Linear Perturbation Theory in LTB Models

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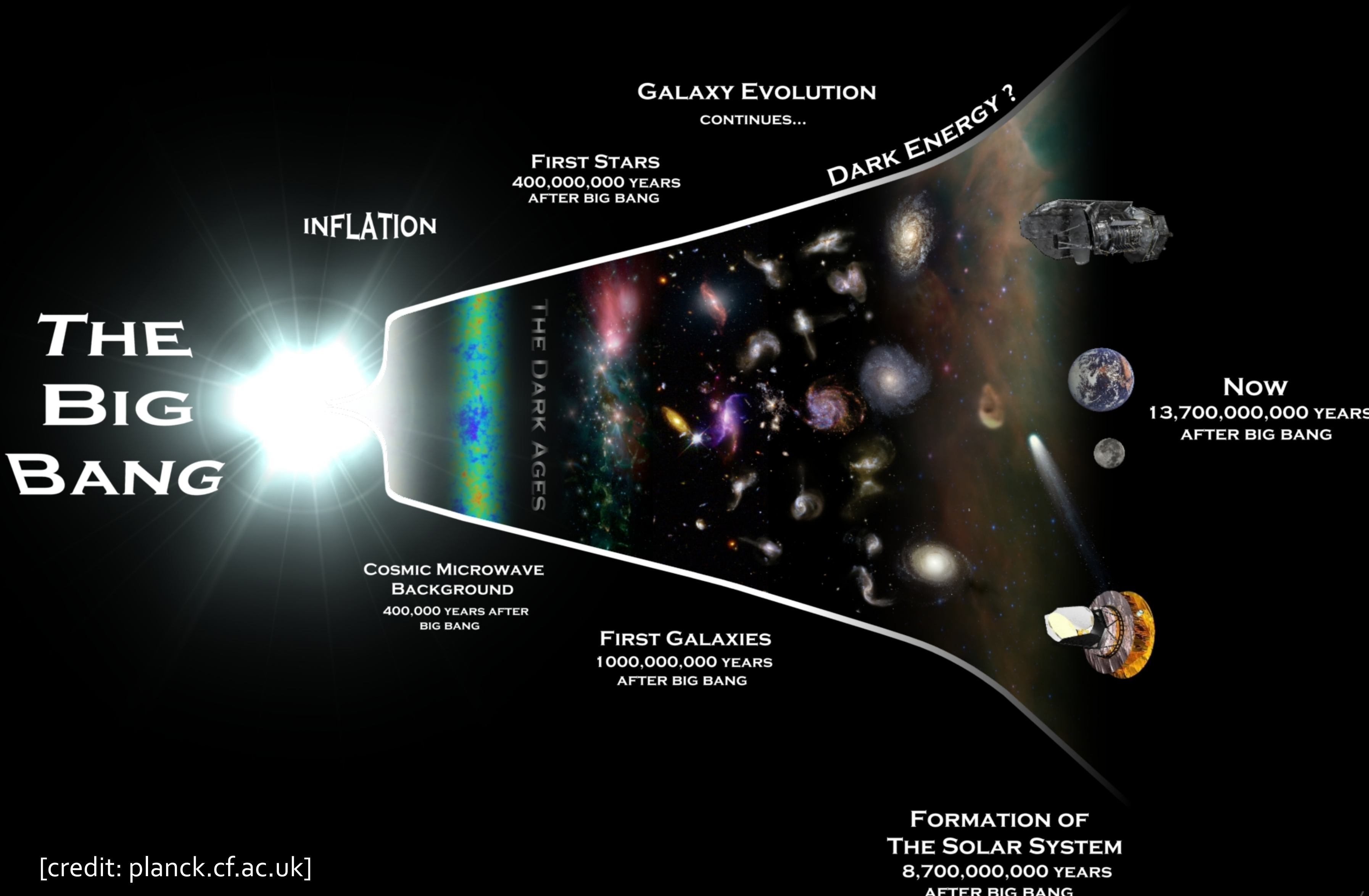
Outline

- Introduction
  - Review of the Concordance Model
  - The LTB Model
- Perturbation Theory in LTB
- Preliminary Results
- Summary & Outlook

Introduction

Review of the Concordance Model

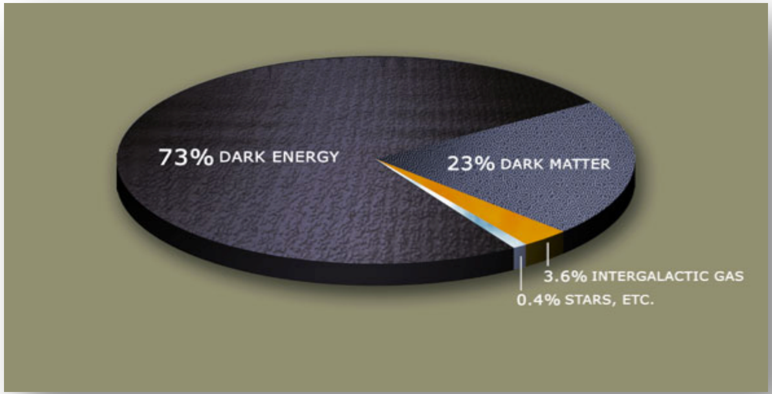


[credit: planck.cf.ac.uk]

