Inflation with turning trajectories:

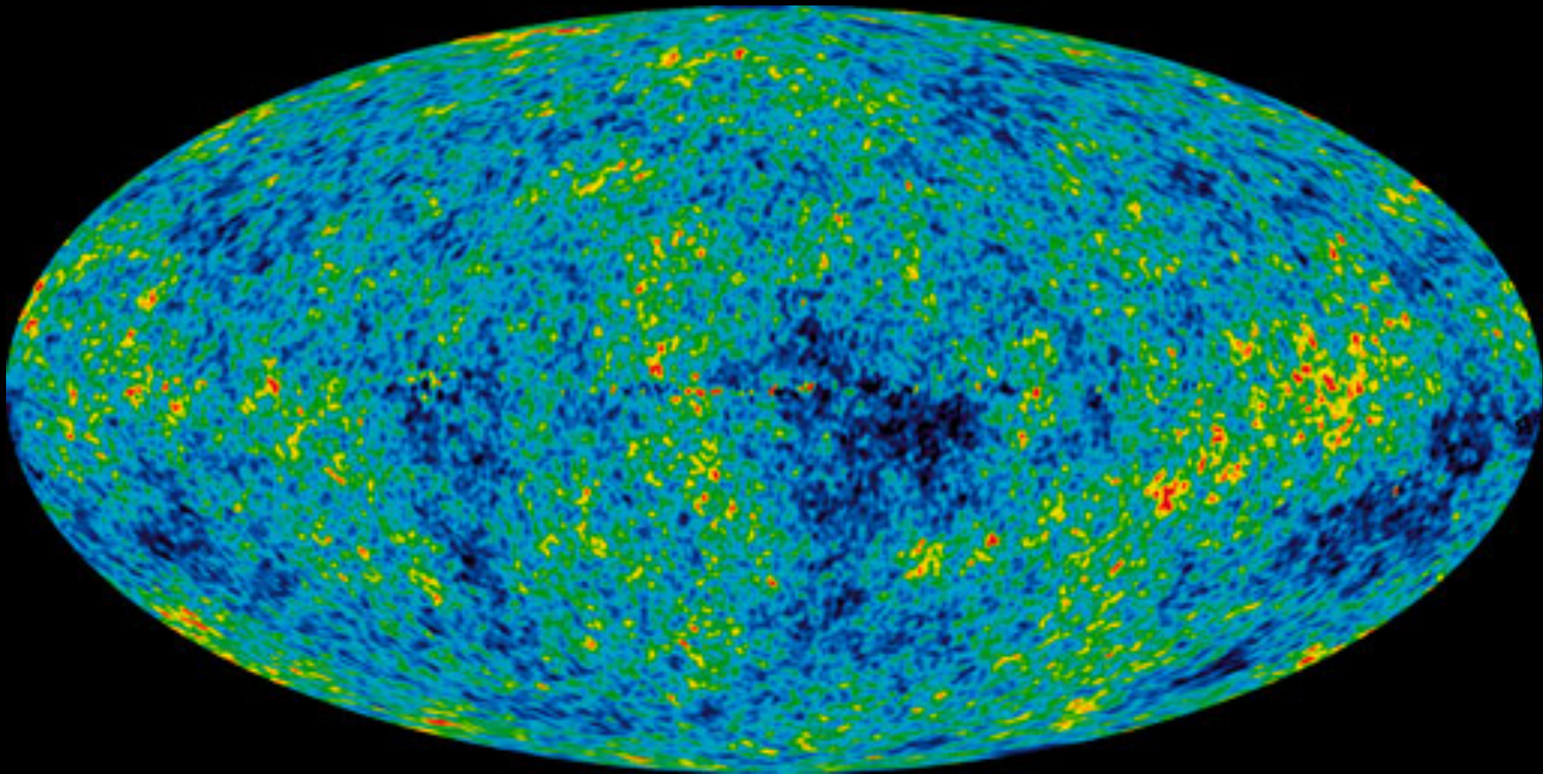
Features of heavy physics in the spectrum of primordial perturbations

with **J-O. Gong, S. Hardeman, G. Palma, S. Patil**
1005.3848 (PRD 2011); 1010.3693 (JCAP 2011); 1201.6342 (JHEP 2012);

and **V. Atal, S. Cespedes 1205.0710**

Ana Achucarro (Leiden / UPV-EHU Bilbao)
Modern Cosmology: Early Universe, CMB and LSS **Benasque, 13/8/12**

Observations of the Cosmic Microwave Background make a convincing case for inflation



Inflation

(accelerated expansion)

-dilutes massive relics (e.g. monopoles)

-solves horizon problem

-solves flatness problem if it lasts long enough (~ 55 e-folds)

-gives mechanism for approximately scale invariant primordial inhomogeneities (from quantum fluctuations)

-produces a background of gravitational waves

Single-field inflation assumes all other fields are decoupled from the inflaton during inflation.

Not so easy to achieve in realistic particle physics models

Extremely difficult in SUGRA and even harder in string theory
(because gravity couples to all fields)

Gravity is strong at those energies

inflation is UV sensitive

It opens the possibility to **detect** heavy fields that interact with the inflaton

Single-field slow-roll inflation with **canonical** kinetic terms
predicts perturbations that are

adiabatic
near scale-invariant
almost gaussian

self-interactions (in the potential) are limited by the slow roll condition

If there are other light fields around, the light isocurvature modes can decay into curvature perturbation

curvature mode not conserved on superhorizon scales
non-gaussianity (squeezed limit)

(multifield inflation, curvaton, modulated reheating)

Multi-field inflation = inflation with several **light** fields

Valle de Benasque

(a turn in the trajectory couples the adiabatic and isocurvature modes)

but: isocurvature modes very tightly constrained by the CMB

Turning trajectories have been studied extensively in the context of inflation with **many light fields**

"multifield inflation"

in the **slow roll** regime under the assumption of **slow/mild turns**.

Gordon Wands Bassett Maartens 2001

Groot Nibbelink van Tent 2001, 2002

Lalak Langlois Pokorski Turzynski 2007

Peterson Tegmark 2011

The effect of the turn is to couple the adiabatic and isocurvature modes.
The curvature perturbation does not remain constant on superhorizon scales, it is sourced by the isocurvature mode.

Chen Wang 2010 (M ~ H, **quasi single field inflation**, constant turn, **equilateral NG**)

Here we are interested in the effect of very heavy fields (M >> H) on the (single) inflaton. In this case, **strong turns are consistent with slow roll**.

The heavy fields are excited and leave an imprint on the primordial spectrum:

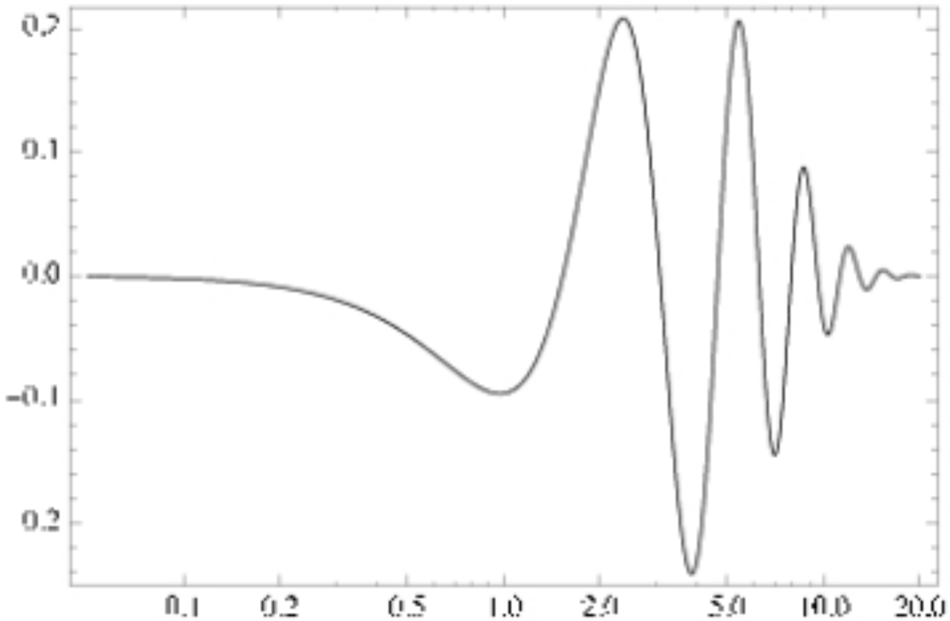
The isocurvature mode is very massive, it decays.

