# A Multi-wavelength View of Galaxy Cluster Scaling Relations







# August (Gus) Evrard

Arthur F. Thurnau Professor Departments of Physics and Astronomy Michigan Center for Theoretical Physics University of Michigan

# <u>Collaborators</u>:

# Eduardo Rozo Eli Rykoff Jim Bartlett

(+ 3 X-ray groups)

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## basic ingredients for cluster cosmology from counts + clustering

I. halo space density (aka, mass function), dn(>M, z)/dV
 well calibrated (~5% in dn) by (dark matter only) simulations

2. two-point spatial clustering of halos (aka, bias function), b(M, z) – similarly well calibrated

3. population model for signal, S, used to identify clusters, p(S | M, z)– power-law with log-normal deviations (typically self-calibrated) – projection effects (signal-dependent)  $S_{observed} \neq S_{intrinsic}$ 

4. selection model for signal, S
– completeness (missed clusters)
– purity (false positives)

# observable signal choices for surveys: pros and cons

| Signal                 | Pros  | Cons  |
|------------------------|---|---|
| X-ray                  | <ul> <li>spatially compact signal<br/>(relative to other methods)</li> <li>hot thermal ICM is unique to<br/>clusters</li> <li>40+ year science history</li> </ul>     | <ul> <li>expensive (space-based)</li> <li>flux confusion from AGN</li> <li>surface brightness dimming</li> <li>most sources will have<br/>moderate S/N</li> </ul> |
| Optical                | <ul> <li>inexpensive (<u>free</u> with any galaxy survey!)</li> <li>old, `red sequence' galaxies reside in massive halos</li> <li>80+ year science history</li> </ul> | <ul> <li>confusion from line-of-<br/>sight projection</li> <li>moderate S/N (Poisson<br/>statistics for N≥10)</li> <li>galaxy formation!</li> </ul>               |
| Sunyaev-<br>Zel'dovich | <ul> <li>inexpensive (<u>free</u> w/ resolved,<br/>multi-band CMB survey)</li> <li>nearly redshift-independent<br/>signal</li> </ul>                                  | <ul> <li>point source confusion</li> <li>I-o-s projected confusion<br/>with low angular resolution</li> <li>moderate S/N for most</li> </ul>                      |

### cluster samples today are sparse relative to massive halos on the sky



Allen, Evrard & Mantz 2011

symbol size scales with median redshift

Halo mass scale is  $M_{200m}$ (h = 0.7)

Allen, Evrard & Mantz 2011

## Abell 1835 (z=0.25) seen in X-ray, optical and mm bands



1.2 Mpc

## Abell 520 (z=0.20) seen in X-ray, optical w/ lensing mass contours



## consistent cosmology from existing optical and X-ray samples



Rozo et al 2010



X-ray: 400d, BCS (lines) ~100 clusters

<u>systematics</u> limited !

#### how hard is counting? Major systematic error sources for cluster cosmology

- I. 3D halo mass is not directly observable
  - what is the form of the intrinsic signal likelihood,  $p(S_{int} | M, z)$ ?
- 2. S<sub>int</sub> is also not directly observable (the universe is a big place!)
  - how does ~Gpc sight-line projection distort **S**?
  - what is the impact on survey selection ?
- 3. Baryons (17% of matter) are dynamically complex on Mpc scales
  - do mergers lead to strong selection biases?
  - does feedback excite decaying modes on quasi-linear scales?

#### surprise from Planck stacking of optically-selected (maxBCG) clusters

#### Planck Collaboration arXiv:1101.2027



SZ decrement in maxBCG cluster sample is smaller than **model prediction** by factor >2

#### Planck model : steps from Ngal to Ysz



- \* masses from stacked weak lensing analysis
- \* optically-selected sample
- \* based entirely on SDSS data

\* masses assume hydrostatic equil'm of hot gas
\* X-ray selected samples
\* based mainly on XMM data
\* assumes Yx = Ysz (Yx = Mgas \*Tx) paper I: comparison of published cluster properties from X-ray observations

#### comparison of published **total mass** (M<sub>500c</sub>) estimates for local galaxy clusters



Rozo et al (2012) arXiv:1204.6301

y-axis shows In(M<sub>A</sub> / M<sub>B</sub>) for samples A–B listed in legend

M10: Mantz et al (2010) V09:Vihklinin et al (2009) P11-LS: Planck Coll. (2011)

median published statistical error ~5%

#### comparison of published gas mass estimates for local galaxy clusters

Rozo et al (2012) arXiv:1204.6301



similar pattern to total mass estimates reflects aperture-induced bias

M10: Mantz et al (2010) V09:Vihklinin et al (2009) P11-LS: Planck Coll. (2011)