Introduction
Review of the Concordance Model

- The concordance model assumes:
  - General Relativity + CDM + Λ
  - The Copernican Principle
  - Isotropy about the Milky Way

→ Friedmann-Lemaitre-Robertson-Walker geometry



- Caveat: no satisfactory theoretical understanding of Λ (or DE) exists….
- In the mean time, worthwhile to question above assumptions…

Introduction
Review of the Concordance Model

- We will assume:
  - General Relativity + CDM + Λ
  ~~- The Copernican Principle~~
  - Isotropy about the Milky Way

→ Lemaitre-Tolman-Bondi (LTB) spacetime

Introduction
The LTB Model

- General line element:

$$ds^2 = -dt^2 + X^2(t,r)dr^2 + A^2(t,r)\left[d\theta^2 + \sin^2\theta\, d\phi^2\right]$$

where $X(t,r) = f(r)\partial_r A(t,r)$.

- Setting $a_\perp(t,r) \equiv A(t,r)/r$, $a_\|(t,r) \equiv \partial_r A(t,r)$, and $f(r) \equiv 1/\sqrt{1-\kappa(r)r^2}$ we can instead write:

$$ds^2 = -dt^2 + \frac{a_\|^2(t,r)}{1-\kappa(r)r^2}dr^2 + a_\perp^2(t,r)r^2 d\Omega^2$$

- Anisotropic expansion:

$$H_\perp(t,r) = \frac{\dot{a}_\perp(t,r)}{a_\perp(t,r)} \quad \text{and} \quad H_\|(t,r) = \frac{\dot{a}_\|(t,r)}{a_\|(t,r)}$$

Introduction
The LTB Model

- Analogue of Friedmann equation:

$$H_\perp^2(t,r) = H_{\perp_0}^2(r)\left[\frac{\Omega_m(r)}{a_\perp^3(t,r)} + \frac{1-\Omega_m(r)}{a_\perp^2(t,r)}\right]$$

  where we set $a_{\perp_0}(r) = 1$.

- The age of the universe is then:

$$t - t_B(r) = \frac{1}{H_{\perp_0}(r)} \int_0^{a_\perp(t,r)} \frac{dx}{\sqrt{\Omega_m(r)x^{-1} + 1 - \Omega_m(r)}}$$

- Free to choose:
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Introduction
The LTB Model

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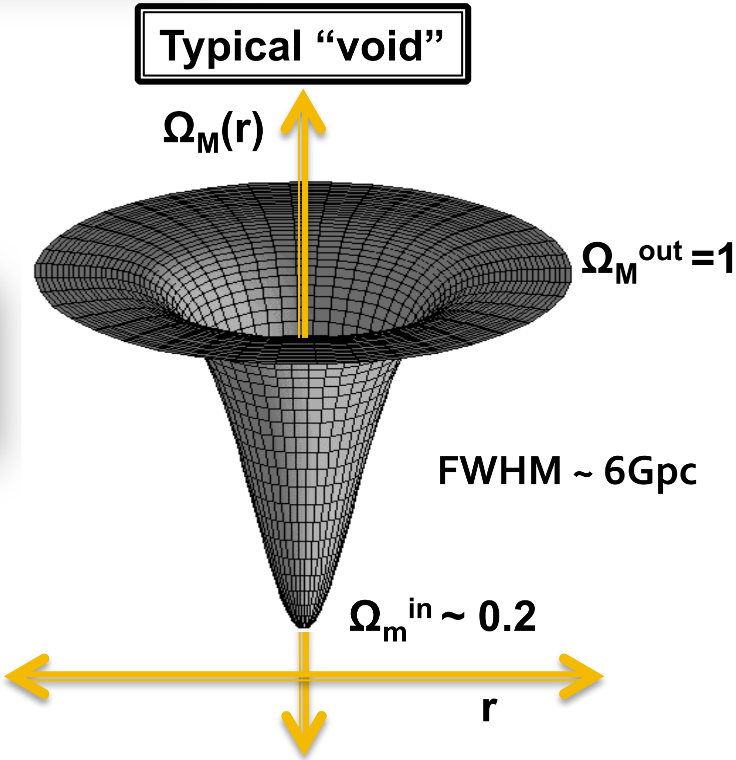
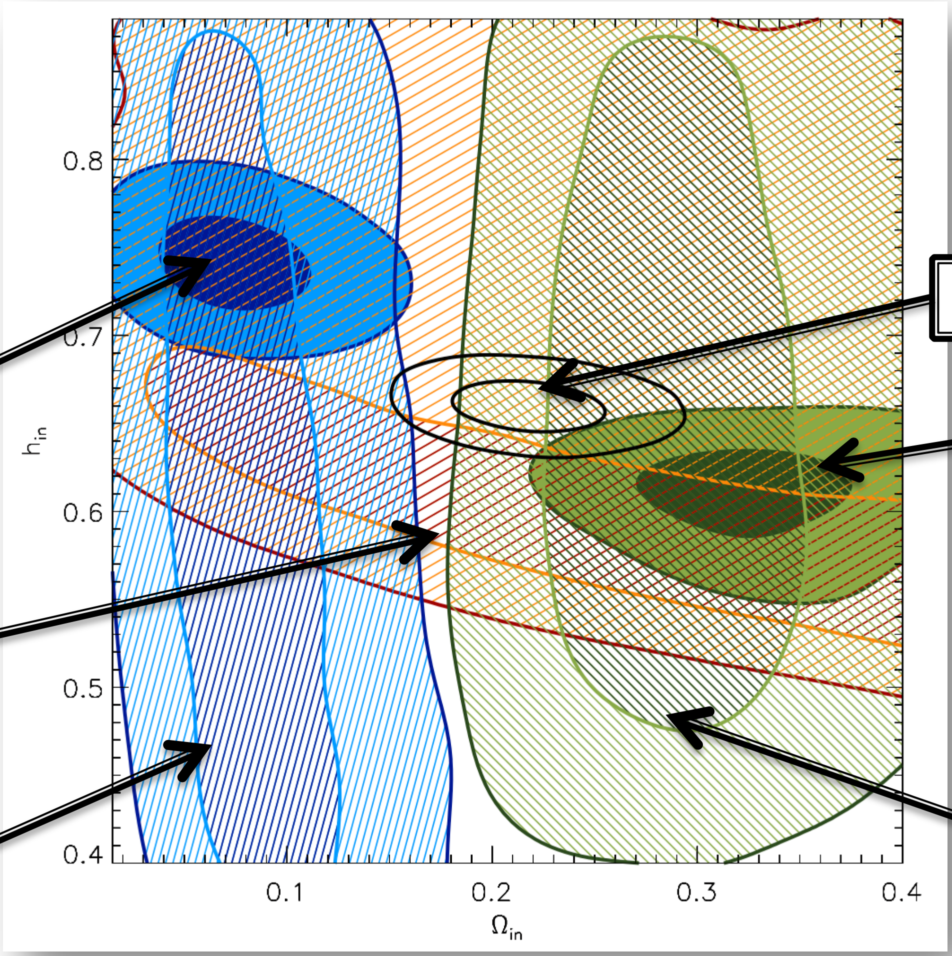


