(PRIMORDIAL) NON-GAUSSIANITY & N-BODY SIMULATIONS

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arXiv: 1008.1730, 1108.3813, 1208.5678, 1209.xxxx

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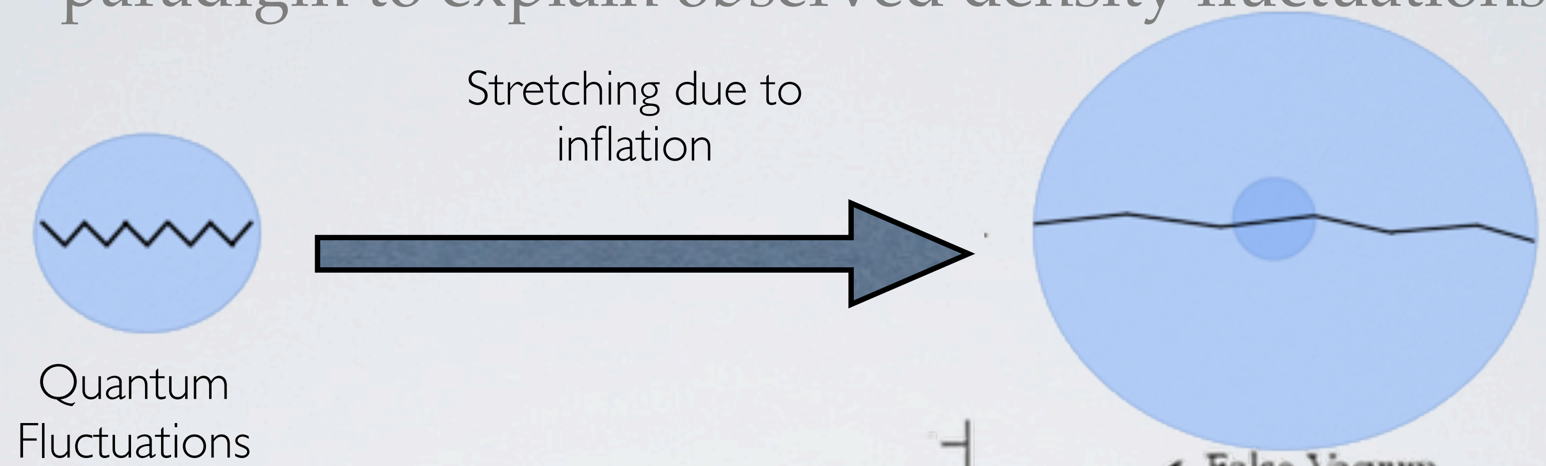
OUTLINE

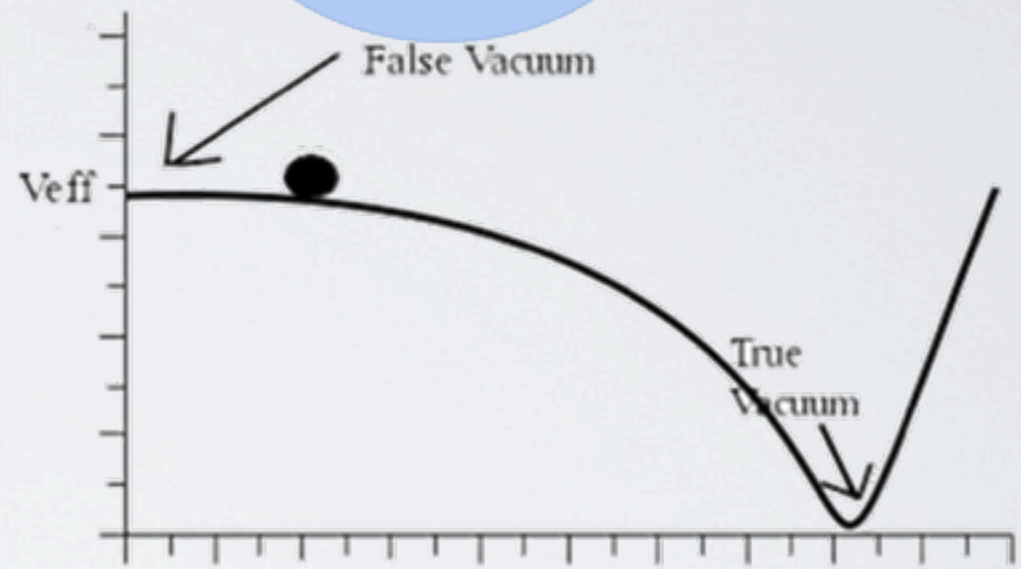
- Non-Gaussianity primer
- Initial Conditions for N-body Simulations
- Analysing results of N-body Simulations
- Comparison to Halo model and 1-loop Corrections (+ simple fitting formula)

@ the primordial level

- Inflation = paradigm to explain observed density fluctuations



- Standard Inflation ... Require slow-roll to satisfy the horizon and flatness problems.



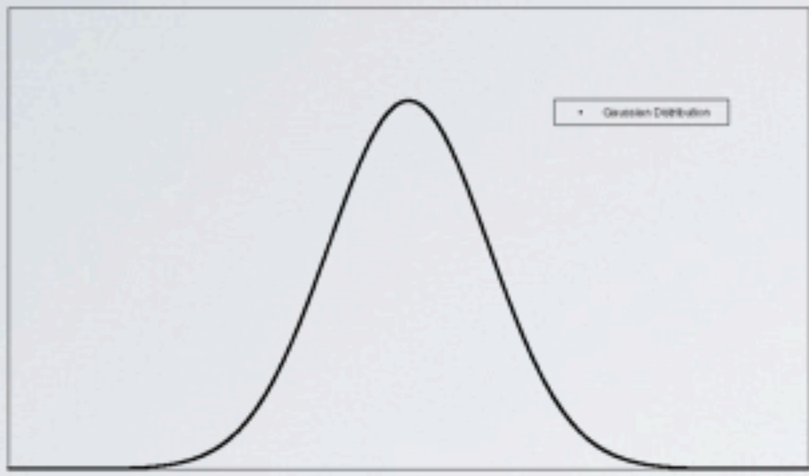
- Prediction... Gaussian primordial fluctuations

- Conditions of standard case:

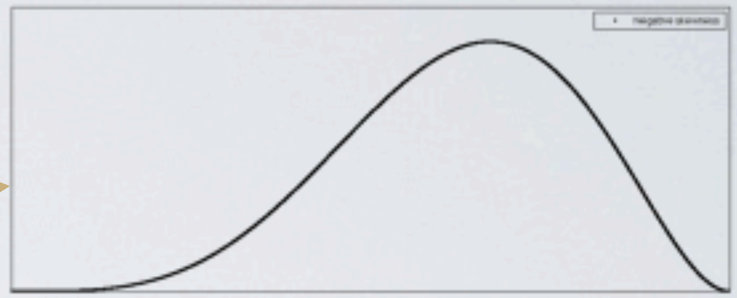
  (a) Single scalar field
  (b) Canonical kinetic energy
  (c) Slow roll
  (d) Standard vacuum state

Break any of these ⟷ *Non-Gaussianities*

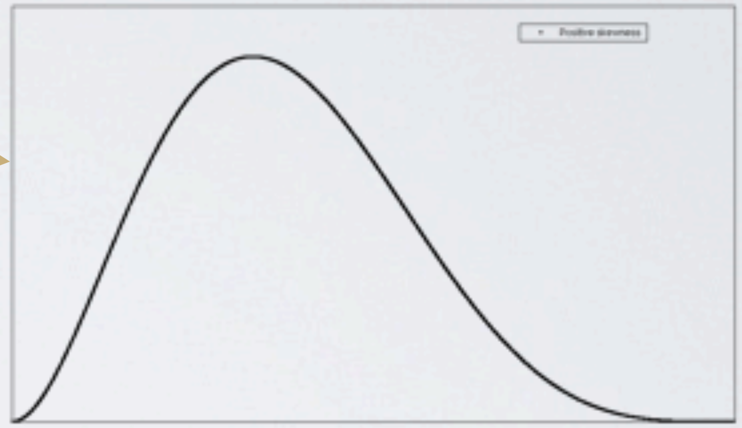
Skewness … Bispectrum



Gaussian distribution

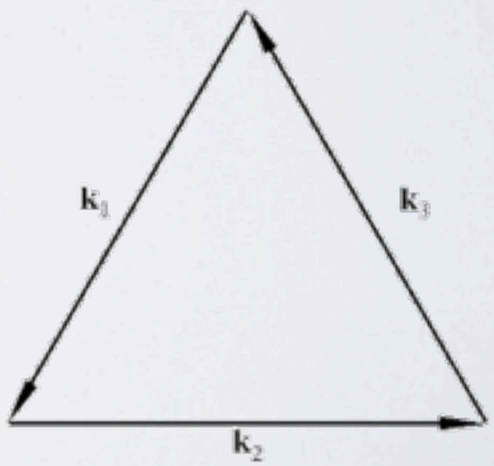


Negatively skewed

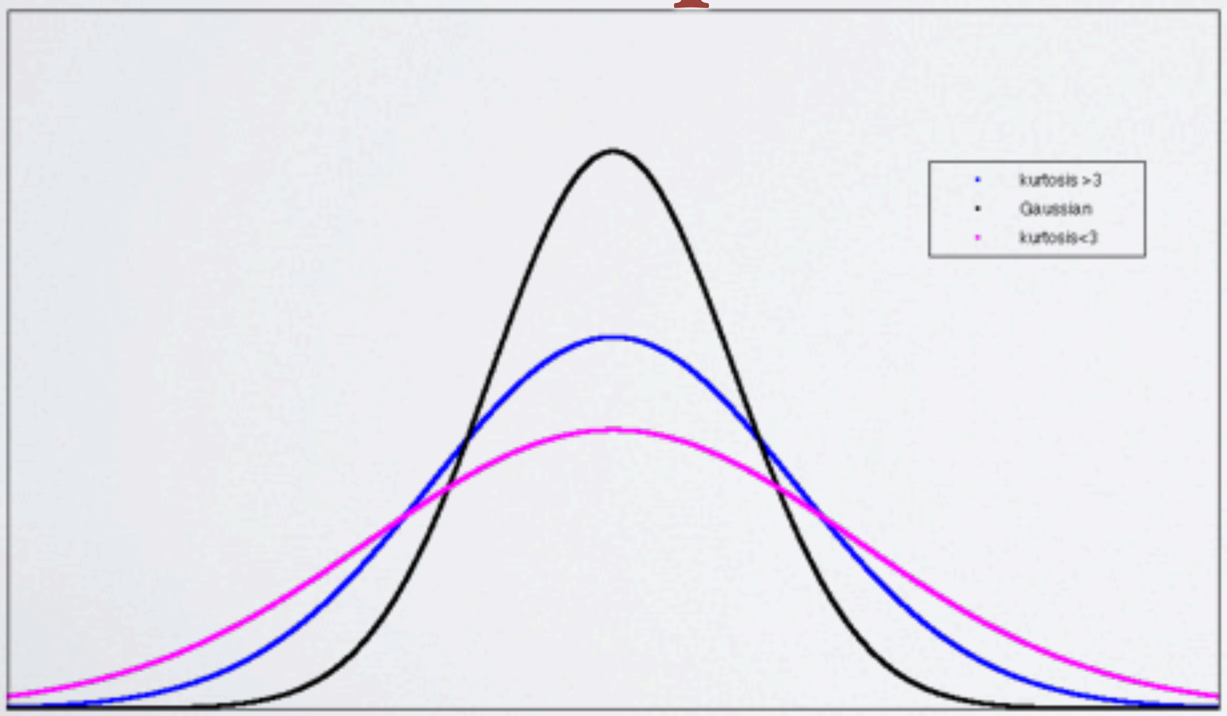


Positively skewed

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(k_1,k_2,k_3)$$



Kurtosis … Trispectrum

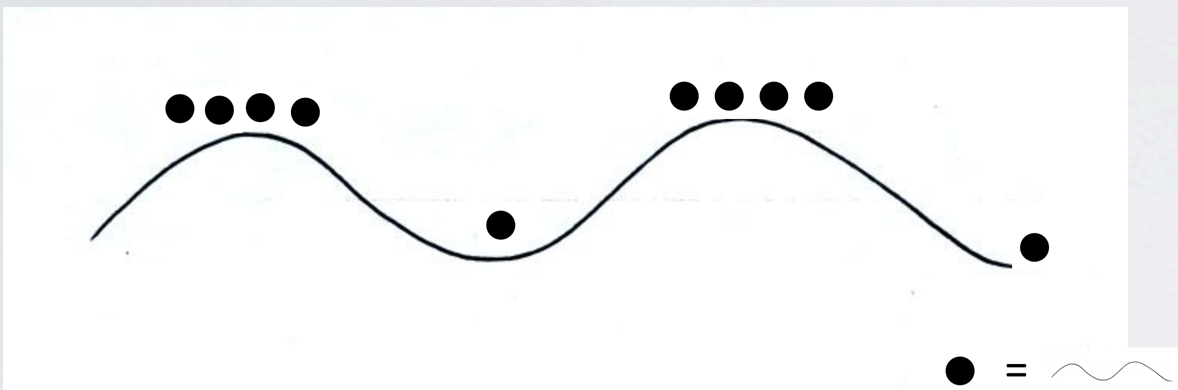


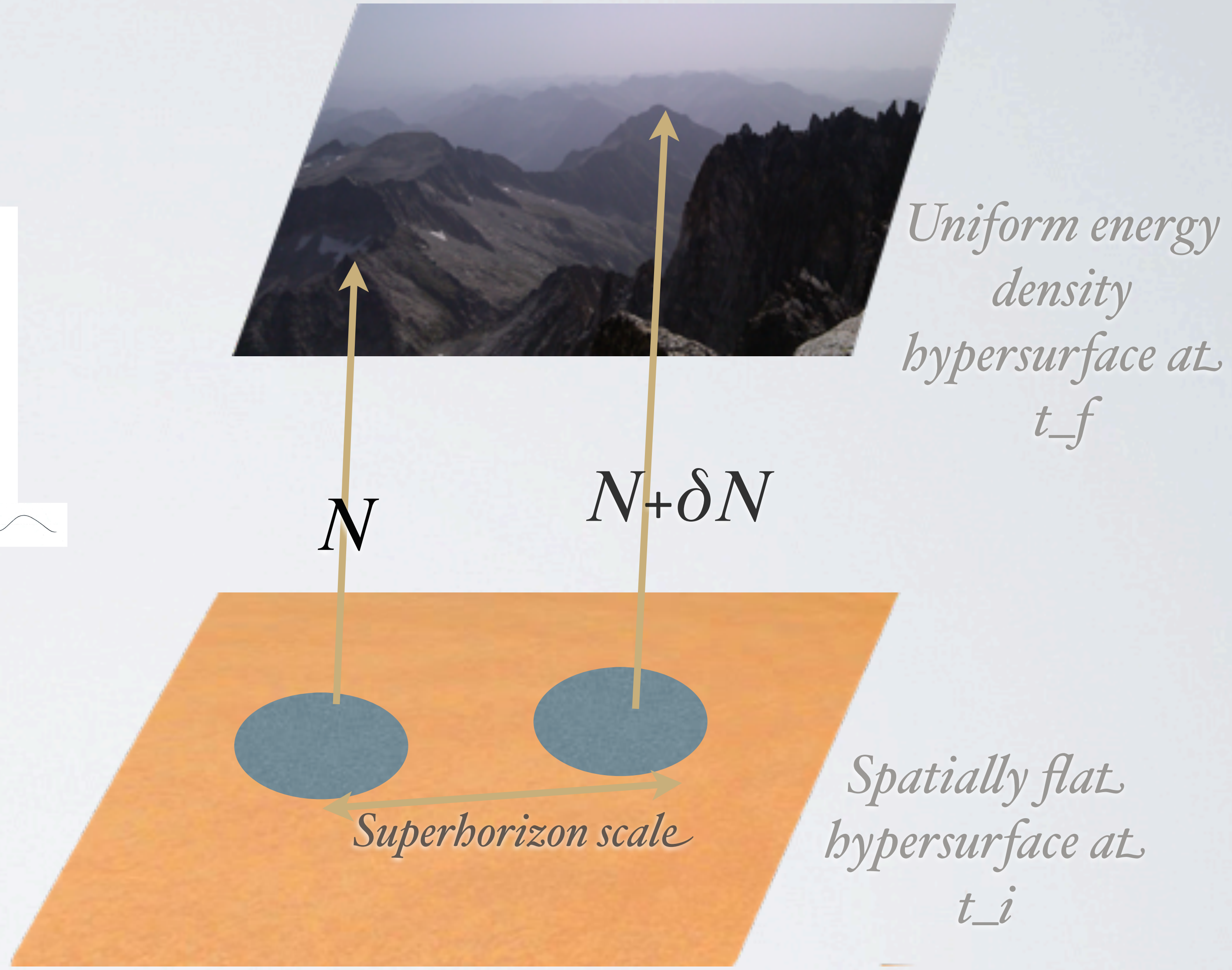
Trispectrum=(connected)4-point correlator