#### comparison of published **gas mass** estimates for local galaxy clusters

Rozo et al (2012) arXiv:1204.6301



good agreement after correcting to common radial aperture

M10: Mantz et al (2010) V09:Vihklinin et al (2009) P11-LS: Planck Coll. (2011)

#### comparison of published **gas temperature** estimates for local galaxy clusters



Rozo et al (2012) arXiv:1204.6301

fewer **independent** estimates of Tx (need long exposures) => no M10-V09 comparison

MI0: Mantz et al (2010) V09:Vihklinin et al (2009) PII-LS: Planck Coll. (2011)

### comparison of published **gas thermal energy** estimates (Yx = Mgas \*Tx)

0.2 0.0 Δ  $\Delta \ln Y_{X}$ 7\' -0.2 Δ Δ Λ -0.4▲ (P11–LS)–V09 (P11–LS)–M10 RXJ0232 -0.60.1 Z

Rozo et al (2012) arXiv:1204.6301

comparison is shown **after** correcting Mgas to common aperture

MI0: Mantz et al (2010) V09:Vihklinin et al (2009) PII-LS: Planck Coll. (2011)

#### comparison of published properties for local galaxy clusters : summary table

Rozo et al (2012) arXiv:1204.6301

MEAN LOG DIFFERENCES IN X-RAY PROPERTIES FOR SAMPLE PAIRS

| Property                  | M10 - V09     | P11-LS - V09   | $	ext{P11-LS} - 	ext{M10} \\ 	ext{Low z} \ (z \leq 0.13) 	ext{}$ | ${ m P11-LS}-{ m M10}\ { m High \ z} \ (z>0.13)$ |
|---------------------------|---------------|----------------|--|--|
| $L_{X}^{a}$               | $0.12\pm0.02$ | $-0.01\pm0.02$ | $-0.12\pm0.02$   | $-0.10\pm0.03$                                   |
| $M_{\rm gas}{}^{a{ m b}}$ | $0.03\pm0.02$ | $0.03\pm0.02$  | $-0.02\pm0.03$   | $-0.04\pm0.02$                                   |
| $T_{\rm X}$               |               | $-0.13\pm0.02$ |  | $-0.14\pm0.05$                                   |
| $Y_{\rm X}{}^{ab}$        |               | $-0.15\pm0.03$ |  | $-0.19\pm0.05$                                   |
| $M_{500}$                 | $0.08\pm0.02$ | $-0.12\pm0.02$ | $-0.16\pm0.07$   | $-0.48\pm0.07$                                   |
| $M_{500}{}^{\mathrm{d}}$  | $0.22\pm0.11$ | $-0.14\pm0.03$ | $-0.25\pm0.03$   | $-0.45\pm0.06$                                   |

<sup>a</sup>Offset computed after outlier removal.

<sup>b</sup>Offset computed after correction to a common aperture.

<sup>c</sup>Relaxed/cool core only.

<sup>d</sup>Non-relaxed/no cool core only.

Note: post-publication, MI0 masses were subsequently adjusted downward by I1% due to Chandra recalibration

M10: Mantz et al (2010) V09: Vihklinin et al (2009) P11-LS: Planck Coll. (2011)

### comparison of published **gas thermal energy** estimates (Yx = Mgas \*Tx)

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Rozo et al (2012) arXiv:1204.6301

comparison is shown **after** correcting Mgas to common aperture

MI0: Mantz et al (2010) V09:Vihklinin et al (2009) PII-LS: Planck Coll. (2011)

#### paper I summary

 X-ray projects derived 3D halo mass <u>implicitly from scaling relations</u> M<sub>gas</sub>-M<sub>tot</sub> (M10) ; Y<sub>X</sub>-M<sub>tot</sub> (V09) ; Y<sub>X</sub>-M<sub>tot</sub> (P11)
 - calibrated by M<sub>hydrostatic</sub> small number (~10's) of relaxed clusters with assumption M<sub>hydrostatic</sub> = M<sub>tot</sub>
 - systematic variations ~few 10's of percent exist in M<sub>tot</sub> estimates

