Classical percolation fingerprints in the high-temperature regime of the quantum Hall effect

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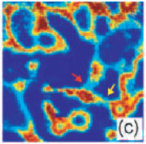

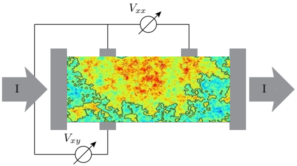

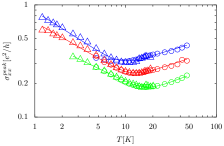

Outline

- High temperature regime of QHE : phenomenology
- Large-scale inhomogeneities : microscopic theory for classical transport critical exponents
- Dominant local dissipation mechanisms
- Longitudinal conductance : scaling law vs experiments

Acknowledgments

- Thierry Champel, LPMMC
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- Benjamin Piot, Duncan Maude, LNCMI

Flöser, Florens & Champel, PRL (2011)
Flöser, Florens & Champel, Int. J. Mod. Phys. (2012)

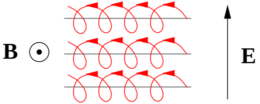
Piot *et al.*, in preparation (2012)

The High Temperature Regime of the Quantum Hall Effect

Classical motion in high perpendicular magnetic field

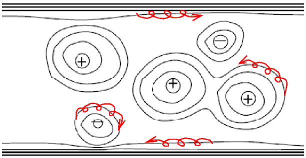
Two "degrees of freedom" with different timescales :

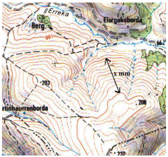
- Fast cyclotron motion : $\frac{d\theta}{dt} = \omega_c = \frac{eB}{m^\star}$
- Slow drift velocity : $\mathbf{v}_d = \frac{\mathbf{E}}{B} \times \hat{\mathbf{z}}$
  $\Longrightarrow$ drift frequency $\omega_d = v_d/\xi$
- Decoupling $(\omega_c > \omega_d)$ for $B_c^{(GaAs)} \simeq 1$ T



Picture : guiding center motion becomes regular

- Disordered bulk : localization on closed equipotential lines
- Sharp edges or percolating bulk states are delocalized

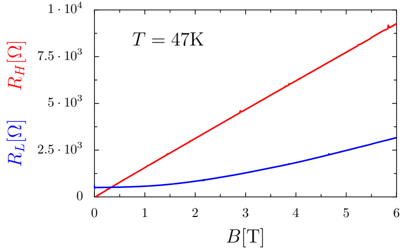


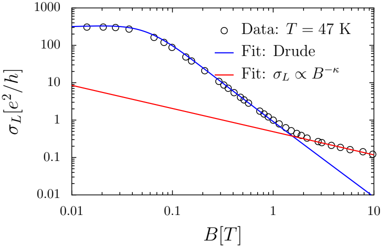


Experimental evidence for (classical) decoupling ?

- High-T regime not well studied curiously

[Renard & al. PRB(2004), B. Piot (unpublished)]

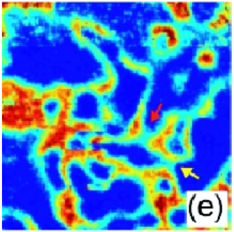


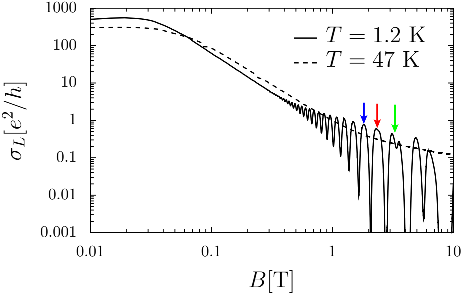


- Classical law $R_H \propto B$ for all B field values
- Drude's law $R_L = \mathrm{Cst}$ and $\sigma_L = \frac{ne^2\tau/m^\star}{1+(\omega_c\tau)^2}$ OK for $B < 1$T
- New regime for $B > 1$T with $\sigma_L \propto B^{-\kappa}$
- Anomalous exponent $\kappa \simeq 0.6 \rightarrow 0.9$