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle_c = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)T_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$

Local Bispectrum ... superhorizon evolution

$$B(k_1, k_2, k_3) = 2f_{\rm NL}\left[P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)\right]$$





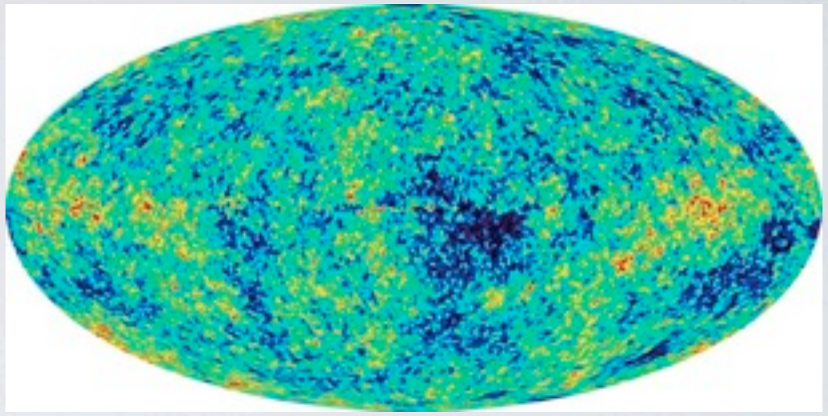
l_2 l_3 l_1

NOTE SQUEEZED SHAPE

Many more examples...

- DBI inflation
- Ghost inflation
- Second order corrections to single field inflation
- Warm inflation
- Inflation with non-standard vacua (&/or higher order kinetic terms)
- Inflation with a step in the potential
- Infation with oscillatory modifications to the potential
- Non-local inflation
- ...

@ the CMB level



a_{lm} ⟷ $\Delta_l(k)$ ⟷ $\Phi(\mathbf{k})$

Primordial to CMB via transfer function

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} \Delta_l(k)\Phi(\mathbf{k})Y^*_{lm}(\hat{\mathbf{k}})$$

CMB Bispectrum ⟷ *Primordial Bispectrum*

$$B^{l_1 l_2 l_3}_{m_1 m_2 m_3} = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}^{l_1 l_2 l_3}_{m_1 m_2 m_3} b_{l_1 l_2 l_3}$$

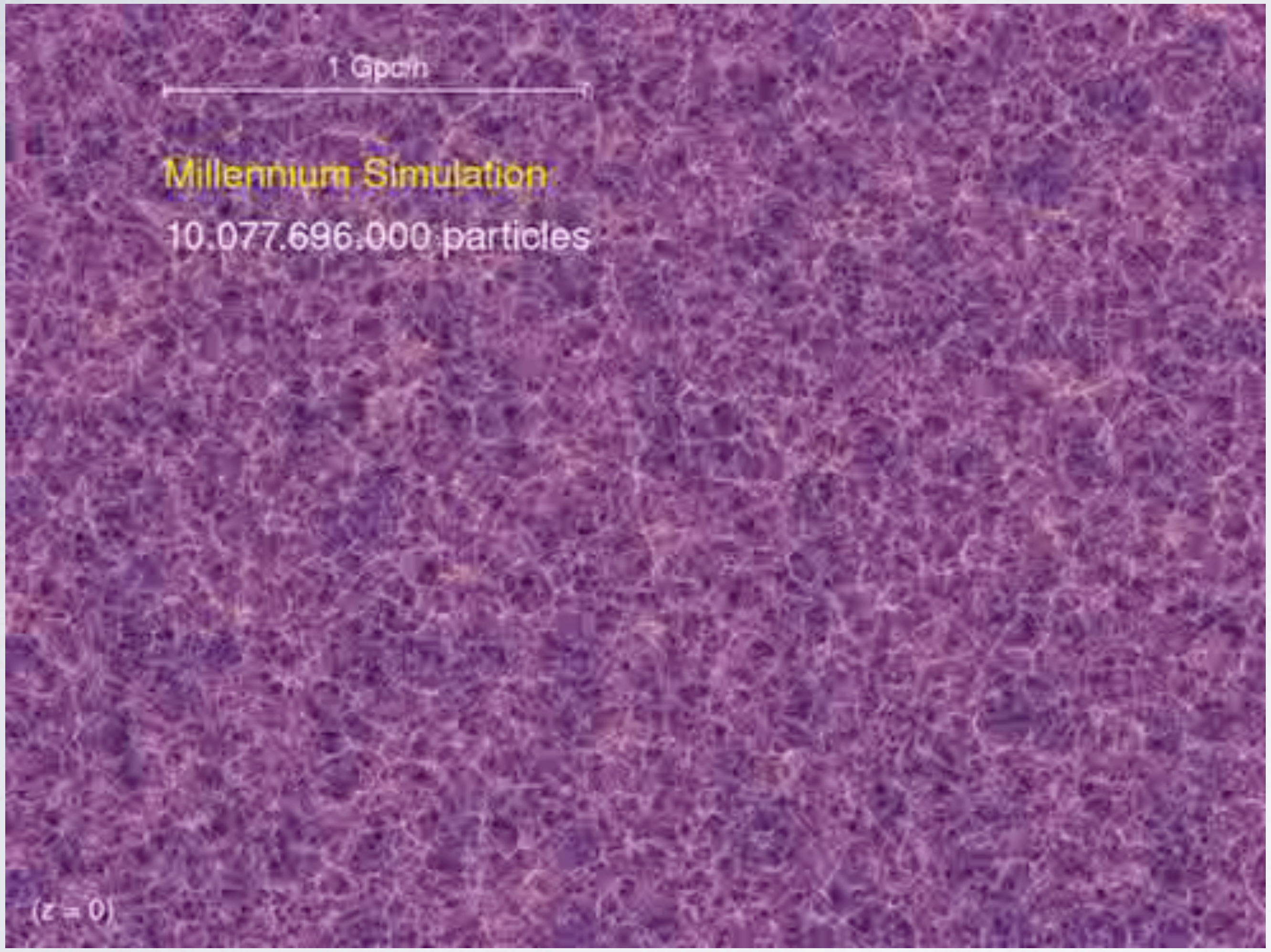
$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\Phi(k_1, k_2, k_3)$$

$$b_{l_1 l_2 l_3} = \Delta^2_\Phi \left(\frac{2}{\pi}\right)^3 \int dk_1 dk_2 dk_3 \Delta_{l_1}(k_1)\Delta_{l_2}(k_2)\Delta_{l_3}(k_3) S(k_1, k_2, k_3)$$
$$\times \int dx x^2 j_{l_1}(k_1 x) j_{l_2}(k_2 x) j_{l_3}(k_3 x),$$

$$S(k_1, k_2, k_3) = \frac{(k_1 k_2 k_3)^2}{\Delta^2_\Phi} B_\Phi(k_1, k_2, k_3)$$

@ the LSS level

At late times fluctuations grow & non-Gaussianity is induced by gravitational evolution



Video: http://www.mpa-garching.mpg.de/galform/virgo/millennium

- Need N-body simulations, e.g. Gadget3

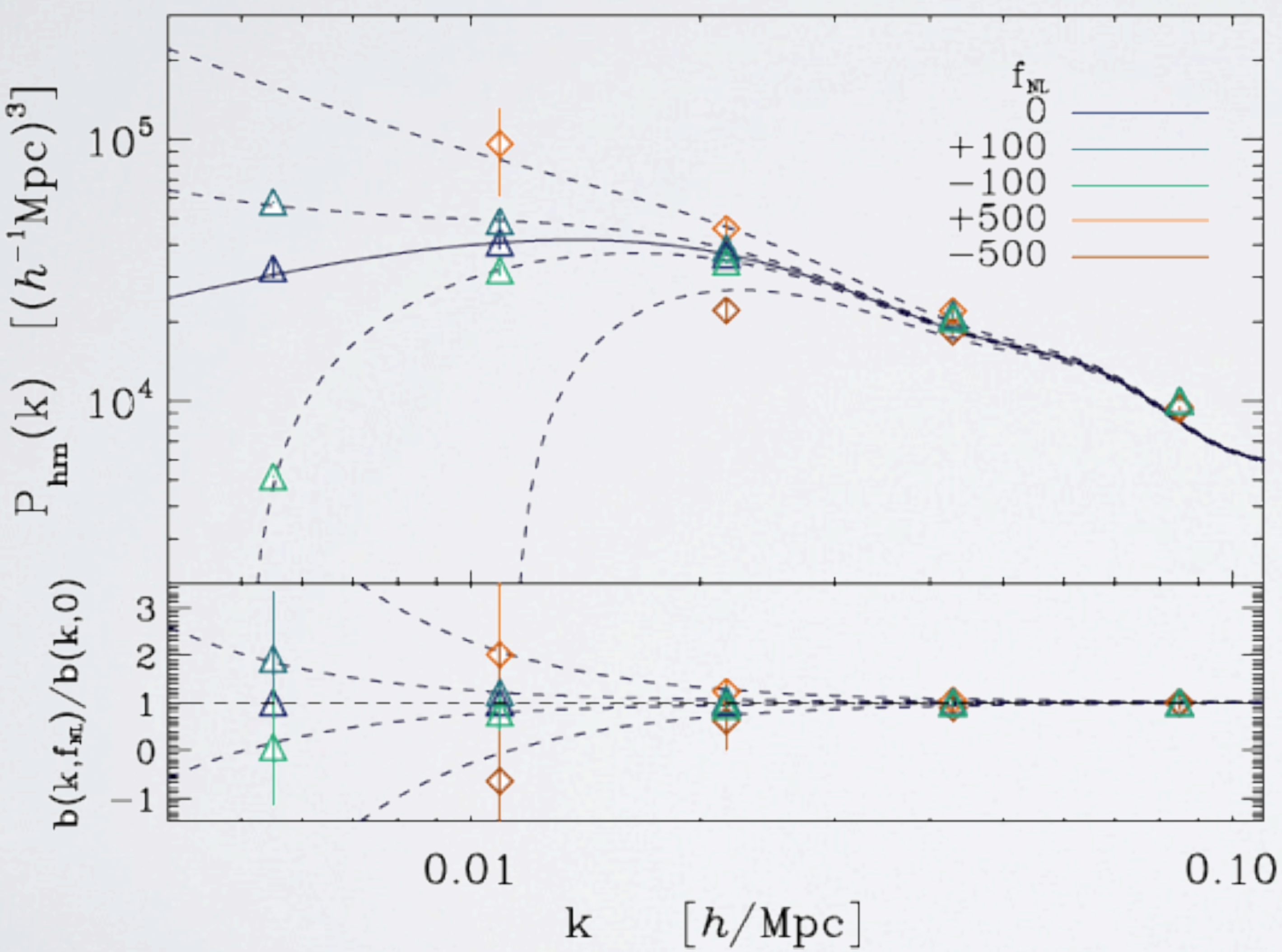
Advantage: More information (z dependence)

Drawback: Non-linear evolution...difficult to distinguish primordial signal

Primordial non-Gaussianity and N-body Simulations

Searching for non-Gaussianity in large scale structure

e.g. Use clustering of rare peaks to get a constraint on f_{NL} (0710.4560 Dalal et al)



The primordial bispectrum gives a contribution to the galaxy power spectrum

State of play of non-Gaussianity in N-body simulations

Until recently only the **local** type was simulated

Then... Verde, Wagner, (Boubekeur) arXiv:1006.5793, arXiv:1102.3229

Scoccimarro, Hui, Manera,Chan arXiv:1108.5512

However, they still require the underlying shape to be **separable**

Also, analysis still is still done on **slices** at particular redshifts.

Difficult to be sure you're seeing everything!

Initial Conditions for N-body Simulations

$$\Phi = \Phi^G + \frac{1}{6} F_{\rm NL} \Phi^B + \frac{1}{24} \tau_{NL} \Phi^T$$

$$\Phi^B(\mathbf{k}) = \int \frac{d^3\mathbf{k}' d^3\mathbf{k}''}{(2\pi)^3} \delta_D(\mathbf{k} - \mathbf{k}' - \mathbf{k}'') \Phi^G(\mathbf{k}') \Phi^G(\mathbf{k}'') \quad \frac{B(k,k',k'')}{P(k)P(k') + P(k)P(k'') + P(k')P(k'')}$$

Complexity of N^6 (N=512!)

Problem is non-separability...

Hint: Try the modal approach arXiv:1008.1730 Fergusson,DMR,Shellard

$$\frac{B(k,k',k'')}{P(k)P(k') + P(k)P(k'') + P(k')P(k'')} = \sum_{rst} \alpha^Q_{rst} q_r(k) q_s(k') q_t(k''),$$

$$\Phi^B(\mathbf{k}) = \sum_n \alpha^Q_n q_{\{r}(k) \int d^3\mathbf{x} e^{i\mathbf{k}.\mathbf{x}} M_s(\mathbf{x}) M_{t\}}(\mathbf{x})$$

$$M_s(\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Phi^G(\mathbf{k}) q_s(k) e^{-i\mathbf{k}.\mathbf{x}}$$ *FFT only!!*

Application to Estimation

$$\mathcal{E} = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\frac{(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)}{P(k_1)P(k_2)P(k_3)}\left[\delta^{obs}_{\mathbf{k}_1}\delta^{obs}_{\mathbf{k}_2}\delta^{obs}_{\mathbf{k}_3} - 3\langle\delta^{sim}_{\mathbf{k}_1}\delta^{sim}_{\mathbf{k}_2}\rangle\delta^{obs}_{\mathbf{k}_3}\right]$$

Modal approach to the rescue!