• observed cluster properties (<R<sub>500</sub>) from 3 groups vary

- mainly due to aperture bias (different  $R_{500}$  estimates)
- M<sub>gas</sub> shows best agreement after aperture correction
- $Y_X$  is worst ( $T_X$  estimates vary in mean)
- largest tensions between PII-LS and MI0 at z>0.13

paper II: comparison of published X-ray scaling relations + a log-normal multivariate model for consistency checks

TABLE 1 INPUT CLUSTER SCALING RELATIONS AT z = 0.23

| Relation  | $\chi_0$                    | a  | α   | $\sigma_{\ln\psi \chi}$   | Citation   | Data Set                     |
|---|-----------------------------|--|---|---|--|------------------------------|
| $L_{ m X}-M_{500} \ L_{ m X}-M_{500} \ L_{ m X}-M_{500} \ L_{ m X}-M_{500}$   | $4.8 \\ 2.0 \\ 10.0 \\ 4.0$ | $\begin{array}{c} 1.16 \pm 0.09 \\ 0.08 \pm 0.08 \\ 2.11 \pm 0.18 \\ 0.98 \end{array}$       | $1.61 \pm 0.14$<br>$1.62 \pm 0.11$<br>$1.34 \pm 0.05$<br>1.52                         | $\begin{array}{c} 0.396 \pm 0.039 \\ 0.411 \pm 0.070 \\ 0.414 \pm 0.044 \\ \end{array}$ | Vikhlinin et al. (2009)<br>Pratt et al. (2009)<br>Mantz et al. (2010b)<br>Reference  | V09<br>P11-LS<br>M10         |
| $M_{500}-Y_{ m X} = M_{500}-Y_{ m X} = M_{500}-Y_{ m X} = M_{500}-Y_{ m X} = M_{500}-Y_{ m X}$  | $3.0 \\ 2.0 \\ 10.0 \\ 4.0$ | $\begin{array}{c} 1.53 \pm 0.04 \\ 1.23 \pm 0.02 \\ 2.25 \pm 0.12 \\ 1.65 \end{array}$       | $\begin{array}{c} 0.57 \pm 0.03 \\ 0.56 \pm 0.02 \\ 0.68 \pm 0.04 \\ 0.6 \end{array}$ | $\leq 0.07^{b}$<br>$\leq 0.09^{c}$<br>$0.072 \pm 0.011$<br>—                            | Vikhlinin et al. (2009)<br>Arnaud et al. (2010)<br>Mantz et al. (2010b)<br>Reference | V09<br>P11-LS<br>M10<br>—    |
| $egin{array}{c} D_A^2 Y_{\mathrm{SZ}} - C Y_{\mathrm{X}} \ D_A^2 Y_{\mathrm{SZ}} - C Y_{\mathrm{X}} \end{array}$ | 8.0<br>10.0<br>10.0<br>10.0 | $\begin{array}{c} 1.877 \pm 0.028 \\ 2.341 \pm 0.038 \\ 2.100 \pm 0.09 \\ 2.303 \end{array}$ | $0.916 \pm 0.035$<br>$0.828 \pm 0.057$<br>1.0<br>1.0                                  | $\begin{array}{c} 0.082 \pm 0.035 \\ 0.167 \pm 0.039 \\ \leq 0.15 \\ \end{array}$       | <u>Rozo et al.</u> (2012a)<br><u>Rozo et al.</u> (2012a)<br>This work<br>Reference   | V09<br>P11-LS(z=0.23)<br>M10 |

<sup>a</sup>In all cases, we assume the  $\psi - \chi$  relation takes the form  $\langle \ln \psi \rangle = a + \alpha \ln(\chi/\chi_0)$ . Our choice of units are  $10^{14} M_{\odot}$  for mass,  $10^{44}$  ergs/s for  $L_X$ ,  $10^{14} M_{\odot}$  keV for  $Y_X$ , and  $10^{-5}$  Mpc<sup>2</sup> for  $D_A^2 Y_{SZ}$  and  $CY_X$ . Unless otherwise noted, we set  $\chi_0$  to the reference scale in the cited work. All scaling relations are evaluated at z = 0.23, the median redshift of the maxBCG cluster sample.