Disgression : onset of Quantum Hall Effect

- The decoupling works also in the quantum world : robust Landau levels with drifting wavefunctions
- Evidence from STM [Hashimoto *et al.*, PRL (2008)]

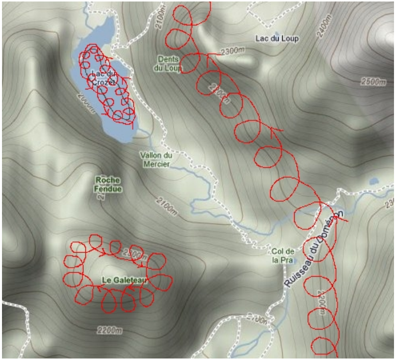




- Breakdown of Drude's law (high T) correlated to onset of QHE (low T) [Fogler, Dobin, Perel, Shklovski, PRB (1997)]

Transport from guiding center alone ?

- Trajectories following closed level lines do not contribute to transport
- Percolating trajectories must go through saddle points ⇒drift velocity $\mathbf{v}_d \propto \nabla V \times \hat{\mathbf{z}}$ vanishes ! (if disorder analytic)
- Conductance is exactly zero



⇒ Extra scattering processes (phonons, “LL-mixing”) are required for longitudinal transport

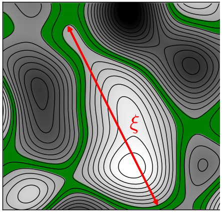
⇒ Introduce small dissipative local conductivity σ_0

Percolating transport at small σ_0

- Classical motion=drift+diffusion
- σ_0 plays the role of cutoff for percolating state

$\Rightarrow$ scaling law for conductance $\boxed{\sigma_L \propto \sigma_0^{1-\kappa}} \gg \sigma_0$

- Conjecture [Isichenko RMP (1992), Simon&Halperin PRL (1994)] :



- $\kappa = 1 - \frac{1}{2+\nu D}$ with ν critical exponent of the correlation length and D fractal dimension of the percolating path.
- $\kappa = 10/13 \simeq 0.7692$

Goal :

- Compute κ microscopically in a direct calculation of σ_L
- Obtain scaling form of $\sigma_L(T, B)$ in order to extract critical exponent from experimental data

The classical percolation transport problem

Local Conductivity Model

- Classical Ohm's law is obeyed locally in space

$$\mathbf{j}(\mathbf{r}) = \hat{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r}) = \begin{pmatrix} \sigma_0 & -\sigma_H(\mathbf{r}) \\ \sigma_H(\mathbf{r}) & \sigma_0 \end{pmatrix} \mathbf{E}(\mathbf{r})$$

  valid for conductors with short coherence length (high-T ok)
- Disorder leads to density fluctuations $\Longrightarrow$ random $\sigma_H(\mathbf{r})$
- Solve continuity equation $\nabla.\mathbf{j}(\mathbf{r}) = 0$

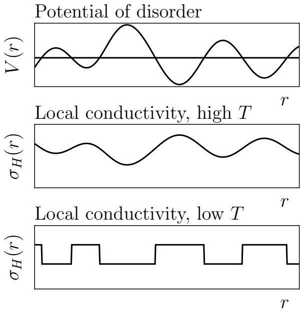
Drift conductivity from semiclassics :

$$\boxed{\sigma_{xy}(\mathbf{r}) = \frac{e^2}{h} \sum_m n_F(E_m + V(\mathbf{r}))}$$

as current density : $\mathbf{j}(\mathbf{R}) = -en(\mathbf{R})\mathbf{v}_{\mathrm{drift}} = \frac{e^2}{h} 2\pi l_B^2 n(\mathbf{R}) \mathbf{E} \times \hat{\mathbf{z}}$
with $n(\mathbf{R})$=density of filled states

