Disordered systems with generic universal finite-disorder fixed points

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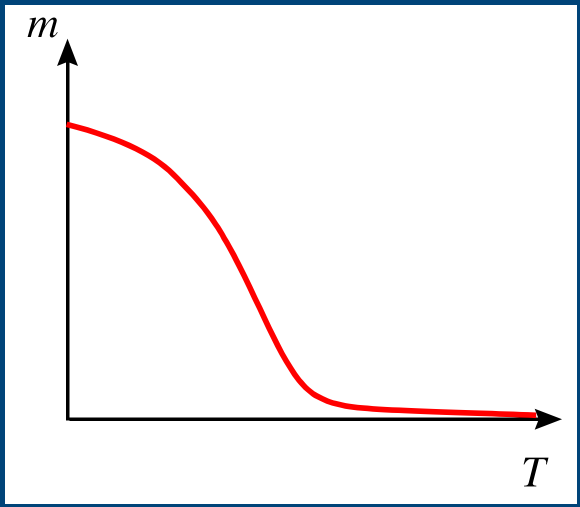
Outline

- Motivation
- The **Strong Disorder Renormalization Group**
  - Universal infinite effective disorder
  - Non-universal finite effective disorder (Griffiths phases)
- The Large Spin phase and its sisters
  - Universal finite effective disorder
- A general scheme of universal finite effective disorder

Randomness in classical systems

- Strong effect on phase transitions
- Harris criterion:
  - $\nu d > 2$: critical behavior is unchanged, disorder is irrelevant (classical Heisenberg model)
  - $\nu d < 2$: disorder is relevant; usually, new critical behavior, new exponents (classical Ising model)
  - smeared phase transitions

(Ising model with planar defects)



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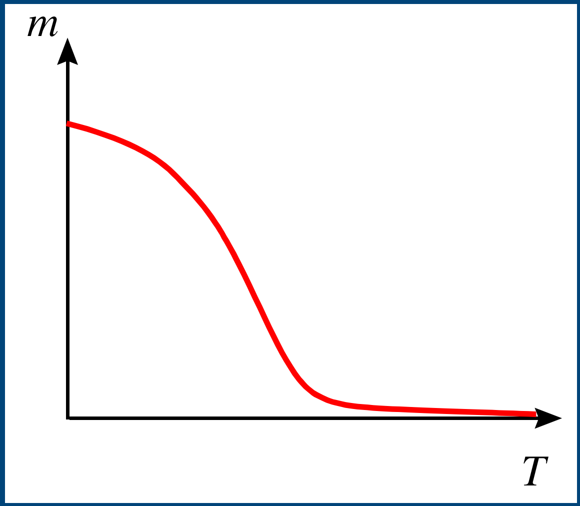

Effects much stronger in quantum systems

Transverse field Ising model: paradigm

$$H_{TFIM} = \sum_j \left(-J\sigma_j^z \sigma_{j+1}^z - h\sigma_j^x \right)$$

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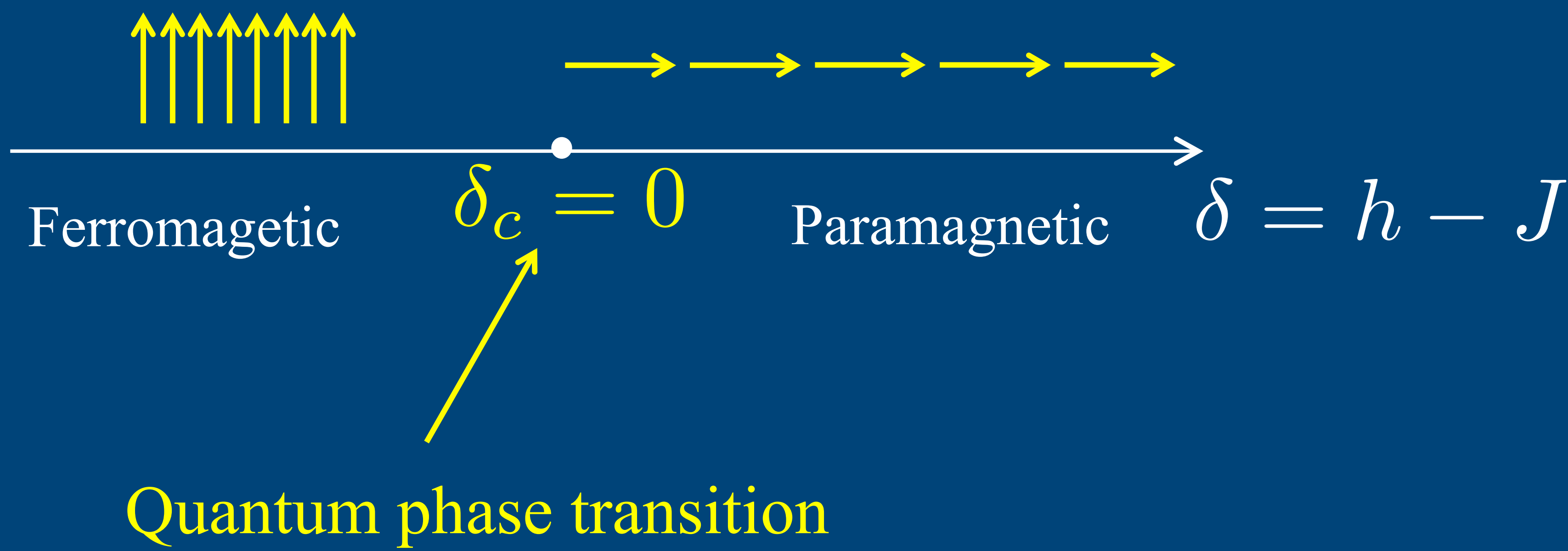

Random transverse field Ising model: RG procedure D. Fisher, PRL **69**, 534 (1992); PRB **51**, 6411 (1995)

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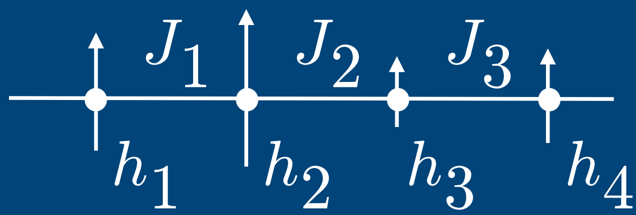

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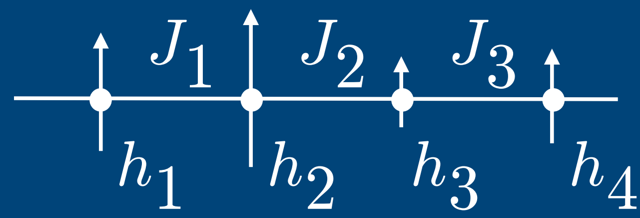

Decimation procedure: Find the largest coupling, $\max(h_i, J_i)$

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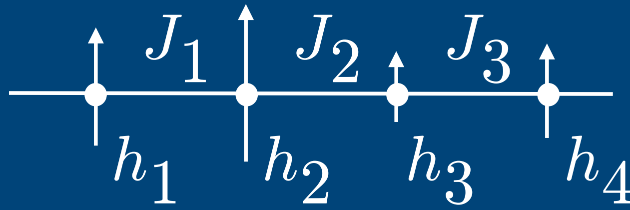

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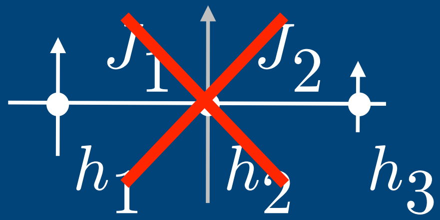
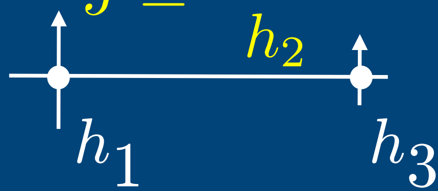
If h_2 = max(h_i,J_i), freeze the spin



2nd order pert. theory

$$\widetilde{J} = \frac{J_1 J_2}{h_2}$$



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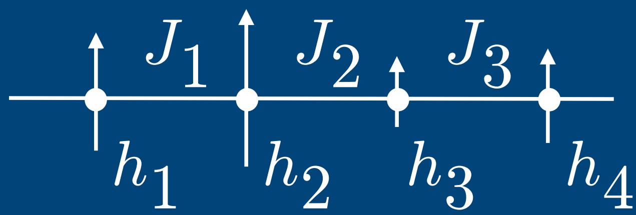