Introduction
The LTB Model

- Current status: “Tension in the Void”, Zumalacárregui et al. (2012)



SNe + H_0

CMB (only)

SNe (only)

SNe + H_0 + BAO + CMB

BAO + CMB

BAO (only)

Introduction
The LTB Model

- Similarly, non-uniform bang-time adiabatic void models are inconsistent with SNIa, H0, CMB and kSZ data [Bull et al. (2011); See also García-Bellido & Haugbølle (2008)].

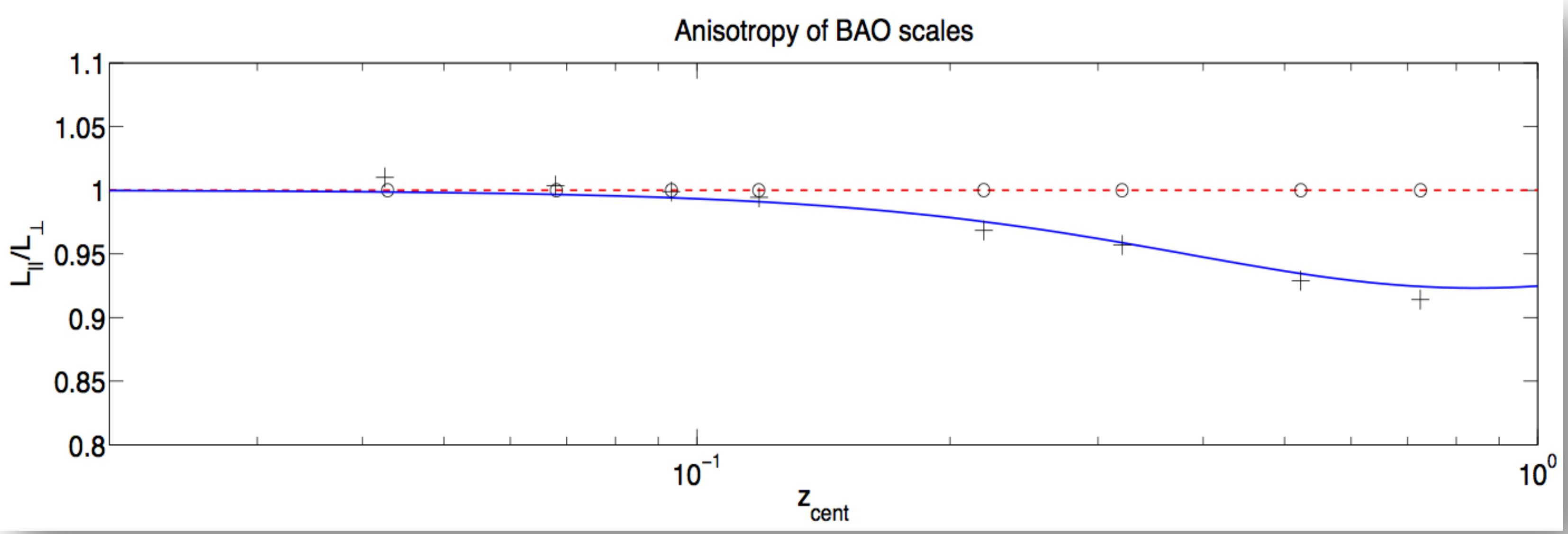
- Perhaps further tweaking is necessary:
  - \`Isocurvature' mode? [See Regis & Clarkson (2010); Clarkson & Regis (2010)]
  - A feature in the primordial power spectrum? [Nadathur & Sakar (2010)]
  - The effects of linear perturbations? [eg. Zibin (2008); Clarkson et al. (2009); Nishikawa et al. (2012)]

Seems to require lots of fine-tuning!

