Cosmological constraints on $f(R)$ gravity

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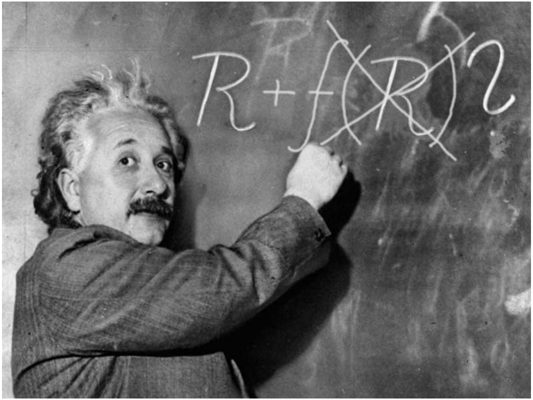
Modern Cosmology

Benasque
Aug 20, 2012

Layout

$f(R)$ gravity

Einstein's biggest blunder?



$f(R)$ gravity action

- Add an arbitrary function of the Ricci scalar R to the Einstein-Hilbert action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}[R + f(R)] + \int d^4x \sqrt{-g}\mathcal{L}_{\mathrm{m}}$$

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- Modified Einstein equation for metric $f(R)$ gravity

$$G_{\mu\nu} + f_R R_{\mu\nu} - \left(\frac{f}{2} - \Box f_R\right) g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R = \kappa^2 T_{\mu\nu}$$

Designer model

- Friedmann equation

$$H^2 - f_R(HH' + H^2) + \frac{f}{6} + H^2 f_{RR} R' = \frac{\kappa^2}{3}\rho$$

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$$H^2 = \Omega_{\mathrm{m}} a^{-3} + (1 - \Omega_{\mathrm{m}})$$

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$$H^2 = \Omega_{\rm m} a^{-3} + (1 - \Omega_{\rm m})$$

- Equating with the matter-dominated Friedmann equation

$$f'' - \left[1 + \frac{H'}{H} + \frac{R''}{R'}\right] f' + \frac{R'}{6H^2} f = -H_0^2 (1 - \Omega_{\rm m}) \frac{R'}{H^2}$$

Layout

Hu-Sawicki model [Hu, Sawicki (2007)]

- Functional form:

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1},$$

where $m^2 \equiv \kappa^2 \bar{\rho}_{\rm m}/3$. For $(n = 1)$ and $(|f_{R0}| \ll 1)$

$$f(R) \simeq -2\Lambda - f_{R0} \frac{\bar{R}_0^2}{R}$$

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- Mimics ΛCDM background history for $|f_{R0}| \ll 1$

Solar-system constraints

Spherically symmetric isotropic metric around $r = 0$

$$ds^2 = -[1 + 2\Psi(r)]dt^2 + [1 + 2\Phi(r)]d\mathbf{x}^2$$

Parametrization of deviation from general-relativistic metric

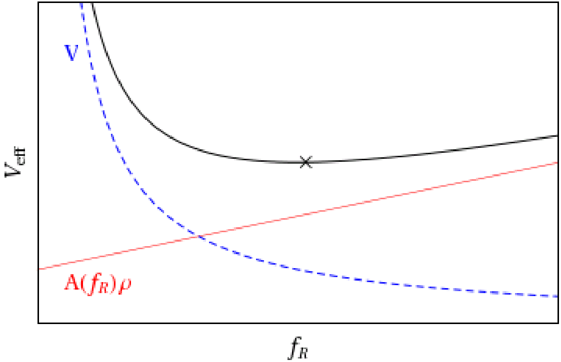
$$\gamma = -\frac{\Phi}{\Psi}$$

Solar-system constraints

$$|f_{Rg}| \lesssim |\gamma_s - 1||\Psi_s| \approx 10^{-5} \times 10^{-6}$$

from Cassini mission.

Chameleon mechanism



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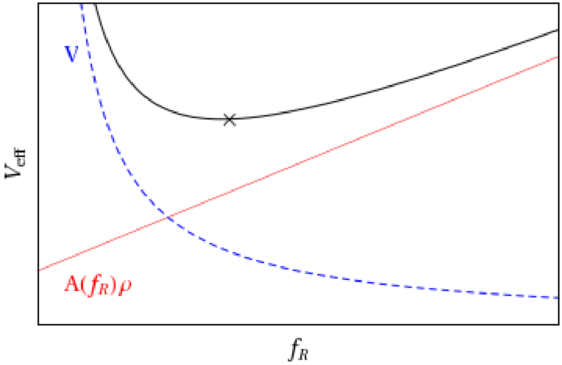

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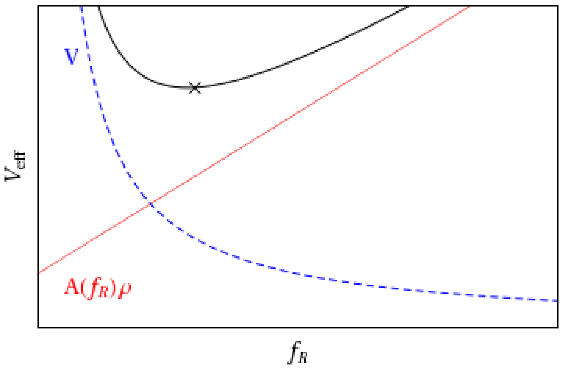

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- Mimics ΛCDM background history for $|f_{R0}| \ll 1$
- Transition between high galactic and low large-scale curvature $\rightarrow |f_{R0}| \lesssim 10^{-6}$.

Layout

N-body simulations

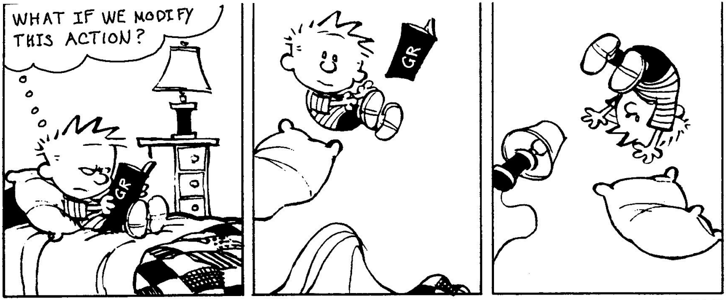
Full chameleon simulations:

$$
\begin{aligned}
\nabla^2 \delta f_R &= \frac{a^2}{3}\left[\delta R(f_R) - \kappa^2 \delta\rho_{\rm m}\right], \\
\nabla^2 \Psi &= \frac{2\kappa^2}{3} a^2 \delta\rho_{\rm m} - \frac{a^2}{6}\delta R(f_R),
\end{aligned}
$$

where $\delta f_R = f_R(R) - f_R(\bar{R})$, $\delta R = R - \bar{R}$, and $\delta\rho_{\rm m} = \rho_{\rm m} - \bar{\rho}_{\rm m}$
Linearized approximation in Fourier space

$$
k^2 \Psi(\mathbf{k}) = -\frac{\kappa^2}{2}\left[\frac{4}{3} - \frac{1}{3}\frac{1}{(\lambda_C k/2\pi a)^2 + 1}\right] a^2 \delta\rho_{\rm m}(\mathbf{k}).
$$

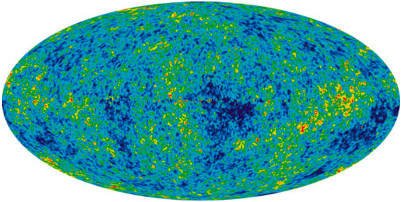
Are $f(R)$ modifications of general relativity really viable?



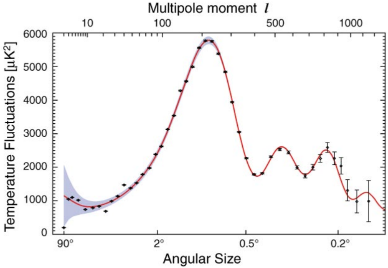
Layout

1. $f(R)$ gravity
   - Hu-Sawicki model
   - N-body simulations
2. Linear structures
   - Cosmic microwave background
   - Galaxy-ISW cross correlations
   - E_G measurement
3. Nonlinear structures
   - Abundance of clusters
   - Density profiles of clusters
4. Conclusion and Outlook

Layout



[NASA]


Multipole moment l
10
100
500
1000
6000
5000
4000
3000
2000
1000
0
Temperature Fluctuations [μK²]
90°
2°
0.5°
0.2°
Angular Size

[NASA]

Late-time ISW

ISW contribution to CMB:

$$C_\ell^{II} \propto \int da\, d\tilde{a}\, dk\, k^2 \frac{dG}{da} \frac{d\tilde{G}}{d\tilde{a}} j_\ell(k\chi) j_\ell(k\tilde{\chi}) P_{\zeta_i}$$

Potential growth rate, $G = \frac{\Phi_-(a,k)}{\Phi_-(a_i,k)} = \frac{\Phi(a,k)-\Psi(a,k)}{\Phi(a_i,k)-\Psi(a_i,k)}$, differs in modified gravity scenarios.