The effect of a localized turn

$$\eta_{max} = 5, \Delta N = 0.5, H^2/M^2 = 1/300$$

Power spectrum (*)

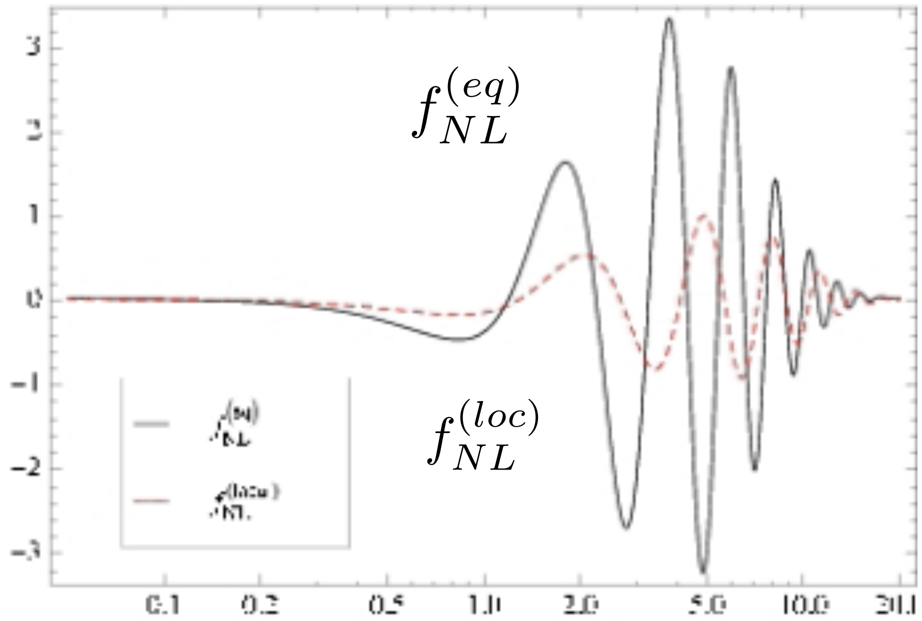
$\Delta P/P$



k/k_*

Bispectrum

f_{NL}



k/k_*

(*) at the end of inflation

inflation

end of inflation

preheating ---- reheating
phase transitions?
defect formation?
...

recombination

today

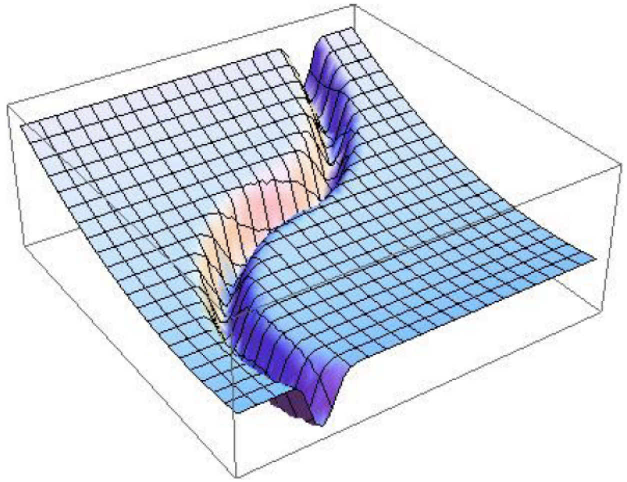
WARNING

All the effects discussed in this talk refer to the primordial spectrum **at the end of inflation --**
but there are no isocurvature modes (they decay very fast)

Single-field inflation in a multi-field landscape:

Congosto

a single light field (inflaton)



all other fields
extremely heavy (M>>H)

The inflaton is not the only field around -
(reheating)

during inflation it must be very “weakly coupled” to everything else

very small couplings (symmetry?)
very massive “spectators”

In (particle) physics we expect heavy degrees of freedom to decouple from the low energy dynamics of light ones

We can integrate out the effect of heavy fields to get an effective action for the light degrees of freedom

Corrections are suppressed by O (k^2 / M^2)

"decoupling"

But sometimes the prefactors are large

On a turning trajectory these terms modify the kinetic terms of the low energy modes (reduced speed of sound)

a partial answer to Tamara's question of
what we might or might not learn about inflation
from the primordial power spectrum and non-gaussianity…

The effective field theory of (the fluctuations of) single-field inflation:

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore JHEP 2008 0709.0293

$$\boxed{S = \int \sqrt{-g}\, d^4x \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} \gamma_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right]} \dots]$$

or

$$c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot{H}}$$

$$S_\pi = \int d^4x\, \sqrt{-g} \left[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) + M_{\rm Pl}^2 \dot{H} \left(1 - \frac{1}{c_s^2} \right) \left(\dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 \dots \right]$$

Π is the Goldstone boson of broken time-translations

$$g_{ij} = a^2(t) \left[(1 + 2\zeta(t, \vec{x}))\delta_{ij} + \gamma_{ij} \right] \qquad \zeta(t, \vec{x}) = -H\pi(t, \vec{x})$$

The effective field theory of (the fluctuations of) single-field inflation:

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore JHEP 2008 0709.0293

$$
\begin{aligned}
S \;=\; \int \mathrm{d}^4x\sqrt{-g}\Big[\; & \tfrac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2 \dot{H} g^{00} - M_{\rm pl}^2(3H^2+\dot{H}) \\
& + \tfrac{1}{2!}M_2^4(t)(\delta g^{00})^2 + \tfrac{1}{3!}M_3^4(t)(\delta g^{00})^3 + \cdots \\
& - \tfrac{1}{2}\bar{M}_1^3(t)\delta g^{00}\delta K^\mu_\mu - \tfrac{1}{2}\bar{M}_2^2(t)(\delta K^\mu_\mu)^2 - \tfrac{1}{2}\bar{M}_3^2(t)\delta K^{\mu\nu}\delta K_{\mu\nu} + \cdots \Big]
\end{aligned}
$$

or

$$
c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot{H}}
$$

$$
S_\pi = \int d^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) + M_{\rm Pl}^2\dot{H}\left(1-\frac{1}{c_s^2}\right)\left(\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 \ldots\right]
$$