Introduction
The LTB Model

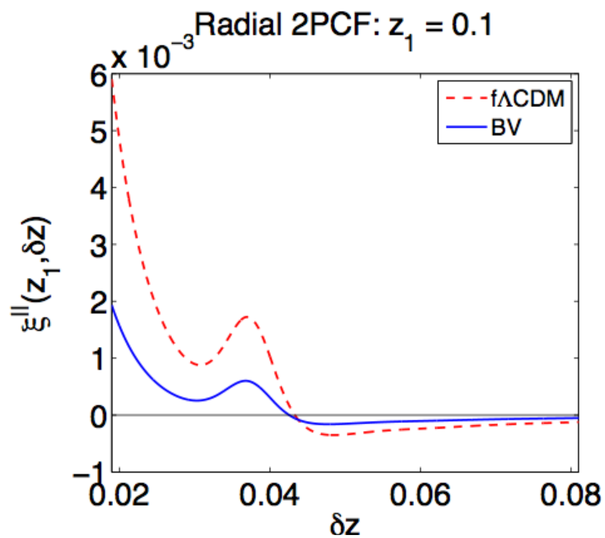
- Recent study of the effects of perturbations on BAO scale:

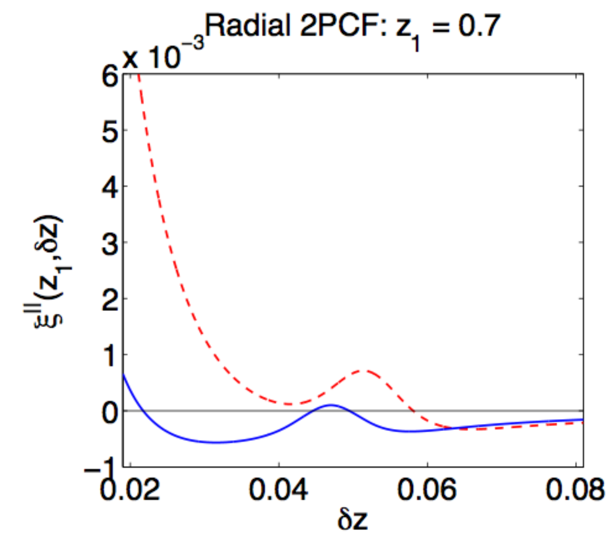
February, Clarkson & Maartens (2012)
arxiv:1206.1602 [astro-ph.CO]



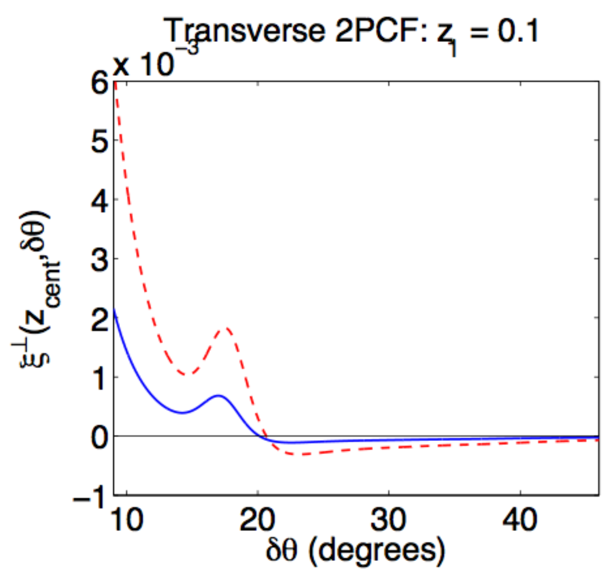
Introduction

The LTB Model

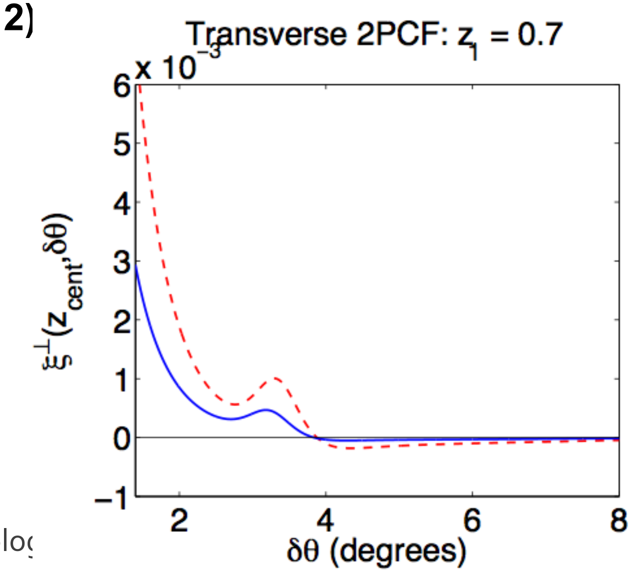




February et al. (2012)



Mimics effects of RSD in FLRW! [Montanari & Durrer (2012)]



