LOCALISATION AND TRANSMISSION IN ACTIVE RANDOM MEDIA WITH CORRELATED DISORDER

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OUTLINE OF THE TALK

- Definition of the model
- The transfer matrix formalism
- General formulae
- Application to specific cases of weak and strong absorption or amplification
- Non-Hermitian disorder (random amplification or absorption)
- Conclusions

DEFINITION OF THE PROBLEM

- Problem: Transmission and localisation of waves in an active 1D system with correlated disorder

- Motivation:

  1. Recognised importance of non-Hermitian models (open systems; physical systems with absorption or amplification, e.g., electronic models with electron-hole recombination or photonic crystals)

  2. Increasing number of applications of correlated disorder in Hermitian models (waveguides, semiconductor superlattices, photonic crystals)

- Purpose: analysis of the interplay of absorption/amplification and spatial correlations of the disorder.

MATHEMATICAL MODEL

Random active barrier sandwiched between two perfect leads.

Schrödinger equation: barrier ($n = 1, \ldots, N$)

$$\psi_{n+1} + \psi_{n-1} + (\varepsilon_n + i\gamma)\,\psi_n = E\psi_n$$

Schrödinger eq.: leads ($n \leq 0$ and $n \geq N+1$)

$$\psi_{n+1} + \psi_{n-1} = E\psi_n,$$

PARAMETERS OF THE MODEL

- γ: average loss/gain coefficient
  ($\gamma > 0$: absorption; $\gamma < 0$: amplification)

- ε_n: random site energies

STATISTICAL PROPERTIES OF DISORDER

- Vanishing average values

$$\langle \varepsilon_n \rangle = 0$$

- Weak disorder

$$\langle \varepsilon_n^2 \rangle \ll 1$$

- Correlated disorder

$$\chi(l) \;=\; \frac{\langle \varepsilon_n \varepsilon_{n+l} \rangle}{\langle \varepsilon_n^2 \rangle}$$

DISORDERED BARRIER WITHOUT AMPLIFICATION/ABSORPTION: EFFECTS OF CORRELATED DISORDER

"Anderson barrier": $\gamma = 0$

$$\psi_{n+1} + \psi_{n-1} + \varepsilon_n \psi_n = E\psi_n$$

Transmission in the localised regime:

$$\langle \ln T_N \rangle \simeq -2\lambda N$$

Inverse localisation length for *weak disorder*

$$\lambda = \frac{\langle \varepsilon_n^2 \rangle}{8 \sin^2(k)} W(k) + O(\langle \varepsilon_n^4 \rangle)$$

Power spectrum

$$W(k) = 1 + 2 \sum_{n=1}^{\infty} \chi(n) \cos(2kn) = \sum_{n=-\infty}^{\infty} \chi(n) e^{i2kn}$$

Note that $\chi(0) = 1 \quad \Rightarrow \quad \int_{-\pi/2}^{\pi/2} W(k) \mathrm{d}k = \pi$

A vanishing power spectrum in selected energy intervals leads to suppression of localisation in those intervals and to enhancement of localisation in the complementary intervals

DESIGNED MOBILITY EDGES

The binary correlator

$$\chi(n) = \frac{1}{2(\kappa_2 - \kappa_1)n} [\sin(2\kappa_2 n) - \sin(2\kappa_1 n)]$$

with $0 < \kappa_1 < \kappa_2 < \pi/2$ corresponds to

$$W(k) = \begin{cases} \dfrac{\pi}{2(\kappa_2 - \kappa_1)} & \text{if} \quad k \in [\kappa_1, \kappa_2] \\ 0 & \text{otherwise} \end{cases}$$

for $k \in [0, \pi/2]$.

Specific long-ranged correlations produce *effective* mobility edges in infinite systems.

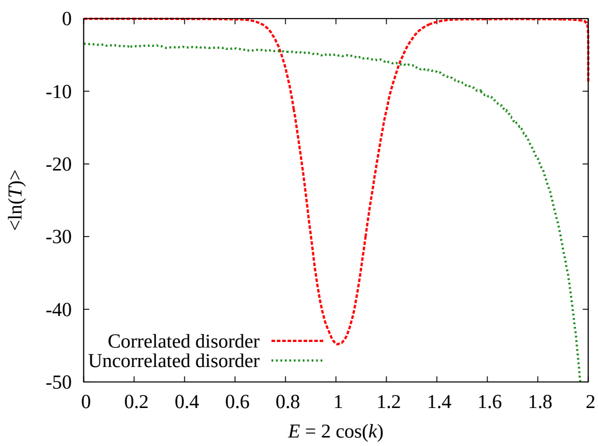
Questions:

- What about finite-size systems?
- What happens if amplification/absorption is present?

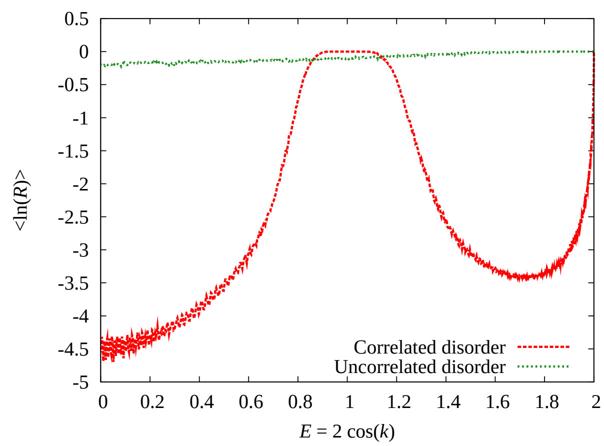
DELOCALISATION EFFECTS ARE ROBUST!