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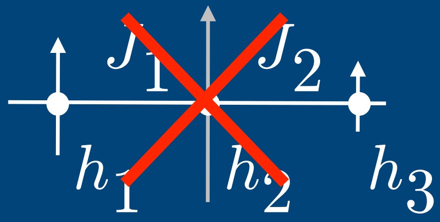

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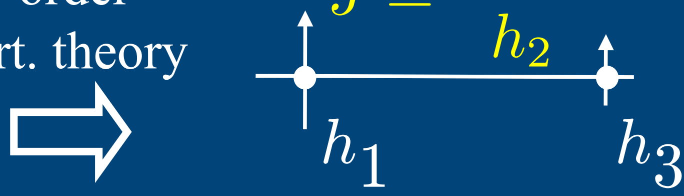

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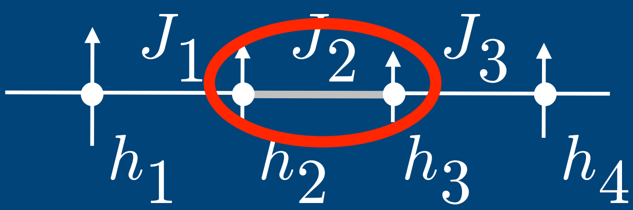

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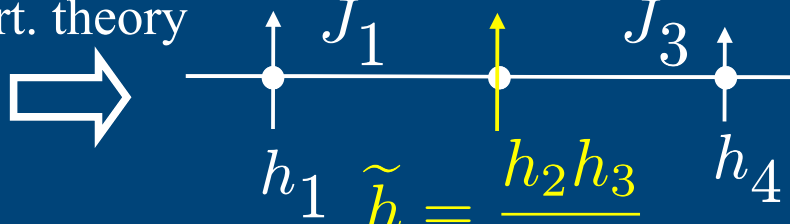
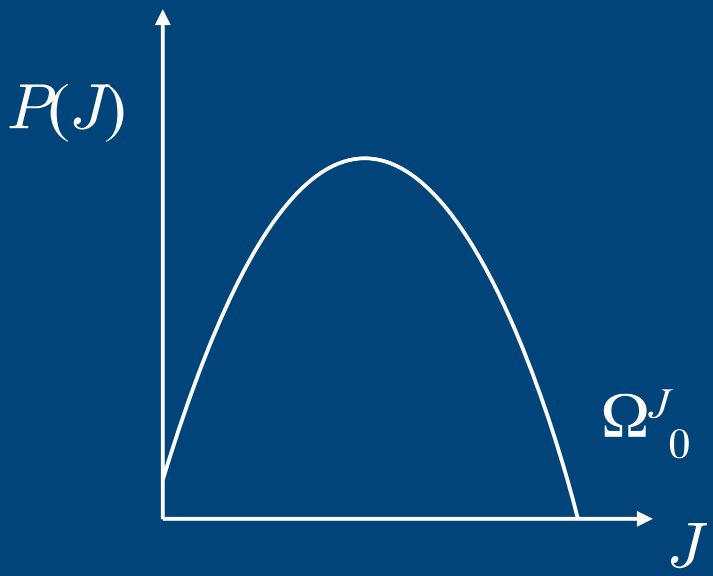
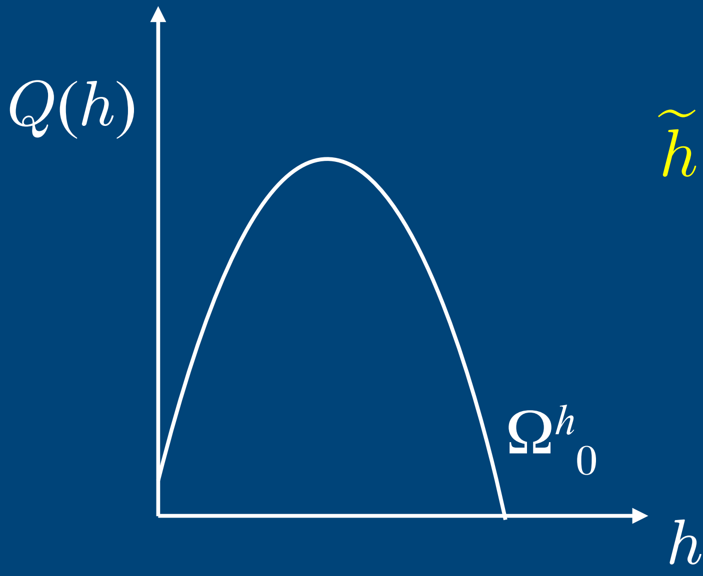


$$\widetilde{h} = \frac{h_2 h_3}{J_2}$$

SDRG flow: decimating bonds





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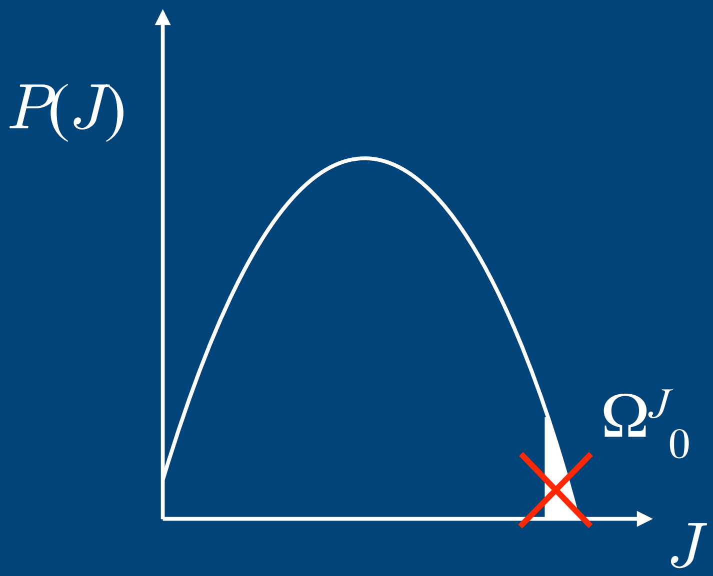

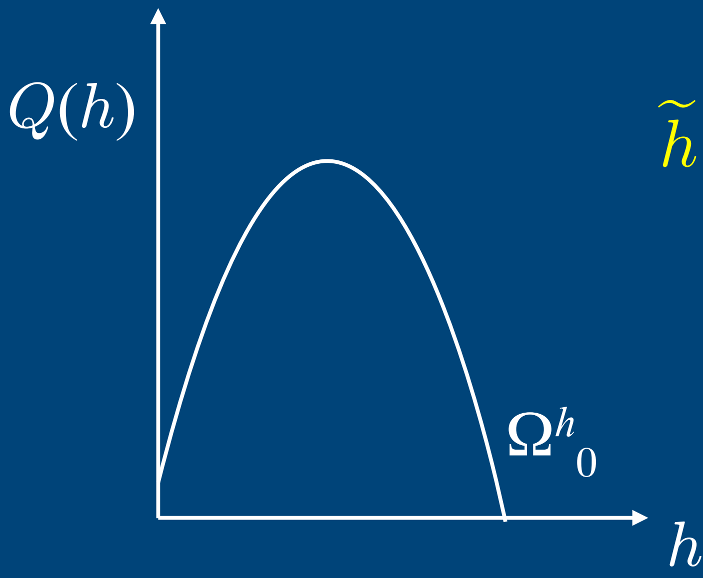

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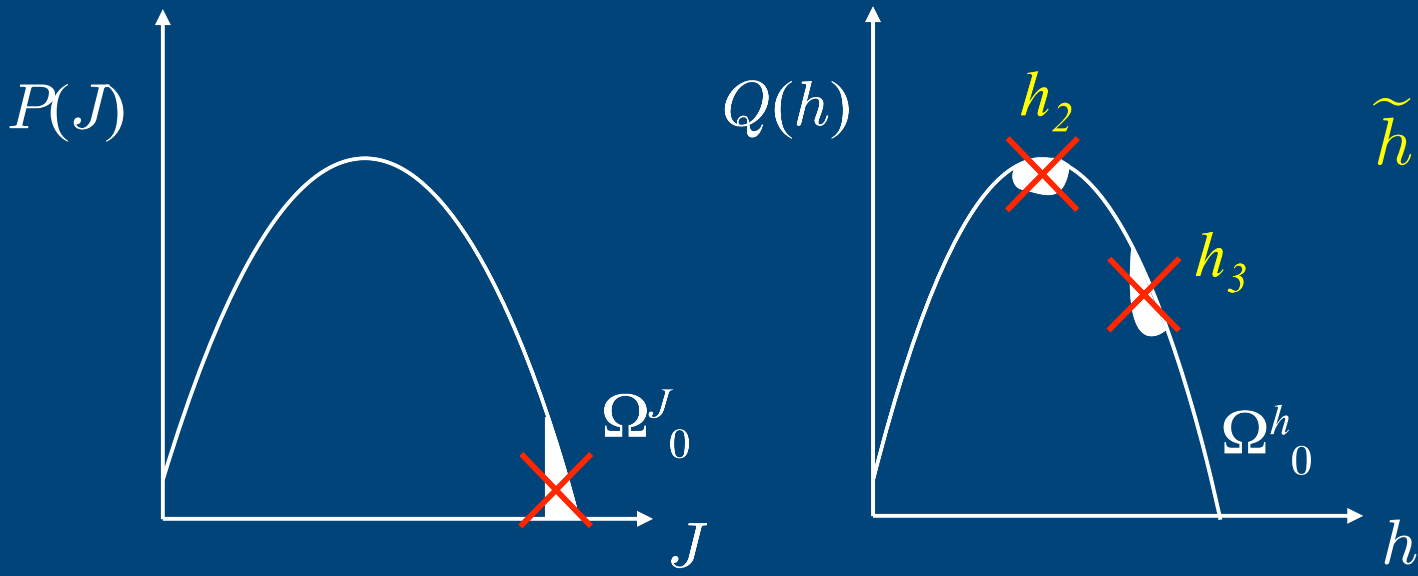

