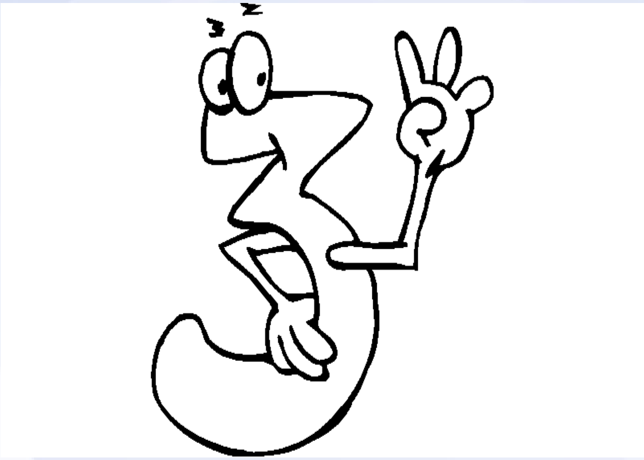
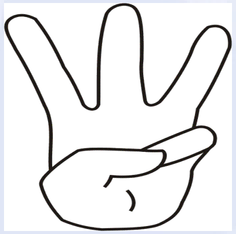
Three-form cosmology

in < 3^3 minutes

- Tomi Koivisto: Three-formal dark energy
- Nelson Nunes: Three-inflation and nongaussianity
- Federico Urban: Three-magnetic fields

BENASQUE, 2012: **Modern Cosmology**: **Early Universe**, **CMB and LSS**





Three-formalities

- Canonical action

$$S_A = \int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{48}F^2 - V(A^2)\right)$$

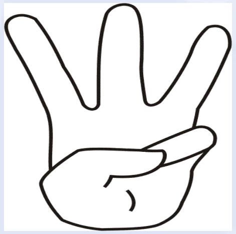
$$F_{\alpha\beta\gamma\delta} = 4\nabla_{[\alpha}A_{\beta\gamma\delta]}$$

- Always dual to a vector, $\star A_\mu = \frac{1}{6}\epsilon_{\mu\alpha\beta\gamma}A^{\alpha\beta\gamma}$

  in some cases reducible to a scalar:

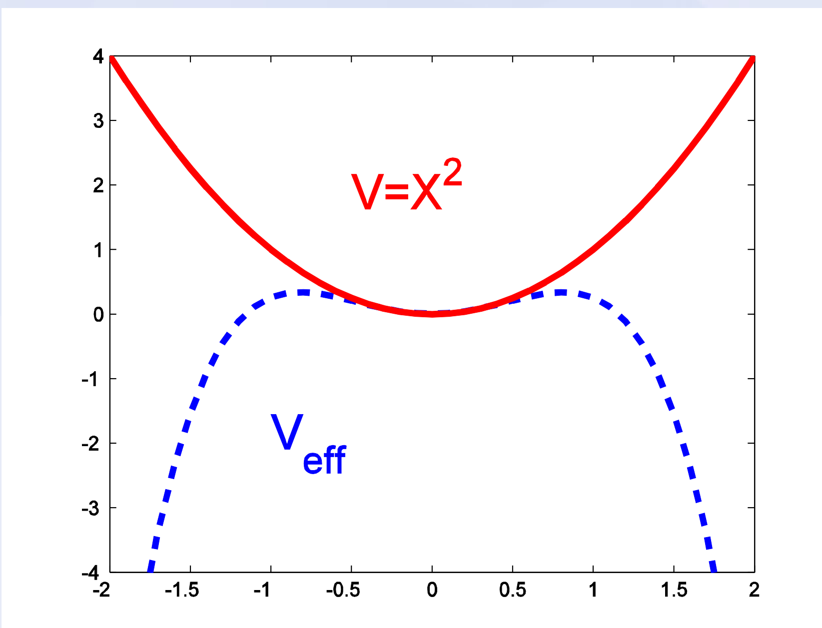
$$(dA, dA) + m^2(A, A) = (\delta \star A, \delta \star A) + m^2(\star A, \star A) \to m^2(\phi, \phi) + (d\phi, d\phi)$$

