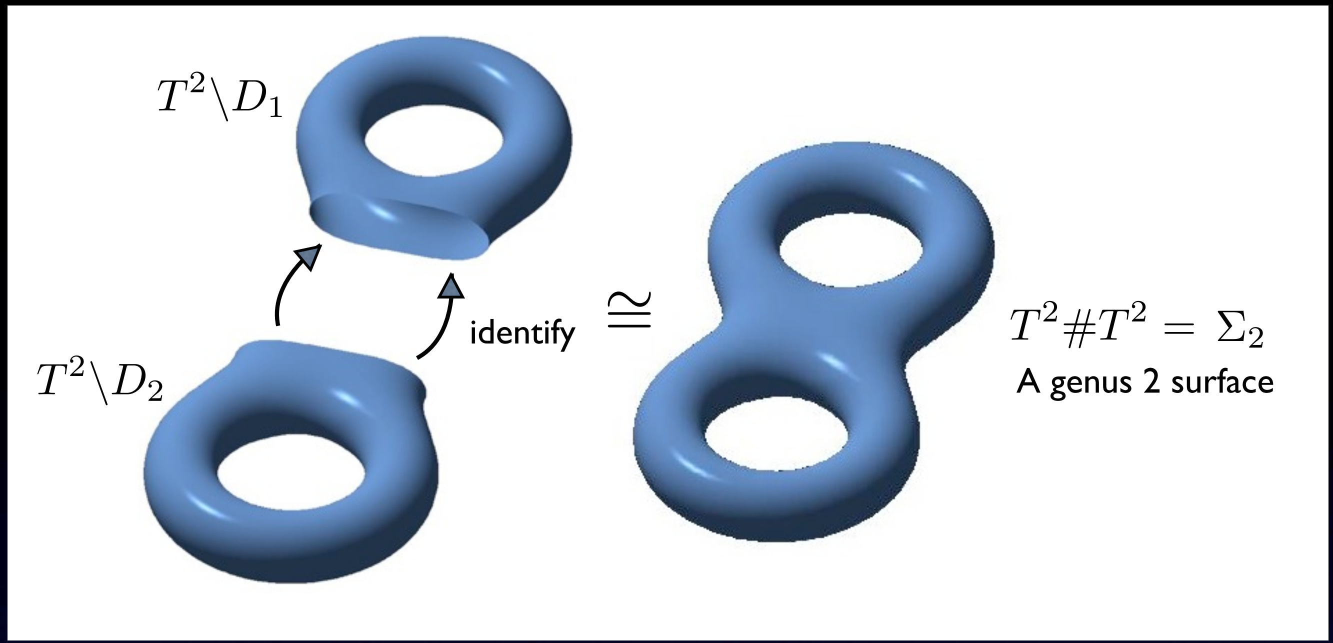

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Spherical Space Forms

$$S^3/D, \quad D \subset SO(4), \quad D \cong \mathbb{Z}_p \times \mathbb{Z}_q \quad L_{p,q} \qquad L_{1,2} \cong L_{2,1} \cong \mathbb{R}P^3$$

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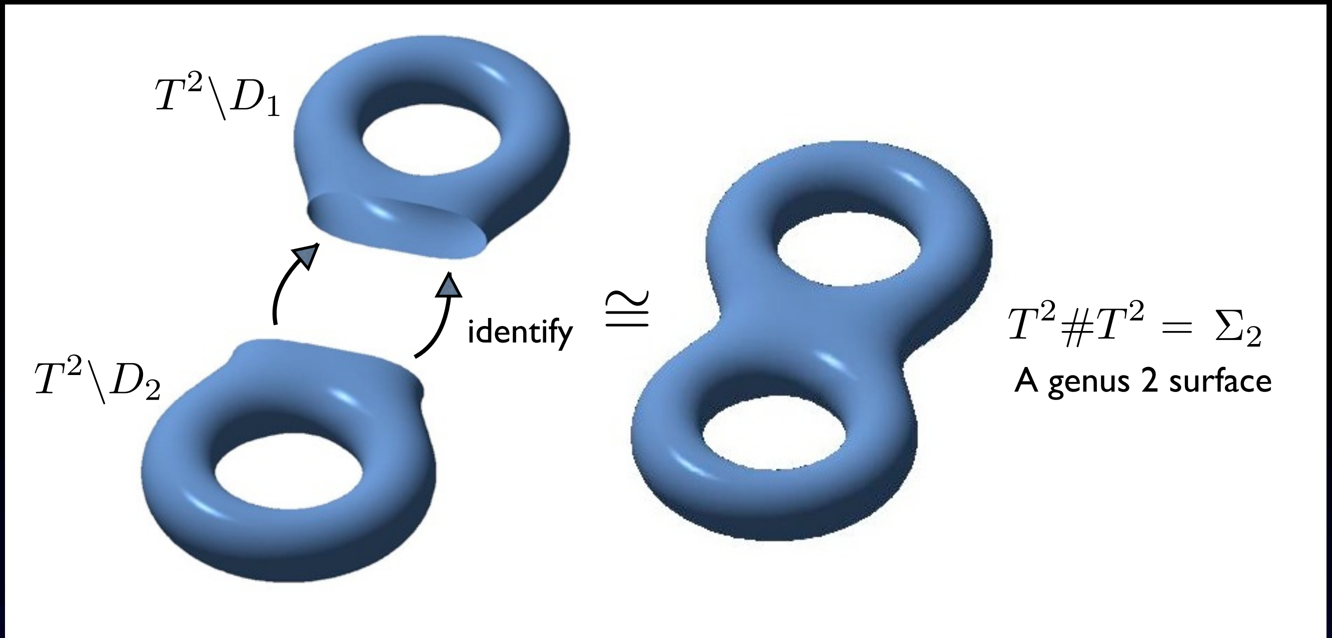