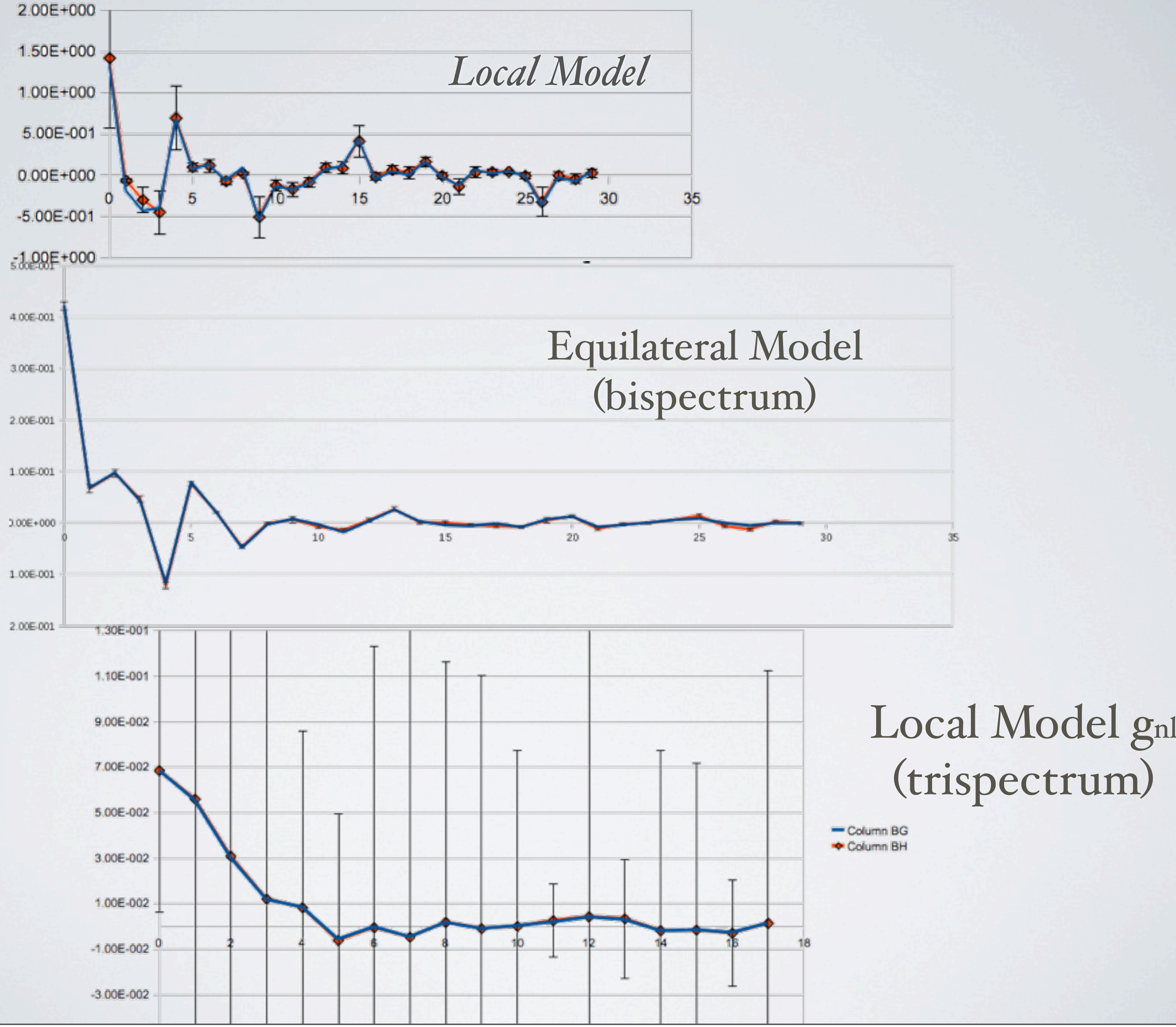
$$\frac{B(k_1,k_2,k_3)\,v(k_1)v(k_2)v(k_3)}{\sqrt{P(k_1)P(k_2)P(k_3)}} = \sum \alpha^{\mathcal{Q}}_n \mathcal{Q}_n(k_1,k_2,k_3)$$

$$\Rightarrow \quad \mathcal{E} = \sum_n \alpha^{\mathcal{Q}}_n\,\beta^{\mathcal{Q}}_n \quad \text{where } \beta^{\mathcal{Q}}_n = \int d^3x\, M_r(\mathbf{x})\, M_s(\mathbf{x})\, M_t(\mathbf{x})$$

$$M_p(\mathbf{x}) = \int d^3k \frac{\delta^{obs}_{\mathbf{k}} q_r(k)\, e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{kP(k)}}$$

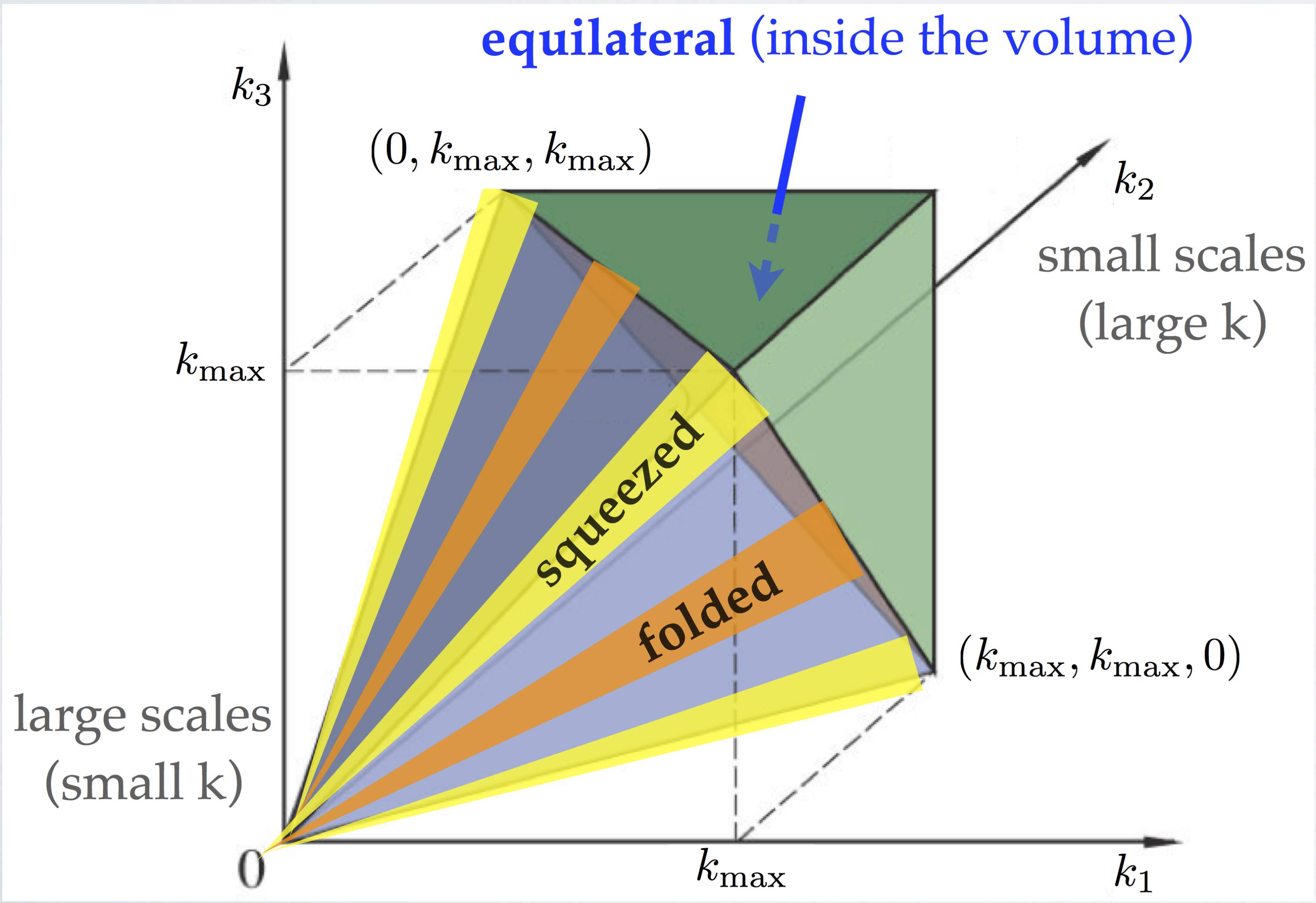
} *FFT & a single 3D integral*

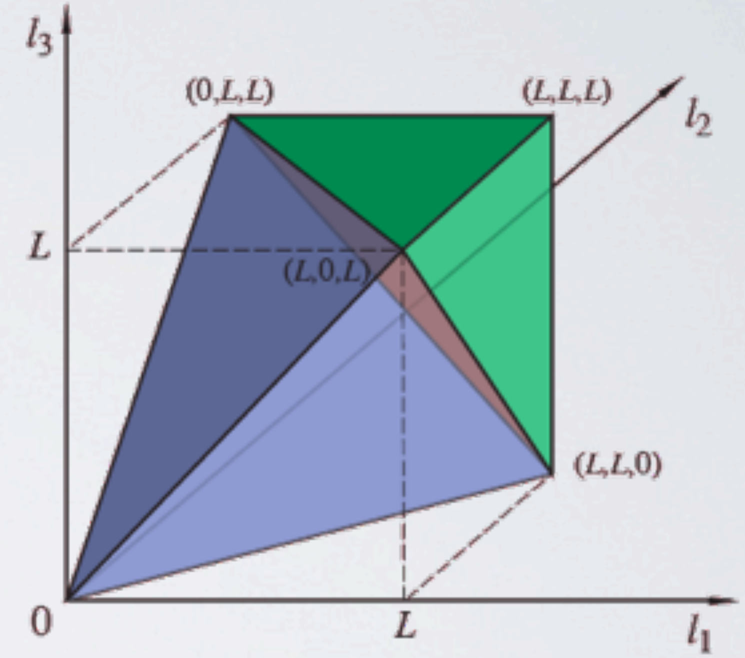
Similarly can define a correlator between shapes


Local Model
2.00E+000
1.50E+000
1.00E+000
5.00E-001
0.00E+000
-5.00E-001
-1.00E+000
0
5
10
15
20
25
30
35
Equilateral Model
(bispectrum)
Local Model g_{nl}
(trispectrum)
Column BG
Column BH

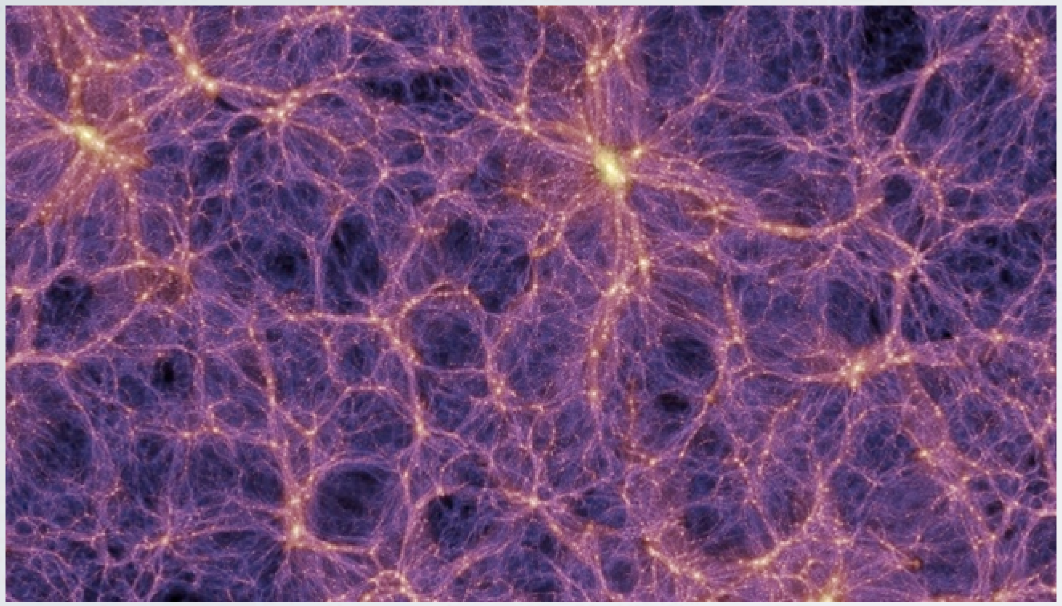
A BIT OF VISUALISATION...

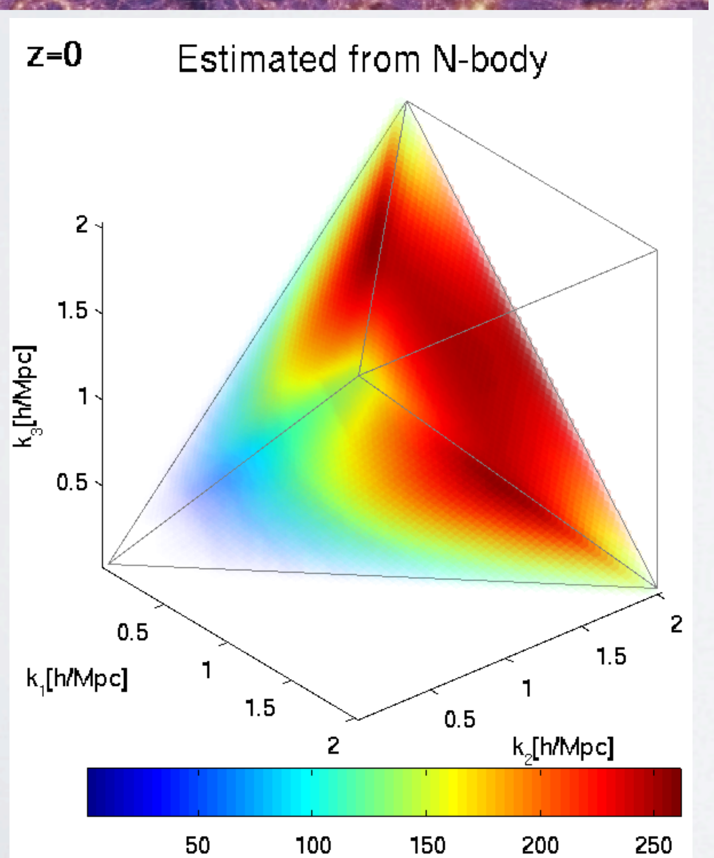
$$S(k_1,k_2,k_3)=\sum_{prs}\alpha_{prs}q_p(k_1)q_r(k_2)q_s(k_3)$$