<sup>b</sup>Vikhlinin et al. (2009) only state that the scatter is undetectable given the errors on hydrostatic mass estimates, but that this is consistent with 7% scatter as predicted by Kravtsov et al. (2006). We implement this scatter in our analysis as a uniform prior in the variance with the maximum value quoted above.

<sup>(Arnaud et al. (2007)</sup> quote a scatter of 0.087, but provide no error bars. We implement this scatter in our analysis as a uniform prior on the variance using the maximum value quoted above.

<sup>d</sup>Uncertainty in the scatter is implemented as a uniform prior on the variance with the maximum value quoted above. The maximum value is chosen to be close to that derived from the <u>Planck Collaboration</u> (2011b) data by Rozo et al. (2012a).

#### published $L_X$ - $M_{500}$ relations from 3 independent groups

Rozo et al (2012) arXiv:1204.6292



evaluated at z=0.23 assuming self-similar redshift evolution

bands show 68% conf. regions from MC of fits params (slope, intercept, scatter) of power-law mean + log-normal scatter

MIO: Mantz et al (2010) V09:Vihklinin et al (2009) PII-LS: Planck Coll. (2011)

#### published $L_X$ - $M_{500}$ relations from 3 independent groups

Rozo et al (2012) arXiv:1204.6292



#### difference view

reference is relation defined by mean slopes and mean intercept at M=4e14

MIO: Mantz et al (2010) V09:Vihklinin et al (2009) PII-LS: Planck Coll. (2011)

#### adjusted L<sub>X</sub>-M<sub>500</sub> relations from 3 independent groups

Rozo et al (2012) arXiv:1204.6292



**adjusted** difference view (same reference)

#### adjustments:

 alignment to PII-LS : offsets in Lx, M estimates (paper I) applied to V09 and MI0\*

2) gas fraction in MIO : fgas = const model changed to fgas~M<sup>0.15</sup>

\* M10 total masses also adjusted by -11% due to Chandra recalibration

#### adjusted M500-Y<sub>X</sub> relations from 3 independent groups

Rozo et al (2012) arXiv:1204.6292



**adjusted** difference view

adjustments: I) alignment to PII-LS : offsets in Lx, M estimates (paper I) applied to V09 and MI0\*

2) gas fraction in MIO : fgas = const model changed to fgas~M<sup>0.15</sup>

\* MI0 total masses also adjusted by -II% due to Chandra recalibration

#### toward a unified model for cluster properties

- scaling relations can be aligned by accounting for aperture (mass) bias and other systematics

 simple "plug-n-play" predictions for multi-property scalings are too simplistic if covariance is appreciable and/or selection effects and/or systematic biases are important



#### 1 A Local Model for Multivariate Counts

Consider a mass function described locally as a power-law in mass with slope  $-\alpha$ . Specifically, using  $\mu \equiv \ln M$ , define the mass function,  $n(\mu, z)$ , as the likelihood of finding a halo at redshift z in the mass range  $\mu$  to  $\mu + d\mu$  within a small comoving volume dV,

$$dp \equiv n(M, z) d\ln M dV = A M^{-\alpha} d\ln M dV = A e^{-\alpha \mu} d\mu dV.$$

The local slope,  $\alpha$ , and amplitude, A, implicitly depend on mass and redshift in a manner dependent on cosmology (*e.g.*, Tinker et al. 2008).

Consider a set of N halo properties,  $S_i \in \{N_{gal}, L_X, T_X, M_{gas}, Y_X, Y_{SZ}, \dots\}$ , let **s** be a vector containing their logarithms,

$$s_i = \ln(S_i) \tag{2}$$

Assume that the mass scaling behavior of these properties are power-laws, so that the mean ln(signal) for a mass-complete sample scales as

$$\bar{\mathbf{s}}(\mu, z) = \mathbf{m}\mu + \mathbf{b}(z). \tag{3}$$

The elements of vector **m** are the slopes of the individual mass-observable relations. (Note that, at some fixed epoch, we can always choose units such that the intercepts  $b_i(z) = 0$ .)