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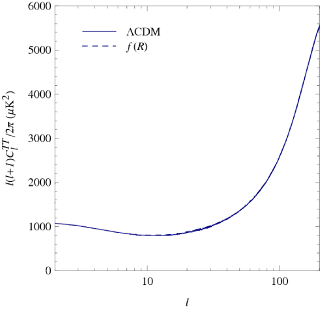
f(R) gravity: enhancement of growth



$|f_{R0}| = 0.00$ (equiv. ΛCDM)

Late-time ISW

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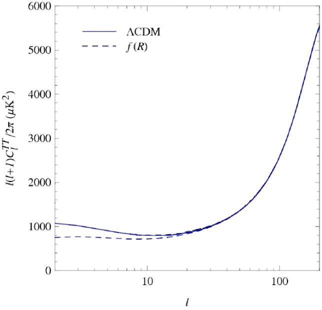
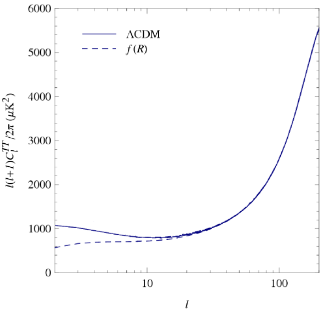


$|f_{R0}| \sim 0.05$

Late-time ISW

f(R) gravity: enhancement of growth



$|f_{R0}| \sim 0.15$

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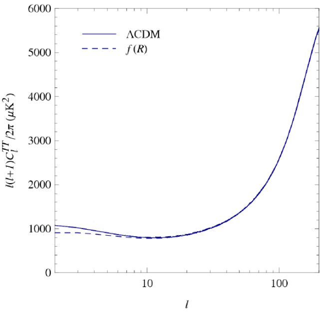

$|f_{R0}| \sim 0.30$

Late-time ISW

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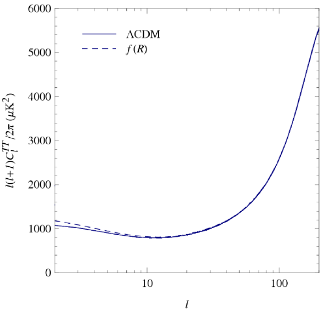

$|f_{R0}| \sim 0.35$

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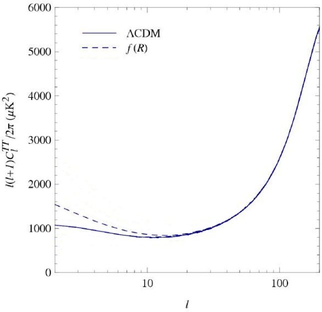

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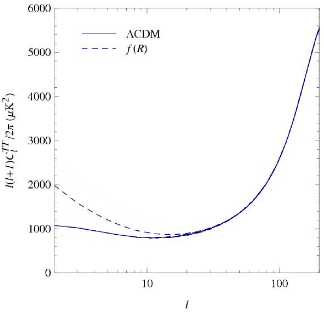
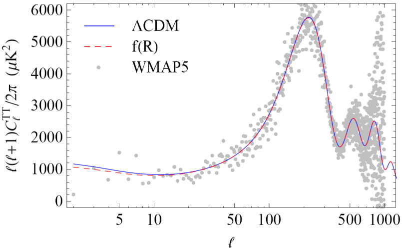


$|f_{R0}| \sim 0.45$

Late-time ISW

f(R) gravity: enhancement of growth



Best fit: $|f_{R0}| = 0.05$ $(\Delta\chi^2 = -1)$

$|f_{R0}| < 0.35$ (95%CL)

[L, Slosar, Seljak, Hu (2010)]

(WMAP5, ACBAR, CBI, VSA, Union, SHOES, BAO)

$|f_{R0}| \lesssim 0.50$ (95%CL)

[Song, Peiris, Hu (2007)]

(WMAP3)

Layout

Galaxy-ISW cross correlations:

$$C_\ell^{g_j I} \propto \int dz\, d\tilde{z}\, dk\, \frac{k^4}{\Omega_\mathrm{m} H_0^2} \frac{dG}{dz} \tilde{D} \tilde{f}_j(z) j_\ell(k\chi) j_\ell(k\tilde{\chi}) P_{\zeta_i}$$

Potential growth rate G and density growth rate, $D = \frac{\Delta(a,k)}{\Delta(a_i,k)} a_i$, differ in modified gravity scenarios.

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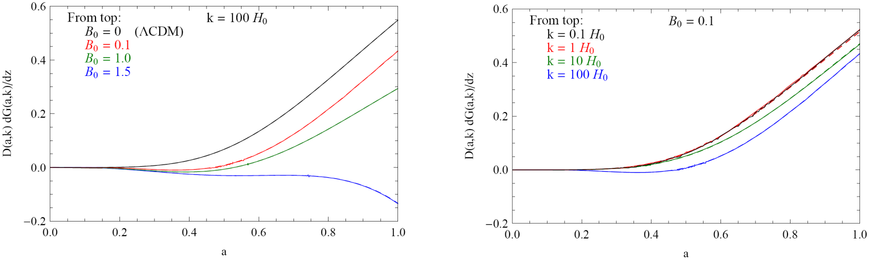
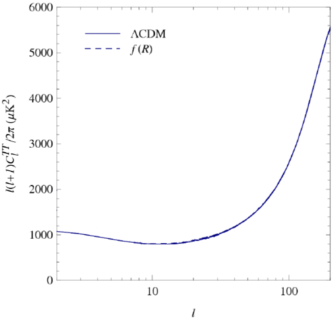
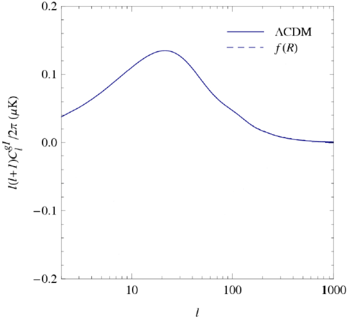
f(R) gravity: suppressed galaxy-ISW cross correlations



ISW vs galaxy-ISW

$$|f_{R0}| = 0.00$$





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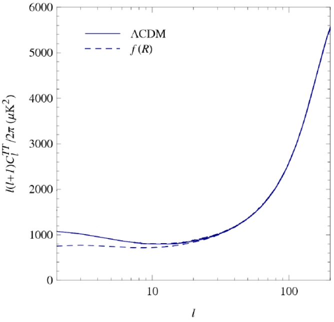
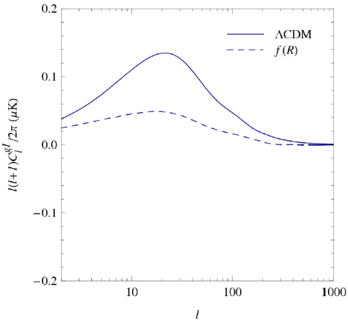
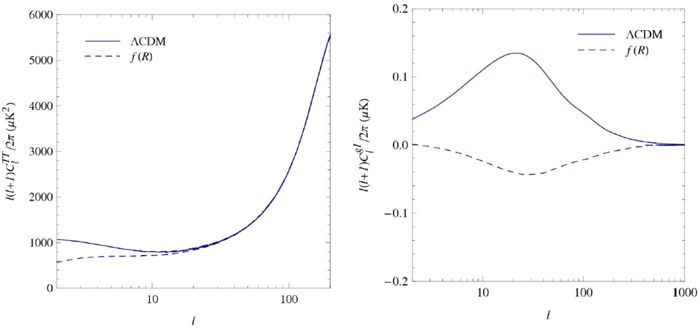