DELOCALISATION EFFECTS SURVIVE IN FINITE SYSTEMS OF LIMITED SIZE

Example:



$\langle \ln(T_N) \rangle$ versus E



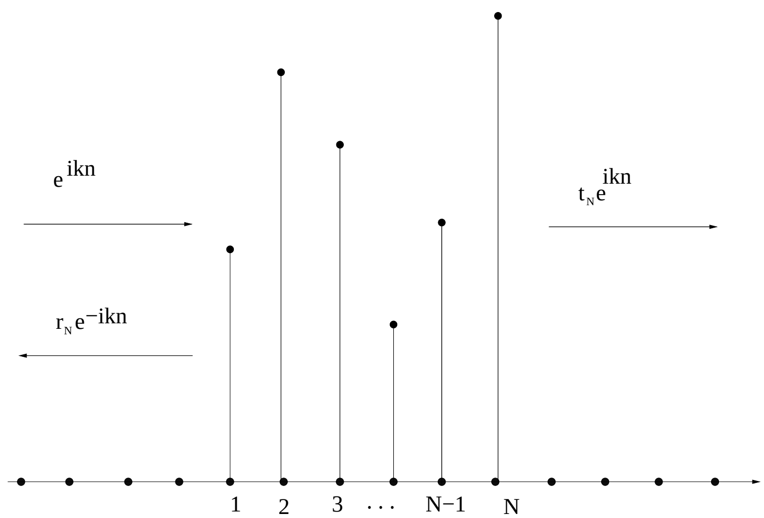
$\langle \ln(R_N) \rangle$ versus E

- Barrier length $N = 300$.
- Disorder strength $\langle \varepsilon_n^2 \rangle = 0.05$.
- For correlated disorder, mobility edges were set at $\kappa_1 = 0.3283\pi$ and $\kappa_2 = 0.3383\pi$

TRANSFER MATRIX APPROACH

Wavefunctions in the leads:

$$\begin{aligned}\psi_n &= e^{ikn} + r_N e^{-ikn} \\ \psi_n &= t_N e^{ikn}\end{aligned}$$



Active random barrier between perfect semi-infinite leads

Schrödinger equation

$$\begin{pmatrix} \psi_{n+1} \\ \psi_n \end{pmatrix} = \mathbf{P}_n \begin{pmatrix} \psi_n \\ \psi_{n-1} \end{pmatrix}$$

Transfer matrix (site representation)

$$\mathbf{P}_n = \begin{pmatrix} E - i\gamma + \varepsilon_n & -1 \\ 1 & 1 \end{pmatrix}$$

TRANSFER MATRIX IN PLANE WAVE REPRESENTATION

Wavefunction amplitudes

$$\begin{aligned}\psi_n &= A_n e^{ikn} + B_n e^{-ikn}\\ \psi_{n-1} &= A_n e^{ik(n-1)} + B_n e^{-ik(n-1)}\end{aligned}$$

Schrödinger equation

$$\begin{pmatrix} A_{n+1}e^{ikn} \\ B_{n+1}e^{-ikn} \end{pmatrix} = \left(\mathbf{Q}^{(0)} + \mathbf{Q}_n^{(1)}\right) \begin{pmatrix} A_n e^{ik(n-1)} \\ B_n e^{-ik(n-1)} \end{pmatrix}$$

Transfer matrix (ordered array)

$$\mathbf{Q}^{(0)} = \begin{pmatrix} \left(1 - \frac{\gamma}{2\sin(k)}\right) e^{ik} & -\frac{\gamma}{2\sin(k)} e^{-ik} \\ \frac{\gamma}{2\sin(k)} e^{ik} & \left(1 + \frac{\gamma}{2\sin(k)}\right) e^{-ik} \end{pmatrix}$$

Additional term due to disorder

$$\mathbf{Q}_n^{(1)} = \frac{i\varepsilon_n}{2\sin(k)} \begin{pmatrix} e^{ik} & e^{-ik} \\ -e^{ik} & -e^{-ik} \end{pmatrix}$$

TRANSMISSION AMPLITUDE

Transfer matrix across the barrier

$$\mathfrak{Q}(N) = \left[\mathbf{Q}^{(0)} + \mathbf{Q}_N^{(1)}\right] \cdots \left[\mathbf{Q}^{(0)} + \mathbf{Q}_1^{(1)}\right]$$

Match between waves in the leads

$$\begin{pmatrix} t_N e^{ikN} \\ 0 \end{pmatrix} = \mathfrak{Q}(N) \begin{pmatrix} e^{-ik} \\ r_N e^{ik} \end{pmatrix}$$

Transmission amplitude

$$t_N = \frac{1}{\mathfrak{Q}_{22}(N)} e^{-ik(N+1)}$$

Reflection amplitude

$$r_N = -\frac{\mathfrak{Q}_{21}(N)}{\mathfrak{Q}_{22}(N)} e^{-i2k}$$

SYMMETRY PROPERTIES

LEFT-RIGHT SYMMETRY: PRESERVED

$$\det\left[\mathbf{Q}^{(0)}+\mathbf{Q}_n^{(1)}\right]=1 \;\Rightarrow\; \begin{cases} |t_N^{(l)}|^2 = |t_N^{(r)}|^2 \\ |r_N^{(l)}|^2 = |r_N^{(r)}|^2 \end{cases}$$