Perturbation Theory in LTB

- Via the “2+2” approach [see Gundlach & Martín-García (2000)]:
  - Rewrite background LTB metric

$$ds^2 = g^{(0)}_{AB} dx^A dx^B + \mathcal{R}^2 \gamma_{ab} dx^a dx^b$$

  - Expand scalars on two-sphere

$$\phi(x^A, x^a) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{m=\ell} \phi^{\ell m}(x^A) Y_{\ell m}(x^a)$$

  - Perturb around this

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g^{polar}_{\mu\nu} + \delta g^{axial}_{\mu\nu}$$

where

$$\delta g^{polar}_{\mu\nu} = \begin{pmatrix} h_{AB} Y & 2h_A Y_{:a} \\ * & \mathcal{R}^2 (K Y \gamma_{ab} + G Y_{:ab}) \end{pmatrix} \qquad \delta g^{axial}_{\mu\nu} = \begin{pmatrix} 0 & \bar{h}_A \bar{S}_a \\ * & 2\bar{h} \bar{S}_{(a:b)} \end{pmatrix}$$

$Y = Y_{\ell m}(\theta,\Phi)$; $Y_{:a} = \partial_a Y$; $\gamma^{ab} Y_{:ab} = -\ell(\ell+1) Y$; $\bar{S}_{:a} = \varepsilon_a{}^b Y_{:b}$

Perturbation Theory in LTB

- General coordinate transformation

$$\tilde{x}^{\mu} = x^{\mu(0)} + \xi^{\mu}$$

where

$$\xi^{\mu} \equiv \left[\xi^{A} Y, \, \xi Y_{:}^{a} + \bar{M} \bar{S}^{a}\right]$$

- Relate variables in 'old' coords to 'new' via active or passive approach.
- Apply Regge-Wheeler gauge.
- For the polar sector we find

$$ds^2 = -\Big[1 + \Big(2\eta - \chi - \varphi\Big)Y\Big]dt^2 - 2\varsigma Y X(t,r) dt dr + \Big[1 + \Big(\chi + \varphi\Big)Y\Big]X^2(t,r)dr^2 + \Big[1 + \varphi Y\Big]A^2(t,r)d\Omega^2$$

$$u_{\mu} = \left[\hat{u}_A + \left(w\hat{n}_A + \frac{1}{2}h_{AB}\hat{u}^B\right)Y, vY_a\right] \quad , \quad \rho = \rho^{\mathrm{LTB}}(1 + \Delta Y)$$

$$\hat{u}^A = \Big(1, 0\Big) \quad , \quad \hat{n}^A = \Big(0, X^{-1}\Big)$$

Perturbation Theory in LTB

- Master equations for $\ell > 1$ ($\eta=0$): (arxiv:0903.5040 [astro-ph.CO])

$$-\ddot{\chi} + \chi'' - 3H_{\|}\dot{\chi} - 2W\chi' + \left[16\pi G\rho - \frac{6M}{a_\perp^3} - 4H_\perp(H_\| - H_\perp) - \frac{(\ell-1)(\ell+2)}{a_\perp^2 r^2}\right]\chi = S_\chi(\varsigma, \varphi)$$

$$\ddot{\varphi} + 4H_\perp\dot{\varphi} - 2\frac{\kappa}{a_\perp^2}\varphi = S_\varphi(\chi, \varsigma)$$