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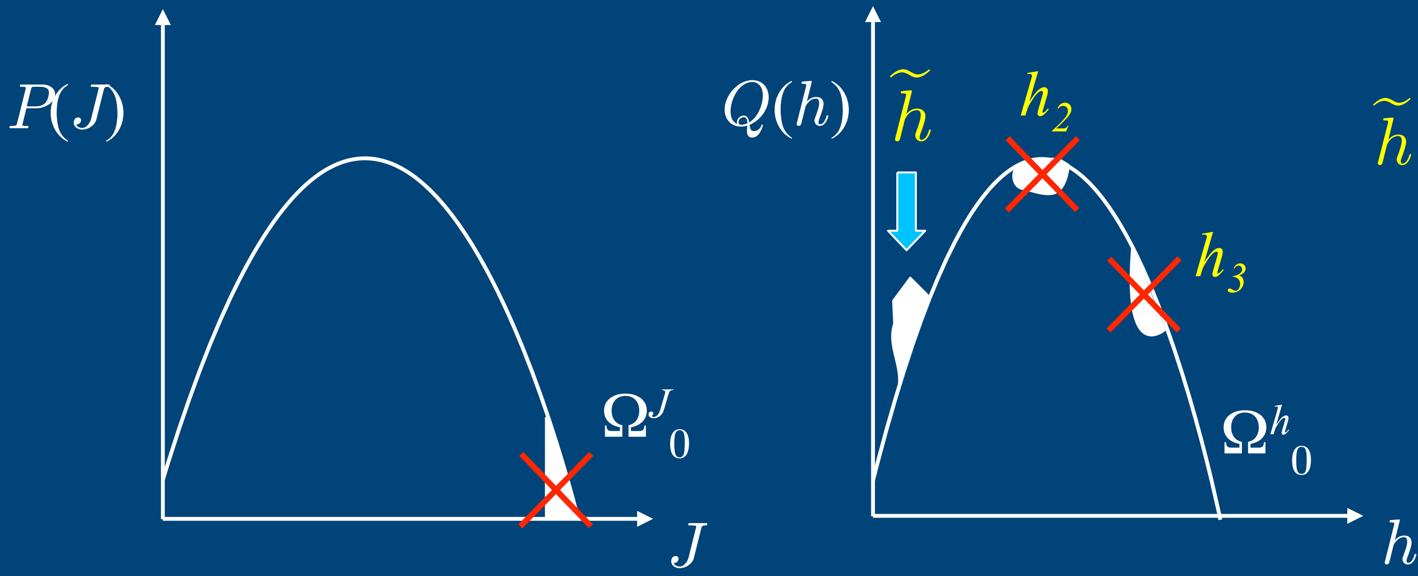

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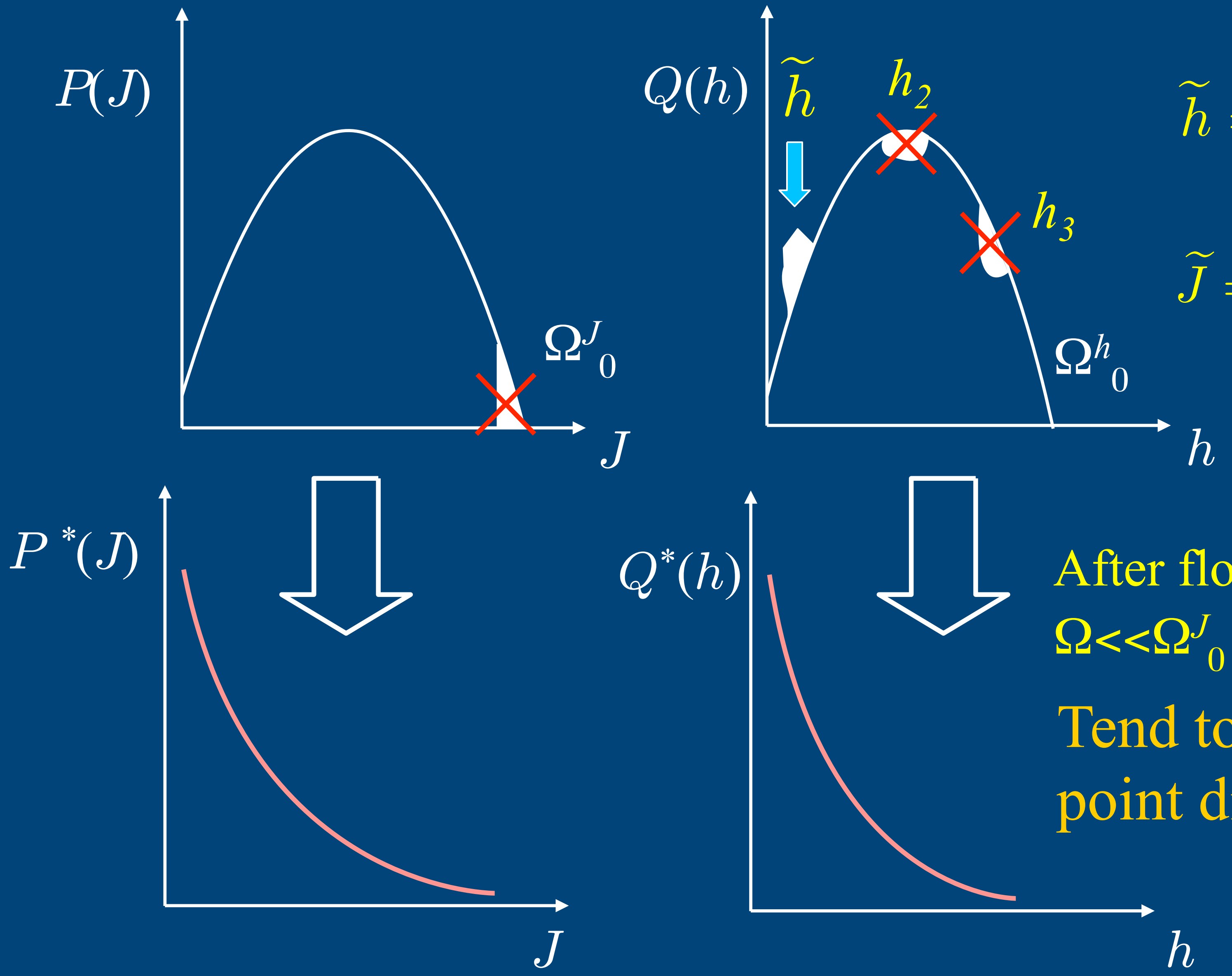

$$\widetilde{h} = \frac{h_2 h_3}{J_2}$$

$$\widetilde{J} = \frac{J_1 J_2}{h_2}$$

After flowing to scale $\Omega << \Omega^J_0$, Ω^h_0

Tend to singular fixed point distributions

Random transverse field Ising model

D. Fisher, PRL **69**, 534 (1992); PRB **51**, 6411 (1995)

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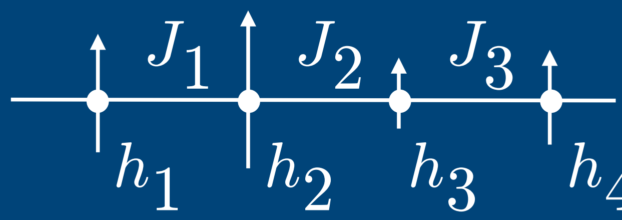