TIME REVERSAL SYMMETRY: BROKEN BY NON-HERMITIAN TERMS

$$\left[\mathbf{Q}^{(0)}+\mathbf{Q}_n^{(1)}\right]^* \neq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\left[\mathbf{Q}^{(0)}+\mathbf{Q}_n^{(1)}\right]\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Downarrow$$

$$|t_N|^2+|r_N|^2\neq 1$$

PERTURBATIVE COMPUTATION OF THE TRANSFER MATRIX

Weak disorder

$\Downarrow$

Transfer matrix can be expanded in powers of ε_n

$$\mathfrak{Q}(N) = \mathfrak{Q}^{(0)}(N) + \mathfrak{Q}^{(1)}(N) + \mathfrak{Q}^{(2)}(N) + O(\varepsilon^3)$$

with

$$\begin{aligned}
\mathfrak{Q}^{(0)}(N) &= \left[\mathbf{Q}^{(0)}\right]^N \\
\mathfrak{Q}^{(1)}(N) &= \sum_{l_1=1}^{N} \left[\mathbf{Q}^{(0)}\right]^{N-l_1} \mathbf{Q}^{(1)}_{l_1} \left[\mathbf{Q}^{(0)}\right]^{l_1-1} \\
\mathfrak{Q}^{(2)}(N) &= \sum_{l_1=1}^{N-1} \sum_{l_2=l_1+1}^{N} \left[\mathbf{Q}^{(0)}\right]^{N-l_2} \mathbf{Q}^{(1)}_{l_2} \\
&\times \left[\mathbf{Q}^{(0)}\right]^{l_2-l_1-1} \mathbf{Q}^{(1)}_{l_1} \left[\mathbf{Q}^{(0)}\right]^{l_1-1}
\end{aligned}$$

Remark: 2nd-order term plays a role only for *correlated* disorder.

PERTURBATIVE COMPUTATION OF THE TRANSFER MATRIX

An explicit expression for the unperturbed transfer matrix can be obtained by diagonalising $\mathbf{Q}^{(0)}$:

$$\mathbf{Q}^{(0)} = \mathbf{V} \begin{pmatrix} e^{iq} & 0 \\ 0 & e^{-iq} \end{pmatrix} \mathbf{V}^{-1}$$

with

$$\mathbf{V} = \begin{pmatrix} \frac{\frac{\gamma}{2\sin(k)} e^{-ik}}{\left(1 - \frac{\gamma}{2\sin(k)}\right) e^{ik} - e^{iq}} & \frac{\frac{\gamma}{2\sin(k)} e^{-ik}}{\left(1 - \frac{\gamma}{2\sin(k)}\right) e^{ik} - e^{-iq}} \\ 1 & 1 \end{pmatrix}$$

and

$$e^{\pm iq} = \cos(k) - i\frac{\gamma}{2} \pm i\sqrt{1 - \left(\cos(k) - i\frac{\gamma}{2}\right)^2}$$

Then one can write

$$\mathfrak{Q}^{(0)}(N) = \left[\mathbf{Q}^{(0)}\right]^N = \mathbf{V} \begin{pmatrix} e^{iqaN} & 0 \\ 0 & e^{-iqaN} \end{pmatrix} \mathbf{V}^{-1}$$

TRANSMISSION COEFFICIENT $T_N = |t_N|^2$: FORMAL EXPRESSIONS

Perturbative expansion

$$\begin{aligned}\ln T_N &= 2\mathrm{Re}\left\{-\ln \mathfrak{Q}_{22}^{(0)}(N) - \frac{\mathfrak{Q}_{22}^{(1)}(N)}{\mathfrak{Q}_{22}^{(0)}(N)}\right.\\ &- \left.\frac{\mathfrak{Q}_{22}^{(2)}(N)}{\mathfrak{Q}_{22}^{(0)}(N)} + \frac{1}{2}\left[\frac{\mathfrak{Q}_{22}^{(1)}(N)}{\mathfrak{Q}_{22}^{(0)}(N)}\right]^2\right\} + O(\varepsilon^3)\end{aligned}$$

with

$$\begin{aligned}\mathfrak{Q}_{22}^{(0)}(N) &= \left(\frac{\gamma}{2\sin(k)} - 1\right) e^{ik}\frac{\sin(qN)}{\sin(q)} + \frac{\sin[q(N+1)]}{\sin(q)}\\ \mathfrak{Q}_{22}^{(1)}(N) &= \sum_{l=1}^{N} \frac{-i\varepsilon_l}{2\sin(k)}\left[e^{-ik}\frac{\sin(ql)}{\sin(q)} - \frac{\sin(q(l-1))}{\sin(q)}\right]\\ &\times \left\{\frac{\sin[q(N-l+1)]}{\sin(q)} - e^{ik}\frac{\sin[q(N-l)]}{\sin(q)}\right\}\\ \mathfrak{Q}_{22}^{(2)}(N) &= \sum_{l_1<l_2} \frac{-i\varepsilon_{l_1}\varepsilon_{l_2}}{\sin(k)}\frac{\sin[q(l_2-l_1)]}{\sin(q)}\\ &\times \left\{e^{-ik}\frac{\sin(ql_1)}{\sin(q)} - \frac{\sin[q(l_1-1)]}{\sin(q)}\right\}\\ &\times \left\{e^{ik}\frac{\sin[q(N-l_2)]}{\sin(q)} - \frac{\sin[q(N-l_2+1)]}{\sin(q)}\right\}\end{aligned}$$