$$\dot{\varsigma} + 2H_\|\varsigma = -\chi'$$

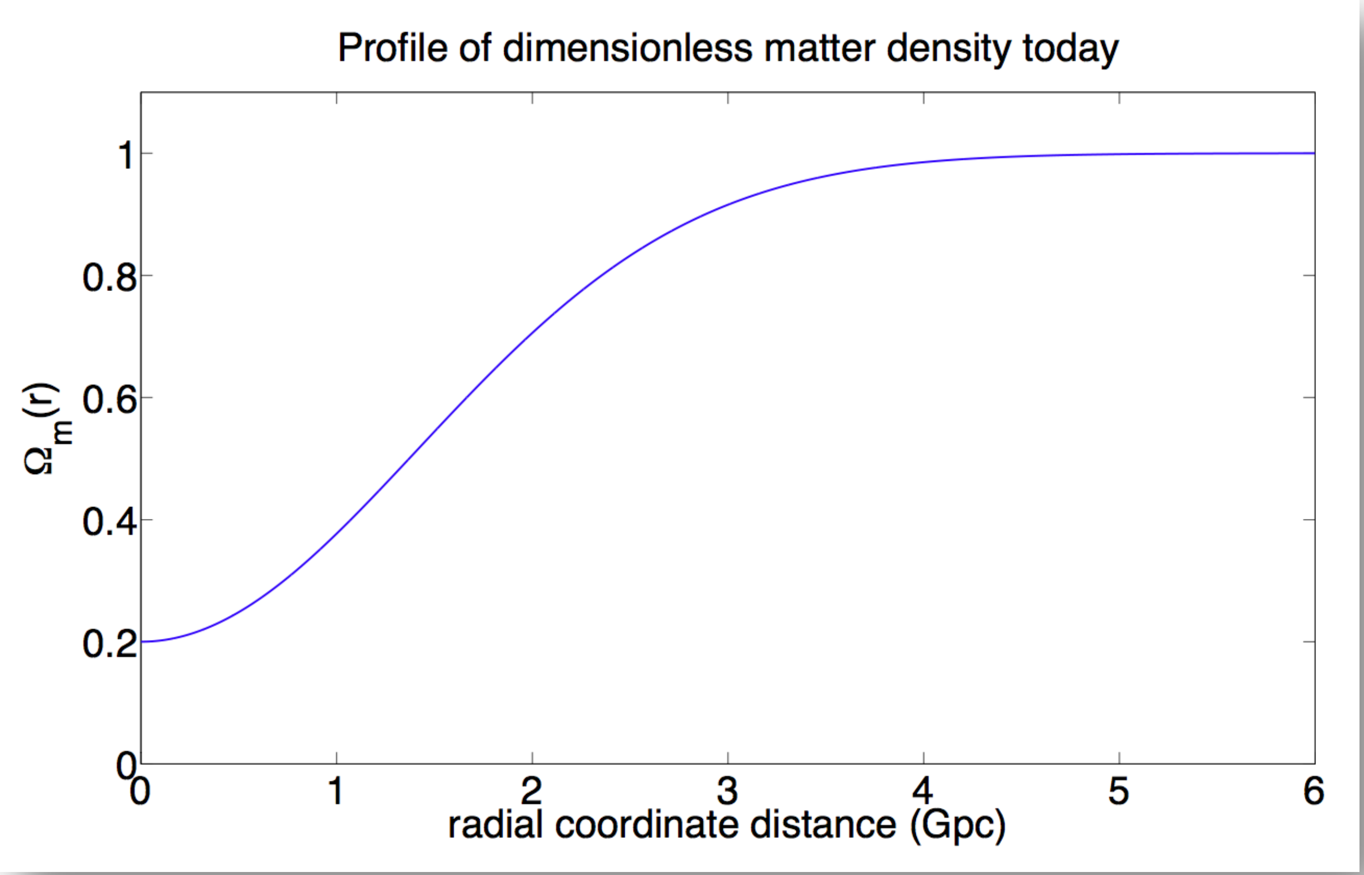
$$\begin{aligned} 8\pi G\rho\Delta = & -\varphi'' - 2W\varphi' + (H_\| + 2H_\perp)\dot{\varphi} + W\chi' + H_\perp\dot{\chi} + \left[\frac{\ell(\ell+1)}{a_\perp^2 r^2} + 2H_\perp^2 + 4H_\| H_\perp - 8\pi G\rho\right](\chi + \varphi) \\ & - \frac{(\ell-1)(\ell+2)}{2a_\perp^2 r^2}\chi + 2H_\perp\varsigma' + 2(H_\| + H_\perp)W\varsigma, \end{aligned}$$

where

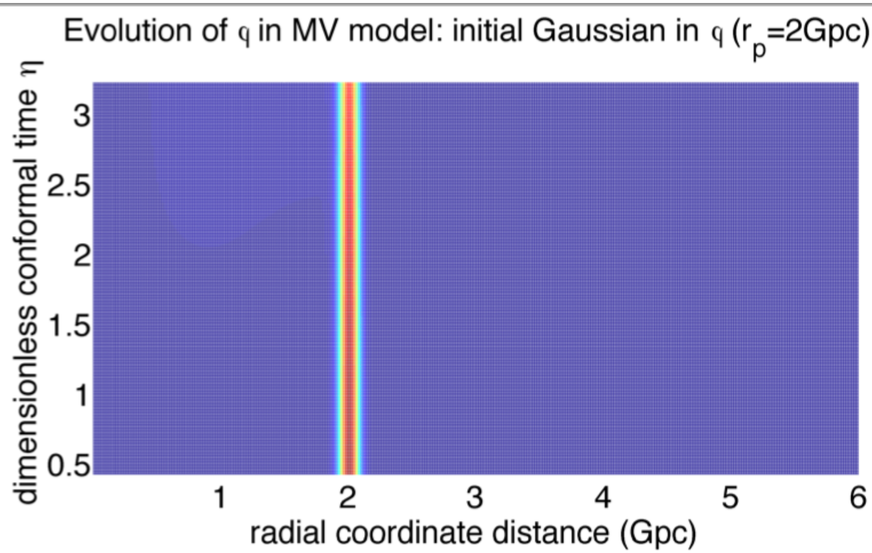
$$X' = \hat{n}^A \nabla_A X \quad , \quad W = \frac{a_\|}{a_\perp r}\hat{n}^r$$