Ferromagetic Paramagnetic

$$\delta = \langle \ln h \rangle - \langle \ln J \rangle$$

$$\delta_c = 0$$

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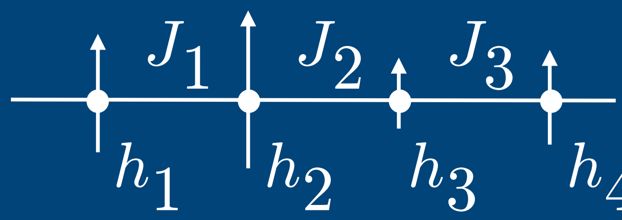

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$$\boxed{\frac{\Delta J}{\langle J \rangle} \sim \sqrt{z}} \qquad \chi(T) \sim \frac{1}{T^{1+1/z}}$$

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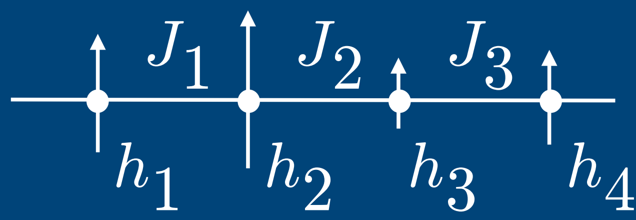

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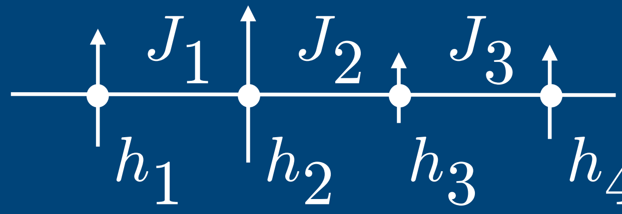

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Away from the critical point, the effective disorder is governed by a non-universal exponent z: quantum Griffiths phase

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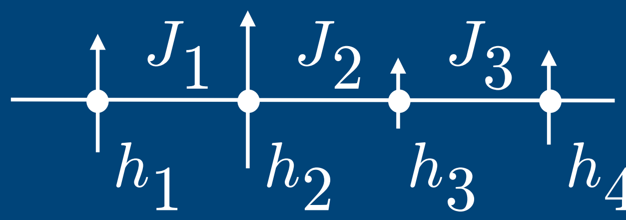

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z diverges at the critical point.

$$z \sim \frac{1}{|\delta|} \to \infty, \text{ as } |\delta| \to 0$$

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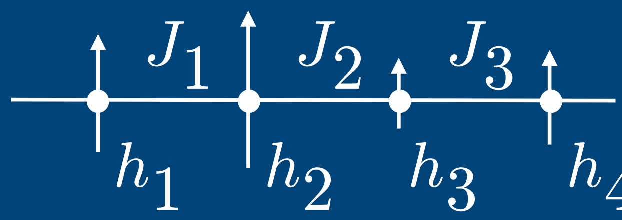

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At the critical (dual) point: Infinite effective disorder

$$P^*(J) \sim \frac{1}{J^{1-\alpha}} \qquad Q^*(h) \sim \frac{1}{h^{1-\alpha}}$$

$$\alpha = \frac{1}{\ln\left(\Omega_0/\Omega\right)} \to 0$$

$$\boxed{\frac{\Delta J}{\langle J \rangle} \sim \frac{1}{\sqrt{\alpha}} \to \infty}$$

$$\chi(T) \sim \frac{\left[\ln\left(1/T\right)\right]^{2\phi-2}}{T} \qquad \phi = \frac{1+\sqrt{5}}{2}$$

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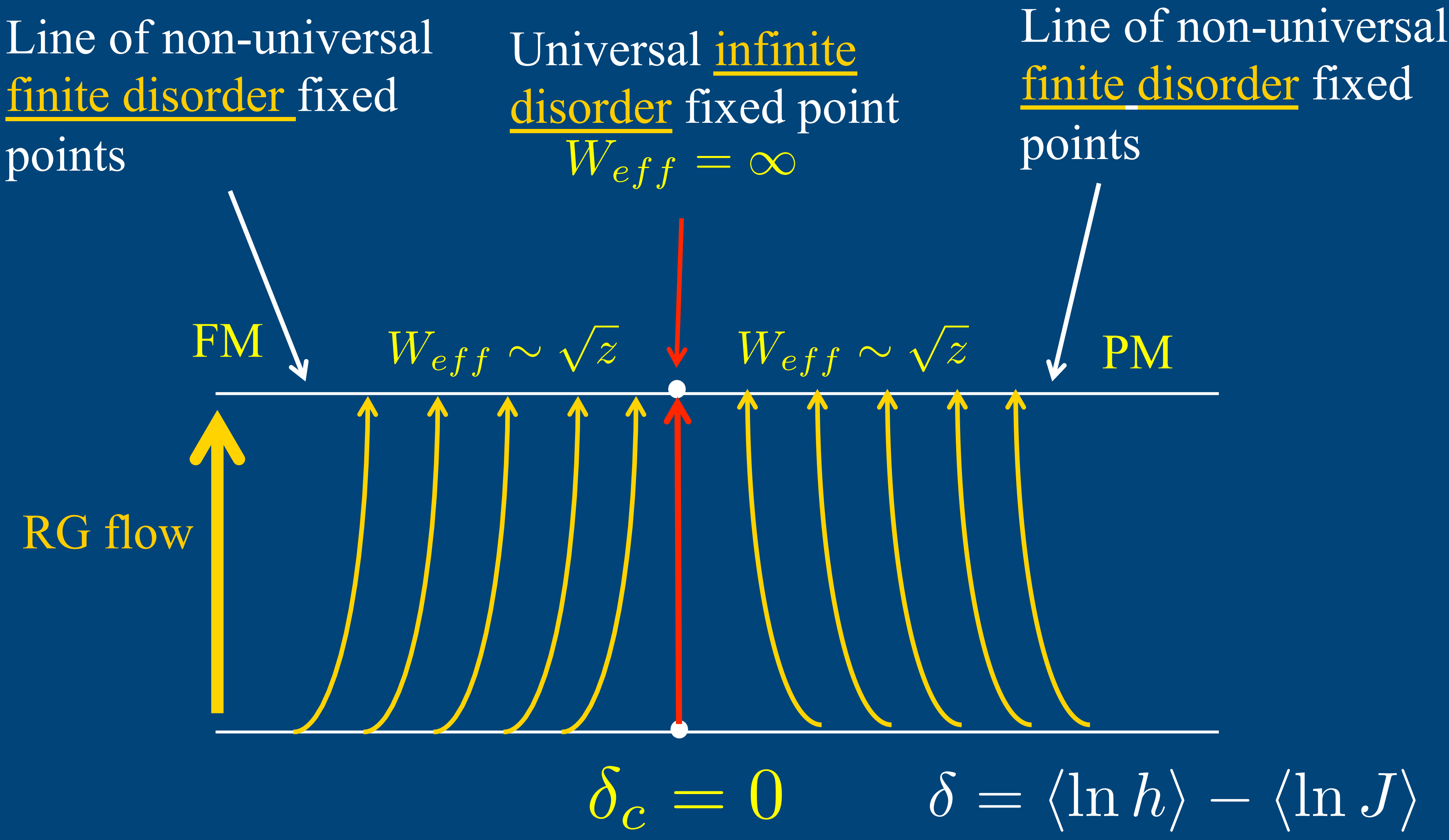

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Line of non-universal finite disorder fixed points