TRANSMISSION COEFFICIENT: NO DISORDER

General formula:

$$T_N = \frac{|\sin(q)|^2}{\left|\left[i\sin(k) - \frac{\gamma}{2}\frac{\cos(k)}{\sin(k)}\right]\sin(qN) - \sin(q)\cos(qN)\right|^2}$$

Strong amplification/absorption $|\gamma| \gg 1$

$$T_N \simeq \frac{4\sin^2(k)}{\gamma^{2N}}$$

Asymptotic behaviour

$$\begin{aligned} \ln T_N &= -2N\left(\ln|\gamma| + \frac{1+\cos^2(k)}{\gamma^2} + O\left(\frac{1}{\gamma^3}\right)\right) \\ &+ O(N^0) \end{aligned}$$

MAIN FEATURES FOR $|\gamma| \gg 1$

- T_N is the same for $\gamma > 0$ and $\gamma < 0$
- Exponential decay of T_N: for $N \gg 1$ the barrier behaves as a reflector
- Very weak dependence on the energy of the impinging wave

Weak amplification/absorption $|\gamma| \ll 1$

$$T_N = \frac{1}{e^{\frac{\gamma N}{\sin(k)}} + \frac{\gamma^2}{8\sin^2(k)}\cos(2kN) + \frac{\gamma^4}{256\sin^4(k)} e^{-\frac{\gamma N}{\sin(k)}}}$$

Small-domain limit $N \ll l_a \sim 1/|\gamma|$

$$T_N \simeq 1 - \frac{\gamma N}{\sin(k)} + \dots$$

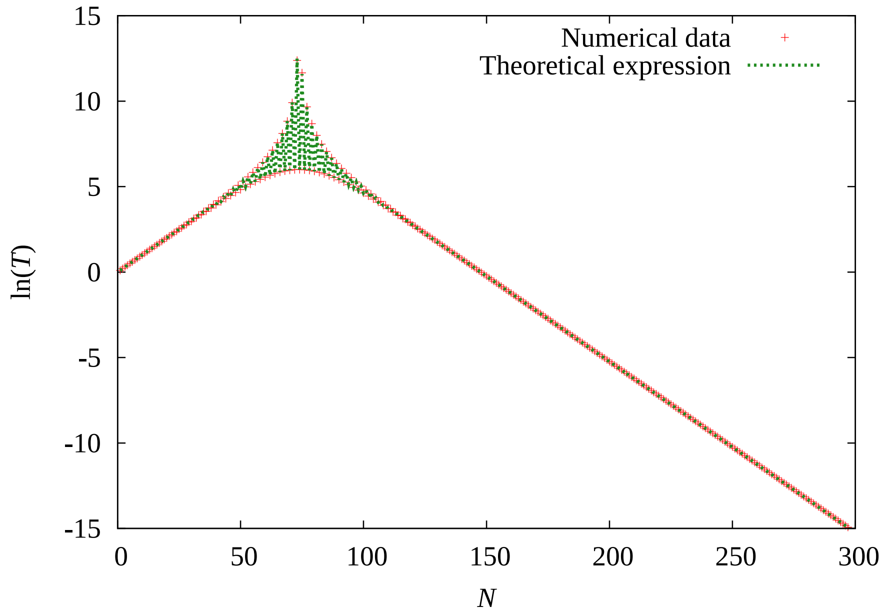
Large-domain limit $N \gg l_a \sim 1/|\gamma|$

$$T_N \simeq \begin{cases} e^{-\frac{\gamma N}{\sin(k)}} & \text{if} \quad \gamma > 0 \\ \left(\frac{4\sin^2(k)}{\gamma}\right)^4 e^{-\frac{|\gamma| N}{\sin(k)}} & \text{if} \quad \gamma < 0 \end{cases}$$

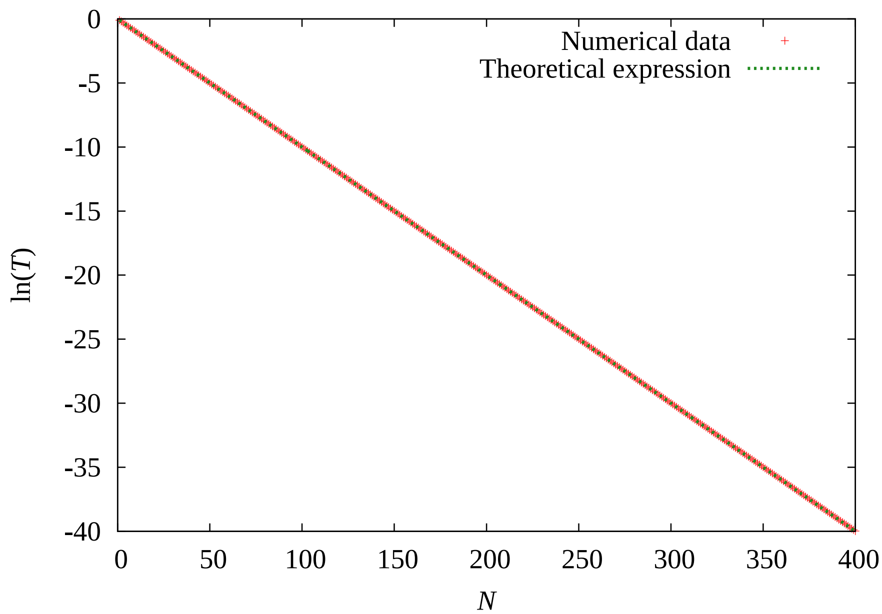
MAIN FEATURES FOR $|\gamma| \ll 1$

- Weak amplification ($\gamma < 0$, $|\gamma| \ll 1$): T_N first increases and then decreases with N (T_N has a maximum)
- Weak absorption ($\gamma > 0$, $|\gamma| \ll 1$): monotonic decrease of T_N with N
- T_N decreases exponentially for $N \gg 1$
- Energy dependence associated to $\gamma/\sin(k)$