Problem : strong fluctuation regime

Modelling conductivity fluctuations :



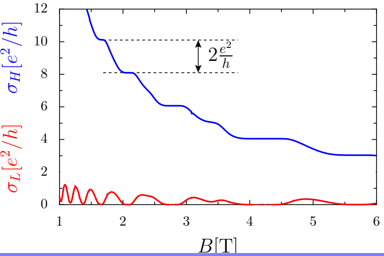
- Gaussian $\sigma_{xy}(\mathbf{r})$ at high T
- Binary $\sigma_{xy}(\mathbf{r})$ at lower T

- Strong conductivity fluctuations :

$$\sqrt{\langle \delta\sigma_H^2 \rangle} \simeq 2\frac{e^2}{h}$$

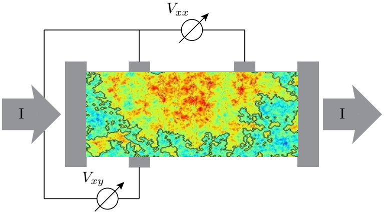
$$\sigma_0 \ll \sigma_L \lesssim \frac{e^2}{h}$$

$\Longrightarrow$ Non-perturbative problem



How to compute transport at small σ_0 ?

Effective conductivity : $\hat{\sigma}^e$ defined after averaging



$$\mathbf{j}(\mathbf{r}) = \hat{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r})$$

spatial average $\langle ... \rangle$

$$\langle \mathbf{j} \rangle = \langle \hat{\sigma}(\mathbf{r})\mathbf{E}(\mathbf{r}) \rangle = \hat{\sigma}^e \langle \mathbf{E} \rangle$$

Problem : a small σ_0 expansion for $\hat{\sigma}^e$ is not possible

Strategy :

- Expand the conductivity perturbatively in powers of $1/\sigma_0$.
- Extrapolate series to small σ_0.

Systematic Calculation of the Effective Conductivity

- Solve continuity equation $\nabla \cdot \mathbf{j} = 0$ with $\mathbf{j}(\mathbf{r}) = \hat{\sigma}(\mathbf{r}) \cdot \nabla \Phi$.
- This leads to boundary value problem [Dreizin & Dykhne JETP (1972), Stroud PRB (1975)] if $\hat{\sigma}(\mathbf{r}) \equiv \hat{\sigma}_0 + \delta\hat{\sigma}(\mathbf{r})$

  $$\nabla \cdot [\hat{\sigma}_0 \nabla \Phi(\mathbf{r})] = -\nabla \cdot [\delta\hat{\sigma}(\mathbf{r}) \nabla \Phi(\mathbf{r})] \text{ in V}$$

  $$\Phi(\mathbf{r}) \equiv \Phi_0(\mathbf{r}) = -\mathbf{E}_0 \cdot \mathbf{r} \text{ on S}$$

  for a sample of volume V bounded by a surface S.
- Introducing the Green's function

  $$\nabla \cdot [\hat{\sigma}_0 \nabla G(\mathbf{r}, \mathbf{r}')] = -\delta(\mathbf{r} - \mathbf{r}') \text{ in V}$$

  $$G(\mathbf{r}, \mathbf{r}') = 0 \text{ for } \mathbf{r} \text{ on S.}$$
- After some algebra we find $\hat{\sigma}^e = \hat{\sigma}_0 + \langle \hat{\chi} \rangle$

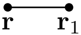
  with $$\boxed{\hat{\chi}(\mathbf{r}) = \delta\hat{\sigma}(\mathbf{r}) + \delta\hat{\sigma}(\mathbf{r}) \int_V d^2 r' \; \hat{\mathcal{G}}_0(\mathbf{r}, \mathbf{r}') \hat{\chi}(\mathbf{r}')}$$

  where $\left[\hat{\mathcal{G}}_0\right]_{ij} = \frac{\partial}{\partial r_i} \frac{\partial}{\partial r_j} G(\mathbf{r}, \mathbf{r}')$. Note : $\hat{\mathcal{G}}_0 \propto 1/\sigma_0$