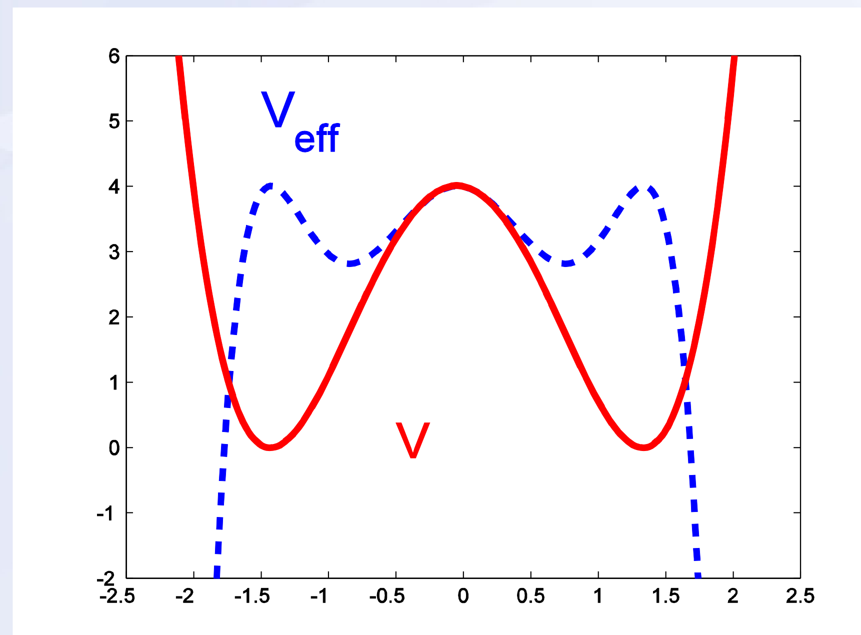
- The simplest case: $F^2 = \Lambda!$



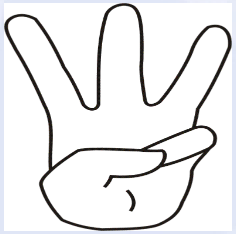
Effective potential

$$V_{effective}(X) = V(X) - \frac{3}{2}\int V'(X)XdX$$





$$V(X^2) = (X^2 - C)^2$$



Three-cosmology

- The FLRW symmetry allows only

$$A = a^3(t)X(t)dx \wedge dy \wedge dz$$

- The field X(t) then behaves as:

$$\rho_X = \frac{1}{2}\left(\dot{X} + 3HX\right)^2 + V(X),$$
$$p_X = -\frac{1}{2}\left(\dot{X} + 3HX\right)^2 + V'(X)X - V.$$

