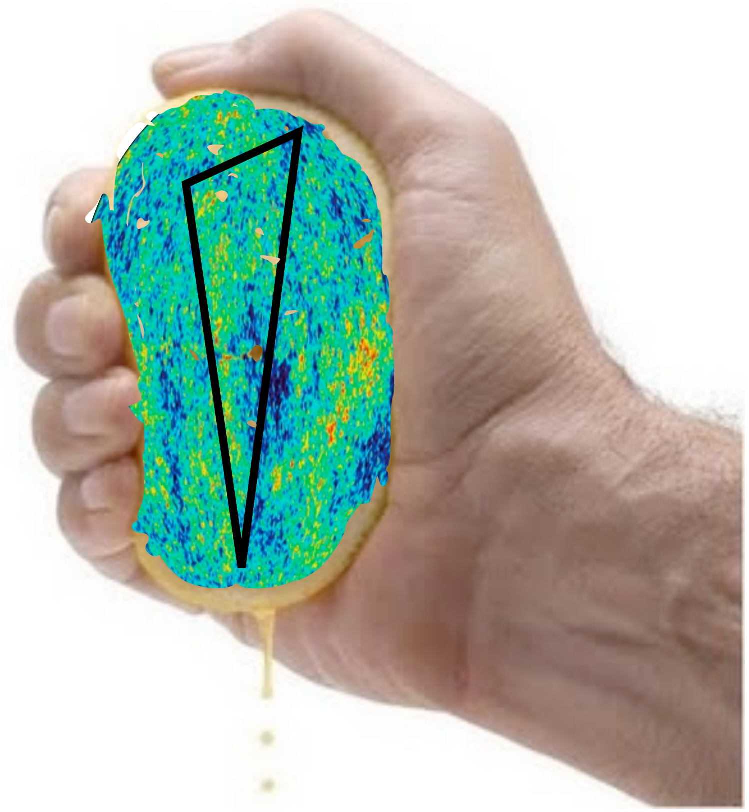
Squeezing the CMB bispectrum

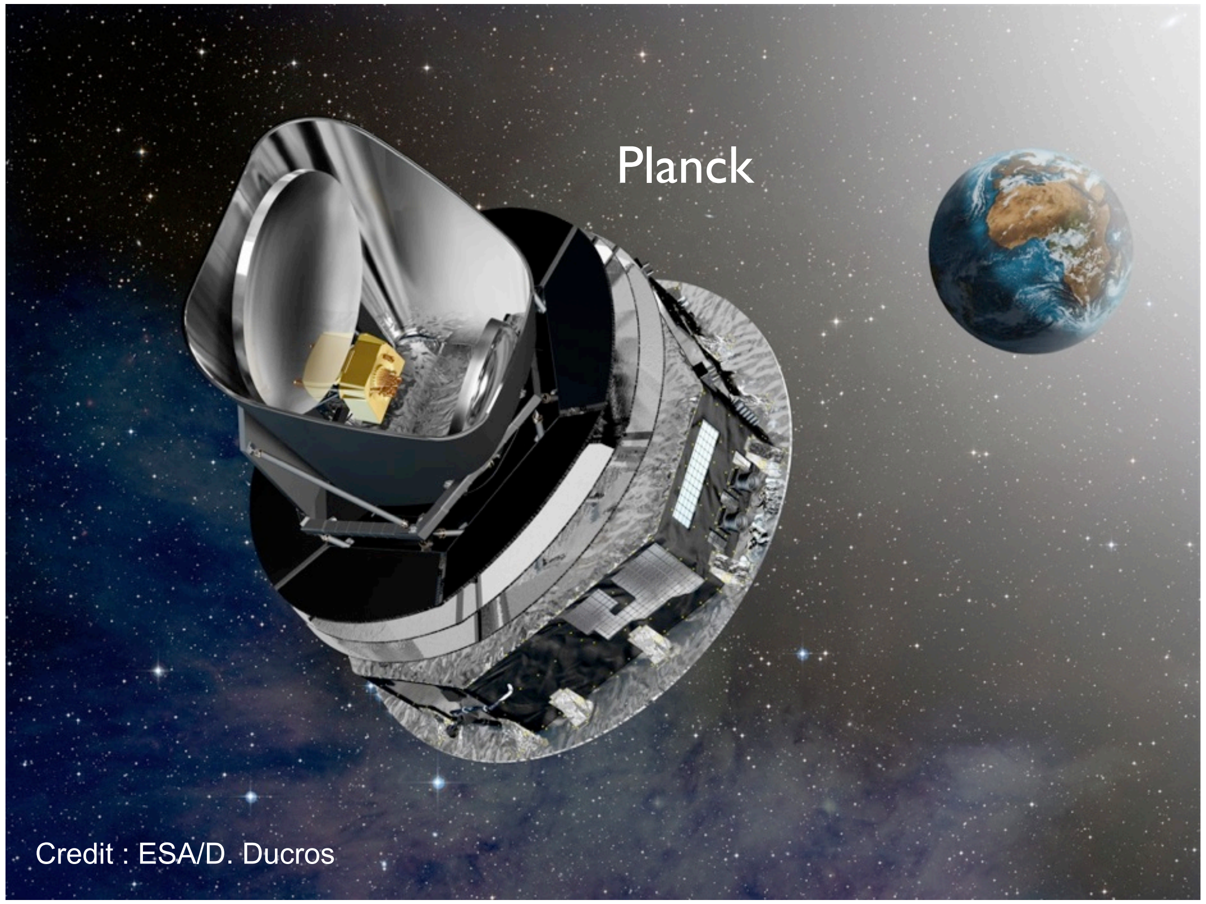


Filippo Vernizzi - IPhT, CEA Saclay

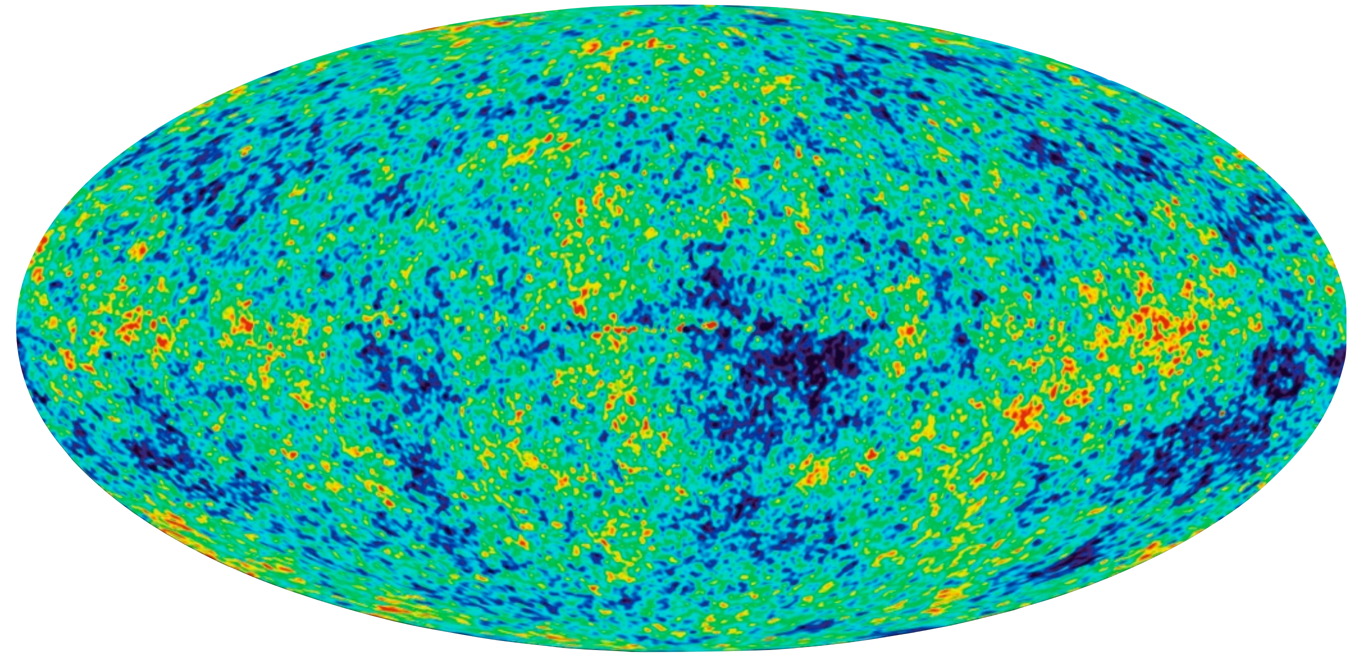
With L. Boubekeur, P. Creminelli, G. D'Amico, J. Noreña,
(arXiv:0806.1016, arXiv:0906.0980)
and P. Creminelli, C. Pitrou (arXiv:1109.1822)

Related talks: Antony Lewis, Guido Pettinari, Takahiro Tanaka, ...

Benasque - 23 August 2012



WMAP



Spherical harmonic expansion of the temperature fluctuations:

$$\frac{\Delta T}{T}(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

If CMB is Gaussian it is fully characterized by the angular 2-point function or power spectrum (~10^3 numbers):

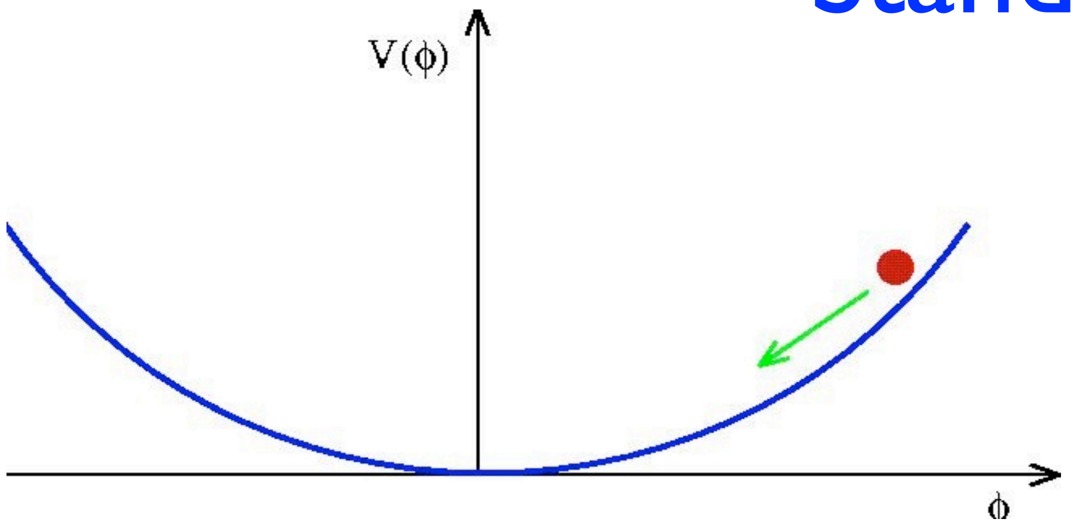
$$\langle \frac{\Delta T}{T}(\hat{n}) \frac{\Delta T}{T}(\hat{n}') \rangle \quad \Rightarrow \quad C_l \equiv \sum_m \langle a_{lm} a_{l'm'} \rangle \delta_{ll'} \delta_{mm'}$$

A CMB map contains ~10^6 pixels. Information could be hidden in higher-order statistics.