$\ln T_N$ versus N: no disorder weak amplification/absorption



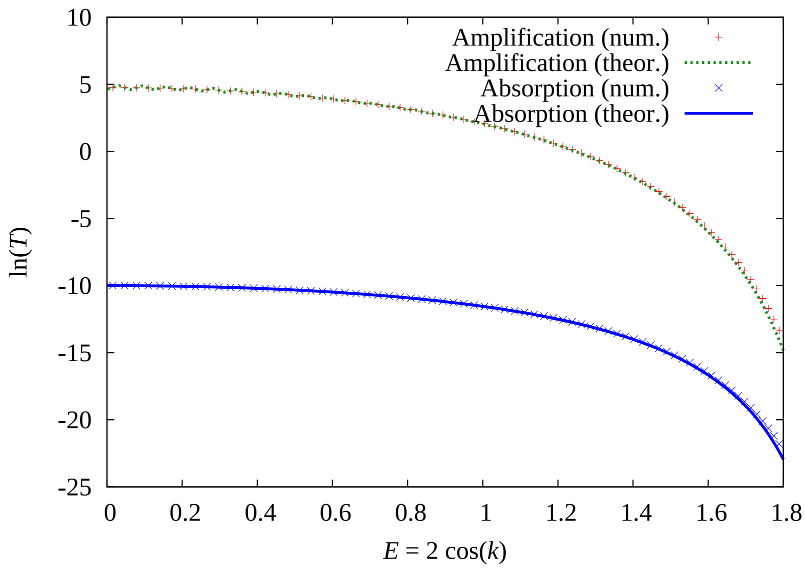
Weak amplification $\gamma = -0.1$; band centre $E = 0$



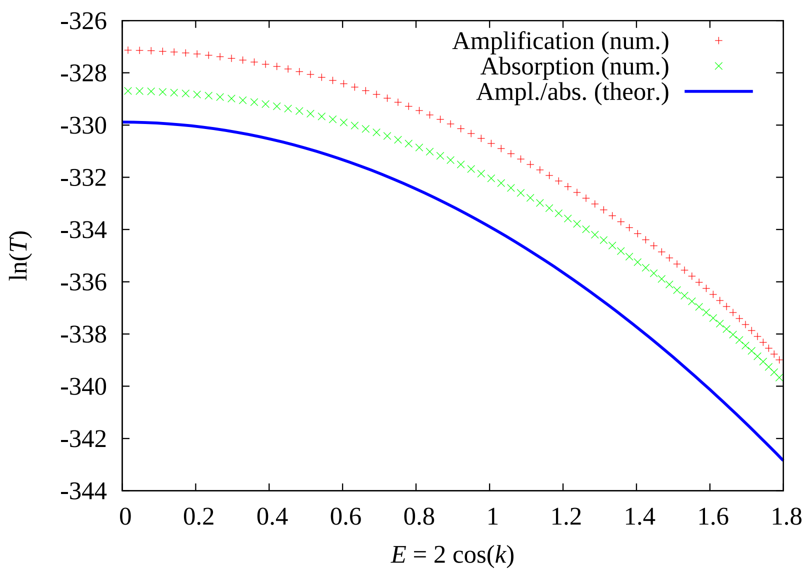
Weak absorption $\gamma = 0.1$; band centre $E = 0$

$\ln T_N$ versus E; no disorder

$T_N(E)$ monotonically decreases away from the band centre



Weak amplification/absorption $\gamma = \pm 0.1$; barrier length $N = 100$



Strong amplification/absorption $\gamma = \pm 5$; barrier length $N = 100$

RANDOM BARRIER
WEAK AMPLIFICATION ABSORPTION; SHORT-DOMAIN LIMIT $N \ll l_a \sim 1/|\gamma|$

Average logarithm of transmission coefficient:

$$\frac{1}{2N}\langle \ln T_N \rangle \simeq -\frac{\gamma}{2\sin(k)} - \frac{\langle \varepsilon_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{N} \chi(n)\left(1 - \frac{n}{N}\right)\cos(2kn)\right]$$

Under the assumptions

$$|\gamma| \sim \frac{1}{N} \quad \text{and} \quad \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} \chi(n) n \cos(2kn) = 0$$

one obtains

$$\begin{aligned}\lambda &= -\lim_{N\to\infty} \frac{1}{2N}\langle \ln T_N \rangle \\ &= \frac{\gamma}{2\sin(k)} + \frac{\langle \varepsilon_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{\infty} \chi(n)\cos(2kn)\right] \\ &= \pm\frac{1}{l_{\text{a}}} + \frac{1}{l_{\text{loc}}}\end{aligned}$$

WEAK AMPLIFICATION/ABSORPTION; SHORT-DOMAIN LIMIT: KEY FEATURES

- Absorption and localisation add up; amplification and disorder have opposite effects: $\lambda = \pm 1/l_{\mathsf{a}} + 1/l_{\mathsf{loc}}$

- Effect of disorder correlation limited by finite-size of the barrier

- Effective mobility edges can arise only if N is not too short (requires small $|\gamma|$).