Diagrammatic Expansion

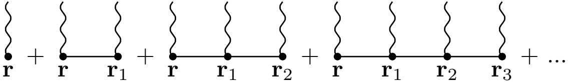
$$\hat{\chi}(\mathbf{r}) = \delta\hat{\sigma}(\mathbf{r}) + \delta\hat{\sigma}(\mathbf{r}) \int_V d^2r' \ \hat{\mathcal{G}}_0(\mathbf{r},\mathbf{r}')\hat{\chi}(\mathbf{r}')$$

Convention :

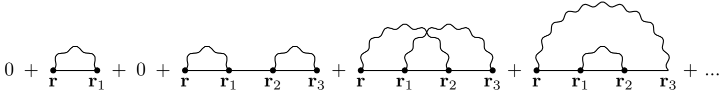
$\hat{\mathcal{G}}_0(\mathbf{r}-\mathbf{r}_1) =$ 

$\delta\hat{\sigma}(\mathbf{r}) =$ 

Systematic expansion of $\hat{\chi}$:

$\hat{\chi}(\mathbf{r}) =$ 

At high-T, Gaussian fluctuations of $\delta\hat{\sigma}(\mathbf{r})$ are averaged with the correlation function $\langle\delta\hat{\sigma}(\mathbf{r})\delta\hat{\sigma}(\mathbf{r}')\rangle$ (curly lines)

$\langle\hat{\chi}(\mathbf{r})\rangle =$ 

Result at Six-Loop Order

- $\langle \hat{\chi}(\mathbf{r}) \rangle$ is purely diagonal :
  - classical Hall law for σ_H
  - non-trivial behavior of σ_L

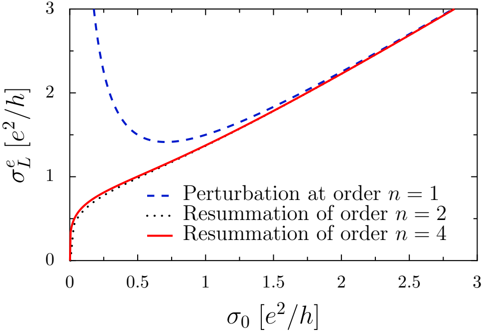
$$\sigma_L^e = \sigma_0 + \langle \chi \rangle = \sigma_0 + \sum_{n=1}^{\infty} a_n \frac{\langle \delta\sigma_H^2 \rangle^n}{\sigma_0^{2n-1}}$$

- We have calculated the diagrams up to six-loop order :

Loop order	Method	Coefficient a_n
1	Analytical	$\frac{1}{2}$
2	Analytical	$\frac{1}{8} - \frac{1}{2}\log(2)$
3	Analytical	0.2034560502
4	Numerical	-0.265 ± 0.001
5	Numerical	0.405 ± 0.001
6	Numerical	-0.694 ± 0.001

- Knowing that $\sigma_L^e \propto \sigma_L \propto \sigma_0^{1-\kappa}$ extrapolate to $\sigma_0 \to 0$.

From Strongly Dissipative to Percolating Regime



- The longitudinal conductance converges well from large to small σ_0.
- Microscopically calculated value for κ is in good agreement with the conjecture.

Loop order	Method	Exponent κ
2	Padé	0.72 ± 0.09
4	Padé	0.779 ± 0.006
4	n-Fit	0.767 ± 0.002
∞	Conjecture	$10/13 \simeq 0.769$

Relevant dissipation mechanisms

Remanent impurity scattering at high magnetic field ?