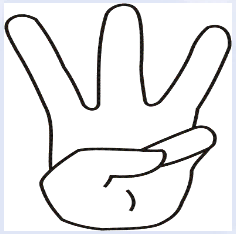
$$w_X = -1 + \frac{V_{,X}X}{\rho_X}$$

$$\ddot{X} = -3H\dot{X} - V_{,X} - 3\dot{H}X\,,$$

$$c_S^2 = \frac{V_{,XX}X}{V_{,X}}$$

- A critical point: X^2=2/3

$$H^2 = \frac{1}{3}\frac{V + \rho_B}{1 - \kappa^2(X' + 3X)^2/6}$$

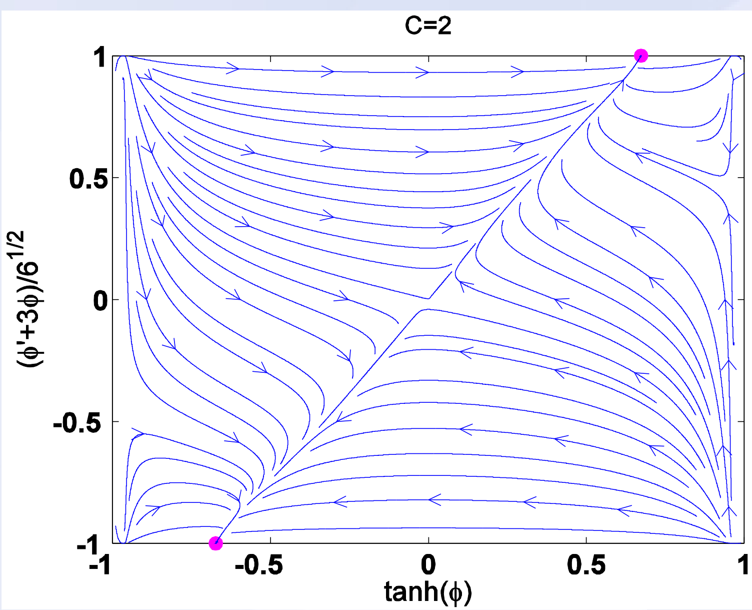


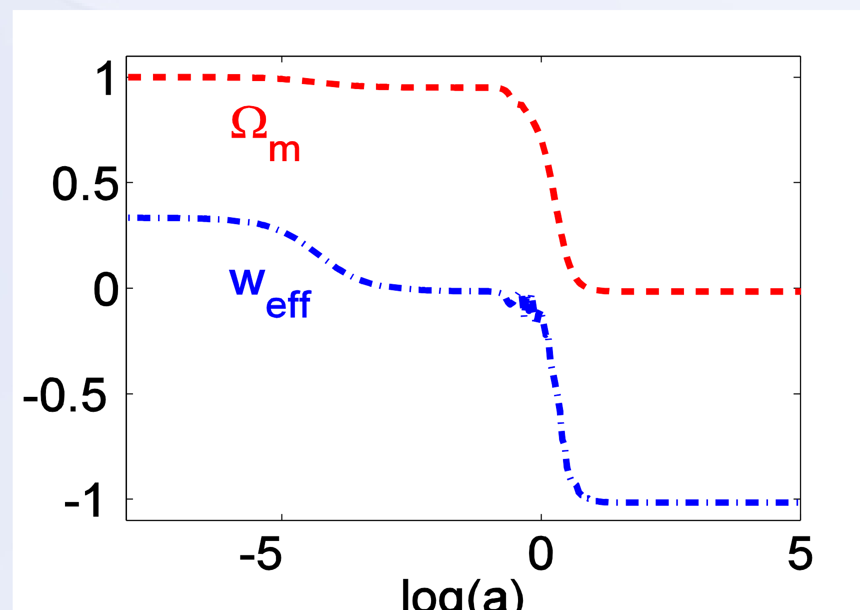
Cosmological dynamics

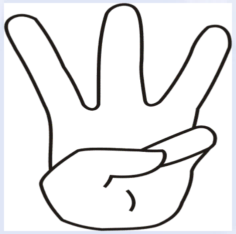
$$x \equiv \kappa X \,, \qquad y \equiv \frac{\kappa}{\sqrt{6}}(X' + 3X)\,, \qquad z^2 = \frac{\kappa^2 V}{3H^2}$$

$$w^2 \equiv \frac{\kappa^2 \rho_B}{3H^2}\,, \qquad \lambda(x) \equiv -\frac{1}{\kappa}\frac{V_{,X}}{V}\,.$$

	x	y	w	$\dot{H}/H^2$	λ	description
A	0	0	± 1	$-3\gamma/2$	any	matter domination
$B_\pm$	$\pm\sqrt{2/3}$	± 1	0	0	any	maximal point
C	$x_{\rm ext}$	$\sqrt{3/2}\,x_{\rm ext}$	0	0	0	potential extremum







Coupled three-form

- Consider coupling to CDM particles

$$\mathcal{L} = -\frac{1}{48}F^2 - V(A^2) - \Sigma_a m_a(A^2)\delta(x - x(\lambda))\sqrt{\dot{x}^2/g}$$

- Contributes to the background pressure

$$2\dot{H} + 3H^2 = 8\pi G\left[\frac{1}{2}\left(\dot{X} + 3HX\right)^2 + V - (V_{,X} + \kappa\rho_m f)X\right]$$

$$f = \frac{1}{\kappa}\log m^2{}_{,X}$$

- The matter overdensity δ evolves as:

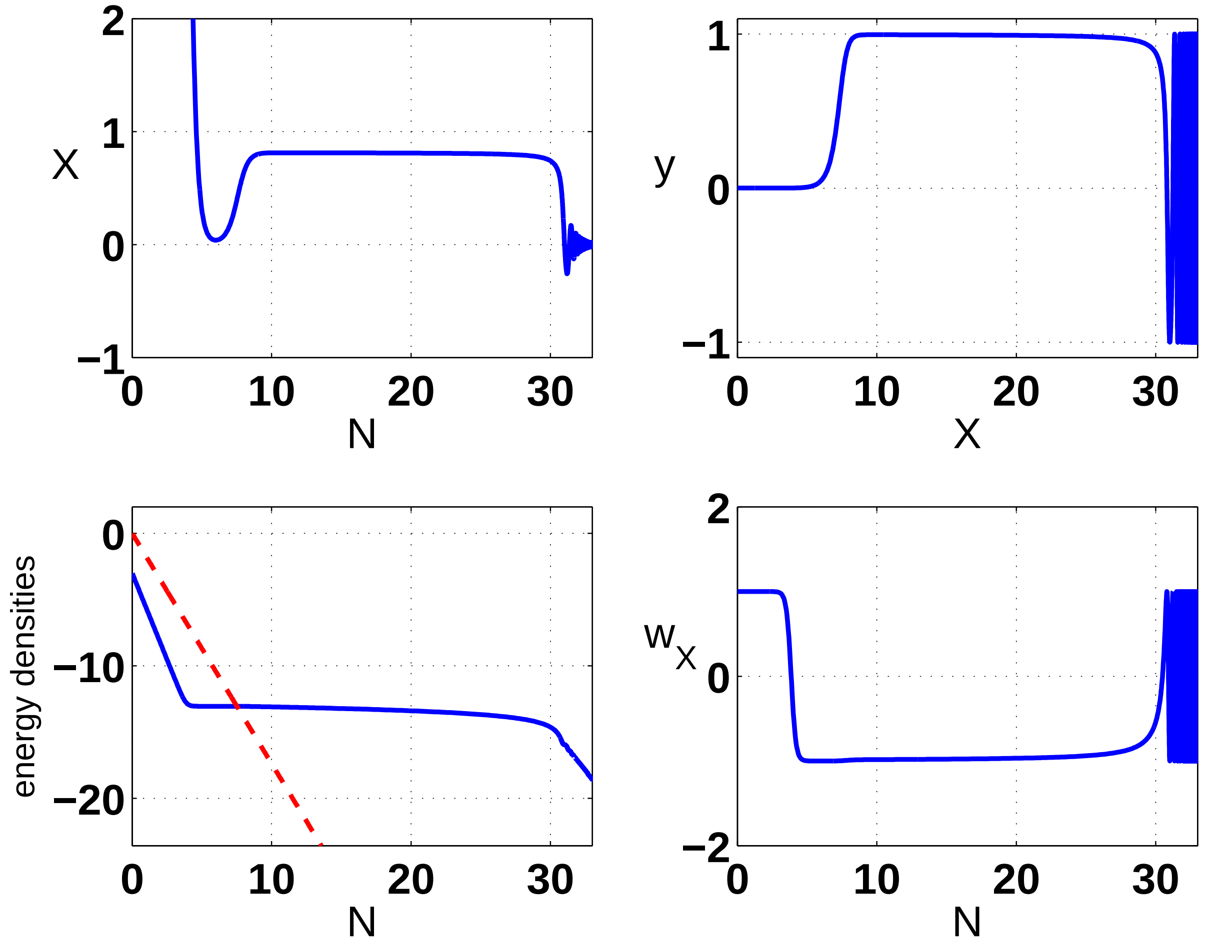
$$\ddot{\delta} + \left[2H + \frac{f\kappa\dot{X} - XF - 2\dot{F}}{1-F}\right]\dot{\delta} = 4\pi G_{eff}\rho\delta - \frac{k^2}{a^2}c^2_{eff}\delta$$