ISW vs galaxy-ISW

$$|f_{R0}| \sim 0.15$$



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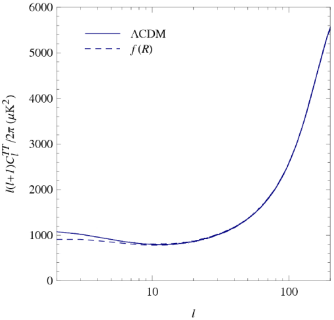

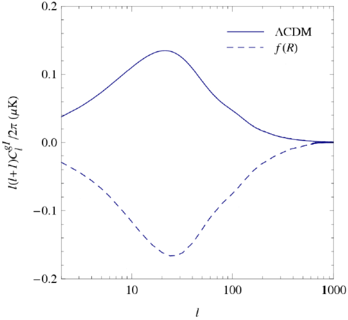

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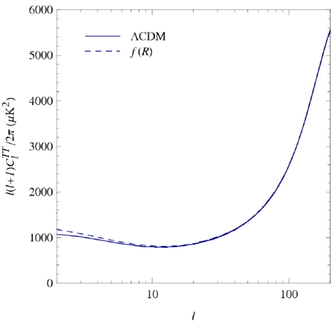

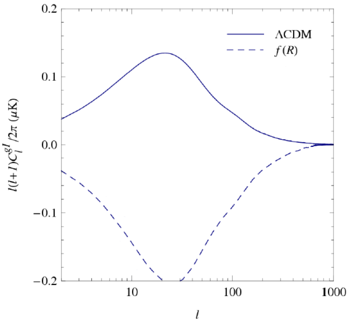

ISW vs galaxy-ISW

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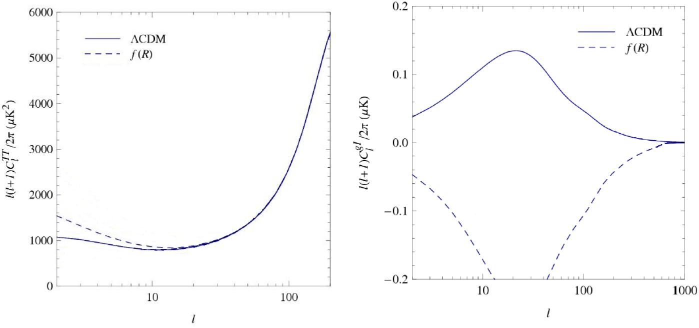

ISW vs galaxy-ISW

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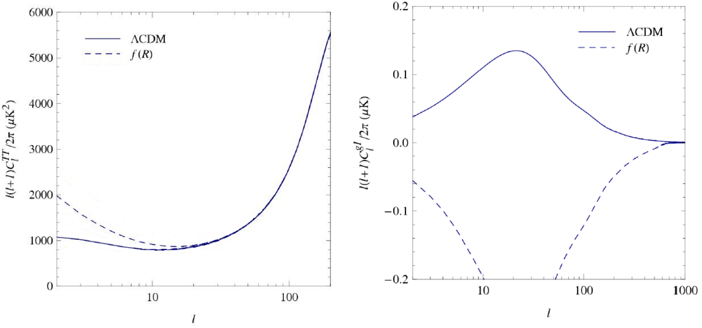
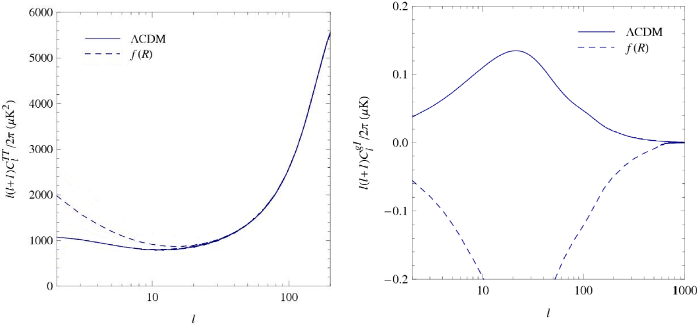
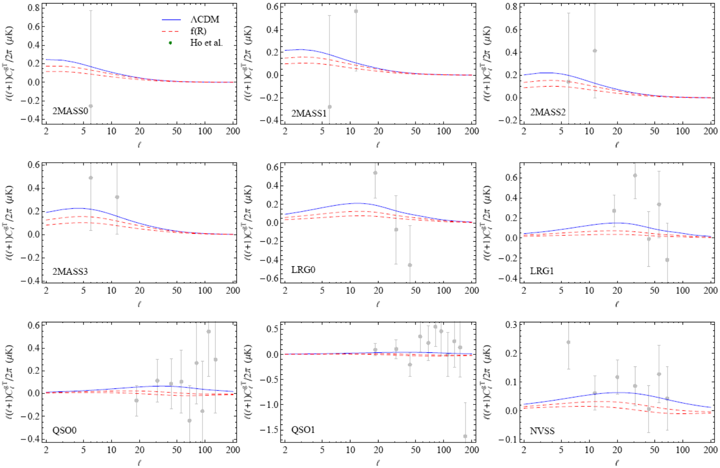


ISW vs galaxy-ISW

$|f_{R0}| \lesssim 0.15$ [Song, Hu, Sawicki (2007)]



Galaxy-ISW cross correlations [Ho *et al.* (2008)]



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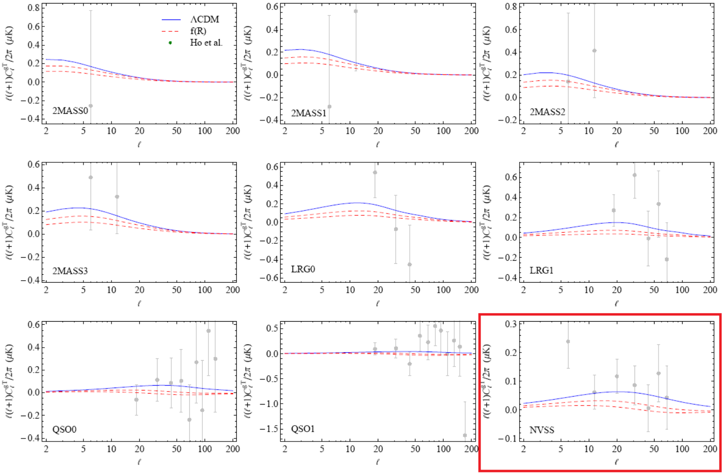

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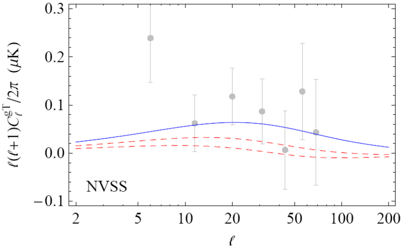

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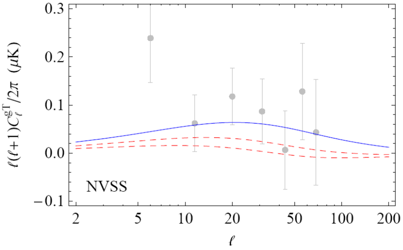

Best fit: $|f_{R0}| \sim 0.000$

constraint: $|f_{R0}| < 0.069$ or $B_0 < 0.43$ (95% C.L.)

[L, Slosar, Seljak, Hu (2010)]

(WMAP5, ACBAR, CBI, VSA, Union, SHOES, BAO, gISW)

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[Giannantonio, Martinelli, Silvestri, Melchiorri (2009)]

[Hojjati, Pogosian, Zhao (2011)]

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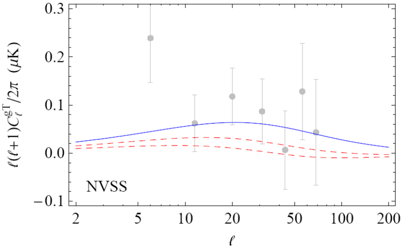

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Layout

E_G probe

Combination of three different probes of large-scale structure:

- Galaxy-galaxy lensing $\rightarrow \Phi - \Psi$
- Galaxy clustering $\rightarrow \Psi$
- Galaxy velocities from galaxy clustering in redshift space $\rightarrow \frac{d \ln \Delta_{\mathrm{m}}}{d \ln a}$

$\rightarrow$ insensitive to bias between galaxy and matter density
$\rightarrow$ insensitive to initial matter fluctuations

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E_G probe

Test of the relationship of weak gravitational lensing around galaxies to their large-scale velocities[1],

$$\frac{P_{\nabla^2(\Phi-\Psi)g}}{P_{g\theta}} = \frac{P_{\nabla^2(\Phi-\Psi)g}}{\beta P_{gg}},$$

yields the estimator

$$E_G = \frac{\Omega_{\rm m}}{(1+f_R)b\beta},$$

where $\beta \equiv b^{-1} d \ln \Delta_{\rm m} / d \ln a$.