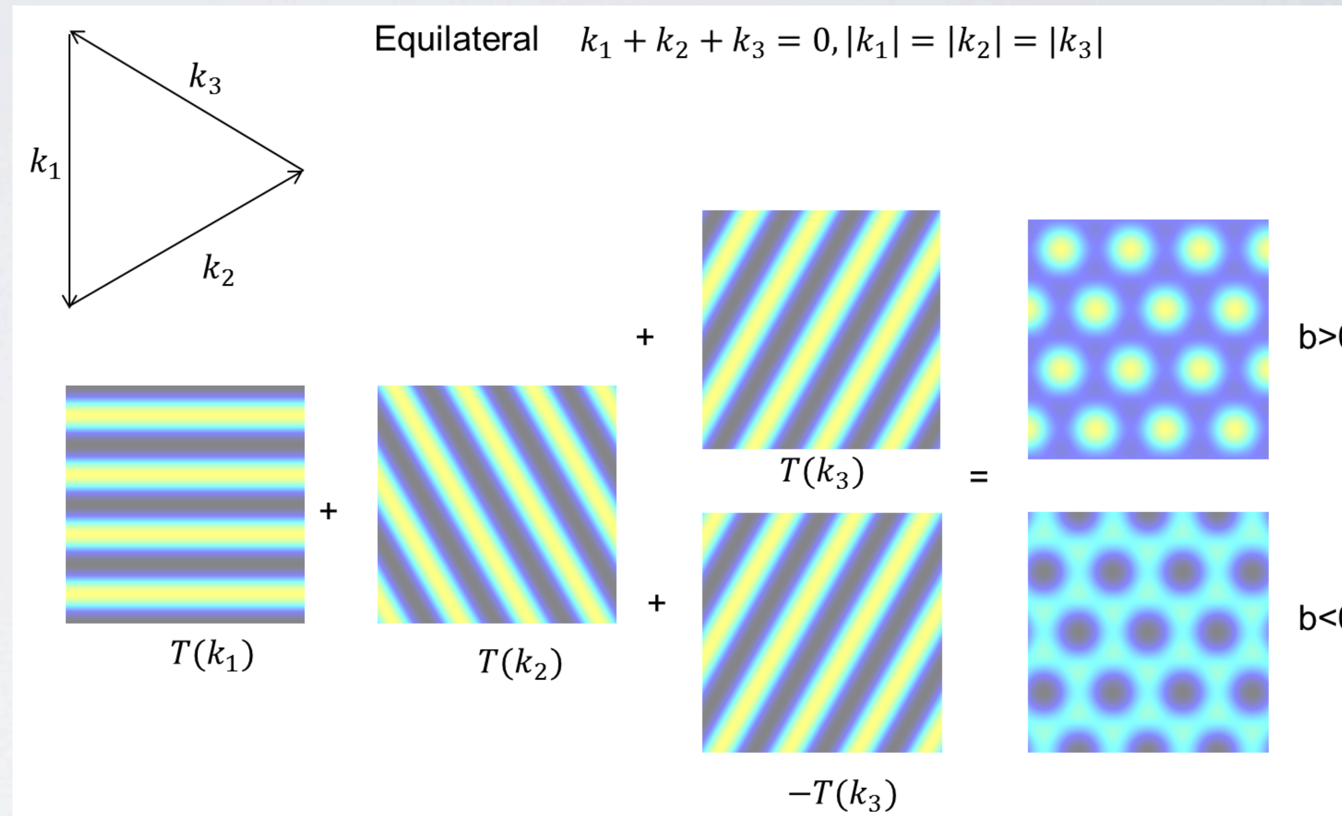




- What does the Gravity Bispectrum look like?







arXiv:1107.5431 Lewis

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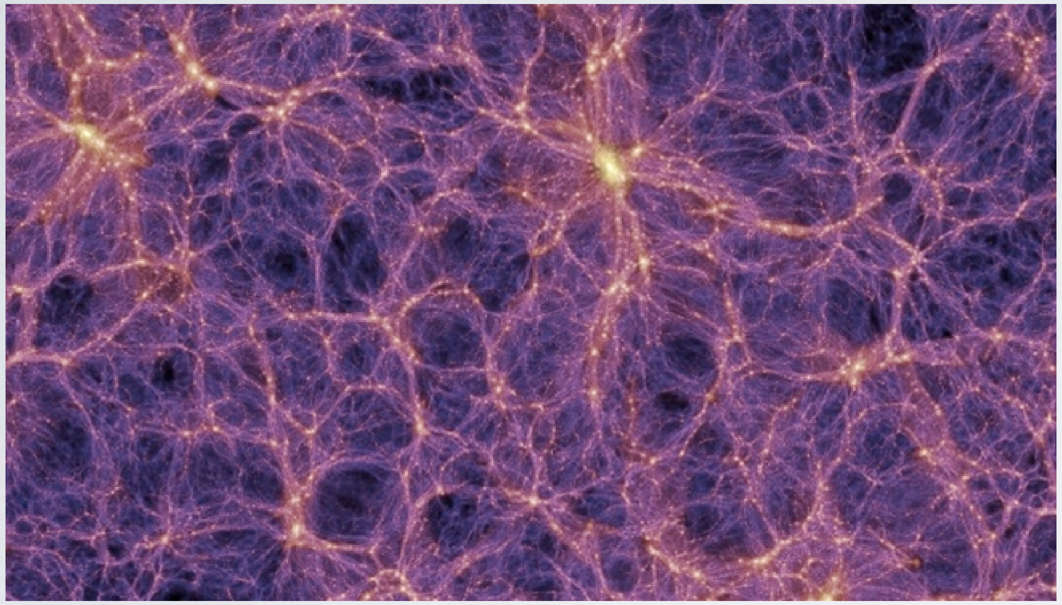
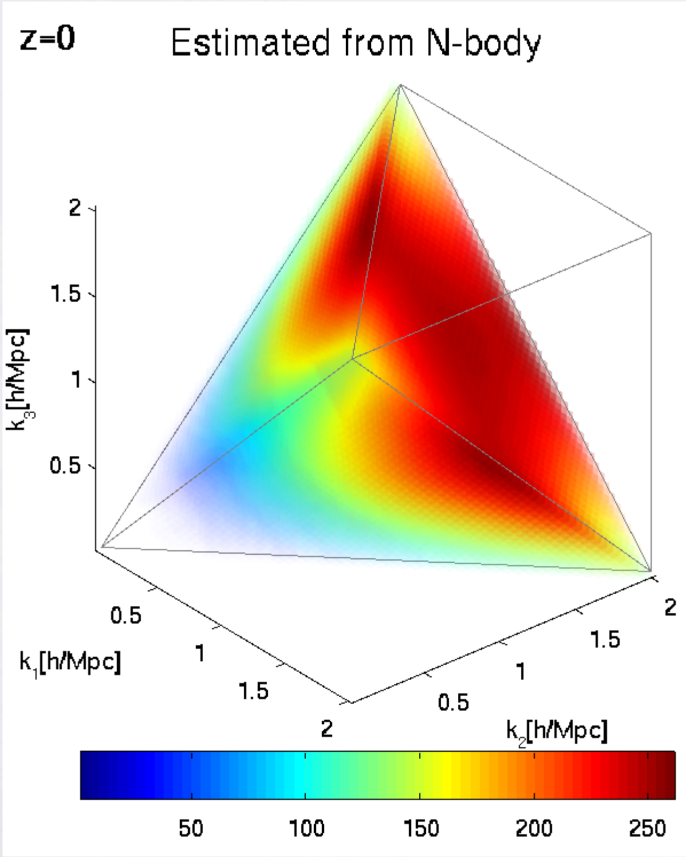
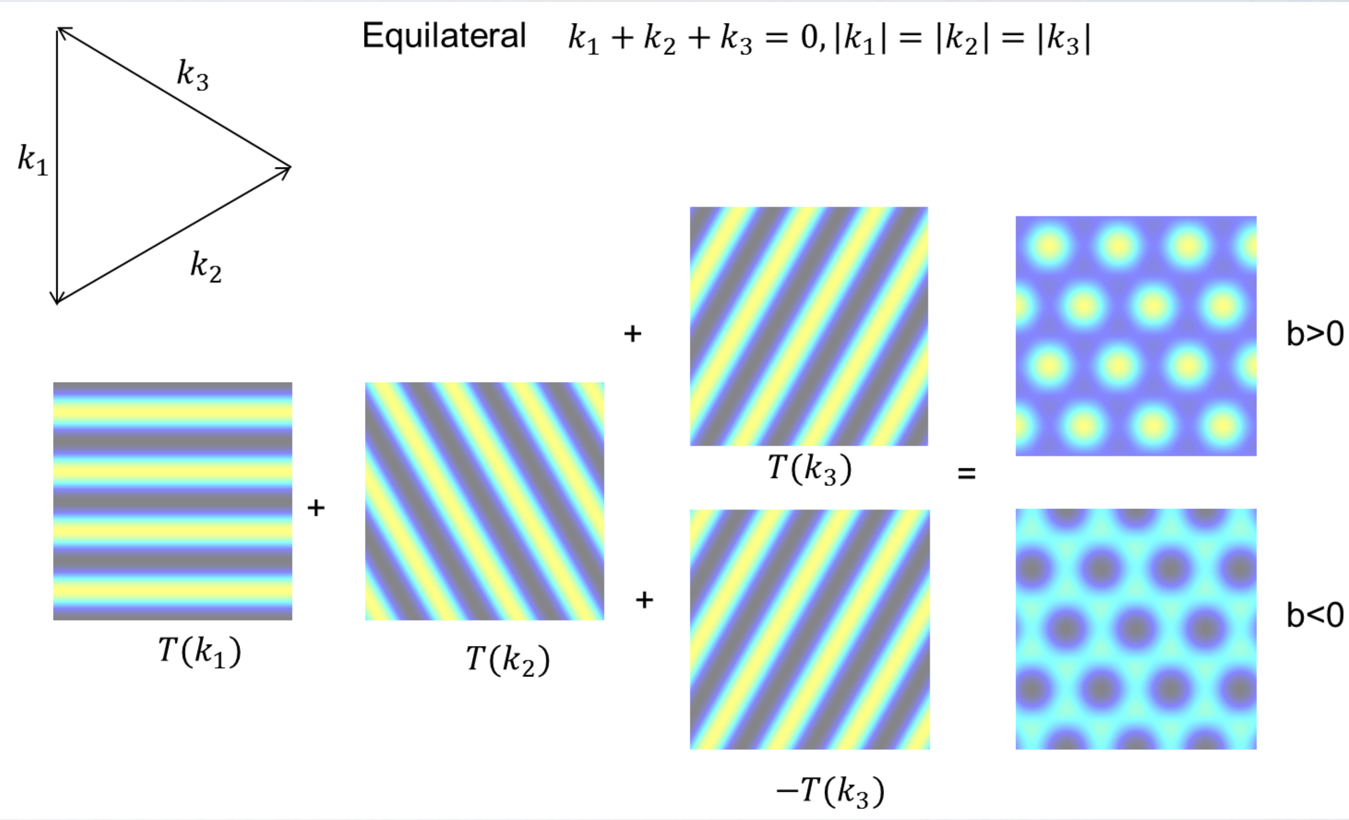






arXiv:1107.5431 Lewis

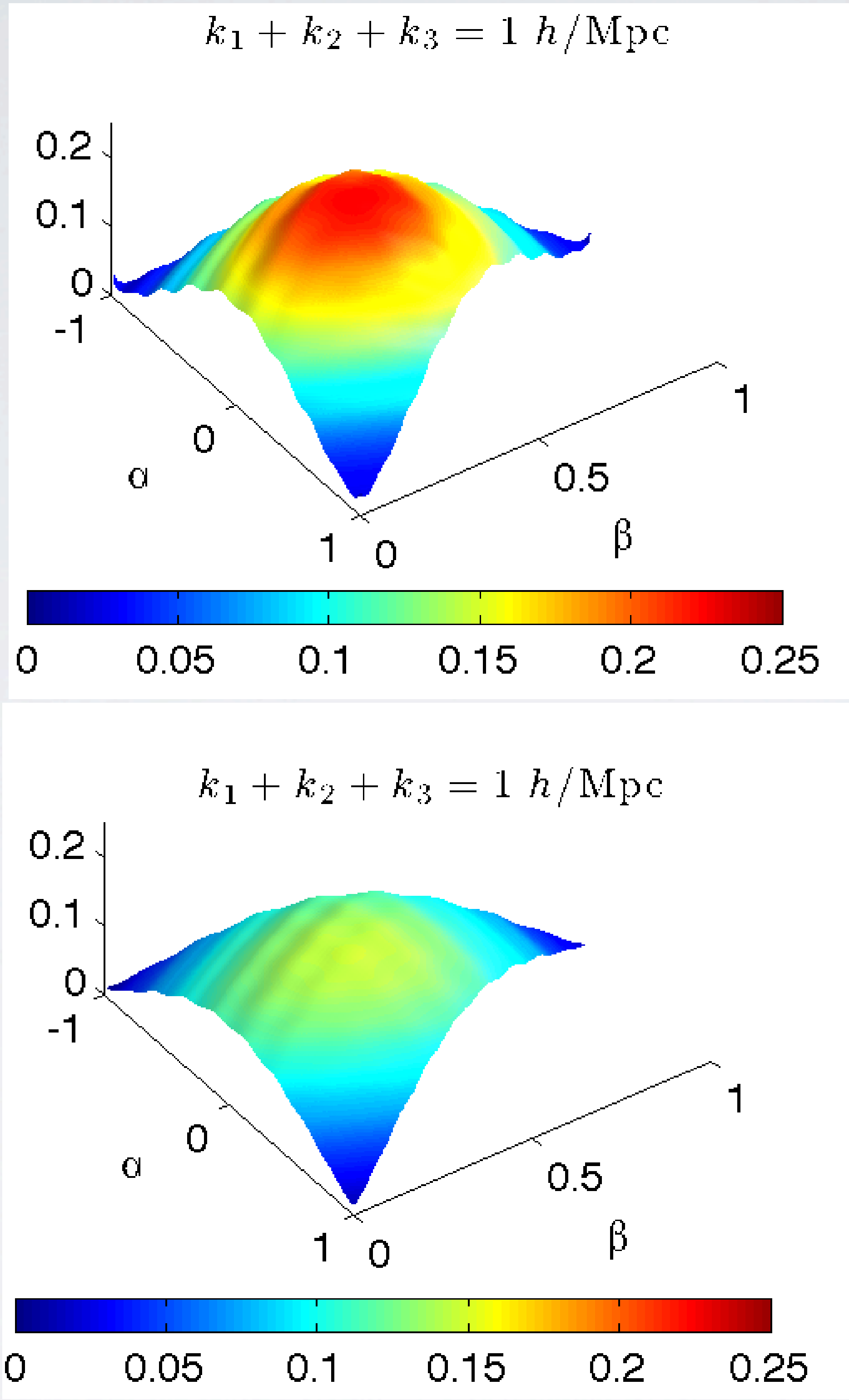


Figure 4. Bispectrum weights $[P_\delta(k_1)P_\delta(k_2) + 2\ \text{perms}]^{-1}$ (top) and $\sqrt{k_1 k_2 k_3 / [P_\delta(k_1)P_\delta(k_2)P_\delta(k_3)]}$ (bottom) evaluated with CAMB [30] at redshift $z = 30$ on slices with $k_1 + k_2 + k_3 = 1h/\mathrm{Mpc}$.