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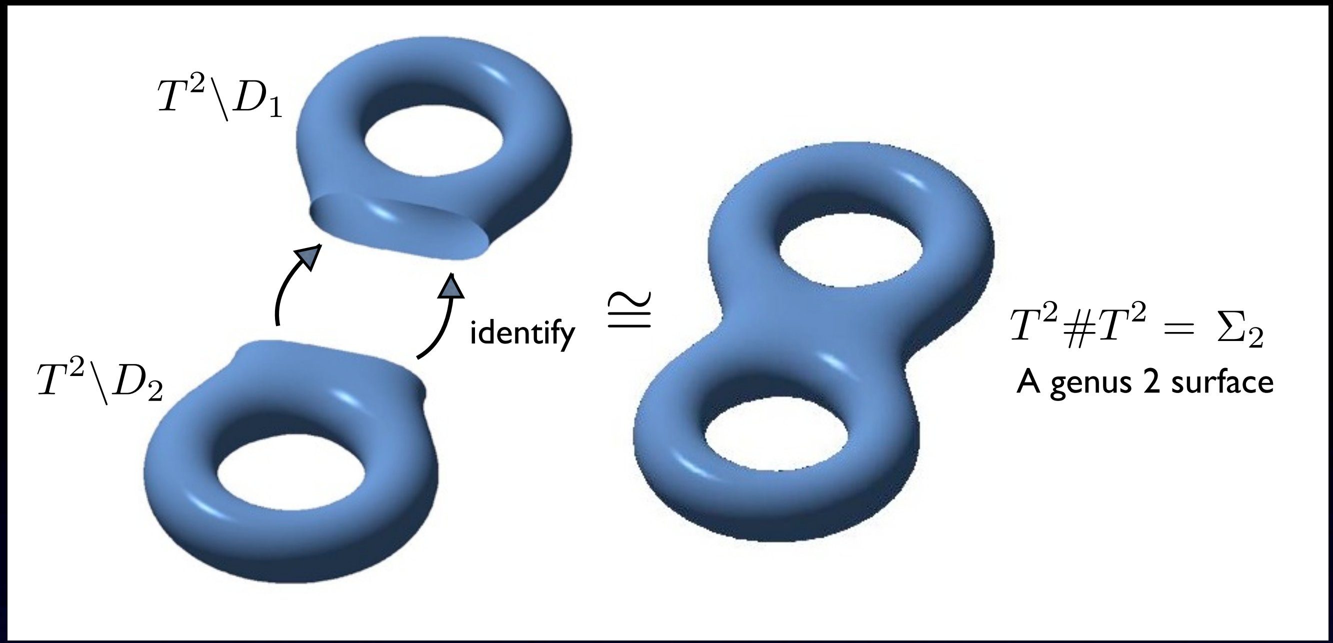

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$$\mathcal{H}^+ = \{x = (x_0, \vec{x}) \in \mathbb{R} \times \mathbb{R}^3 \cong \mathbb{R}^4 : (x_0)^2 - (\vec{x})^2 = 1, x_0 > 0\}$$