[1] [Zhang, Liguori, Bean, Dodelson (2007)]

E_G probe

Annular differential surface density ($R \approx \theta D_l$)

$$\Upsilon(R) \equiv \Delta\Sigma(R) - \frac{R_0^2}{R^2}\Delta\Sigma(R_0)$$

Differential surface mass density

$$\Delta\Sigma_{\rm gm}(R) = -\Sigma_{\rm gm}(R) + \frac{2}{R^2}\int_0^R \Sigma_{\rm gm}(R')dR'$$

Projected surface mass density

$$\Sigma_{\rm gm}(R) = \frac{2H^2\Omega_{\rm m}}{8\pi G}\int_{\mathbb{R}} g_l(\chi)\left[1 + \xi_{\rm gm}\left(\sqrt{R^2+\chi^2}\right)\right]d\chi$$

E_G probe

- Measurement of $E_G(R) = \frac{1}{\beta}\frac{\Upsilon_{\rm gm}(R)}{\Upsilon_{\rm gg}(R)}$
- 70'205 luminous red galaxies (LRGs) from SDSS at $z = 0.32$
- $\beta = b^{-1} \left.\frac{d\ln\Delta_{\rm m}}{d\ln a}\right|_{z=0.32} = 0.309 \pm 0.035$ from anisotropy in power spectra[2]
- E_G averaged over $R = (10 - 50)h^{-1}$ Mpc

[2] [Tegmark *et al.* (2006)]

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- $\beta = b^{-1} \left.\frac{d\ln\Delta_{\rm m}}{d\ln a}\right|_{z=0.32} = 0.309 \pm 0.035$ from anisotropy in power spectra[2]
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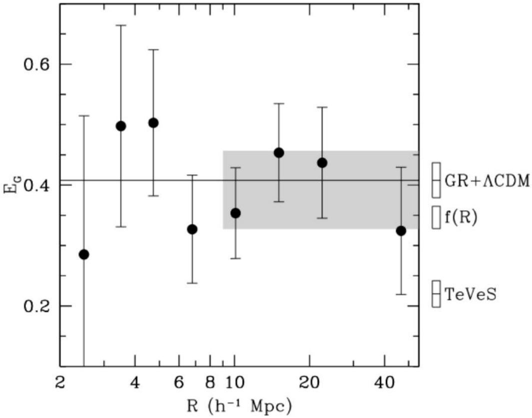
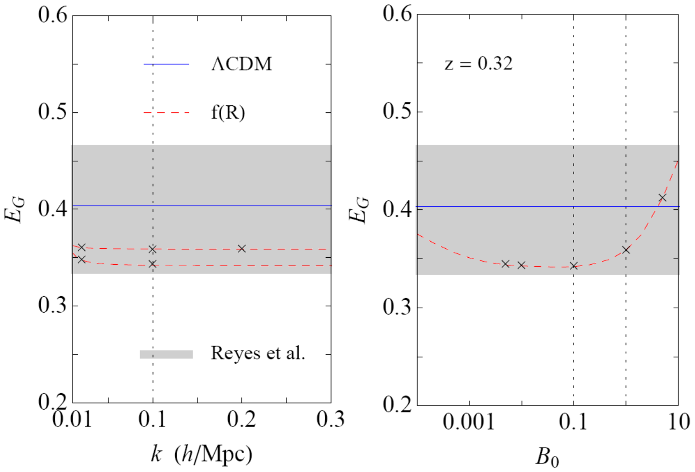


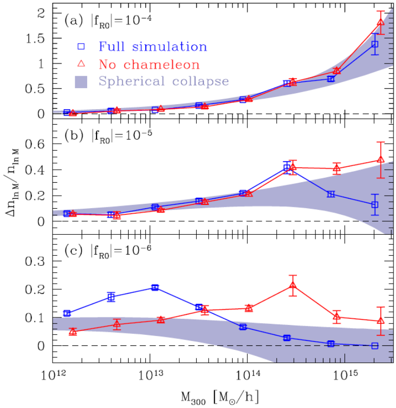
[Reyes, Mandelbaum, Seljak, Baldauf, Gunn, L, Smith (2010)]

E_G probe

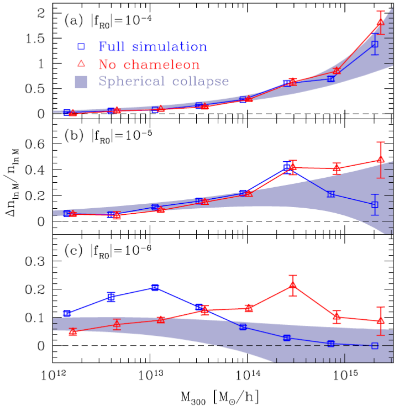

0.6
0.5
0.4
0.3
0.2
E_G
ΛCDM
f(R)
Reyes et al.
0.01
0.1
0.2
0.3
k (h/Mpc)
z = 0.32
0.001
0.1
10
B_0

Layout

Layout



[Schmidt, Lima, Oyaizu, Hu (2009)]



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Galaxy-galaxy lensing signal from MaxBCG catalog

Massive galaxy clusters identified by overdensity of bright, uniformly red galaxies

[Koester *et al.* (2007)]

Calibration from halo mass function ($z = 0.18,\ 0.25$)

$\bar{n} = 2.5 \times 10^{-7}\ (\mathrm{Mpc}/h)^{-3}$

$\bar{n} = 2.0 \times 10^{-6}\ (\mathrm{Mpc}/h)^{-3}$

$\bar{n} = 1.8 \times 10^{-5}\ (\mathrm{Mpc}/h)^{-3}$

$$\sigma_8 \left(\frac{\Omega_\mathrm{m}}{0.25}\right)^{0.40} = 0.844 \pm 0.036$$

[SDSS MaxBCG clusters]

$$\sigma_8 \left(\frac{\Omega_\mathrm{m}}{0.25}\right)^{0.47} = 0.813$$

$$\pm 0.013(\mathrm{stat}) \pm 0.024(\mathrm{sys})$$

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[SDSS MaxBCG clusters]

$$B_0 < 3.3 \times 10^{-3} \text{ (95\%CL)}$$
$$|f_{R0}| < 4.8 \times 10^{-4} \text{ (95\%CL)}$$

[L, Slosar, Seljak, Hu (2010)]

(WMAP5, ACBAR, CBI, VSA, Union, BAO, SHOES, CA)

$$\sigma_8 \left(\frac{\Omega_{\rm m}}{0.25}\right)^{0.47} = 0.813$$
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[Vikhlinin *et al.* (2009)]

$$|f_{R0}| \lesssim 1.4 \times 10^{-4} \text{ (95\%CL)}$$

[Schmidt, Vikhlinin, Hu (2009)]

(WMAP5, SNIa, SHOES, BAO, CCCP)

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$$B_0 < 1.1 \times 10^{-3} \text{ (95\%CL)}$$
$$|f_{R0}| < 1.9 \times 10^{-4} \text{ (95\%CL)}$$

[L, Slosar, Seljak, Hu (2010)]