$$\frac{G_{eff}}{G} = \frac{1 + \frac{\ddot{F} + (2H+X)\dot{F}}{4\pi G\rho_m}}{1-F}$$

$$c^2_{eff} = \frac{F}{1-F}$$

$$\frac{-f^2\kappa^2\rho}{V_{,XX} + f_{,X}\kappa\rho}\delta \equiv F\delta$$

Inflation



$$H^2 = \frac{1}{3}\,\frac{V+\rho}{1-(X_{,N}+3X)^2/6}$$

$$V_{\mathrm{eff},X} = V_{,X}\left(1-\frac{3}{2}X^2\right)-\frac{3}{2}\gamma\rho_B X$$

How much inflation?

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{3}{2}\frac{V_{,X}X}{V}\left(1-\frac{3}{2}X^2\right)$$

For $V = X^n$

$$\Delta N \approx \frac{2}{9n}\,\frac{1}{2/3 - X_{\rm init}^2}$$

More inflation for $X^2 \to 2/3$

Power spectrum and consistency relation

$$\mathcal{P}_\zeta = \left(\frac{k}{aH}\right)^{3-2\nu} c_S \left(\frac{c_S}{1+\sigma}\right)^{1-2\nu} \frac{1}{32\pi^2} 2^{2\nu-1} (1-\epsilon)^{2\nu-1} \frac{\Gamma(\nu)^2}{\Gamma(3/2)^2} \frac{H^2}{|\epsilon|}$$

$$r \equiv \frac{A_T^2}{A_\zeta^2} = 16\, c_S\, |\epsilon|$$

For $V = X^n$ and 60 e-folds

$$n_s \approx 0.97$$

independent of the value of power n!