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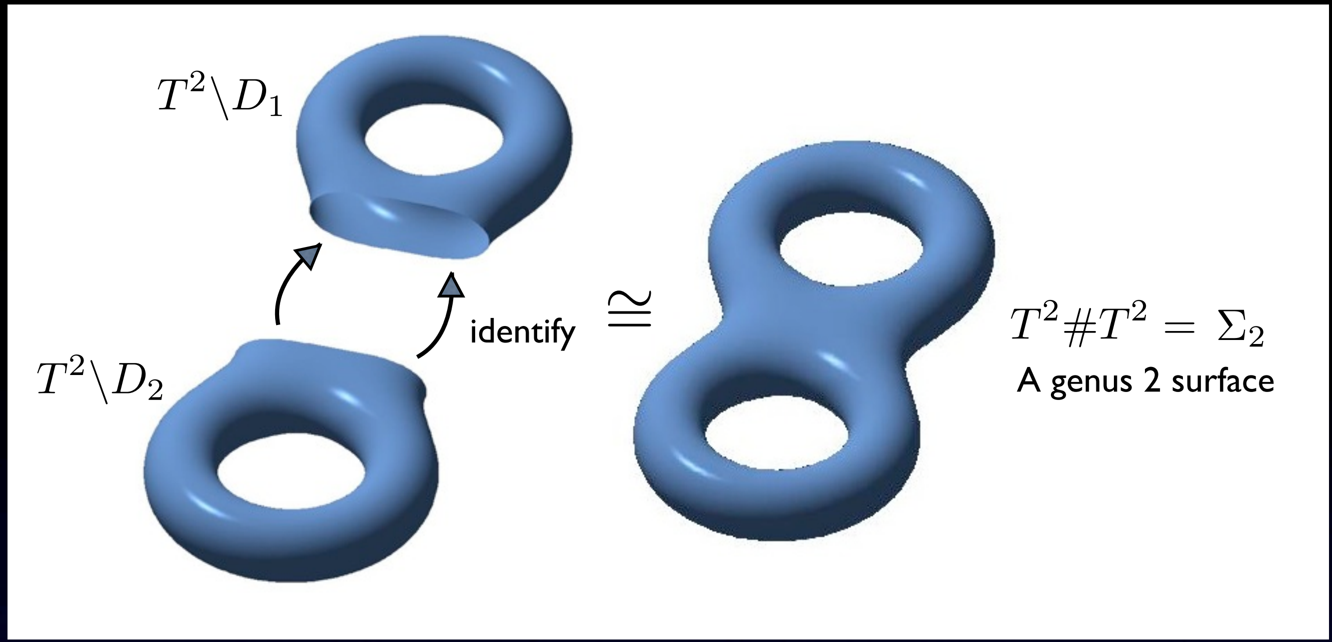

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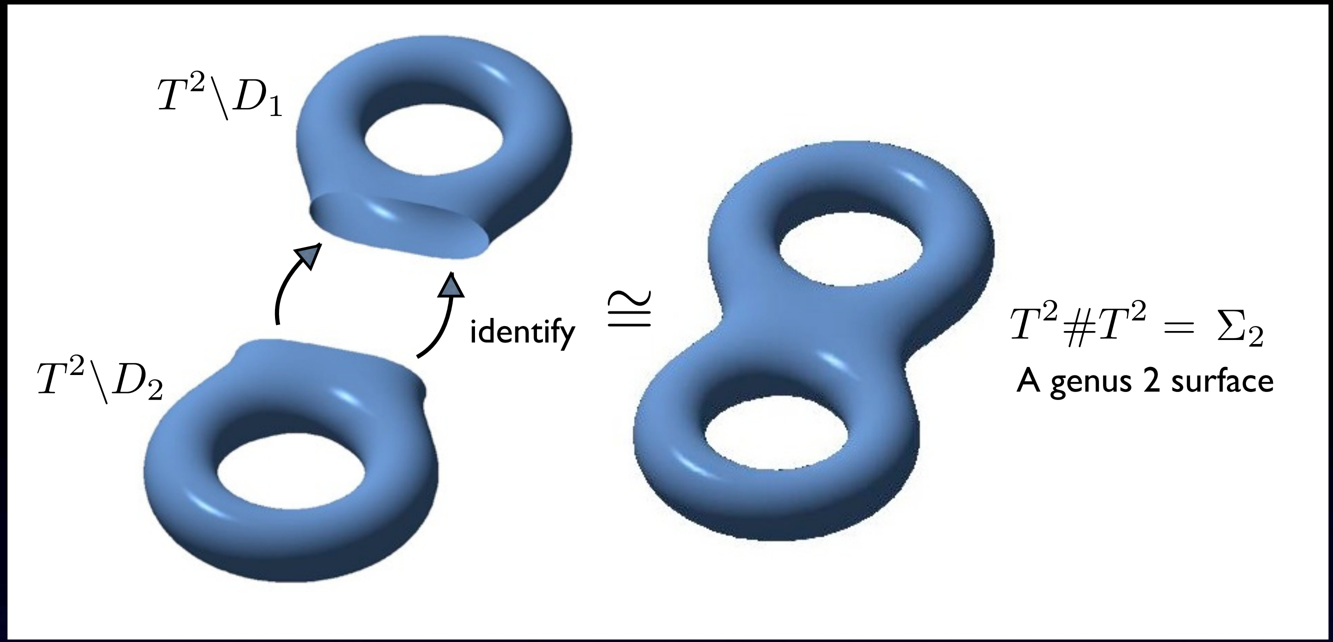