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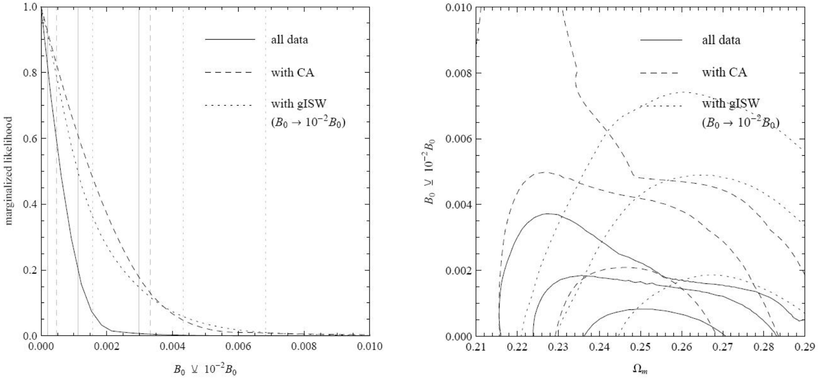
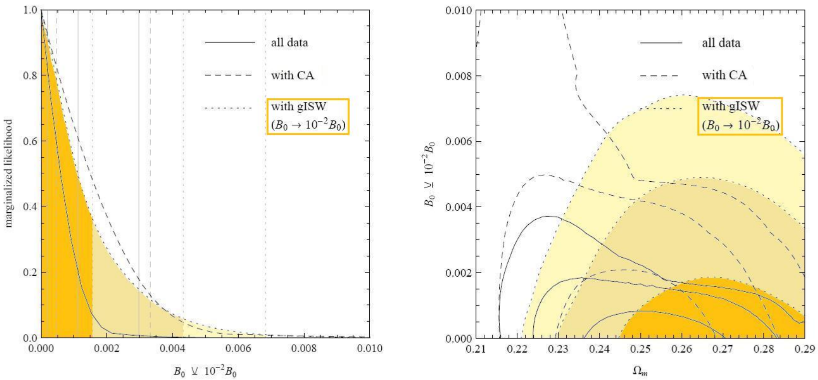
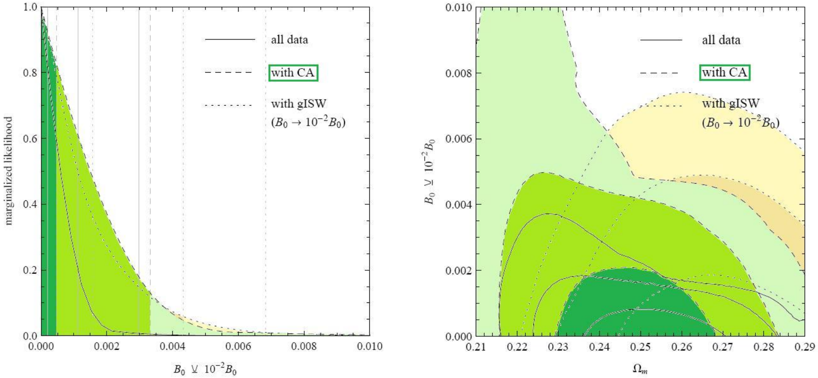
Marginalized likelihoods and contours



Marginalized likelihoods and contours



Marginalized likelihoods and contours



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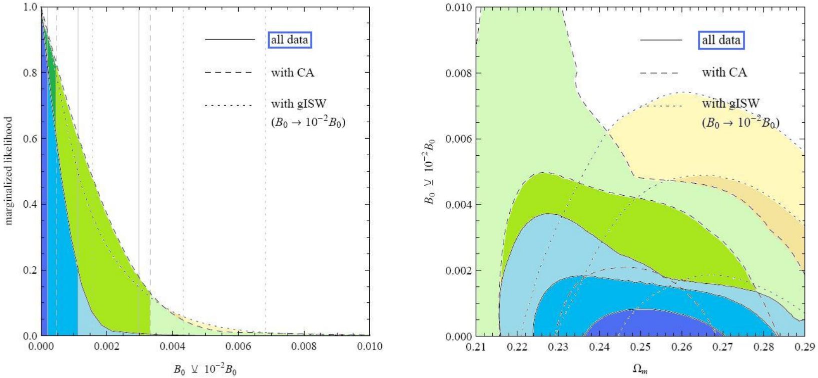
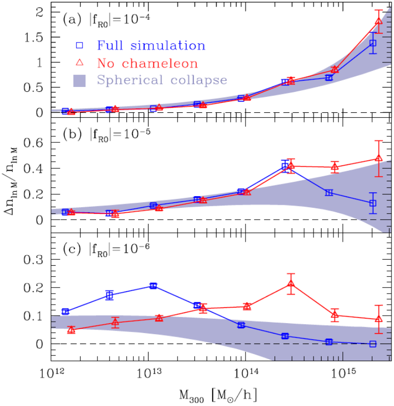
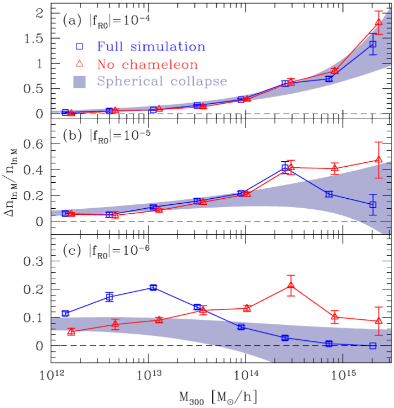




[Schmidt, Lima, Oyaizu, Hu (2009)]



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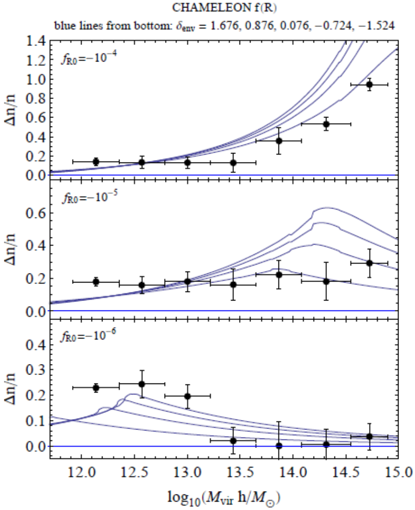
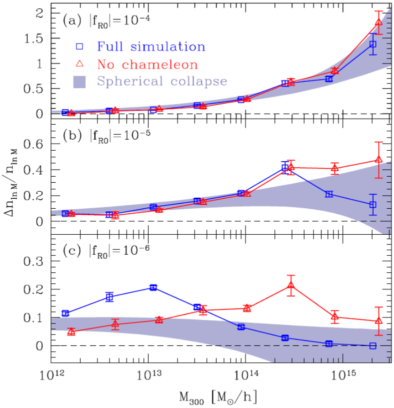


[L, Li, Zhao, Koyama (in prep.)]



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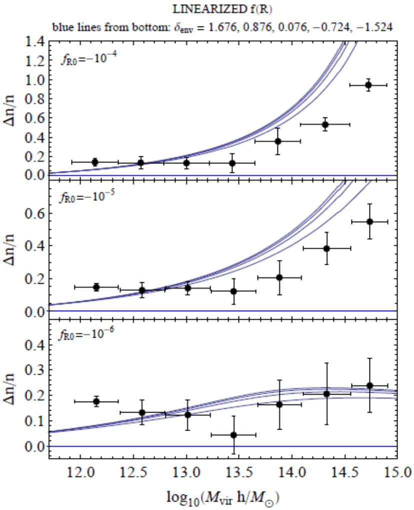

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Layout

1. $f(R)$ gravity
   - Hu-Sawicki model
   - N-body simulations
2. Linear structures
   - Cosmic microwave background
   - Galaxy-ISW cross correlations
   - E_G measurement
3. Nonlinear structures
   - Abundance of clusters
   - Density profiles of clusters
4. Conclusion and Outlook

- Hu-Sawicki model
  $f(R) \simeq -2\Lambda - f_{R0}\frac{\bar{R}_0^2}{R}$
- *Linearized* $f(R)$ simulations (256^3 on $64, 128h^{-1}\mathrm{Mpc}$)
  $|f_{R0}| = 0, 10^{-4}, 10^{-3}, 10^{-2}$
- ZHORIZON ΛCDM simulations (750^3 on $1.5h^{-1}\mathrm{Gpc}$)
  $\Omega_\mathrm{m} = 0.2, 0.25, 0.3$
  $\sigma_8 = 0.7, 0.8, 0.9$
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- 5 891 maxBCG clusters from SDSS with background sources

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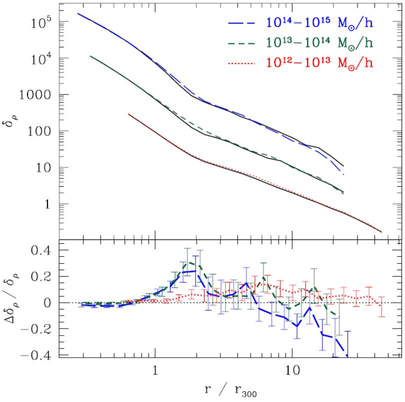
Density profiles of clusters

$$|f_{R0}| = 10^{-4}$$



[Schmidt, Lima, Oyaizu, Hu (2009)]