Non-gaussianity

Speed of sound

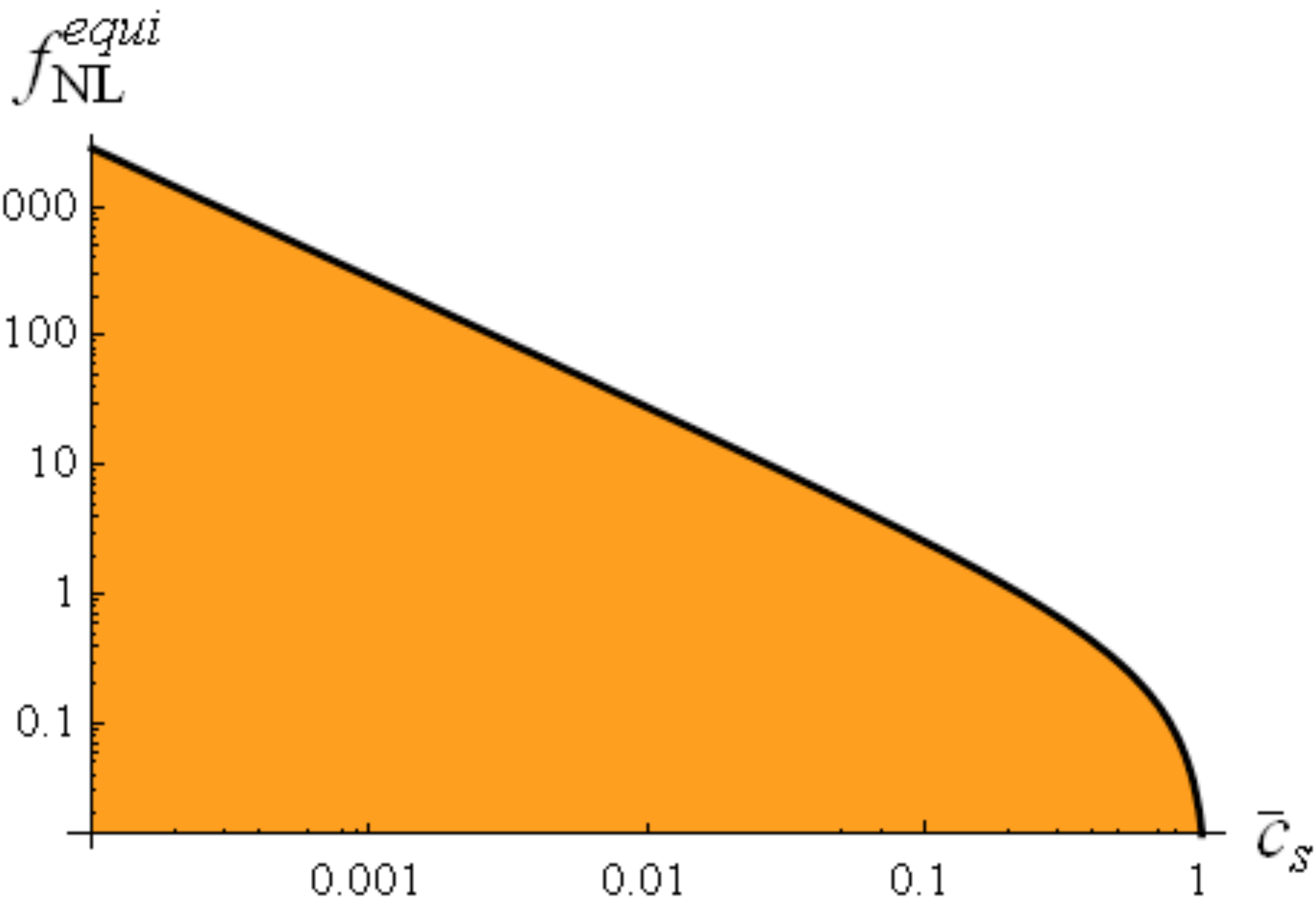
$$c_s^2 = \frac{V_{,XX} X}{V_{,X}}$$

For power law $V = X^n$

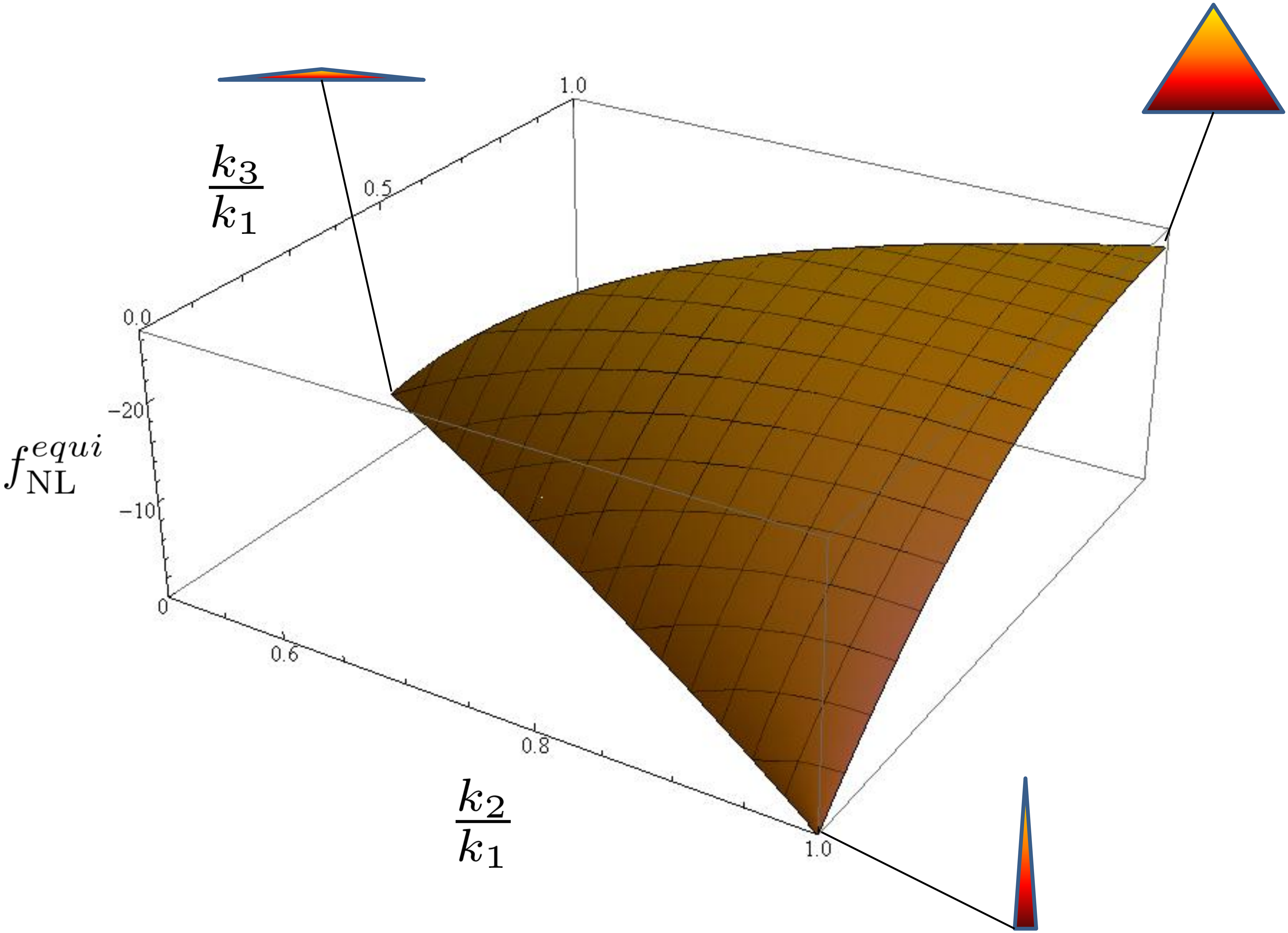
$$c_s^2 = n - 1$$