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Manifolds with one asymptotic region $\qquad \mathcal{M}_\infty = \mathbb{R}^d \#_\alpha \mathcal{P}_\alpha$

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Space-time topology $\mathcal{M}_\infty \times \mathbb{R}$ $\quad D^\infty \subset \mathrm{Diff}^\infty(\mathcal{M}_\infty)$

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Quantization of non-simply connected configuration spaces

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$$\mathcal{H}, \quad \rho\colon \pi_1(\mathcal{Q}) \to \mathcal{U}(\mathcal{H}) \qquad \mathcal{H} \cong \bigoplus_{l \in \widehat{\pi_1(\mathcal{Q})}} \mathcal{H}_l$$

$$\mathcal{E} = \widetilde{\mathcal{Q}} \times \mathcal{H}/\pi_1(\mathcal{Q}) = \widetilde{\mathcal{Q}} \times_{\pi_1(\mathcal{Q})} \mathcal{H}$$

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$$\mathcal{F} = \Gamma(\mathcal{E}) = \bigoplus_l \mathcal{F}_l$$ Fock space

$$\mathcal{F}_l = \Gamma(\mathcal{E}_l); \quad \mathcal{E}_l = \widetilde{Q} \times_{\pi_1} \mathcal{H}_l$$

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$$\Psi \in \mathcal{F}_l$$

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Quantum geons are state vectors of a quantum gravity theory over a topologically non-trivial space-time, selected by the UIRR's of

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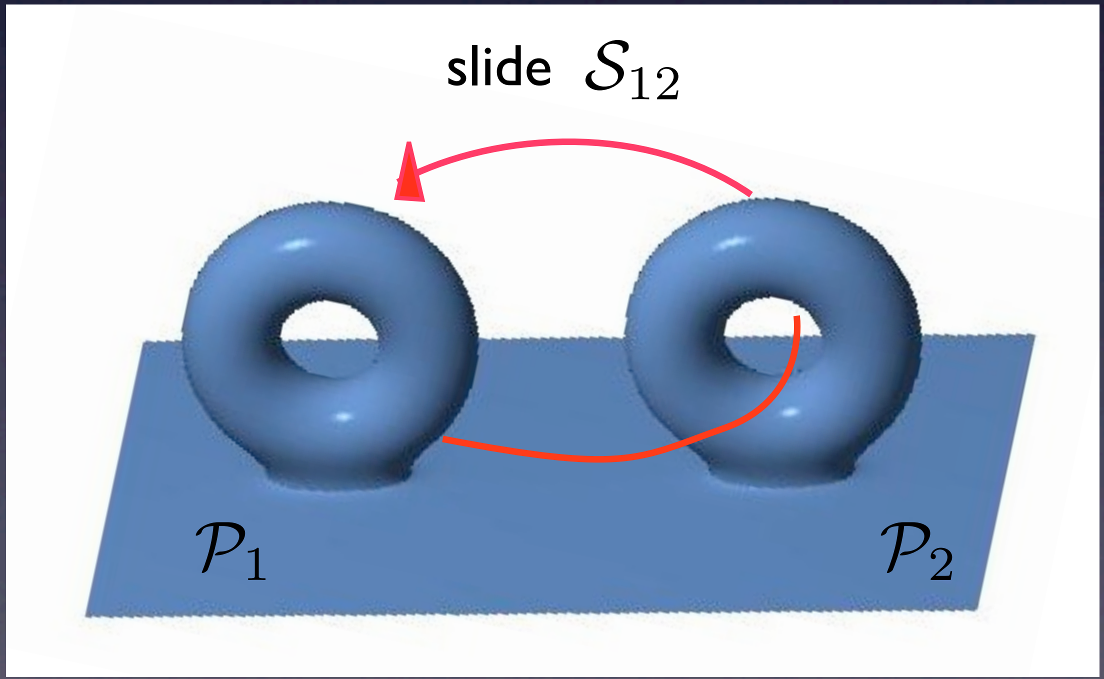