RANDOM BARRIER
WEAK AMPLIFICATION/ABSORPTION
$|\gamma| \ll 1$; LONG-DOMAIN LIMIT: $|\gamma| N \gg 1$

Average logarithm of transmission coefficient:

$$\frac{1}{2N}\langle \ln T_N \rangle \simeq -\frac{|\gamma|}{2\sin(k)} - \frac{1}{2N}\left(1 - \frac{\gamma}{|\gamma|}\right) \ln \frac{\gamma^2}{16\sin^4(k)}$$
$$-\frac{\langle \varepsilon_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{N} \chi(n)\left(1 - \frac{n}{N}\right) e^{-\frac{|\gamma| n}{\sin(k)}} \cos(2kn)\right]$$

Taking the limit for $N \to \infty$ one obtains

$$\begin{aligned}
\lambda &= -\lim_{N\to\infty} \frac{1}{2N}\langle \ln T_N \rangle \\
&= \frac{\langle \varepsilon_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{\infty} \chi(n) e^{-\frac{|\gamma| n}{\sin(k)}} \cos(2kn)\right] \\
&+ \frac{|\gamma|}{2\sin(k)} + O(\varepsilon^2 \gamma)
\end{aligned}$$

One has

$$\lambda = \frac{1}{l_{\mathrm{a}}} + \frac{1}{l_{\mathrm{loc}}}$$

with l_{loc} modified with respect to the standard Izrailev-Krokhin formula.

WEAK AMPLIFICATION/ABSORPTION; LONG DOMAIN: KEY FEATURES

- Amplification/absorption effects and localisation add up: $\lambda = 1/l_{\mathrm{a}} + 1/l_{\mathrm{loc}}$

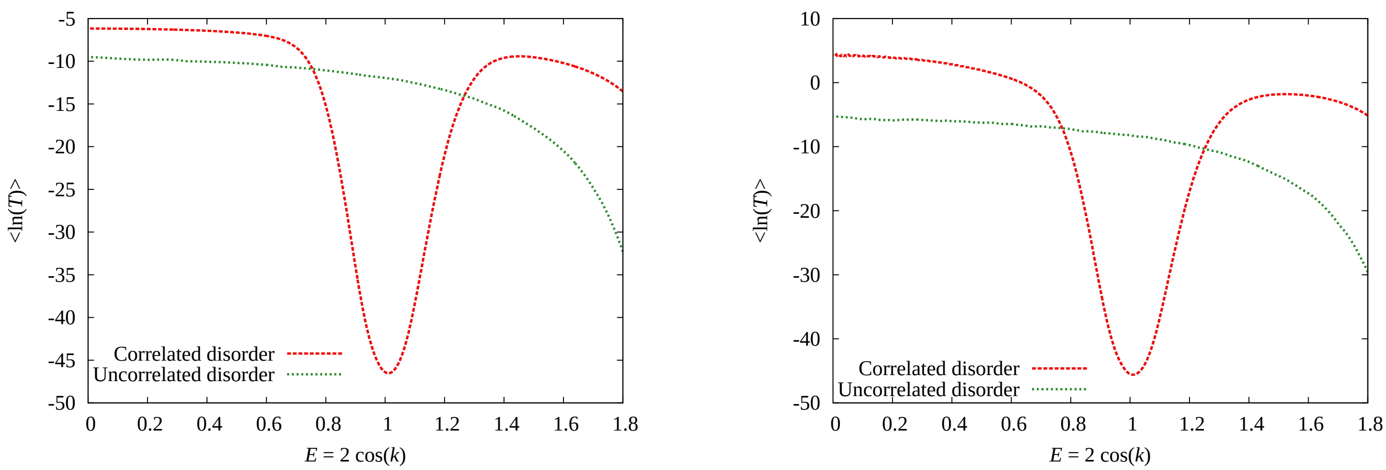
- l_{loc} is given modified Izrailev-Krokhin formula

- Disorder correlations are felt, but gain/loss mechanism introduces an effective correlation length $l_{\mathrm{c}} \sim l_{\mathrm{a}} \sim \sin(k)/|\gamma|$

- Effective mobility edges can arise only if l_{c} is not too short.

DELOCALISATION EFFECTS SURVIVE IN MODERATELY ACTIVE MEDIA

$\langle \ln T_N \rangle$ versus E:



Absorption ($\gamma = 0.02$) Amplification ($\gamma = -0.02$)

- Disorder strength $\langle \varepsilon_n^2 \rangle = 0.05$
- barrier length $N = 300$
- Mobility edges for $\kappa_1 = 0.3283\pi$ and $\kappa_2 = 0.3383\pi$

RANDOM BARRIER
STRONG AMPLIFICATION/ABSORPTION
$|\gamma| \gg 1$

Asymptotic behaviour ($N \gg 1$) of $\ln T_N$:

$$\begin{aligned}\lambda &= -\lim_{N\to\infty} \frac{1}{2N}\langle \ln T_N \rangle \\ &= \ln|\gamma| + \frac{1+2\cos^2(k)}{\gamma^2} \\ &+ \frac{\langle \varepsilon_n^2 \rangle}{2\gamma^2} + o\left(\frac{1}{\gamma^2}\right)\end{aligned}$$

- Any effect of disorder correlations is suppressed

- Localisation itself tends to be destroyed.