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Suppression of chaotic dynamics and localization of two-dimensional electrons by a weak magnetic field

M. M. Fogler, A. Yu. Dobin,* V. I. Perel,* and B. I. Shklovskii
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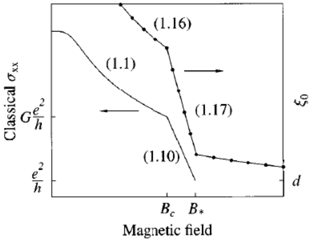
(Received 19 February 1997)



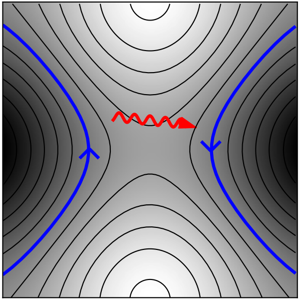
- $\sigma_{xx} \simeq e^{-B/B_c}$ at large B
- Exponential decoupling of guiding center and cyclotron motion

Phonon Scattering ?

- Huge reduction of Debye temperature T_D at high field due to weaker phase space constraints

  [Zhao & Feng, PRL(1993), Gantmakher & Levinson, Elsevier (1987)]

- T_D reduced from 10K at $B = 0$ down to 1K (or less) at 1T



- Fermi's Golden Rule calculation for the scattering rate :

$$\boxed{\frac{1}{\tau} \propto TB^2}$$

Estimation of σ_0 from Phonon Scattering

Coarse-grained approach :

- Drude's formula with electron-phonon scattering time τ applies on short length scales :

$$\sigma_0 = \frac{ne^2\tau/m^*}{1+\omega_c^2\tau^2} \propto \frac{1}{\omega_c^2\tau} \text{ with } \frac{1}{\tau} \propto TB^2.$$

- Independent of the magnetic field $\boxed{\sigma_0(B) \propto \mathrm{const.}}$
- Linear temperature dependence $\boxed{\sigma_0(T) \propto T}$

Scaling law vs experiments

Scaling of longitudinal conductivity

Percolation scaling law : $\sigma_{xx} \propto \sigma_0^{1-\kappa} \left\langle \delta\sigma_{xy}^2 \right\rangle^{\kappa/2}$

Fluctuation of Hall component : at high-T linearize

$\sigma_{xy}(\mathbf{r}) \simeq \frac{e^2}{h} \sum_m [n_F(E_m) + V(\mathbf{r}) n'_F(E_m)]$

At plateau transition : $\delta\sigma_{xy}(\mathbf{r}) \simeq \frac{e^2}{h} \left[\frac{1}{4T} + \frac{1}{\hbar\omega_c}\right] V(\mathbf{r})$

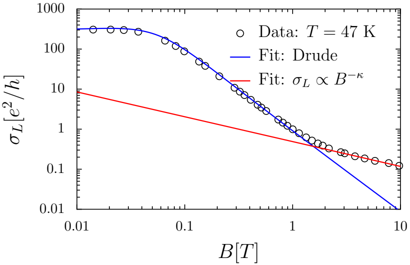
Scaling function : $\sigma_{xx} \propto [\sigma_0(T)]^{1-\kappa} \left[\frac{e^2}{h}\sqrt{\left\langle V^2 \right\rangle}\right]^{\kappa} \left[\frac{1}{4T} + \frac{1}{\hbar\omega_c}\right]^{\kappa}$

- $\sigma_{xx} \propto T^{1-2\kappa} \simeq T^{-0.5}$ at $T < \hbar\omega_c/4$
- $\sigma_{xx} \propto T^{1-\kappa} \simeq T^{0.2}$ at $T > \hbar\omega_c/4$

  σ_{xx} should go through a minimum at $T \simeq \hbar\omega_c/4$
- $\sigma_{xx} \propto B^{-\kappa}$ at $T > \hbar\omega_c/4$ slower than Drude

  [Polyakov & Shklovskii, PRB(1994)]

Magnetoconductance σ_L at $T = 47$K



[B. Piot (unpublished)]