Assume that ln(signal) deviations about the mean are Gaussian, described by a likelihood

$$p(\mathbf{s}|\mu) = \frac{1}{(2\pi)^{N/2} |\Psi|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{s} - \bar{\mathbf{s}})^{\dagger} \Psi^{-1}(\mathbf{s} - \bar{\mathbf{s}})\right],$$
(4)

where the covariance matrix has elements

$$\Psi_{ij} \equiv \langle (s_i - \bar{s}_i)(s_j - \bar{s}_j) \rangle, \tag{5}$$

and the brackets denote an ensemble average over a (large) mass-complete sample.

piecewise power-law mass function; alpha is local slope d(logn)/d(logM)

(1)

assumed form of property-mass relation

#### 1.1 Multivariate Space Density

The space density as a function of the multivariate properties, **s**, is found by the convolution,  $n(\mathbf{s}) = \int d\mu n(\mu) p(\mathbf{s}|\mu)$ . Using equations (1) and (4), the result is

$$n(\mathbf{s}) = rac{A\Sigma}{(2\pi)^{(N-1)/2} |\Psi|^{1/2}} \exp{\left[-rac{1}{2}(\mathbf{s}^{\dagger}\Psi^{-1}\mathbf{s} - rac{ar{\mu}^2(\mathbf{s})}{\Sigma^2})
ight]},$$

where  $\Sigma^2$  is the **multi-property mass variance** defined by

$$\Sigma^2 = (\mathbf{m}^{\dagger} \Psi^{-1} \mathbf{m})^{-1},$$

and the mean mass is

$$\bar{\mu}(\mathbf{s}) = \frac{\mathbf{m}^{\dagger} \Psi^{-1} \mathbf{s}}{\mathbf{m}^{\dagger} \Psi^{-1} \mathbf{m}} - \alpha \Sigma^{2}, \qquad (8)$$
$$\equiv \bar{\mu}_{0}(\mathbf{s}) - \alpha \Sigma^{2}. \qquad (9)$$

The first term,  $\bar{\mu}_0(\mathbf{s})$ , is the mean mass for the case of a flat mass function,  $\alpha = 0$ , which corresponds to the mass expected from inverting the input log-mean relation.

The second term,  $\alpha \Sigma^2$ , represents the mass shift induced by asymmetry in the convolution when  $\alpha > 0$ . (Low mass halos scattering up outnumber high mass systems scattering down.) Note that **the magnitude of this effect scales with the variance**, not the rms deviation.

Applying Bayes' theorem in the form  $p(\mu|\mathbf{s}) = p(\mathbf{s}|\mu)n(\mu)/n(\mathbf{s})$  leads to the result that the set of masses selected by a specific set of properties is Gaussian in the log with mean given by equation (9) and variance, equation (7).

**exact** form for multi-property space density

(6)

(7)

mean mass selected by signals is biased low (Malmquist bias)

#### 1.1.1 Explicit expressions for the one-variable case

For a single property,  $s \equiv \ln(S)$ , with slope, m, and logarithmic scatter at fixed mass,  $\sigma$ , the mass variance at fixed S is

$$\Sigma^2 = \left(\frac{\sigma}{m}\right)^2. \tag{10}$$

The mean mass for a sample complete in S is

$$\bar{\mu}(s) = \frac{s}{m} - \alpha \Sigma^2. \tag{11}$$

The property space density function is

$$n(s) ds = (A/m) \exp\{-\alpha \left(\frac{s}{m} - \alpha \Sigma^2/2\right)\} ds,$$
(12)

which is a power-law in the original property,  $n(S) \propto S^{-(\alpha/m)}$ .

Note that the effective shift in mass,  $\alpha \Sigma^2/2$ , is half that in the expression above. These expressions are consistent, in that they address different questions. Equation (11) gives the mean ln(mass) of a signal-selected sample while equation (12) gives the ln(mass) value that matches the local space density – in number per volume per ln(S) – of halos with property value, S.

cosmology astrophysics

#### two properties

#### 1.1.2 Explicit expressions for the two-variable case

For two properties, we introduce the correlation coefficient,  $r \equiv \langle \delta_1 \delta_2 \rangle$ , of the normalized deviations,  $\delta_i \equiv (s_i - \bar{s}_i)/\sigma_i$ , and write the covariance matrix,

 $\Psi = \left( egin{array}{cc} \sigma_1^2 & r\sigma_1\sigma_2 \ r\sigma_1\sigma_2 & \sigma_2^2 \end{array} 
ight),$ 

and its inverse,

$$\Psi^{-1} = (1 - r^2)^{-1} \begin{pmatrix} \frac{1}{\sigma_1^2} & -\frac{r}{\sigma_1 \sigma_2} \\ -\frac{r}{\sigma_1 \sigma_2} & \frac{1}{\sigma_2^2} \end{pmatrix}.$$

The mass variance is now a harmonic mixture

$$\Sigma^{-2} = (1-r^2)^{-1} \left( \sigma_{\mu 1}^{-2} + \sigma_{\mu 2}^{-2} - 2r\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1} 
ight),$$

where  $\sigma_{\mu i} = \sigma_i/m_i$  is the mass scatter at fixed signal  $S_i$ .