RANDOM AMPLIFICATION/ABSORPTION: NON-HERMITIAN DISORDER

Barrier with random amplification/absorption
$(n = 1, \dots, N)$

$$\psi_{n+1} + \psi_{n-1} + i\gamma_n\psi_n = E\psi_n$$

STATISTICAL PROPERTIES OF THE GAIN/LOSS COEFFICIENT

- $\langle\gamma_n\rangle = 0$: zero average
- $\langle\gamma_n^2\rangle \ll 1$: weak disorder
- $\chi_2(l) = \frac{\langle\gamma_{n+l}\gamma_n\rangle}{\langle\gamma_n^2\rangle}$: known binary correlator.

NET EFFECT: AMPLIFICATION OR ABSORPTION?

FLUCTUATING GAIN/LOSS COEFFICIENT: RESULTS

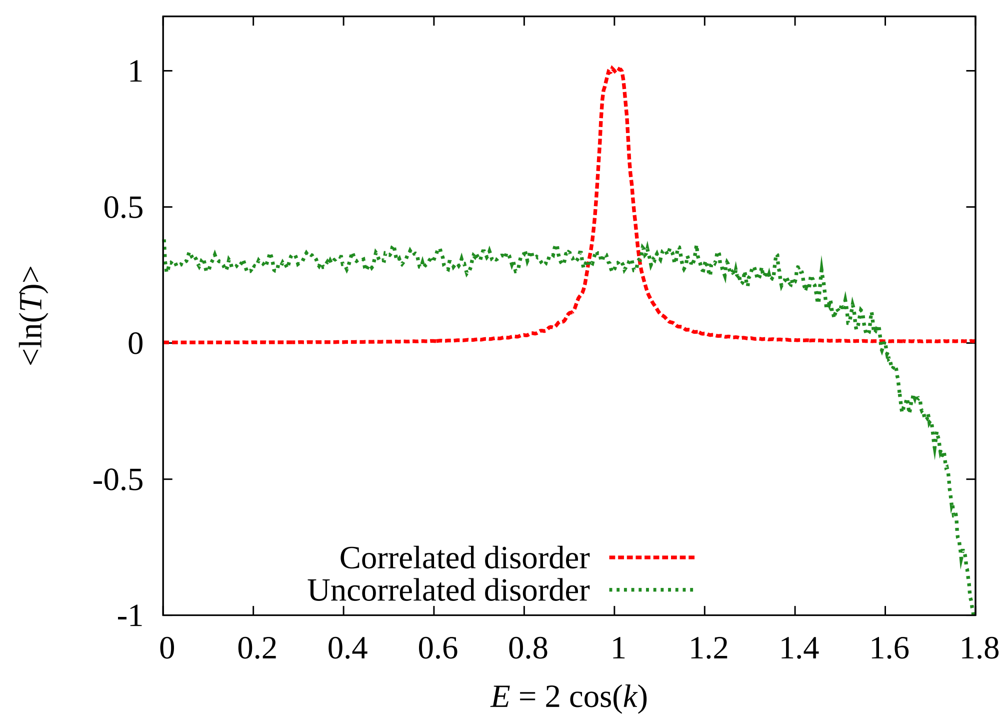
Average logarithm of transmission coefficient:

$$\frac{1}{2N}\langle \ln T_N \rangle \simeq +\frac{\langle \gamma_n^2 \rangle}{8\sin^2(k)}\left[1+2\sum_{n=1}^{N}\chi_2(n)\left(1-\frac{n}{N}\right)\cos(2kn)\right]$$

In the limit $N \to \infty$

$$\lambda = -\lim_{N\to\infty}\frac{1}{2N}\langle \ln T_N \rangle = -\frac{\langle \gamma_n^2 \rangle}{8\sin^2(k)}\left[1+2\sum_{n=1}^{\infty}\chi_2(n)\cos(2kn)\right]$$

Fluctuations of the gain/loss coefficient lead to enhancement of transmission



$\langle \gamma_n^2 \rangle = 0.005, \; N = 300$

ACTIVE RANDOM BARRIER: GENERAL CASE

Active random barrier ($n = 1, \ldots, N$)

$$\psi_{n+1} + \psi_{n-1} + [\varepsilon_n + i(\gamma + \gamma_n)]\,\psi_n = E\psi_n$$

Statistical properties

- Vanishing average values

$$\langle \varepsilon_n \rangle = \langle \gamma_n \rangle = 0$$

- Weak disorder

$$\langle \varepsilon_n^2 \rangle \ll 1 \quad \text{and} \quad \langle \gamma_n^2 \rangle \ll 1$$

- Correlated disorder

$$\begin{aligned} \chi_1(l) &= \frac{\langle \varepsilon_n \varepsilon_{n+l} \rangle}{\langle \varepsilon_n^2 \rangle} \\ \chi_2(l) &= \frac{\langle \gamma_n \gamma_{n+l} \rangle}{\langle \gamma_n^2 \rangle} \\ \chi_3(l) &= \frac{\langle \varepsilon_n \gamma_{n+l} + \varepsilon_{n+l} \gamma_n \rangle}{2 \langle \varepsilon_n \gamma_n \rangle}. \end{aligned}$$