- Clear crossover from low field Drude behavior

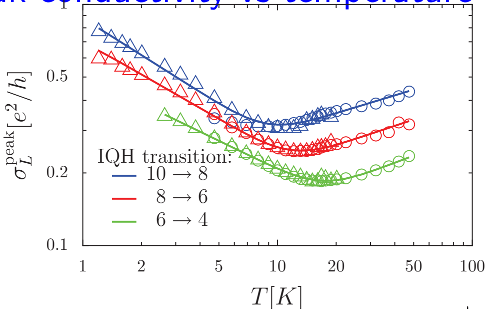
$$\sigma_L \propto \frac{\sigma(B=0)}{1+(\omega_c\tau)^2} \propto B^{-2}$$

to high field regime

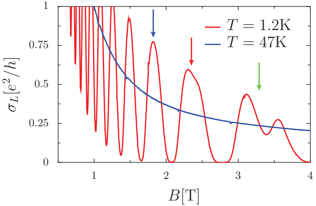
$$\sigma_L \propto B^{-\kappa}$$

- Fit gives $\kappa = 0.62$ (smaller than theoretical expected value $\kappa = 0.77$).
- But exponent extraction difficult as only small B-field range (onset of oscillations at $B \gtrsim 8$T).

Peak conductivity vs temperature



Examine $\sigma_{xx}^{\rm peak}(T)$ for three IQHE transitions :

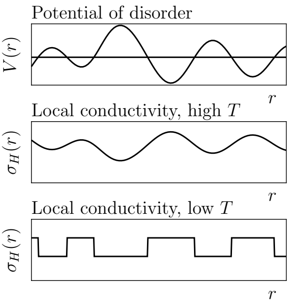


- Two power-laws : $T^{1-2\kappa} = T^{-0.5}$ (low T)
  $T^{1-\kappa} = T^{0.2}$ (high T)

  $\Rightarrow$ crossover from classical to quantized cyclotron motion
- The minimum appears as predicted ($\hbar\omega_c$ is not fitted)
- Quantitative agreement with the scaling function for $\sigma_{xx}(T)$
  We extract : $\kappa = 0.73 \pm 0.03$

Low Temperature Behavior $k_B T \ll \sqrt{\langle V^2(\mathbf{r})\rangle}$

Binary model : neglects quantum tunneling, interference...



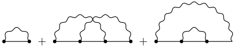
- $$\sigma_H = \frac{e^2}{h}\sum_m n_F(E_m + V(\mathbf{r}) - \mu)$$
  $$\xrightarrow[T\to 0]{} \frac{e^2}{h}[\nu + \Theta(-V(\mathbf{r}) + \mu)]$$
- [Dykhne & Ruzin, PRB (1994)] :
  $$\sigma_L^{\text{peak}} = \sqrt{\left(\frac{e^2}{2h}\right)^2 + \sigma_0^2}$$
  exact from duality mapping

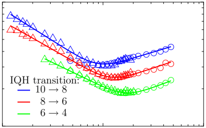
Note : σ_L^{peak} finite at $\sigma_0 \to 0$ [possible as non-analytic $\sigma_H(\mathbf{r})$]

We recover precisely this formula from resummed perturbation theory in $1/\sigma_0$ (from two loops and on)

$\Longrightarrow$ Computation of full crossover for $\sigma_{xx}(T)$ possible

Summary



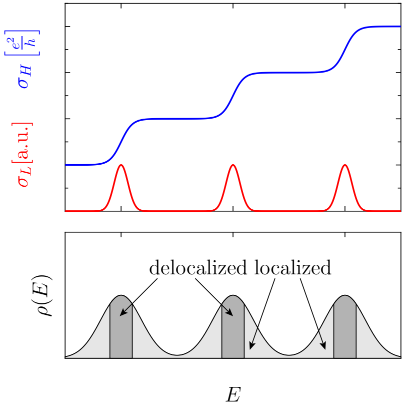


- Diagrammatic method allows microscopic calculation of the conductivity and the critical transport exponent κ
- Scaling laws compatible with experiment :
  - role of classical percolation
  - phonons are dominant dissipation mechanism
- Prospects :
  - Find better sample or system (spin gap problem)
  - Compute complete scaling law (with quantum corrections)
  - Investigate phonon scattering with disorder at high field