Universal infinite disorder fixed point
$W_{eff} = \infty$

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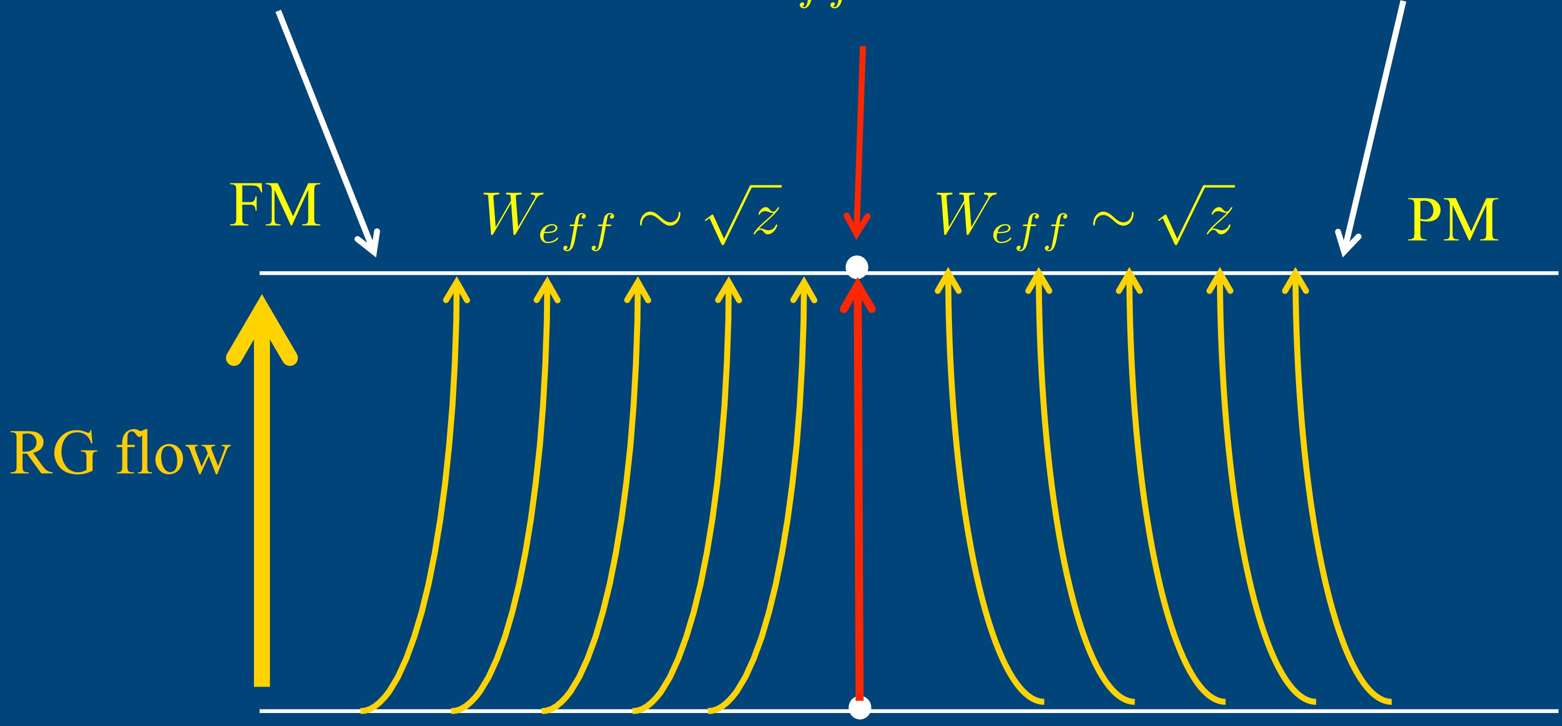

$\delta_c = 0$ $\qquad$ $\delta = \langle \ln h \rangle - \langle \ln J \rangle$

Numerical confirmation (Young, Rieger, Iglói); true also for 2D and 3D (Motrunich, Mau, Huse & Fisher; Pich, Young, Rieger, Kawashima).

Energy vs. length scales

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Excitations of size L (clusters of spins) have energy Ω such that:

Along the line of finite disorder fixed points $\boxed{\Omega \sim L^{-z}}$

Conventional scaling (similar to clean systems) $\omega \sim k^z$

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At the infinite disorder critical point: $\boxed{\Omega \sim e^{-L^\psi}, \ \psi = \frac{1}{2}}$

Activated dynamical scaling (dynamical exponent is formally = ∞)

Related behavior in other systems

Finite disorder: $z < \infty$

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This structure, lines of non-universal finite-disorder fixed points ending at universal infinite-disorder fixed points, is seen in many models:

1. Other discrete models: Potts, clock, Ashkin-Teller (Senthil & Majumdar; Carlon, Lajkó, Chatelain, Berche, Juhász, Iglói)
2. Heisenberg AFM models with higher spins (Hyman & Yang, Monthus, Golinelli, Jolicoeur; Saguia, Boechat, Continentino; Refael, Kehrein, Fisher, Iglói)
3. Heisenberg AFM models with explicit dimerization (Hyman, Yang, Bhatt, Girvin; Iglói, Juhász, Rieger, Lajkó)
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In all cases: $$\boxed{\tilde{\Delta} = \# \frac{\Delta_1 \Delta_2}{\Omega}}$$

- 2nd order pert. theory
- multiplicative structure
- activated dynamics

Other possibilities: chains with AFM + FM

Westerberg *et al*., Phys. Rev. B **55**, 12578 (1997); Hikihara *et al*., Phys. Rev. B **60**, 12116 (1990)

$$H = \sum_i J_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}; \; J_i < 0 \,\text{or}\, > 0$$

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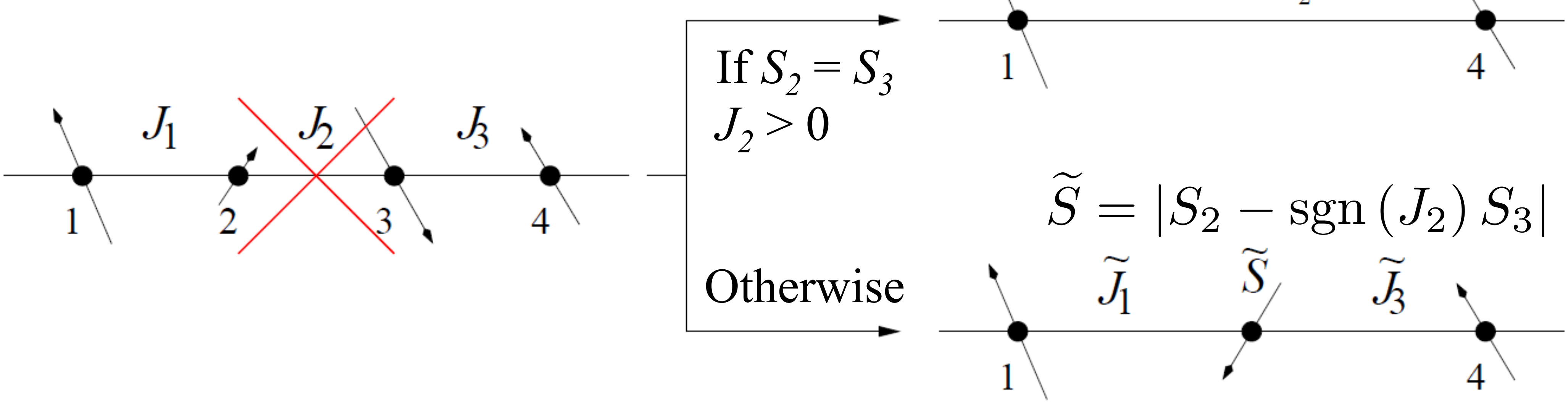
$$\tilde{J} = \frac{2J_1 J_3 S_2 (S_2 + 1)}{3J_2}$$



$$\tilde{S} = |S_2 - \mathrm{sgn}(J_2) S_3|$$

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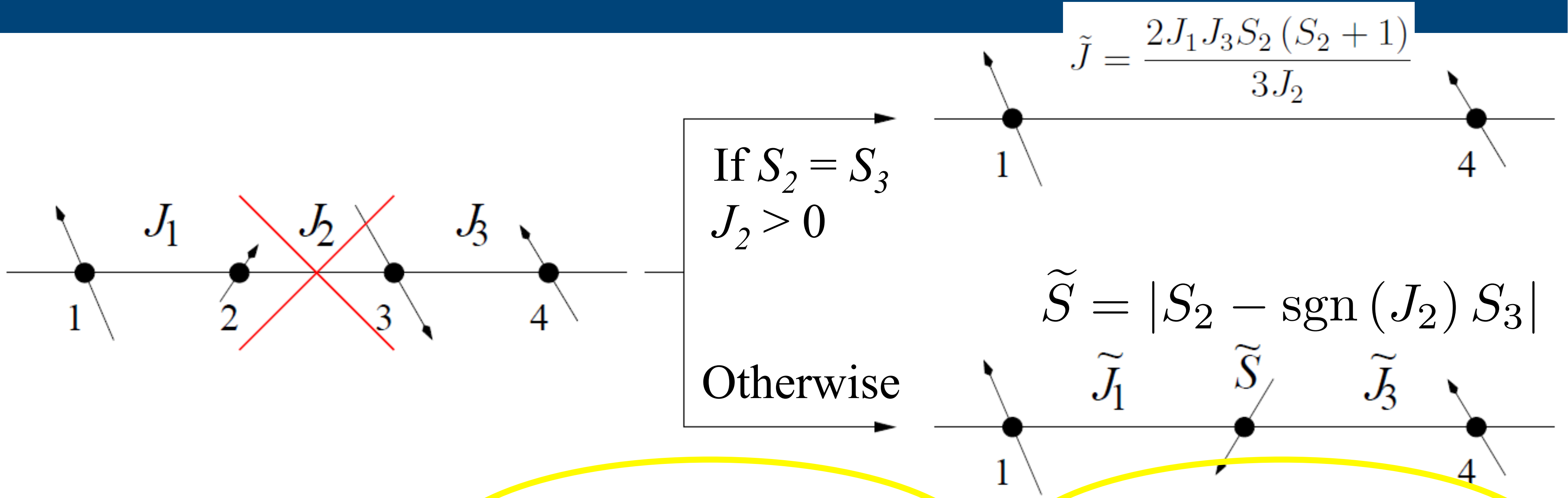

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1st order in perturbation theory – NO multiplicative structure
Generates higher spins

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- For stronger disorder, $z > 2.27$ is non-universal and varies continuously with the disorder strength (like in more conventional Griffiths phases).