Bispectrum: angular 3-point function

$$\langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \frac{\Delta T}{T}(\hat{n}_3) \rangle \quad \Rightarrow \quad \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3} b_{l_1 l_2 l_3}$$

Standard picture



$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

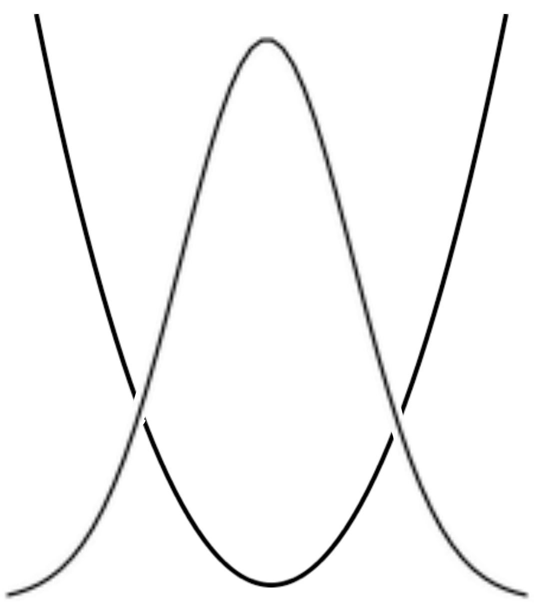
Curvature fluctuations of a constant inflaton field hypersurface:

$$\phi = \phi(t)$$

$$g_{ij} = a^2(t)e^{2\zeta}\delta_{ij}$$

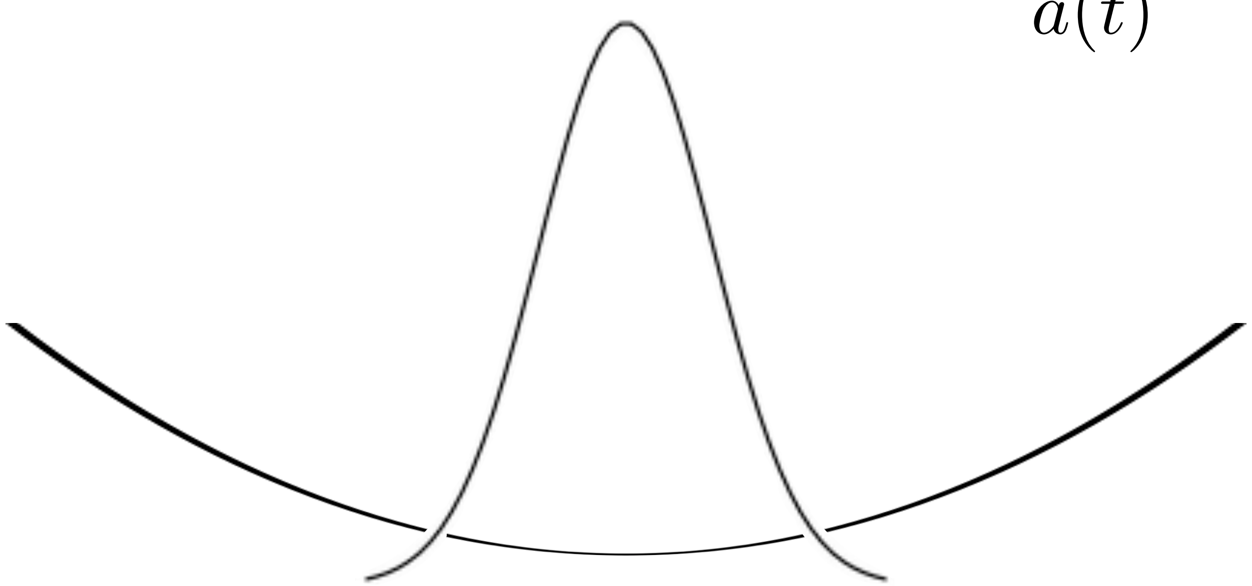
$$S = \frac{1}{2}\int dtd^3xa^3\frac{\dot{\phi}^2}{H^2}\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial\zeta)^2\right]$$

initial potential



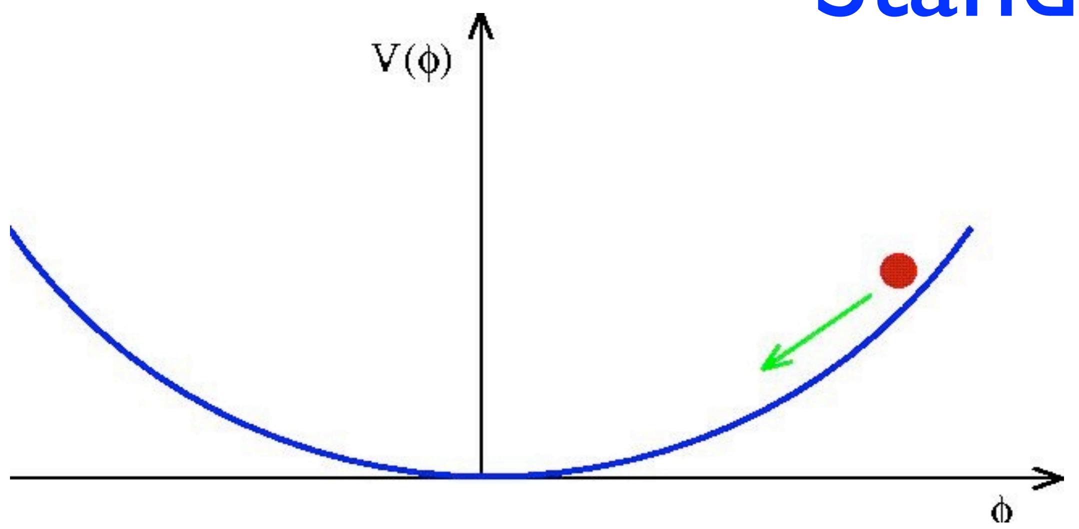
final potential

$$m = \frac{k}{a(t)} \qquad H \equiv \frac{\dot{a}}{a}$$



Each curvature/inflaton Fourier mode behaves as an harmonic oscillator with time dependent mass

Standard picture



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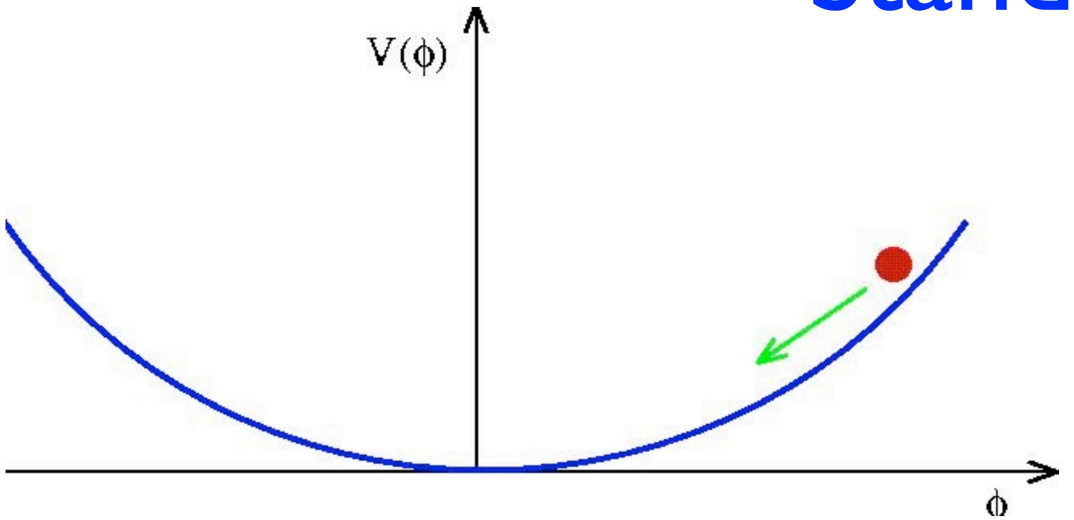
Each mode exits the Hubble radius, $k/a(t) \ll H$, and get frozen out the horizon with an almost scale-invariant spectrum:

$$\langle\zeta_{\vec{k}}\zeta_{\vec{k}'}\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')P_\zeta(k)$$

$$P_\zeta(k) = \frac{1}{2k^3}\left.\frac{H^4}{\dot{\phi}^2}\right|_{k=aH} = \frac{A_\zeta}{k^3}\left(\frac{k}{k_*}\right)^{n_s-1} \qquad n_s - 1 \simeq -0.04$$

These modes re-enter the horizon and we observe them today...

Standard picture



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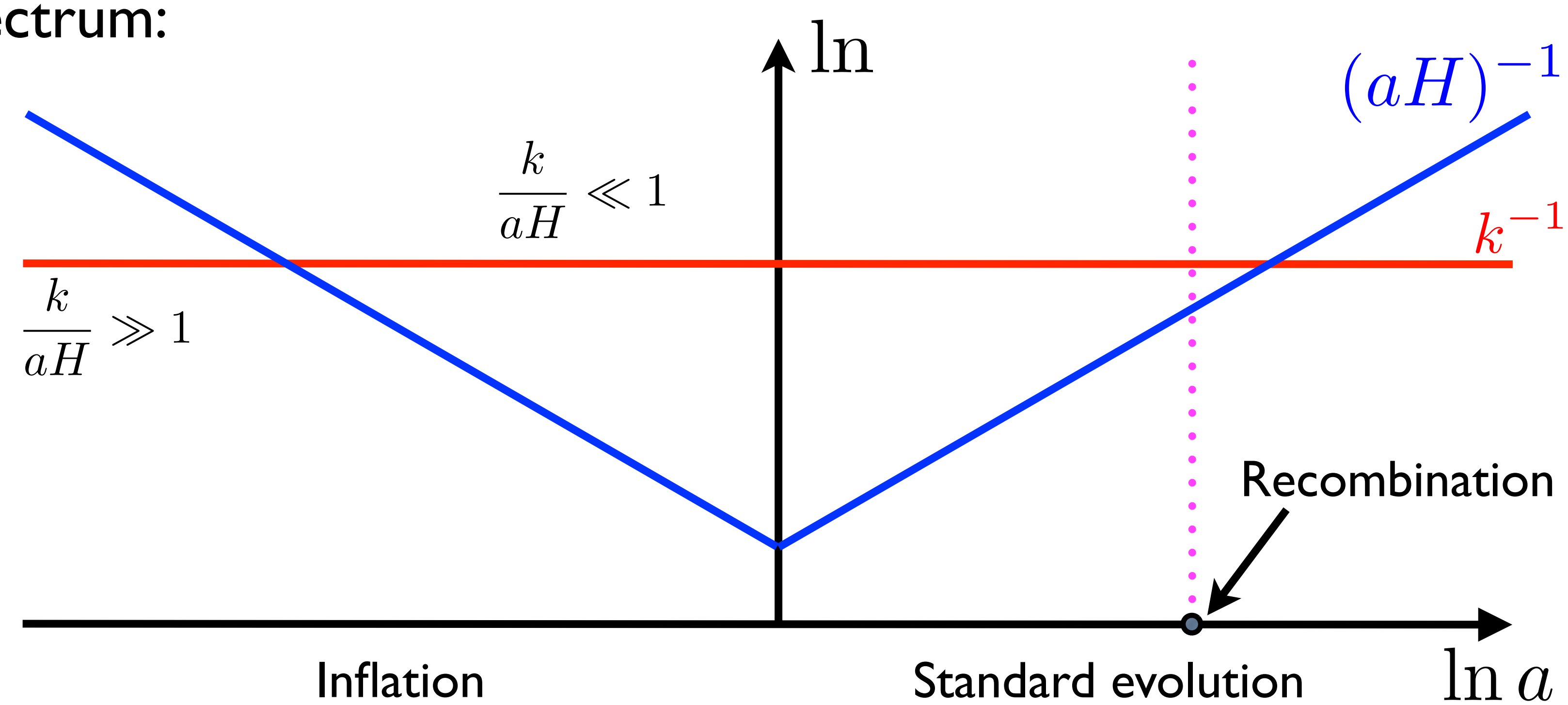
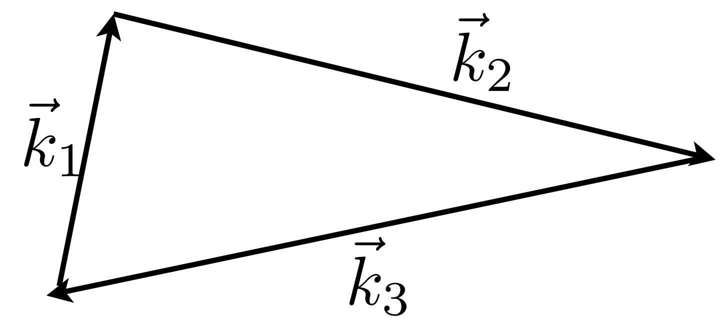


Beyond Gaussianity

Are there correlations between these modes?