The zero-slope mean mass is

$$\bar{\mu}_0(s_1, s_2) = \frac{(s_1/m_1)\sigma_{\mu 1}^{-2} + (s_2/m_2)\sigma_{\mu 2}^{-2} - r(s_1/m_1 + s_2/m_2)\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1}}{\sigma_{\mu 1}^{-2} + \sigma_{\mu 2}^{-2} - 2r\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1}},$$
(14)

and the joint space density is

$$n(s_1, s_2) = \frac{A\Sigma}{\sqrt{2\pi(1 - r^2)}\sigma_1\sigma_2} \exp\left[-\alpha\bar{\mu}_0 + \frac{\Sigma^2}{2}\left(\alpha^2 - \frac{(s_1/m_1 - s_2/m_2)^2}{\sigma_{\mu_1}^2\sigma_{\mu_2}^2}\right)\right].$$
 (15)

The first two terms in the exponent are analogous to those in the 1D expression, equation (12). For "reasonable" choices of  $(S_1, S_2)$  pairs — meaning values that pick out comparable mass scales,  $s_1/m_1 \sim s_2/m_2$  — the space density remains effectively power-law. The third term in the exponent suppresses the number density for unreasonable pairings of  $s_1/m_1$  and  $s_2/m_2$ , those lying out in the wings of the bivariate Gaussian.

anti-correlated signals best for mass selection

(13)

# mass scatter for two-property joint selection



#### 1.2 Property-selected samples

For a halo sample selected with some property,  $s_1$ , we can now use Bayes' theorem to find the joint probability of those halos having a second property,  $s_2$ , and mass,  $\mu$ . The result can be expressed as a bivariate Gaussian in terms of the two-element vector,  $\mathbf{t} = [s_2 \ \mu]$ ,

$$p(\mathbf{t}|s_1) = \frac{1}{(2\pi)|\tilde{\Psi}|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{t}-\bar{\mathbf{t}})^{\dagger}\tilde{\Psi}^{-1}(\mathbf{t}-\bar{\mathbf{t}})\right],$$
(16)

where the mean mass,  $\bar{\mu}(s_1)$ , is defined by equation (11) and the mean of the non-selection property is given by

$$\bar{s}_2(s_1) = m_2 \left( \bar{\mu}(s_1) + \alpha r \sigma_{\mu 1} \sigma_{\mu 2} \right).$$
(17)

Note that, if r < 0, the non-selected property mean can be "doubly" biased low relative to a simple  $m_2(s_1/m_1)$  expectation, with one shift coming from the extra  $(-\alpha \Sigma^2)$  term in the mean mass and the second coming from the second term in the above expression.

The covariance in  $s_2$  and  $\mu$  at fixed  $s_1$  is given by

$$\tilde{\Psi} = \begin{pmatrix} \sigma_{21}^2 & \tilde{r}\sigma_{21}\sigma_{\mu 2} \\ \tilde{r}\sigma_{21}\sigma_{\mu 2} & \sigma_{\mu 2}^2 \end{pmatrix},$$

where the variance in  $s_2$  at fixed  $s_1$  is

$$\sigma_{21}^2 \;=\; m_2^2 \left( \sigma_{\mu 1}^2 + \sigma_{\mu 2}^2 - 2 r \sigma_{\mu 1} \sigma_{\mu 2} 
ight).$$

<u>future program</u>: combine large samples to extract signal covariance

mean of 2nd property for sample selected on 1st property

variance in 2nd property

(18)

## RASS X-ray stacking analysis of maxBCG sample

Rykoff et al 2008

variance in Lx at fixed Ngal



#### model exercise to derive Ysz-M scaling



Rozo et al (2012) arXiv:1204.6292

Use model to combine <u>published</u> relations for <M |Yx> and <Ysz |Yx> to derive <Ysz | M>

difference view using reference w/ self-similar slope (5/3) and mean amplitude of 3 works