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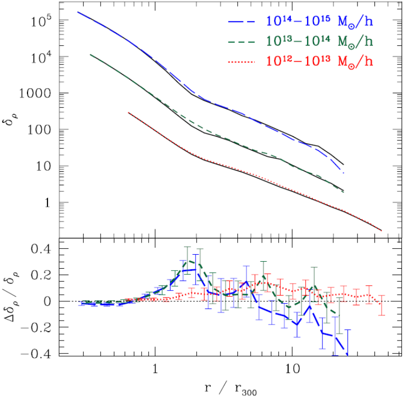
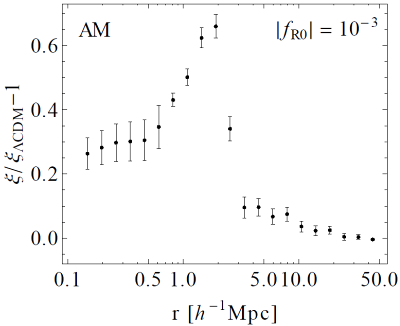


[Schmidt, Lima, Oyaizu, Hu (2009)]

$\bar{n} = 1.8 \times 10^{-5} h^{-3}$ Mpc^3



Density profiles of clusters

Halo-mass correlation function with 1-halo and 2-halo terms

$$\begin{aligned}\xi_{\rm hm}(r) &\equiv \frac{\langle\rho_{\rm h}(r)\rangle}{\bar{\rho}_{\rm m}} - 1\\ &= \frac{\rho_{\rm NFW}(r)}{\bar{\rho}_{\rm m}}\\ &\quad + b_L(M_v)\int\frac{d^3k}{(2\pi)^3}I(k)P_L(k)e^{-i\mathbf{k}\cdot\mathbf{x}},\end{aligned}$$

where

$$I(k) = \int d\ln M_v\, n_{\ln M_v}\frac{M_v}{\bar{\rho}_{\rm m}}y(k,M_v)b_L(M_v).$$

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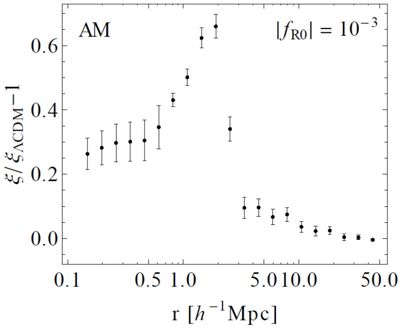

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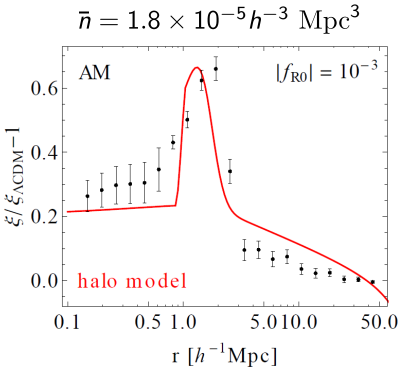

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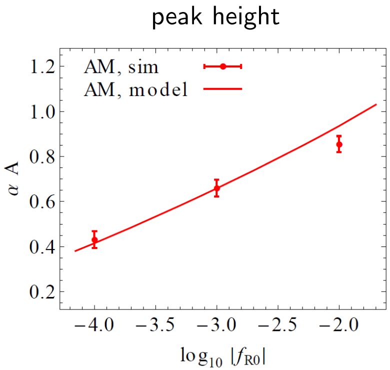
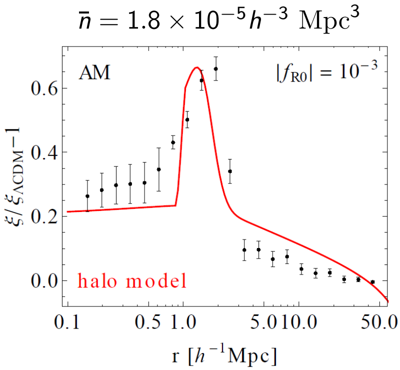
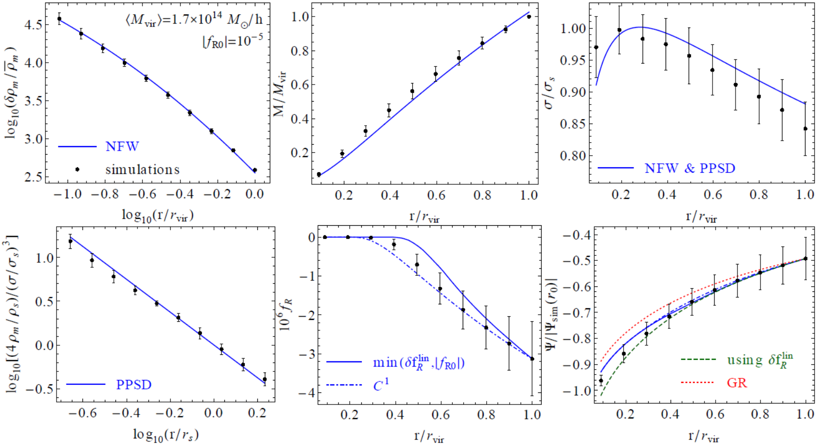




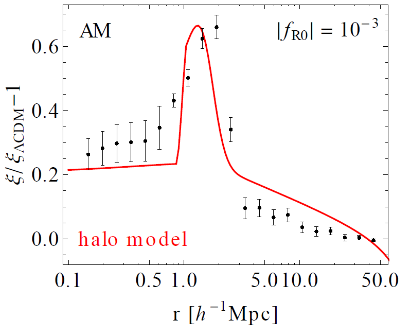
NFW profile in $f(R)$ gravity



[L, Koyama, Zhao, Li (2012)]

Cluster-galaxy lensing

abundance: $\bar{n} = 1.8 \times 10^{-5}\ (\mathrm{Mpc}/h)^{-3}$



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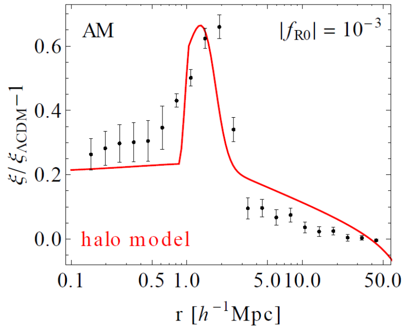

Excess surface mass density ($r_\perp \approx \theta D_l$)

$$\Delta\Sigma_{\mathrm{gm}}(r_\perp) = \bar{\Sigma}_{\mathrm{gm}}(r_\perp) - \Sigma_{\mathrm{gm}}(r_\perp),$$

where

$$\bar{\Sigma}_{\mathrm{gm}}(r_\perp) = \frac{2}{r_\perp^2}\int_0^{r_\perp} \Sigma_{\mathrm{gm}}(r'_\perp)dr'_\perp$$

$$\Sigma_{\mathrm{gm}}(r_\perp) = \frac{2H^2\Omega_{\mathrm{m}}}{8\pi G}\int_{\mathbb{R}} g_l(\chi) \times \left[1 + \xi_{\mathrm{gm}}\left(\sqrt{r_\perp^2 + \chi^2}\right)\right] d\chi$$

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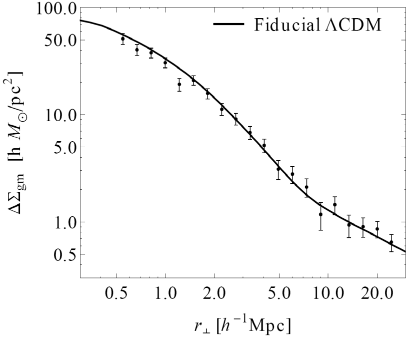

maxBCG clusters (SDSS)

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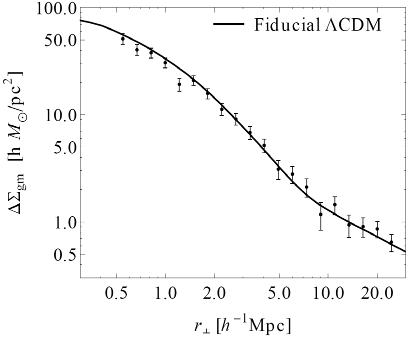