As the cluster spins grow, the probability of occurrence of 2nd order processes becomes negligible: only 1st order decimations survive

Other systems with similar behavior

Universal minimum finite disorder: z_{min}

Non-universal tunable finite disorder: $z > z_{min}$

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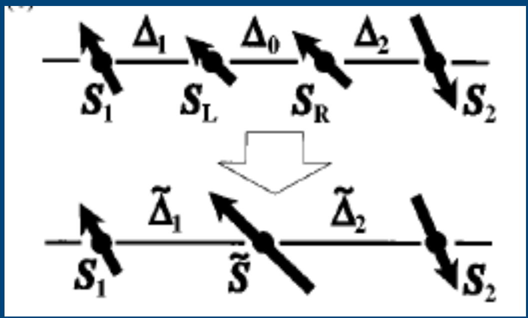
Why is this often observed?
Is there a general scheme?

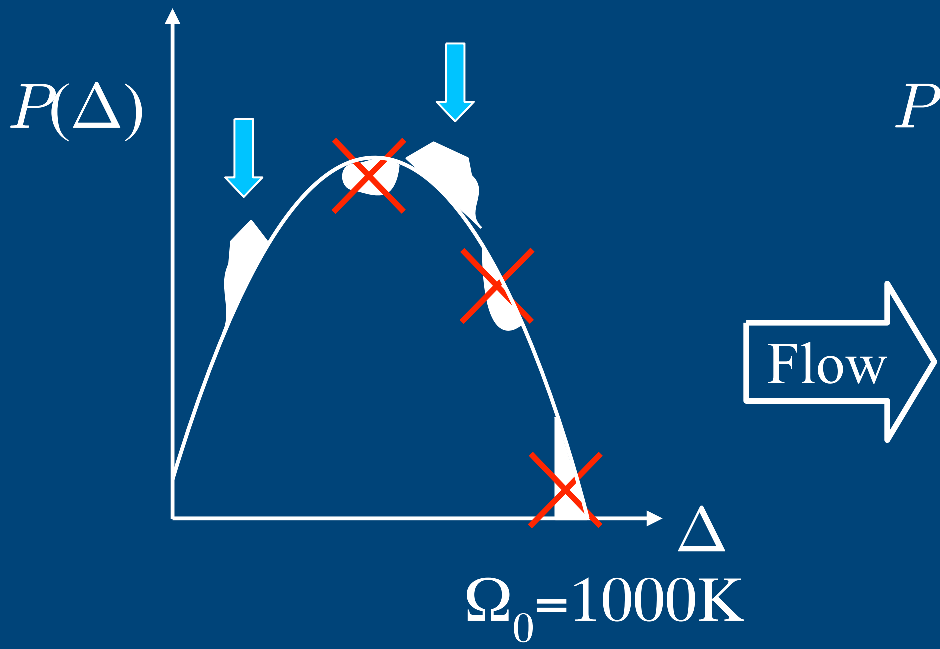
What do they all have in common?

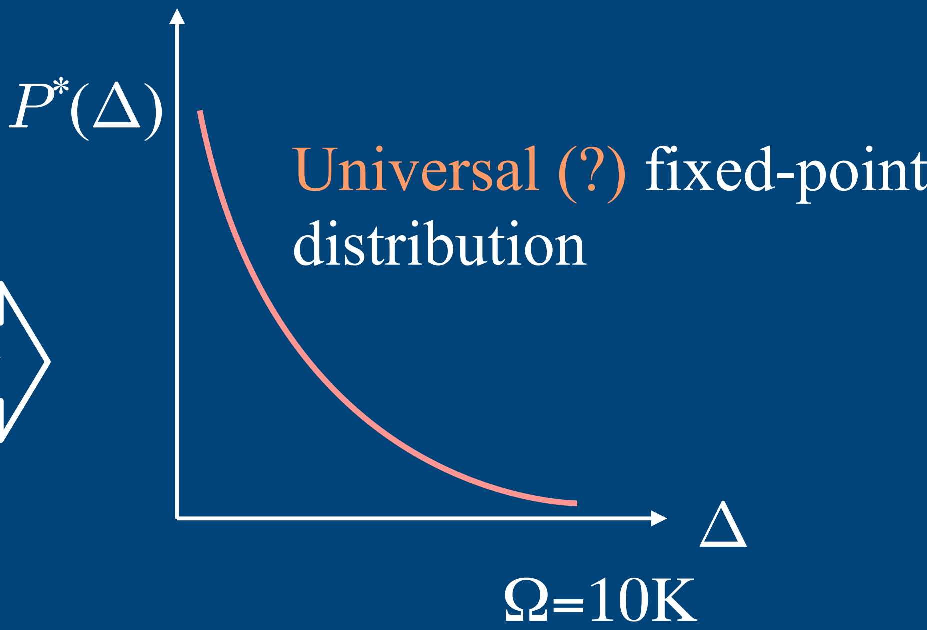
They are all dominated by 1st order processes!



Simplification

$$\boxed{\tilde{\Delta}_i = \alpha_i \Delta_i}$$





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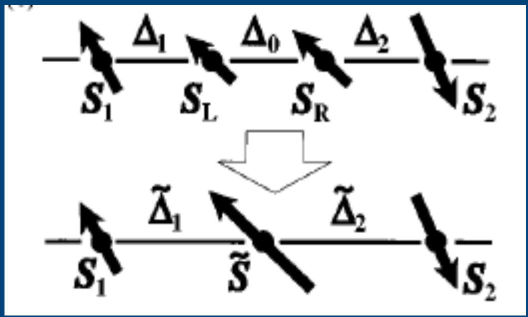

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Generic flow equation:

$$\frac{\partial P(\Delta)}{\partial \Omega} = P(\Omega) P(\Delta) - 2P(\Omega) \int d\Delta' d\alpha P(\Delta') A(\alpha) \delta(\Delta - \alpha \Delta')$$

where $A(\alpha)$ is the distribution function of the multiplicative prefactors.

General solution of the flow equation

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Here, z is the dynamical exponent, which can be shown to be given by:

$$z = \frac{1}{\Omega P(\Omega)} = \frac{1}{\rho(0)}$$

Minimum z in our solution

Low energy behavior is set by the largest negative real pole of $\tilde{\rho}\left(x\right)$

$$\tilde{\rho}\left(x\right) \approx \frac{1}{x + x_m} \Rightarrow \rho(\zeta) \sim e^{-x_m \zeta} \Rightarrow P(\Delta) \sim \frac{1}{\Delta^{1-x_m}}$$