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) F_\zeta(k_1, k_2, k_3)$$



Gravity induces interactions which are suppressed by slow-roll:

$$S = \int d^4 x a^3 \frac{\dot\phi^2}{2H^2} \left[\dot\zeta^2 - \frac{1}{a^2}(\partial\zeta)^2 + \frac{2}{H} \frac{\partial_i}{\partial^2} \dot\zeta \partial_i \zeta \frac{\partial^2}{a^2} \zeta + \ldots \right]$$

$$F_\zeta(k_1, k_2, k_3) \sim \mathcal{O}(\epsilon, \eta) P(k_1) P(k_2) + \ldots \qquad \Rightarrow \qquad \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{\frac{3}{2}}} \sim 0.01 \times 10^{-5}!!$$

Present bounds consistent with this small signal. Any modification from this standard picture enhances non-Gaussianity:

➡ Modified inflaton Lagrangian: higher-derivative terms, DBI, ghost inflation, etc...

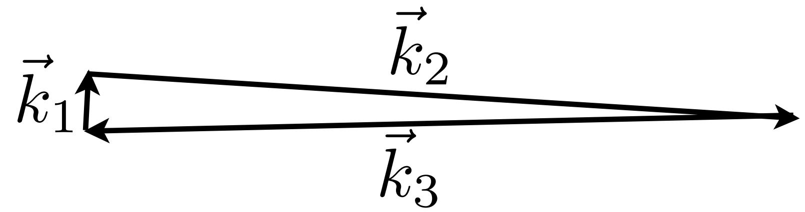
➡ Multi-field inflation; curvaton or varying decay rate reheating

➡ Alternative to inflation

$F_\zeta(k_1, k_2, k_3)$ potentially contains a wealth of information about the source of perturbations

Squeezed limit

$k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3$



Maldacena's consistency relation - for any single-field mode, in the squeezed limit:

$$\langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle = -(n_s - 1) P(k_L) P(k_S)$$

Maldacena '02

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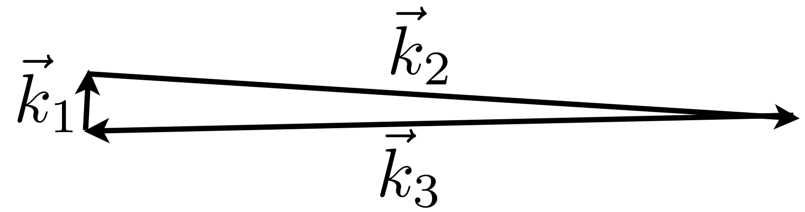

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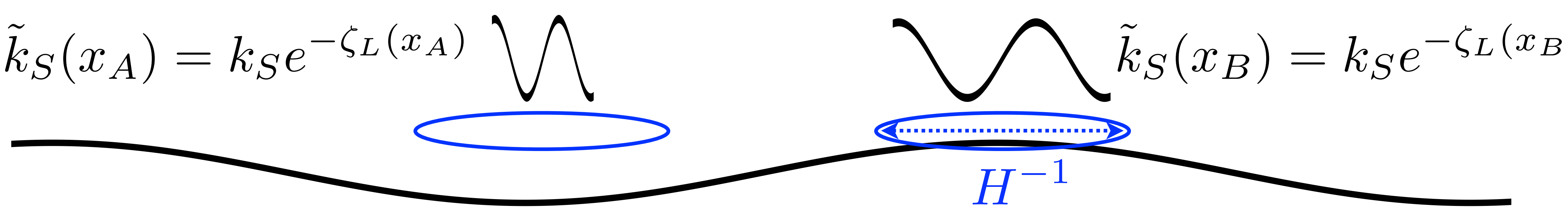
The effect of the long mode translates into a rescaling of the momenta:

$$g_{ij} dx^i dx^j = a^2(t) e^{2\zeta_L(\vec{x})} d\vec{x}^2 = a^2(t) d\tilde{\vec{x}}^2 \quad \Rightarrow \quad \tilde{k} = k e^{-\zeta_L}$$

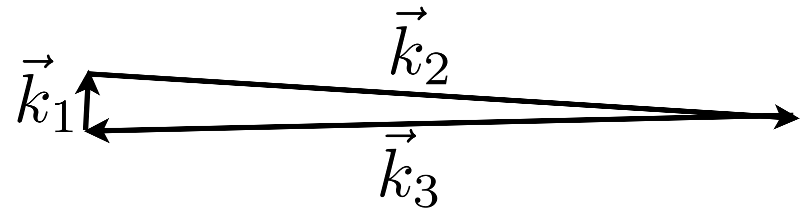
$$\langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle = \langle \zeta_{\vec{k}_L} \langle \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle_{\zeta_L} \rangle \approx \langle \zeta_{\vec{k}_L} P(\tilde{k}_S) \rangle = -\frac{d \ln (k_S^3 P_{k_S})}{d \ln k_S} P(k_S) P(k_L)$$



Flat spectrum n_s-1=0

Squeezed limit

$$k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3$$



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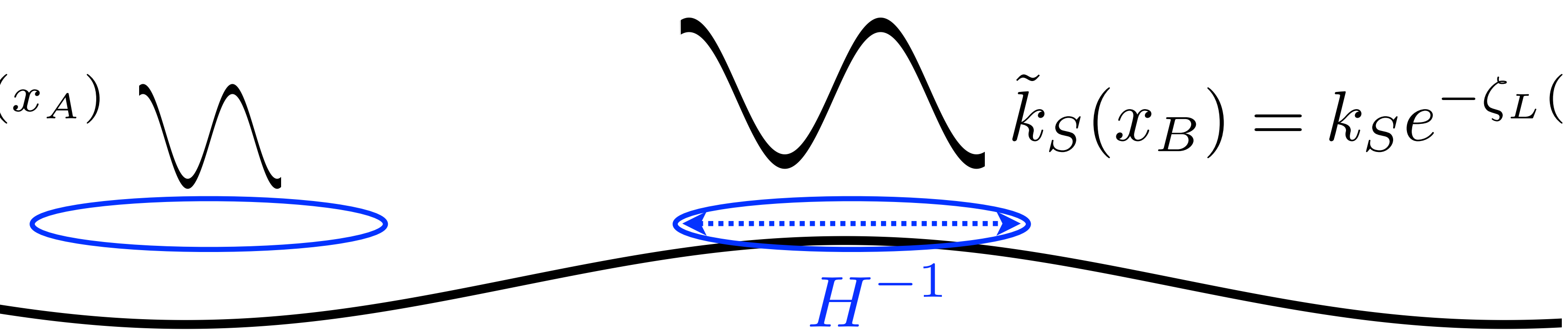
$$\tilde{k}_S(x_A) = k_S e^{-\zeta_L(x_A)}$$

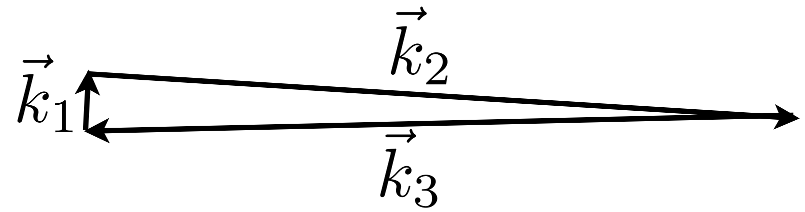


$$\tilde{k}_S(x_B) = k_S e^{-\zeta_L(x_B)}$$

Red spectrum n_s-1<0

Squeezed limit

$$k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3$$



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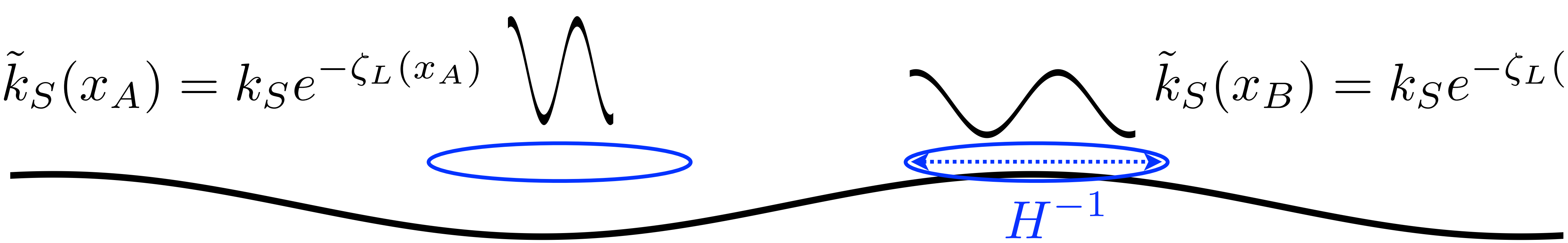
$$\tilde{k}_S(x_A) = k_S e^{-\zeta_L(x_A)}$$

$$\tilde{k}_S(x_B) = k_S e^{-\zeta_L(x_B)}$$



Blue spectrum n_s-1>0

All single-field models predict negligible NG in the squeezed limit: a detection of **local** NG rules out all single-field models!

CMB non-Gaussianity

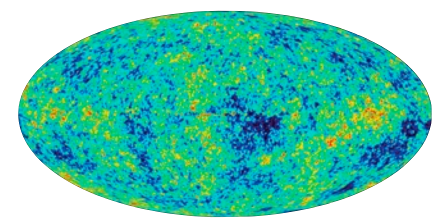
Even in absence of primordial non-Gaussianity, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = 0$, the CMB is non-Gaussian!