$\Rightarrow$ non-gaussianity larger for smaller c_s^2

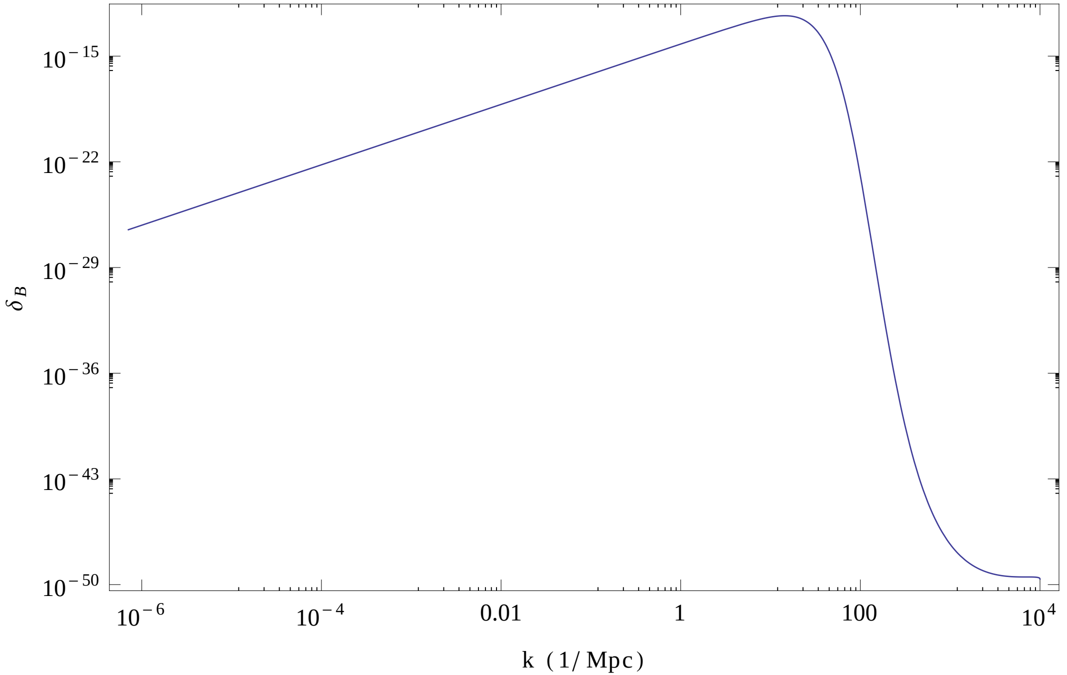
$$f_{NL} \propto \frac{1}{c_s^2}$$



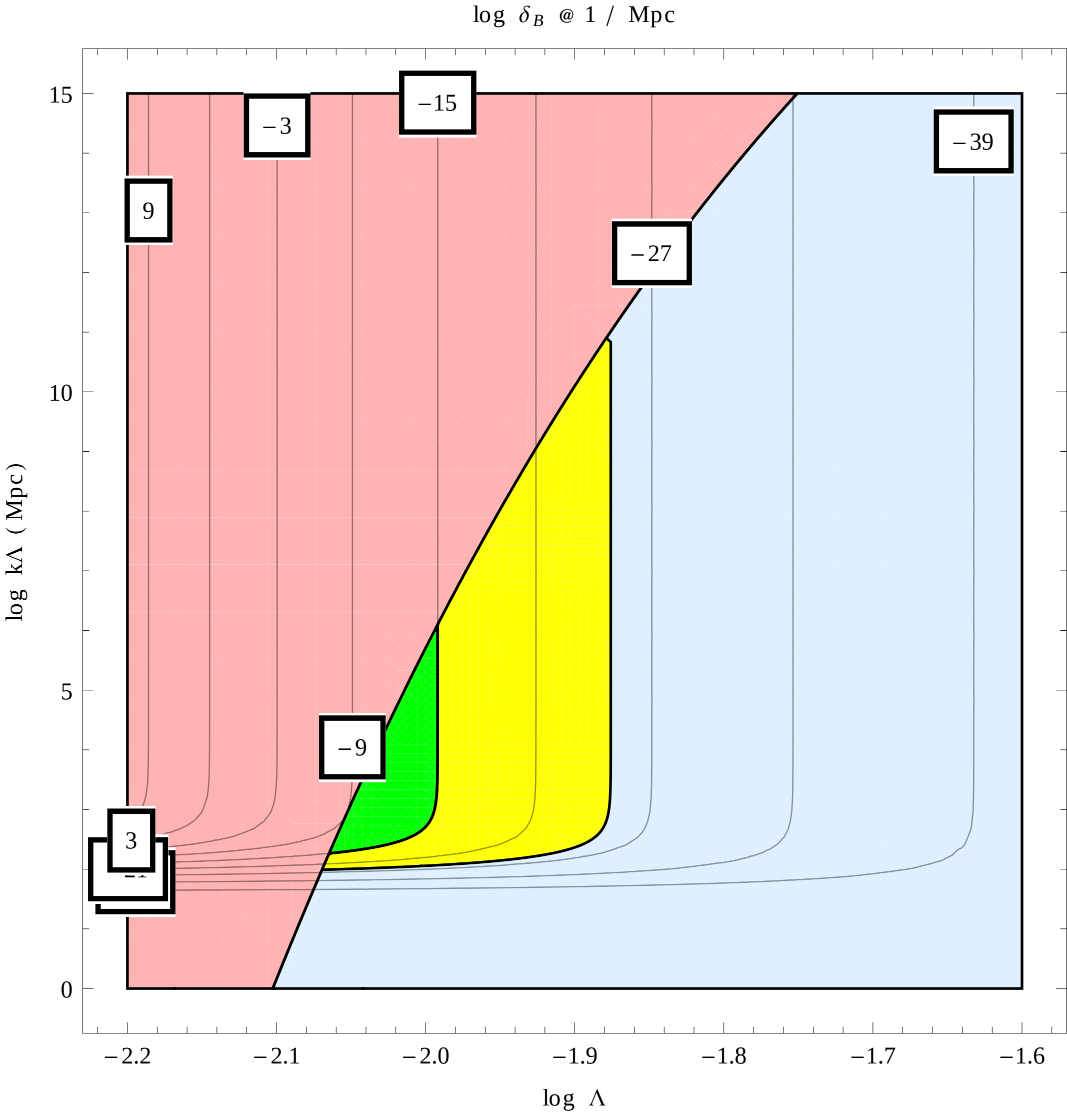
$f_{\rm NL}^{equi}$