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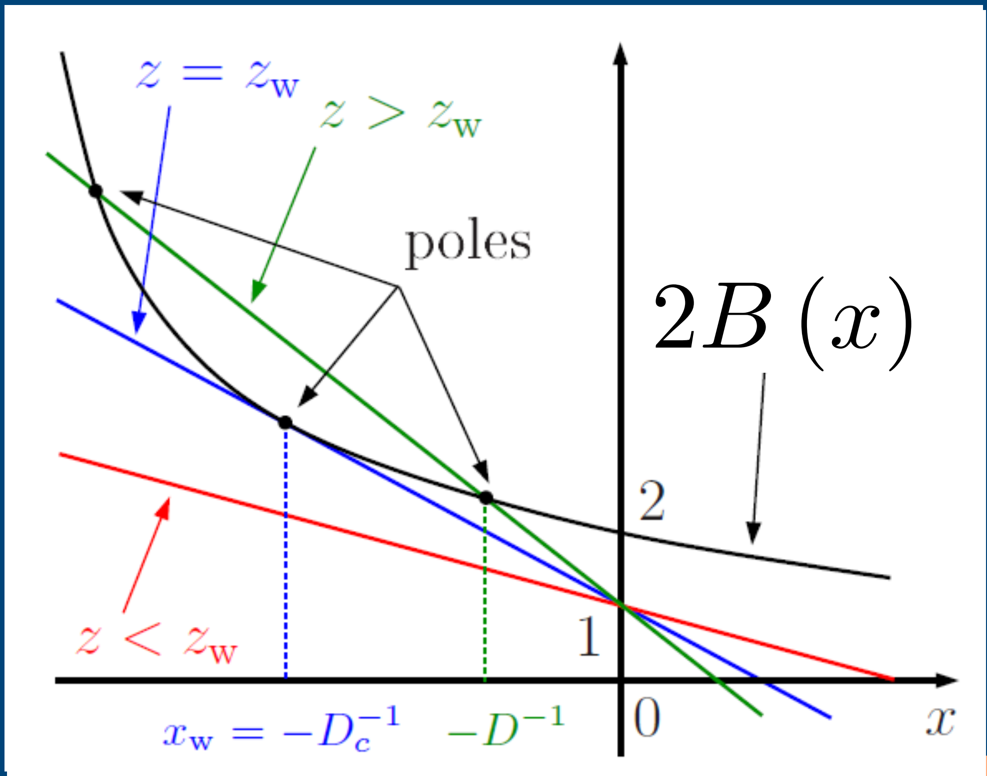

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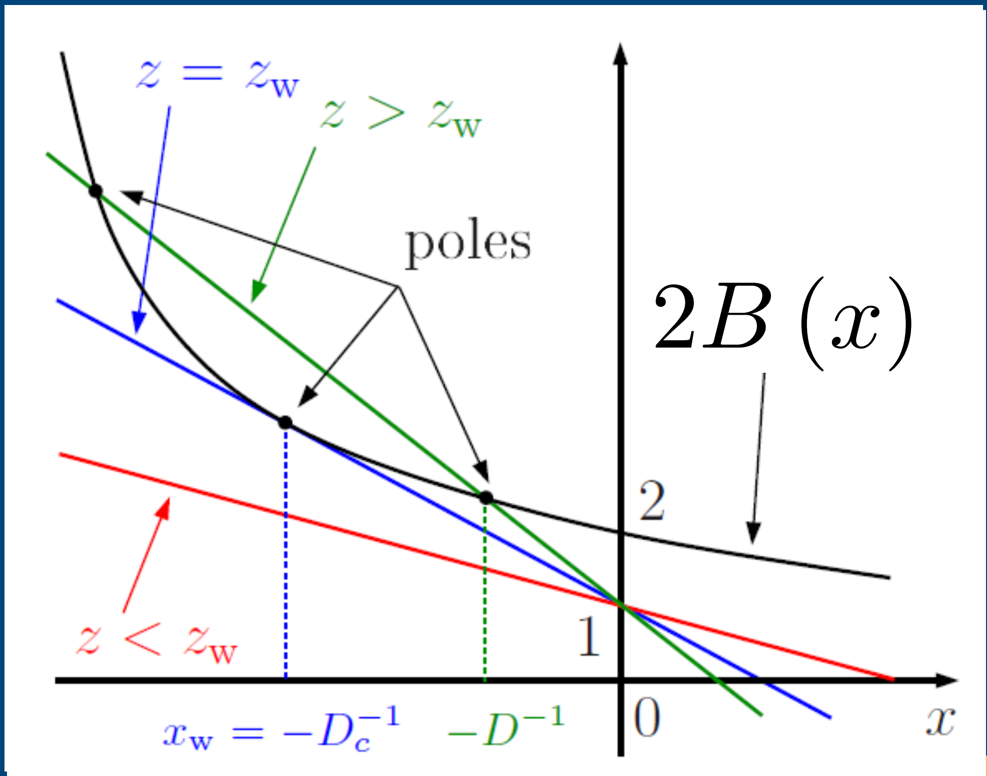

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For sufficiently small z, there is no solution.

There is a minimum dynamical exponent z

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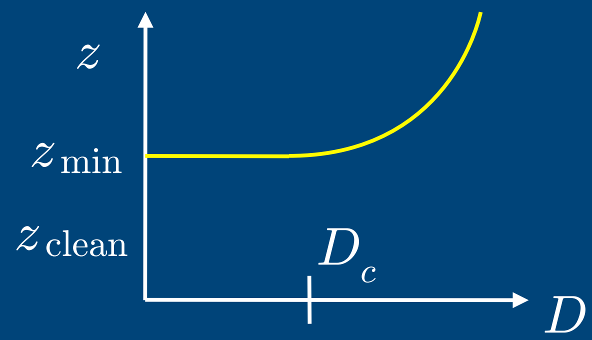

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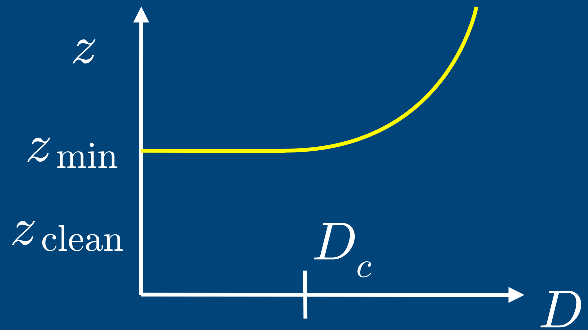

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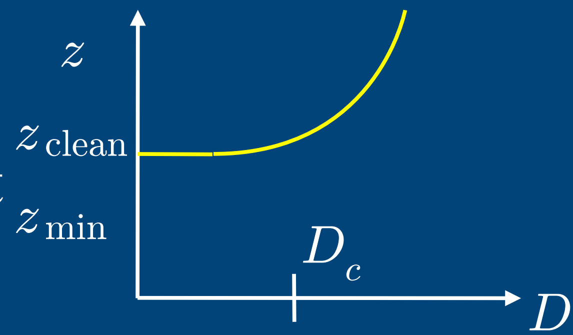

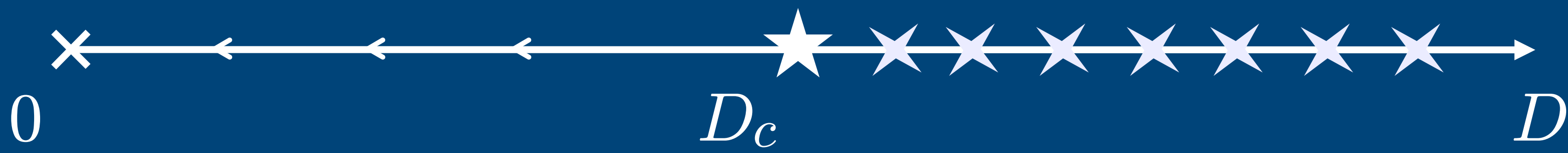

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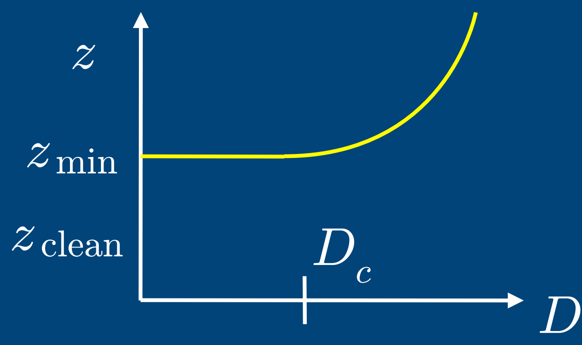

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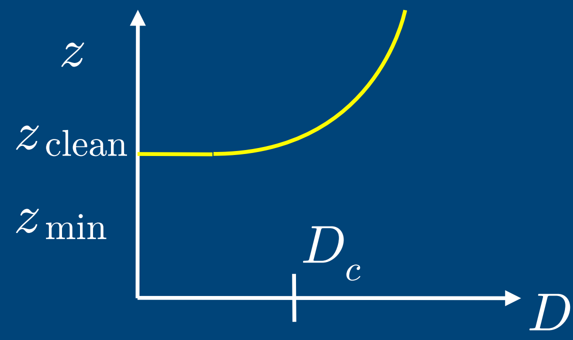