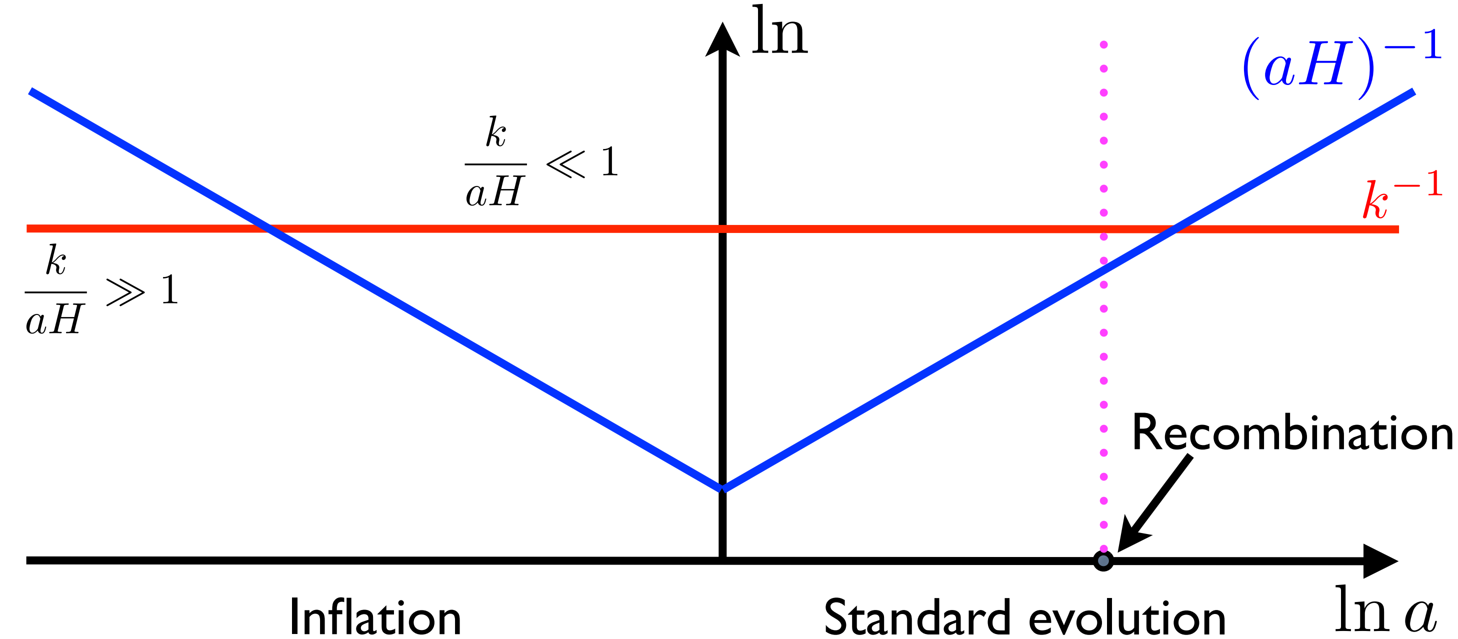
2nd-order effects induce NG:

- at recombination: 2nd-order perturbations in the fluid + GR nonlinearities;
- late time: ISW-lensing (extensively studied in the literature).

Nonlinear relation

$$\zeta \quad \Rightarrow \quad \frac{\delta T}{T}$$





$$\delta = \delta^{(1)} + \delta^{(2)} \quad \Rightarrow \quad \begin{aligned} D[\delta^{(1)}] &= 0 \\ D[\delta^{(2)}] &= S[\delta^{(1)2}] \end{aligned} \quad \Rightarrow \quad f_{\rm NL} \sim \frac{\langle \delta^{(2)} \delta^{(1)} \delta^{(1)} \rangle}{\langle \delta^{(1)} \delta^{(1)} \rangle^2} \sim \text{few}$$

All these effects are order ~few: important to interpret Planck data!

Recent progress

Complete Boltzmann calculation at 2nd-order is extremely challenging. Complicated non-linearities in baryon-photon fluid & from GR.

- Super-Hubble scales ⇒ 2nd-order GR-only problem:

$$f_{\rm NL}^{\rm local} = -(\frac{1}{6} + \cos 2\theta) \qquad (\cos\theta = \hat{l}_{\rm long} \cdot \hat{l}_{\rm short})$$

Bartolo, Matarrese, Riotto '04; Boubekeur et al, '09

- Sub-Hubble scales at recombination:

➡ Small-scale growth of nonlinear perturbations in the dark matter fluid:

$$\frac{\delta T}{T} = \frac{\delta T_{rec}}{T_{rec}} + \Phi \qquad \Rightarrow \qquad f_{\rm NL}^{\rm equil} \sim \mathcal{O}(10)$$

Bernardeau, Pitrou, Uzan '08; Bartolo, Riotto '08

➡ Perturbed recombination, free electron density (100<l<3000):

$$\frac{\delta n_e}{n_e} \approx \frac{\dot{n}_e}{n_e}\delta t \approx 5\frac{\delta n_b}{n_b} \qquad \Rightarrow \qquad f_{\rm NL}^{\rm local} \sim -3.5/-1$$

Senatore, Tassev, Zaldarriaga '09; Khatri, Wandelt '09

- All scales, numerical Boltzmann code:

➡ 2nd-order perts from product of 1st-order (not gauge invariant!): $f_{\rm NL}^{\rm local} \approx 1$

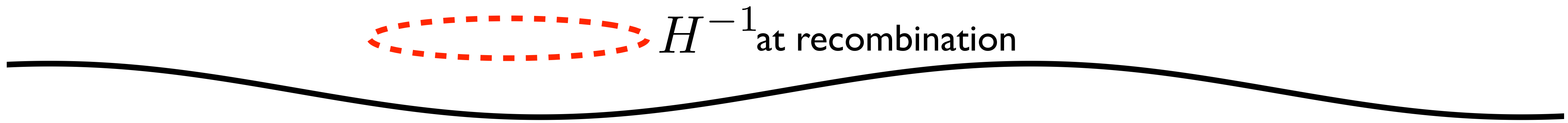
Nitta et al, '09

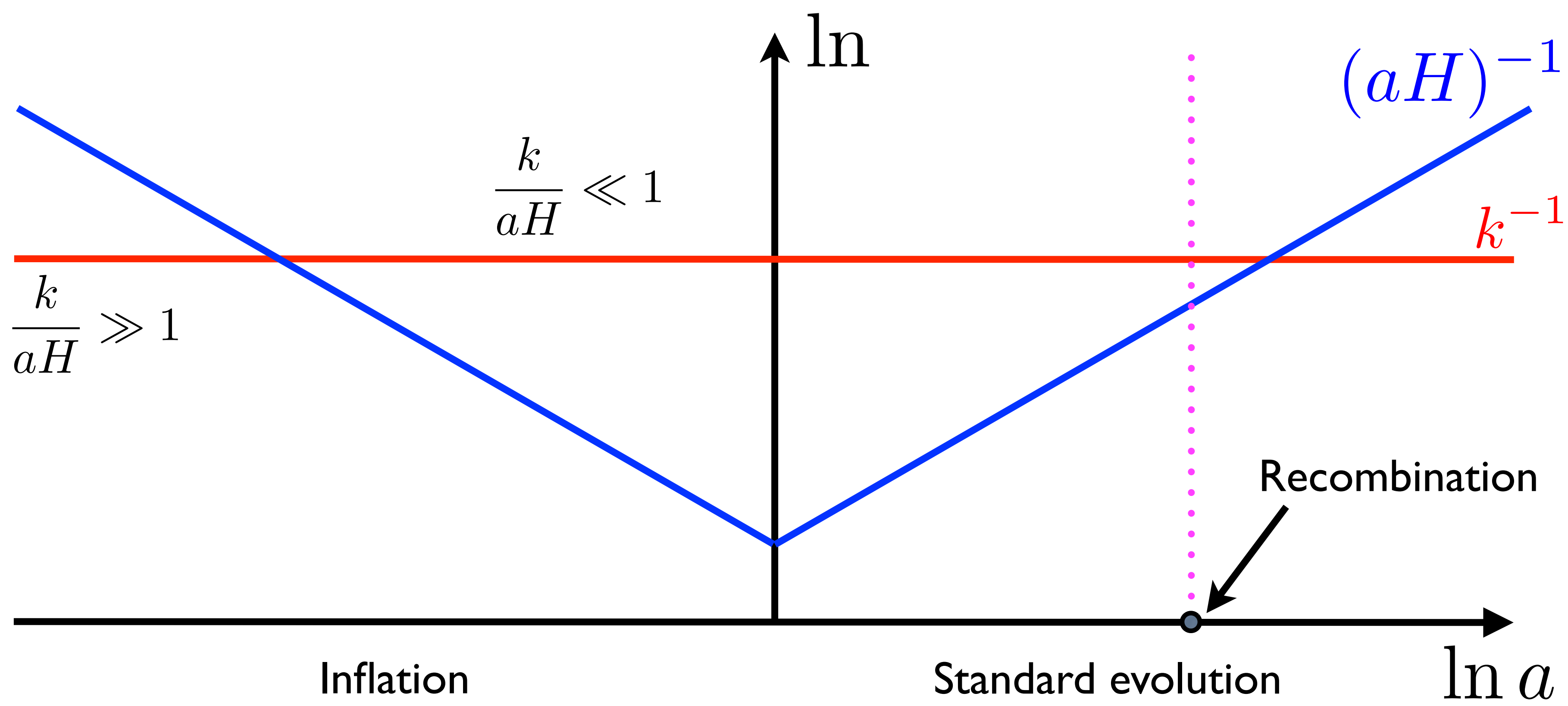
➡ Full Boltzmann code of Cyril Pitrou: http://www2.iap.fr/users/pitrou/cmbquick.htm $f_{\rm NL}^{\rm local} \approx 5$

Pitrou, Uzan, Bernardeau, '08, '09, '10; Beneke, Fidler, '10

Particular squeezed limit

One of the angles must subtend a scale longer than Hubble radius at recombination (but smaller than Hubble radius today):