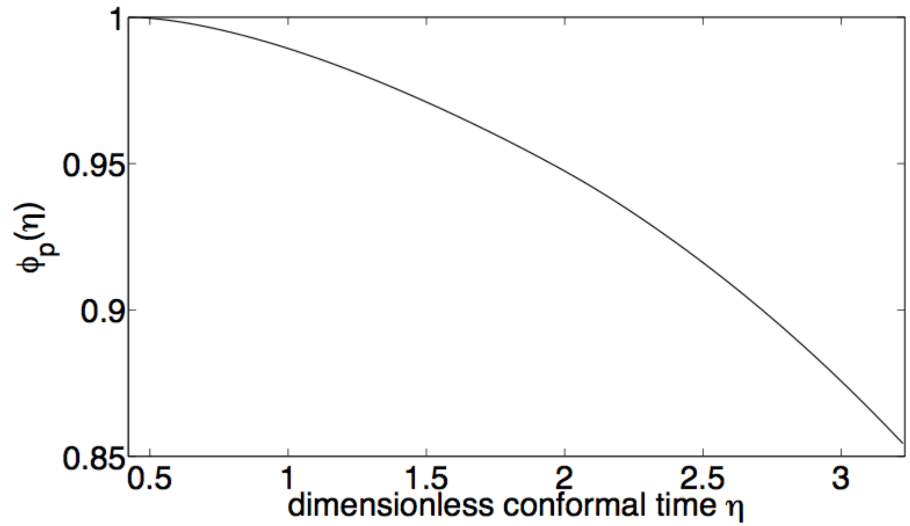
Preliminary Results

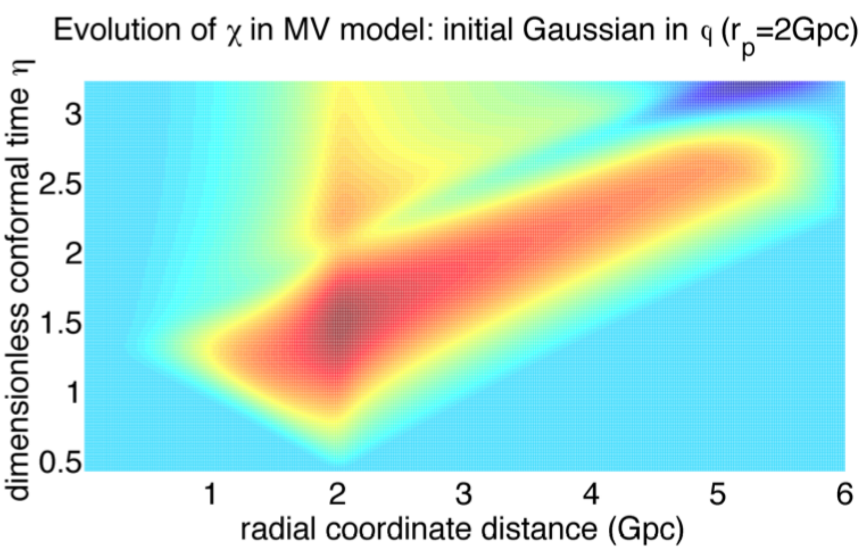
- For the following void model:

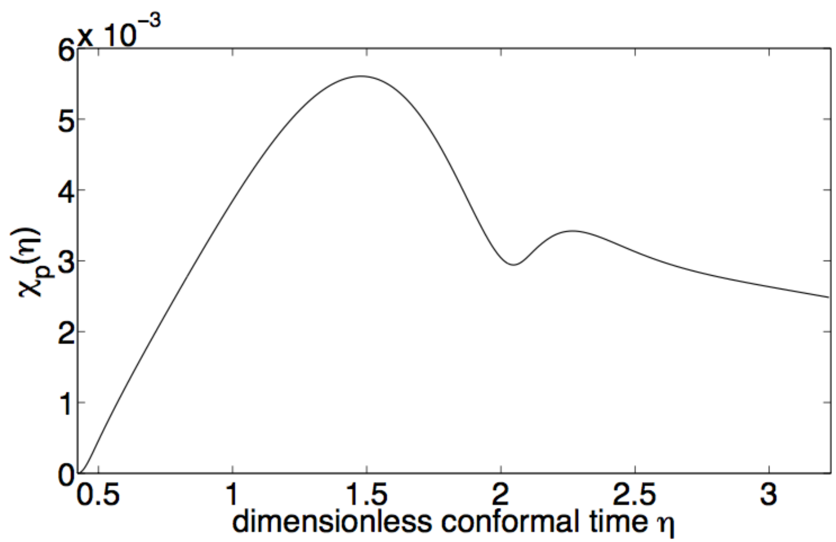


Preliminary Results









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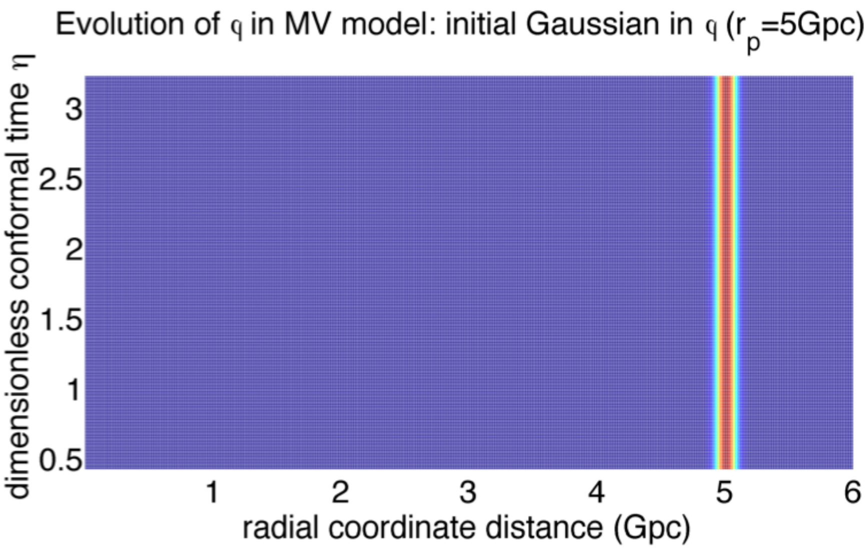