1000
100
10
1
0.1
0.001
0.01
0.1
1
$\bar{c}_s$



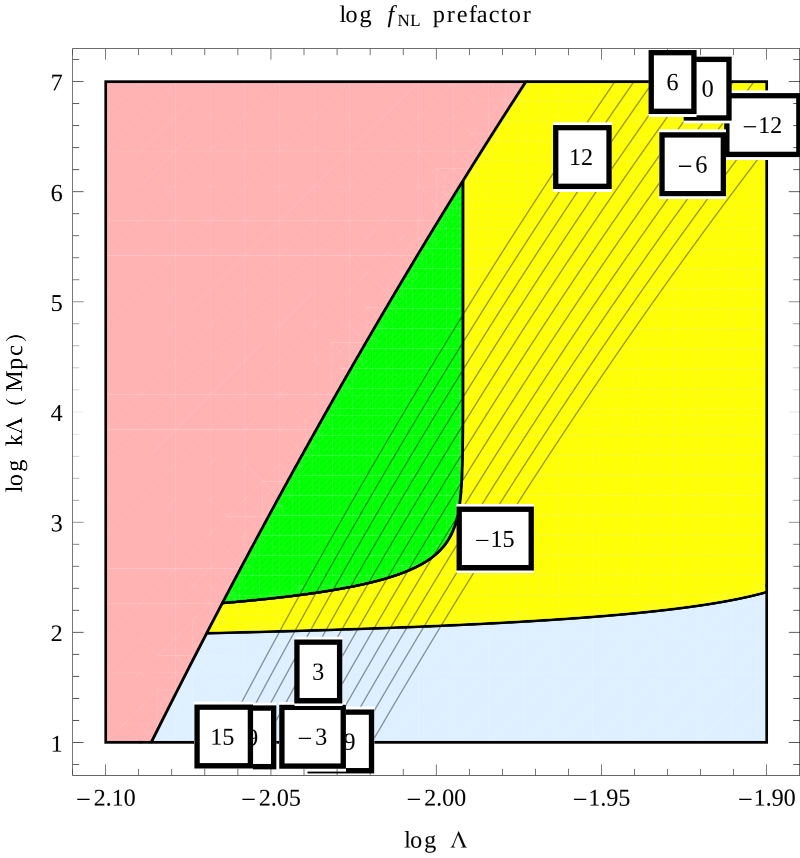
$\frac{k_3}{k_1}$
1.0
0.5
0.0
$f_{\rm NL}^{equi}$
−20
−10
0
0.6
0.8
1.0
$\frac{k_2}{k_1}$