Lots of results - *the evolving bispectrum*

Getting the N-body initial conditions

(A) Poisson equation to convert to density perturbations

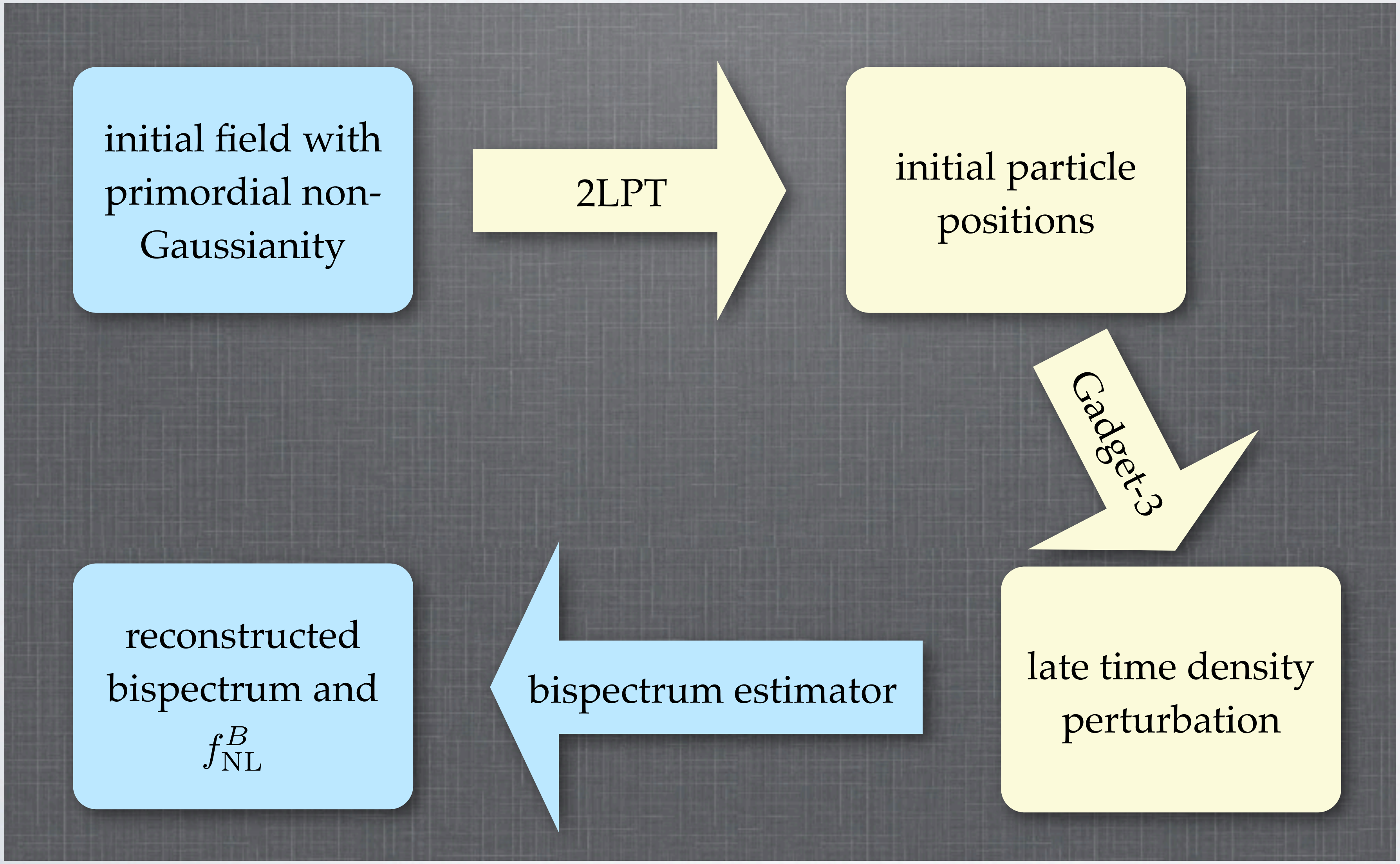
$$\delta_{\mathbf{k}}(a) = M(k;a)\Phi_{\mathbf{k}} \qquad M(k;a) = -\frac{3}{5}\frac{k^2T(k)}{\Omega_m H_0^2}D_+(a)$$

(B) 2nd order Lagrangian perturbation theory to get the **positions** and **velocities** of the particles... must decide whether to use a glass or a grid configuration.

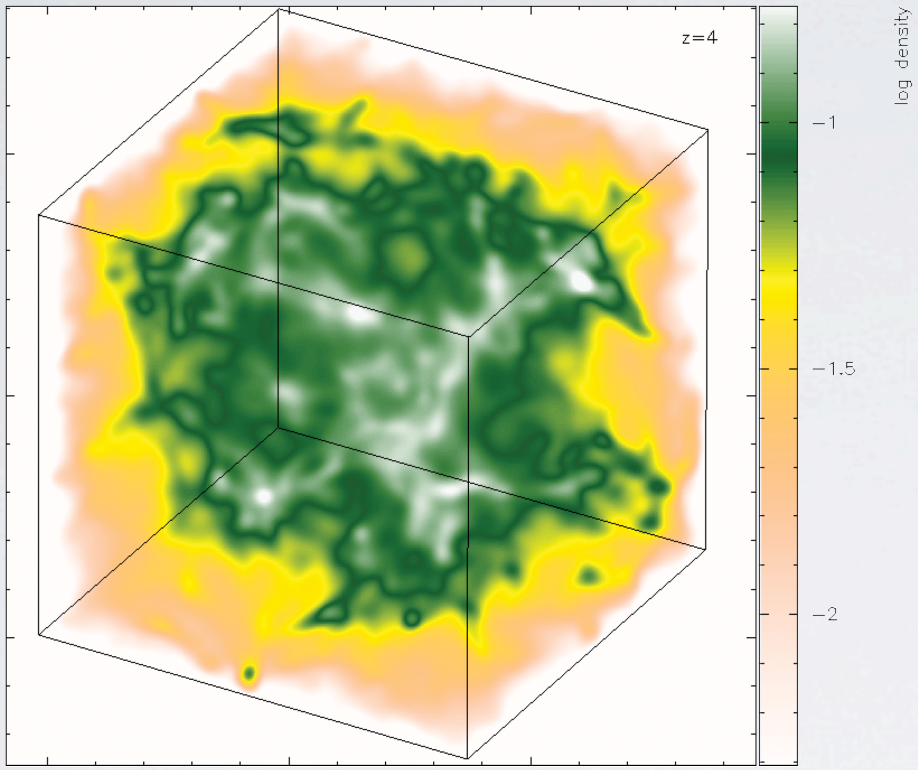
(C) Other things to choose:
- Boxsize
- Number of particles
- Initial redshift
- plus the 'gas softening length'

Name	NG shape	$f_{\rm NL}$	$L[\frac{\rm Mpc}{h}]$	N_p	z_i	$L_s[\frac{\rm kpc}{h}]$	N_r	glass
G512	–	–	1600	512	49	156	3	no
G512g	–	–	1600	512	49	156	3	yes
G768	–	–	2400	768	19	90	3	no
G1024	–	–	1875	1024	19	40	2	no
Loc10	local	10	1600	512	49	156	3	no
Loc10g	local	10	1600	512	49	156	3	yes
Eq100	equil	100	1600	512	49	156	3	no
Eq100g	equil	100	1600	512	49	156	3	yes
Orth100	orth	100	1600	512	49	156	3	no
Orth100g	orth	100	1600	512	49	156	3	yes
Orth100$^-$	orth	-100	1600	512	49	156	3	no
Flat10	flat	10	1600	512	49	156	3	no