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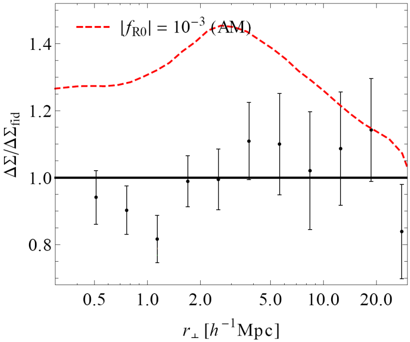

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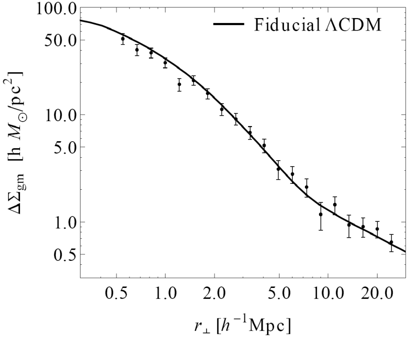

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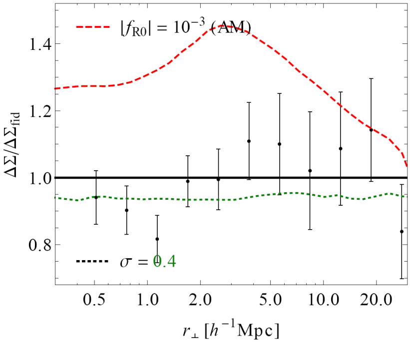

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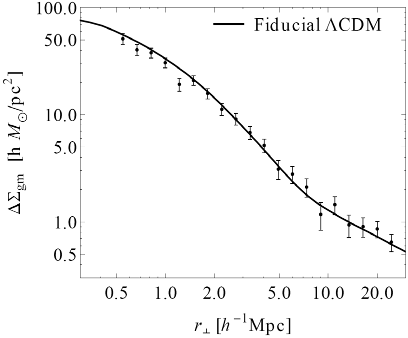

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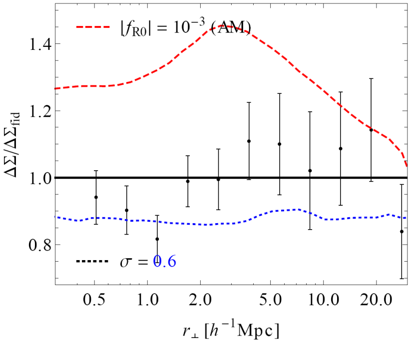

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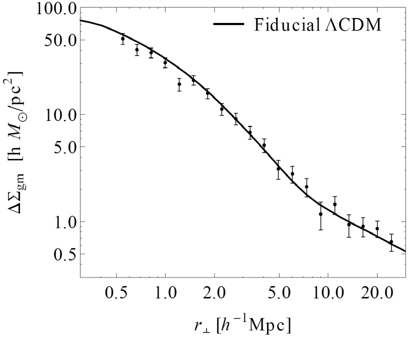

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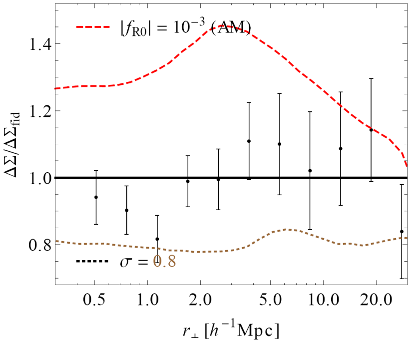

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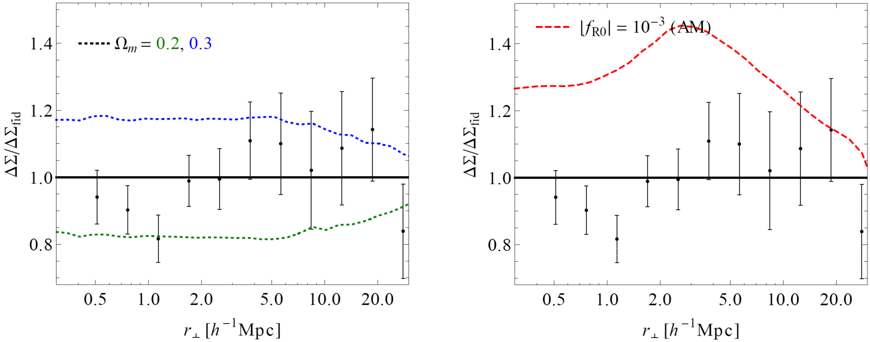

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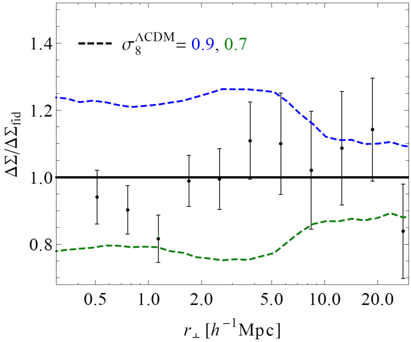

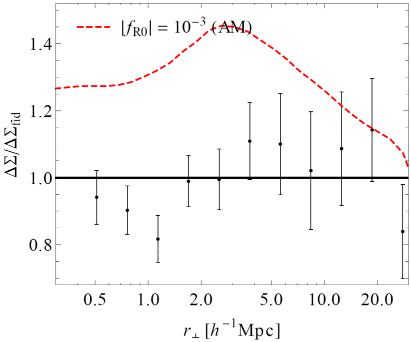

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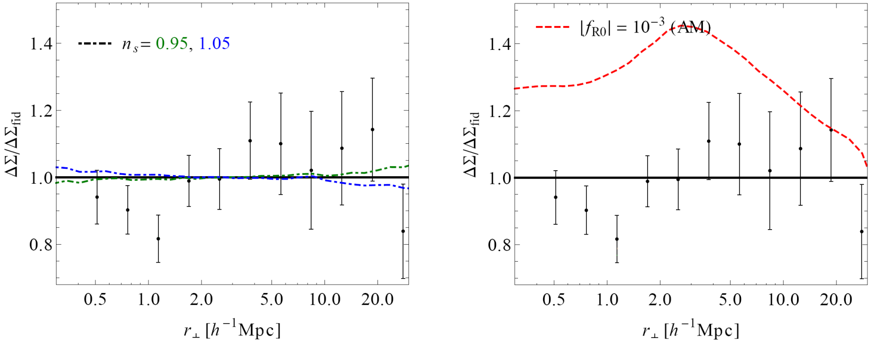
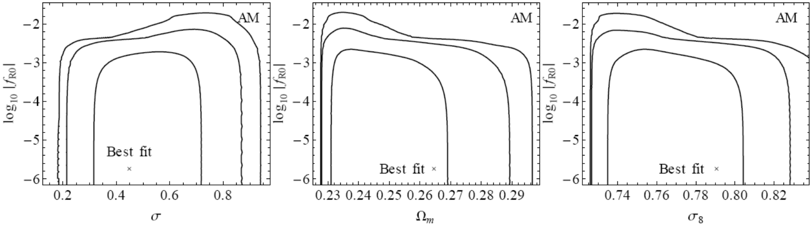
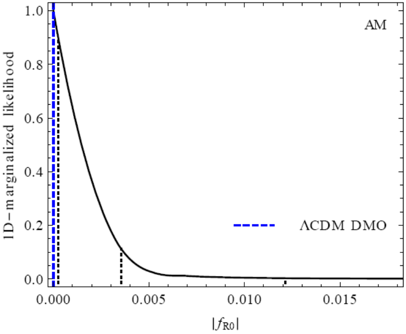


Constraints



Constraints

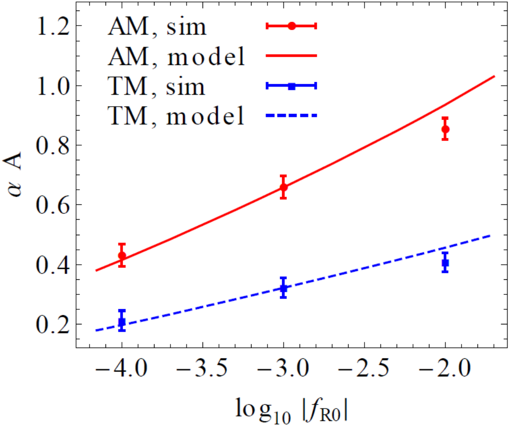


Best fit: $|f_{R0}| = 1.7 \times 10^{-6}$

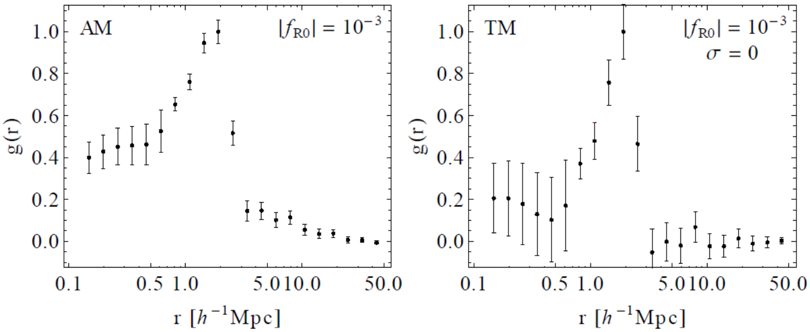
$|f_{R0}| < 3.5 \times 10^{-3}$ (95%CL)