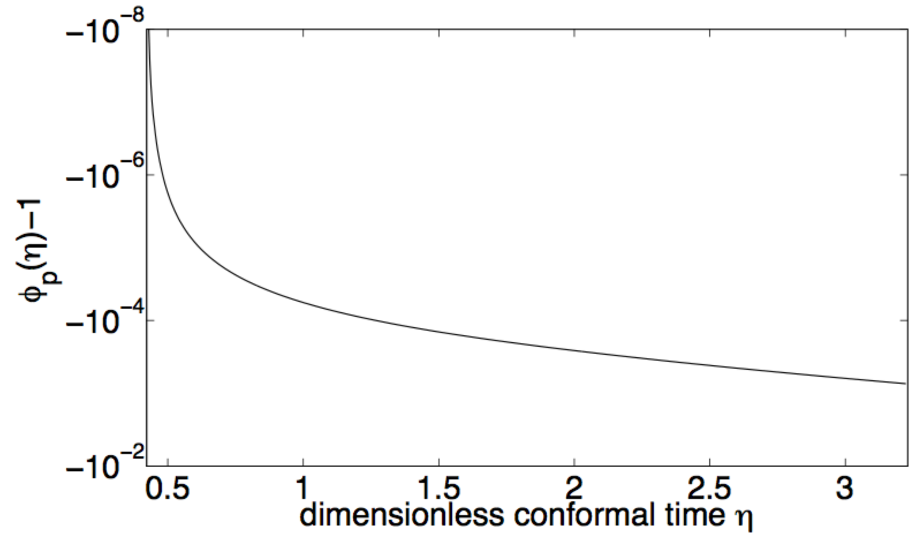

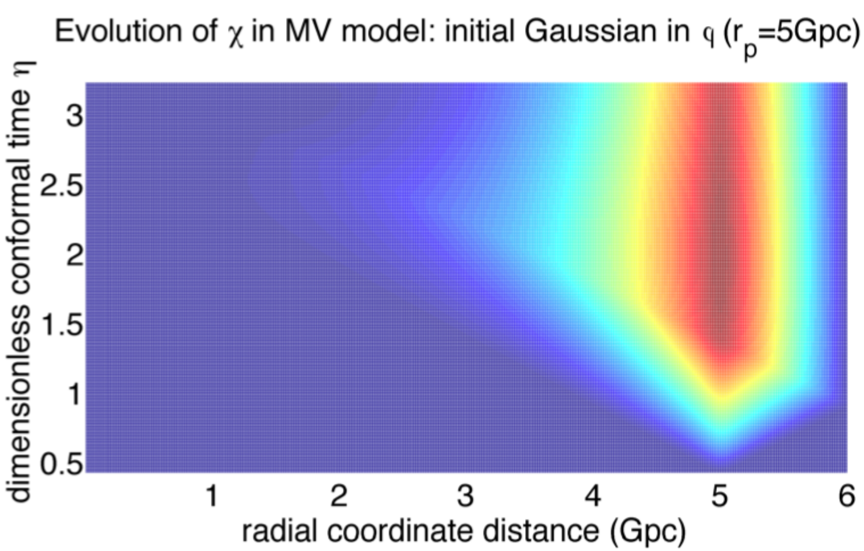

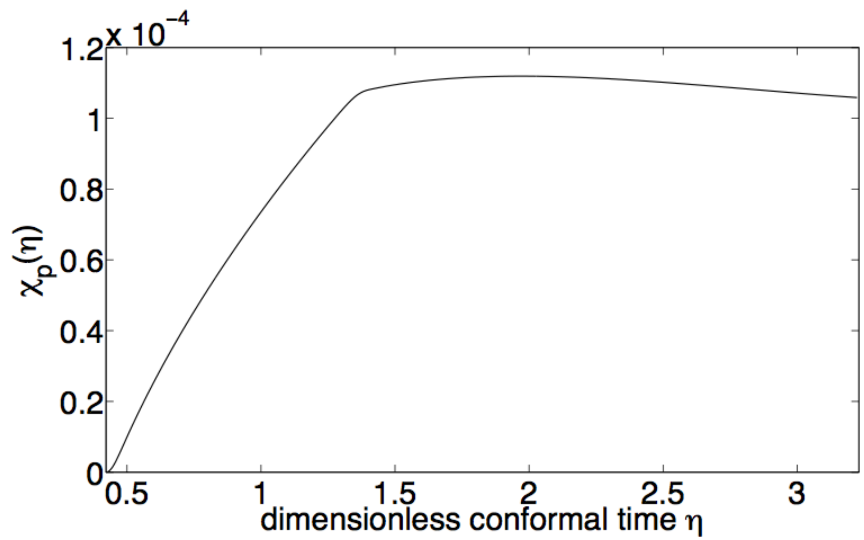

Summary & Outlook

- LTB models require enormous fine-tuning to accommodate observations.
- Introduced linear perturbation theory to address structure formation in a spherically symmetric, inhomogeneous background.
- Found:
  - adding scalar mode to 2PCF calculation produces negligible corrections to BAO predictions, but noticeable differences seen in amplitude of peak.
  - evolution of full system of perturbations so far makes sense: shearing inside void causes larger amplitude of 'GWs' than outside.

Where to from here?

- GWs corrections can be used to address validity of Newtonian theory (N-body), astrophysical applications etc...

Watch this space!