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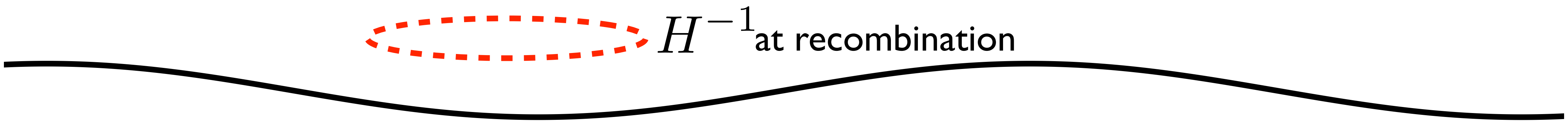
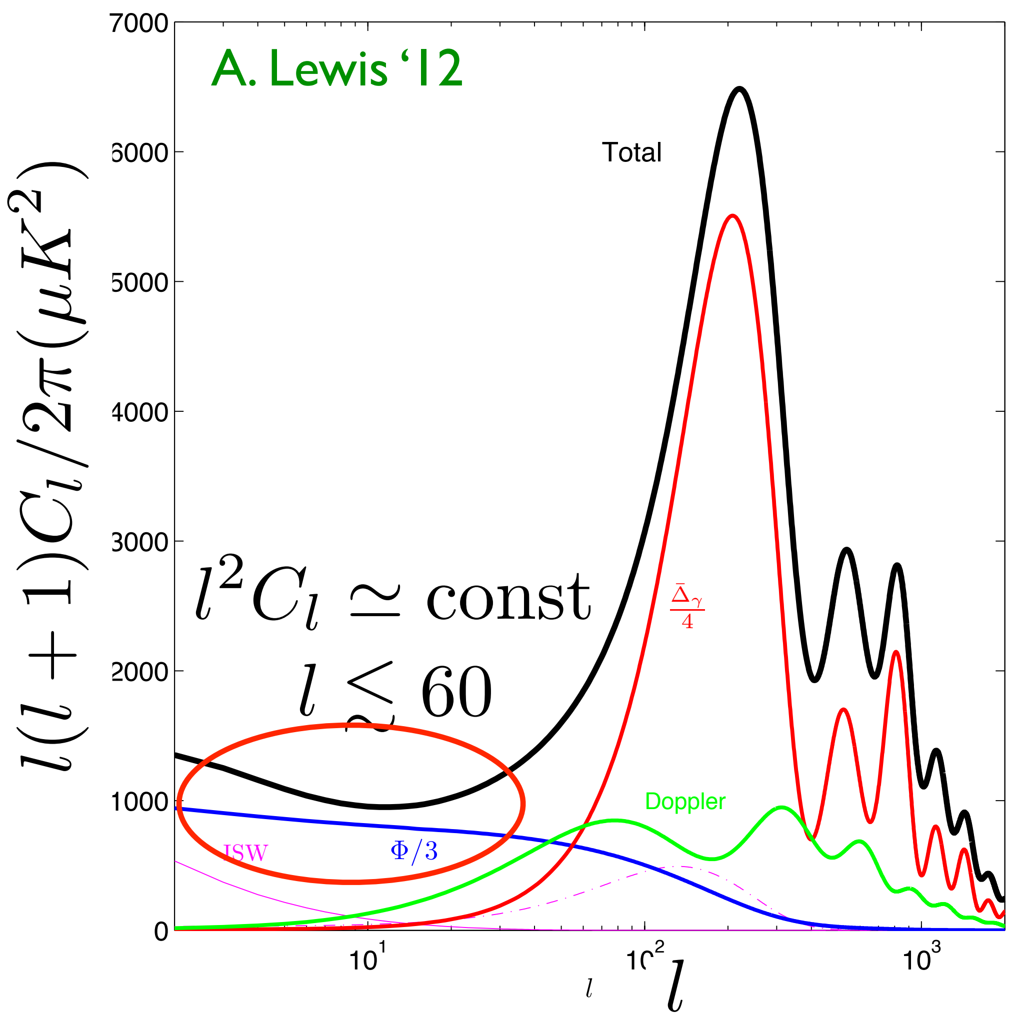


For the bispectrum: $B_{l_1 l_2 l_3}\ , \quad l_1 \ll l_2 \simeq l_3 \quad \& \quad l_1 \ll 200$



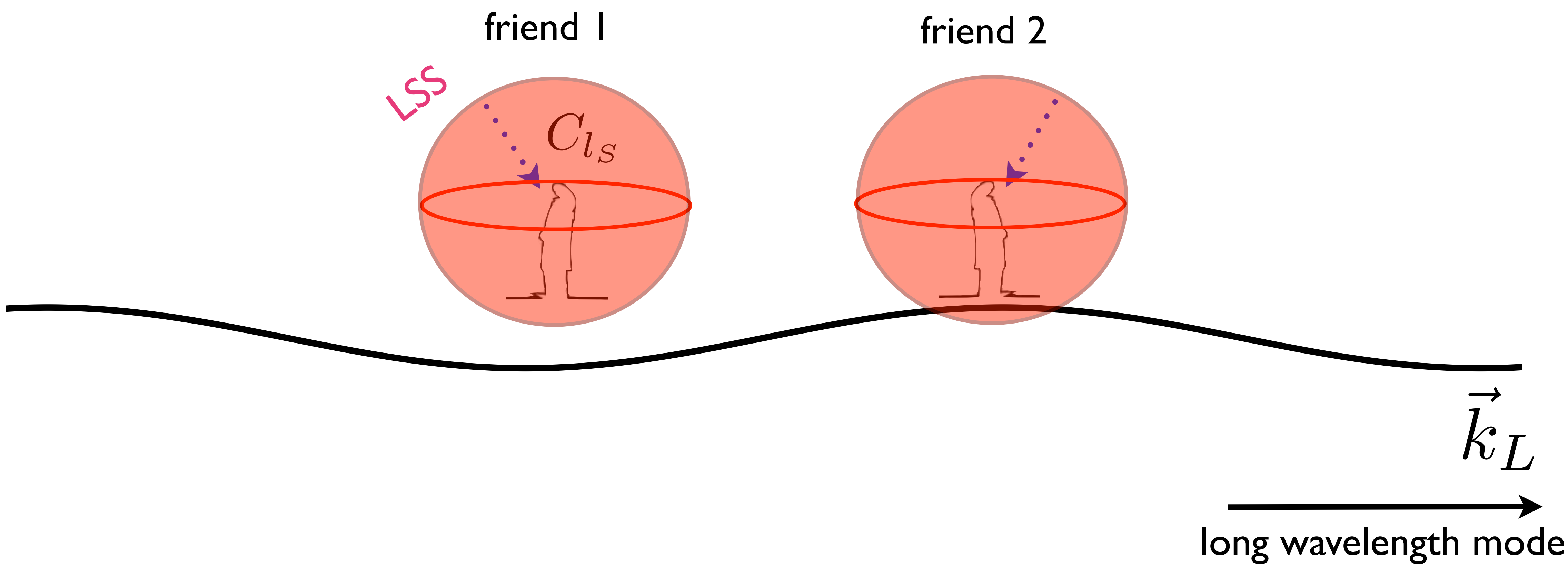
Sachs-Wolfe effect:

$$\frac{\delta T}{T} = \frac{1}{3}\Phi(\vec{x}_{\rm rec}) = -\frac{1}{5}\zeta(\vec{x}_{\rm rec}) \quad \Rightarrow$$

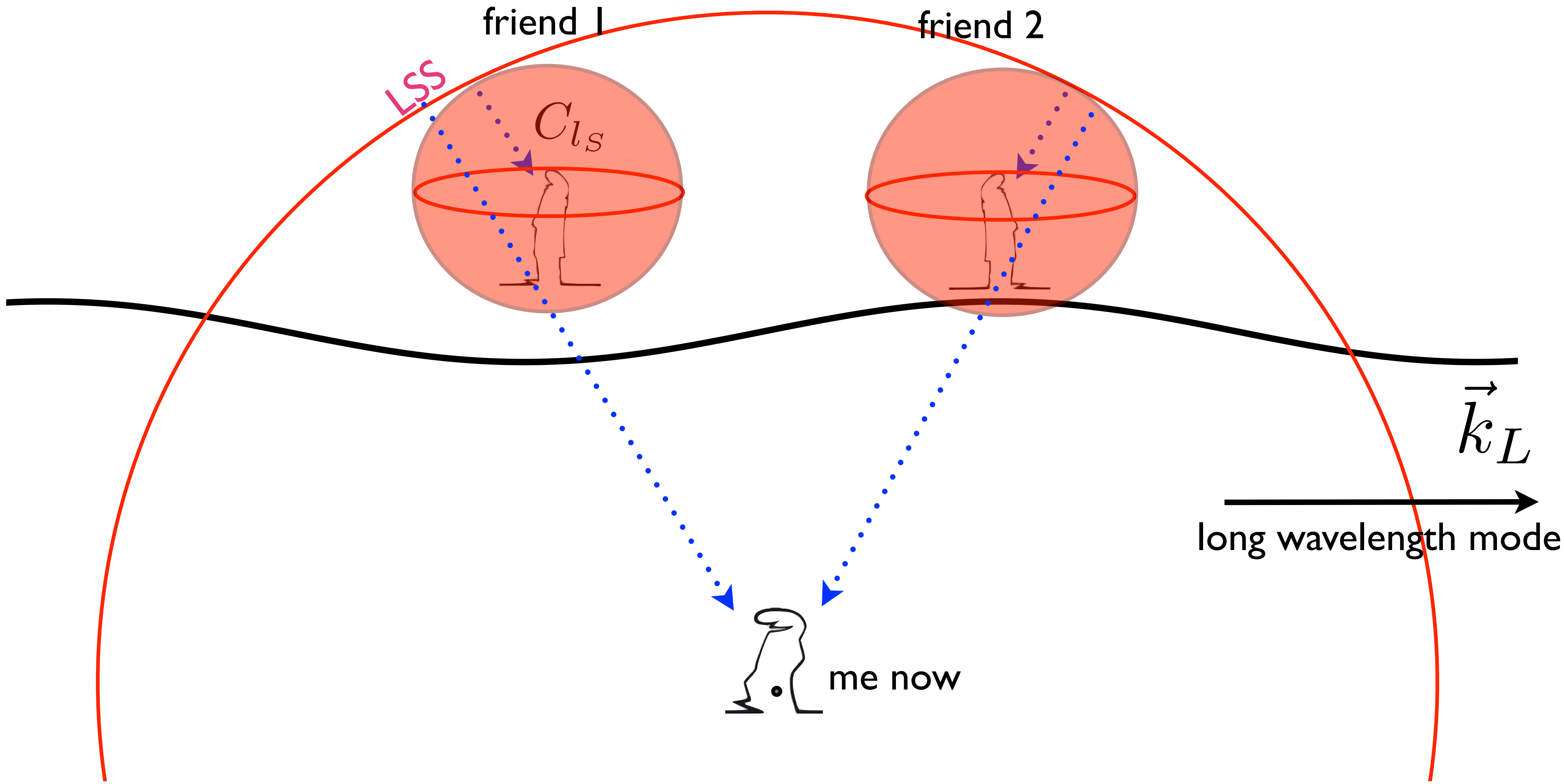
$$C_l \simeq \frac{A_T}{l^2}\left(\frac{l}{l_*}\right)^{n_s-1}$$

Physical argument

Creminelli, Zaldarriaga, '04



Two friends at different positions receive photons a bit after recombination at the same physical temperature or Hubble time. The long mode is out of their horizon: they will see the same CMB anisotropies.