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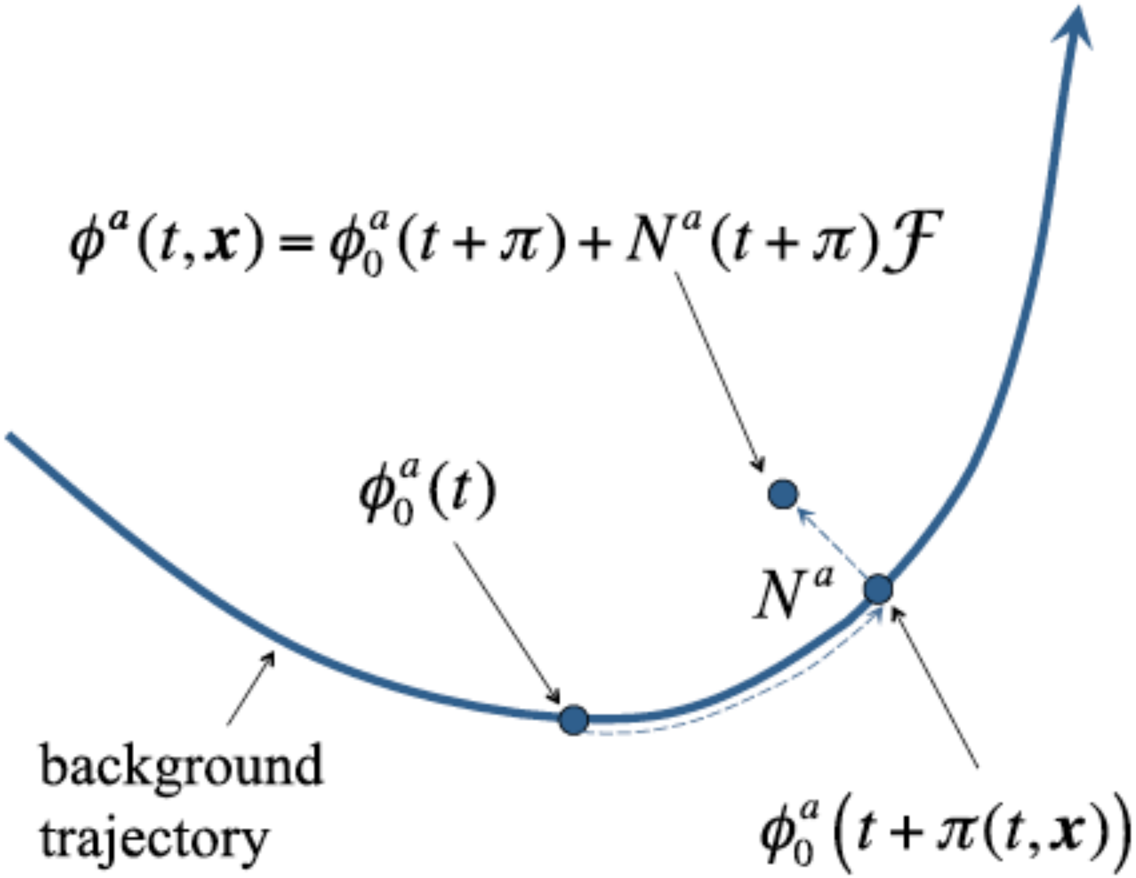
$$

The effective field theory of inflation is an expansion – observations constrain the values of the coefficients of each term

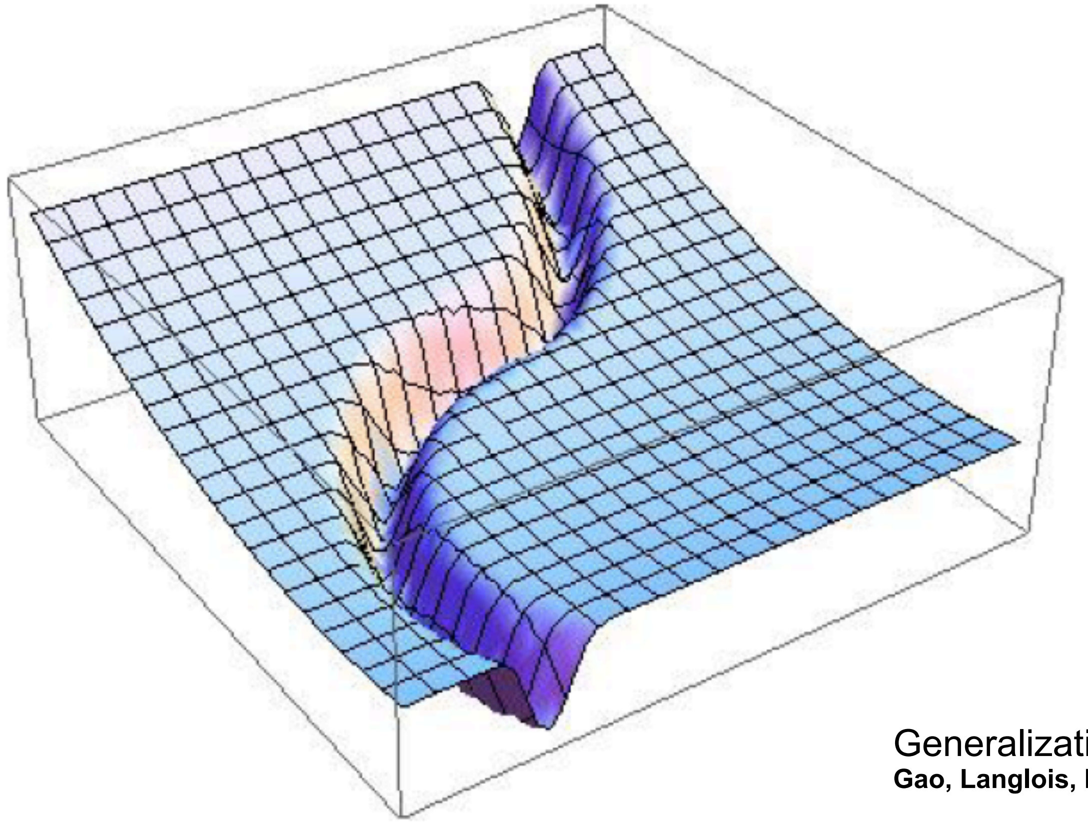
For a particular particle physics model, the coefficients can be calculated

Models of inflation that give the same coefficients cannot be distinguished observationally – even in principle


$\phi^a(t,\boldsymbol{x}) = \phi_0^a(t+\pi) + N^a(t+\pi)\mathcal{F}$
$\phi_0^a(t)$
N^a
background trajectory
$\phi_0^a\left(t+\pi(t,\boldsymbol{x})\right)$

An inflationary trajectory with bends

(straight = geodesic w.r.t. metric in field space)



Generalization to many fields:

Gao, Langlois, Mizuno 1205.5275

One field much heavier than the other.
Light field rolls slowly down a valley with steep walls.

c.f. Chen & Wang PRD 2010 0909.0496; Tolley & Wyman PRD 2010 0910.1853 ;
Peterson & Tegmark PRD 2010, PRD 2011; Cremonini, Lalak, Turzynski JCAP 2011 1010.3021;
Shiu Xu PRD 2011 1108.0981; Baumann Green JCAP 2011 1102.5343 Cespedes, Atal, Palma JCAP 2012 1201.4848
Avgoustidis e.a. JCAP 2012 1203.0016; Chen Wang 1205.0160; Pi, Sasaki 1205.0161,

Slow roll, with strong turns -

Enforce adiabatic evolution: $\frac{1}{H}\dot{X} << X$

In a multifield context, everyone agrees about

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}_0^2}{2M_{\rm Pl}^2 H^2} << 1$$

near-de Sitter background (adiabatic evolution)

beyond that, it is letter soup…

(**WARNING:**
even $\eta, \ \eta_{||}, \ \eta_{\perp}$ mean different things to different authors !)