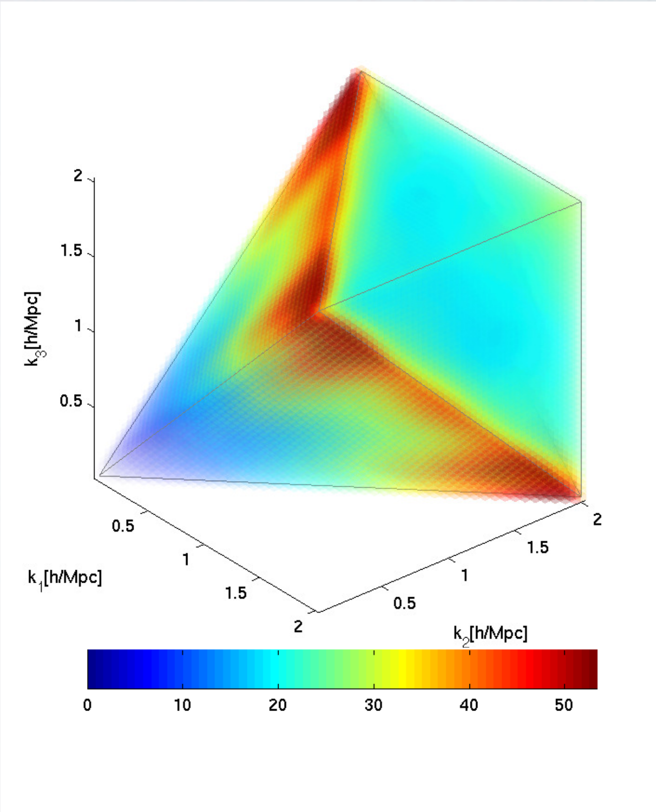
Gravity only



Evolving 'Gaussian' Bispectrum

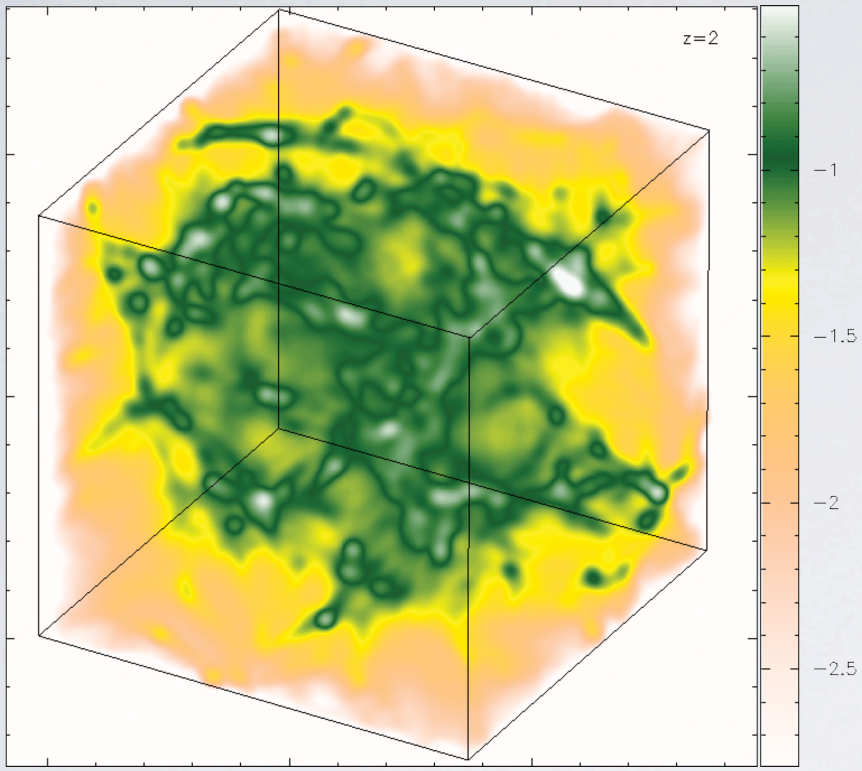


(a) Dark matter, $z = 4$

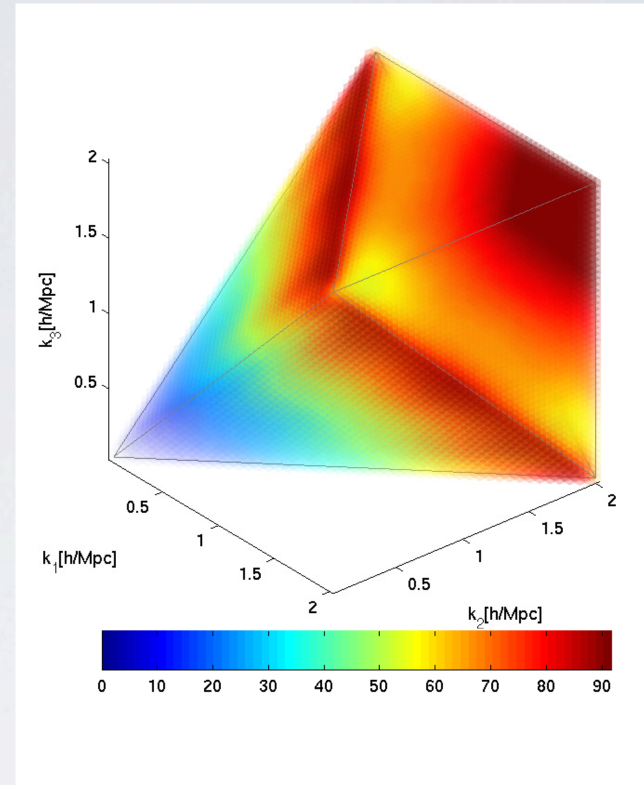


(b) Bispectrum signal, $z = 4$

Early blob-like structure → Flattened bispectrum

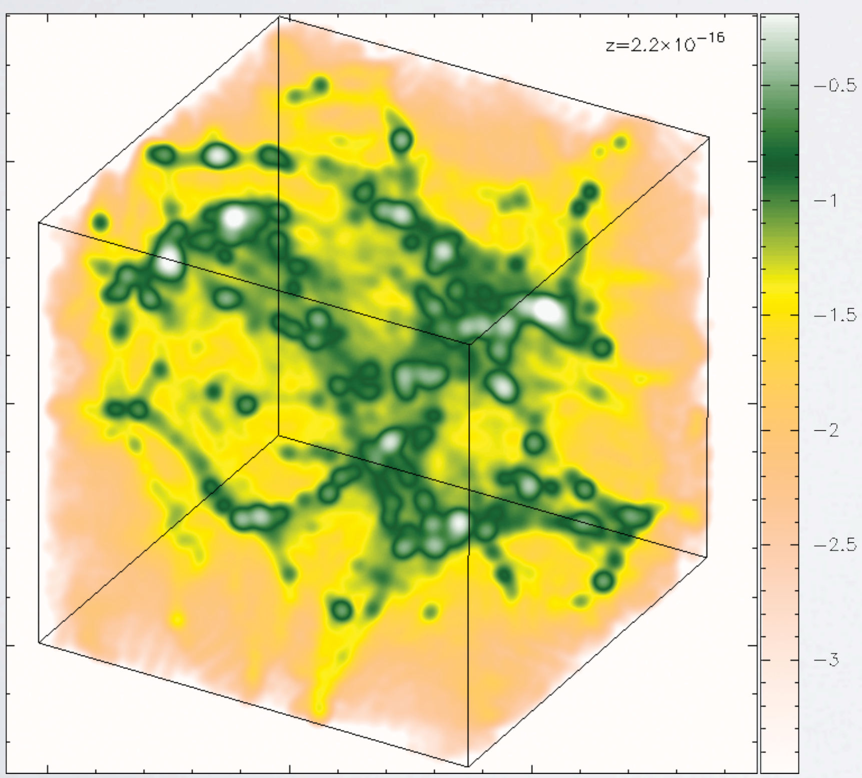


(c) Dark matter, $z = 2$

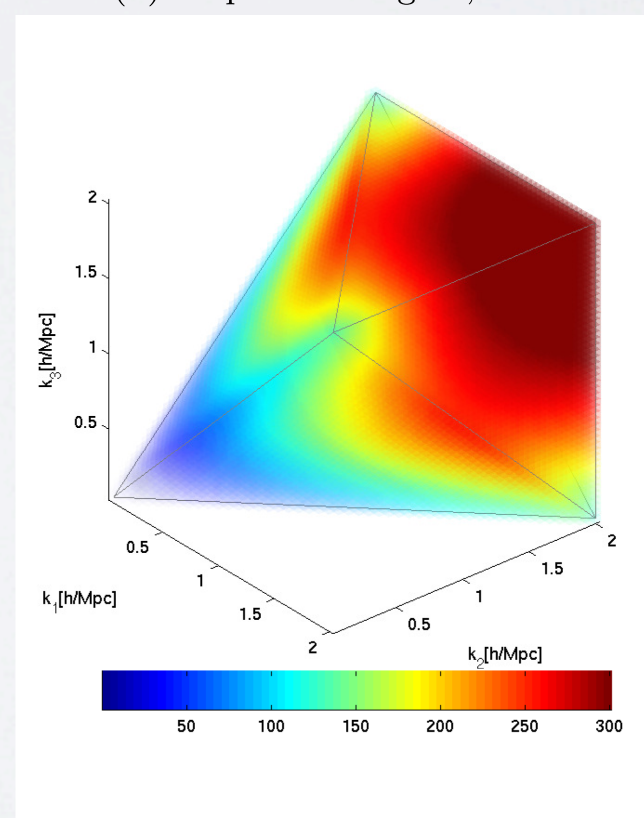


(d) Bispectrum signal, $z = 2$

Filaments and clusters appearing

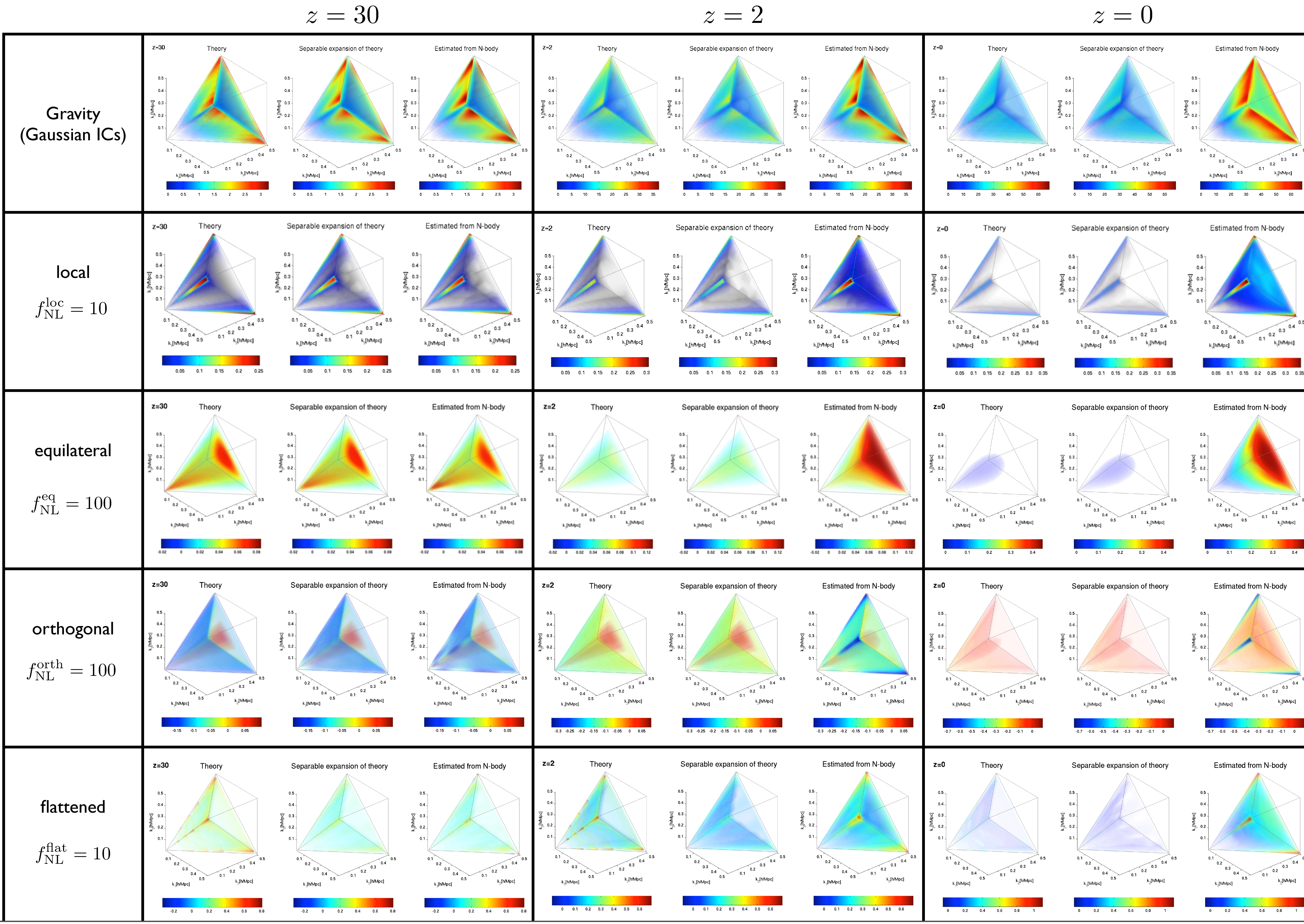


(e) Dark matter, $z = 0$

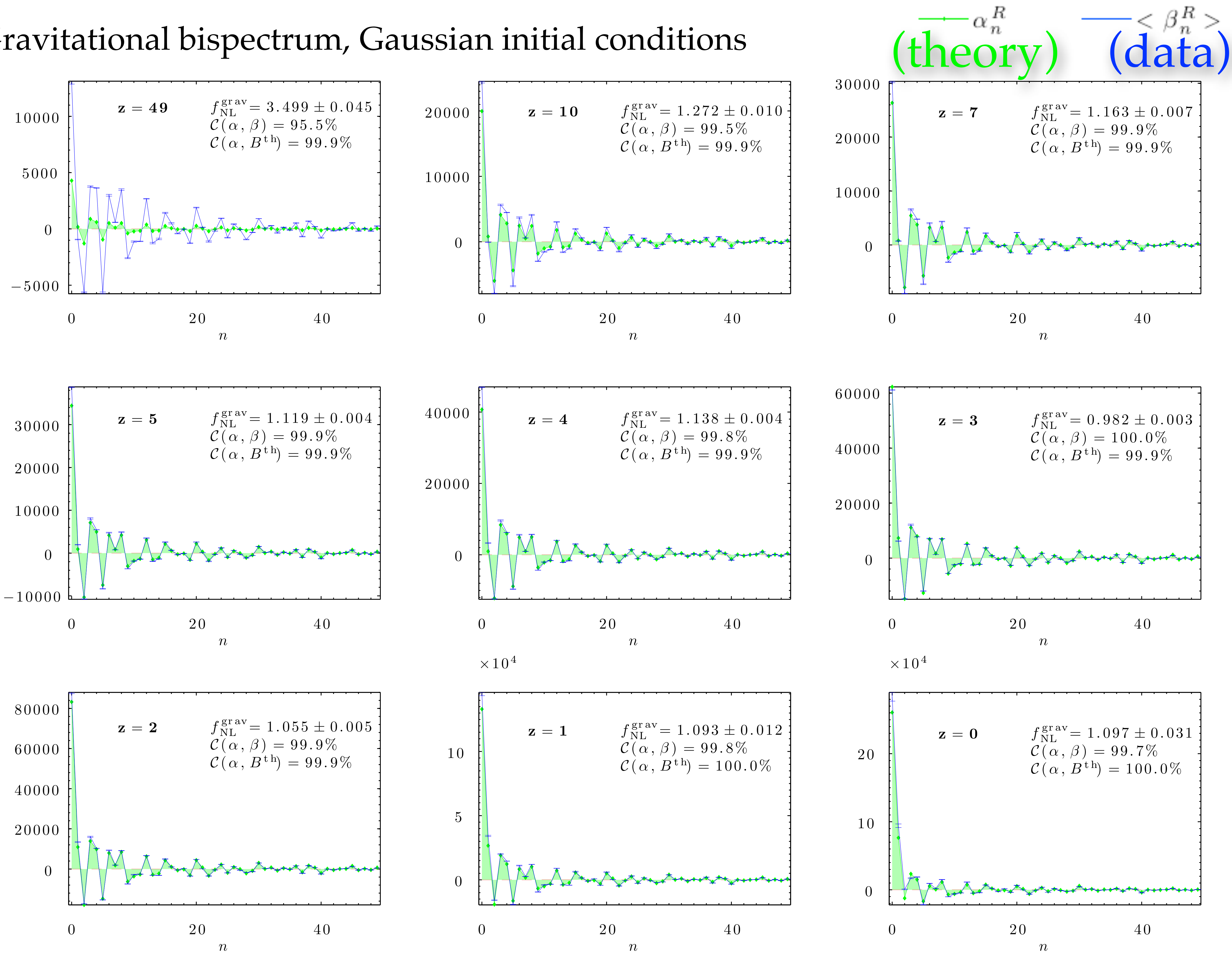


(f) Bispectrum signal, $z = 0$

Looking more and more equilateral

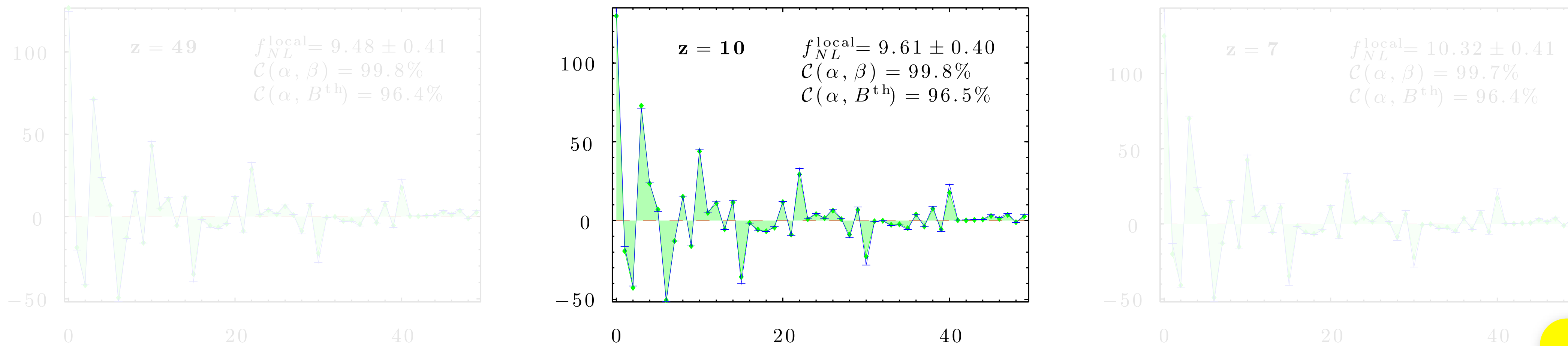

$z = 30$
$z = 2$
$z = 0$
Gravity (Gaussian ICs)
local
$f_{\rm NL}^{\rm loc} = 10$
equilateral
$f_{\rm NL}^{\rm eq} = 100$
orthogonal
$f_{\rm NL}^{\rm orth} = 100$
flattened
$f_{\rm NL}^{\rm flat} = 10$
Theory
Separable expansion of theory
Estimated from N-body

Gravitational bispectrum, Gaussian initial conditions



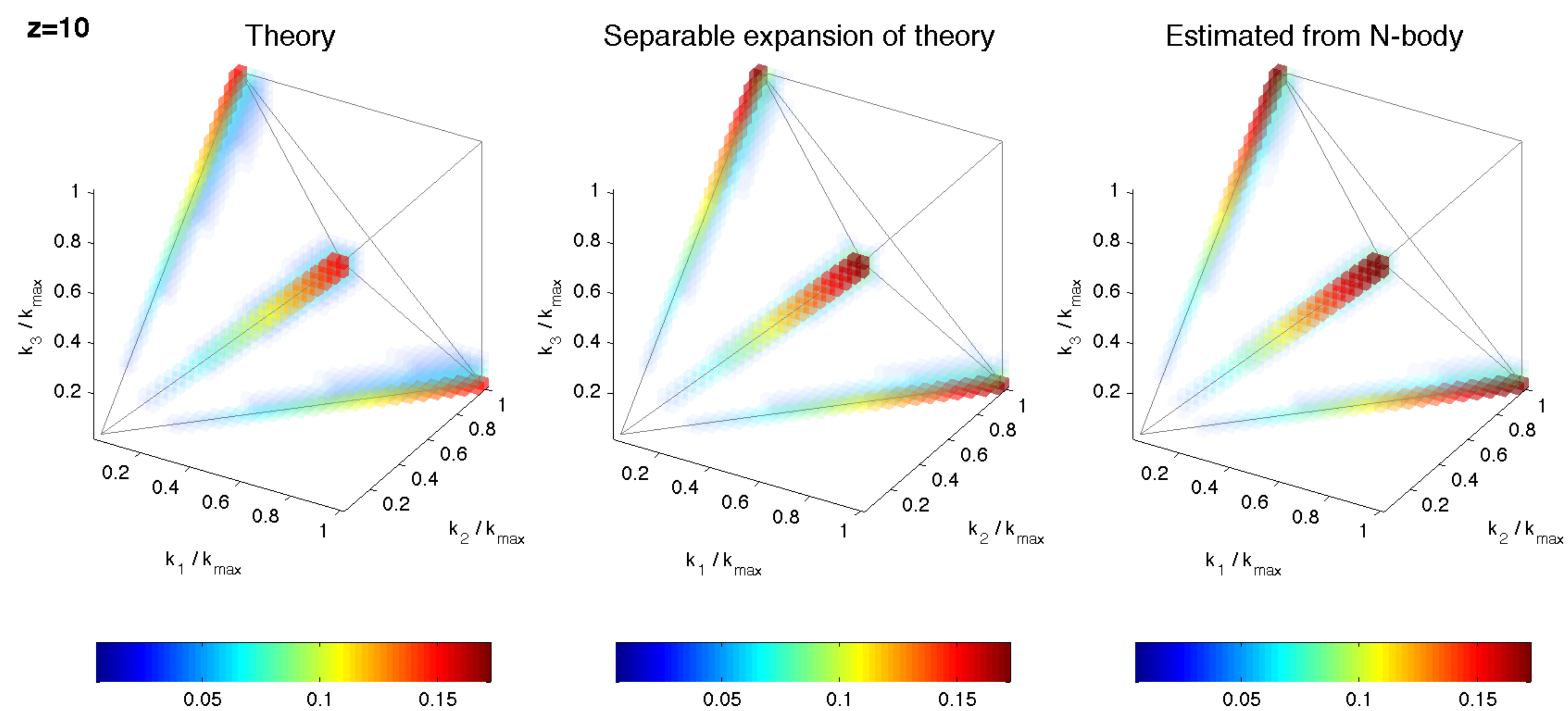
3 realisations of 512^3 particles in a $L = 1600$ Mpc/h box with $z_{\rm init} = 49$ and $k = 0.004 - 0.5$ h/Mpc

Non-Gaussian initial conditions $f_{\rm NL}^{\rm local} = 10$, plot $(\alpha_n^R)^{\rm primordial}_{\rm tree}$ and $\langle \beta_n^R \rangle - \langle (\beta_n^R)_{\rm Gaussian} \rangle$



z=10

plot $\sqrt{k_1 k_2 k_3} B_\delta(k_1, k_2, k_3) / \sqrt{P_\delta(k_1) P_\delta(k_2) P_\delta(k_3)}$