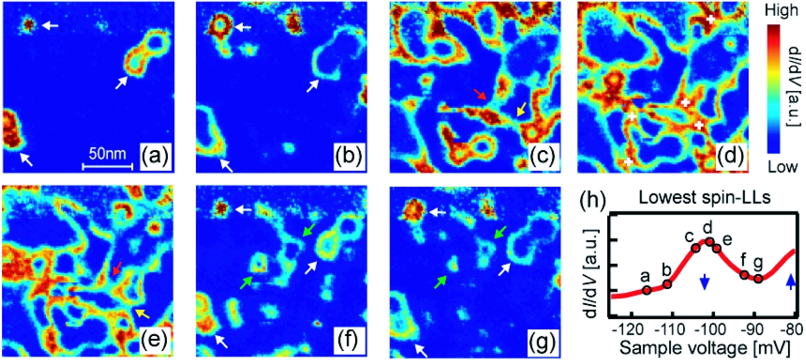
Extra Slides

Ingredients for the QHE : Landau Levels + Disorder

- $\mathcal{H}_0 = \frac{1}{2m^*}\left(\mathbf{p} - e\mathbf{A}(\mathbf{r})\right)^2$
  - Landau levels (LL) :
    $E_n = \hbar\omega_c \left(n + \frac{1}{2}\right)$
    with $\omega_c = \frac{eB}{m^*}$
  - typical length scale :
    $l_B = \sqrt{\frac{\hbar}{|e|B}}$
- Disorder Potential $V(\mathbf{r})$
  with correlation length ξ
  $\mathcal{H} = \mathcal{H}_0 + V(\mathbf{r})$
  - localized states
  - if $l_B \ll \xi$
    $E_n(\mathbf{r}) \approx \hbar\omega_c(n + \frac{1}{2}) + V(\mathbf{r})$



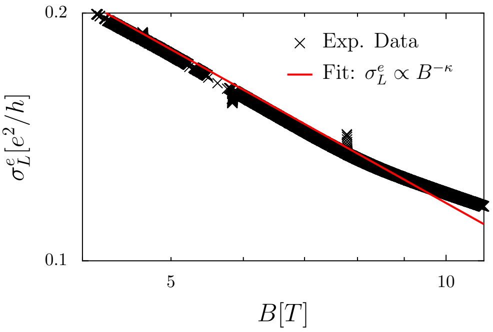
STM Images : Density of States Maps Disorder Landscape



(InSb, $T = 0.3$K, $B \geq 12$T), [Hashimoto *et al.*, PRL (2008)]

- Low temperature : Quantum percolation physics plays a key role at the plateau transitions.
- High temperature : Role of classical percolation ?

Upper B-field Limit



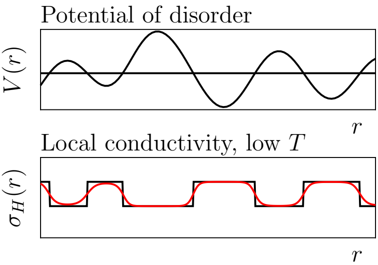
- For $B > 8\,T$ deviations from the $B^{-\kappa}$ law.
- $k_B T > \hbar\omega_c$ no longer valide.

- Zeeman splitting $E_z = \pm\frac{1}{2} g \mu_B B$
- Problem : Interaction effects between the spin species leads to unknown large g_{eff}.