Rule of thumb: slow roll is needed only in tangential direction / derivatives

Action

$$S = \int \sqrt{-g}\, d^4x \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2}\gamma_{ab} g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi) \right]$$

Equations of motion

$$\Box\phi^a + \Gamma^a_{bc} g^{\mu\nu}\partial_\mu\phi^b\partial_\nu\phi^c = V^a$$

Background

$$\phi^a = \phi_0^a(t)$$

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

$$\dot\phi_0^2 \equiv \gamma_{ab}\dot\phi_0^a\dot\phi_0^b$$

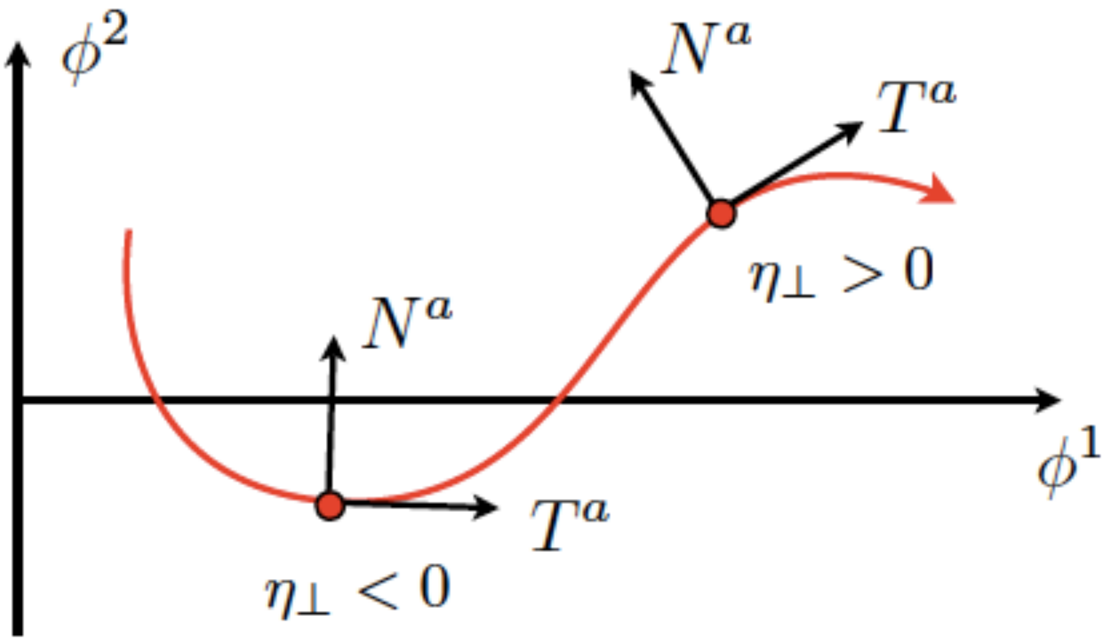
$$\frac{D}{dt}\dot\phi_0^a + 3H\dot\phi_0^a + V^a = 0$$

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[\frac{1}{2}\dot\phi_0^2 + V\right]$$

$$\dot H = -\frac{1}{2M_{\rm Pl}^2}\dot\phi_0^2$$

$$T^a \equiv \frac{\dot{\phi}_0^a}{\dot{\phi}_0}$$

$$\frac{DT^a}{dt} = -\dot{\theta} N^a$$



$$\eta_\perp = \dot{\theta} H$$

Eqs. of motion -- tangential projection

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_\phi = 0,$$

(single-field inflation)

$T^a V_a$ tangential projection

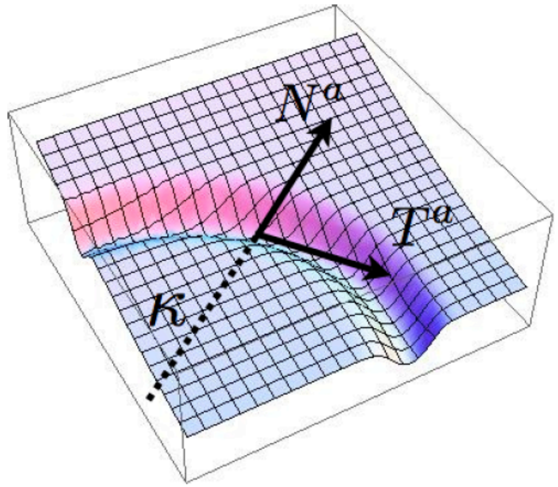
Eqs. of motion -- normal projection

2 x kinetic energy
$2\epsilon H^2$

$$\frac{\dot{\phi}_0^2}{\kappa} = N^a V_a$$

radius of turn

normal projection



$$T^a \equiv \frac{\dot{\phi}_0^a}{\dot{\phi}_0}$$

$$\frac{DT^a}{d\phi_0} = -\frac{1}{\kappa} N^a$$

$$\frac{D}{dt} \equiv \dot{\phi}_0 T^a \nabla_a = \dot{\phi}_0 \nabla_\phi.$$

Slow roll parameters

$$\dot{\phi}_0 = \sqrt{2\epsilon} H M_{\rm Pl}$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}_0^2}{2M_{\rm Pl}^2 H^2}$$ << 1

$$\eta^a \equiv -\frac{1}{H\dot{\phi}_0}\frac{D\dot{\phi}_0^a}{dt}$$

$$\eta_{||} = -\frac{\ddot{\phi}_0}{H\dot{\phi}_0}$$ << 1

Project: $\eta^a = \eta_{||} T^a + \eta_\perp N^a$

$$\boxed{\eta_\perp = \sqrt{2\epsilon}\frac{M_{\rm Pl}}{\kappa}}$$

Not necessarily small

(strong turns are consistent with slow roll inflation)

$$\eta_\perp = \dot{\theta} H$$

Eqs. of motion -- tangential projection

$$T^a \equiv \frac{\dot\phi_0^a}{\dot\phi_0}$$

$$\ddot\phi_0 + 3H\dot\phi_0 + V_\phi = 0,$$

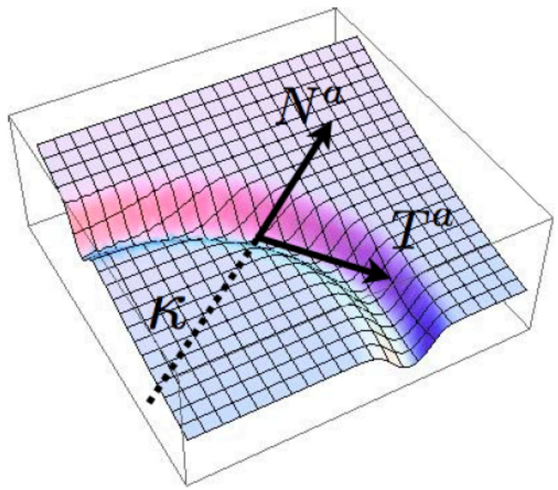
(single-field inflation)

Eqs. of motion -- normal projection

$$\frac{DT^a}{dt} = -\dot\theta N^a$$

$$\frac{\dot\phi_0^2}{\kappa} = N^a V_a$$

$$\frac{\dot\phi_0}{\kappa} = \dot\theta \equiv \frac{V_N}{\dot\phi_0}$$ turning rate



$$\frac{D}{dt} \equiv \dot\phi_0 T^a \nabla_a = \dot\phi_0 \nabla_\phi.$$

If $M^2 >> H^2$,
a sufficiently heavy field can **still** be integrated out --

Get an effective single-field theory with a **reduced speed of sound** for the adiabatic mode

$$c_s^{-2} = 1 + \frac{4\dot{\theta}^2}{M_{\mathrm{eff}}^2}$$

effective mass of heavy field at turn

$$M_{\mathrm{eff}}^2 = M^2 - \dot{\theta}^2$$

Ricci scalar of field space metric

mass of heavy field on straight trajectory

$$M^2 \equiv V_{NN} + \epsilon H^2 \mathbb{R}$$

$$V_{NN} \equiv N^a N^a \nabla_a \nabla_b V$$

mass of heavy field at a point

Reduced speed of sound requires large turning rate

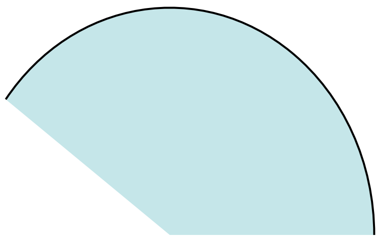
effective mass of heavy field is reduced

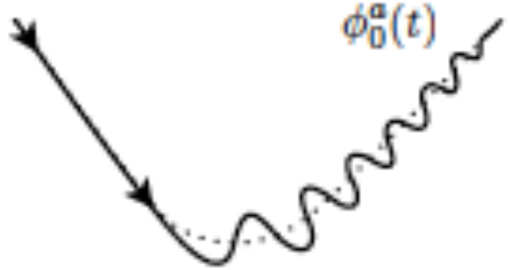
is this consistent??
(can the heavy field still be integrated out?)