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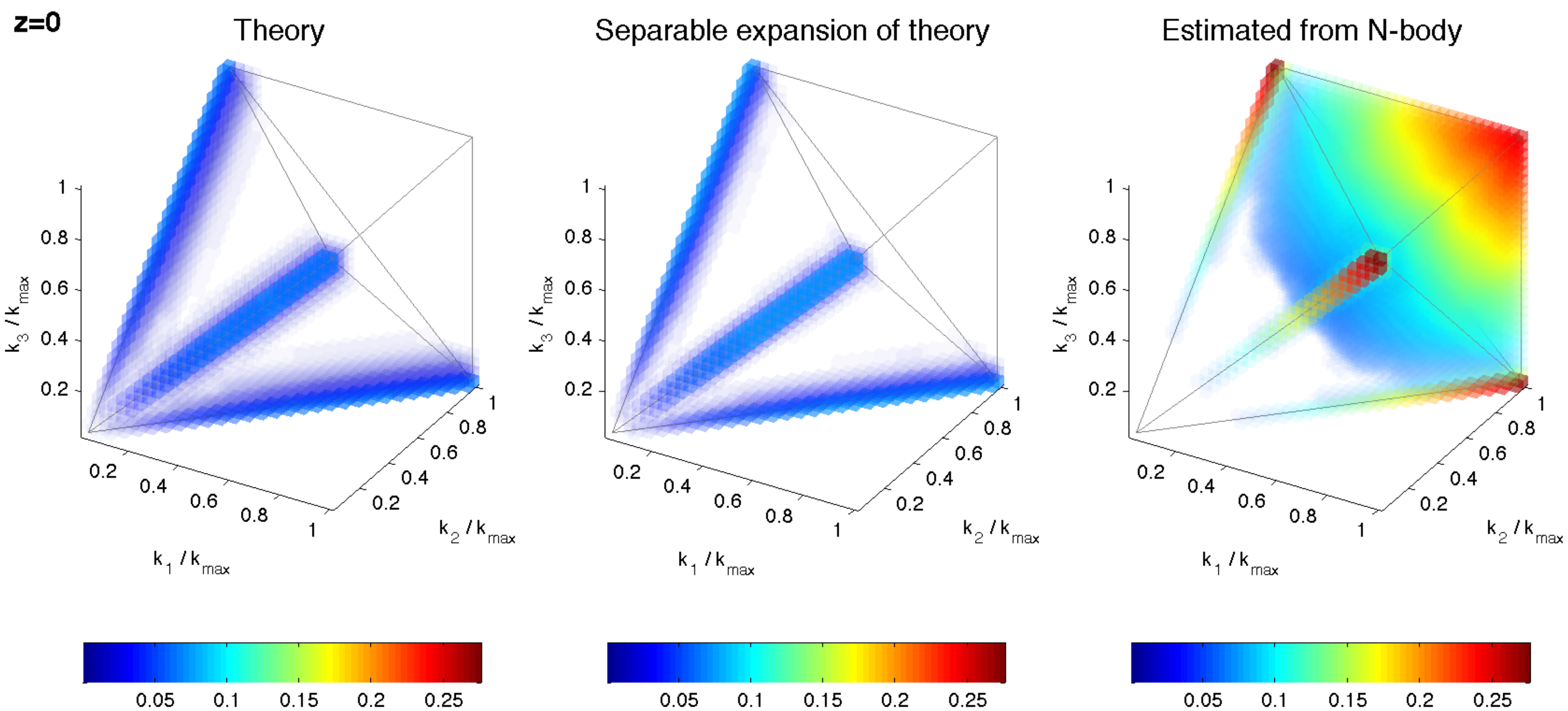
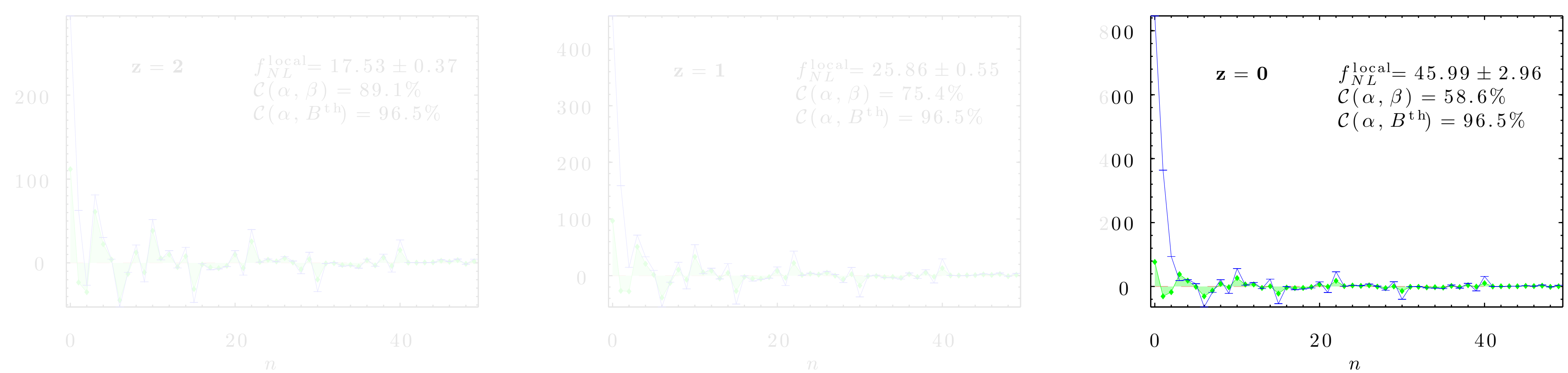


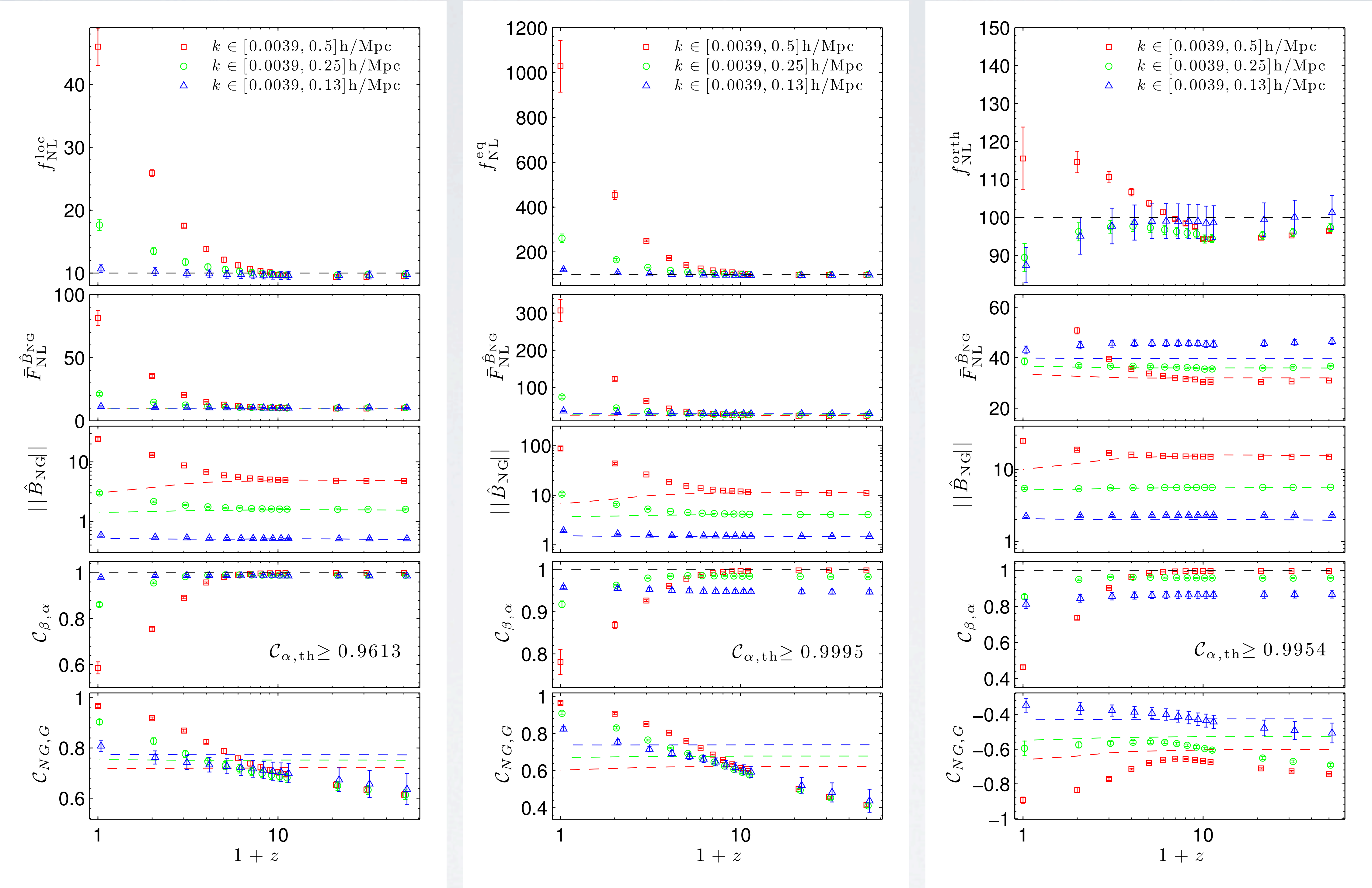
z=0

plot $\sqrt{k_1 k_2 k_3} B_\delta(k_1, k_2, k_3) / \sqrt{P_\delta(k_1) P_\delta(k_2) P_\delta(k_3)}$



3 realisations of 512^3 particles in a $L = 1600$ Mpc/h box with $z_{\rm init} = 49$ and $k = 0.004 - 0.5$ h/Mpc

Comparing to the tree level result



(a) Local (b) Equilateral (c) Orthogonal

COMPARISON AGAINST LOOP LEVEL RESULTS & PHENOMENOLOGICAL MODELS

Gaussian case

Tree level

$$B_\delta^{\rm grav}(k_1,k_2,k_3) = 2P_\delta^L(k_1)P_\delta^L(k_2)F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + 2 \text{ perms}$$

where $$F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2$$

Phenomenological fit (Gil-Marin et al arXiv:1111.4477)

$$\begin{aligned} F_2^{(s)\,\rm eff}(\mathbf{k}_1,\mathbf{k}_2) = {} & \frac{5}{7}a(n_1,k_1)a(n_2,k_2) \\ & + \frac{1}{2}\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)b(n_1,k_1)b(n_2,k_2) \\ & + \frac{2}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2 c(n_1,k_1)c(n_2,k_2)\,, \end{aligned}$$

99% correlation to N-body result

Non-Gaussian case

Tree level $B_\delta^{\rm prim}(k_1,k_2,k_3;z) =$
$$M(k_1,z)M(k_2,z)M(k_3,z)B_\Phi(k_1,k_2,k_3)$$

... < 60% correlation for k>0.1 h/Mpc

Loop Level Result (Sefusatti et al arXiv:1111.6966) $B = B^{II}_{112} + B^{I}_{122} + B^{II}_{122} + B^{I}_{113} + B^{II}_{113}$

$$B^{II}_{112} = \int \frac{d^3q}{(2\pi)^3} F_2(\mathbf{q},\mathbf{k}_3-\mathbf{q}) T^L_\delta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathbf{k}_3-\mathbf{q})\,, \tag{17}$$

$$B^{I}_{112} = F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2)[P^L_\delta(k_1)P_{12}(k_2) + k_1 \leftrightarrow k_2] + 2\,\text{perms}, \tag{18}$$

$$B^{II}_{122} = 4\int \frac{d^3q}{(2\pi)^3} F_2^{(s)}(\mathbf{q},\mathbf{k}_2-\mathbf{q}) F_2^{(s)}(\mathbf{k}_1+\mathbf{q},\mathbf{k}_2-\mathbf{q}) \times B^L_\delta(k_1,q,|\mathbf{k}_1+\mathbf{q}|)P^L_\delta(|\mathbf{k}_2-\mathbf{q}|) + 2\,\text{perms}\,, \tag{19}$$

$$B^{I}_{113} = 3B^L_\delta(k_1,k_2,k_3)\int \frac{d^3q}{(2\pi)^3} F_3^{(s)}(\mathbf{k}_3,\mathbf{q},-\mathbf{q})P^L_\delta(q) + 2\,\text{perms}, \tag{20}$$

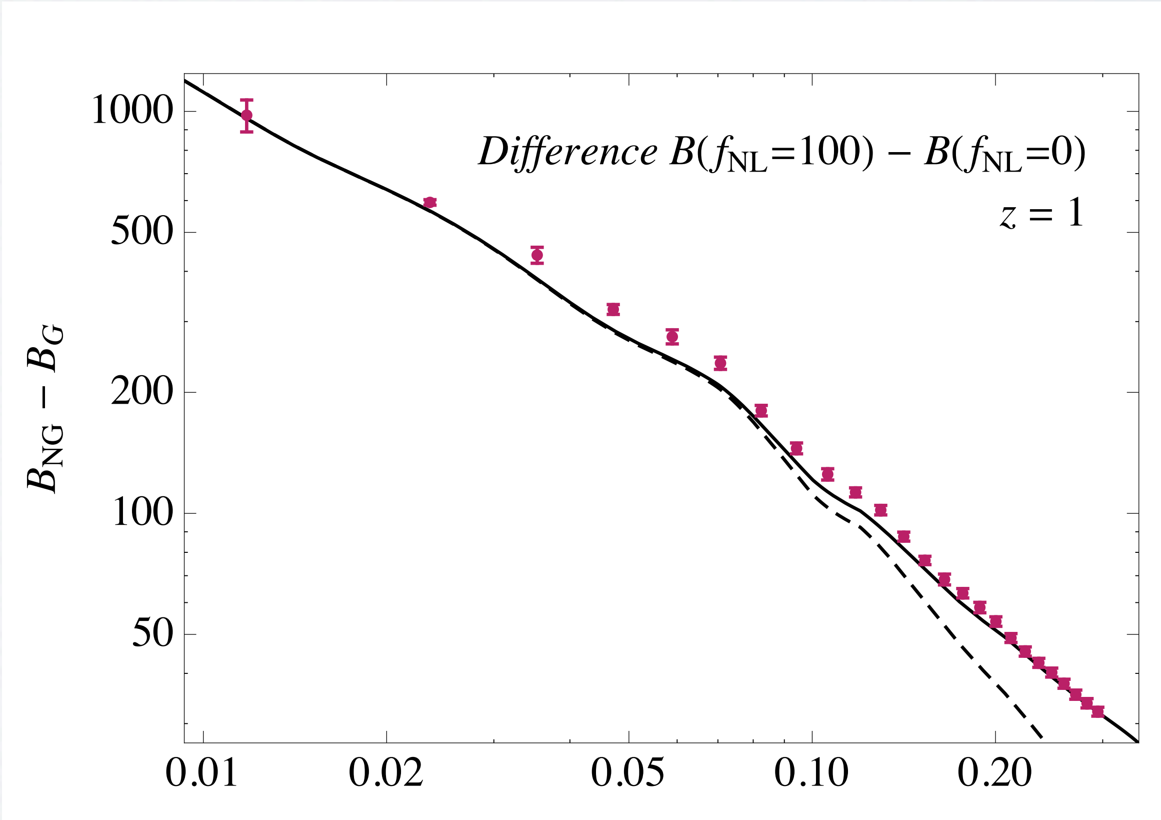
$$B^{II}_{113} = 3P^L_\delta(k_1)\int \frac{d^3q}{(2\pi)^3} F_3^{(s)}(\mathbf{k}_1,\mathbf{q},\mathbf{k}_2-\mathbf{q}) \times B^L_\delta(k_2,q,|\mathbf{k}_2-\mathbf{q}|) + (k_1 \leftrightarrow k_2) + 2\,\text{perms}, \tag{21}$$

where

$$P_{12}(k) = 2\int \frac{d^3q}{(2\pi)^3} F_2^{(s)}(\mathbf{q},\mathbf{k}-\mathbf{q})B^L_\delta(k,q,|\mathbf{k}-\mathbf{q}|)\,.$$