RANDOM BARRIER
WEAK AMPLIFICATION ABSORPTION; SHORT-DOMAIN LIMIT $N \ll l_a \sim 1/|\gamma|$

Average logarithm of transmission coefficient:

$$
\begin{aligned}
\frac{1}{2N}\langle \ln T_N \rangle &= -\frac{\gamma}{2\sin(k)} \\
&- \frac{\langle \varepsilon_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{N} \chi_1(n)\left(1-\frac{n}{N}\right)\cos(2kn)\right] \\
&+ \frac{\langle \gamma_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{N} \chi_2(n)\left(1-\frac{n}{N}\right)\cos(2kn)\right] \\
&+ \frac{\langle \varepsilon_n \gamma_n \rangle}{2\sin^2(k)}\left[\sum_{n=1}^{N} \chi_3(n)\left(1-\frac{n}{N}\right)\sin(2kn)\right] + \ldots
\end{aligned}
$$

Under the assumptions

$$
|\gamma| \sim \frac{1}{N} \quad \text{and} \quad \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} \chi_i(n) n e^{i2kn} = 0
$$

one obtains for $\lambda = -\lim_{N\to\infty}(1/2N)\langle \ln T_N \rangle$

$$
\begin{aligned}
\lambda &= \frac{\gamma}{2\sin(k)} + \frac{\langle \varepsilon_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{\infty} \chi_1(n)\cos(2kn)\right] \\
&- \frac{\langle \gamma_n^2 \rangle}{8\sin^2(k)}\left[1 + 2\sum_{n=1}^{\infty} \chi_2(n)\cos(2kn)\right] \\
&- \frac{\langle \varepsilon_n \gamma_n \rangle}{2\sin^2(k)}\sum_{n=1}^{\infty} \chi_3(n)\sin(2kn) + O(\eta^2\gamma)
\end{aligned}
$$

RANDOM BARRIER WEAK AMPLIFICATION/ABSORPTION $|\gamma| \ll 1$; LONG-DOMAIN LIMIT: $|\gamma| N \gg 1$

Average logarithm of transmission coefficient:

$$\frac{1}{2N}\langle \ln T_N \rangle = -\frac{|\gamma|}{2\sin(k)} - \frac{1}{2N}\left(1-\frac{\gamma}{|\gamma|}\right)\ln\frac{\gamma^2}{16\sin^4(k)}$$
$$-\frac{\langle \varepsilon_n^2\rangle}{8\sin^2(k)}\left[1+2\sum_{n=1}^{N}\chi_1(n)\left(1-\frac{n}{N}\right)e^{-\frac{|\gamma|n}{\sin(k)}}\cos(2kn)\right]$$
$$+\frac{\langle \gamma_n^2\rangle}{8\sin^2(k)}\left[1+2\sum_{n=1}^{N}\chi_2(n)\left(1-\frac{n}{N}\right)e^{-\frac{|\gamma|n}{\sin(k)}}\cos(2kn)\right]$$
$$+\frac{\langle \varepsilon_n\gamma_n\rangle}{2\sin^2(k)}\left[\sum_{n=1}^{N}\chi_3(n)\left(1-\frac{n}{N}\right)e^{-\frac{|\gamma|n}{\sin(k)}}\sin(2kn)\right]+\ldots$$

Taking the limit for $N \to \infty$ one obtains

$$\lambda = -\lim_{N\to\infty}\frac{1}{2N}\langle \ln T_N\rangle = \frac{|\gamma|}{2\sin(k)}$$
$$+\frac{\langle \varepsilon_n^2\rangle}{8\sin^2(k)}\left[1+2\sum_{n=1}^{\infty}\chi_1(n)e^{-\frac{|\gamma|n}{\sin(k)}}\cos(2kn)\right]$$
$$-\frac{\langle \gamma_n^2\rangle}{8\sin^2(k)}\left[1+2\sum_{n=1}^{\infty}\chi_2(n)e^{-\frac{|\gamma|n}{\sin(k)}}\cos(2kn)\right]$$
$$-\frac{\langle \varepsilon_n\gamma_n\rangle}{2\sin^2(k)}\sum_{n=1}^{\infty}\chi_3(n)e^{-\frac{|\gamma|n}{\sin(k)}}\sin(2kn)+O(\eta^2\gamma)$$

RANDOM BARRIER
STRONG AMPLIFICATION/ABSORPTION
$|\gamma| \gg 1$

Asymptotic behaviour ($N \gg 1$) of $\ln T_N$:

$$\begin{aligned}
\lambda &= -\lim_{N\to\infty} \frac{1}{2N}\langle \ln T_N \rangle \\
&= \ln|\gamma| + \frac{1+2\cos^2(k)}{\gamma^2} \\
&+ \frac{\langle \varepsilon_n^2 \rangle - \langle \gamma_n^2 \rangle}{2\gamma^2} + o\left(\frac{1}{\gamma^2}\right)
\end{aligned}$$

STRONG AMPLIFICATION/ABSORPTION
DESTROY LOCALISATION

CONCLUSIONS

- For moderate amplification and absorption, spatial correlations of the disorder display strong effects. Strong amplification or absorption destroy influence of correlations (and localisation itself).

- Specific long-range correlations of the disorder can create energy windows of high or low transmittivity in finite barriers with moderate amplification or absorption.

- Fluctuations of the gain/loss coefficient increase transmission.