Need to distinguish between

STRONG and **SUDDEN** turns.

small, constant c / **large changes in c**





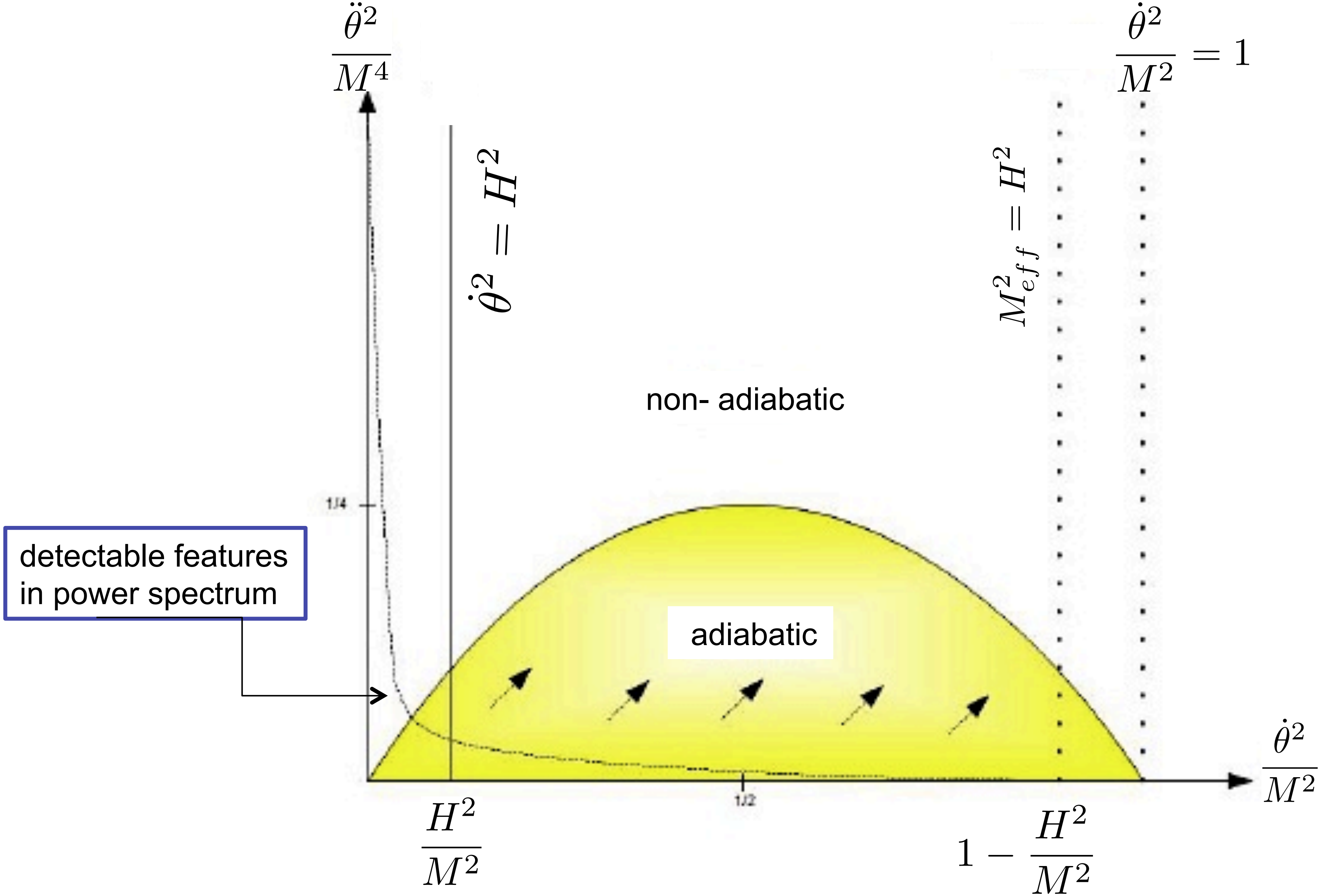
	STRONG	SUDDEN
adiabatic evolution		x
no heavy particle production		x
consistent with slow roll		x
EFT works		x

Adiabaticity condition is

$$\left|\frac{\ddot{\theta}}{\dot{\theta}}\right| \ll M_{\rm eff}$$

Cespedes Atal Palma 2011

ADIABATICITY PLOT



At a strong turn, the effective mass of heavy field goes down, however **hierarchy** between **fast** and **slow** modes **INCREASES**: single-field EFT works !

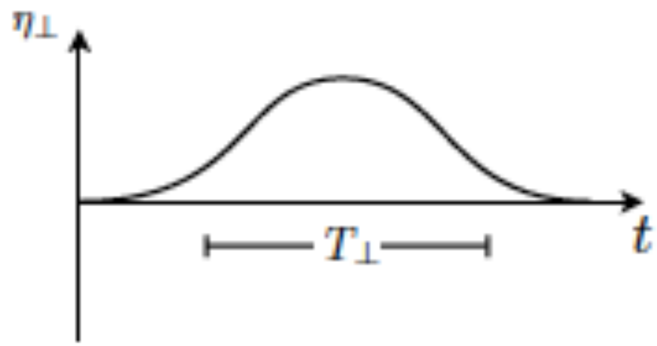
$$\omega^2_{heavy} = M^2_{eff} c_s^{-2} = M^2 + 3\dot{\theta}^2$$

opens window for observable features in power spectrum

An almost constant strong turn is **consistent with current observations.**
Power spectrum renormalized, no features,
potentially observable equilateral non-gaussianity?? (scenario can be disproved)