Model	z	Tree $k_{\max} = 0.5$	Tree+Loop (0.5)	Tree $k_{\max} = 0.25$	Tree+Loop (0.25)	Tree $k_{\max} = 0.125$	Tree+Loop (0.125)
Local	0	58%, 66.6	79%, 12.3	84%	84%	94%, 11	98%, 10.2
	1	87%, 29.2	87%, 8.5	95%	98%	96%, 10.6	97%, 10.5
	2	89%, 25.9	92%, 13.8	98%	99%	96%, 10.5	97%, 10.4
	3	96%, 20.8	96%, 13.8	99%	100%	96%, 10.3	97%, 10.3
Equil	0	78%	84%	91%	95%	96%	96%
	1	87%	91%	96%	97%	95%	97%
	2	92%	96%	98%	98%	95%	95%
	3	96%	98%	98%	99%	95%	95%
Flat	0	48%, 47.45	88%, 11.1	76%	98%	90%, 10.3	95%, 9.8
	1	66%, 30.3	94%, 11.8	90%	98%	92%, 10.2	93%, 10.1
	2	83%, 22.9	98%, 12.1	94%	97%	94%, 10.3	94%, 10.1
	3	92%, 19.1	99%, 12.2	95%	97%	92%, 10.2	93%, 10.1
Orthog	0	47%	70%	87%	87%		
	1	74%	85%	95%	96%		
	2	90%	95%	96%	97%		
	3	96%	98%	96%	96%		

Previous analysis only particular slices=>



- Beyond perturbation theory... The Halo Model

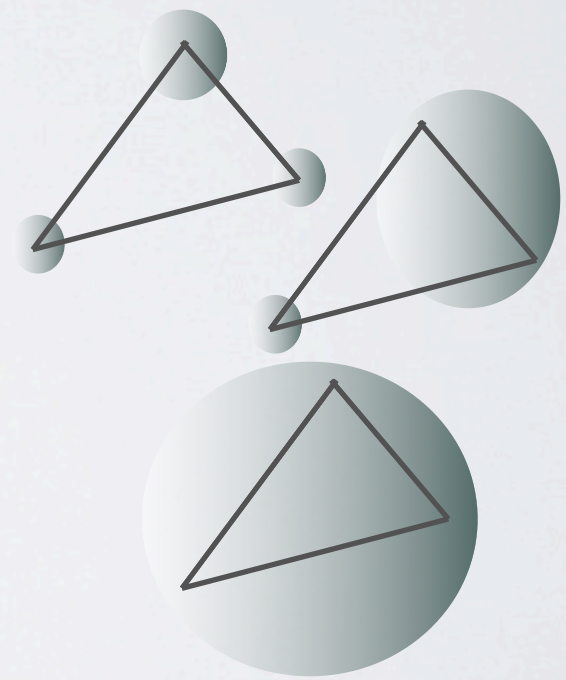
Assumes all dark matter resides in halos

Ingredients: (1) Halo mass function $n(m,z)$
(2) Halo density profile $\hat{\rho}(k_1, m, z)$
(3) Halo bias functions $b_1(m_1)$

(Figueroa et al arXiv:1205.2015)

$$B(k_1,k_2,k_3) = B_{3h}(k_1,k_2,k_3) + B_{2h}(k_1,k_2,k_3) + B_{1h}(k_1,k_2,k_3)$$

$$\begin{aligned}
B_{3h}(k_1,k_2,k_3,z) &= \frac{1}{\bar{\rho}^3}\left[\prod_{i=1}^{3}\int d\,m_i\, n(m_i,z)\,\hat{\rho}(m_i,z,k_i)\right] B_h(k_1,m_1;k_2,m_2;k_3,m_3;z)\,,\\
B_{2h}(k_1,k_2,k_3,z) &= \frac{1}{\bar{\rho}^3}\int d\,m\, n(m,z)\,\hat{\rho}(m,z,k_1)\int d\,m'\, n(m',z)\,\hat{\rho}(m',z,k_2)\,\hat{\rho}(m',z,k_3)\\
&\quad \times P_h(k_1,m,m',z) + \text{cyc.}\,,\\
B_{1h}(k_1,k_2,k_3,z) &= \frac{1}{\bar{\rho}^3}\int d\,m\, n(m,z)\,\hat{\rho}(k_1,m,z)\,\hat{\rho}(k_2,m,z)\,\hat{\rho}(k_3,m,z)\,.
\end{aligned}$$



where
$$\begin{aligned}
B_h(k_1,m_1;k_2,m_2;k_3,m_3;z) &= b_1(m_1)\,b_1(m_2)\,b_1(m_3)\,B(k_1,k_2,k_3)\\
&\quad + [b_1(m_1)\,b_1(m_2)\,b_2(m_3)\,P(k_1)\,P(k_2) + \text{cyc.}]
\end{aligned}$$

Gaussian case: Correlation > 99.2% out to k=2 h/Mpc (z=0,1,2)

Local non-Gaussian case:

Correlation > 97.5% out to k=2 h/Mpc (z=0,1,2)

but ... overestimates amplitude beyond k=0.5h/Mpc. Need to divide the 1-halo term by factor of 4.

Simple phenomenological fitting formulae

$$B_\delta^{\text{fit}}(k_1,k_2,k_3) \equiv B_{\delta,\text{NL}}^{\text{grav}} + c_1 \tilde{D}(z)^{d_1} B_\delta^{\text{const}}$$

where $B_\delta^{\text{const}}(k_1,k_2,k_3) \equiv (k_1+k_2+k_3)^{-1.7}$

99.3% correlation to halo model at 2h/Mpc

Simulation	c_1	d_1	$\min_{z\leq 20}(\mathcal{C}_{\beta,\alpha})$	$\mathcal{C}_{\beta,\alpha}(z=0)$
G_{400}^{512}	1.0×10^7	8	99.8%	99.8%
G512g	4.1×10^6	7	99.8%	99.8%
Loc10	2.4×10^3	6	99.7%	99.7%
Eq100	8.6×10^2	6	97.9%	99.4%
Flat10	1.1×10^4	6	98.8%	98.9%
Orth100	-2.6×10^2	6	90.5%	90.5%

Primordial Non-Gaussianity

Time shifted model? $B^{\rm grav}_{\delta,\rm const}(k_1,k_2,k_3)|_z \equiv c_1\,\bar{D}^{n_h}(z)\,(k_1+k_2+k_3)^\nu$

Constant component of primordial bispectrum ⟷ One-Halo term

➡ Faster growth of structure than tree-level

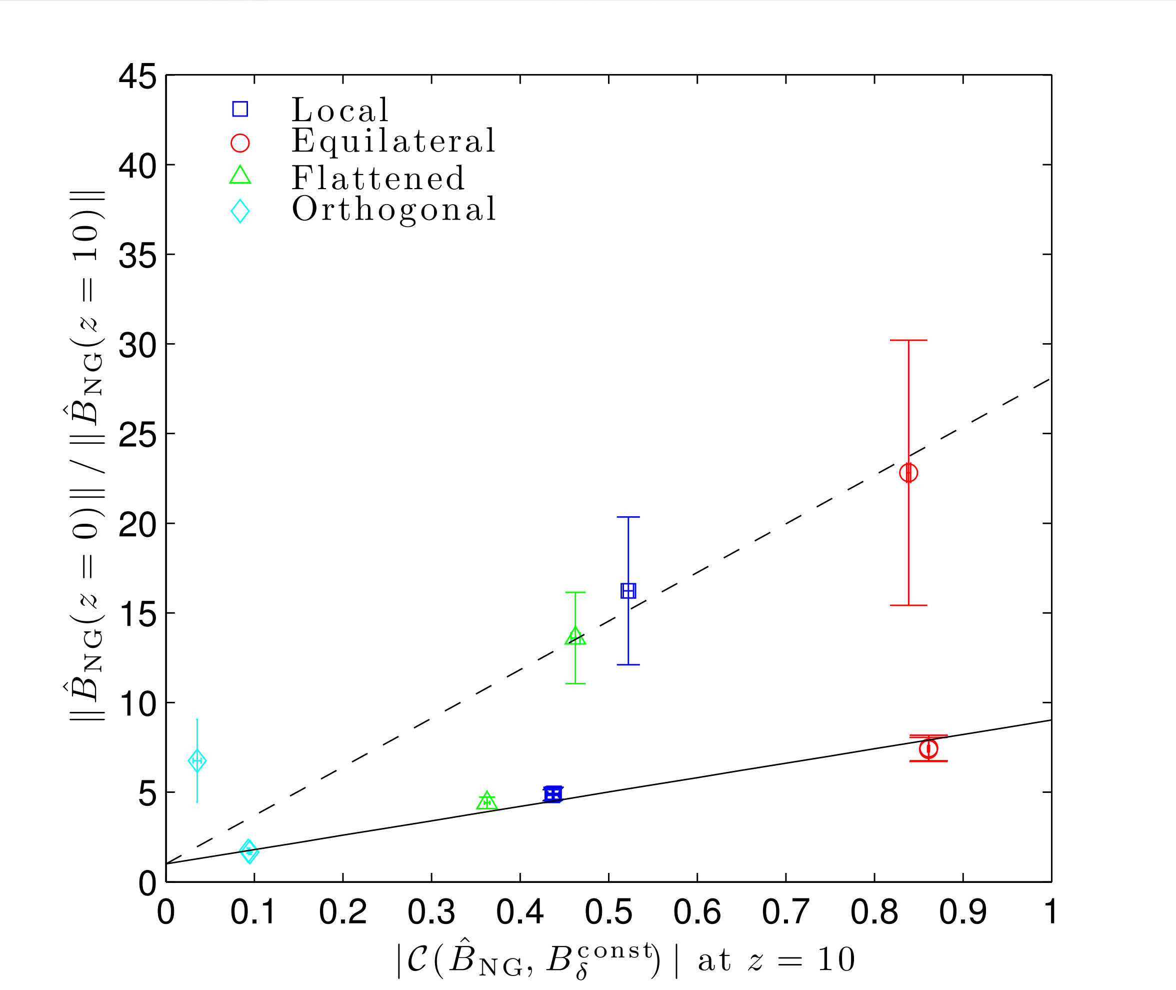
$$\bar{D}^{n_h}(z+\Delta z) \approx \bar{D}^{n_h}(z) + n_h \bar{D}^{n_h-1}(z)\frac{d\bar{D}(z)}{dz}\Delta z.$$

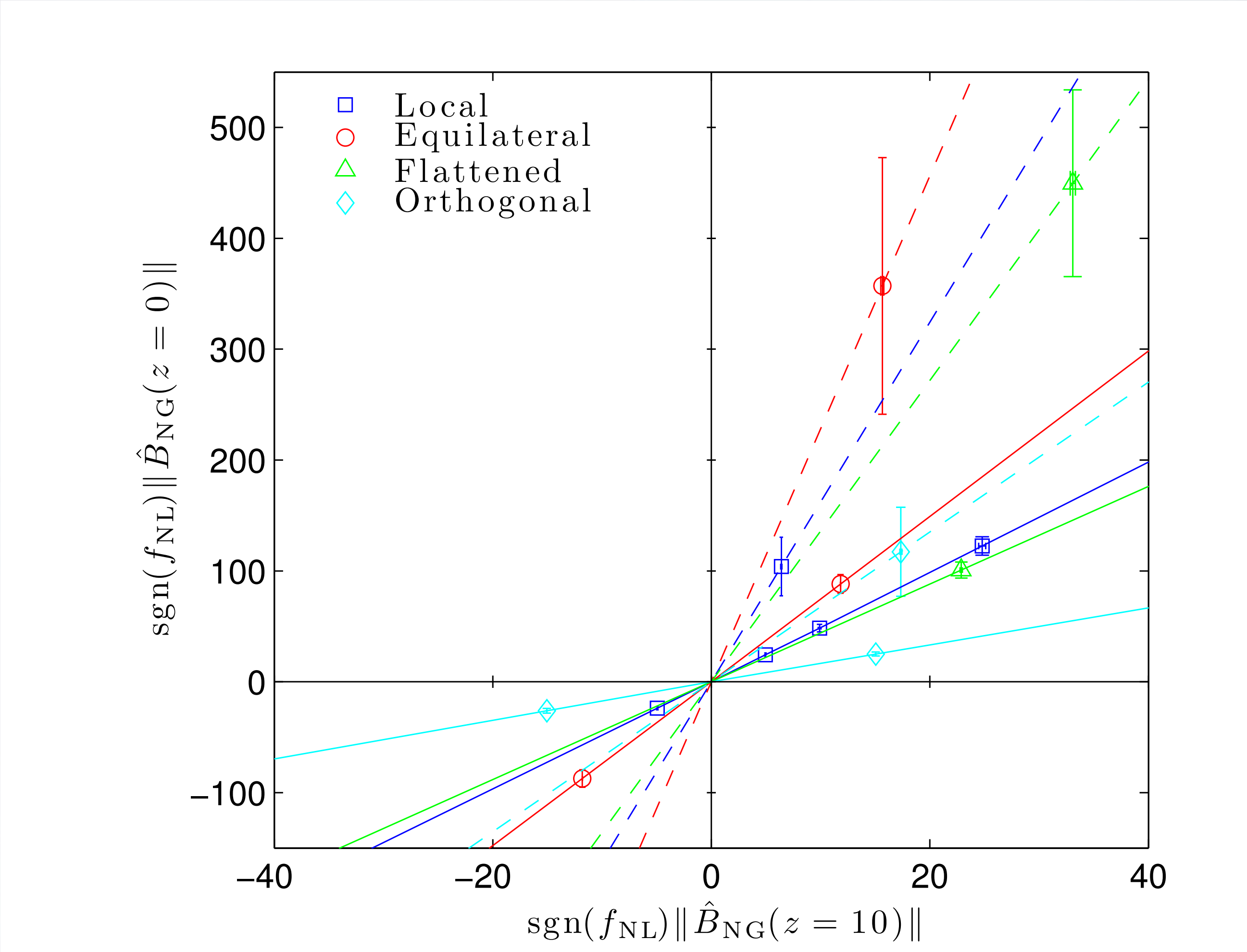
$$(\hat{f}^{\rm const}_{\rm NL})_{\rm Gauss}(z+\Delta z) = (\hat{f}^{\rm const}_{\rm NL})_{\rm NG}(z)$$

➡ $B^{\rm fit}_{\rm NG}(k_1,k_2,k_3) \equiv f_{\rm NL}\left[B^{\rm prim}_{\delta,\rm NL} + B^{\rm prim}_{\delta,\rm const}\right]$

$$B^{\rm prim}_{\delta,\rm const}(k_1,k_2,k_3) \equiv c_2\,\bar{D}^{n_h^{\rm prim}}(z)\;(k_1+k_2+k_3)^\nu$$

$$\|B_{\mathrm{NG}}(z_{\mathrm{late}})\| \propto \mathcal{C}(B_{\mathrm{NG}}(z_{\mathrm{early}}), B^{\mathrm{grav}}_{\delta,\mathrm{const}}(z_{\mathrm{early}})) \times \|B_{\mathrm{NG}}(z_{\mathrm{early}})\|.$$





Message: To predict the relative *growth* of the *bispectrum* only need to correlate to *constant* model at *early* times

CONCLUSIONS

- Can efficiently produce reliable general non-Gaussian **initial conditions** for N-body simulations
- Can more accurately analyse the results of N-body simulations
- More complete comparison to tree-level, loop-level and phenomenological predictions
- Next step... application to the galaxy bispectrum