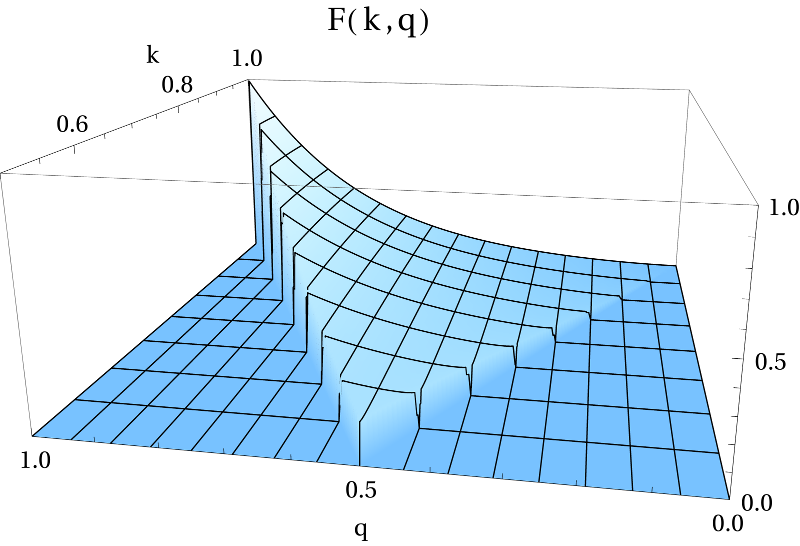


10^{-15}
10^{-22}
10^{-29}
10^{-36}
10^{-43}
10^{-50}
δ_B
10^{-6}
10^{-4}
0.01
1
100
10^{4}
k (1/ Mpc)

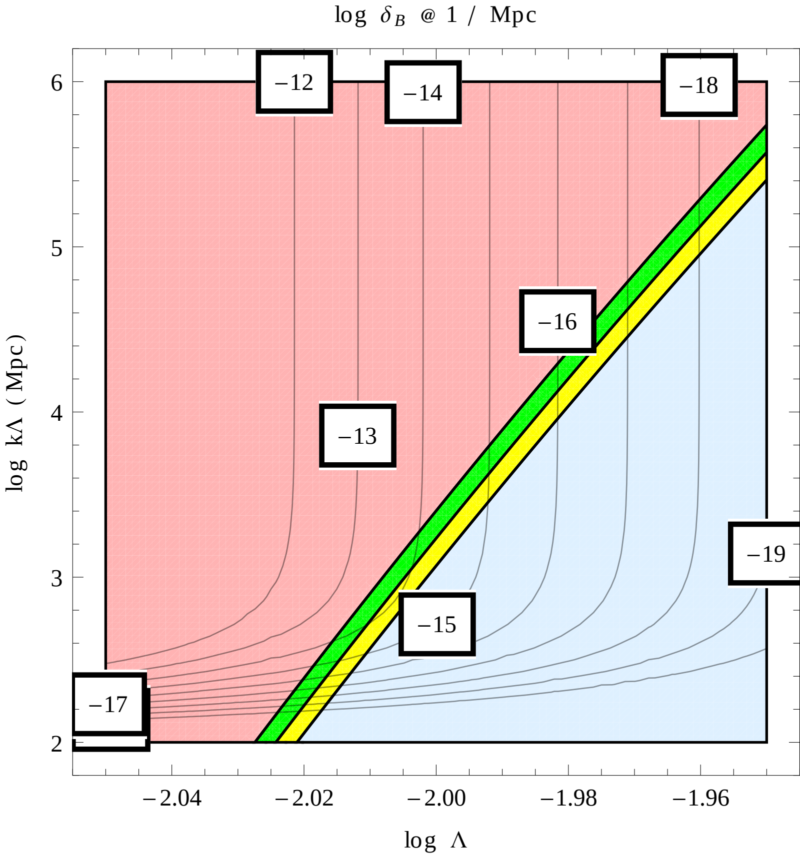


log δ_B @ 1 / Mpc
log kΛ (Mpc)
log Λ
15
10
5
0
−2.2
−2.1
−2.0
−1.9
−1.8
−1.7
−1.6
9
−3
−15
−27
−39
−9
3


log f_{NL} prefactor
log kΛ (Mpc)
log Λ
−2.10
−2.05
−2.00
−1.95
−1.90
1
2
3
4
5
6
7
6
0
−12
12
−6
−15
3
15
−3
9



F(k , q)
k
1.0
0.8
0.6
1.0
0.5
0.0
1.0
0.5
q
0.0


log δ_B @ 1 / Mpc
−12
−14
−18
−16
−13
−19
−15
−17
log kΛ (Mpc)
6
5
4
3
2
−2.04
−2.02
−2.00
−1.98
−1.96
log Λ