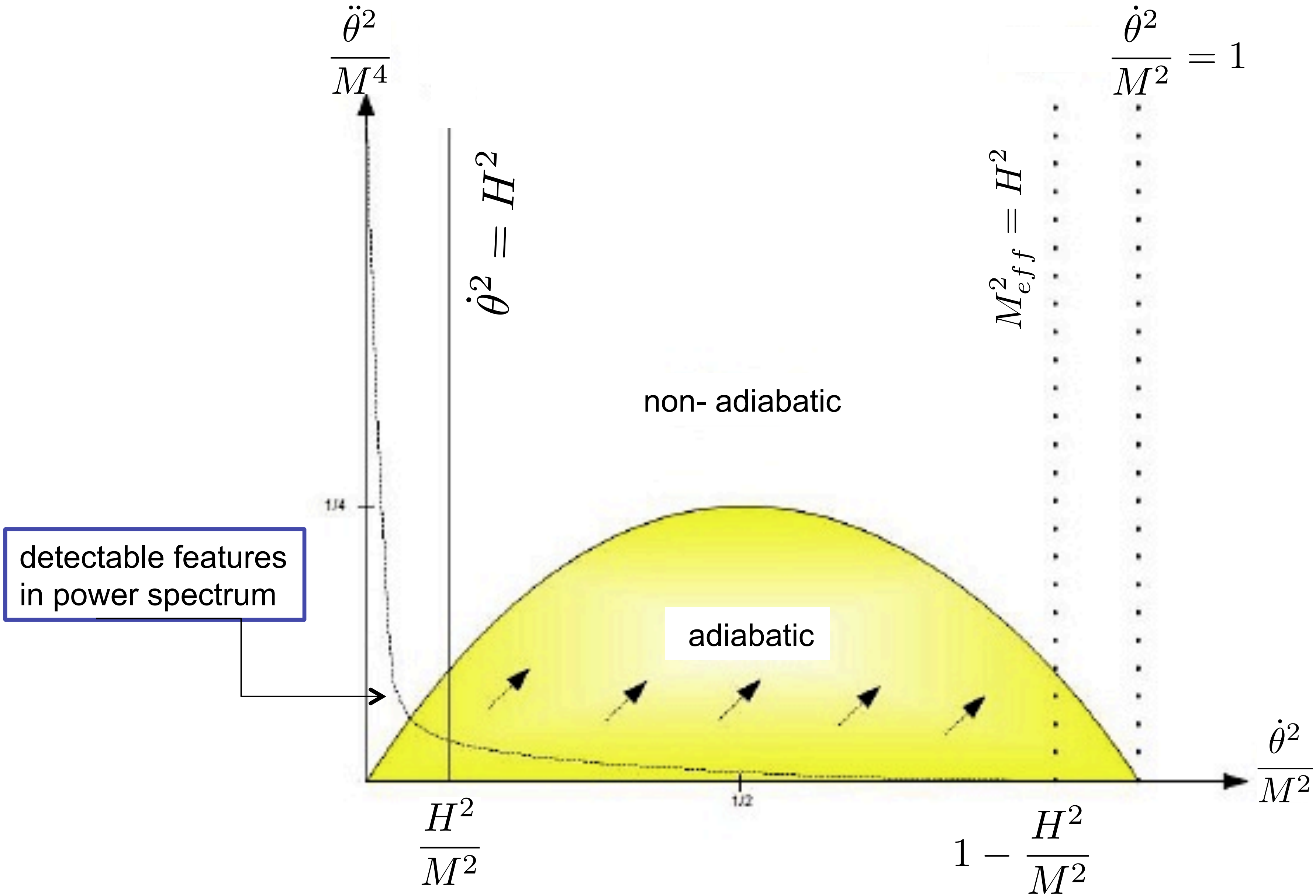
The sweet spot in between…

$$\Delta c_s / c_s \sim 0.1$$



O(1) efold

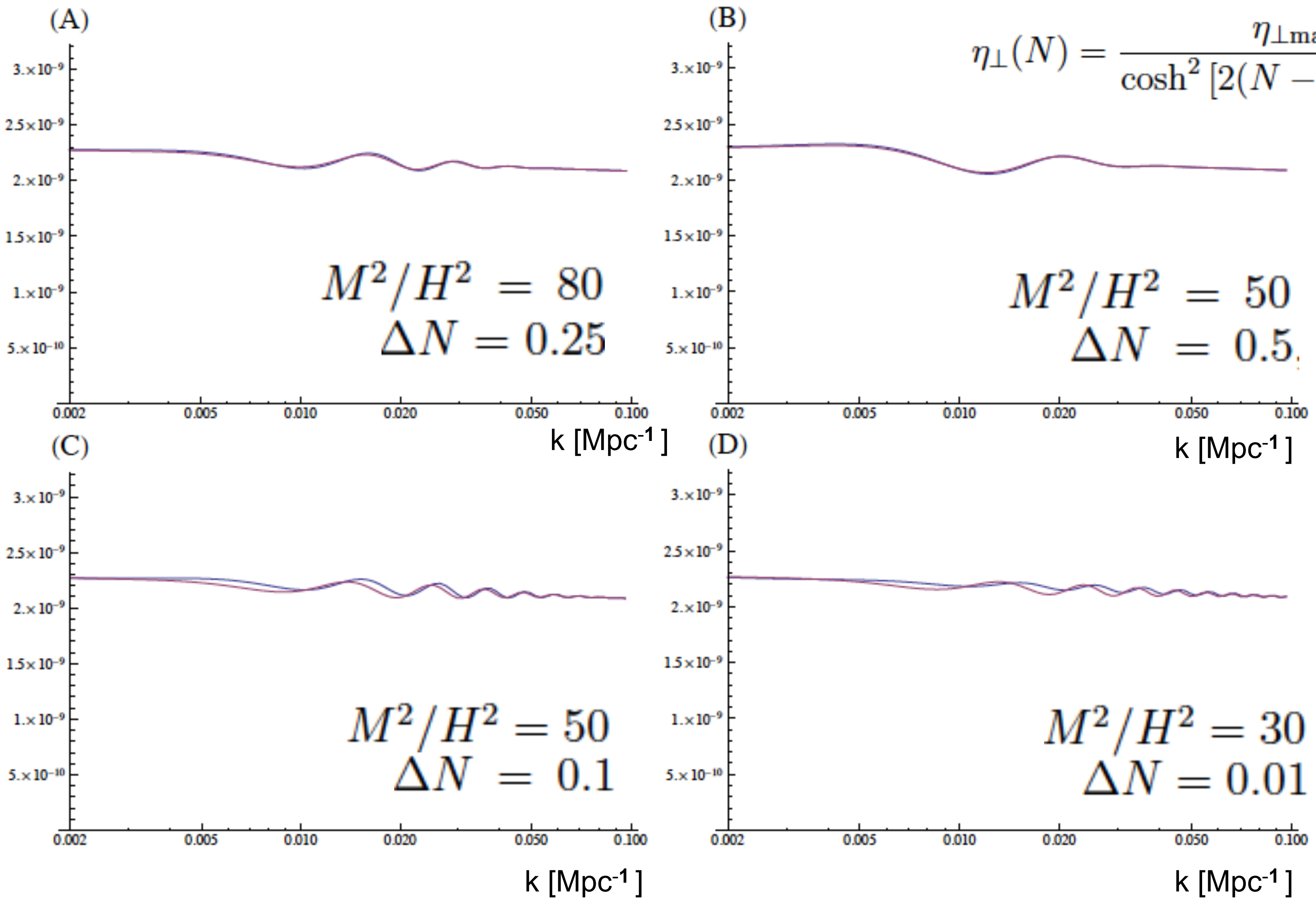
ADIABATICITY PLOT



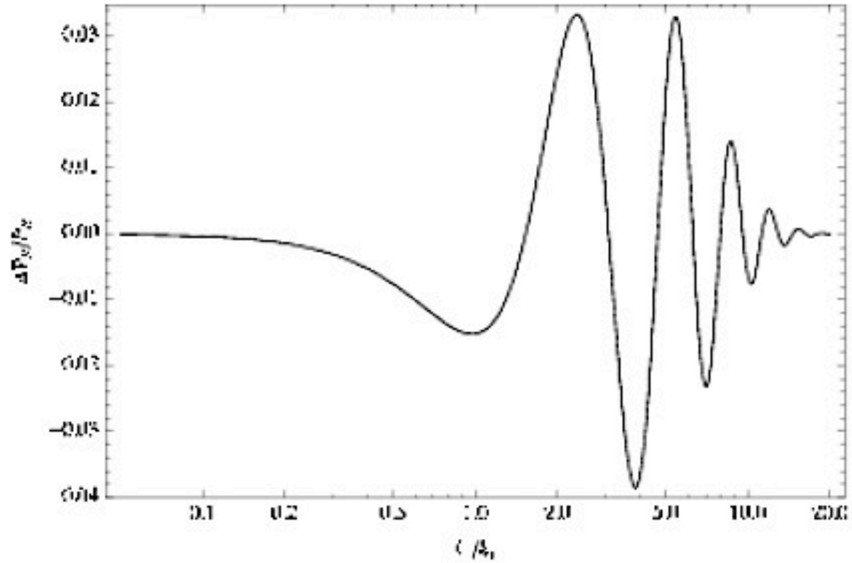
Primordial spectra - $P_{\mathcal{R}}(k)$

blue:two-field system red: effective theory (slowroll) $\epsilon = 0.022$ $\eta_{||} = 0.034$