PII-LS (z=0.23) uses Ysz-Yx for 0.13<z<0.3 only (maxBCG z-range) magenta line gives full sample result

#### additional constraint from number counts (`abundance matching')

Rozo et al (2012) arXiv:1204.6292

![](_page_35_Figure_2.jpeg)

Use WMAP7 halo mass function convolved with <u>published</u> Lx-M relations

Compare to X-ray luminosity function published for the local REFLEX sample (median z = 0.08)  published X-ray scalings have moderate tension that improves after aperture + other bias adjustments

• we introduce a power-law plus log-normal covariance model for multiple cluster properties (including true mass) as an improvement to direct substitution of mean relations

- corrections for means may depend on local slope of mass function
- scatter in property B includes covariance with selection property A

• abundance constraint adds additional constraining power

– largest tension with PII-LS Lx-M scaling

paper III: closing the loop a return to the Planck maxBCG result

#### mis-centering of optical clusters is a 10-30% effect

Rozo et al (2012) arXiv:1204.6305

#### Biesiadzinski, T. et al (2012) arXiv:1201.1282B

#### TABLE 1 $Y_{SZ}-N_{200}$ Data

| $N_{200}$   | $D_A^2 Y_{ m SZ}/(10^{-5}~{ m Mpc}^{-2})$   | Centering Correction  |
|---|---|---|
| $10-13 \\ 14-17 \\ 18-24 \\ 25-32 \\ 33-43 \\ 44-58 \\ 59-77 \\ 78-104$ | $\begin{array}{c} 0.058 \pm \ 0.012 \\ 0.107 \pm \ 0.020 \\ 0.222 \pm \ 0.028 \\ 0.394 \pm \ 0.044 \\ 0.692 \pm \ 0.074 \\ 1.205 \pm \ 0.130 \\ 1.876 \pm \ 0.241 \\ 4.594 \pm \ 1.009 \end{array}$ | $\begin{array}{c} 0.74 \pm \ 0.13 \\ 0.77 \pm \ 0.11 \\ 0.80 \pm \ 0.08 \\ 0.82 \pm \ 0.08 \\ 0.84 \pm \ 0.07 \\ 0.86 \pm \ 0.07 \\ 0.87 \pm \ 0.07 \\ 0.89 \pm \ 0.09 \end{array}$ |

<sup>a</sup> The data in the first two columns is from Planck Collaboration (2011c, P11-opt), after being corrected for the effects of cluster miscentering following Biesiadzinski et al. (2012) (third column). The uncertainty in the corrections is added in quadrature to the observational errors. maxBCG lensing analysis includes estimate of miscentering derived from ADDGALS-based mock catalogs

Biesiadzinski et al use this model to simulate effect of mis-centering on SZ measurements

#### Ysz-N200 scalings using scaling results from paper II

Rozo et al (2012) arXiv:1204.6305

![](_page_39_Figure_2.jpeg)

- Planck measurements corrected for mis-centering

- reference used in RHS is published Planck observed relation

- all models are in tension with the observations

#### Ysz-N200 scalings : potential resolution

Rozo et al (2012) arXiv:1204.6305

![](_page_40_Figure_2.jpeg)

# Proposed resolution: mass estimate biases

15% (21% at R500) bias in
 hydrostatic masses (estimates are biased low)

– 10% reduction in maxBCG
lensing masses measurements
published in Rozo et al (2009)
(~Isigma systematic error)

#### recent hydrostatic mass bias from gas dynamic simulations

Sembolini et al (2012) arXiv:1207.4438

![](_page_41_Figure_2.jpeg)

## Marenostrum-MultiDark SImulations (MUSIC)

histogram of fractional error (M<sub>HS</sub> - M<sub>true</sub>) / M<sub>true</sub> using 3D information

ALL simulations studies have shown this effect (since Evrard 1990 with ~500 particles/halo !)