[L, Schmidt, Baldauf, Mandelbaum, Seljak, Smith (2011)]

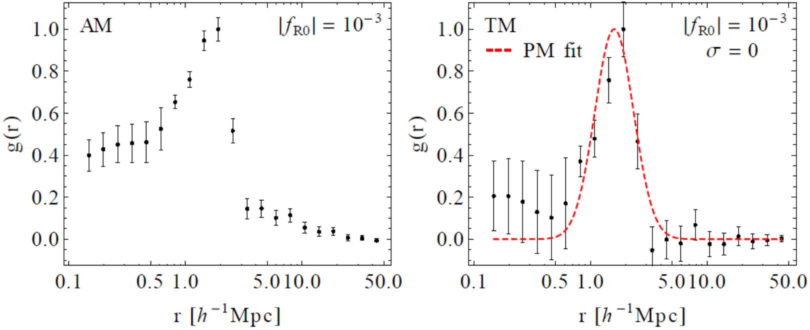
Cluster abundance



Cluster abundance



Cluster abundance



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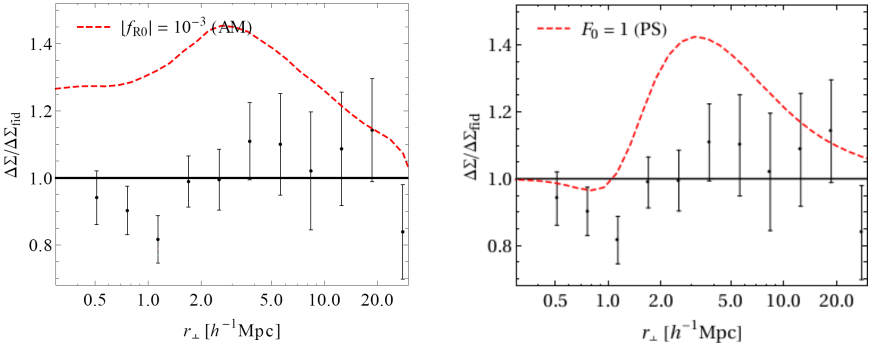

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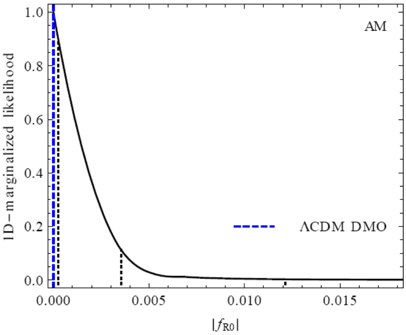
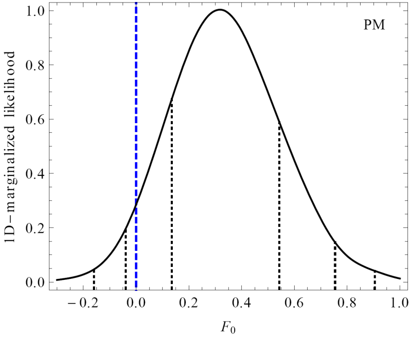
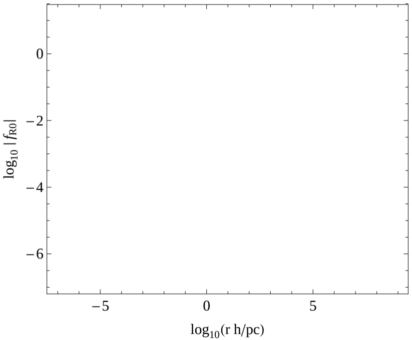




Layout

Current constraints on $f(R)$ gravity



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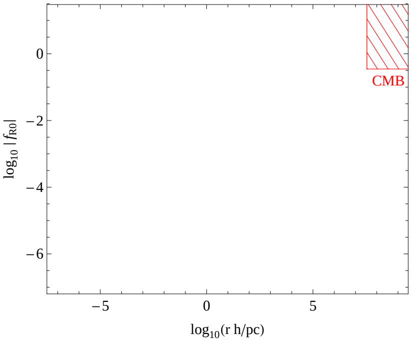

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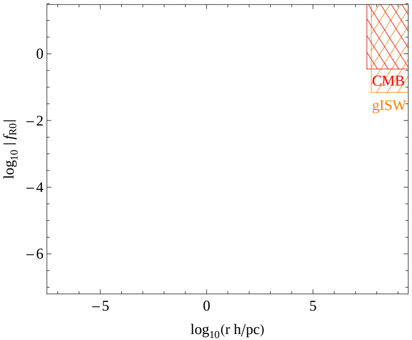

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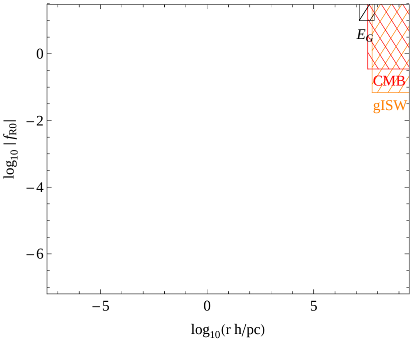

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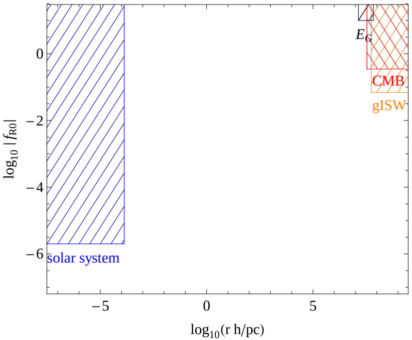

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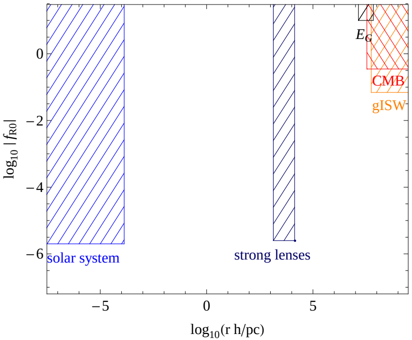

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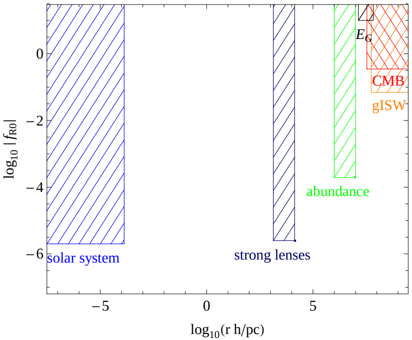

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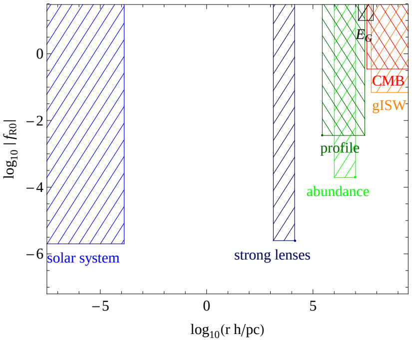

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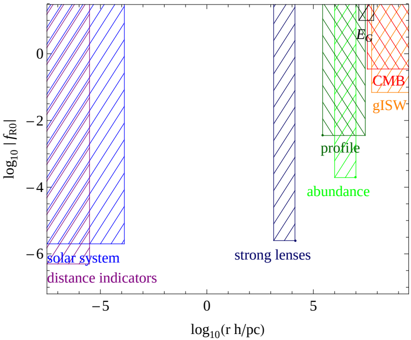

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Thank you!