$$\eta_{\perp}(N) = \frac{\eta_{\perp \max}}{\cosh^2 [2(N - N_0)/\Delta N]}$$



$\eta_{\max} = 2,\ \Delta N = 0.5,\ H^2/M^2 = 1/300$



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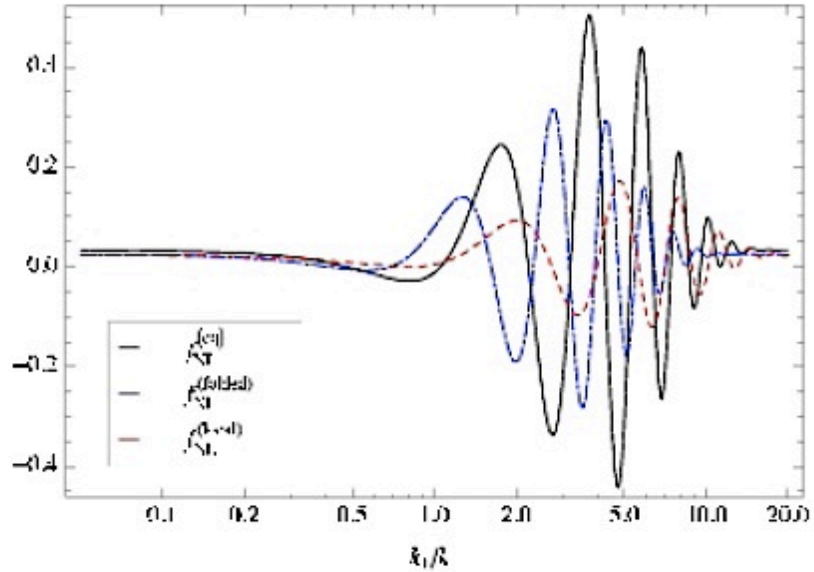
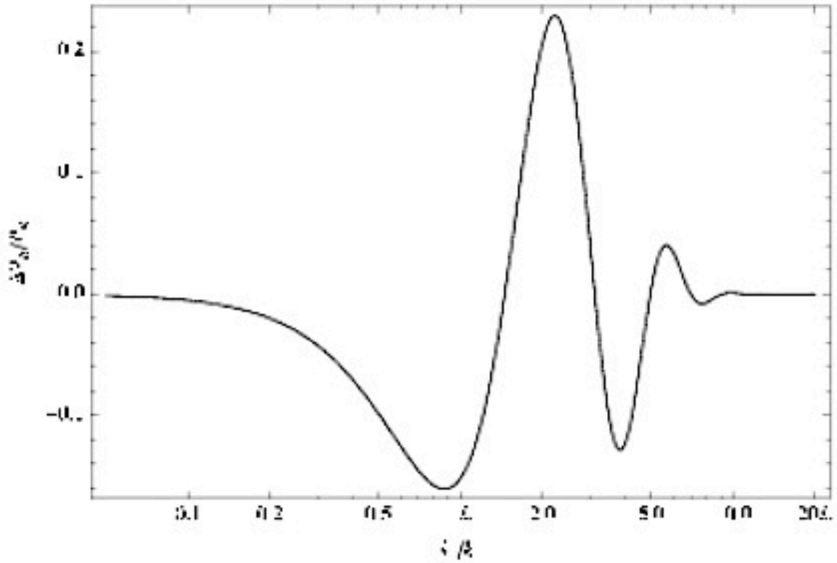


$\eta_{\max} = 5,\ \Delta N = 1,\ H^2/M^2 = 1/300$



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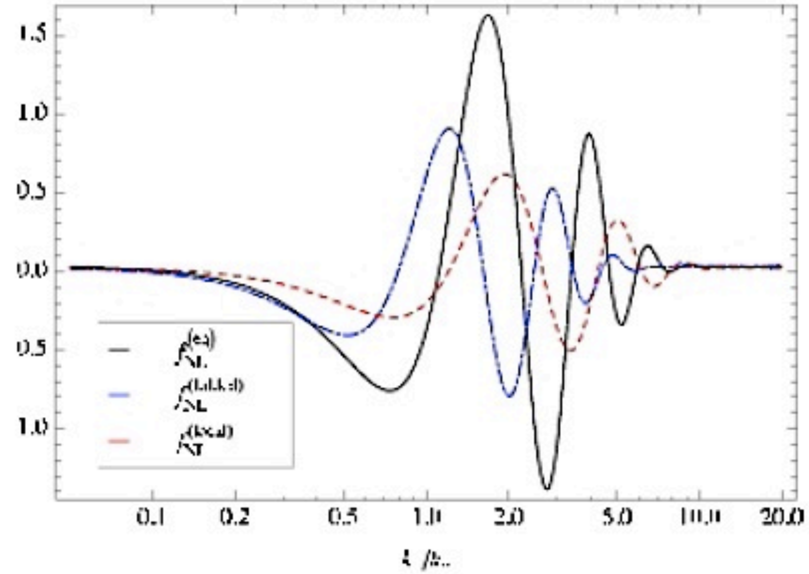