| TABLE 4   |     |    |         |           |  |  |
|-----------|-----|----|---------|-----------|--|--|
| Preferred | Set | OF | SCALING | Relations |  |  |

| Relation   | $\chi_0$                                | Amplitude $(a_{\psi \chi})$   | $lpha_{\psi \chi}$   | $\sigma_{\ln\psi \chi}$  | Sample                     |
|--|---|---|--|--|----------------------------|
| $L_{ m X}\!\!-\!\!M \ D_A^2 Y_{ m SZ}\!\!-\!\!M$       | $\begin{array}{c} 4.4\\ 4.4\end{array}$ | $\begin{array}{c} 0.72\pm 0.07 (ran)\pm 0.16(sys) \ 0.87\pm 0.06(ran)\pm 0.17(sys) \end{array}$ | $\begin{array}{c} 1.55 \pm 0.09 \\ 1.71 \pm 0.08 \end{array}$              | $\begin{array}{c} 0.39 \pm 0.03 \\ 0.15 \pm 0.02 \end{array}$                  | V09+maxBCG<br>V09+maxBCG   |
| $M{-}N_{200}\ L_{ m X}{-}N_{200}\ Y_{ m SZ}{-}N_{200}$ | 40<br>40<br>40                          | $egin{array}{c} 0.75 \pm 0.10 \ 0.04 \pm 0.10 \ -0.24 \pm 0.20 \end{array}$                     | $egin{array}{c} 1.06 \pm 0.11 \ 1.63 \pm 0.08 \ 1.97 \pm 0.10 \end{array}$ | $\begin{array}{c} 0.45 \pm 0.10 \\ 0.83 \pm 0.10 \\ 0.70 \pm 0.15 \end{array}$ | maxBCG<br>maxBCG<br>maxBCG |
| $Y_{ m SZ}-L_{ m X}$                                   | 1.0                                     | $-0.29\pm0.06$  | $1.10\pm0.03$  | $0.40\pm0.05$  | P11-X                      |

<sup>a</sup> X-ray luminosity is measured in the [0.1, 2.4] keV band in units of  $10^{44}$  ergs/s.  $D_A^2 Y_{SZ}$  is in units of  $10^{-5}$  Mpc<sup>2</sup>. The maxBCG scaling relations are bias-corrected, while the V09+maxBCG relations are the joint constraint from the bias-corrected V09 and maxBCG samples. Scaling relations involving mass include a  $\pm 10\%$  systematic uncertainty in the mass. The error in the amplitude of the  $Y_{SZ}-L_X$  relation is larger than that quoted in P11-X because we include the uncertainty in our systematic corrections. This set of scaling relations is fully self-consistent.

#### abundance test of preferred scalings

Rozo et al (2012) arXiv:1204.6305

![](_page_43_Figure_2.jpeg)

#### consistency check:

 maxBCG number counts convolved with Lx-N200 relation

halo mass function
convolved with V09
(adjusted) Lx-M relation

#### Rozo et al (2012) arXiv:1204.6305

| TABLE 6             |        |         |     |           |        |     |        |
|---------------------|--------|---------|-----|-----------|--------|-----|--------|
| Mass                | Offset | Between | OUR | Predicted | MASSES | AND | VALUES |
| FROM THE LITERATURE |        |         |     |           |        |     |        |

| Work                        | $\ln{\langle M L_X\rangle} - \ln{M_{lit}}$ | No. of Clusters<br>in Sample |
|-----------------------------|--|------------------------------|
| V09 ( $z \le 0.3$ )         | $0.11 \pm 0.04$                            | 49                           |
| V09 $(z \ge 0.3)$           | $0.22 \pm 0.05$                            | 36                           |
| M10                         | $0.02 \pm 0.03$                            | 95                           |
| P11 $(z \le 0.13)$          | $0.09 \pm 0.05$                            | 24                           |
| P11 $(z > 0.13)$            | $0.21 \pm 0.04$                            | 38                           |
| Okabe et al. (2010)         | $0.53 \pm 0.07$                            | 21                           |
| Mahdavi et al. (2008)       | $0.22 \pm 0.12$                            | 11                           |
| Hoekstra et al. (2011)      | $0.13 \pm 0.13$                            | 25                           |
| <u>Umetsu et al.</u> (2011) | $-0.42\pm0.14$                             | 5                            |

<sup>a</sup> Mean mass offset between our predicted masses using  $L_X$ , and those reported in the literature. All means are inverse-variance weighted.

• the Planck Ysz-N200 stacked scaling discrepancy can be resolved by a combination of

- biases in hydrostatic (~20%) and lensing (~10%) mass estimates
- mis-centering of optically selected clusters from halo/gas centers

• a set of compromise scaling relations (power-law plus log-normal scatter) are proposed for several relations involving {M, Lx, N200, Ysz}

• abundance constraint demonstrates internal consistency of these relations

• tensions with some published Lx-M relations are identified