Sample Characteristics

- Density : $4 \cdot 10^{11}\mathrm{cm}^{-2}$
- Mobility at 1.2K : $3.3 \cdot 10^{5}\mathrm{cm}^{2}\mathrm{V}^{-1}\mathrm{s}^{-1}$
- Aspect ration : $L_x/L_y = 6$
- Spacer : 102Å

Low Temperature Behavior $k_B T \ll \sqrt{\langle V^2(\mathbf{r}) \rangle}$

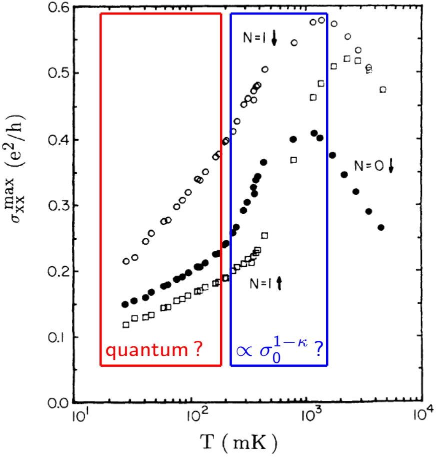


- But conductance saturation is not observed experimentally.
- LL wave function spread is missing [Champel, Florens & Canet, PRB(2008)] :

$$\sigma_H(\mathbf{r}) = \frac{e^2}{h} \int \mathrm{d}^2 R \sum_m |\Psi_{m,\mathbf{R}}(\mathbf{r})|^2 \, n_F(E_m + V(\mathbf{r}) - \mu)$$

$\Rightarrow$Width of transition between $\sigma_H = \frac{e^2}{h}\nu$ and $\sigma_H = \frac{e^2}{h}(\nu + 1)$ no longer given by T, but by l_B.

Low Temperature Behavior $k_B T \ll \sqrt{\langle V^2(\mathbf{r})\rangle}$

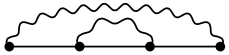
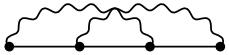
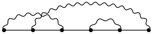
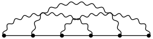
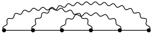
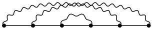
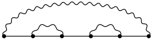
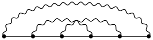
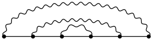


[Wei, Lin, Tsui, Pruisken, PRB (1992)]

- $\sqrt{\langle \delta\sigma_H^2(\mathbf{r})\rangle}$ dominated by l_B $\Rightarrow$temperature independent at low T.

$\Rightarrow$ again $\sigma_L^e \propto \sigma_0^{1-\kappa}$ at $k_B T \ll \sqrt{\langle V^2(\mathbf{r})\rangle}$.

- Qualitative explanation of the drop of σ_L at $T < 1K$.

Diagram	Multi-plicity	Analytical Value	Decimal Value
Two loops			
	1	$-\frac{1}{4}\log(2)$	-0.173
	1	$\frac{1}{8}(1-\log(4))$	-0.0483
Three loops			
	1	$\frac{1}{96}\left(3-\pi^2+3\log[3](-3+\log[9])+12\mathrm{Li}_2\left[\frac{2}{3}\right]\right)$	0.00504
	2	$\frac{1}{32}\log\left[\frac{27}{16}\right]$	0.0164
	1	$\frac{1}{16}\left(2\log[2]^2-3\log[3]+\log[8]+\mathrm{Li}_2\left[\frac{1}{4}\right]\right)$	0.000760
	2	$\frac{1}{384}(2+100\log[2]-63\log[3])$	0.00547
	1	$\frac{1}{8}\log\left[\frac{32}{27}\right]$	0.0212
	1	$\frac{1}{8}\log\left[\frac{27}{16}\right]$	0.0654
	1	$-\frac{1}{48}-\frac{\log[2]}{6}+\frac{9\log[3]}{64}$	0.0181
	1	$\frac{3}{16}\log\left[\frac{4}{3}\right]$	0.0539

Multiplicity and analytical values of the diagonal elements of the non-zero second and three loop order diagrams. Li_2 is the dilogarithm defined by $\mathrm{Li}_2(z)=\int_z^0 dt\,\frac{\log(1-t)}{t}$