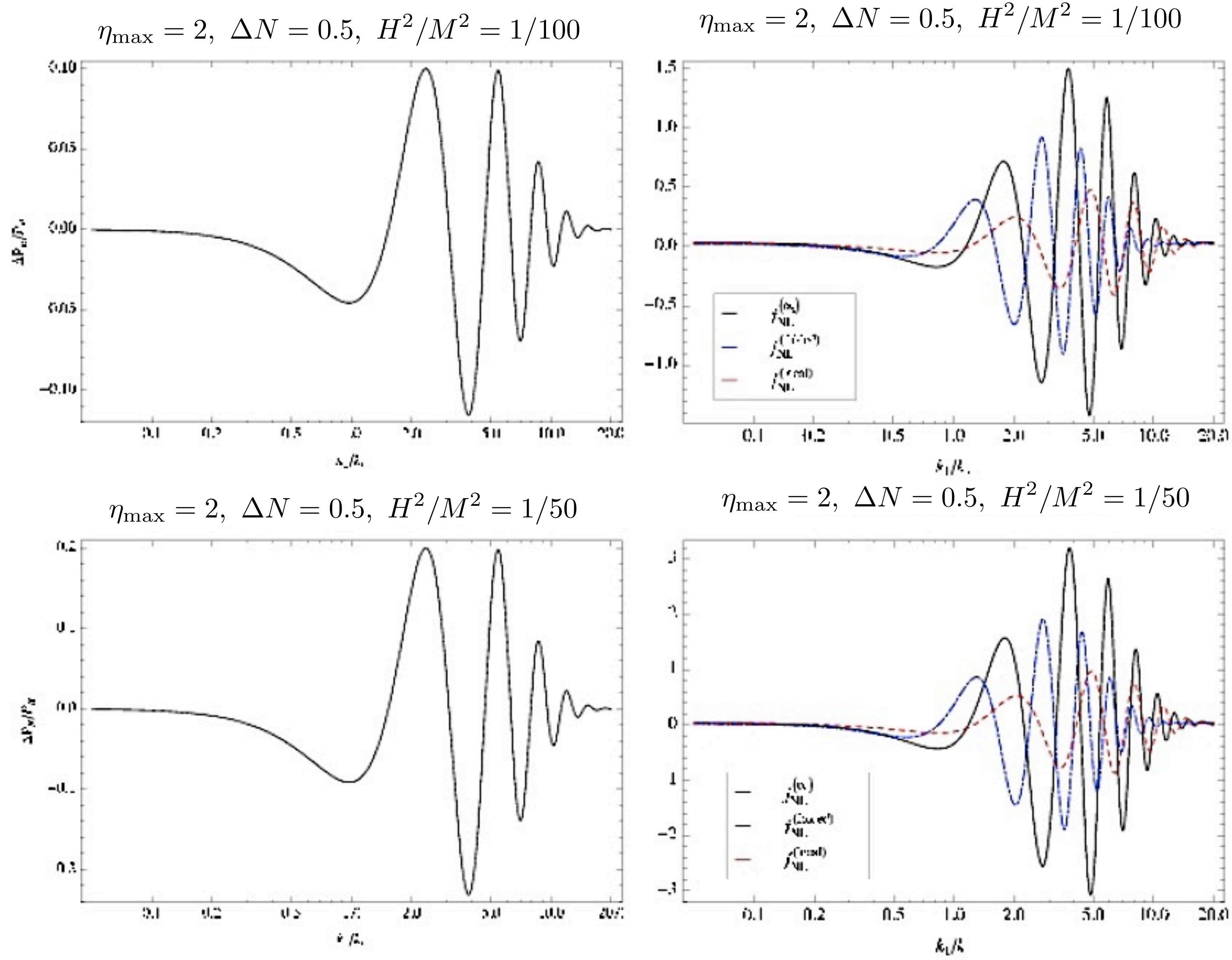

$\eta_{\max} = 2,\ \Delta N = 0.5,\ H^2/M^2 = 1/100$
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Summary

(0) Single field inflation is succesful (so far) but difficult to achieve in particle physics models

(1) Truncating (as opposed to integrating out) heavy fields at their minima misses the coupling effect of turns (curvature and isocurvature perturbations coupled, masses are not given by $\nabla_a \nabla_b V$)

$$M_{\text{eff}}^2 = M^2 - \dot{\theta}^2$$

(2) Turns are **generic** (SUGRA , String Theory)

(3) They are **consistent with slow roll** unless too sudden.

(4) If the heavy fields are very heavy (M>>H) they can be integrated out. The net effect is a **reduced speed of sound** for the adiabatic perturbations

$$c_s^{-2} = 1 + \frac{4\dot{\theta}^2}{M_{\text{eff}}^2}$$

(5) The signature of a turn is a **feature** in the power spectrum **correlated** with **(equilateral) non-gaussianity on the same scales**.

Truncating heavy fields at their critical points is a dangerous approximation to make in supergravity inflation. Even if masses and couplings look safe, it misses the effects of turns in the inflationary trajectory -- which are generic

unless the critical points are **supersymmetric along the whole trajectory (*)**

-- protected from SUSY breaking !

e.g. Sgoldstino inflation 1203.1907

Consistent decoupling requires (counterintuitive) direct couplings in the superpotential

Turns are defined with respect to the geodesics of the sigma model metric, irrespective of whether the kinetic terms are canonical or not

If the critical points of the potential (the "valley") track a geodesic, these non-decoupling effects from turns disappear. In this case, truncation of the heavy fields is a much better approximation

(*) even in this case stability cannot be taken for granted

Summary – cont.

(6) Need to distinguish between **STRONG** and **SUDDEN** turns.

small, constant c / **large changes in c**

Strong turns: adiabatic evolution, no heavy particle production, slow roll, EFT works.
Sudden turns: not !

(7)) Adiabaticity condition is $$\left|\frac{\ddot{\theta}}{\dot{\theta}}\right| \ll M_{\rm eff}$$

(8) At a strong turn, the effective mass of heavy field goes down, however **hierarchy** between **fast** and **slow** modes **INCREASES**: single-field EFT works !

opens window for observable features in power spectrum

(9) A strong turn is **consistent with current observations.**
Power spectrum renormalized, no features,
potentially observable equilateral non-gaussianity (model can be disproved)

Hlosek et al 2011 (ACT)
Still no evidence of features in the power spectrum…

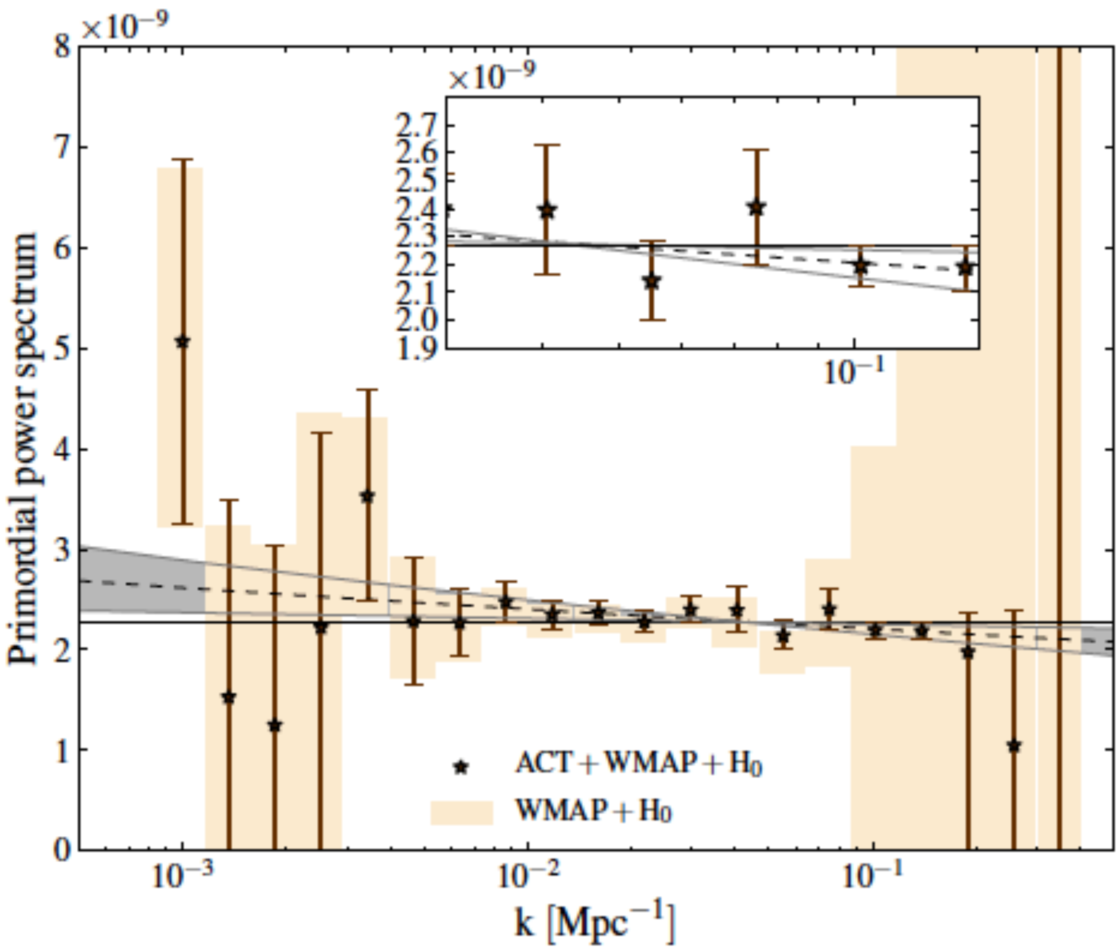

FIG. 2.— Primordial power constraints: the constraints on the primordial power spectrum from the ACT data in addition to WMAP data compared to the WMAP constraints alone. In both cases, a prior on the Hubble parameter from Riess et al. (2009) was included. Where the marginalised distributions are one-tailed, the upper errorbars show the 95% confidence upper limits. On large scales the power spectrum is constrained by the WMAP data, while at smaller scales the ACT data yield tight constraints up to $k = 0.19$ Mpc^{-1}. The horizontal solid line shows a scale-invariant spectrum, while the dashed black line shows the best-fit ΛCDM power-law with $n_s = 0.963$ from Dunkley et al. (2010), with the spectra corresponding to the 2σ variation in spectral index indicated by solid band. The constraints are summarized in Table 1.