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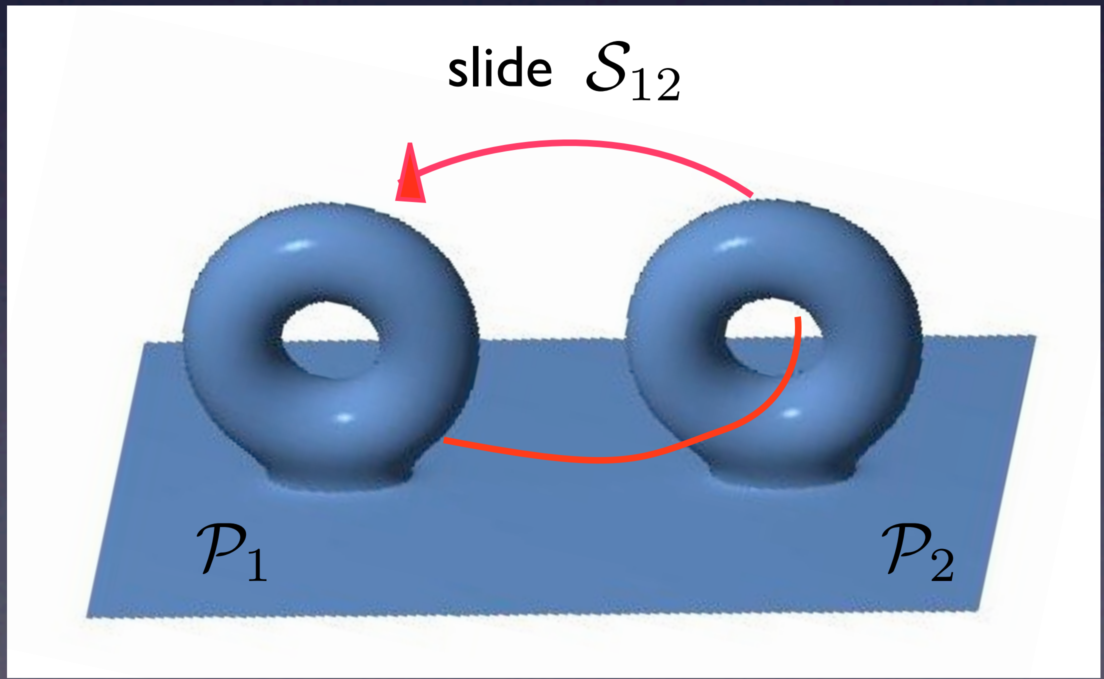

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jueves 6 de octubre de 2011

II. Twists of geon spacetimes

II.1 Drinfel'd twist

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Poincaré group algebra

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$$f \star g = f \mathrm{e}^{-\frac{i}{2}\overleftarrow{\mathcal{P}_\mu}\theta^{\mu\nu}\overrightarrow{\mathcal{P}}_\nu} g, \quad f, g \in \mathcal{A}_\theta(\mathbb{R}^d)$$

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We will twist geon spacetimes by using $D^{(1)\infty}/D_0^{(1)\infty}$ which is consistent with DFR argument

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Geon quantum field

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$$[g]\varphi_0 = U([g])\varphi_0 U([g])^\dagger, \quad [g] \in D^{(1)}/D_0^{(1)\infty}$$

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II.4 Twisted covariant geons

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$$F_\sigma = \sum_{\varrho,\varrho'} \sigma(\varrho, \varrho') \mathbb{P}_\varrho \otimes \mathbb{P}_{\varrho'}, \quad \sigma(\varrho, \varrho') \in \mathbb{C}$$

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II.5 Twisting nonabelian discrete groups

Abelian twists imply associative spacetimes

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$$(\varphi_\theta \star \varphi_\theta) \star \varphi_\theta \neq \varphi_\theta \star (\varphi_\theta \star \varphi_\theta)$$

jueves 6 de octubre de 2011

That's all for today ... congratulations Manolo!