The long mode is inside my horizon. Have to take into account of the modulation due to it. This induces a bispectrum.

$$C_{l_S} \to C_{l_S} + \Theta_L \frac{d}{d\Theta_L} C_{l_S} \qquad \Theta \equiv \Delta T/T$$

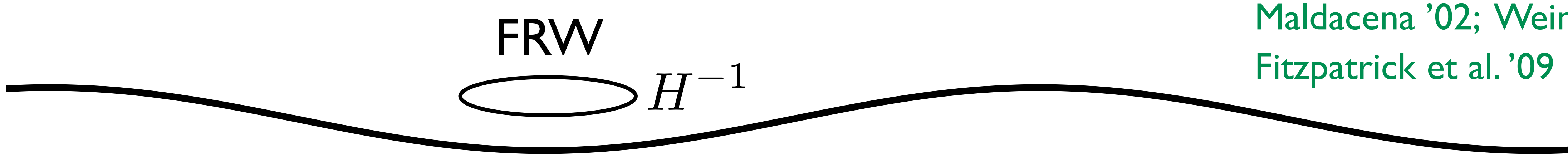
Rescaling of spatial coords ⇒ rescaling of angles: $C_{l_S} \to C_{l_S} - \Theta_L (5\hat{n} \cdot \nabla_{\hat{n}} C_{l_S})$

$$B_{l_L l_S l_S} = \langle \Theta_L C_{l_S} \rangle = 5 \frac{C_{l_L}}{l_S^2} \frac{d(l_S^2 C_{l_S})}{d \ln l_S} \qquad (\Theta_L = -\frac{1}{5}\zeta)$$

Long mode changes the local average temperature: $B_{l_L l_S l_S} = 2 C_{l_L} C_{l_S}$ $\qquad f_{\rm NL}^{\rm loc} = -\frac{1}{6}$

Lensing close to last scattering displaces the 2-p function

2^{nd}-order evolution as a coord change



Maldacena '02; Weinberg '03; Fitzpatrick et al. '09

Locally, possible to rewrite a perturbed FRW metric as an unperturbed one by reabsorbing the long mode with a coordinate transformation. Ex, in matter dominance:

$$ds^2 = a^2(\eta)\left[-(1+2\Phi_L)d\eta^2 + (1-2\Phi_L)dx^2\right] \quad \Rightarrow \quad ds^2 = a^2(\tilde{\eta})\left[-d\tilde{\eta}^2 + d\tilde{x}^2\right]$$

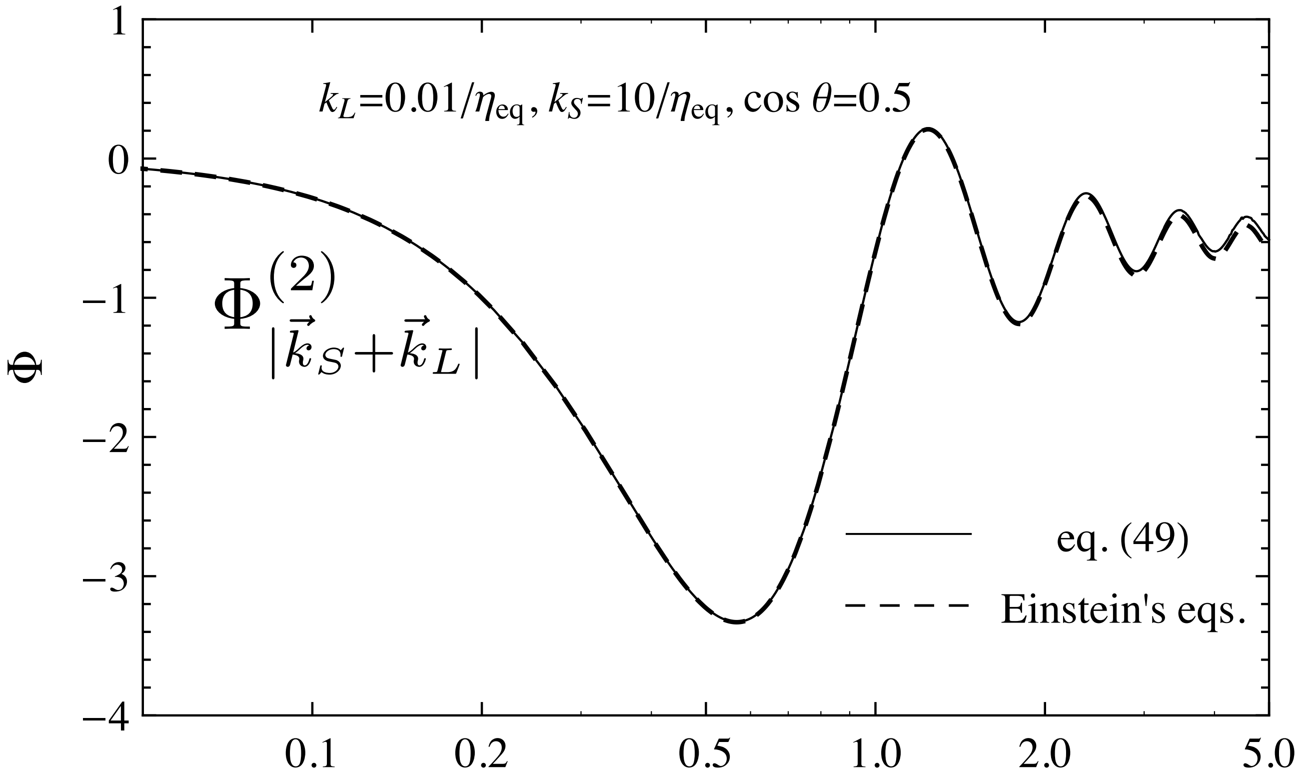
$$\tilde{\eta} = \eta(1+\Phi_L/3) \qquad \tilde{x}^i = x^i(1-5\Phi_L/3) \qquad \left(\zeta = -\frac{5}{3}\Phi_L\right)$$

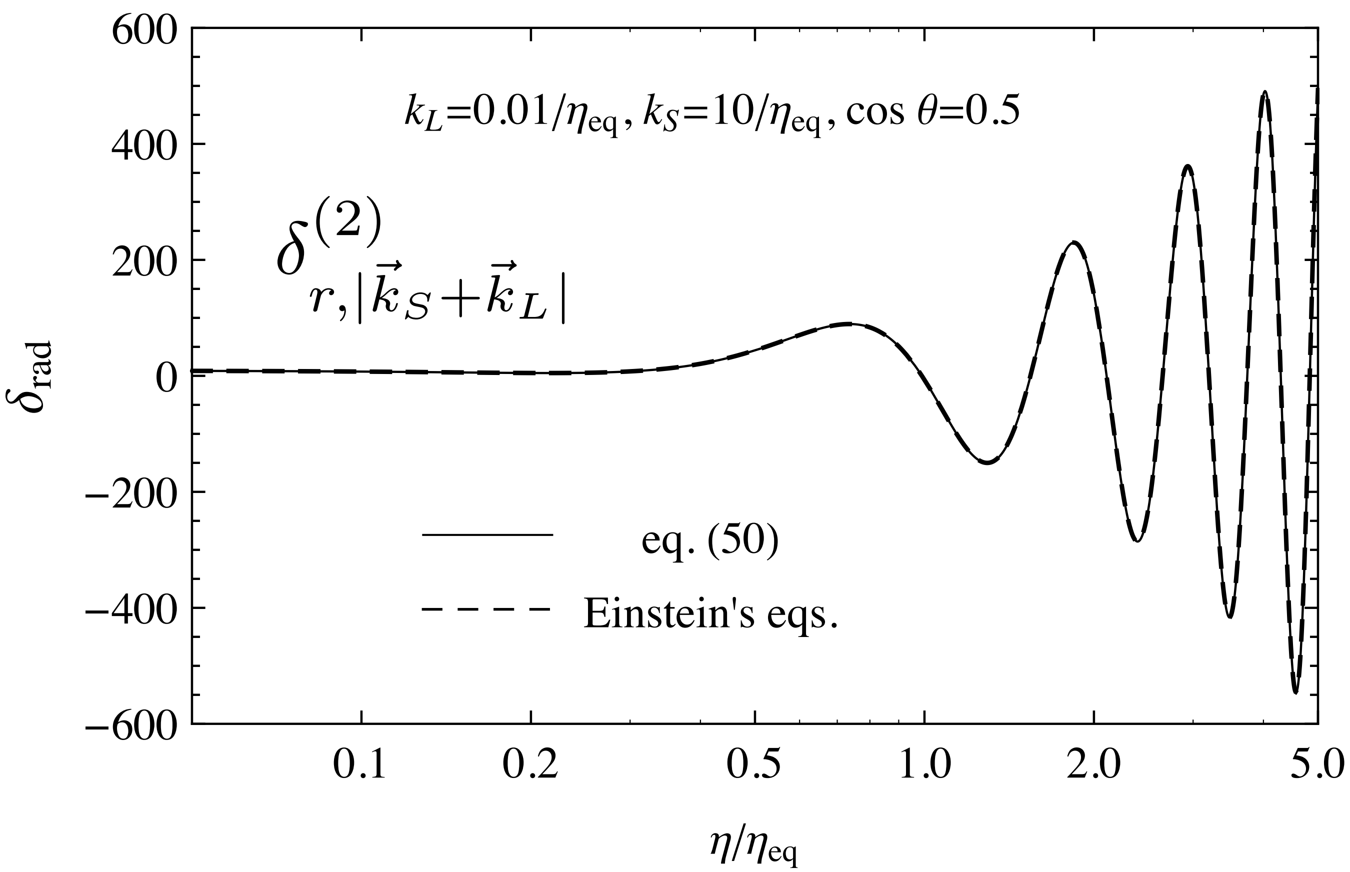
Conversely, start from a perturbed metric at 1^{st}-order and "generate" 2^{nd}-order couplings between short and long modes by the inverse coordinate transformation:

$$ds^2 = a^2(\tilde{\eta})\left[-(1+2\tilde{\Phi}_S)d\tilde{\eta}^2 + (1-2\tilde{\Psi}_S)d\tilde{x}^2\right] \Rightarrow ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + e^{2\Psi}dx^2\right]$$

$$\Phi = \tilde{\Phi}_S + \Phi_L + \frac{1}{3}\Phi_L\frac{\partial\tilde{\Phi}_S}{\partial\ln\eta} - \frac{5}{3}\Phi_L x^i\frac{\partial\tilde{\Phi}_S}{\partial x^i}$$

2^{nd}-order evolution as a coord change





Example: radiation-to-matter transition:

Solution of Einstein's eqs. at 2^{nd} order

Coordinate transformation:

$$\Phi_{k_S} \to \Phi_{k_S} - \frac{5}{3}\Phi_{k_L}\left(f(\eta)\frac{\partial \Phi_{k_S}}{\partial \ln \eta} - \frac{\partial \Phi_{k_S}}{\partial \ln k}\right)$$

$$f(\eta) = -\frac{20 + 15\alpha\eta + 3\alpha^2\eta^2}{5(2+\alpha\eta)}$$

$$\alpha = (\sqrt{2}-1)/\eta_{eq}$$

$$\delta_{k_S} \to \delta_{k_S} - \frac{5}{3}\Phi_{k_L}\left(f(\eta)\frac{\partial \delta_{k_S}}{\partial \ln \eta} - \frac{\partial \delta_{k_S}}{\partial \ln k}\right)$$

Coordinate transformation on the CMB

Apply this coordinate transformation to the observed CMB: $\Theta_{\rm obs}(\hat{n}) = \dfrac{T_{\rm obs}(\hat{n}) - \langle T_{\rm obs}\rangle}{\langle T_{\rm obs}\rangle}$

$$\Theta_{\rm obs} = [\Theta + \Phi - \hat{n}\cdot\vec{v}](\eta_{\rm rec}, \vec{x}_{\rm rec})$$

In pure matter dominance and instantaneous recombination

Coordinate transformation on the CMB

Apply this coordinate transformation to the observed CMB: $\Theta_{\rm obs}(\hat{n}) = \dfrac{T_{\rm obs}(\hat{n}) - \langle T_{\rm obs}\rangle}{\langle T_{\rm obs}\rangle}$

$$\Theta_{\rm obs} = \Theta_{{\rm obs},S} + \Theta_{{\rm obs},L} + \Theta_{{\rm obs},L}\left(1 + \frac{\partial}{\partial \ln \eta_{\rm rec}} - 5\hat{n}\cdot\nabla_{\hat{n}}\right)\Theta_{{\rm obs},S}$$

$$\Theta_{{\rm obs},S} = [\Theta_S + \Phi_S - \hat{n}\cdot\vec{v}_S](\eta_{\rm rec}, \vec{x}_{\rm rec}) \qquad \Theta_{{\rm obs},L} = \frac{1}{3}\Phi_L(\eta_{\rm rec}, \vec{x}_{\rm rec})$$

Holds also when including radiation/matter transition (early Sachs-Wolfe) and finite recombination.