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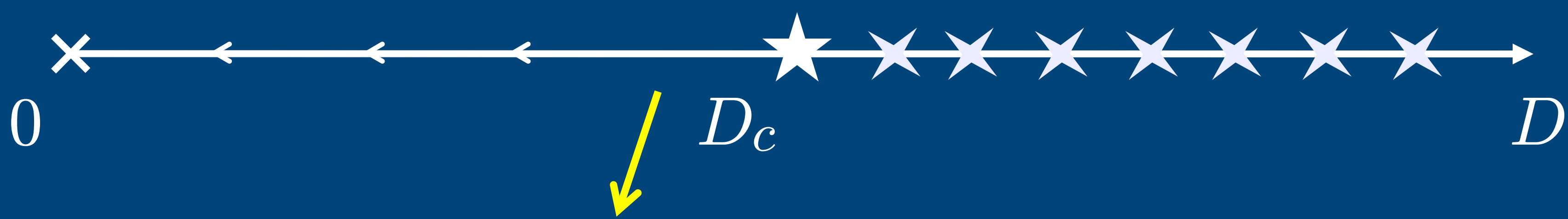
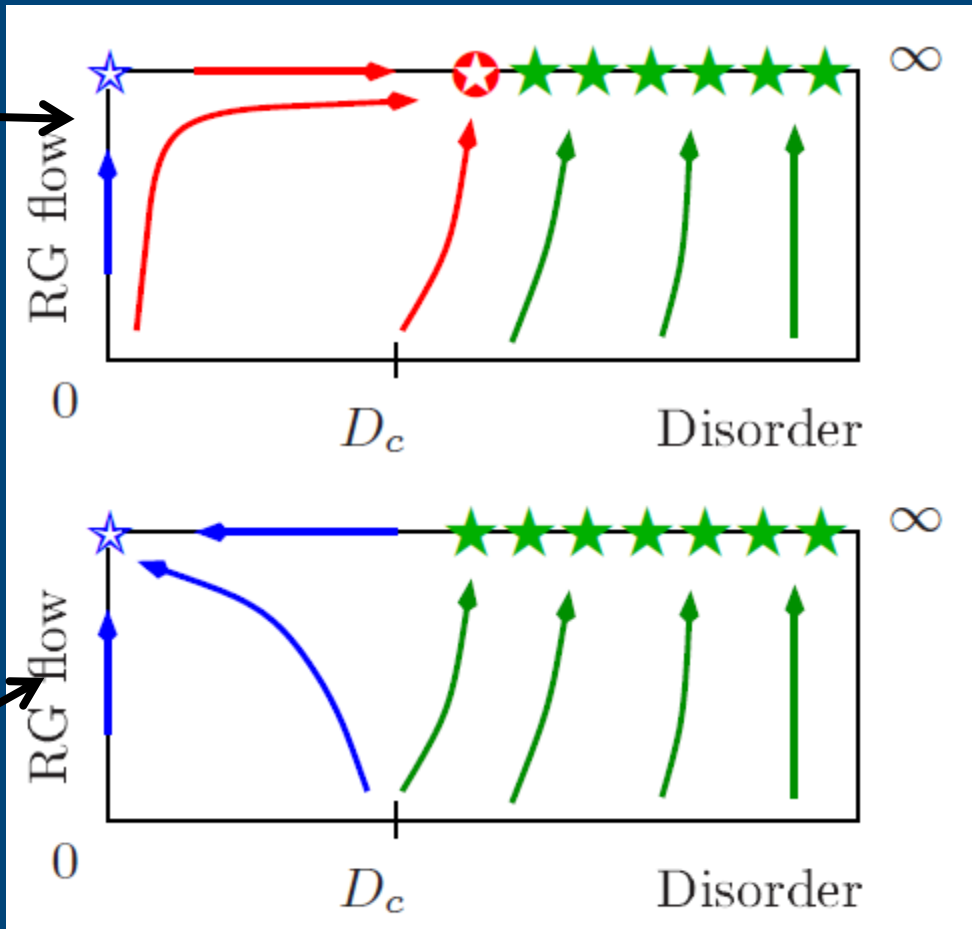


RTFIM with correlated disorder ($h_j = J_j$), q>4 clock model,…(?)

Posssible RG flows

- AFM+FM chain, zigzag ladder, SU(∞)

$$\tilde{\Delta}_i = \alpha_i \Delta_i$$

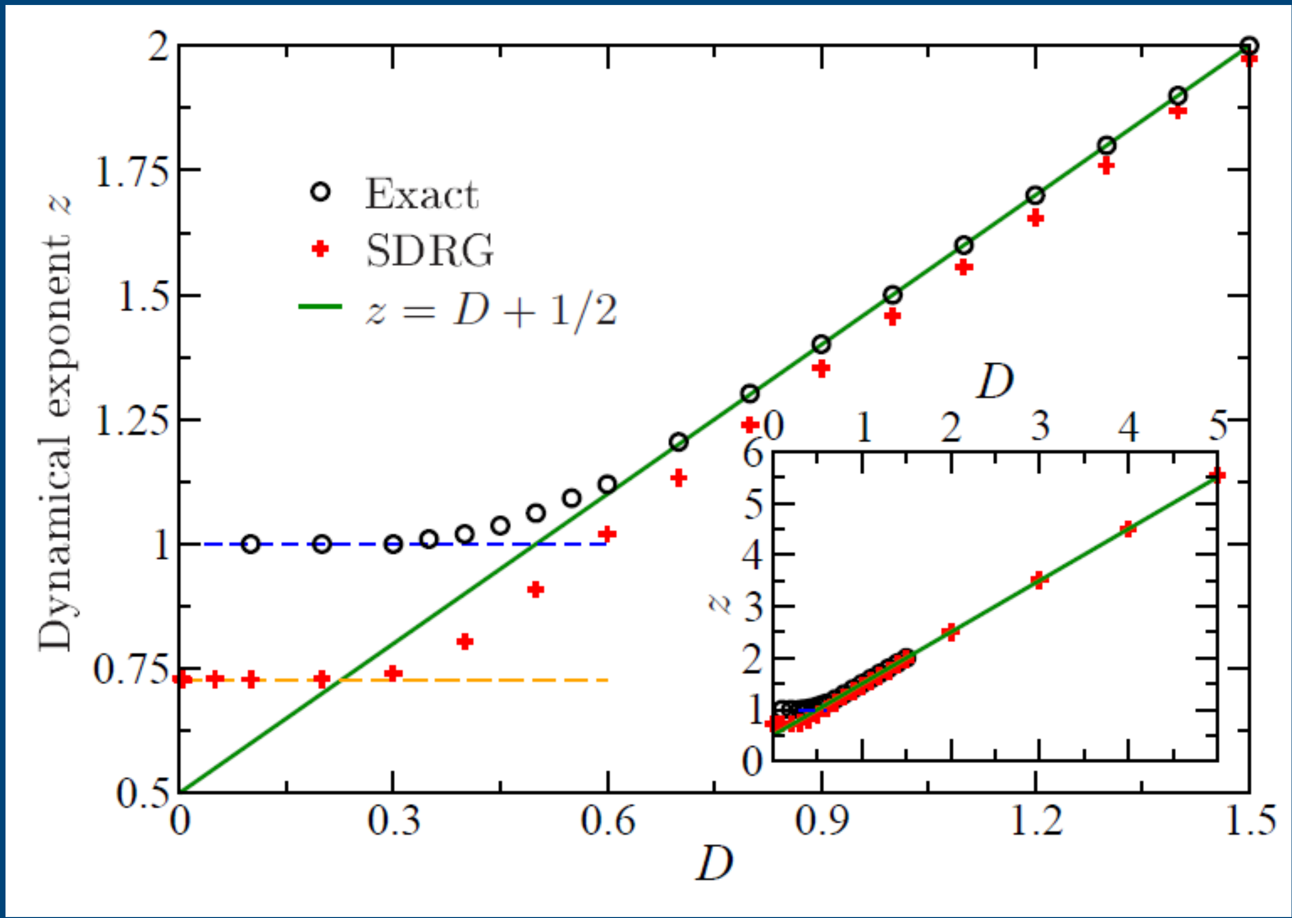


- RTFIM with correlated disorder, q>4 clock model, ….

Check with other methods

Compare with exact diagonalization of $H_{RTFIM} = -\sum_j J_j \left(\sigma_j^z \sigma_{j+1}^z \sigma_j^x\right)$

Hoyos, Laflorencie, Vieira & Vojta, EPL **93**, 30004 (2011)
Hoyos & Miranda, in preparation



Conclusions

1. Generic analytical treatment for a class of disordered systems

   Existence of a minimum z

   Relevance of disorder

   Line of finite-disorder fixed points

2. 2^{nd} order *vs* 1^{st} order decimation processes

   Multiplicative *vs* Simple structure

   Exponential activated *vs* Conventional power-law dynamics

$$\tilde{\Delta} = \# \frac{\Delta_1 \Delta_2}{\Omega} \qquad vs \qquad \widetilde{\Delta}_i = \alpha_i \Delta_i$$

Non-degenerate *vs* Degenerate "local" ground states