Check: a mode out of Hubble radius today is unobservable. Cancels out from this expression.

Time derivative is geometrically suppressed as $\sim \dfrac{\eta_{\rm rec}}{\eta_{\rm obs}}$

Bispectrum: $$B_{l_L l_S l_S} = C_{l_L} C_{l_S}\left(2 + 5\frac{d\ln(l_S^2 C_{l_S})}{d\ln l_S}\right)$$

Extension of the Maldacena relation. Cf: $$B_{k_L k_S k_S} = -P_{k_L} P_{k_S}\frac{d\ln(k_S^3 P_{k_S})}{d\ln k_S}$$

Boltzmann code: CMBquick

This relation can be used as consistency check of Boltzmann codes based on a physical limit

There have been several contributions to development of Boltzmann numerical code at 2nd order.

Bartolo, Matarrese, Riotto '06; Bernardeau, Pitrou, Uzan '08; Pitrou '08; Bartolo, Riotto '08; Khatri, Wandelt '08; Senatore, Tassev, Zaldarriaga '09; Nitta et al. '09, Beneke and Fidler '10

One of the most complete code is Pitrou's CMBquick (http://www2.iap.fr/users/pitrou/).

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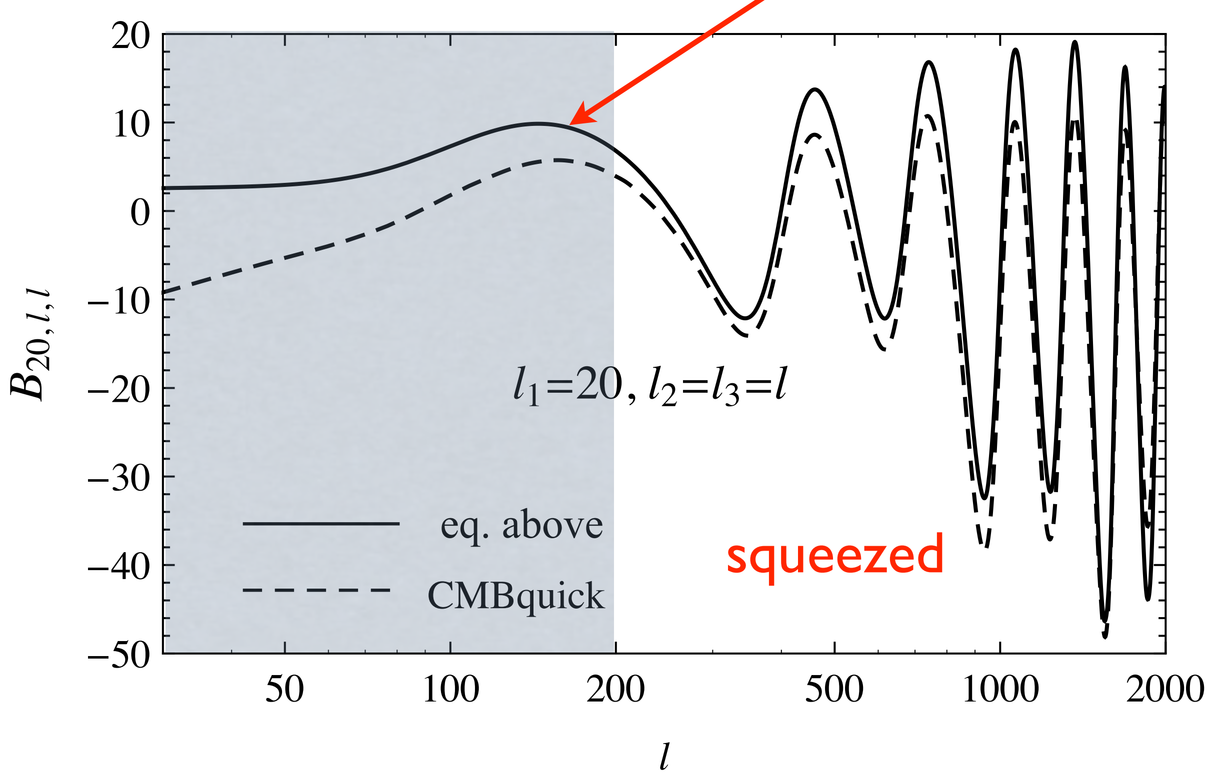
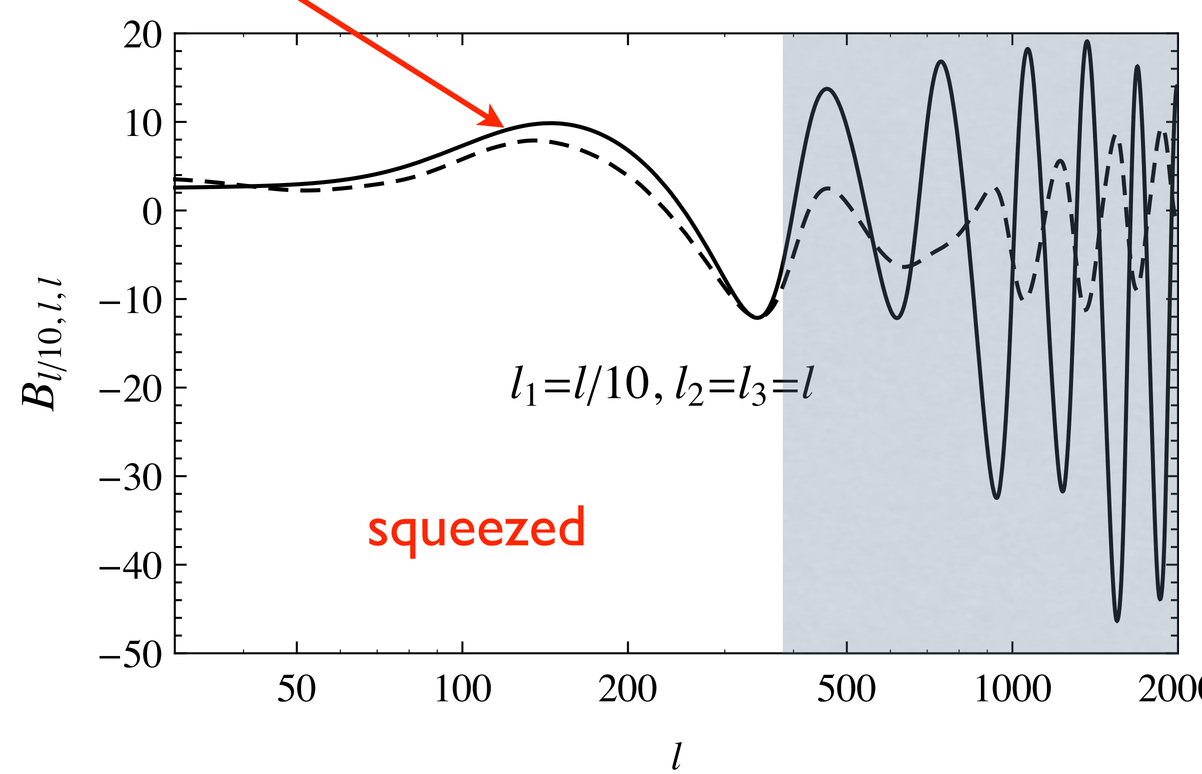




The check is nontrivial! Even though analytically the squeezed limit is easy, in the code all 2nd-order effects must conspire to reproduce the simple analytical formula.

Final result

Coordinate and average temperature redefinition

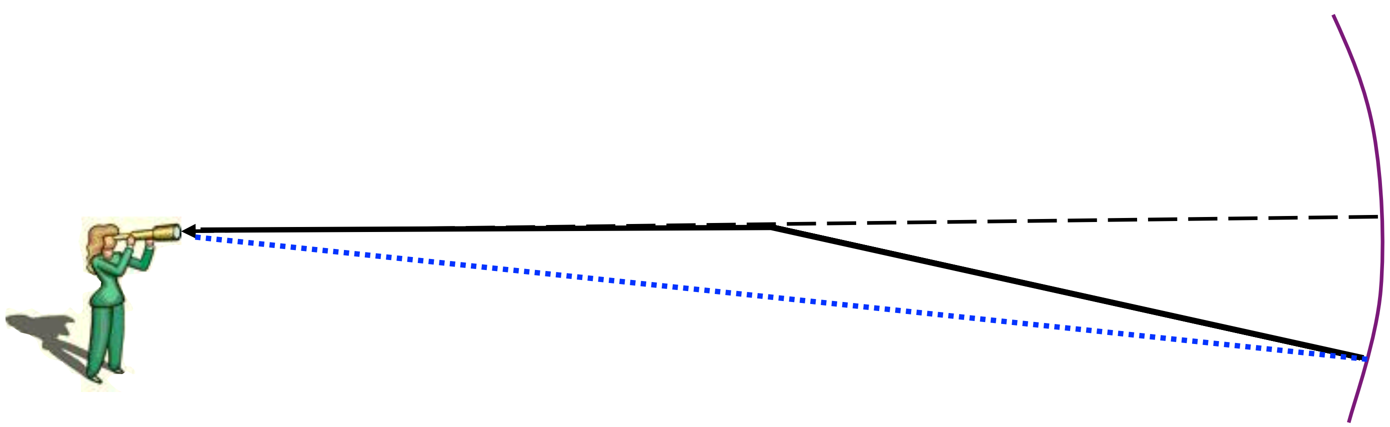
Lensing

$$B_{l_L l_S l_S} = C_{l_L} C_{l_S} \left(2 + 5 \frac{d \ln(l_S^2 C_{l_S})}{d \ln l_S} \right) + 6 C_{l_L} C_{l_S} \left[2\cos 2\theta - (1 + \cos 2\theta) \frac{d \ln(l_S^2 C_{l_S})}{d \ln l_S} \right]$$

Boubekeur et al. '09

$$(\cos\theta = \hat{l}_L \cdot \hat{l}_S)$$

Lensing due to correlation between temperature at recombination and transverse displacement:



$$\delta\vec{x}^{\perp} = -2 \int_{\eta_{\rm rec}}^{\eta_0} (1 - \frac{\eta_{\rm rec}}{\eta}) \vec{\nabla}_{\hat{n}} \Phi(\vec{x}) d\eta$$

$$\Theta_{\rm obs}^{\rm lensed} = \Theta_{{\rm obs},S}(\eta_{\rm rec}, \vec{x}_* + \delta\vec{x}^{\perp}) = \Theta_{{\rm obs},S}(\eta_*, \vec{x}_*) + \delta\vec{x}^{\perp} \cdot \vec{\nabla}\Theta_{{\rm obs},S}(\eta_*, \vec{x}_*)$$

Final result

Coordinate and average temperature redefinition — Lensing

$$B_{l_L l_S l_S} = C_{l_L} C_{l_S} \left(2 + 5\frac{d\ln(l_S^2 C_{l_S})}{d\ln l_S}\right) + 6 C_{l_L} C_{l_S}\left[2\cos 2\theta - (1+\cos 2\theta)\frac{d\ln(l_S^2 C_{l_S})}{d\ln l_S}\right]$$

Combining the two contributions:

$$\boxed{B_{l_L l_S l_S} = C_{l_L} C_{l_S}(1 + 6\cos 2\theta)\left(2 - \frac{d\ln(l_S^2 C_{l_S})}{d\ln l_S}\right)}$$

Partial cancellation between (isotropic) lensing convergence and space redefinition. Overdense regions (negative potential) give positive convergence, moving the spectrum towards larger angles, while coordinate redefinition shrink it.

The bispectrum vanishes for a white noise spectrum $C_{l_S} \propto \mathrm{const}$

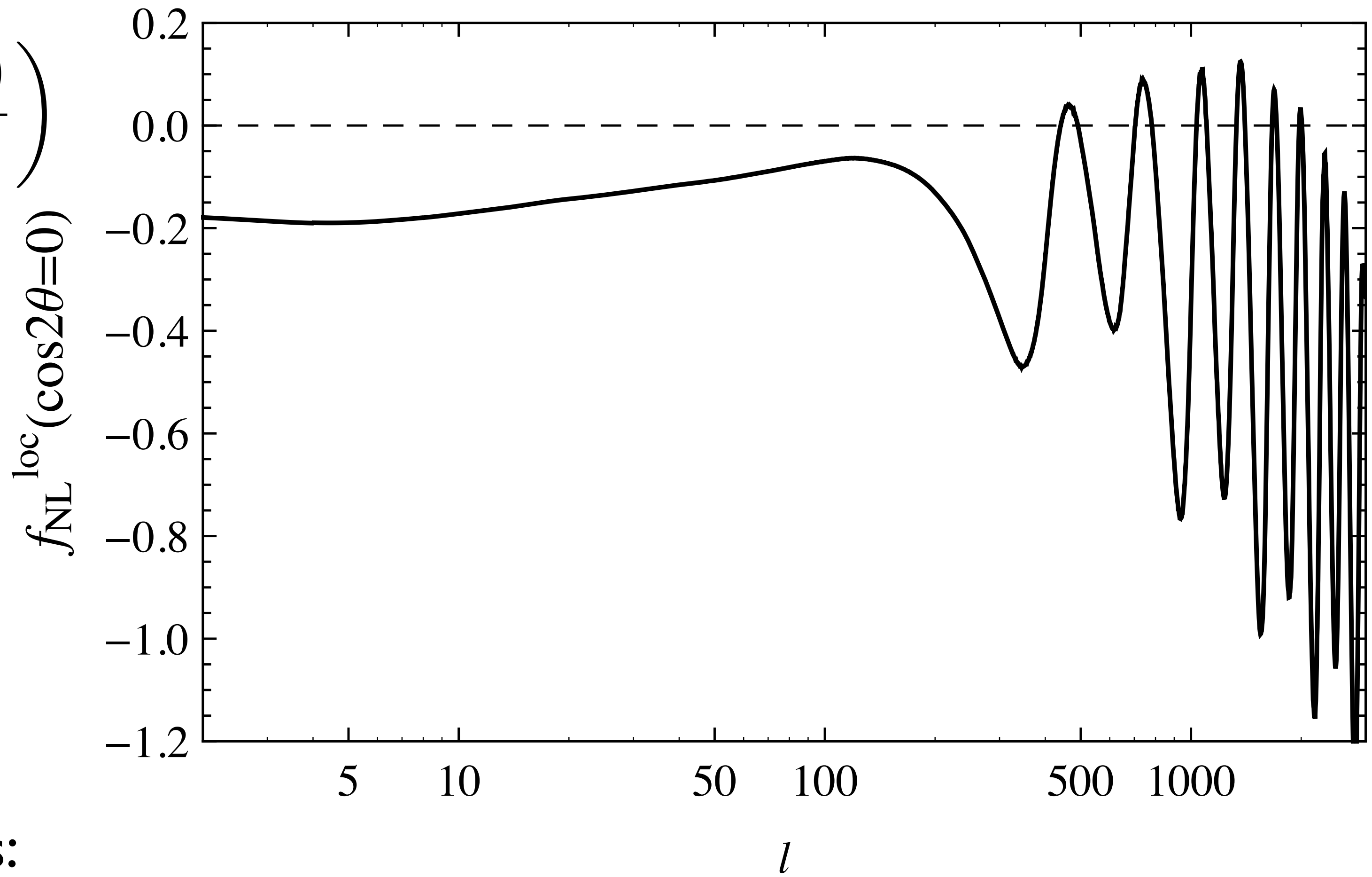
We expect this for anisotropic lensing: shear just remaps the points preserving the area, so white noise remains the same. Zaldarriaga, '99

We do not have a simple argument for the cancellation of the isotropic part. However, the above equation is correct only at lowest order in the scale invariance of the long mode.

Contamination

$$f_{\rm NL}^{\rm local} = -(\frac{1}{6} + \cos 2\theta)\left(1 - \frac{1}{2}\frac{d\ln(l_S^2 C_{l_S})}{d\ln l_S}\right)$$

In the limit of scale invariance: NG computed in Boubekeur et al, '09



We can define the contamination to $f_{\rm NL}^{\rm loc}$ as:

$$f_{\rm NL}^{\rm loc} = \left(\int \frac{d^2l_2 d^2l_3}{(2\pi)^2}\frac{[B_{\rm loc}(l_1,l_2,l_3)]^2}{6C_{l_1}C_{l_2}C_{l_3}}\right)^{-1} \cdot \int \frac{d^2l_2 d^2l_3}{(2\pi)^2}\frac{B_{\rm loc}(l_1,l_2,l_3)B_X(l_1,l_2,l_3)}{6C_{l_1}C_{l_2}C_{l_3}}$$

Integrating only in the squeezed limit ($100<l<l_{max}$): $f_{\rm NL}^{\rm loc} = -0.39\ ,\qquad l_{\rm max} = 2000$

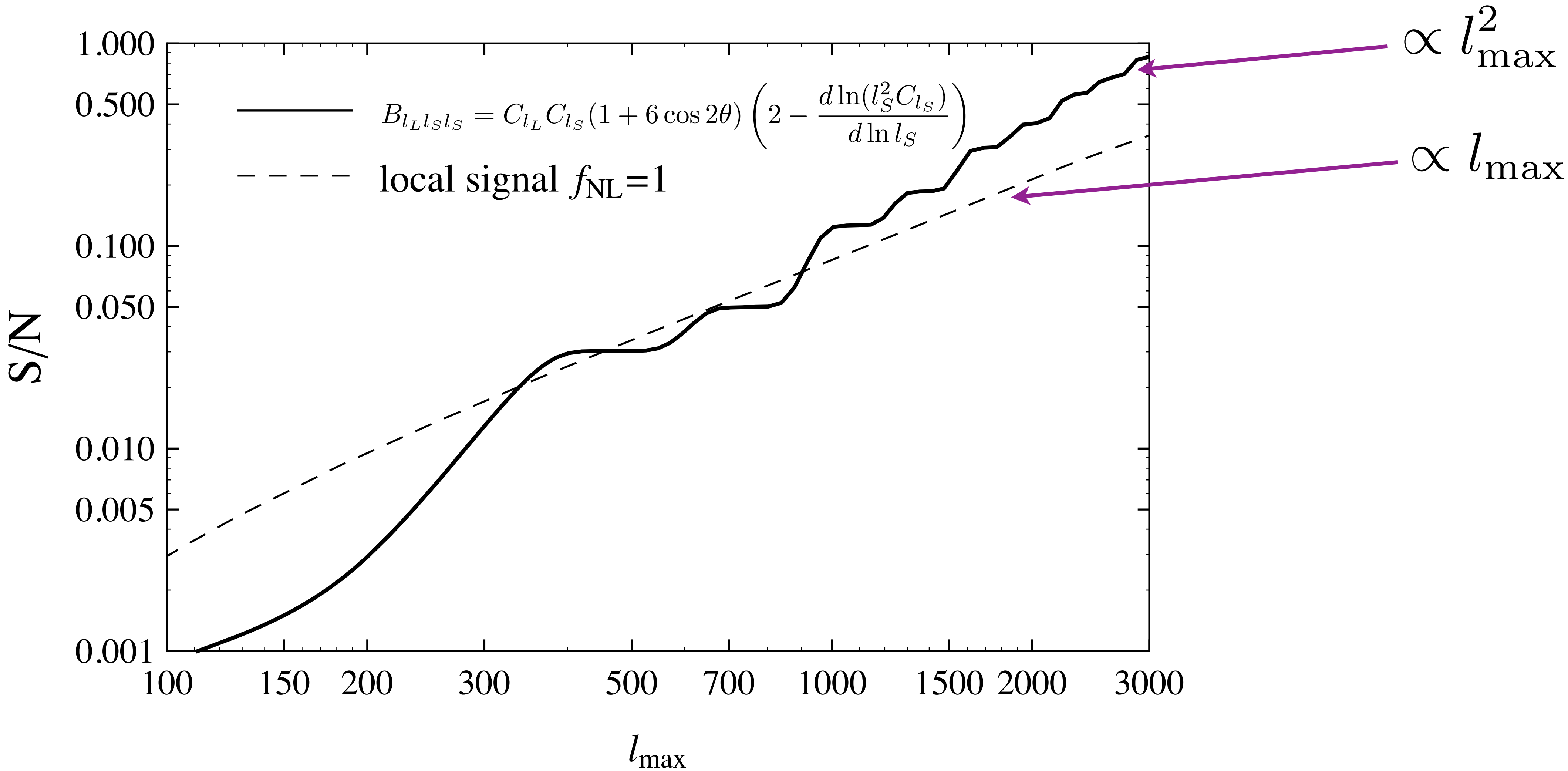
Negligible contamination to Planck.

See also Bartolo, Riotto '11

Observability

Can we observe this signal? Signal-to-noise ratio is:

$$\left(\frac{S}{N}\right)^2 = \frac{1}{\pi}\int \frac{d^2l_2 d^2l_3}{(2\pi)^2}\frac{[B(l_1,l_2,l_3)]^2}{6C_{l_1}C_{l_2}C_{l_3}}$$



We are integrating only in the squeezed limit and we do not include polarization. Boltzmann code would give a better estimate. Possibly measurable effect.

Conclusion

In the squeezed limit (one mode longer than horizon at recombination), it is possible to compute the CMB bispectrum exactly.

$$B_{l_L l_S l_S} = C_{l_L} C_{l_S} (1 + 6\cos 2\theta) \left(2 - \frac{d \ln(l_S^2 C_{l_S})}{d \ln l_S} \right)$$

Valid for adiabatic (single clock) perturbations. Already takes into account NG from single-field models. It is a consistency relation on the observable (CMB temperature) in the squeezed limit.

- Planck will not be biased by 2^{nd}-order effects at recombination (detectability?). Late time effects important (A. Lewis' talk).

- Test Boltzmann codes at 2^{nd} order. Reasonable agreement with Pitrou's CMBquick code. Code reliable in the regime that we studied.

➡ Improve numerical reliability of the code and compute contamination (G. Pettinari's talk).

The value of the contamination by Pitrou may be overestimated: $f_{\rm NL}^{\rm local} \approx 5 \longrightarrow f_{\rm NL}^{\rm local} \approx 3$

- Is it compatible with these results? Yes if large signal comes from l_L>100.
- Is it compatible with Senatore & Zaldarriaga's results f_{NL} ~ -3.5?